



# CASAL2 User Manual

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## 1. Introduction

### 1.1. About CASAL2

CASAL2 is NIWA's open-source integrated assessment tool for modelling population dynamics of marine species, including fishery stock assessments. CASAL2 expands functionality and maintainability over its predecessor, CASAL. CASAL2 can be used for quantitative assessments of marine populations, including fish, invertebrates, marine mammals and seabirds.

CASAL2 software implements a generalised age-structured marine population model that allows a great deal of choice in specifying the population dynamics, parameter estimation, and model outputs. CASAL2 is designed for flexibility. It can implement an age-structured model for a single population or multiple populations using user-defined categories such as area, sex and maturity. However, these structural elements are generic and not predefined, but are easily constructed. CASAL2 models can be used for a single population with a single anthropogenic event (in a fish population model this would be a single fishery), or for multiple species and populations, areas, and/or anthropogenic or exploitation methods, and including predator-prey interactions.

The time period and annual cycle of CASAL2 is defined by the user. Observational data used can be from many different sources, for example removals-at-size or -age from an anthropogenic or exploitation event (e.g., fishery or other human impact), research survey and other biomass indices, and mark-recapture data. Model parameters can be estimated using penalised maximum likelihood or Bayesian methods.

As well as generating point estimates of the parameters of interest, CASAL2 can calculate likelihood or posterior profiles and can generate Bayesian posterior distributions using Monte Carlo Markov Chain methods. CASAL2 can project population status into the future using deterministic or stochastic population dynamics, or simulate observations from a set of given model structures

### 1.2. Citing CASAL2

A suitable reference for this document is S. Rasmussen, C. Marsh, I. Doonan, A. Dunn, K. Large (2019). CASAL2 User Manual, v2019-12-11 (rev. 69b27ff). National Institute of Water & Atmospheric Research Ltd. *NIWA Technical Report 139*. 213 p.

And the peer-reviewed journal article reference for CASAL2 is (Doonan et al., 2016).

### 1.3. Software license

This program and the accompanying materials are made available under the terms of the GNU General Public License version 2 which accompanies this software (see Section 17).

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### 1.4. Where to get CASAL2

In the first instance, see <http://www.niwa.co.nz/> for information about CASAL2. The CASAL2 source code is hosted on github, and can be found at <https://github.com/NIWAFisheriesModelling/CASAL2>.

A Microsoft Windows bundle includes the binary, manual, examples and other help guides. It can be downloaded at <ftp://ftp.niwa.co.nz/Casal2/windows/Casal2.zip> for the Microsoft Windows version. The Linux bundle which includes a binary, manual, examples and other help guides can be downloaded at <ftp://ftp.niwa.co.nz/Casal2/linux/Casal2.tar.gz>.

### 1.5. System requirements

CASAL2 is available for most IBM compatible machines running 64-bit Linux and Microsoft Windows operating systems.

Several of CASAL2's tasks are highly computer intensive and a fast processor is recommended. Depending on the model implemented, some of the CASAL2 tasks can take a considerable amount of processing time (minutes to hours), and in extreme cases may take several days to complete an MCMC estimate.

The program itself requires only a few megabytes of hard-disk space but output files can consume large amounts of disk space. Depending on the number and type of user output requests, the output could range from a few hundred kilobytes to several hundred megabytes. When estimating model fits, several hundred megabytes of RAM may be required, depending on the spatial size of the model, number of categories, and complexity of processes and observations. For extremely large models, several gigabytes of RAM may occasionally be required.

### 1.6. Necessary files

For both 64-bit Linux and Microsoft Windows, only the binary executable `casal2` or `casal2.exe` is required to run CASAL2. No other software is required. We do not provide a version for 32-bit operating systems.

However, CASAL2 offers little in the way of post-processing of model output, and a package available that allows tabulation and graphing of model outputs is recommended. We suggest software such as **R** (R Core Team, 2014) to assist in the post processing of CASAL2 output. We provide the CASAL2 **R** package for importing the CASAL2 output into **R** (see Section 15).

### 1.7. Getting help

CASAL2 is distributed as unsupported software. The Development Team would appreciate being notified of any problems or errors in CASAL2, please contact the `Casal2` Development Team. See Section 16.2 for the recommended template for reporting issues.

### 1.8. Technical details

CASAL2 was compiled on Linux using `gcc` (<http://gcc.gnu.org>), the C/C++ compiler developed by the GNU Project (<http://gcc.gnu.org>). The 64-bit Linux version was compiled using `gcc` version 5.2.1 20151010 Ubuntu Linux (<http://www.ubuntu.com/>). The Microsoft Windows (<http://www.microsoft.com>) version was compiled using MingW (<http://www.mingw.org>) `gcc` 8.1.0 (<http://gcc.gnu.org>). The Microsoft Windows (<http://www.microsoft.com>) installer was built using the Inno Setup 5 (<http://www.jrsoftware.org/isdl.php>).

CASAL2 includes number of different minimisers — Different minimisers may be better at some models than others. The first three are non-differentiation based minimisers: the first is closely

based on the main algorithm of Dennis Jr and Schnabel (1996), and which uses finite difference gradients; the second is an implementation of the differential evolution solver (Storn and Price, 1995), and based on code by Lester E. Godwin of PushCorp, Inc.; and the third is Dlib (King, 2009). The three differentiation based minimisers are: ADOLC, an auto differentiation minimiser (Walther et al., 1996); CPPAD an auto differentiation minimiser similar to ADOLC (Wächter and Biegler, 2006); and the third is a modified version of an older version of ADOL-C (v1.8.4) that was used as the auto differentiation minimiser in the first version of CASAL (Bull et al., 2012).

The random number generator used by CASAL2 uses an implementation of the Mersenne twister random number generator (Matsumoto and Nishimura, 1998). This, the command line functionality, matrix operations, and a number of other functions use the BOOST C++ library (Version 1.58.0).

Note that the output from CASAL2 may differ slightly on the different platforms due to different precision arithmetic or other platform dependent implementation issues. The source code for CASAL2 is available in the windows bundle or on the github repository at <https://github.com/NIWAFisheriesModelling/CASAL2>.

Unit tests of the underlying CASAL2 code are carried out at build time, using the GOOGLE mock and unit testing framework. The unit test framework aims to cover a significant proportion of the key functionality within the CASAL2 code base. The unit test code for CASAL2 is available as a part of the underlying source code.





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## 2. Model overview

### 2.1. Introduction

CASAL2 is an age-structured population dynamics model. It implements a statistical catch-at-age population dynamics, using a discrete time-step state-space model that represents a cohort-based population age structure.

CASAL2 is run from the console window in Microsoft Windows or from a terminal window in Linux. CASAL2 gets its information from input data files, the main one of which is the *input configuration file*. Commands and subcommands in the input configuration file are used to define the model structure, provide observations, define parameters, and define the outputs (reports) for CASAL2. Command line switches tell CASAL2 the run mode and where to direct its output. See Section 3 for details.

We define the model in terms of the *state*. The state consists of two parts, the *partition*, and any *derived quantities*. The state will typically change in each *time-step* of every year, depending on the *processes* defined for those time-steps in the model.

The *partition* is a representation of the population at an instance in time, and can be considered a matrix of the numbers of individuals within each category and at each age.

A *derived quantity* is a summary of the abundance or biomass in a selected part of the partition at some instance in time. Unlike the partition (which is updated as each new process is applied), a derived quantity records a single value for each year of the model run. Hence, derived quantities build up a vector of values over the time period represented by the model. For example, the total biomass of individuals in categories labelled, say, ‘mature’ at some instance in the annual cycle may be a derived quantity. The derived quantity is then available as an *addressable parameter* to the model — to be reported, or to be an input into another process (e.g. recruitment) at some instance in the model in a subsequent year.

The state at some instance in time is the term for the combination of the partition and any derived quantities at that instance in time. Throughout the model, changes to the state occur from the application of *processes*. This state then provides the basis for the generation of expected values for *observations*, as well as for reports and other outputs.

Running of the model consists of two steps — first the model state is initialised for a number of iterations (years), then the model runs over a range of predefined years.

Initialisation can be in one or more phases, and for each phase, the processes that occur in each year, and the order in which they are applied, need to be defined. The processes that occur is controlled by the *annual cycle*. This defines what processes happen in each model year and in what sequence. Further, the processes in each year are split up into one or more time-steps (with at least one process occurring in each time-step). You can think of each time-step as representing a particular part of the calendar year, or you can just treat them as an abstract sequence of events.

The division of the year into an arbitrary number of time-steps allows the user to specify the exact order in which processes occur, and how/when observations are evaluated. The user specifies the time-steps, their order, and the processes within each time-step. If more than one process occurs in the same time-step, then they occur in the order that they are specified.

Observations are always linked to a time-step, and are evaluated by the model in the time-step in which they occur. Hence, time-steps can be used to break processes into groups, and assist in defining the timing of the observations within the annual cycle. The manner in which observations are evaluated and how the expected values are calculated by the model is described later in Section

6.

The population structure of CASAL2 follows the usual population modelling conventions and is similar to those implemented in, for example, CASAL (Bull et al., 2012). The model records the numbers of individuals by category and age (e.g., numbers of males and females at age). In general, cohorts are added via a recruitment event, are aged annually, and are removed from the population via various forms of mortality. The population is assumed to be closed (i.e., no immigration or emigration from the modelled area).

A model is implemented in CASAL2 using an input configuration file, which provides a complete description of the model structure (i.e., population structure, initialisation, and the subsequent population processes), observations, estimation methods, and reports (outputs) requested. CASAL2 runs from a console window on Microsoft Windows or from a text terminal on Linux. A model can be either *run*, estimable parameters can be *estimated* or *profiled*, *MCMC* distributions calculated, and these estimates can be *projected* into the future or used by CASAL2 as parameters of an operating model to *simulate* observations.

A model in CASAL2 is specified by an input configuration file, comprising four main components. These are the population section that defines the model structure, population dynamics, etc.; the estimation section that defines the methods of estimation (minimisation methods or MCMC algorithms) and the model parameters to be estimated; the observation section that defines the observational data and associated likelihoods; and the report section that defines the printouts and reports from the model and where these are saved. The input configuration file completely describes a model implemented in CASAL2. See Sections 8, 9, 10, and 11 for details and specification of CASAL2's command and subcommand syntax within the input configuration file.

## 2.2. The population section

The population section (Section 4) defines the model of the population dynamics. It describes the model structure (i.e. the population structure), initialisation method and phases, run and projection years (model period), population processes (for example, recruitment, migration, and mortality), selectivities, and key population parameters.

## 2.3. The estimation section

The estimation section (Section 5) specifies the parameters to be estimated, estimation methods, penalties and priors. Estimation is based on an objective function (e.g., negative log posterior). Depending on the run mode, the estimation section is used to specify the methods for finding a point estimate (i.e., the set of parameter values that minimizes the objective function), doing profiles, or MCMC methods and options, etc.

Further, the estimation section specifies the parameters to be estimated within each model run and the estimation methods. The estimation section specifies the choice of estimation method, which model parameters are to be estimated, priors, starting values, and minimiser control values.

Penalties and priors act as constraints on the estimation. They can either encourage or discourage (depending on the specific implementation) parameter estimates that are 'near' some value, and hence influence the estimation process. For example, a penalty can be included in the objective function to discourage parameter estimates that lead to models where the recorded catch was unable to be fully taken.

## **2.4. The observation section**

Types of observations, their values, and the associated error structures are defined in the observation section (Section 6). Observations are data which allow us to make inferences about unknown parameters. The observation section specifies the observations, their errors, likelihoods, and when the observations occur. Examples include relative or absolute abundance indices, proportions-at-age and tag recapture observations. Estimation generates values for each of the estimated parameters that are the best fit to the data, i.e., where each expected value is ‘close’ (in some mathematical sense) to the corresponding observed value.

