Q_{10} error

Assume Q_{10} is computed as

$$Q_{10} = \left(\frac{\kappa_1}{\kappa_2}\right)^{10^\circ\mathrm{C}/(T_2 - T_1)}, \tag{1}$$

where κ_1, κ_2 are rate constants, estimated as the inverse of time constants τ_1^{-1}, τ_2^{-1}, at temperature T_1, T_2 respectively. Assuming linear error and denote $10^\circ\mathrm{C}/(T_2 - T_1)$ as ΔT, we can apply error propagation to obtain

$$\begin{aligned}
\delta Q_{10} &= \sqrt{\delta\kappa_1^2 \left(\frac{\partial Q_{10}}{\partial \kappa_1}\bigg|_{\hat{Q}_{10}}\right)^2 + \delta\kappa_2^2 \left(\frac{\partial Q_{10}}{\partial \kappa_2}\bigg|_{\hat{Q}_{10}}\right)^2 + \delta T_1^2 \left(\frac{\partial Q_{10}}{\partial T_1}\bigg|_{\hat{Q}_{10}}\right)^2 + \delta T_2^2 \left(\frac{\partial Q_{10}}{\partial T_2}\bigg|_{\hat{Q}_{10}}\right)^2} \\
&= \sqrt{\delta\kappa_1^2 \left(\hat{\Delta T}\frac{\hat{k}_1^{\hat{\Delta T}-1}}{\hat{k}_2^{\hat{\Delta T}}}\right)^2 + \delta\kappa_2^2 \left(-\hat{\Delta T}\frac{\hat{k}_1^{\hat{\Delta T}}}{\hat{k}_2^{\hat{\Delta T}+1}}\right)^2 + \delta T_1^2 \left(\hat{Q}_{10}\ln\frac{\hat{k}_1}{\hat{k}_2}\frac{10^\circ\mathrm{C}}{(\hat{T}_2 - \hat{T}_1)^2}\right)^2 + \delta T_2^2 \left(\hat{Q}_{10}\ln\frac{\hat{k}_1}{\hat{k}_2}\frac{(-10^\circ\mathrm{C})}{(\hat{T}_2 - \hat{T}_1)^2}\right)^2} \\
&= \sqrt{\left(\delta\kappa_1 \frac{\hat{\Delta T}}{\hat{\kappa}_1}\hat{Q}_{10}\right)^2 + \left(\delta\kappa_2 \frac{\hat{\Delta T}}{\hat{\kappa}_2}\hat{Q}_{10}\right)^2 + \left(\delta T_1 \hat{Q}_{10}\ln\frac{\hat{\kappa}_1}{\hat{\kappa}_2}\frac{\hat{\Delta T}^2}{10^\circ\mathrm{C}}\right)^2 + \left(\delta T_2 \hat{Q}_{10}\ln\frac{\hat{\kappa}_1}{\hat{\kappa}_2}\frac{\hat{\Delta T}^2}{10^\circ\mathrm{C}}\right)^2}
\end{aligned}$$

where δx denotes the standard deviation of the variable x, and $\hat{x}$ denotes its mean estimator.