FDP Assignment
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Data Pack for the Jetstream 31

Dr Alastair K. Cooke
School of Engineering
Cranfield University
Cranfield
Bedford MK43 0AL
a.cooke@cranfield.ac.uk

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$(a_1)_t$	tailplane lift-curve slope
$(a_1)_w$	wing lift-curve slope
$(a_1)_{w_b}$	wing lift-curve slope under influence of body
$(a_1)_{tw_b}$	wing lift-curve slope with flaps deployed under influence of body
$(a_2)_t$	elevator lift-curve slope in viscous compressible flow
A	propeller disc area
A_t	aspect ratio of tailplane
A_w	aspect ratio of wing
b	wing span
$\bar{\bar{c}}$	wing mean aerodynamic chord
c_0	chord at fuselage centreline
c_r	root chord
c_t	tip chord
C_{D0}	zero-lift drag coefficient
C_{L0}	zero-incidence lift coefficient
C_{L1}	basic lift coefficient ($\equiv$ lift coefficient of clean wing)
$(C_L)_f$	lift coefficient for fuselage
$(C_L)_t$	lift coefficient for tailplane
$(C_L)_{w_b}$	lift coefficient for a wing under influence of body ($\equiv C_{L1}$)
C_M	pitching moment coefficient - $\mathscr{M}/\bar{q}S_w\bar{\bar{c}}$
C_{M_f}	fuselage pitching moment coefficient
C_{M_Γ}	fuselage and nacelle pitching moment coefficient due to circulation
$(C_{M_0})_w$	wing zero-lift pitching moment coefficient
C_{n_i}	yawing moment coefficient due to induced drag from ailerons
C_{n_p}	yawing moment coefficient due to profile drag from ailerons
C_N	fuselage normal force coefficient
C_P	power coefficient - $P/\rho n^3 D^5$
C_Q	torque coefficient - $Q/\rho n^2 D^5$
C_T	thrust coefficient - $T/\rho n^2 D^4$
C_Y	propeller sideforce coefficient - $Y/\rho n^2 D^4$
C_Z	propeller normal force coefficient - $Z/\rho n^2 D^4$
D	propeller diameter
D_f	fuselage diameter
i_t	tailplane setting angle relative to chordline of wing
J	advance ratio - V/nD
$\mathscr{L}$	rolling moment
L_p	aeronormalised rolling moment derivative due to roll rate, $(\partial\mathscr{L}/\partial p)\,/\,\left(\frac{1}{2}\rho V S_w b^2\right)$
L_r	aeronormalised rolling moment derivative due to yaw rate, $(\partial\mathscr{L}/\partial r)\,/\,\left(\frac{1}{2}\rho V S_w b^2\right)$
L_{r0}	aeronormalised rolling moment derivative due to yaw rate, incompressible flow
L_T	distance between tailplane aerocentre and reference axes centre
L_v	aeronormalised rolling moment derivative due to sideslip, $(\partial\mathscr{L}/\partial v)\,/\,\left(\frac{1}{2}\rho V S_w b\right)$
L_ζ	aeronormalised rolling moment derivative due to rudder, $(\partial\mathscr{L}/\partial\zeta)\,/\,\left(\frac{1}{2}\rho V^2 S_w b\right)$
L_{prop}	total powerplant rolling moment about the body axes centre - less slipstream effects
L_P	free-air propeller rolling moment
$\mathscr{M}$	pitching moment
M	flight Mach number
M_{prop}	total powerplant pitching moment about the body axes centre - less slipstream effects
M_P	free-air propeller pitching moment

M_q	aeronormalised pitching moment derivative due to pitch rate, $(\partial \mathscr{M}/\partial q) \,/\, \left(\frac{1}{2}\rho V S_w \bar{\bar{c}}^2\right)$
n	propeller speed (revolutions/s)
$\mathscr{N}$	yawing moment
$\mathscr{N}_\xi$	yawing moment arising from aileron deflection
N_{prop}	total powerplant yawing moment about the body axes centre - less slipstream effects
N_p	aeronormalised yawing moment derivative due to roll rate, $(\partial \mathscr{N}/\partial p) \,/\, \left(\frac{1}{2}\rho V S_w b^2\right)$
N_P	free-air propeller yawing moment
N_r	aeronormalised yawing moment derivative due to yaw rate, $(\partial \mathscr{N}/\partial r) \,/\, \left(\frac{1}{2}\rho V S_w b^2\right)$
N_v	aeronormalised yawing moment derivative due to sideslip, $(\partial \mathscr{N}/\partial v) \,/\, \left(\frac{1}{2}\rho V S_w b\right)$
N_ζ	aeronormalised yawing moment derivative due to rudder, $(\partial \mathscr{N}/\partial \zeta) \,/\, \left(\frac{1}{2}\rho V^2 S_w b\right)$
N_ξ	aeronormalised rolling moment derivative due to aileron, $(\partial \mathscr{N}/\partial \xi) \,/\, \left(\frac{1}{2}\rho V^2 S_w b\right)$
$\bar{q}$	dynamic pressure - $\frac{1}{2}\rho V^2 \equiv \frac{1}{2}\rho_0 V_e^2$
S_t	gross tailplane area
S_w	gross wing area
T_P	free-air axial propeller force (aligned with body axes)
V	velocity of aircraft relative to air
V_e	equivalent air speed
$\bar{V}_T$	tail volume coefficient, $(S_t l_T) \,/\, (S_w \bar{\bar{c}})$
$\bar{x}_0$	distance of aero-centre aft of apex
x_f	position of aerocentre of fuselage forward of apex
x_h	position of aerocentre of wing aft of apex
x_p	longitudinal location of propeller from body axes centre
X_P	free-air propeller X-force
X_{prop}	total powerplant X-force about the body axes centre - less slipstream effects
y_p	lateral location of propeller from body axes centre
Y	Y-force
Y_p	aeronormalised sideforce derivative due to roll rate, $(\partial Y/\partial p) \,/\, \left(\frac{1}{2}\rho V S_w b\right)$
Y_P	free-air propeller Y-force
Y_{prop}	total powerplant Y-force about the body axes centre - less slipstream effects
Y_r	aeronormalised sideforce derivative due to yaw rate, $(\partial Y/\partial r) \,/\, \left(\frac{1}{2}\rho V S_w b\right)$
Y_v	aeronormalised sideforce derivative due to sideslip, $(\partial Y/\partial v) \,/\, \left(\frac{1}{2}\rho V S_w\right)$
Y_ζ	aeronormalised sideforce derivative due to rudder, $(\partial Y/\partial \zeta) \,/\, \left(\frac{1}{2}\rho V^2 S_w\right)$
z_p	vertical location of propeller from body axes centre
Z_P	free-air normal propeller force (aligned with body axes)
Z_{prop}	total powerplant Z-force about the body axes centre - less slipstream effects
$(\alpha_0)_{w_b}$	wing zero-lift incidence under influence of body
$(\alpha_0)_{tw_b}$	wing zero-lift incidence under influence of body with flaps deployed
α_b	angle of attack of fuselage (body)
α_e	effective angle of attack of installed propeller ($\approx \alpha_b$)
α_t	tailplane angle of attack
α_w	angle of attack of wing
β	sideslip angle
β_0	blade angle (taken to equal the pitch at 0.7R, that is $\theta_{.7}$)
γ	flight path angle
Γ	dihedral angle, circulation strength
δ_f	flap setting
$\Delta C'_{D_f}$	vortex drag coefficient due to flap deployment
$\Delta C_{L_{tw}}$	increment in lift coefficient due to flap deployment
$\Delta C_{M_{\alpha 0}}$	increment in pitching moment coefficient at zero incidence due to flap

$(\Delta C_{M_0})_b$	fuselage influence on wing zero lift pitching moment coefficient
Δh_1	aft shift of wing/body aerocentre caused flap deployment
Δx_h	forward shift of wing/body aerocentre caused by presence of fuselage
$\bar{\varepsilon}$	average downwash across tailplane
$\bar{\varepsilon}_0$	average downwash across tailplane at zero wing incidence
$\bar{\varepsilon}_{fT}$	average downwash across tailplane with flaps deployed
ζ	rudder deflection
η	elevator deflection
η_i	spanwise location of inboard limit of control surface as a fraction of semi-span
η_o	spanwise location of outboard limit of control surface as a fraction of semi-span
ρ	ambient air density
ρ_0	air density at sea-level ISA standard day
ξ	mean aileron deflection
ξ_p	deflection of port aileron
ξ_s	deflection of starboard aileron
σ_α	sidewash factor
Φ_i	part-span factor corresponding to η_i
Φ_o	part-span factor corresponding to η_o
ψ_e	effective sideslip of installed propeller ($\approx \beta$)

1 Introduction

The following sections provide estimates of the aerodynamic of a standard Jetstream 31. Aero-derivatives, control derivatives, products and moments of inertia are given about the body axes centre which is located at the intersection of the 30% wing chordline and the fuselage centreline. The material presented here is based partly on Cooke [2006].

2 Lift Estimates

2.1 Wing

For the clean wing under the influence of the body, that is with $\delta_f = 0°$:

$$C_{L_w} = 1.265 \cos\alpha_e \left\{ (a_1)_w \frac{\sin 2\alpha_e}{2} + C_{N_{aa}} \sin\alpha_e \left|\sin\alpha_e\right| \right\} \tag{1}$$

where, with reference to Tables 2.1 and 2.2:

$$\alpha_e = \alpha_w - (\alpha_0)_{w_b} \qquad \text{and} \qquad C_{N_{aa}} = (C_{N_{aa}})_{\mathsf{ref}} + \Delta C_{N_{aa}}$$

and:

$$\Delta C_{N_{aa}} = \begin{cases} 2.82 & \text{if } \tan\alpha_e / \tan(\alpha_e)_{C_{L\mathsf{max}}} \leq 0.6 \\ 7.05 - 7.05 \tan\alpha_e / \tan(\alpha_e)_{C_{L\mathsf{max}}} & \text{otherwise} \end{cases} \tag{2}$$

given that:

$$\alpha_w = \alpha_b + \left(\frac{\bar{\bar{c}} - c_t}{c_r - c_t}\right) 2° = \alpha_b + 1.33° = \alpha_b + 0.0232 \text{ rad}$$

When required to find a trim solution for Mach numbers greater that 0.4 it is safe to assume that $\tan\alpha_e / \tan(\alpha_e)_{C_{L\mathsf{max}}} < 0.6$ and so $\Delta C_{N_{aa}} = 2.82$.