## **2.5. The report section**

The report section (Section 7) specifies the model outputs. It defines the quantities and model summaries to be output to external files or to the standard output. While CASAL2 will communicate helpful and informative messages generated from the source code to the screen as the application runs, CASAL2 will only produce model estimates, population states, and other data as requested by the report section. Note that if no reports are specified, then no output will be produced.



---

### 3. Running CASAL2

CASAL2 is run from a console window (i.e., the command line) on Microsoft Windows or from a terminal window on Linux. CASAL2 uses information from input data files -- the *input configuration file* being the main input file.

The input configuration file is required and defines the model structure and processes, the observations and parameters (both the fixed parameters and the parameters to be estimated), and the reports (outputs) requested.

The following sections describe how to construct the CASAL2 configuration file. By convention, the name of the input configuration file ends with the suffix `.csl2`. However, any file name is acceptable. Note that the input configuration file can include other files as a part of its syntax. Collectively, these are called the input configuration file.

Other input files can be included depending on the run mode and what information is required. For example, an input file can define the initial parameter values for estimation, or values from which to simulate observations, or values from which to run projections.

Command line arguments are used to specify the actions or *tasks* of CASAL2, e.g., to run a model with a set of parameter values, to estimate parameter values (either point estimates or MCMC), to project quantities, or to simulate observations. Hence, the *command line arguments* define the *task*. For example, `-r` is the *run* mode, `-e` is the *estimation* mode, and `-m` is the *MCMC* mode. The *command line arguments* are described in Section 3.4.

#### 3.1. Using CASAL2

To use CASAL2, open a console window (i.e. the command prompt) window on Microsoft Windows or a terminal window on Linux. Navigate to the directory where the model input configuration files are located. Then enter `casal2` with arguments for a specific mode to start the CASAL2 mode running; see Section 3.4 for the list of possible arguments. CASAL2 will print output to the screen.

The CASAL2 executable and shared libraries (files with extension `.dll` or `.so`) must either be in the same directory as the input configuration files or in one of the directories in your operating system's `PATH` environment variable. The CASAL2 installer will update `PATH`; see your operating system documentation for help displaying or modifying `PATH`.

#### 3.2. The input configuration file

The input configuration file is made up of four broad sections:

- the description of the population structure and parameters (the population section),
- the estimation methods and estimated variables (the estimation section),
- the observations and their associated properties and likelihoods (the observation section), and
- the output values and reports that CASAL2 will output (the report section).

The input configuration file is made up of a number of commands, many with subcommands, which specify various options for each of these components.

The command and subcommand definitions in the input configuration file can be extensive, particularly if you have a model that has many partitions or timesteps, and can result in a input configuration file that is long and difficult to navigate.

To aid in making the model configuration more readable and flexible, the input configuration file command `!include "filename"` can be used (Figure 3.1). This command specifies that another file, `filename`, be read and processed, exactly as if its contents had been inserted into the main input configuration file at that point. The file name must be the complete file name with extension, and can use either the relative or absolute path as part of its name. Included files can also contain `!include` commands. See Section 12 for more detail.



**Figure 3.1:** Example of using the input configuration file command `!include "filename"`.

### 3.3. Redirecting standard output

CASAL2 uses the standard output stream to display runtime information. The standard error stream is used by CASAL2 to output the program exit status and runtime errors. We suggest redirecting both the standard output and standard error into files.

With the bash shell (on Linux systems), you can do this using the command structure

```
(casal2 [arguments] > run.out) >& run.err &
```

It may be useful to redirect the standard input, especially if you're using CASAL2 inside a batch job, i.e.

```
(casal2 [arguments] > run.out < /dev/null) >& run.err &
```

On Microsoft Windows systems, you can redirect to standard output using

```
casal2 [arguments] > run.out
```

And, on some Microsoft Windows systems (e.g., Windows 10), you can redirect to both standard output and standard error, using the syntax

```
casal2 [arguments] > run.out 2> run.err
```

CASAL2 outputs header information to the output (Figure 3.2). The header consists of the program name and version, the arguments passed to CASAL2 from the command line, the date and time that the program was called (derived from the system time), the user name, and the machine name (including the operating system and the process identification number). This information can be used to track outputs as well as identifying the version of CASAL2 used to run the model.



Figure 3.2: Example of output file header information.

### 3.4. Command line arguments

CASAL2 is called using:

```
casal2[-c config_file] [task] [options]
```

where

**-c *config\_file*** Define the input configuration file for CASAL2 (if this argument is omitted, the default input configuration file is `config.csl2`)

and where *task* must be one of the following ([ ] indicates a secondary label to call the task, e.g. **-h** will execute the same task as **--help**),

**-h [--help]** Display help (this page)

**-l [--licence]** Display the reference for the software license (GPL v2)

**-v [--version]** Display the CASAL2 version number

**-r [--run]** Run the model once using the parameter values in the input configuration file, or optionally with the starting parameter values from the file specified with argument **-i filename**

**-e [--estimate]** Do a point *estimate* using the values in the input configuration file as the starting point for the parameters to be estimated, or optionally with the starting parameter values from the file specified with the argument **-i filename**

**-p [--profiling]** Do a likelihood *profile* using the parameter values in the input configuration file as the starting point, or optionally with the starting parameter values from the file specified with the argument **-i filename**

**-m [--mcmc]** Do an *MCMC* chain using the values in the input configuration file as the starting point for the parameters to be estimated, or optionally with the starting parameter values from the file specified with the argument **-i filename**

**-f [--projection]** Project the model *forward* in time using the parameter values in the input configuration file as the starting point for the estimation, or optionally with the starting parameter values from the file specified with the argument **-i filename**

**-s [--simulation] *number*** Simulate the *number* of observation sets using values in the input configuration file as the parameter values, or optionally with the parameter values from the file specified with the argument **-i filename**

and where the following optional arguments [*options*] may be specified

- i** [**--input**] *filename* *Input* one or more sets of free (estimated) parameter values from *filename* (see Section 11 for details about the format of *filename*)
- o** [**--output**] *filename* *Output* a report of the free (estimated) parameter values in a format suitable for **-i** *filename* (see Section 11 for details about the format of *filename*)
- g** [**--seed**] *seed* Initialise the random number *generator* with *seed*, a positive (long) integer value (note, if **-g** is not specified, then CASAL2 will generate a random number seed based on the computer clock time)
- loglevel** *arg* = {trace, finest, fine, medium} (see Section 7)
- tabular** Run with **-r** or **-f** command to print @report in tabular format (see Section 7)
- single-step** Run with **-r** to pause the model and ask the user to specify parameters and their values to use for the next iteration (see Section 3.6)
- q** [**--query**] *object type* *Query* an object type to print an extract of the object description and parameter definitions. An object can be defined as *block.type*, e.g. `casal2 --query process.recruitment_constant` will query the constant recruitment block.

### 3.5. Constructing the CASAL2 input configuration files

The model definition, characteristics, parameters, observations, and reports are specified in input configuration files:

- Population input (Section 4) specifies the model structure, population dynamics, and other associated parameters;
- Estimation input (Section 5) defines the objective function, parameters of the model, and the method of estimation (point estimates, Bayesian posteriors, profiles, etc.);
- Observation input (Section 6) specifies the observations data used in the model and describes how the observed values should be formatted, how CASAL2 calculates the expected values, and the likelihoods applied for each set of observations; and
- Report input (Section 7) specifies the output.

The command and subcommand syntax to be used in each of these configuration sections are listed in Sections 8 (Population), 9 (Estimation), 10 (Observation) and 11 (Report).

#### 3.5.1. Commands

CASAL2 has a range of commands that define the model structure, processes, parameters, observations, and how tasks are carried out. There are three types of commands

- Commands that have an argument and do not have subcommands (for example, `!include filename`)
- Commands that have a label and subcommands (for example @process must have a label and has subcommands)
- Commands that do not have either a label or argument, but have subcommands (for example @model)



Commands that have a label must have a unique label, i.e., the label cannot be used on more than one command of that type. The labels can contain alpha numeric characters, period ('.'), underscore ('\_') and dash ('-'). Labels must not contain whitespace (tabs or spaces) or other characters that are not letters, numbers, dash, period, or an underscore. For example,

```
@process NaturalMortality
or
!include MyModelSpecification.csl2
```

### 3.5.2. Subcommands

CASAL2 subcommands define options and parameter values related to a particular command. Subcommands always take an argument which is one of a specific *type*. The argument *types* acceptable for each subcommand are defined in Section 12, and are summarised below.

Like commands (@command), subcommands and their arguments are not order specific, except that that all subcommands of a given command must appear before the next @command block. CASAL2 may report an error if they are not supplied in this way. However, in some circumstances a different order may result in a valid, but unintended, set of actions, leading to possible errors in the expected results.

The argument type for a subcommand can be:

<b>switch</b>	true/false
<b>integer</b>	an integer number
<b>integer vector</b>	a vector of integer numbers
<b>integer range</b>	a range of integer numbers separated by a colon, e.g. 1994:1996 is expanded to an integer vector of values (1994 1995 1996)
<b>constant</b>	a real number (i.e., a double)
<b>constant vector</b>	a vector of real numbers (i.e., a vector of doubles)
<b>estimable</b>	a real number that can be estimated (i.e., a double)
<b>estimable vector</b>	a vector of real numbers that can be estimated (i.e., a vector of doubles)
<b>addressable</b>	a real number that can be referenced but not estimated (i.e., an addressable double)
<b>addressable vector</b>	a vector of real numbers that can be referenced but not estimated (i.e., a vector of addressable doubles)
<b>string</b>	a categorical (string) value
<b>string vector</b>	a vector of categorical values.

Switches are characteristics which are either true or false. Enter *true* as true or t, and *false* as false or f.

Integers must be entered as whole numbers without decimal points (i.e., if *year* is an integer then it is specified as 2008, not 2008.0)

Arguments of type integer vector, constant vector, estimable vector, addressable vector, or categorical vector must contain one or more entries on a row, separated by whitespace (tabs or spaces). Arguments of type integer range must contain a colon (:) and no whitespace (tabs or spaces).

Parameters are defined in the population section and can be specified as estimable with the subcommand type *estimable* or *estimable vector*. These parameters will be estimated if specified as such in the estimation section. If an estimable parameter is not specified in the estimation section it will instead be treated as a constant (or constant vector). In other words, only estimable parameters can be estimated and the parameter command must explicitly specify that the parameter

is estimable with the `estimable` or `estimable vector` subcommand type.

Parameters defined as addressable with the subcommand type `addressable` or `addressable vector` are usually derived quantities and are not directly estimable. As such, they can be referenced by various processes, or have priors and/or penalties associated with them, but they do not directly contribute to any estimation within the model.

### 3.5.3. The command block format

Each command block consists of a single command which starts with the symbol `@` and, for most commands, a unique label or an argument. Each command is then followed by its subcommands and their arguments, e.g.,

<code>@command</code>	<code>@command argument</code>	<code>@command label</code>
<code>subcommand argument</code>	<code>subcommand argument</code>	<code>subcommand argument</code>
<code>subcommand argument</code>	<code>subcommand argument</code>	<code>subcommand argument</code>
<code>.</code>	<code>.</code>	<code>.</code>
<code>.</code>	<code>.</code>	<code>.</code>
<code>etc.</code>	<code>etc.</code>	<code>etc.</code>

Blank lines are ignored, as is extra whitespace (tabs and spaces) between arguments. However, to start command block the `@` character must be the first character on the line and must not be preceded by any whitespace. Each input file must end with a carriage return.

There is no indicator of the end of a command block. Each command block is delimited by the end of the file, the end of the section, or the start of the next command block (which is marked by the `@` on the first character of a line). The `!include` command is the only exception to this rule (see Section 12 for details of the use of `!include`).

Commands, subcommands, and arguments in the input configuration files are not case sensitive. However, labels and variable values are case sensitive. On Linux, filenames and paths are case sensitive (i.e., when using `!include filename`, the argument `filename` will be case sensitive).

### 3.5.4. Commenting out lines

Text on a line that starts with the symbol `#` is considered to be a comment and is ignored. To comment out a group of commands or subcommands, use `#` at the beginning of each line to be ignored.

Alternatively, to comment out an entire block or section, use `/*` at the beginning of a line to start the comment block, then end the block with `*/`. All lines (including line breaks) between `/*` and `*/` inclusive are ignored.

```
# This line is a comment and will be ignored
@process NaturalMortality
m 0.2
/*
This block of text
is a comment and
will be ignored
*/
```

### 3.5.5. Determining CASAL2 parameter names

When CASAL2 processes the input configuration file it translates each command block and each subcommand block into a CASAL2 object, each with a unique parameter name. For commands, this parameter name is simply the command label. For subcommands, the parameter name format is either:

```
command[label].subcommand if the command has a label, or
command.subcommand if the command has no label, or
command[label].subcommand{i} if the command has a label and the subcommand arguments
    are a vector, and we are accessing the ith element of that vector.
command[label].subcommand{i:j} if the command has a label, and the subcommand arguments
    are a vector, and we are accessing the elements from i to j (inclusive) of that vector.
```

The unique parameter name is used to reference that parameter when, e.g., estimating, applying a penalty, projecting, time varying, or profiling. For example, the parameter name of the Natural Mortality rates subcommand `m` of the command `@process` with the label `NaturalMortality` is category related and so, the syntax to reference all `m` related categories is

```
process[NaturalMortality].m
```

The syntax to specify a single category to which the natural mortality process is applied is

```
process[NaturalMortality].m{male}
```

All labels (parameter names) are user specified. As such, naming conventions are non-restrictive and can be model specific.

## 3.6. Single-stepping CASAL2

Single-stepping means CASAL2 can ‘pause’ after each year in the annual cycle during a model run, write reports, then wait and process user input of updated estimable parameters for the next year (see the command line argument `--single-step`).

This enables CASAL2 to implement models for management simulations or scenarios that require feedback and can be used, for example, in operational management procedures (OMPs). The single-stepping process can be automated using **R**, so that CASAL2 may be used with **R** to update input harvest values (e.g., catches from a fishery in a fisheries model) to evaluate a particular harvest control rule.

## 3.7. CASAL2 exit status values

When CASAL2 is run, it will complete its task successfully or output errors. CASAL2 will return a single exit status value ‘completed’ to the standard output. Error messages will be printed to the console. When input file configuration errors are found, CASAL2 will print error messages, along with the associated filename(s) and line number(s) where the errors were identified, for example,

```
#1: At line 15 in Reports.csl2: Parameter '{' is not supported
```



---

## 4. The population section

### 4.1. Introduction

The population section specifies the model of the population dynamics. It describes the model structure (population structure), defines the population processes (e.g., recruitment, migration, and mortality), the selectivities, and associated parameters.

The population section consists of several components, including:

- The population structure (categories and ages in an age-based model);
- Model initialisation (i.e., the state of the partition at the start of the first year);
- The years over which the model runs (i.e., the start and end years of the model)
- The annual cycle (time-steps and processes that are applied in each time-step);
- The specification and parameters of the population processes (i.e., processes that add, remove individuals to or from the partition, or shift numbers between ages and categories in the partition);
- Selectivities;
- Parameter values and their definitions; and
- Derived quantities, required as parameters for some processes (e.g. mature biomass to resolve any density dependent processes, such as the spawner-recruit relationship in a recruitment process).

### 4.2. Population structure

The basic structure of the population section of a CASAL2 model is defined in terms of an annual cycle, time steps, states, and transitions.

The annual cycle defines what processes happen in each model year, and in what sequence. CASAL2 runs on an annual cycle rather than, for example, a 6-monthly cycle.

Each year is split into one or more time steps, with at least one process occurring in each time step. Each time step can be thought of as representing a particular part of the calendar year, or time steps can be treated as an abstract sequence of events. Note that mortality processes are grouped into a mortality block — in every time step, there exists a mortality block (defined from a group of consecutive mortality-based processes) where individuals are removed from the partition (see Section 4.4.2).

The state is the current status of the population at any given time. The state can change one or more times in each time step of every year. The state object must contain sufficient information to figure out how the underlying population changes over time (given a model and a complete set of parameters).

The state can undergo a number of possible changes during the annual cycle, called transitions. Transitions are accomplished by processes, for example, recruitment, natural mortality, anthropogenic mortality, ageing, migration, tagging events, or maturation. These are repeated for each year of the model (although some processes can be specified to only occur in a subset of years).

The division of the year into an arbitrary number of time steps allows the user to specify the exact order in which processes and observations occur throughout the year. The user needs to specify the

time step in which each process occurs. If more than one process occurs in the same time step, the order in which to apply each process is specified in the `@time_step` block.

The key element of the state is the partition. The partition divides the total number of individuals in the current population into different categories. The partition records numbers of individuals in each age or length, and category. The individuals are grouped into categories, for example, sex, maturity state, area, and species. CASAL2 has no predefined categories — these are defined by the user (this differs from CASAL (Bull et al., 2012) that has only pre-defined partition categories). It is up to the user to define these.

The resulting partition can be conceptualised as a matrix, where each row is represented by a category and the columns are the age classes, shown in Figure 4.1. Each row represents the number of individuals for that category in that age class.

The names of categories are user defined, but there must be at least one category defined for any model. The ages are defined as a sequence from  $age_{min}$  to  $age_{max}$ , with the last age optionally a plus group. In order to calculate biomass, the age-length relationship for each category must also be defined for an age based model (but could be defined as ‘none’). An example of how this is specified for four categories based on sex and area is as follows,

```
@categories
format mature.sex
names spawn.male spawn.female nonspawn.male nonspawn.female
age_lengths male_AL female_AL male_AL female_AL
```

For an example of these ideas, consider a model of a fish population with a mature and non-spawning fishery. We assume that the non-spawning fishery happens over most of the year (say 10 months) in the non-spawning area. The mature fish then migrate to the spawning area, where the spawning fishery operates. At the end of spawning, these fish, along with the recruits from the previous year, migrate back to the non-spawning area. The modeller decides that fish will be divided in the partition by age, sex, maturity, and area (spawning and non-spawning grounds). So the partition has 8 rows, 2 sexes  $\times$  (mature or immature)  $\times$  2 areas, and one column per age class.

Spawning male immature				
Spawning male mature				
Spawning female immature				
Spawning female mature				
Non-spawning male immature				
Non-spawning male mature				
Non-spawning female immature				
Non-spawning female mature				

**Figure 4.1: A visual representation of a partition.**

For this example four time steps are defined and labelled 1 through 4: `step1` includes the non-spawning fishery, `step2` includes the migration to the spawning area, `step3` includes the spawning

fishery, and `step4` includes recruitment and migration back to the non-spawning area. Note that the default order of processes within a time step places migrations before fisheries, therefore a single step could have been specified in place of steps 2 and 3. Other details that define population structure are also specified in time steps, such as proportions of natural mortality occurring in each time step and where observations occur.

This structure can be used to implement complex models, with intermingling of separate species and stocks, with complex migration patterns occurring over multiple areas, and including multiple sources of anthropogenic impact using different methods that cover different areas and times. However, the specification of stock structure is constrained by the available data and we note that there is little point in using a complex structure to model a population when there are no observations to support that structure. For information on how to define categories and using CASAL2's shorthand syntax see Section 14.2.

The model is run from an initial year up to the final (current) year. It can also be run past the final year to make projections — to estimate the state of the population sometime in the future — up to the final projection year.

An example, to specify a model with 2 categories (male and female) with ages 1-20 (with the last age a plus group) and an age-length relationship defined with the label `male_growth` and `female_growth`, then the `@model` block is specified as:

```
@model
start_year
final_year
min_age 1
max_age 20
age_plus_group True
initialisation_phases iphase
time_steps step1 step2 step 3
```

### 4.3. The state object and the partition

The key component of the state object is the partition, a matrix that stores numbers of individuals at age or length for each category. A category represents a group of individuals that have the same specific attributes, examples of such attributes include life histories and growth rates, etc. For example, categories may include labels such as:

- Sex (male or female)
- Area (any number of areas, named by the user)
- Maturity (immature or mature)
- Growth-path (any number of growth-paths)
- Tag (any number of tagging events)
- Species

A stock can be thought of as a population of individuals which recruits to that population. Maturity can either be defined as a separate category in the partition, or calculated from the population at the time required — see Section 4.11 for the treatment of maturity when maturity is not a category in the partition.

CASAL2 requires the following:

- The minimum and maximum age classes in an age-based model
- Whether there is an age-plus group
- The names of all categories

Age classes are always one year wide, except for the maximum age group which can (optionally) be a plus group. Users need to choose the minimum and maximum age classes.

CASAL2 allows categories of the partition to exist for certain years of the model. This aids computational efficiency for when models contain a large number of categories that do not persist over all model years. This is beneficial when a model contains a process that does a one-off transition of individuals from one category into another that is in a subset of the model initialisation phases or years (e.g. tagging events). Excluding categories for certain years can save a considerable amount of computational time as CASAL2 does not need to initialise empty categories or implement processes in time periods where they will have no effect. The structure of the partition is defined in a configuration block via the `@categories` as discussed earlier in Section 4.2.

Derived quantities are another important component of the state object. Derived quantities include mature biomass, i.e., in fisheries models the mid-spawning season biomass of spawning fish (SSB) for either one, or the sum over more than one, category. CASAL2 calculates such a quantity through the command `@derived_quantity`, and this may need to be specified for some processes (e.g., in fisheries models, a recruitment process that specifies a stock recruitment relationship requires the definition of a derived quantity that specifies the mid-season spawning stock biomass).

#### 4.4. Time sequences

The time sequence of the model is defined in the following parts;

- Annual cycle
- Mortality blocks
- Initialisation
- Model run years
- Projection years

##### 4.4.1. Annual cycle

The annual cycle is implemented as a set of processes that occur, in a user-defined order, within each year. Time-steps are used to break the annual cycle into separate components, and allow observations to be associated with different time periods and processes. Any number of processes can occur within each time-step, in any order (although there are limitations around mortality based processes — see Section 4.4.2) and can occur multiple times within each time-step. Note that time-steps are not implemented during the initialisation phases (effectively, there is only one time-step), and that the annual cycle in the initialisation phases can, optionally, be different from that which is applied during the model years.

##### 4.4.2. Mortality blocks

For every time step in an annual cycle there is an associated *mortality block*. Mortality blocks are a key concept in CASAL2.