Now with flaps deployed and $\alpha_w < \alpha^*$:

$$C_{L_w} = 1.265 \, (a_1)_{tw} \left[\alpha_w - (\alpha_0)_{tw}\right] \cos^N \left[\alpha_w - (\alpha_0)_{tw}\right] \tag{3}$$

see Table 2.3. Above α^*:

$$C_{L_w} = 1.265 \left\{ C_{L\mathsf{max}} - 0.30 \left[1 - \frac{a_1^*(\alpha_w - \alpha^*)}{0.39}\right]^{1.3} \right\} \tag{4}$$

where α^*, a_1^*, and $C_{L\mathsf{max}}$ are given in Table 2.2. The wing, body and nacelle lift are assumed

Mach	$(a_1)_w$ rad^{-1}	$(\alpha_0)_{w_b}$ rad	$\Delta x_h / \bar{\bar{c}}$
0.05	4.326	-0.03217	0.1467
0.10	4.386	-0.03308	0.1437
0.15	4.438	-0.03350	0.1417
0.20	4.490	-0.03382	0.1401
0.25	4.546	-0.03398	0.1388
0.30	4.613	-0.03405	0.1373
0.35	4.688	-0.03409	0.1360
0.40	4.782	-0.03411	0.1342
0.45	4.896	-0.03408	0.1318
0.50	5.046	-0.03392	0.1283
0.55	5.242	-0.03366	0.1234

Table 2.1: Lift Data - Clean Wing

(δ_f)		Mach 0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40
0°	$(C_{N_{aa}})_{\text{ref}}$ rad^{-1}	0.767	0.764	0.752	0.501	0.280	-0.002	-0.255	-0.514
	$(\alpha_e)_{C_{L_{\max}}}$ rad	0.3571	0.3561	0.3543	0.3322	0.3145	0.2947	0.2791	0.2652
10°	α^* rad	0.2953	0.2954	0.2939	0.2686	0.2491			
	a_1^* rad^{-1}	3.621	3.754	3.855	4.029	4.170			
	$C_{L_{\max}}$	1.821	1.856	1.877	1.802	1.745			
20°	α^* rad	0.2864	0.2869	0.2855	0.2591	0.2388			
	a_1^* rad^{-1}	3.465	3.613	3.724	3.921	4.079			
	$C_{L_{\max}}$	1.968	2.010	2.035	1.963	1.908			
35°	α^* rad	0.2990	0.2997	0.2981	0.2697	0.2481			
	a_1^* rad^{-1}	3.221	3.391	3.518	3.747	3.928			
	$C_{L_{\max}}$	2.140	2.190	2.220	2.151	2.099			

Table 2.2: High Lift Data

	Mach	0.05	0.10	0.15	0.20	0.25
flap setting (δ_f)	N	0.83	0.73	0.67	0.61	0.56
10°	$(a_1)_{tw}$ rad^{-1}	4.357	4.417	4.469	4.522	4.579
	$(\alpha_0)_{tw}$ rad	-0.0748	-0.0755	-0.0758	-0.0760	-0.0762
	$\Delta C_{L0_{tw}}$	0.245	0.248	0.251	0.254	0.257
20°	$(a_1)_{tw}$ rad^{-1}	4.370	4.430	4.482	4.535	4.592
	$(\alpha_0)_{tw}$ rad	-0.1238	-0.1244	-0.1246	-0.1248	-0.1249
	$\Delta C_{L0_{tw}}$	0.456	0.463	0.468	0.474	0.480
35°	$(a_1)_{tw}$ rad^{-1}	4.365	4.426	4.478	4.530	4.587
	$(\alpha_0)_{tw}$ rad	-0.1631	-0.1636	-0.1637	-0.1638	-0.1638
	$\Delta C_{L0_{tw}}$	0.622	0.631	0.638	0.646	0.654

Table 2.3: Lift Data - Wing with Flap Deployed

to be located at single aerocentre x_h[a]. Now the aero-centre of the wing alone, $\bar{x}/\bar{\bar{c}}$, is invariant with Mach number and equals 0.246. The forward shift caused by the fuselage, $\Delta x_h/\bar{\bar{c}}$, is given in Table 2.1, thus:

$$\frac{x_h}{\bar{\bar{c}}} = \frac{\bar{x}}{\bar{\bar{c}}} - \frac{\Delta x_h}{\bar{\bar{c}}} \qquad \text{or} \qquad \frac{x_h}{\bar{\bar{c}}} = \frac{\bar{x}}{\bar{\bar{c}}} - \frac{\Delta x_h}{\bar{\bar{c}}} + \Delta h_1 \tag{5}$$

where Δh_1 is given in Table 4.3.

2.2 Tailplane

The tailplane lift coefficient is given by:

$$\begin{aligned} C_{L_t} &= (a_1)_t\,\alpha_t + (a_2)_t\,\eta = (a_1)_t\,\{\alpha_w - \bar{\varepsilon} + i_t\} + (a_2)_t\,\eta \\ &= (a_1)_t\left[\alpha_w - \bar{\varepsilon}_0 - \frac{d\bar{\varepsilon}}{d\alpha}\,\{\alpha_w - (\alpha_0)_{wb}\} + i_t\right] + (a_2)_t\,\eta \end{aligned}$$

[a]Fuselage drag will be applied at the Body Axes Centre and nacelle drag at the engine mass centre.

that is, with reference to Table 2.4:

$$C_{L_t} = (a_1)_t \left[\alpha_b - \bar{\varepsilon}_0 - \frac{d\bar{\varepsilon}}{d\alpha} \left\{ \alpha_w - (\alpha_0)_{wb} \right\} \right] + (a_2)_t \, \eta \tag{6}$$

			$d\bar{\varepsilon}/d\alpha$ flap setting (°)				$\bar{\varepsilon}_0$ (rad) flap setting (°)		
Mach	$(a_1)_t$ (rad^{-1})	$(a_2)_t$ (rad^{-1})	0	10	20	35	10	20	35
0.05	2.134	1.800	0.3295	0.3403	0.3213	0.3060	0.0256	0.0404	0.0510
0.10	2.142	1.806	0.3289	0.3434	0.3253	0.3099	0.0261	0.0410	0.0518
0.15	2.156	1.818	0.3286	0.3464	0.3285	0.3130	0.0264	0.0415	0.0524
0.20	2.176	1.834	0.3284	0.3504	0.3317	0.3160	0.0268	0.0421	0.0529
0.25	2.202	1.856	0.3283	0.3539	0.3348	0.3188	0.0272	0.0425	0.0535
0.30	2.235	1.884	0.3282						
0.35	2.276	1.919	0.3289						
0.40	2.326	1.961	0.3302						
0.45	2.387	2.013	0.3325						
0.50	2.461	2.075	0.3355						
0.55	2.552	2.152	0.3400						

Table 2.4: Aerodynamic Properties of Tailplane

2.3 Fuselage and Nacelles

The lift generated by the fuselage is given by the following coefficient (based on wing area S_w):

$$C_{L_f} = 0.218(\alpha_b - 0.0349) + 0.0171(\alpha_b - 0.0349)^2 \tag{7}$$

and for each nacelle:

$$C_{L_n} = 0.0243\alpha_b + 0.0143\alpha_b^2 \tag{8}$$

3 Drag Estimates

3.1 Profile Drag

The profile drag characteristics of the major airframe components are given in the form of 'drag areas' ($D_0/\bar{q}$) in Table 3.1. The effect of deployment of the control surfaces on the profile drag[b] are given in Table 3.2. Flap increments for $D_0/\bar{q}$ were estimated as 0.0778, 0.2634 and 0.7099 for 10°, 20° and 35° respectively. Undercarriage drag can estimated using a ΔC_{D_0} of 0.0343.

Mach	Wing	Fuselage	Tailplane	Fin	Nacelles	Total
0.05	0.26755	0.47045	0.09366	0.05739	0.08662	0.97567
0.10	0.24113	0.44593	0.08309	0.05112	0.07838	0.89966
0.15	0.22536	0.43304	0.07765	0.04789	0.07408	0.85803
0.20	0.20739	0.42438	0.07405	0.04574	0.07122	0.82278
0.25	0.19661	0.41786	0.07137	0.04413	0.06908	0.79907
0.30	0.19993	0.41260	0.07136	0.04416	0.06736	0.79542
0.35	0.20226	0.40816	0.07131	0.04416	0.06592	0.79182
0.40	0.20437	0.40428	0.07123	0.04413	0.06468	0.78869
0.45	0.20555	0.40081	0.07111	0.04408	0.06356	0.78511
0.50	0.20728	0.39764	0.07096	0.04400	0.06254	0.78242
0.55	0.20954	0.39471	0.07077	0.04391	0.06160	0.78053

Table 3.1: Drag Areas, $D_0/\bar{q}$, for Aircraft

The drag obtained using this coefficient should be distributed according to the relative masses of the nose and main gear. The centre of pressure for these components can be assumed to be coincident with the relevant cg.

3.2 Lift Dependent Drag

The vortex drag of the tailplane is given by:

$$(C_{Dv})_t = \left[\frac{1+\delta+\pi A_t K_1}{\pi A_t}\right] C_{L_t}^2 \tag{9}$$

where $(1+\delta)$ and K_1 are shown in Table 3.3. The lift dependent drag of the wing is given by:

$$(C_{Dv})_w = \frac{1}{\pi A_w}\left[k_1 C_{L1}^2 + k_f \Delta C_{L0_{tw}}^2 + k_{1f} C_{L1} \Delta C_{L0_{tw}}\right] + C_{DV0} + K_{\text{visc}}\left(C_{L1} - C_{L_{\mathsf{min}}}\right)^2$$

where C_{L1} represents the basic lift coefficient due to incidence, camber and twist, that is C_{L_w} for $\delta_f = 0°$, and $\Delta C_{L0_{tw}}$ is shown in Table 2.3, see also Tables 3.4, 3.5 and 3.6. Note that k_{1f} is essentially constant with flap angle. Values of $k_1, C_{DV0}, K_{\text{visc}}$ and $C_{L_{\mathsf{min}}}$ for the plain wing are shown in Table 3.6. The lift dependent drag of the fuselage and nacelles is given by:

$$(C_{Dv})_f = C_{L_f}(\alpha_b - 0.0349) \qquad \text{and} \qquad (C_{Dv})_n = C_{L_n}\alpha_b$$

[b]To account for non-symmetric deflection the increments given for the aileron are per surface

deflection (deg)	(rad)	elevator	rudder	aileron
-30	-0.5236	0.58993	0.23164	0.22077
-25	-0.4363	0.44106	0.17319	0.14592
-20	-0.3491	0.29951	0.11761	0.08077
-15	-0.2618	0.17630	0.06923	0.03033
-10	-0.1745	0.08091	0.03177	-0.00156
-5	-0.0873	0.02062	0.00810	-0.01246
0	0.0000	0.00000	0.00000	0.00000
5	0.0873	0.02062	0.00810	0.03644
10	0.1745	0.08091	0.03177	0.09408
15	0.2618	0.17630	0.06923	0.16854
20	0.3491	0.29951	0.11761	0.25408
25	0.4363	0.44106	0.17319	0.34405
30	0.5236	0.58993	0.23164	0.43130

Table 3.2: Drag Area Increments, $D_0/\bar{q}$, due to Control Deflection

Mach	K_1	$(1+\delta)$
0.05	0.00431	1.00286
0.10	0.00377	1.00284
0.15	0.00347	1.00280
0.20	0.00328	1.00275
0.25	0.00318	1.00268
0.30	0.00307	1.00260
0.35	0.00300	1.00250
0.40	0.00296	1.00239
0.45	0.00295	1.00225
0.50	0.00295	1.00210
0.55	0.00296	1.00192

Table 3.3: Variation of Lift Dependent Drag Factors for Tailplane

α_w	10°	20°	35°
0°	2.141	2.141	2.142
2°	2.095	2.095	2.095
4°	2.072	2.072	2.072
6°	2.058	2.058	2.058
8°	2.048	2.049	2.049
10°	2.042	2.042	2.042
12°	2.037	2.037	2.037

Table 3.4: Variation of Vortex Drag Factor (k_{1f}) with Incidence - Flaps Deployed

Mach	10°	20°	35°
0.05	2.021	2.024	2.036
0.10	2.018	2.021	2.034
0.15	2.014	2.017	2.029
0.20	2.008	2.011	2.023
0.25	2.000	2.003	2.016

Table 3.5: Variation of Vortex Drag Factor (k_f) with Mach Number - Flaps Deployed

Mach	k_1	C_{DV0}	K_{visc}	$C_{L_{\text{min}}}$
0.05	1.01052	0.000113	0.0224	0.357
0.10	1.01045	0.000123	0.0196	0.332
0.15	1.01034	0.000130	0.0180	0.343
0.20	1.01019	0.000136	0.0170	0.345
0.25	1.00999	0.000142	0.0164	0.350
0.30	1.00975	0.000147	0.0158	0.354
0.35	1.00947	0.000155	0.0156	0.359
0.40	1.00914	0.000163	0.0155	0.365
0.45	1.00877	0.000174	0.0155	0.371
0.50	1.00836	0.000184	0.0156	0.379
0.55	1.00790	0.000201	0.0165	0.385