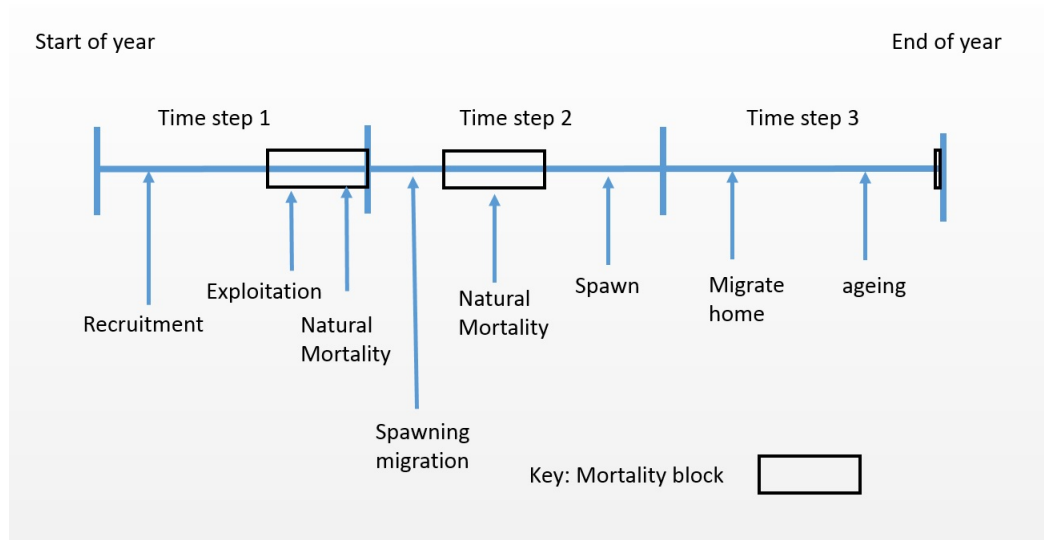


Mortality blocks are used to define the ‘point’ in the model time sequence when observations (see Section 6) are evaluated, and derived quantities (see Section 4.6) are evaluated.

A mortality block is defined as a consecutive sequence of mortality processes within a time step. The processes that are mortality processes are all pre-defined in CASAL2, and cannot be modified. These mortality processes are described in subsection 4.5.3.

CASAL2 requires that each time step has exactly one mortality block. To achieve this, either all the mortality processes in a time step must be sequential (i.e., there can not be a non-mortality process between any two mortality processes within any one time step); or if no mortality processes occur in a time step then the mortality block is defined to occur at the end of the time step.

CASAL2 will error out if more than one mortality block occurs in a single time step. Use separate time-steps to sequence mortality blocks if required.



**Figure 4.2: A visual representation of a hypothetical sequence for an annual cycle.**

#### 4.4.3. Initialisation

Initialisation is the process of determining the model starting state, whether it be equilibrium/steady state or some other initial state for the model (e.g exploited), prior to the start year of the model. This can be computationally expensive if a plus group is present in the partition.

There are multiple methods to initialise a partition in CASAL2, including iterative, fixed, derived, and Cinitial.

Model initialisation can also occur in several phases, each of which can be a different method. These are carried out in sequence. At the end of all of the initialisations, CASAL2 then runs the model years carrying out the user-defined processes in each time step in the annual cycle.

The multi-phased initialisation allows the user to choose a number of initialisations that may assist with optimising the models for speed, initialise a non-equilibrium starting state, or resolve simple processes before introducing more complex ones.

Each phase of the initialisation defaults to have the same processes and in the same order as defined in the annual cycle. Although, an initialisation phase can involve any number of processes using the `insert_processes` subcommand.

In each initialisation phase, the processes defined for that phase are carried out and used as the

starting point for the following phase or, if it is the last phase, the starting point for the years that the model is run over.

Note that the *first* initialisation phase is always initialised with each element (i.e., each age and category) set at zero. Care must be taken when using complex category inter-relationships or density dependent processes that depend on a previously calculated state, as they may fail when used in the first phase of an initialisation.

Multi-phase iterations can also be used to determine if the initialisation has converged. Here, add a second initialisation phase for, say, 1 year (with the same processes applied). Then report the state at the end of the first and second phase. If these states are identical, then its likely that the initialisation has converged to an equilibrium state.

The subcommands for including or excluding processes are `insert_processes` and `exclude_processes`.

For the `insert_processes` the syntax is:

```
insert_processes time_step_label(process_label_in_annual_cycle) = label_new_process
```

For example, this could be used in an `@time_step` labelled `Oct_Nov`, and including the `@process` labelled `predationIni`, and before the `@process` labelled `Instantaneous_Mortality`,

```
insert_processes Oct_Nov(Instantaneous_Mortality)=predationIni
```

and to include a process at the end of the time step you can use the following syntax,

```
insert_processes Oct_Nov()=predationIni
```

To exclude a process from an initialisation phase the syntax is much simpler, for example, using the subcommand `exclude_processes` in `@initialisation_phase`,

```
exclude_processes Instantaneous_Mortality
```

The command above will remove the process labelled `Instantaneous_Mortality` during that particular initialisation phase.

#### 4.4.3.1. Iterative Initialisation

The iterative initialisation is a general solution for initialising the model. The iterative method can be slow to converge, depending on the nature of the problem being resolved, but will work on even complex structured models that may be difficult or impossible to implement using analytic approximations.

The number of iterations in the iterative initialisation can effect the model output, and these should be chosen to be large enough to allow the population state to fully converge. We recommend that a period of about two generations to ensure convergence. `CASAL2` can be requested to report a number of convergence statistics that can assist the user in determining the level of convergence.

In addition, the iterative initialisation phase can optionally be stopped early if some user defined convergence criteria is met. For a list of supplied years in the initialisation phase, convergence is

defined as met if the proportional absolute summed difference between the state in year  $t - 1$  and the state in year  $t$  ( $\hat{\lambda}$ ) is less than a user defined  $\lambda$  where,

$$\hat{\lambda} = \frac{\sum_i \sum_j |\text{element}(i, j)_t - \text{element}(i, j)_{t-1}|}{\sum_i \sum_j \text{element}(i, j)_t} \quad (4.1)$$

Hence, for an iterative initialisation you need to define:

- The initialisation phases,
- The number of years in each phase, and
- the processes to apply in each (default is the annual cycle).

An example of the syntax to implement this would be,

```
@model
...
initialisation_phases Iterative_initialisation

@initialisation_phase Iterative_initialisation
type iterative
years 50
lambda 0.0001
convergence_years 20 40
```

#### 4.4.3.2. Derived Initialisation

Derived initialisation is an analytical solution that calculates the equilibrium age structure and the plus group using a geometric series solution. The benefit of this method is it can be solved in  $\text{max\_age} - \text{min\_age} + 1$  years, so is computationally faster than the iterative initialisation phase. Users should be warned that we have found under some process combinations (e.g. one-way migrations) that this solution does not reach the exact equilibrium partition. We recommend, if using this method, users confirm the partition has reached an equilibrium state by either comparing with an iterative initialisation, or by adding a second iterative initialisation phase of a limited number of iterations to confirm convergence.

An example of the syntax to implement this would be,

```
@model
...
initialisation_phases Equilibrium_initialisation

@initialisation_phase Equilibrium_initialisation
type derived
```

#### 4.4.3.3. Cinitial Initialisation

This initialisation is only available as a second or greater phase initialisation, and can only be applied after derived or iterative initialisation phases. The Cinitial factors that can be estimated to shift the initial population away from an equilibrium state prior to the start year. If there is known exploitation before data exists for a population this can be a solution for estimating a non-equilibrium population. Note that it may be advisable to include an observation of age composition data for the first year of the model in order to estimate the non-equilibrium population state.

An example of the syntax to implement this would be,

```
@model
...
initialisation_phases Cinitial

@initialisation_phase Cinitial
type cinitial
categories spawn.male+nonspawn.male  spawn.female+nonspawn.female
table n
spawn.male+nonspawn.male    5e7 5e7 7e6 6e6 5e6 4e6 3e6 2e6 1e6 1e6 1e1 1e1 1e1 1e1
spawn.female+nonspawn.female 5e7 5e7 7e6 6e6 5e6 4e6 3e6 2e6 1e6 1e6 1e1 1e1 1e1 1e1
end_table
```

The Cinitial factors can also be estimated using the following syntax

```
@estimate cinit_male
parameter initialisation_phase[Cinitial].spawn.male+nonspawn.male
same initialisation_phase[Cinitial].spawn.female+nonspawn.female
lower_bound 2e2 2e2 2e2 2e2 2e2 2e2 2e2 2e2 2e2 2e2 2e0 2e0 2e0 2e0
upper_bound 2e9 2e9 2e9 2e9 2e9 2e9 2e9 2e9 2e9 2e9 2e9 2e9 2e9 2e9
type uniform
```

### 4.4.3.4. Fixed Initialisation

Fixed initialisation takes a user defined table to be the initial partition prior to the start year. Users have the ability to initialise models by specify the numbers-at-age for each category. When initialising models with this type, undefined behaviour may be result if the model applies processes that require derived quantities to be calculated in the initialisation phase.

An example of the syntax to implement this would be,

```
@model
...
initialisation_phases Fixed

@initialisation_phase Fixed
type state_category_by_age
categories male female
min_age 3
max_age 10
table n
male    1000 900 800 700 600 500 400 700
female 1000 900 800 700 600 500 400 700
end_table
```

### 4.4.4. Model run years

Following initialisation, the model then runs over a number of user-defined years from (initial\_year to final\_year). For this part of the model, the annual cycle can be broken into separate time-steps, and observations can be associated with the state of the model at the end of any time-step, i.e., likelihoods for particular observations are evaluated, if required, within each time-step.

Processes are carried out in the order specified within each time-step. These can be the same or different to the processes in initialisation phases of the model.

The run years define the years over which the model is to run and the annual cycle within each year. The model runs from the start of year `initial` to the end of year `final`. The projection then extends the run time up to the end of year `project_final_year`. The following must be specified in the model,

- The time-steps and the processes applied in each
- The initial year (i.e., the model start year)
- The final year (i.e., the model end year)
- The projection final year (i.e., the model projection end year)

and an example of how the syntax would look,

```
@model
start_year 1972
final_year 2016
projection_final_year 2021
min_age 1 ## Define the age bins "columns" in the partition
max_age 30
age_plus true
base_weight_units tonnes
initialisation_phases Equilibrium_state
time_steps Sep_Feb Mar_May Jun_Aug ## Defines the annual cycle

## Define the "rows" in the partition
@categories
format stock ## This is a single sex and area population
names HAK4
age_lengths age_size

@initialisation_phase Equilibrium_state
type derived

## Define the processes in the Annual Cycle
## A list of labels in each time step that correspond to a process
@time_step Sep_Feb
processes Recruitment Instantaneous_Mortality

@time_step Mar_May
processes Instantaneous_Mortality

@time_step Jun_Aug
processes Ageing Instantaneous_Mortality
```

#### 4.4.5. Projection years

Projecting is the process of running the model forwards into the future, using stochastic and/or deterministic values for population dynamic parameters, such as recruitments and catches. The CASAL2 command to run the model in projection mode is `casal2 -f 1`. The number that follows the `-f` parameter indicates how many projections to undertake for each set of parameters supplied. This allows the exploration of many future scenarios for a single set of parameters. The value should

only be greater than 1 if applying a projection type that is stochastic. When running projections after a Bayesian analysis, the `-tabular` parameter should be added into the command. This will output a tabular report (see Section 7.21) which is easier to manipulate and run diagnostics on in the **R** environment.

Projection years occur immediately after the model run years, and are defined as the `final_year + 1` up to and including `final_projection_year`.

For a projection run in CASAL2 a model is initialised and run through the model years from `start_year` to `final_year`. During this phase CASAL2 stores all parameter values so that projection classes can (if the users chooses) allow parameters before `final_year` to be projected (See end of this paragraph for an example of when to do this). The model then is re-run from `start_year` to `projection_final_year`, where any parameter can be either fixed or, if specified, drawn from a stochastic distribution or process during that time period. An example of when users may want to begin projecting a parameter before the projection phase has started is for year class parameters. Usually, the last few year class parameters are poorly estimated (if they are estimated at all), although this depends on the quality and coverage of the compositional data that would inform these parameters, or the presence of a recruitment index. Therefore, users may wish to assume such parameters are unknown and apply projection methods as for future values.

CASAL2 does not have any default projections for when parameters are specified by year. These must be specified using the `@project` command block. This is important for parameters that may vary from year to year (e.g. year class strength parameters). CASAL2 should error out if run in projection mode without a `@project` block for the `yces_values` parameter being specified.

CASAL2 allows any estimable parameter to be specified in a `@project` block and then used in a projection. The available projection types for these parameters include: constant, lognormal, empirical-lognormal, empirical re-sampling, or user-defined. To see all the projection classes available, and examples of their syntax, see Section 4.13.

Common to all projection methods are the subcommands `years`, `parameter` and `multiplier`, as a constant is multiplied to the projected value after it has been derived by its respective method.

**Important note** for the specific year class parameter: the definition of year applies to the `yces_years`, not the model years. As defined in Section 4.5.1.2, `yces_years` are offset between time of spawning and when they enter the partition.

## 4.5. Population processes

Population processes are those processes that change the model state. Processes produce changes in the model partition by adding and removing individuals, or by moving individuals between ages and/or categories. The population processes include: recruitment, ageing, growth, maturation, mortality events (e.g., natural and anthropogenic) and category transition processes (i.e., processes that move individuals between categories while preserving their age structure).

There are two types of processes, processes that occur across multiple time steps in the annual cycle e.g `mortality_constant_rate` and `mortality_instantaneous`. There are also processes that only occur within the time step they are defined. Each of these processes are carried out in the user-defined prescribed order when initialising the model, and then for a user-defined order in each year in the annual cycle.

### 4.5.1. Recruitment

Recruitment processes are defined as a process that introduces new individuals into the partition. CASAL2 currently implements two types of recruitment process, constant recruitment and Beverton-Holt recruitment (Beverton and Holt, 1957).

In the recruitment processes, a number of individuals are added to a single age class within the partition, with the amount defined by the type of recruitment process specified and its function. If more than one category is defined, then the proportion of recruiting individuals to be added to each category is specified by the `proportions` parameter, or multiple recruitment processes can be defined. For example, if recruiting to categories labelled `male` and `female`, then you might set the proportions as 0.5 and 0.5 respectively, to denote that half of the recruits recruit to the male category and the remaining half to the female category.

An important note here is that recruitment can differ between a spawning event or the creation of a cohort/year class. In a fisheries context, recruitment usually refers to individuals 'recruiting' to the fishery. This is done for a few reasons, one being that often there is not a lot of information relating to age classes between spawning and data collection i.e. an information gap exists. Once again, in a fisheries context this information gap can refer to the time between spawning and being vulnerable to a survey or fishery for data collection. Thus, users may only want to model the population for which data is available. This offset between spawning and recruitment is parameterised either by the recruitment variable `age`, or `min_age` (the default value for the `age` parameter in the recruitment process). (Note for CASAL users: the CASAL2 parameter `age` is the same as the CASAL parameter `y_enter`.)

For the constant and Beverton-Holt recruitment processes, the number of individuals following recruitment in year  $y$  is,

$$N_{y,a,j} \leftarrow N_{y,a-1,j} + p_j(R_{y,a}) \quad (4.2)$$

where  $N_{y,a,j}$  is the numbers in category  $j$  at age  $a$ ,  $p_j$  is the proportion to category  $j$ , and  $R_{y,a}$  is the number of recruits for year  $y$ . See below for how  $N_{y,a,j}$  is determined in each of these cases.

#### 4.5.1.1. Constant recruitment

In the constant recruitment process the total number of recruits added in each year  $y$  in age  $a$  is  $R_{y,a}$ , and is simply  $p_j(R_0)$ , i.e.,

$$R_{y,a,j} = p_j(R_0) \quad (4.3)$$

Constant recruitment recruits a constant number of individuals each year. It is equivalent to a Beverton-Holt recruitment process with steepness set equal to one (i.e.,  $h = 1$ ).

For example, to specify a constant recruitment process, where individuals are added to male and female immature categories at  $age = 1$  evenly (`proportions = 0.5`), and the number to add is  $R_0 = 5 \times 10^5$ , the syntax is,

```
@process Recruitment
type constant_recruitment
categories male.immature female.immature
proportions 0.5 0.5
r0 500000
age 1
```

**4.5.1.2. Beverton-Holt recruitment**

In the Beverton-Holt recruitment process the total number of recruits added each year is  $R_y$ .  $R_y$  is the product of the average recruitment  $R_0$ , the annual year class strength multiplier  $YCS$ , and the stock-recruit relationship  $SR$ , i.e.,

$$R_{y,a,j} = p_j(R_0 \times YCS_{y_{cs\_year}} \times SR(SSB_{y_{cs\_year}})) \quad (4.4)$$

where,

$$y_{cs\_year} = y - ssb\_offset \quad (4.5)$$

and where,  $a$  is the parameter age,  $p_j$  is the proportion of recruits to enter category  $j$ , and  $ssb\_offset$  is the lag between spawning and recruitment.

As stated earlier, recruitment refers to the recruitment into the population and may differ from the spawning event. See two paragraphs below on more information about  $ssb\_offset$ , but in general this parameter shouldn't be specified by the user.

$SR$  is the Beverton-Holt stock-recruit relationship parametrised by the steepness  $h$  based on Mace and Doonan (1988) parametrisation,

$$SR(SSB_y) = \frac{SSB_y}{B_0} / \left( 1 - \frac{5h-1}{4h} \left( 1 - \frac{SSB_y}{B_0} \right) \right) \quad (4.6)$$

Note that the Beverton-Holt recruitment process requires a value for  $B_0$  and  $SSB_y$  to resolve the stock-recruitment relationship. Here, a derived quantity (see Section 4.6) must be defined that provides the annual  $SSB_y$  for the recruitment process.  $B_0$  is then defined as the value of the  $SSB$  at the end of one of the initialisation phases, this initialisation phase is defined by the parameter `b0_initialisation_phase`. During initialisation the  $YCS$  multipliers are assumed to be equal to one, and recruitment that happens in the initialisation phases that occur before and during the phase when  $B_0$  is determined is assumed to have steepness  $h = 1$  (i.e. in those initialisation phases, recruitment is simply equal to  $R_0$ ). Recruitment in the initialisation phases after the phase where  $B_0$  was determined follow the Beverton-Holt stock-recruit relationship defined above.  $R_0$  and  $B_0$  have a direct relationship when there are no density dependent processes in the annual cycle, for this reason users can choose to initialise models using  $B_0$  or  $R_0$ .

`ssb_offset` should not be manually specified by the user; as is commonplace, CASAL2 determines `ssb_offset` by the order of ageing, recruitment, spawning and the recruitment parameter age accordingly,

- if annual time-step order is recruitment, ageing, spawning, then `ssb_offset` should equal `age + 1`, or
- if annual time-step order is spawning, ageing, recruitment, then `ssb_offset` should equal `age - 1`, or
- else `ssb_offset = age`

There may be certain scenarios where the user will manually want to input these values, e.g. if there are multiple ageing processes in the annual cycle. CASAL2 has not been coded to deal with this situation and in this case `ssb_offset` needs to be manually defined.



There are two variants of this process, and they refer to how the stock recruitment residuals or  $YCS_{y_{cs\_year}}$  are parametrised as. This can either be as natural multipliers denotes as year class strength (YCS) multipliers or in log space usually termed recruitment deviations. Due to the difference in terminology we decided to split these two parametrisations into two separate processes `recruitment_beverton_holt` and `recruitment_beverton_holt_with_deviations` respectively. The difference is discussed in the below sections.

### YCS ( $YCS_y$ )

The YCS parameter (`y_{cs\_years}`) is defined in Equation (4.5). The parameter `y_{cs\_values}` should always be referenced by the `y_{cs\_years}` parameter and is important to note when defining `@estimate`, `@project` and `@time_varying` blocks for the parameter `y_{cs\_values}`. An example is given at the end of the section.

A common practice when estimating YCS is to standardise using the Haist parametrisation suggested by V. Haist. Note that CASAL2 will only standardise if users define the sub command `standardise_y_{cs\_years}`. When standardising the model parameter `y_{cs\_values}` is a vector  $\mathbf{Y}$ , covering the set of years from `start_year - ssb_offset` to `final_year - ssb_offset`, as defined by the parameter `y_{cs\_years}`. The resulting year class strengths are calculated by  $YCS_i = Y_i / \bar{Y}$ , where the mean is calculated over the user-specified years `standardise_y_{cs\_years}`. Then,

$$YCS_i = \begin{cases} Y_i / \text{mean}_{y \in S}(Y_y) & : y \in S \\ Y_i & : y \notin S \end{cases}$$

where  $S$  is the set of years from `standardise_y_{cs\_years}`. One effect of this parametrisation is that  $R_0$  is then defined as the mean estimated recruitment over the set of years  $S$ , because the mean year class multiplier over these years will always be one.

Typically, the user will define `standardise_y_{cs\_years}` to span the years for which they expect to have reasonable estimates of YCSs. Often, the user will wish to force  $Y_y = 1$  for some or all years  $y \in S$  (this is equivalent to forcing  $R_y = R_0 \times SR$ ) by setting the lower and upper bounds of these  $Y$ s to be 1. An exception to this might occur for the most recent YCSs, which the user may want to estimate but not include in the definition of  $R_0$  (because the estimates are based on too few data). Note that, optionally, the user may exclude one or more years within the range from the averaging process of the Haist parametrisation.

The advantage of the Haist parametrisation is that the user need no longer use a large penalty to force the mean of the YCS parameter to be 1 (though they should still use a small penalty to stop the mean of  $\mathbf{Y}$  from drifting). This may improve MCMC performance. Projected YCS are not affected by this feature. A disadvantage with this parametrisation in a Bayesian analysis is that the prior refers to the  $Y$ s, not the YCS.

The following syntax is an example of a the configuration of the Beverton-Holt recruitment process, where individuals are added to the category ‘immature’ at  $age = 1$ , and the number to add is  $R_0 = 5 \times 10^5$ . `SSB_derived_quantity` is a derived quantity that specifies the total spawning stock biomass that contributed to the year class, with  $B_0$  the value of the derived quantity at the end of the initialisation phase labelled `phase1`. The YCS are standardised to have mean one in the period 1995 to 2004, and recruits enter into the model two years following spawning.

```
@process Recruitment
type recruitment_beverton_holt
categories immature
proportions 1.0
```

```

r0 500000
b0_initialisation_phase phase1
steepness 0.75
age 1
ssb SSB_derived_quantity
standardise_ycs_years 1995:2004
ycs_years 1994 1995 1996 1997 1998 1999 2000 2001 2002 2003 2004 2005 2006
ycs_values 0.65 0.87 1.6 1.13 1.0235 0.385 2.653 1.35 1 1 1 1 1

```

### Recruitment deviations ( $\epsilon_y$ )

Recruitment deviations model the stock recruitment residuals in log space, with the following link between  $YCS_y$  and  $\epsilon_y$  shown in equation 4.7.

$$YCS_y = \exp^{-0.5b_y\sigma_R^2 + \epsilon_y} \quad (4.7)$$

where  $\sigma_R^2$  is the expected variance of the stock recruitment residuals,  $b_y$  is a bias correction defined by Methot Jr and Taylor (2011) and equation 4.8. There is a constraint when using this parametrisation which is that  $\epsilon_y \sim N(0, \sigma_R^2)$ . That is  $\epsilon_y$  are normally distributed in log space and thus log normal when back-transformed to the resulting stock recruitment relationship  $YCS_y$ . Recent work has found that this transformation does not technically lead to *a priori* assumption that the resulting  $YCS_y$  are lognormal, see appendix A for more discussion.