Table 3.6: Variation of Lift Dependent Drag Factors for Wing

4 Pitching Moment Estimates

The zero-lift pitching moment for the aircraft[c] is given by:

$$C_{M_0} = (C_{M_0})_w + (C_{M_0})_f + \Delta C_{M_0} + \Delta C_{M_{\alpha 0}} \tag{10}$$

where $(C_{M_0})_w$ and $(C_{M_0})_f$ are given in Table 4.1 as is ΔC_{M_0}, the influence of the fuselage on the wing. The shift in the aero-centre, Δh_1, caused by the deployment of flap[d] is shown in Table 4.3. This must be applied when flaps are deployed as discussed in Section 2. The extra nose-down zero-lift pitching moment due to the camber caused by flap deployment, $\Delta C_{M_{\alpha 0}}$, is also given in Table 4.1. Due to wing circulation the fuselage and nacelles generate an additional

					δ_f	
				10°	20°	35°
Mach	$(C_{m_0})_w$	$(C_{m_0})_f$	ΔC_{m_0}	$\Delta C_{M_{\alpha 0}}$	$\Delta C_{M_{\alpha 0}}$	$\Delta C_{M_{\alpha 0}}$
0.05	-0.06981	-0.14577	-0.014727	-0.0629	-0.1202	-0.1655
0.10	-0.07006	-0.14662	-0.014744	-0.0631	-0.1206	-0.1660
0.15	-0.07048	-0.14701	-0.014752	-0.0634	-0.1212	-0.1668
0.20	-0.07109	-0.14731	-0.014758	-0.0639	-0.1221	-0.1680
0.25	-0.07190	-0.14746	-0.014761	-0.0645	-0.1232	-0.1696
0.30	-0.07293	-0.14752	-0.014762	-	-	-
0.35	-0.07420	-0.14757	-0.014763	-	-	-
0.40	-0.07576	-0.14758	-0.014763	-	-	-
0.45	-0.07765	-0.14755	-0.014763	-	-	-
0.50	-0.07995	-0.14741	-0.014760	-	-	-
0.55	-0.08274	-0.14716	-0.014755	-	-	-

Table 4.1: Pitching Moment Data

pitching moment given by, see Table 4.2:

$$(\Delta C_m)_\Gamma = \alpha_w \left[(C_{m_\Gamma})_f + 2\,(C_{m_\Gamma})_n \right] \tag{11}$$

and with flaps deployed:

$$(\Delta C_m)_\Gamma = m_\Gamma \alpha_w^2 + c_\Gamma \alpha_w + 2\alpha_w\,(C_{m_\Gamma})_n \tag{12}$$

where $m_\Gamma = 0.196$, 0.172, or 0.151 and $c_\Gamma = 0.2086$, 0.2170 or 0.2223 for the flap angles, δ_f, of 10°, 20° and 35° respectively. Trimmed pitching moments arising from other parts of the airframe can be estimated by determining appropriate locations for the X and Z forces with reference to the chosen axes centre.

[c]The effect of aileron deflection on the pitching moment has been ignored.
[d]Defined as positive for aft movement and given as a fraction of $\bar{\bar{c}}$.

	$(C_{m_\Gamma})_f$	$(C_{m_\Gamma})_n$ flap setting (°)			
Mach	0	0	10	20	35
0.05	0.24684	0.01220	0.00998	0.00963	0.00934
0.10	0.24713	0.01231	0.01003	0.00973	0.00943
0.15	0.24706	0.01238	0.01007	0.00979	0.00949
0.20	0.24667	0.01241	0.01012	0.00982	0.00953
0.25	0.24615	0.01242	0.01013	0.00984	0.00955
0.30	0.24547	0.01242			
0.35	0.24459	0.01239			
0.40	0.24346	0.01237			
0.45	0.24214	0.01234			
0.50	0.24100	0.01233			
0.55	0.23926	0.01234			

Table 4.2: Free Moment Coefficients (C_{m_Γ}) [rad^{-1}]

flap		Mach 0.05	0.10	0.15	0.20	0.25
10°	α_w (rad)			Δh_1		
	0.03491	0.00935	0.00887	0.00849	0.00818	0.00790
	0.06981	0.00930	0.00882	0.00844	0.00814	0.00786
	0.10472	0.01005	0.00954	0.00913	0.00881	0.00851
	0.13963	0.01104	0.01048	0.01004	0.00969	0.00937
	0.17453	0.01213	0.01153	0.01105	0.01067	0.01032
	0.20944	0.01328	0.01263	0.01211	0.01170	0.01132
	0.24435	0.01447	0.01376	0.01320	0.01276	0.01234
20°	α_w (rad)			Δh_1		
	0.03491	0.02282	0.02172	0.02085	0.02016	0.01951
	0.06981	0.02003	0.01905	0.01827	0.01766	0.01708
	0.10472	0.01960	0.01864	0.01788	0.01728	0.01671
	0.13963	0.01993	0.01895	0.01818	0.01757	0.01700
	0.17453	0.02059	0.01959	0.01880	0.01817	0.01757
	0.20944	0.02145	0.02040	0.01958	0.01893	0.01831
	0.24435	0.02241	0.02133	0.02047	0.01979	0.01915
35°	α_w (rad)			Δh_1		
	0.03491	0.05350	0.05106	0.04914	0.04762	0.04618
	0.06981	0.04421	0.04218	0.04058	0.03931	0.03811
	0.10472	0.04091	0.03903	0.03754	0.03637	0.03525
	0.13963	0.03956	0.03773	0.03629	0.03515	0.03408
	0.17453	0.03911	0.03730	0.03588	0.03475	0.03369
	0.20944	0.03916	0.03735	0.03593	0.03480	0.03373
	0.24435	0.03953	0.03770	0.03626	0.03513	0.03405

Table 4.3: Shift of Aero-Centre Position (Δh_1) with Mach Number and Flap Setting

5 Longitudinal Aeroderivatives

This section is included for completeness. Note that an evaluation of the longitudinal derivatives is *not* required as the assessment is focused on lateral and directional characteristics.

The determination of the longitudinal derivatives is problematic as the aerodynamic properties of the aircraft have been expressed in the form of component equations for lift, drag and pitching moment. The solution is therefore to use the approximate relationships given in Cook [2013] and represent the lift and drag properties of the whole aircraft, at zero control deflection by:

$$C_L = a_1\alpha_b + C_{L0} \qquad \text{and} \qquad C_D = C_{D0} + kC_L^2$$

The methodology requires values for $\partial C_L/\partial\alpha$, $\partial C_L/\partial M$, $\partial C_D/\partial\alpha$, $\partial C_D/\partial M$, $\partial C_M/\partial\alpha$ and $\partial C_M/\partial M$. Hence:

$$\frac{\partial C_L}{\partial\alpha} = a_1 \qquad \frac{\partial C_L}{\partial M} = \alpha_b\frac{\mathrm{d}a_1}{\mathrm{d}M} + \frac{\mathrm{d}C_{L0}}{\mathrm{d}M} \qquad \frac{\partial C_D}{\partial\alpha} = 2kC_L\frac{\partial C_L}{\partial\alpha} = 2kC_La_1$$

$$\frac{\partial C_D}{\partial M} = \frac{\mathrm{d}C_{D0}}{\mathrm{d}M} + C_L^2\frac{\mathrm{d}k}{\mathrm{d}M} + 2kC_L\frac{\partial C_L}{\partial M} = \frac{\mathrm{d}C_{D0}}{\mathrm{d}M} + C_L^2\frac{\mathrm{d}k}{\mathrm{d}M} + 2kC_L\left(\alpha_b\frac{\mathrm{d}a_1}{\mathrm{d}M} + \frac{\mathrm{d}C_{L0}}{\mathrm{d}M}\right)$$

See Table 5.1 for the variation of key variables with Mach number. Now the pitching moment about the body axes centre, x positive forward, can be written as:

$$M = M_{0w} + M_{0t} + x_h(L_w + L_b) + x_{cg}mg + x_tL_t$$

or:

$$C_M = C_{M_{0w}} + C_{M_{0t}} + \bar{x}_h\left[(C_L)_{w_b} + C_N\frac{\pi D_f^2}{4S_w}\right] + \bar{x}_{cg}\frac{mg}{\bar{q}S_w} + \bar{V}_tC_{L_t}$$

where:

$$\bar{V}_t = \frac{S_tx_t}{S_w\bar{\bar{c}}}$$

Noting that $\bar{x}_{cg}$, $\bar{x}_b$, C_N and $\bar{V}_t$ are essentially constant with Mach number gives:

$$\frac{\partial C_M}{\partial M} = \frac{\partial C_{M_{0w}}}{\partial M} + \frac{\partial C_{M_{0t}}}{\partial M} + \left[(C_L)_{w_b} + C_N\frac{\pi D_f^2}{4S_w}\right]\frac{\partial\bar{x}_h}{\partial M} + \bar{x}_h\left[\frac{\partial\,(C_L)_{w_b}}{\partial M} + \frac{\partial C_N}{\partial M}\frac{\pi D_f^2}{4S_w}\right] + \bar{V}_t\frac{\mathrm{d}C_{L_t}}{\mathrm{d}M}$$

But:

$$C_{M_{0w}} = C_{M_0} + \Delta C_{M_{\alpha0}} + (\Delta C_{M_0})_b \qquad \text{and} \qquad C_{M_{0t}} = -0.652k_f\eta$$

then as $(\Delta C_{M_0})_b$ is a constant:

$$\frac{\partial C_{M_{0w}}}{\partial M} = \frac{\mathrm{d}C_{M_0}}{\mathrm{d}M} + \frac{\mathrm{d}\,(\Delta C_{M_{\alpha0}})}{\mathrm{d}M} \qquad \text{and} \qquad \frac{\partial C_{M_{0t}}}{\partial M} = -0.652\eta\frac{\mathrm{d}k_f}{\mathrm{d}M}$$

which can be estimated from Table 4.1. Now:

$$(C_L)_{w_b} = (a_1)_{w_b}\left[\alpha - (\alpha_0)_{w_b}\right] \qquad \text{or} \qquad (C_L)_{w_b} = (a_1)_{tw_b}\left[\alpha - (\alpha_0)_{tw_b}\right]$$

So:

$$\frac{\partial\,(C_L)_{w_b}}{\partial M} = \frac{\mathrm{d}\,(a_1)_{w_b}}{\mathrm{d}M}\left[\alpha - (\alpha_0)_{w_b}\right] - (a_1)_{w_b}\frac{\mathrm{d}\,(\alpha_0)_{w_b}}{\mathrm{d}M}$$

or, with flaps deployed:

$$\frac{\partial\left(C_L\right)_{w_b}}{\partial M} = \frac{\mathrm{d}\left(a_1\right)_{tw_b}}{\mathrm{d}M}\left[\alpha - \left(\alpha_0\right)_{tw_b}\right] - \left(a_1\right)_{tw_b}\frac{\mathrm{d}\left(\alpha_0\right)_{tw_b}}{\mathrm{d}M}$$

Also:

$$\bar{x}_h = \frac{c_0}{\bar{\bar{c}}}\frac{x_h}{c_0} = \frac{c_0}{\bar{\bar{c}}}\left[\frac{\bar{\bar{x}}_0}{c_0} - \frac{\Delta x_h}{c_0}\right] + \Delta h_1$$

thus:

$$\frac{\partial\bar{x}_h}{\partial M} = -\frac{\mathrm{d}}{\mathrm{d}M}\frac{\Delta x_h}{c_0} \qquad \text{clean} \qquad \frac{\partial\bar{x}_w}{\partial M} = \frac{\mathrm{d}\Delta h_1}{\mathrm{d}M} - \frac{\mathrm{d}}{\mathrm{d}M}\frac{\Delta x_h}{c_0} \qquad \text{flaps deployed}$$

Similarly, at a fixed elevator position:

$$C_{L_t} = \left(a_1\right)_t\alpha_t + \left(a_2\right)_t\eta$$

$$\frac{\partial C_{L_t}}{\partial M} = \alpha_t\frac{\mathrm{d}\left(a_1\right)_t}{\mathrm{d}M} + \left(a_1\right)_t\frac{\partial\alpha_t}{\partial M} + \eta\frac{\mathrm{d}\left(a_2\right)_t}{\mathrm{d}M}$$

where:

$$\frac{\partial\alpha_t}{\partial M} = \frac{\mathrm{d}}{\mathrm{d}M}\left\{\alpha_w - \bar{\varepsilon} + i_t\right\} = -\frac{\mathrm{d}\bar{\varepsilon}}{\mathrm{d}M}$$

hence with flaps deployed:

$$\frac{\mathrm{d}\bar{\varepsilon}}{\mathrm{d}M} = \frac{\mathrm{d}\bar{\varepsilon}_{fT}}{\mathrm{d}M}$$

or for the clean configuration:

$$\frac{\mathrm{d}\bar{\varepsilon}}{\mathrm{d}M} = \frac{\mathrm{d}}{\mathrm{d}M}\left[\bar{\varepsilon}_0 + \frac{d\bar{\varepsilon}}{d\alpha}\left\{\alpha_w - \left(\alpha_0\right)_{w_b}\right\}\right] = \left\{\alpha_w - \left(\alpha_0\right)_{w_b}\right\}\frac{\mathrm{d}(d\bar{\varepsilon}/d\alpha)}{\mathrm{d}M} - \frac{d\bar{\varepsilon}}{d\alpha}\frac{\mathrm{d}\left(\alpha_0\right)_{w_b}}{\mathrm{d}M}$$

Likewise:

$$\frac{\partial C_M}{\partial\alpha} = \bar{x}_b\frac{\mathrm{d}C_{L_b}}{\mathrm{d}\alpha_b} + \left(C_L\right)_{w_b}\frac{\mathrm{d}\bar{x}_w}{\mathrm{d}\alpha_b} + \bar{x}_w\frac{\mathrm{d}\left(C_L\right)_{w_b}}{\mathrm{d}\alpha_b} - \bar{V}_t\frac{\mathrm{d}C_{L_t}}{\mathrm{d}\alpha_b}$$

Now:

$$\frac{\mathrm{d}C_{L_b}}{\mathrm{d}\alpha_b} = 0.218 + 0.346\alpha_b$$

and:

$$\frac{\mathrm{d}\left(C_L\right)_{w_b}}{\mathrm{d}\alpha_b} = \frac{\mathrm{d}\left(C_L\right)_{w_b}}{\mathrm{d}\alpha_w} = \left(a_1\right)_{w_b} \qquad \text{clean} \qquad \frac{\mathrm{d}\left(C_L\right)_{w_b}}{\mathrm{d}\alpha_b} = \left(a_1\right)_{tw_b} \qquad \text{flaps deployed}$$

and:

$$\frac{\mathrm{d}\bar{x}_w}{\mathrm{d}\alpha_b} = 0 \qquad \text{clean} \qquad \frac{\mathrm{d}\bar{x}_w}{\mathrm{d}\alpha_b} = \frac{\mathrm{d}}{\mathrm{d}\alpha_w}\Delta h_1 \qquad \text{flaps deployed}$$

Finally:

$$\frac{\mathrm{d}C_{L_t}}{\mathrm{d}\alpha_b} = \left(a_1\right)_t\frac{\mathrm{d}\alpha_t}{\mathrm{d}\alpha_w} = \frac{\mathrm{d}}{\mathrm{d}\alpha_w}\left\{\alpha_w - \bar{\varepsilon} - i_t\right\} = \left[1 - \frac{\mathrm{d}\bar{\varepsilon}}{\mathrm{d}\alpha_w}\right]$$

where:

$$\frac{\mathrm{d}\bar{\varepsilon}}{\mathrm{d}\alpha_w} = \frac{d\bar{\varepsilon}}{d\alpha} \qquad \text{clean} \qquad \frac{\mathrm{d}\bar{\varepsilon}}{\mathrm{d}\alpha_w} = \frac{\mathrm{d}\bar{\varepsilon}_{fT}}{\mathrm{d}\alpha_b} \qquad \text{flaps deployed}$$

flap	Mach	k	C_{L0}	a_1	C_{D0}
0°	0.05	0.03403	0.2688	5.1865	0.03890
	0.10	0.03402	0.2801	5.2089	0.03587
	0.15	0.03401	0.2872	5.2474	0.03421
	0.20	0.03399	0.2937	5.2823	0.03281
	0.25	0.03397	0.2993	5.3203	0.03186
	0.30	0.03394	0.3049	5.3664	0.03172
	0.35	0.03391	0.3100	5.4309	0.03157
	0.40	0.03388	0.3159	5.5013	0.03145
	0.45	0.03384	0.3225	5.5798	0.03130
	0.50	0.03379	0.3291	5.6680	0.03120
	0.55	0.03375	0.3349	5.7559	0.03112
10°	0.05	0.04118	0.5451	5.5454	0.04200
	0.10	0.04093	0.5545	5.6227	0.03897
	0.15	0.04078	0.5611	5.6833	0.03731
	0.20	0.04066	0.5676	5.7415	0.03591
	0.25	0.04057	0.5740	5.8004	0.03496
20°	0.05	0.04651	0.7727	5.5719	0.04940
	0.10	0.04619	0.7827	5.6492	0.04637
	0.15	0.04601	0.7903	5.7101	0.04471
	0.20	0.04583	0.7983	5.7687	0.04331
	0.25	0.04568	0.8067	5.8283	0.04236
35°	0.05	0.04952	0.9523	5.5927	0.06721
	0.10	0.04921	0.9627	5.6071	0.06418
	0.15	0.04899	0.9711	5.7312	0.06252
	0.20	0.04880	0.9804	5.7902	0.06111
	0.25	0.04864	0.9904	5.8502	0.06017

Table 5.1: Lift and Drag Properties of the Whole Aircraft

6 Sideforce Estimates

The following derivatives estimates are *all* wind axes referenced.

6.1 Wing-Body Contributions

The wing/body contribution to Y_v was estimated to be constant at -0.2627. The wing planform contribution to Y_p is dependent on lift coefficient as indicated below:

$$(Y_p)_w = C_{L_w}\left[\frac{(Y_p)_w}{C_L}\right]$$

Where $(Y_p)_w / C_L$ is given in Table 6.1. The contribution to Y_p arising from wing dihedral can be obtained using the following formula:

$$\frac{(Y_p)_\Gamma}{(L_p)_w} = 0.3924$$

The contribution to Y_r from the fuselage is -0.0283.

6.2 Engine Nacelle Contributions

The contribution to Y_v arising from the engine nacelles was found to be constant at -0.0458:

6.3 Fin Contributions

The fin contribution to Y_v is as given in Table 6.1 likewise the ventral fin contribution to Y_v. The contribution to Y_p from the fin is a function of body incidence, α_b, and sidewash, which is itself dependent on incidence. Thus:

$$(Y_p)_f = -1.377\,(0.060\cos\alpha_b - 0.444\sin\alpha_b - 0.18 - \sigma_\alpha)$$

Where σ_α is obtained from Table 6.2. The fin contribution to Y_r is given by:

$$(Y_r)_f = \frac{-(Y_v)_f}{1.26}\left[0.099\sin\alpha_b + 0.400\cos\alpha_b\right]$$

6.4 Rudder Contributions

The rudder sideforce derivative, Y_ζ, is given in Table 6.1

6.5 Flap Contributions

The flap contribution to Y_v is given by:

$$(Y_v)_{tw} = -\Delta C'_{D_f} + \left[0.045\,(Y_v)_f + 0.003\right]\Delta C_{L0_{tw}}$$

Mach	$(Y_p)_w/C_L$	$(a_1)_f$ rad^{-1}	$(Y_v)_f$	$(a_1)_v$ rad^{-1}	$(Y_v)_v$	Y_ζ
0.05	0.05000	2.710	-0.698	1.335	-0.0779	0.260
0.10	0.04995	2.739	-0.706	1.336	-0.0779	0.273
0.15	0.04988	2.759	-0.711	1.338	-0.0780	0.280
0.20	0.04980	2.776	-0.715	1.340	-0.0782	0.284
0.25	0.04970	2.793	-0.720	1.343	-0.0783	0.288
0.30	0.04956	2.812	-0.725	1.346	-0.0785	0.291
0.35	0.04940	2.834	-0.730	1.351	-0.0788	0.294
0.40	0.04918	2.857	-0.736	1.356	-0.0791	0.297
0.45	0.04890	2.882	-0.743	1.362	-0.0794	0.300
0.50	0.04860	2.911	-0.750	1.369	-0.0798	0.303
0.55	0.04825	2.944	-0.759	1.378	-0.0804	0.307

Table 6.1: Variation of Sideforce Derivatives with Mach Number

$0.060(1-\cos\alpha_b)+0.444\sin\alpha_b$	0.00	0.05	0.10	0.15	0.20	0.25
σ_α	0.00	0.07	0.15	0.25	0.35	0.45

Table 6.2: Variation of Sidewash Factor with Body Incidence

7 Rolling Moment Estimates

The following derivatives estimates are *all* wind axes referenced.

7.1 Wing-Body Contributions

The contribution to L_v arising from wing dihedral is dependent on Mach number as shown in Table 7.1 as is the case for the wing planform contribution to L_p. The wing planform contribution to L_v is expressed in the form $(L_v)_w/C_L$, see Table 7.1. As the effect of flaps is covered separately only the basic wing lift coefficient is required here. The fuselage contribution to L_v is given by:

$$(L_v)_b = -0.00145\alpha_b$$

where α_b is measured in degrees. Similarly, the contribution arising from wing-body interference was estimated to be equal to a constant value of 0.0445. The dihedral contribution to L_p is obtained using the following formula:

$$\frac{(L_p)_\Gamma}{(L_p)_w} = -0.0466$$

The wing contribution to L_r is given by the following formula for incompressible flow. Account can be made of compressibility effects by using the scaling factor, L_r/L_{r0}, given in Table 7.1

$$(L_{r0})_w = 0.109\,(C_L)_{w_b} - 0.0037$$

7.2 Nacelle Contributions

The contribution to L_v arising from the engine nacelles is 0.0165. Note that is this aeronormalised derivative is invariant with Mach number and incidence.

7.3 Aileron Contributions

The rolling moment due to aileron deflection, L_ξ, is based on an estimate of lift curve slope with control deflection. Thus for *each* aileron[e] L_ξ is as given in Table 7.1:

7.4 Fin Contributions

The fin contribution to L_v is given by the following formula:

$$(L_v)_f = (Y_v)_f\,[0.099\cos\alpha_b - 0.400\sin\alpha_b]$$

Similarly the fin contribution to L_p is given by:

$$(L_p)_f = -\,(Y_p)_f\,[0.060\cos\alpha_b - 0.444\sin\alpha_b]$$

Likewise, the fin contribution to L_r is given by

$$(L_r)_f = (Y_r)_f\,[0.099\cos\alpha_b - 0.400\sin\alpha_b]$$

7.5 Rudder Contributions

The rudder rolling moment derivative, L_ζ, is found using the following formula:

$$L_\zeta = Y_\zeta\,[0.095\cos\alpha_b - 0.431\sin\alpha_b]$$

[e] Positive surface deflection is starboard down and port up

7.6 Flaps

The contribution to L_v arising from flap deployment is given by:

$$(L_v)_{tw} = 0.0322\Delta C_{L0_{tw}}$$

The flap contribution to L_r for incompressible flow is given in Table 7.2. Account can be made of compressibility effects by using the scaling factor presented in Table 7.1

Mach	$(L_v)_\Gamma$	$(L_v)_w/C_L$	$(L_p)_w$	L_r/L_{r0}	L_ξ
0.05	-0.1058	0.0263	-0.2572	1.005	-0.1182
0.10	-0.1068	0.0263	-0.2593	1.010	-0.1233
0.15	-0.1077	0.0263	-0.2610	1.015	-0.1251
0.20	-0.1087	0.0263	-0.2628	1.020	-0.1265
0.25	-0.1097	0.0263	-0.2648	1.030	-0.1264
0.30	-0.1109	0.0263	-0.2671	1.045	-0.1258
0.35	-0.1122	0.0263	-0.2700	1.060	-0.1253
0.40	-0.1140	0.0263	-0.2740	1.080	-0.1243
0.45	-0.1161	0.0270	-0.2790	1.105	-0.1233
0.50	-0.1190	0.0284	-0.2857	1.135	-0.1211
0.55	-0.1219	0.0303	-0.2935	1.170	-0.1175

Table 7.1: Variation of Rolling Moment Derivatives with Mach Number

δ_f	10°	20°	35°
$(L_{r0})_{tw}$	-0.0096	-0.0180	-0.0245

Table 7.2: Flap Contribution to Rolling Moments

8 Yawing Moment Estimates

The following derivatives estimates are *all* wind axes referenced.