$$b_y = \begin{cases} 0, & \text{for } y \leq y_1^b \\ b_{max}(1 - \frac{y - y_1^b}{y_2^b - y_1^b}), & \text{for } y_1^b < y < y_2^b \\ b_{max}, & \text{for } y_2^b \leq y \leq y_3^b \\ b_{max}(1 - \frac{y_3^b - y}{y_4^b - y_3^b}), & \text{for } y_3^b < y < y_4^b \\ 0, & \text{for } y_4^b \leq y \end{cases} \quad (4.8)$$

The ramp function described above for the bias correction has the additional subcommands controlling the ramp as follows,

- $y_1^b = \text{last\_year\_with\_no\_bias}$
- $y_2^b = \text{first\_year\_with\_bias}$
- $y_3^b = \text{last\_year\_with\_bias}$
- $y_4^b = \text{first\_recent\_year\_with\_no\_bias}$
- $b_{max} = \text{b\_max}$

```

@process Recruitment
type recruitment_beverton_holt_with_deviations
categories immature
proportions 1.0
r0 500000
last_year_with_no_bias 1940
first_year_with_bias 1950
last_year_with_bias 2016
first_recent_year_with_no_bias 2018

```

```

b_max 0.85
b0_initialisation_phase phase1
steepness 0.75
age 1
ssb SSB_derived_quantity
deviation_years 1994 1995 1996 1997 1998 1999 2000 2001 2002 2003 2004 2005 2006
deviation_values 0 -0.2 0.4 0 0 0 0 0 0 0 0 0 0

```

So, to specify a Beverton-Holt recruitment for each stock, the following information is required:

1. *YCS*, starting from year (`start_year - ssb_offset`) and extending up to year (`final_year - ssb_offset`)
2. the value of age (a.k.a. `y_enter` to CASAL users)
3. the steepness parameter `h`
4. in a multi category model, the proportion of recruits for each category
5. a label for the derived quantity

Also note when an `@initialisation_phase` (Section 4.4.3) `type = derived` is specified and the recruitment is defined by `b0`, then all categories must be specified in the `@recruitment` block. Usually in a recruitment processes you only need to define the categories that receive recruits. Say, for example, a population had a spawning area different from the area that recruits enter the population, then an area specific model could be specified containing spawning categories and recruiting categories. recruiting categories would be specified in the subcommand `categories`, as these will be the categories receiving recruits. **If** you have `@initialisation_phase, type=derived` you need also to specify all categories that are a part of that recruitment process, e.g.,

```

@process Recruitment
type recruitment_beverton_holt
categories recruits.male recruits.female spawn.male spawn.female
proportions 0.5 0.5 0.0 0.0
r0 500000
ssb SSB
....

```

The proportions = 0.0 are needed due to the way the derived initialisation phase works, i.e., it finds a solution for when `r0 = 1.0` based on an infinite geometric series for the plus group, and scales the initial partition by `r0`. Thus, if all categories are not specified, then those that are missed would not be initialised to true values and this could lead to inaccurate model outputs. This caution extends to multiple-stock fisheries models where all the categories that make up the stock need to be listed.

#### 4.5.2. Ageing

The ageing process ‘ages’ individuals — simply, moving all individuals in the named categories  $j$  from one age class  $a$  to the next  $a + 1$ , or accumulates them if the last age class is a plus group.

The ageing process is defined as,

$$\text{element}(a, j) \leftarrow \text{element}(a + 1, j) \quad (4.9)$$

except in the case of the plus group (if defined),

$$\text{element}(a_{\max}, j) \leftarrow \text{element}(a_{\max}, j) + \text{element}(a_{\max-1}, j). \quad (4.10)$$

For example, to apply ageing to the categories `immature` and `mature`, then the syntax is,

```
@process Ageing
type ageing
categories immature mature
```

**Note** that ageing is *not* applied by CASAL2 by default. As with other processes, CASAL2 will not apply a process unless it is defined and specified as a process within the annual cycle. Hence, it is possible to specify a model where a category is not aged. *CASAL2 will not check or otherwise warn if there is a category defined where ageing is not applied.*

### 4.5.3. Mortality

Eight types of mortality processes are permissible in CASAL2: constant rate, event, biomass-event, instantaneous, instantaneous retained, Hollings, initialisation, and a density-dependent relationship based on prey suitability. These processes remove individuals from the partition, either as a rate, as a total number (abundance), as a biomass of individuals or as a mixture of these. Note that CASAL2 does not (yet) implement the Baranov catch equation. However, instantaneous mortality is considered an approximation to the Baranov catch equation. To apply both natural and biomass-event mortality `mortality_instantaneous` can be specified. Note that all mortality processes occur within a mortality block of a time step see Section 4.4.2 for more information and definitions on mortality blocks.

#### 4.5.3.1. Constant mortality rate

To specify a constant annual mortality rate (e.g.  $M = 0.2$ ) for categories ‘male’ and ‘female’, then,

```
@process NaturalMortality
type mortality_constant_rate
categories male female
selectivities One One
m 0.2 0.2
```

$$D_{j,t} = \sum_a N_{a,j} (1 - \exp S_{a,j} M_j p_t) \quad (4.11)$$

Where,  $D_{j,t}$  is the number of deaths in category  $j$  in time step  $t$ ,  $N_{a,j}$  is the number of individuals in category  $j$  at age  $a$ .  $S_{a,j}$  is the selectivity value for age  $a$  in category  $j$ ,  $M_j$  is the mortality rate for category  $j$ , and  $p_t$  is the proportion of the mortality rate to apply in time step  $t$ .

Note that the mortality rate process requires a selectivity. To apply the same mortality rate over all age classes, use a selectivity defined as  $S_j = 1.0$  for all ages  $j$ , e.g.,

```
@selectivity One
type constant
c 1
```

You can also apply age specific mortality rates. For example consider the hypothesis that mortality is higher for younger and older individuals and lowest when individuals are at their optimal fitness. This could be done using a double exponential selectivity (see Section 4.12 for what that selectivity looks like);

```

@selectivity age_specific_M
type double_exponential
x0 7.06524
x1 1
x2 17
y0 0.182154
y1 1.43768
y2 1.57169
alpha 1.0

@process NaturalMortalityByAge
type mortality_constant_rate
categories male female
selectivities age_specific_M age_specific_M
m 1.0 1.0

```

In the above syntax you can see how you set  $m$  to zero and allow the rate to be described through the selectivity. This concept can be applied in other mortality methods such as `instantaneous_mortality`.

#### 4.5.3.2. Event and biomass-event mortality

The event mortality and biomass-event mortality processes act in a similar manner, except that they remove a specified abundance (number of individuals) or biomass respectively. These can be used to include anthropogenic mortality where numbers of removals are known, e.g. fishing in a fisheries model, rather than applying mortality as a rate.

In these cases, the abundance or biomass removed is also constrained by a maximum exploitation rate. CASAL2 removes as many individuals, or as much biomass, as it can while not exceeding the maximum exploitation rate. When minimising, event mortality processes require a penalty to discourage parameter values that will not allow the defined number of individuals to be removed. Here, the model penalises those parameter estimates that result in an too low a number of individuals in the defined categories (after applying selectivities) to allow for removals at the maximum exploitation rate (and similarly for biomass). See Section 5.8 for more information on how to specify penalties.

For example, the event mortality applied to user-defined categories  $i$ , with the numbers removed at age  $j$  determined by a selectivity-at-age  $S_j$  is applied as follows:

First, calculate the vulnerable abundance for each category  $j$  in  $1 \dots J$  for ages  $a = 1 \dots A$  that are subject to event mortality,

$$V(a, j) = S(a)N(a, j) \quad (4.12)$$

and, hence, define the total vulnerable abundance  $V_{total}$  as,

$$V_{total} = \sum_j \sum_a V(a, j) \quad (4.13)$$

Hence the exploitation rate to apply is

$$U = \begin{cases} C/V_{total}, & \text{if } C/V_{total} \leq U_{max} \\ U_{max}, & \text{otherwise} \end{cases} \quad (4.14)$$

And the number removed  $R$  from each age  $a$  in category  $j$  is,

$$R(a, j) = UV(a, j) \quad (4.15)$$

For example, to specify an **abundance based** fishing mortality process in a fisheries model, with catches given for a set of specific years, over categories ‘immature’ and ‘mature’, with selectivity ‘FishingSel’, and assuming a maximum possible exploitation rate of 0.7, the syntax is,

```
@process Fishing
type event_mortality
categories immature mature
years 2000 2001 2002 2003
U_max 0.70
selectivities FishingSel FishingSel
penalty event_mortality_penalty
```

and specified similarly for a **biomass based** fishing mortality process,

```
@process Fishing
type mortality_event_biomass
categories immature mature
years 2000 2001 2002 2003
U_max 0.70
selectivities FishingSel FishingSel
penalty event_mortality_penalty
```

#### 4.5.3.3. Instantaneous mortality

The instantaneous mortality process combines both natural mortality and event biomass mortality into a single process. This allows the simultaneous application of both natural mortality and anthropogenic mortality to occur across multiple time steps. This process applies half the natural mortality in each time step, then the mortalities from all the concurrent removals instantaneously, then the remaining half of the natural mortality. In fisheries models this is the most commonly used mortality process. It allows for multiple removal events in the case of a fisheries model multiple fisheries/fleets. There are a few constraints that a removal method can only occur in one time step, but you can have multiple removals to cover events during the year.

When instantaneous mortality is applied the following equations are used.

- An exploitation rate (actually a proportion) is calculated for each fishery, as the catch divided by the selected-and-retained biomass,

$$U_f = \frac{C_f}{\sum_a \bar{w}_a S_{f,a} n_a e^{-0.5tM_a}}$$

- The mortality pressure associated with method  $f$  is defined as the maximum proportion of fish taken from any element of the partition in the area affected by the method  $f$ ,

$$U_{f,obs} = \max_a \left( \sum_k S_{k,a} U_k \right)$$

where the maximum is over all partition elements affected by fishery  $f$ , and the summation is over all methods  $k$  which affect the  $j$ th partition element in the same time step as fishery  $f$ .

In most cases the mortality pressure will be equal to the exploitation rate (i.e.,  $U_{f,obs} = U_f$ ), but can be different if: (a) there is another removal method operating in the same time step as removal method  $f$  and affecting some of the same partition elements, and/or (b) the selectivity  $S_{f,a}$  does not have a maximum value of 1.

There is a maximum mortality pressure limit of  $U_{f,max}$  for each method of removal  $f$ . So, no more than proportion  $U_{f,max}$  can be taken from any element of the partition affected by removal method  $f$  in that time step. Clearly,  $0 \leq U_{max} \leq 1$ . It is an error if two removal methods, which affect the same partition elements in the same time step, do not have the same  $U_{max}$ .

For each  $f$ , if  $U_{f,obs} > U_{f,max}$ , then  $U_f$  is multiplied by  $U_{f,max}/U_{f,obs}$  and the mortality pressures are recalculated. In this case the catch actually taken from the population in the model will differ from the specified catch,  $C_f$ .