8.1 Wing-Body Contributions

The wing/body contribution to N_v is -0.0719. Note that this aeronormalised derivative is invariant with Mach number and incidence. The wing planform contribution to N_p is dependent on lift coefficient as indicated below:

$$(N_p)_w = (C_L)_{w_b}\left[\frac{(N_p)_w}{C_L} + 0.000038\,(a_1)_w\right]$$

Where $(N_p)_w / C_L$ is given in Table 8.1 and $(a_1)_w$ is $(a_1)_{w_b}$ given in Table **??** or $(a_1)_{tw_b}$ given in Table **??** depending whether flaps are deployed. The contribution to N_p arising from wing dihedral is given by the following formula:

$$\frac{(N_p)_\Gamma}{(L_p)_w} = 0.0019$$

The wing contribution to N_r can be estimated using from:

$$(N_r)_w = -0.1238\left[(C_{D0})_w + 0.305\Delta C_{D0_f}\right] - 0.0053\,(C_L)^2_{w_b}$$

The contribution to N_r from the fuselage, $(N_r)_b$, is constant at -0.0060.

8.2 Nacelle Contributions

The contribution to N_v arising from the engine nacelles is -0.0005. Note that this aeronormalised derivative is also invariant with Mach number and incidence.

8.3 Aileron Contributions

The yawing moment arising from aileron deflection, $\mathscr{N}_\xi$, is caused by antisymmetric changes in wing drag. So:

$$\mathscr{N}_\xi = \bar{q}S_w b\left(C_{n_i} + C_{n_p}\right) = \bar{q}S_w b\left(F(\eta_i) - F(\eta_o) + C_{n_p}\right)$$

So, using the average aileron deflection, ξ:

$$N_\xi = \frac{dF(\eta_i)}{d\xi} - \frac{dF(\eta_o)}{d\xi} + \frac{dC_{n_p}}{d\xi}$$

However for small deflections of the ailerons $|\xi_s| \approx |\xi_p|$, therefore $C_{n_p} = 0$. Also:

$$\frac{dF(\eta)}{d\xi} = \left[\frac{H(\eta)}{A}\left(2.8 + 18\Delta C_{L_{tw}}\right) - G(\eta)\,(C_L)_{tw}\right] L_\xi$$

Give that $\Delta C_{L_{tw}} \approx \Delta C_{L0_{tw}}$, this simplifies to:

$$N_\xi = a + b\,(C_L)_{wb} \qquad \text{or} \qquad N_\xi = a + b_f\,(C_L)_{tw}$$

see Table 8.1 for a and b or b_f.

8.4 Fin Contributions

The fin contribution to N_v is given by:

$$(N_v)_f = -(Y_v)_f\left[0.099\sin\alpha_b + 0.400\cos\alpha_b\right]$$

Using an analogous method the ventral fin contribution to N_v is given by:

$$(N_v)_v = -(Y_v)_v\left[0.400\cos\alpha_b - 0.033\sin\alpha_b\right]$$

Similarly the fin contribution to N_p is given by:

$$(N_p)_f = -(Y_p)_f\left[0.059\sin\alpha_b + 0.444\cos\alpha_b\right]$$

Likewise, the fin contribution to N_r is given by:

$$(N_r)_f = -(Y_r)_f\left[0.099\sin\alpha_b + 0.400\cos\alpha_b\right]$$

8.5 Rudder Contributions

The rudder yawing moment derivative, N_ζ, can be found from:

$$N_\zeta = -Y_\zeta\left[0.431\cos\alpha_b + 0.095\sin\alpha_b\right]$$

8.6 Flap Contribution

The flap contribution to N_v is given by:

$$(N_v)_{tw} = 0.196\Delta C'_{D_f} + 0.045\,(N_v)_f\,\Delta C_{L0_{tw}}$$

					δ_f	
				10°	20°	35°
Mach	$(N_p)_w/C_L$	a	b	b_f	b_f	b_f
0.05	-0.03896	0.00317	-0.00118	-0.00332	-0.00516	-0.00661
0.10	-0.03890	0.00351	-0.00124	-0.00347	-0.00540	-0.00691
0.15	-0.03884	0.00376	-0.00125	-0.00353	-0.00549	-0.00703
0.20	-0.03873	0.00401	-0.00127	-0.00358	-0.00559	-0.00716
0.25	-0.03861	0.00421	-0.00127	-0.00360	-0.00562	-0.00720
0.30	-0.03842	0.00439	-0.00126	-	-	-
0.35	-0.03822	0.00457	-0.00125	-	-	-
0.40	-0.03799	0.00473	-0.00124	-	-	-
0.45	-0.03765	0.00489	-0.00123	-	-	-
0.50	-0.03725	0.00500	-0.00121	-	-	-
0.55	-0.03686	0.00503	-0.00118	-	-	-

Table 8.1: Variation of Yawing Moment Derivative Factors with Mach Number

9 Body Axes Referenced Derivatives

The lateral and directional derivative estimates given above are all wind axes referenced and therefore need to be converted to a body axes reference. Likewise the methodology used in Cook [2013] for the longitudinal derivatives assumes wind axes. So:

$$
\begin{aligned}
\mathring{\mathrm{Y}}_{v_b} &= \mathring{\mathrm{Y}}_{v_w} \\
\mathring{\mathrm{Y}}_{\zeta_b} &= \mathring{\mathrm{Y}}_{\zeta_w} \\
\mathring{\mathrm{Y}}_{p_b} &= \mathring{\mathrm{Y}}_{p_w} \cos\alpha_b - \mathring{\mathrm{Y}}_{r_w} \sin\alpha_b \\
\mathring{\mathrm{Y}}_{r_b} &= \mathring{\mathrm{Y}}_{r_w} \cos\alpha_b + \mathring{\mathrm{Y}}_{p_w} \sin\alpha_b \\
\mathring{\mathrm{L}}_{v_b} &= \mathring{\mathrm{L}}_{v_w} \cos\alpha_b - \mathring{\mathrm{N}}_{v_w} \sin\alpha_b \\
\mathring{\mathrm{N}}_{v_b} &= \mathring{\mathrm{N}}_{v_w} \cos\alpha_b + \mathring{\mathrm{L}}_{v_w} \sin\alpha_b \\
\mathring{\mathrm{L}}_{\zeta_b} &= \mathring{\mathrm{L}}_{\zeta_w} \cos\alpha_b - \mathring{\mathrm{N}}_{\zeta_w} \sin\alpha_b \\
\mathring{\mathrm{N}}_{\zeta_b} &= \mathring{\mathrm{N}}_{\zeta_w} \cos\alpha_b + \mathring{\mathrm{L}}_{\zeta_w} \sin\alpha_b \\
\mathring{\mathrm{L}}_{\xi_b} &= \mathring{\mathrm{L}}_{\xi_w} \cos\alpha_b - \mathring{\mathrm{N}}_{\xi_w} \sin\alpha_b \\
\mathring{\mathrm{N}}_{\xi_b} &= \mathring{\mathrm{N}}_{\xi_w} \cos\alpha_b + \mathring{\mathrm{L}}_{\xi_w} \sin\alpha_b \\
\mathring{\mathrm{L}}_{p_b} &= \mathring{\mathrm{L}}_{p_w} \cos^2\alpha_b + \mathring{\mathrm{N}}_{r_w} \sin^2\alpha_b - \left(\mathring{\mathrm{L}}_{r_w} + \mathring{\mathrm{N}}_{p_w}\right) \sin\alpha_b \cos\alpha_b \\
\mathring{\mathrm{N}}_{p_b} &= \mathring{\mathrm{N}}_{p_w} \cos^2\alpha_b - \mathring{\mathrm{L}}_{r_w} \sin^2\alpha_b + \left(\mathring{\mathrm{L}}_{p_w} - \mathring{\mathrm{N}}_{r_w}\right) \sin\alpha_b \cos\alpha_b \\
\mathring{\mathrm{L}}_{r_b} &= \mathring{\mathrm{L}}_{r_w} \cos^2\alpha_b - \mathring{\mathrm{N}}_{p_w} \sin^2\alpha_b + \left(\mathring{\mathrm{L}}_{p_w} - \mathring{\mathrm{N}}_{r_w}\right) \sin\alpha_b \cos\alpha_b \\
\mathring{\mathrm{N}}_{r_b} &= \mathring{\mathrm{N}}_{r_w} \cos^2\alpha_b + \mathring{\mathrm{L}}_{p_w} \sin^2\alpha_b + \left(\mathring{\mathrm{L}}_{r_w} + \mathring{\mathrm{N}}_{p_w}\right) \sin\alpha_b \cos\alpha_b \\
\mathring{\mathrm{Z}}_{u_b} &= \mathring{\mathrm{Z}}_{u_w} \cos^2\alpha_b - \mathring{\mathrm{X}}_{w_w} \sin^2\alpha_b + \left(\mathring{\mathrm{X}}_{u_w} - \mathring{\mathrm{Z}}_{w_w}\right) \sin\alpha_b \cos\alpha_b \\
\mathring{\mathrm{Z}}_{w_b} &= \mathring{\mathrm{Z}}_{w_w} \cos^2\alpha_b + \mathring{\mathrm{X}}_{u_w} \sin^2\alpha_b + \left(\mathring{\mathrm{X}}_{w_w} + \mathring{\mathrm{Z}}_{u_w}\right) \sin\alpha_b \cos\alpha_b
\end{aligned}
$$

$$
\begin{aligned}
\mathring{\mathrm{X}}_{u_b} &= \mathring{\mathrm{X}}_{u_w} \cos^2\alpha_b + \mathring{\mathrm{Z}}_{w_w} \sin^2\alpha_b - \left(\mathring{\mathrm{X}}_{w_w} + \mathring{\mathrm{Z}}_{u_w}\right) \sin\alpha_b \cos\alpha_b \\
\mathring{\mathrm{X}}_{w_b} &= \mathring{\mathrm{X}}_{w_w} \cos^2\alpha_b - \mathring{\mathrm{Z}}_{u_w} \sin^2\alpha_b + \left(\mathring{\mathrm{X}}_{u_w} - \mathring{\mathrm{Z}}_{w_w}\right) \sin\alpha_b \cos\alpha_b \\
\mathring{\mathrm{M}}_{u_b} &= \mathring{\mathrm{M}}_{u_w} \cos\alpha_b - \mathring{\mathrm{M}}_{w_w} \sin\alpha_b \\
\mathring{\mathrm{M}}_{w_b} &= \mathring{\mathrm{M}}_{w_w} \cos\alpha_b + \mathring{\mathrm{M}}_{u_w} \sin\alpha_b \\
\mathring{\mathrm{X}}_{q_b} &= \mathring{\mathrm{X}}_{q_w} \cos\alpha_b - \mathring{\mathrm{Z}}_{q_w} \sin\alpha_b \\
\mathring{\mathrm{Z}}_{q_b} &= \mathring{\mathrm{Z}}_{q_w} \cos\alpha_b + \mathring{\mathrm{X}}_{q_w} \sin\alpha_b \\
\mathring{\mathrm{X}}_{\dot{w}_b} &= \mathring{\mathrm{X}}_{\dot{w}_w} \cos^2\alpha_b - \mathring{\mathrm{Z}}_{\dot{w}_w} \sin\alpha_b \cos\alpha_b \\
\mathring{\mathrm{Z}}_{\dot{w}_b} &= \mathring{\mathrm{Z}}_{\dot{w}_w} \cos^2\alpha_b + \mathring{\mathrm{X}}_{\dot{w}_w} \sin\alpha_b \cos\alpha_b \\
\mathring{\mathrm{X}}_{\eta_b} &= \mathring{\mathrm{X}}_{\eta_w} \cos\alpha_b - \mathring{\mathrm{Z}}_{\eta_w} \sin\alpha_b \\
\mathring{\mathrm{Z}}_{\eta_b} &= \mathring{\mathrm{Z}}_{\eta_w} \cos\alpha_b + \mathring{\mathrm{X}}_{\eta_w} \sin\alpha_b \\
\mathring{\mathrm{M}}_{\dot{w}_b} &= \mathring{\mathrm{M}}_{\dot{w}_w} \cos\alpha_b \\
\mathring{\mathrm{M}}_{q_b} &= \mathring{\mathrm{M}}_{q_w} \\
\mathring{\mathrm{M}}_{\eta_b} &= \mathring{\mathrm{M}}_{\eta_w}
\end{aligned}
$$