- The partition is updated using

$$n'_a = n_a \exp(-tM_a) \left[ 1 - \sum_f S_{f,a} U_f \right]$$

As an example, to apply natural mortality of 0.20 across three time steps on both male and female categories, and with two methods of removals (fisheries) FishingWest FishingEast with respective catches (kg) known for years 1975:1977 (the catches are given in the catches table and information on selectivities, penalties and maximum exploitation rates are given in the method table), the syntax is,

```
@process instant_mort
type mortality_instantaneous
m 0.20
time_step_ratio 0.42 0.25 0.33
selectivities One
categories male female
units kgs

table catches
year FishingWest FishingEast
1975 80000 111000
1976 152000 336000
1977 74000 1214000
end table

table method
method      category  selectivity u_max  time_step penalty
FishingWest  stock      westFSel   0.7   step1    CatchPenalty
FishingEast  stock      eastFSel   0.7   step1    CatchPenalty
end_table
```

and for referencing catch parameters for use in projecting, time\_varying and estimating, the syntax is,

```
parameter process[mortality_instantaneous].method_"method_label"{2018}
```

where "method\_label" is lower case from the catch or method table and continuing the example,

```
parameter process[instant_mort].method_FishingWest{2018}
```

If you want weight to be calculated by empirical weight at age matrices as described in Section 4.9 the method table has an additional column that can call weight at age objects. For example

```
@age_weight jan_weight_at_age
type data
table data
year 1 2 3 4
1980 3.4 5.6 7.23 8.123
end_table

table method
method category selectivity u_max time_step penalty age_weight
FishingWest stock westFSel 0.7 step1 CatchPenalty
FishingEast stock eastFSel 0.7 step1 CatchPenalty
end_table
```

#### 4.5.3.4. Instantaneous mortality with retained catch and discards

The instantaneous mortality retained process builds on the instantaneous mortality process (4.5.3.3) which has simultaneous applications of fishing and natural mortality, but with all catch-at-sea being landed, i.e., no discarding. The process `mortality_instantaneous_retained` allows for retained catch, discards, and also a mortality to be applied to discards, i.e., some are allowed to survive. The method for taking catch from the partition and constraints used are the same as in `mortality_instantaneous`.

The process was created to solve problems in the pot fishery for blue cod which has a minimum legal size and so some catch must be discarded at sea and some of these discards are expected to survive (based on some experimental work). There are length data taken at sea so that the catch selectivity can be estimated, and length and age data from the landed catch (retained) so that the retention selectivity can also be estimated. In this version of the process, discard mortality is specified by the user by defining a selectivity to represent mortality by age or length (e.g., constant or reverse logistic) and it is intended that it would not be estimated since there is no observation class for it yet. If discard mortality is not provided, it is assumed that all discards die. Landed catch, and both retained and catch selectivities must be specified.

Extending the example shown in instantaneous mortality process (4.5.3.3) to use retained weight instead of catch, the input commands would be:

```
@process FishingRetainedCatch
type mortality_instantaneous_retained
m 0.20
#natural mortality
time_step_ratio 0.42 0.25 0.33 # ratio of natural mortality in each of the three time steps
selectivities One
#for natural mortality by age
categories male female
units kgs

table catches # two fisheries, West and East
year FishingWest FishingEast
1975 80000 111000 # these are now landed catch
1976 152000 336000
1977 74000 1214000
end table
```



```

table method #all discards die
method      category selectivity retained_selectivity u_max time_step penalty
FishingWest stock    westFSel    westRetainedSel    0.7  step1    CatchPenalty
FishingEast stock    eastFSel    eastRetainedSel    0.7  step1    CatchPenalty
end_table

```

If discard mortality is less than 1.0 is required, then use:

```

table method # 50% discard mortality
method      category selectivity retained_selectivity discard_mortality u_max time_step penalty
FishingWest stock    westFSel    westRetainedSel    DisMort    0.7  step1    CatchPenalt
FishingEast stock    eastFSel    eastRetainedSel    DisMort    0.7  step1    CatchPenalt
end_table

@selectivity DisMort
Type constant
c 0.5 #50% mortality of discards returned to the sea

```

See instantaneous mortality process (4.5.3.3) for referencing catch parameters and calculating weight using empirical weight by age matrices.

The report gives total catch, actual landed catch, and discards, without and with discard mortality; use the following

```

@report Mortality
type process
process Instantaneous_Mortality_Retained

```

In the following, fisheries are indexed by  $f$ , and  $a$  indexes both age and category combinations.

The total catch is found using by using a selectivity,  $S_{f,a}$ , in the same way as in the instantaneous mortality process. Retention,  $R_{f,a}$ , is defined by specifying a selectivity and it can be a function of length or age. The retained catch is the product of these two values,  $R_{f,a} S_{f,a}$ . If sex is in the partition, then there are potentially two retention curves, one for each sex. In general, there is a retention curve for each category in the partition. It does not apply to surveys. Discard mortality is also specified as a selectivity,  $D_{f,a}$ . The fraction of dead fish from fishing activity is  $S_{f,a} * (R_{f,a} + (1.0 - R_{f,a}) * D_{f,a})$ . If  $D_{f,a}$  is 1.0, then all selected fish are dead, but if it is 0.0, then only retained fish are dead.

When the `mortality_instantaneous_retained` process is applied, the following equations are used:

- Total catch (catch-on-board),  $C_f$ , is calculated by (retained catch) \* VF / VR, where VF is vulnerable retained biomass,  $j$  indexes categories and  $t$  is the proportion of M in the time-step, and VR is the full vulnerable biomass,  $VF = \sum_{a,j} \bar{w}_a S_{f,a} n_{a,j} \exp(-0.5tM_{a,j})$ .
- An exploitation rate (actually a proportion) is calculated for each fishery, as the total catch (retained + discards) divided by the selected biomass (VF above) using selectivity  $S_{f,a}$ ,

$$U_f = \frac{C_f}{\sum_a \bar{w}_a S_{f,a} n_a \exp(-0.5tM_a)}$$

- The mortality pressure associated with method  $f$  is defined as the maximum proportion of fish taken from any element of the partition in the area affected by the method  $f$ ,

$$U_{f,obs} = \max_a \left( \sum_k S_{k,a} U_k \right)$$

where the maximum is over all partition elements affected by fishery  $f$ , and the summation is over all methods  $k$  which affect the  $j$ th partition element in the same time step as fishery  $f$ .

In most cases the mortality pressure will be equal to the exploitation rate (i.e.,  $U_{f,obs} = U_f$ ), but can be different if: (a) there is another removal method operating in the same time step as removal method  $f$  and affecting some of the same partition elements, and/or (b) the selectivity  $S_{f,a}$  does not have a maximum value of 1.

There is a maximum mortality pressure limit of  $U_{f,max}$  for each method of removal  $f$ . So, no more than proportion  $U_{f,max}$  can be taken from any element of the partition affected by removal method  $f$  in that time step. Clearly,  $0 \leq U_{max} \leq 1$ . It is an error if two removal methods, which affect the same partition elements in the same time step, do not have the same  $U_{max}$ .

For each  $f$ , if  $U_{f,obs} > U_{f,max}$ , then  $U_f$  is multiplied by  $U_{f,max}/U_{f,obs}$  and the mortality pressures are recalculated. In this case the catch actually taken from the population in the model will differ from the specified catch,  $C_f$ .

- Discard numbers-at-age (including their share of natural mortality) is  $S_{a,j}(1 - R_{a,j})n_{a,j} \exp(-0.5tM_{a,j})$ , and those that die at the end of the time step (updating the partition) are  $D_{a,j}S_{a,j}(1 - R_{a,j})n_{a,j} \exp(-tM_{a,j})$ , where  $D_{f,a}$  is the fraction that die on return to the sea.
- The partition is updated by removing landed catch, natural mortality, and discard mortality

$$n'_a = n_a \exp(-tM_a) \left[ 1 - \sum_f S_{f,a} U_f (R_{f,a} + D_{f,a}(1 - R_{f,a})) \right]$$

#### 4.5.3.5. Holling's mortality rate

The density-dependent Hollings mortality process applies the Holling Type II and Type III functions (Holling, 1959), but is generalised using the Michaelis-Menten equation (Michaelis and Menten, 1913). The function removes a number or biomass from a set of categories according to the total (selected) abundance (or biomass) and some 'predator' abundance (or biomass), and constrained by a maximum exploitation rate.

For example, the mortality applied to user-defined categories  $k$ , with the numbers removed at age  $l$ , determined by a selectivity-at-age  $S(l)$  is applied as follows:

First, calculate the total predator abundance (or biomass) over all predator categories  $k$  in  $1 \dots K$  and ages  $l = 1 \dots L$  that are applying the mortality,

$$P(k, l) = S_{predator}(l) N_{predator}(k, l) \quad (4.16)$$

And define the total predator abundance (or biomass)  $P_{total}$  as,

$$P_{total} = \sum_K \sum_L P(k, l) \quad (4.17)$$

Then, calculate the total vulnerable abundance (or biomass) over all prey categories  $k$  in  $1 \dots K$  and ages  $l = 1 \dots L$  that are subject to the mortality,

$$V(k, l) = S_{prey}(l) N_{prey}(k, l) \quad (4.18)$$

And hence, define the total vulnerable abundance (or biomass)  $V_{total}$  as,

$$V_{total} = \sum_K \sum_L V(k, l) \quad (4.19)$$

and then, the the number to remove is determined by,

$$R_{total} = P_{total} \frac{aV_{total}^{x-1}}{b + V_{total}^{x-1}} \quad (4.20)$$

where  $x = 2$  for Holling type II function,  $x = 3$  for Holling type III function, or any value of  $x \geq 1$  for the generalised Michaelis-Menten function, and  $a > 0$  and  $b > 0$  are the Holling function parameters.

Hence, the exploitation rate to apply is,

$$U = \begin{cases} R_{total}/V_{total}, & \text{if } R_{total}/V_{total} \leq U_{max} \\ U_{max}, & \text{otherwise} \end{cases} \quad (4.21)$$

And the number removed  $R$  from each age  $l$  in category  $k$  is,

$$R(k, l) = UV(k, l) \quad (4.22)$$

The density-dependent Holling mortality process is applied either as a biomass or an abundance depending on the value of the `is_abundance` switch.

For example, a biomass Holling type II mortality process on `prey` by our predator `predator` would have syntax,

```
@process HollingMortality
type Holling_mortality_rate
is_abundance F
a 0.08
b 10000
x 2
categories prey
selectivities One
predator_categories predator
predator_selectivities One
u_max 0.8
```

#### 4.5.3.6. Initialisation-event mortality

Initialisation event mortality is a specific process that only can occur in the initialisation phase. It allows users to apply abundance or biomass mortality events specifically in initialisation phases. This can be useful if you wanted to deviate a model from equilibrium before model start. This process applies a single catch for all iterations within the initialisation phase, and mortality will not be applied outside of the initialisation phase. This process should not be embedded in the annual cycle. We advise that this process be used in conjunction with the `insert_processes` command in the `@initialisation_phase` block. Example syntax to implement such a scenario,

```
initialisation_phases Equilibrium_state Predation_state
time_steps Oct_Nov Dec_Mar
```

```
@initialisation_phase Equilibrium_state
type derived

@initialisation_phase Predation_state
type iterative
insert_processes Oct_Nov()=predation_initialisation

@process predation_initialisation
type initialisation_mortality_event
categories male.HOKI female.HOKI
catch 90000
selectivities Hakesl Hakesl

time_step Oct_Nov
processes Mgl Instantaneous_Mortality

@time_step Dec_Mar
processes Recruitment Instantaneous_Mortality
```

Note how the `initialisation_mortality_event` has been specified in the initialisation phase `Predation_state` but not in the annual cycle.

#### 4.5.3.7. Prey-suitability mortality

The density-dependent prey-suitability process applies predation mortality from a predator group to its prey groups simultaneously. It removes an abundance (or biomass) from each prey group according to the total (selected) abundance (or biomass) of each prey group, the total (selected) abundance (or biomass) of the other prey groups, some 'predator' abundance (or biomass), and the preference (electivity) of the predator to each prey group, but constrained by a maximum exploitation rate. The predator-prey suitability functions were based on the multispecies Virtual Population Analysis (MSVPA) functions described by (Jurado-Molina et al., 2005).