10 Propeller Derivatives

Neglecting any slipstream effects and ignoring the cross-coupling moments generated by incidence and sideslip gives, for the Jetstream 31:

$$\frac{dC_Z}{d\alpha_e} = \frac{dC_Y}{d\psi_e} = -\frac{4.25\sigma_e}{1+2\sigma_e}\sin\left(\beta_0+3\right)\frac{\pi J^2}{8}\left[1+\frac{3T_c}{8\sqrt{1+\frac{2}{3}T_c}}\right]$$

where:

$$T_c = \frac{8}{\pi J^2}C_T \qquad \sigma_e = 0.0954$$

At a fixed propeller speed, and advance ratio, only variations in blade angle ($\beta_0 = \theta_{.7}$) will lead to any change in propeller forces. The constant speed unit fitted to the Jetstream 31 maintains propeller speed by changes in blade pitch as the shaft power varies. Figures 10.1 and 10.2 show that blade angle is the link between power coefficient, advance ratio and thrust coefficient. However when disturbed from trim at a given power setting and blade angle the propeller thrust coefficient will only vary with advance ratio. Now the forces at each propeller hub are given, in steady flight, by:

$$\begin{aligned}
X_P &= \rho n^2 D^4 C_T \\
Y_P &= \rho n^2 D^4 \left(\frac{\delta C_Y}{\delta \psi_e}\right)\psi_e = \rho n^2 D^4 \left(\frac{\delta C_Z}{\delta \alpha_e}\right)\beta \\
Z_P &= \rho n^2 D^4 \left(\frac{\delta C_Z}{\delta \alpha_e}\right)\alpha_e = \rho n^2 D^4 \left(\frac{\delta C_Z}{\delta \alpha_e}\right)\alpha_b
\end{aligned}$$

From Figure 10.3 the powerplant forces and moments about the body axes centre are given by:

$$\begin{aligned}
X_{\text{prop}} &= X_{P_1} + X_{P_2} \\
Y_{\text{prop}} &= Y_{P_1} + Y_{P_2} \\
Z_{\text{prop}} &= Z_{P_1} + Z_{P_2} \\
L_{\text{prop}} &= -\left(Z_{P_1} - Z_{P_2}\right) y_p - \left(Y_{P_1} + Y_{P_2}\right) z_p \\
M_{\text{prop}} &= +\left(X_{P_1} + X_{P_2}\right) z_p - \left(Z_{P_1} + Z_{P_2}\right) x_p \\
N_{\text{prop}} &= +\left(X_{P_1} - X_{P_2}\right) y_p + \left(Y_{P_1} + Y_{P_2}\right) x_p
\end{aligned}$$

Now during a manoeuvre the following relationships apply at the propeller spinner:

$$U = U_e + u - r y_p + q z_p \qquad \alpha_e = \frac{W_e + w - q x_p + p y_p}{V_0} \qquad \psi_e = \frac{v + r x_p - p z_p}{V_0}$$

10.1 Longitudinal Derivatives

Note that use of this section is not required for the assessment: it has been included for completeness. From above $X_P = \rho n^2 D^4 C_T$ and given that $J = V_0/nD \approx U/nD$:

$$\frac{\mathrm{d}X_P}{\mathrm{d}u} = \rho n^2 D^4 \frac{\mathrm{d}C_T}{\mathrm{d}u} = \rho n^2 D^4 \frac{\mathrm{d}C_T}{\mathrm{d}J}\cdot\frac{\mathrm{d}J}{\mathrm{d}u} = \rho n^2 D^4 \frac{\mathrm{d}C_T}{\mathrm{d}J}\cdot\frac{1}{nD} = \rho n D^3 \frac{\mathrm{d}C_T}{\mathrm{d}J}$$

Thus:

$$\left(\mathring{\mathrm{X}}_u\right)_{\text{prop}} = 2\rho n D^3 \frac{\mathrm{d}C_T}{\mathrm{d}J}$$

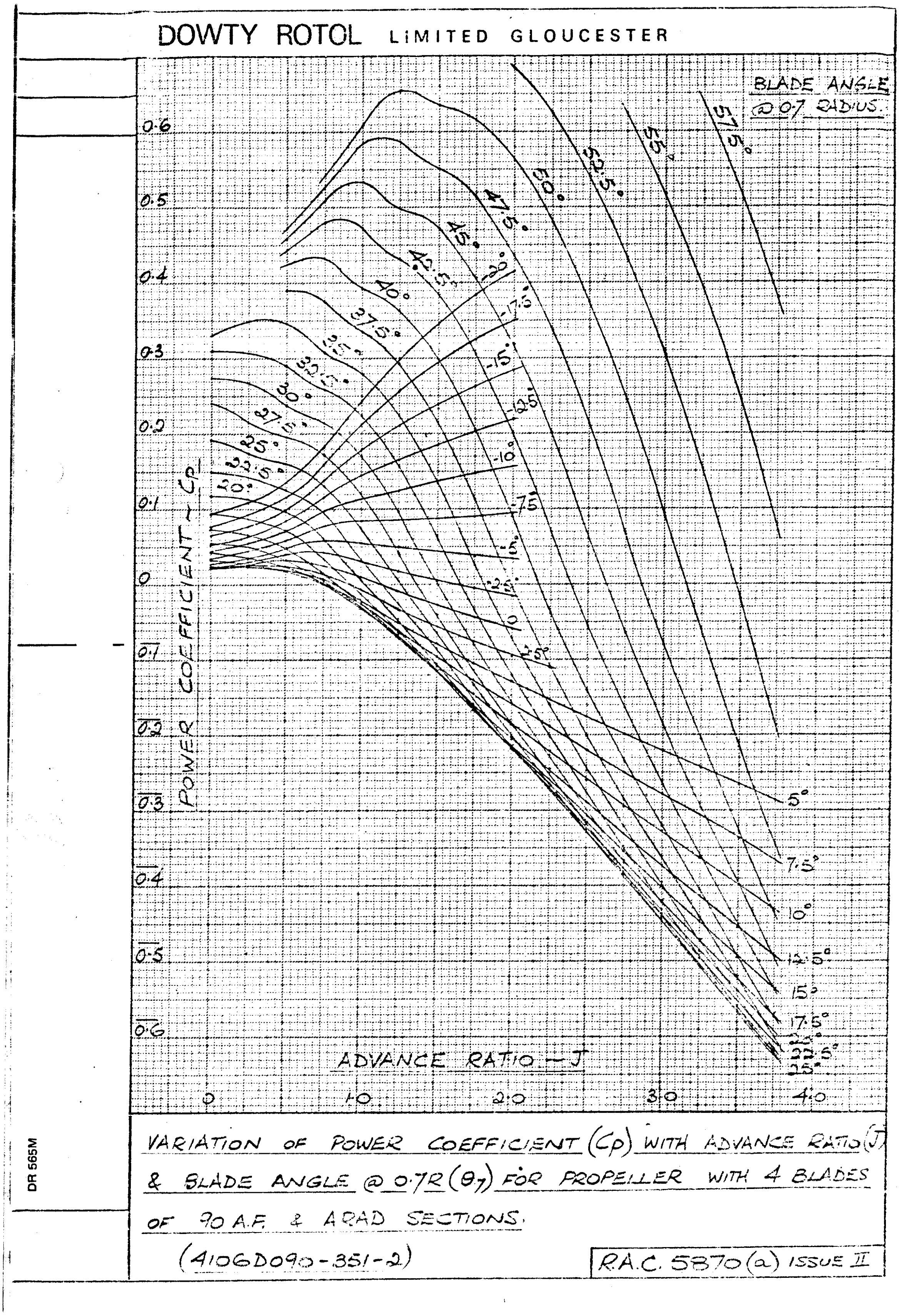

Figure 10.1: Variation of Power Coefficient with Advance Ratio and Blade Angle

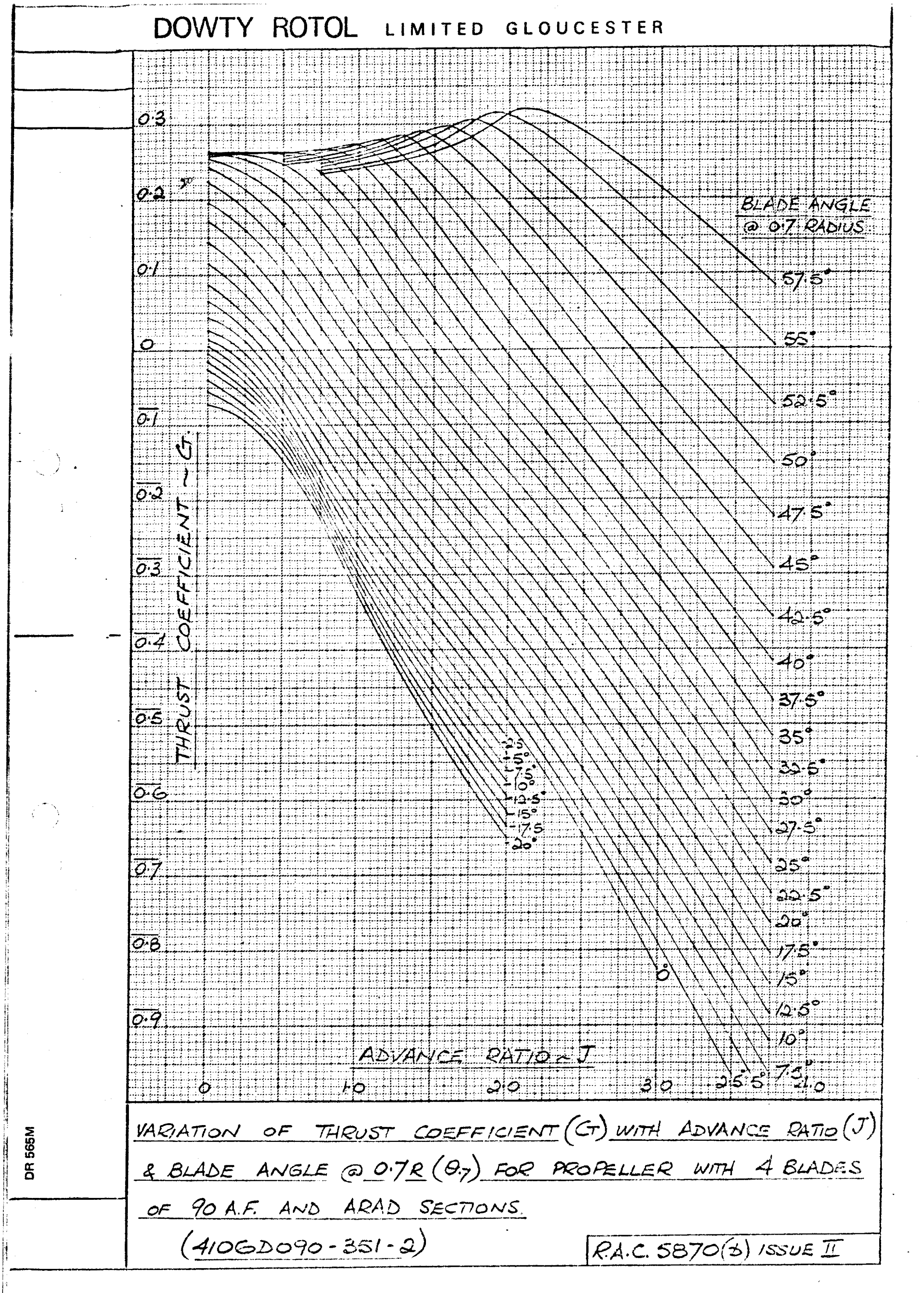

Figure 10.2: Variation of Thrust Coefficient with Advance Ratio and Blade Angle

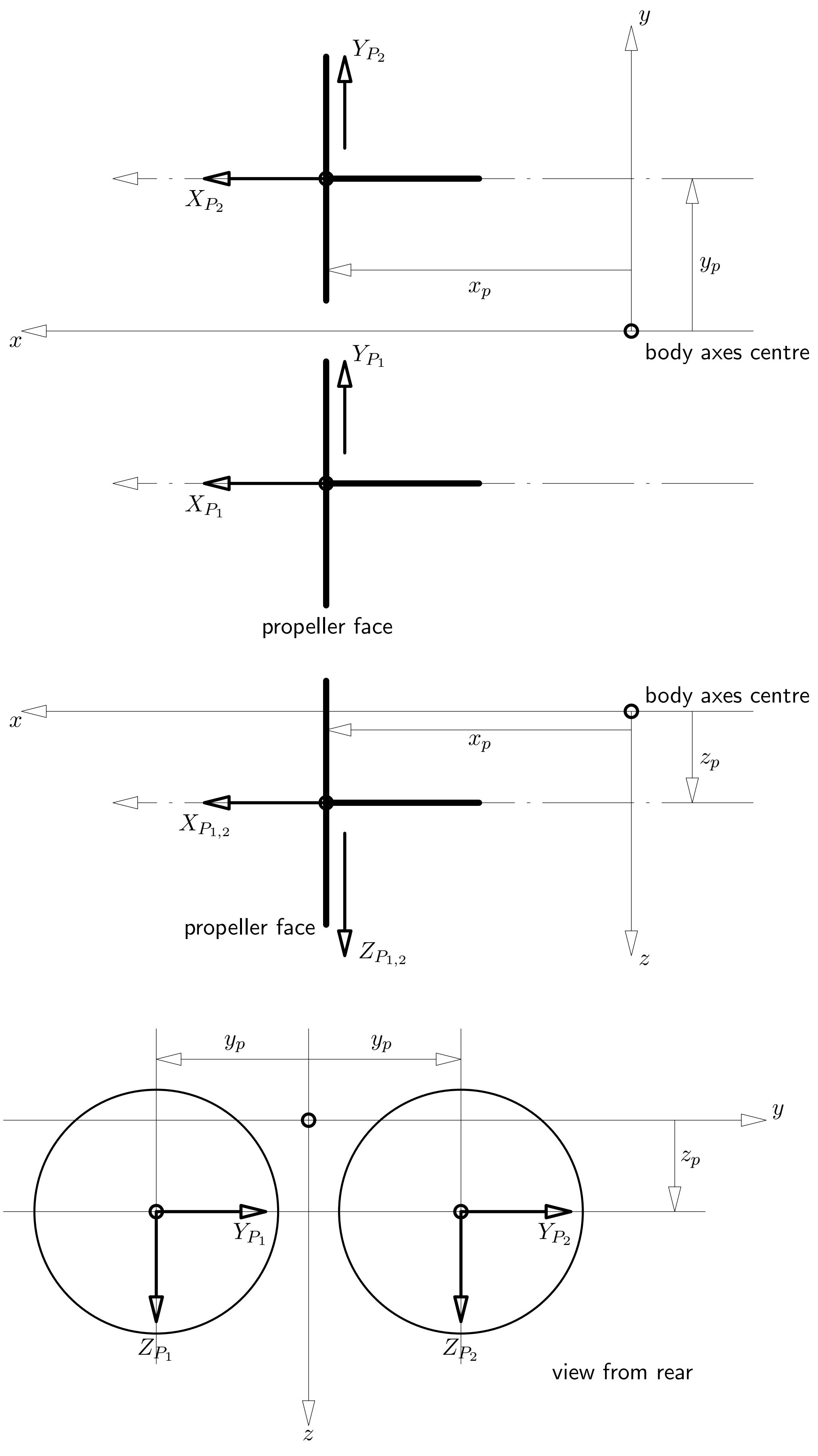

Figure 10.3: Body Axes Referencing of Propeller Forces

Also:

$$\frac{\mathrm{d}Z_P}{\mathrm{d}w} = \rho n^2 D^4 \left(\frac{\delta C_Z}{\delta \alpha_e}\right) \frac{\mathrm{d}\alpha_e}{\mathrm{d}w} = \rho n^2 D^4 \left(\frac{\delta C_Z}{\delta \alpha_e}\right) \frac{1}{V_0} = \rho \frac{nD^3}{J} \left(\frac{\delta C_Z}{\delta \alpha_e}\right)$$

Hence:

$$\left(\mathring{\mathrm{Z}}_w\right)_{\text{prop}} = 2\rho \frac{nD^3}{J} \left(\frac{\delta C_Z}{\delta \alpha_e}\right)$$

Similarly:

$$\frac{\mathrm{d}X_P}{\mathrm{d}q} = \rho n^2 D^4 \frac{\mathrm{d}C_T}{\mathrm{d}q} = \rho n^2 D^4 \frac{\mathrm{d}C_T}{\mathrm{d}J} \cdot \frac{\mathrm{d}J}{\mathrm{d}q} = \rho n^2 D^4 \frac{\mathrm{d}C_T}{\mathrm{d}J} \cdot \frac{z_p}{nD} = \rho z_p n D^3 \frac{\mathrm{d}C_T}{\mathrm{d}J}$$

And:

$$\left(\mathring{\mathrm{X}}_q\right)_{\text{prop}} = +2\rho z_p n D^3 \frac{\mathrm{d}C_T}{\mathrm{d}J} = +z_p \left(\mathring{\mathrm{X}}_u\right)_{\text{prop}}$$

Likewise:

$$\left(\mathring{\mathrm{Z}}_q\right)_{\text{prop}} = -2\rho x_p \frac{nD^3}{J} \left(\frac{\delta C_Z}{\delta \alpha_e}\right) = -x_p \left(\mathring{\mathrm{Z}}_w\right)_{\text{prop}}$$

Finally:

$$\left(\mathring{\mathrm{M}}_u\right)_{\text{prop}} = z_p \left(\mathring{\mathrm{X}}_u\right)_{\text{prop}} \qquad \left(\mathring{\mathrm{M}}_w\right)_{\text{prop}} = -x_p \left(\mathring{\mathrm{Z}}_w\right)_{\text{prop}}$$

$$\left(\mathring{\mathrm{M}}_q\right)_{\text{prop}} = z_p \left(\mathring{\mathrm{X}}_q\right)_{\text{prop}} - x_p \left(\mathring{\mathrm{Z}}_q\right)_{\text{prop}} = z_p^2 \left(\mathring{\mathrm{X}}_u\right)_{\text{prop}} + x_p^2 \left(\mathring{\mathrm{Z}}_w\right)_{\text{prop}}$$

10.2 Lateral/directional Derivatives

Now:

$$\frac{\mathrm{d}Y_P}{\mathrm{d}v} = \rho n^2 D^4 \left(\frac{\delta C_Y}{\delta \psi_e}\right) \frac{\mathrm{d}\psi_e}{\mathrm{d}v} = \rho n^2 D^4 \left(\frac{\delta C_Z}{\delta \alpha_e}\right) \frac{1}{V_0} = \rho \frac{nD^3}{J} \left(\frac{\delta C_Z}{\delta \alpha_e}\right)$$

So:

$$\left(\mathring{\mathrm{Y}}_v\right)_{\text{prop}} \equiv \left(\mathring{\mathrm{Z}}_w\right)_{\text{prop}}$$

Likewise:

$$\frac{\mathrm{d}Y_P}{\mathrm{d}r} = \rho n^2 D^4 \left(\frac{\delta C_Y}{\delta \psi_e}\right) \frac{\mathrm{d}\psi_e}{\mathrm{d}r} = \rho x_p \frac{nD^3}{J} \left(\frac{\delta C_Z}{\delta \alpha_e}\right)$$

Thus:

$$\left(\mathring{\mathrm{Y}}_r\right)_{\text{prop}} = -\left(\mathring{\mathrm{Z}}_q\right)_{\text{prop}} = x_p \left(\mathring{\mathrm{Z}}_w\right)_{\text{prop}}$$

Similarly:

$$\frac{\mathrm{d}Y_P}{\mathrm{d}p} = \rho n^2 D^4 \left(\frac{\delta C_Y}{\delta \psi_e}\right) \frac{\mathrm{d}\psi_e}{\mathrm{d}p} = -\rho z_p \frac{nD^3}{J} \left(\frac{\delta C_Z}{\delta \alpha_e}\right)$$

Thus:

$$\left(\mathring{\mathrm{Y}}_p\right)_{\text{prop}} = -z_p \left(\mathring{\mathrm{Z}}_w\right)_{\text{prop}}$$

In addition:

$$\frac{\mathrm{d}X_P}{\mathrm{d}r} = \rho n^2 D^4 \frac{\mathrm{d}C_T}{\mathrm{d}r} = \rho n^2 D^4 \frac{\mathrm{d}C_T}{\mathrm{d}J} \cdot \frac{\mathrm{d}J}{\mathrm{d}r} = -\rho n^2 D^4 \frac{\mathrm{d}C_T}{\mathrm{d}J} \cdot \frac{y_p}{nD} = -\rho y_p n D^3 \frac{\mathrm{d}C_T}{\mathrm{d}J}$$

Assuming the effects on U due to yawing are equal and opposite then the derivative contributions will additive, hence:

$$\left(\mathring{\mathrm{N}}_r\right)_{\text{prop}} = -2\rho y_p^2 n D^3 \frac{\mathrm{d}C_T}{\mathrm{d}J} + x_p \left(\mathring{\mathrm{Y}}_r\right)_{\text{prop}} = x_p^2 \left(\mathring{\mathrm{Z}}_w\right)_{\text{prop}} - y_p^2 \left(\mathring{\mathrm{X}}_u\right)_{\text{prop}}$$

Also:

$$\frac{\mathrm{d}Z_P}{\mathrm{d}p} = \rho n^2 D^4 \left(\frac{\delta C_Z}{\delta \alpha_e}\right) \frac{\mathrm{d}\alpha_e}{\mathrm{d}p} = \rho n^2 D^4 \left(\frac{\delta C_Z}{\delta \alpha_e}\right) \frac{y_p}{V_0} = \rho y_p \frac{nD^3}{J} \left(\frac{\delta C_Z}{\delta \alpha_e}\right)$$

Again assuming the effects on incidence due to rolling are equal and opposite, gives:

$$\left(\mathring{\mathrm{L}}_p\right)_{\text{prop}} = -2\rho y_p^2 \frac{nD^3}{J} \left(\frac{\delta C_Z}{\delta \alpha_e}\right) - z_p \left(\mathring{\mathrm{Y}}_p\right)_{\text{prop}} = \left(\mathring{\mathrm{Z}}_w\right)_{\text{prop}} \left[z_p^2 - y_p^2\right]$$

Finally it can be shown that:

$$\left(\mathring{\mathrm{L}}_r\right)_{\text{prop}} \equiv \left(\mathring{\mathrm{N}}_p\right)_{\text{prop}} = x_p z_p \left(\mathring{\mathrm{Z}}_w\right)_{\text{prop}}$$

Also note that:

$$\left(\mathring{\mathrm{L}}_v\right)_{\text{prop}} = -z_p \left(\mathring{\mathrm{Y}}_v\right)_{\text{prop}} \qquad \text{and} \qquad \left(\mathring{\mathrm{N}}_v\right)_{\text{prop}} = x_p \left(\mathring{\mathrm{Y}}_v\right)_{\text{prop}}$$

References

M. Cook. *Flight Dynamics Principles: a linear systems approach to aircraft stability and control.* Elsevier Aerospace Engineering. Butterworth-Heinemann, 3rd edition, 2013. ISBN 978-0-08-098242-7.

A. Cooke. A simulation model of the NFLC Jetstream 31. Technical Report COA - 0402, Cranfield University, 2006.

A Geometric Details

A.1 Wing

span	52	ft	15.850	m
gross area	270	ft^2	25.084	m^2
aerodynamic mean chord ($\bar{\bar{c}}$)	5.63	ft	1.717	m
centreline chord (c_0)	7.80	ft	2.377	m
root chord (c_r)	7.20	ft	2.195	m
tip chord (c_t)	2.60	ft	0.792	m
position of 30% chordline (aft of datum)	18.60	ft	5.669	m
position of centreline chordline (below datum)	3.20	ft	0.929	m
aspect ratio	10.0			
taper ratio - tip/centreline	0.333			
sweep of 30% chordline	0.0°			
root aerofoil section	NACA 63A418			
tip aerofoil section	NACA 63A412			
twist	-2° washout			
setting angle	2° nose up			
dihedral angle	7°			

A.2 Fuselage

maximum diameter	6.5	ft	1.981	m
length	43.8	ft	13.35	m
plan area	227	ft^2	21.076	m^2
side area	227	ft^2	21.076	m^2

A.3 Tailplane

span	21.67	ft	6.604	m
gross area	83.8	ft^2	7.785	m^2
aerodynamic mean chord	4.11	ft	1.253	m
root chord	5.5	ft	1.676	m
position of root leading edge aft of datum	36.91	ft	11.25	m
position of tailplane above fuselage datum	4.71	ft	1.440	m
aspect ratio	5.6			
taper ratio - tip/centreline	0.409			
sweep of 60% chordline	0.0°			
root aerofoil section	NACA 0012			
tip aerofoil section	NACA 0010			
quarter chord sweep	7.1°			
setting angle	0°			

A.4 Fin

area	50.96	ft^2	4.734	m^2
tip chord	2.917	ft	0.889	m
root chord	9.07	ft	2.764	m
height	8.50	ft	2.592	m
position of aero-centre aft of datum	39.37	ft	12.00	m
position of aero-centre above fuselage datum	5.15	ft	1.570	m

aspect ratio	2.838
taper ratio	0.322
root aerofoil section	NACA 0012
tip aerofoil section	NACA 0010
quarter chord sweep	42.7°

A.5 Powerplant

length	9.426	ft	2.873	m
lateral offset	8.775	ft	2.675	m
propeller diameter	8.832	ft	2.692	m
datum propeller speed	1591	rpm		

location of propeller spinner: [140.0, ±107.0, 11.5]

A.6 Maximum Control Surface Deflections

aileron	25°	up	15°	down
elevator	22°	up	28°	down
rudder	25°	left	25°	right

B Mass Properties

B.1 Fixed Structure

The empty fuselage of a Standard J31, with no crew or passengers, has a mass of 3975 lb with the cg located at [202.3, 0, 0]. The moments of inertia about the body axes centre at [223, 0, 0] are:

$$\begin{aligned}(I_{xx})_f &= 6046127 \text{ lb.in}^2\\ (I_{yy})_f &= 55297854 \text{ lb.in}^2\\ (I_{zz})_f &= 55297854 \text{ lb.in}^2\end{aligned}$$

The wing has a mass of 2090 lb with the cg located at [228.5, 0, 24.4]. The moments and products of inertia about the body axes centre are:

$$\begin{aligned}(I_{xx})_w &= 34066713 \text{ lb.in}^2\\ (I_{yy})_w &= 1524258 \text{ lb.in}^2\\ (I_{zz})_w &= 32733657 \text{ lb.in}^2\\ (I_{xz})_w &= -261511 \text{ lb.in}^2\end{aligned}$$

The Cranfield Jetstream J31 (G-NFLA) has a different set of mass properties when compared with a standard aircraft. This is due partly to modifications fitted during its service life prior being acquired by the University and partly as a result of the instrumentation suite fitted for its role as a flying classroom. The changes have been accounted for by adding a point mass of 240 lb at FS 337 and by distributing an additional 1334 lb within the cabin structure, as a result the fuselage mass properties become:

$$\begin{aligned}(I_{xx})_f &= 7078355 \text{ lb.in}^2\\ (I_{yy})_f &= 64738605 \text{ lb.in}^2\\ (I_{zz})_f &= 64738605 \text{ lb.in}^2\end{aligned}$$

with a cg located at [209.1, 0, 0]. Other parts of the aircraft are assumed to be point masses, located as indicated in Table B.1.

Item	Mass (lb)	CG Location up	CG Location down
Nose Undercarriage	147	[34.0, 0.0, 24.0]	[61.0, 0.0, 54.0]
Main Undercarriage	221	[241.2, 72.0, 26.8]	[243.5, 109.6, 52.2]
Main Undercarriage	221	[241.2, -72.0, 26.8]	[243.5, -109.6, 52.2]
Tail Unit	588	[474.9, 0.0, -53.0]	
Engine	894	[168.1, 105.3, 10.1]	
Engine	894	[168.1, -105.3, 10.1]	

Table B.1: Location of Main Components

B.2 Variable Masses

The fuel is stored in a set of wing tanks of rather complex shape, however the Type Record for the aircraft gives a tank-by-tank breakdown for the maximum weight configuration. Using this data the variation in cg location and inertia can be determined as function of fuel mass as shown in Table B.2. The passengers and crew are situated at fixed locations within the fuselage as indicated by the seat positions given in Table B.3.

Fuel Mass (lb)	CG Position	I_{xx} (lb.in^2)	I_{yy} (lb.in^2)	I_{zz} (lb.in^2)	I_{xz} (lb.in^2)
104.0	[214.3, 0.0, 30.3]	366036	103403	278376	27423
298.0	[214.6, 0.0, 29.0]	1392752	272618	1162606	73061
527.4	[216.3, 0.0, 27.9]	3132715	436890	2747588	100292
796.4	[218.1, 0.0, 26.7]	5929524	597618	5384445	108173
1134.8	[221.0, 0.0, 25.4]	10575415	775329	9866945	73227
1521.6	[223.4, 0.0, 24.1]	17575971	956121	16731392	13729
1858.8	[224.6, 0.0, 23.0]	25374079	1083213	24437370	-30044
2153.2	[225.3, 0.0, 22.0]	33856112	1170501	32859341	-61713
2410.0	[225.8, 0.0, 21.1]	42883714	1228582	41849899	-84080
2629.6	[226.0, 0.0, 20.3]	52140939	1265023	51086494	-99100
2806.6	[226.2, 0.0, 19.6]	60957918	1285519	59894078	-108208
2935.0	[226.3, 0.0, 19.1]	68421772	1295517	67355423	-113047
3020.4	[226.4, 0.0, 18.7]	74152432	1299768	73086449	-115165
3066.2	[226.4, 0.0, 18.4]	77666920	1301259	76601829	-115762
3072.0	[226.4, 0.0, 18.4]	78171665	1301399	77106713	-115772

Table B.2: Fuel Load - CG Location and Inertias about Body Axes Centre

Seat	Location
Pilot	[111.0, +15.60, 8.36]
Copilot	[111.0, -15.60, 8.36]
Attendant	[376.0, 0.0, 0.0]
1A	[152.7, -22.48, +2.61]
1B	[152.7, +6.88, +2.61]
1C	[152.7, +22.48, +2.61]
2A	[182.7, -22.48, +2.61]
2B	[182.7, +6.88, +2.61]
2C	[182.7, +22.48, +2.61]
3A	[212.7, -22.48, +2.61]
3B	[212.7, +6.88, +2.61]
3C	[212.7, +22.48, +2.61]
4A	[242.7, -22.48, +2.61]
4B	[251.7, +6.88, +2.61]
4C	[251.7, +22.48, +2.61]
5A	[272.7, -22.48, +2.61]
5B	[281.7, +6.88, +2.61]
5C	[281.7, +22.48, +2.61]
6A	[302.7, -22.48, +2.61]
6B	[311.7, +6.88, +2.61]
6C	[311.7, +22.48, +2.61]
Baggage	[390.0, 0.0, 0.0]

Table B.3: Seating Positions

C Consolidated Flight Test Data

It is possible to estimate the characteristics of the actual aircraft by combining data from numerous flight tests.

C.1 Clean Aircraft

C.1.1 Body Angle of Attack

Now:

$$C_L = [0.344 \pm 0.003] + [0.1017 \pm 0.0008]\,\alpha_b \qquad (\alpha_b \text{ in degrees})$$

where:

$$C_L = \frac{2\,(mg_0 \cos\gamma - T \sin\alpha_b)}{\rho_0 V_e^2 S_w}$$

C.1.2 Elevator Angle to Trim

Now:

$$\eta_{\mathsf{CLEAN}} = 2.128 - 14.663 C_L + 27.196 h C_L + 1.535 C_L^2 \pm 0.5 \qquad (\eta \text{ in degrees})$$

where:

$$C_L = \frac{2mg_0}{\rho_0 V_e^2 S_w}$$

and h is the CG location as a fraction of $\bar{\bar{c}}$: for example h = 28% = 0.28.

C.2 Effect of Landing Gear

C.2.1 Body Angle of Attack

The deployment of the landing gear into the propeller slipstream or ‘propwash’ has an effect on the lift characteristics of the wing, thus:

$$C_L = [0.430 \pm 0.008] + [0.096 \pm 0.002]\,\alpha_b \qquad (\alpha_b \text{ in degrees})$$

C.2.2 Elevator Angle to Trim

With the landing gear lowered the elevator angle to trim is slightly more ‘nose-down’, that is:

$$\eta_{\mathsf{UC\ DWN}} = \eta_{\mathsf{CLEAN}} + 0.4^\circ$$

C.3 Effect of Flap

C.3.1 Body Angle of Attack

The deployment of flap improves the lift performance of the wing, thus:

$$C_L = a_0 + a_1 \alpha_b$$

where values for a_0 and a_1 are given in Table C.1.

C.3.2 Elevator Angle to Trim

The deployment of flap, based on the limited testing conducted so far, appears to add only a trailing edge down bias to the trimmed elevator angle, thus:

$$\eta_{\mathsf{flap}} = \eta_{\mathsf{CLEAN}} + \Delta\eta_f \pm 0.5 \qquad (\eta \text{ in degrees})$$

where $\Delta\eta_f$ (°) is given in Table C.2.

C.4 Combined effects

In the landing configuration both gear and flap (35°) are deployed. The lift characteristics are given by:

$$C_L = [0.99 \pm 0.02] + [0.11 \pm 0.01]\,\alpha_b \qquad (\alpha_b \text{ in degrees})$$

and the trimmed elevator deflection is given by:

$$\eta_{\mathsf{LND}} = 6.128 - 14.663C_L + 27.196hC_L + 1.535C_L^2 \pm 1.0 \qquad (\eta \text{ in degrees})$$

C.5 Effect of Climb Power

The extra thrust required in climbing flight generates a 'static' pitching moment and a more intense slipstream effect which cause changes in the elevator angle required for trim. It is suggested, based on the testing conducted to date, that this effect can be accounted for by adding an increment to the elevator deflection required in level fight. So in climbing flight the elevator angle is given by:

$$\eta_{\mathsf{climb}} = \eta_{\mathsf{level}} + \Delta\eta_\gamma = \eta_{\mathsf{level}} + 0.17\gamma \qquad (\eta \text{ and } \gamma \text{ in degrees})$$

flap angle	10°	20°	35°
a_0	0.48±0.01	0.65±0.01	0.10±0.01
a_1	0.111±0.004	0.111±0.004	0.115±0.005

Table C.1: Effect of Flap on Lift Coefficient - flight test results

flap angle	10°	20°	35°
$\Delta\eta_f$	1.5°	2.6°	4.6°

Table C.2: Effect of Flap on Elevator Angle to Trim - flight test results