For example, the mortality applied to the user-defined prey group  $g$  of category  $k$ , with the numbers removed at age  $l$  determined by a selectivity-at-age  $S(l)$  is applied as follows:

First, calculate the total predator abundance (or biomass) over all predator categories  $k$  in  $1 \dots K$  and ages  $l = 1 \dots L$  that are applying the mortality,

$$P(k, l) = S_{predator}(l) N_{predator}(k, l) \quad (4.23)$$

And define the total predator abundance (or biomass)  $P_{total}$  as,

$$P_{total} = \sum_K \sum_L P(k, l) \quad (4.24)$$

Then, given the total vulnerable abundance (or biomass) of prey group  $g$  over all categories  $k$  in  $1 \dots K$  and ages  $l = 1 \dots L$  that are subject to the mortality,

$$V(g, k, l) = S_{prey}(l) N_{prey}(k, l) \quad (4.25)$$

And define the total vulnerable abundance (or biomass) of each prey group  $V(g)_{total}$  as,

$$V(g)_{total} = \sum_K \sum_L V(g, k, l) \quad (4.26)$$

And the total availability  $A(g)_{total}$  for each prey group as,

$$A(g)_{total} = \frac{V(g)_{total}}{\sum_G V(i)_{total}} \quad (4.27)$$

The vulnerable abundance (or biomass) and availability every prey group  $g$  in  $1 \dots G$  is calculated simultaneously. Then the abundance (or biomass) to remove from each prey group  $g$  is a function of its electivity  $E(g)$ , the availability of all other prey groups  $i$  in  $1 \dots G$ , the electivity of the predator for each prey group  $E(i)$ , and the total consumption rate of the predator  $CR$  and its abundance (or biomass)  $P_{total}$ ,

$$R(g)_{total} = P_{total} CR \frac{A(g)_{total} E(g)}{\sum_G A(i)_{total} E(i)} \quad (4.28)$$

Hence the exploitation rate to apply to each prey group  $g$  is

$$U(g) = \begin{cases} R(g)_{total}/V(g)_{total}, & \text{if } R(g)_{total}/V(g)_{total} \leq U_{max} \\ U_{max}, & \text{otherwise} \end{cases} \quad (4.29)$$

And the number removed  $R(g)$  in each prey group  $g$  from each age  $l$  in category  $k$  is,

$$R(g, k, l) = U(g) V(g, k, l) \quad (4.30)$$

Note that prey suitability choice occurs only between prey groups specified by the process, and the total predator consumption rate represents the consumption of the predator on those prey groups alone. Also note that the electivities must sum to one. Further, the consumption rate can be modified by a layer, to be cell specific.

The density-dependent prey-suitability process is applied as either a biomass or an abundance depending on the value of the `is_abundance` switch.

Individual categories can be aggregated into prey groups using the '+' symbol. To indicate that two (or more) categories are to be aggregated, separate them with a '+' symbol. For example, to specify two prey groups of two species made up of the males and females in each, then the subcommand would be,

```
prey_categories maleSpeciesA + femaleSpeciesA maleSpeciesB + femaleSpeciesB
```

This would indicate that there are two prey groups, `maleSpeciesA + femaleSpeciesA` and `maleSpeciesB + femaleSpeciesB`, with each group having its own electivity. For example, a biomass prey-suitability mortality process with an overall consumption rate of 0.8 of species A and species B (modelled as males and females) by our predator `predatorSpecies` with electivities between species A and species B of 0.18 and 0.82 would have syntax,

```
@process PreySuitabilityMortality
type prey-suitability_predation
is_abundance F
consumption_rate 0.8
categories maleSpeciesA + femaleSpeciesA maleSpeciesB + femaleSpeciesB
electivities 0.18 0.82
selectivities One One One One
predator_categories predatorSpecies
predator_selectivities One
u_max 0.8
```

#### 4.5.4. Transition By Category

This process moves individuals between categories. The CASAL2 partition is user-defined, and this type of process is used to move individuals between categories, and is used to specify processes such as maturation (move individuals from an immature to mature state) or migration (move individuals from one area to another).

##### 4.5.4.1. Annual transition by category

A special case is annual transition by category, which allows a transition to occur in a specific subset of years only, where each year can have a different rate.

In both cases, there has to be a one to one relationship between the ‘from’ category and the ‘to’ category, i.e., for every source category there is one target category,

$$N_{a,j} = N_{a,i} \times P_i \times S_{a,i} \quad (4.31)$$

where  $N_{a,j}$  is the number of individuals that have moved to category  $j$  from category  $i$  in age  $a$  and  $N_{a,i}$  is the number of individuals in category  $i$ .  $P_i$  is the proportion parameter for category  $i$  and  $S_{a,i}$  is the selectivity at age  $a$  for category  $i$ .

Note, to merge categories just repeat the ‘to’ category multiple times.

An example, to specify a simple spawning migration of mature males from a western area to an eastern (spawning) area, the syntax is,

```
@process Spawning_migration
type transition_category
from West.males
to East.males
selectivities MatureSel
proportions 1
```

where `MatureSel` is a selectivity that describes the proportion of age or length classes that are mature and thus move to the eastern area.

#### 4.5.5. Tag Release events

Tagging processes can be age or length based processes, where-by numbers of individuals are moved from an untagged category to a tagged category that the user has defined from the `@categories` block. Tag release processes can also account for initial tag-induced mortality on individuals. Age-based tag release events take a known number of individuals tagged for each age and do a straightforward category transition, along with extra mortality. Individuals are deducted from the non-tagged categories and shifted into tagged categories. Often, the ages of tagged individuals are not known and length based tagging is the most commonly used tagging process.

Length-based tag release processes are more complicated, as CASAL2 needs to calculate the age-length matrix and the exploitation by each length-bin to then move the correct numbers-at-age based on the known lengths of release. CASAL2 also allows for initial tag loss. The algorithm that CASAL2 follows,

For each length bin ( $l$ ) of the input vector of numbers at length ( $\tilde{N}_l$ ) do the following

$$N_{l,j} = \sum_{a=1} N_{a,l,j} * S_a$$

where  $N_{a,l,j}$  is the numbers at age and length for category  $j$  in the partition, and  $S_a$  is the selectivity at age  $a$ .

calculate the total numbers at length ( $T_l$ ) across all source categories at length  $l$  taking into account the selectivities

$$T_l = \sum_{j=1} N_{l,j}$$

Calculate the transition rate for this length bin ( $u_l$ )

$$u_l = \tilde{N}_l / T_l$$

Check that we don't exceed some threshold, usually called a  $u_{max}$  which is analogous to the  $u_{max}$  in a mortality processes.

$$u_l = \begin{cases} u_{max}, & \text{if } u_l > u_{max} \text{ flag a penalty} \\ u_l, & \text{otherwise} \end{cases}$$

Calculate the numbers at age in this category that we are going to move by multiplying across the age-length matrix and storing this info by age, for each age we want to accumulated these for all length bins. then move the necessary

$$N_{a,j+} = N_{a,l} * u_l$$

The syntax for an example of tag release by length process in CASAL2 follows,

```
@process 2005Tags_shelf
type tag_by_length
years 2005
from male.untagged+female.untagged
to male.2005 female.2005
selectivities ShelfselMale ShelfselFemale
penalty tagging_penalty
initial_mortality 0.1
table proportions
year 30 40 50 60 70 80 90 100 110 120 130 140 150 160 170 180 190 200 210 220
2005 0 0 0.0580 0.1546 0.3380 0.1981 0.1643 0.0531 0.0242 0.0097 0 0 0 0 0 0 0 0 0 0
end_table
n 207
U_max 0.999
```

The above process will move 207 individuals from a combination of male.untagged and female.untagged categories, based on the combination of growth rates and selectivity into tagged male and tagged female categories based on selectivities and growth rates between.

#### 4.5.6. Tag Loss

Tag Loss is the process which accounts for tags being lost from tagged individual over time due to, for example, tag failure or tags getting knocked off. This process is applied as an instantaneous migration rate that can happen over multiple time steps in the annual cycle. This method assumes that when tags are lost the fish are transferred from the `from` category to the `to` category. How the tag loss rate is applied depends on whether the fish were only tagged with a single tag (`tag_number_per_animal = 1`), double tagged (`tag_number_per_animal = 2`) or *n* tagged (`tag_number_per_animal = n`). The syntax for this relationship is,

```
@process Tag_loss
type tag_loss
categories tagged_fish
tag_loss_rate 0.02
time_step_ratio 0.25 0.75
selectivities One
tag_loss_type single
year 1985
```

#### 4.6. Derived quantities

Some processes require, as arguments, a population value derived from the population state. These are termed *derived quantities*. Derived quantities are values, calculated by CASAL2 at the end of a specified time-step in every year, and hence have a single value for each year of the model. Derived quantities can be calculated as either an abundance or as a biomass. Abundance derived quantities are simply the count or sum over the categories (after applying a selectivity). Similarly for biomass Biomass derived quantities. Derived quantities are also calculated during the initialisation phases. Therefore, the time-step during each phase must also be specified. If the initialisation time-steps are not specified, CASAL2 will calculate the derived quantity during the initialisation phases in every year, at the end of the annual cycle.

Derived quantities are required by some processes, e.g. the Beverton-Holt recruitment process. The Beverton-Holt recruitment process can require an equilibrium biomass ( $B_0$ ) and annual spawning stock biomass values ( $SSB_y$ ) to resolve the stock-recruit relationship. Here, these would be defined as the abundance or biomass of a part of the population at some point in the annual cycle for selected ages and categories, and would be calculated as a derived quantity.

Derived quantities are associated with a mortality block see section 4.4.2 for more detail on mortality blocks. Users can ask for derived quantities partway through mortality blocks. Currently, two methods are implemented in CASAL2 to interpolate derived quantities part-way through a mortality block, these are `weighted_sum` and `weighted_product`, and are defined as,

- `weighted_sum`: after proportion  $p$  of the mortality block, the partition elements are given by  $n_{p,j} = (1 - p)n_j + p'n'_j$
- `weighted_product`: after proportion  $p$  of the mortality block, the partition elements are given by  $n_{p,j} = n_j^{1-p} n_j'^p$

where,  $n_{p,j}$  is the derived quantity at proportion  $p$  of the mortality block for category  $j$ ,  $n_j$  is the quantity at the beginning of the mortality block and  $n'_j$  is the quantity at the end of the mortality block.

As an example, to define a biomass derived quantity (say spawning stock biomass, *SSB*), evaluated at the end of the first time-step (labelled `step_one`), over all 'mature' male and female categories



and halfway through the mortality block using the `weighted_sum` method, we would use the syntax,

```
@derived_quantity SSB
type biomass
time_step step_one
categories mature.male mature.female
selectivities One
time_step_proportion 0.5
time_step_proportion_method weighted_sum
```

## 4.7. Age-length relationship

The age-length relationship defines the length at age (and the weight at length, see Section 4.8) of individuals at age/category within the model. There are three length-age relationships available in CASAL2. The first is the naive no relationship (where each individual has length 1 irrespective of age). The second and third are the von-Bertalanffy and Schnute relationships respectively. The length-at-age relationship is used to determine the length frequency, given age, and then with the length-weight relationship, a weight-at-age of individuals within an age/category. When defining length-at-age in CASAL2, you must also define a length-weight relationship (see Section 4.8 below). The model can incorporate changes in length-at-age during the year – i.e., growth between fish birthdays – by incrementing age as specified by the `time_step_proportions` parameter.

### 4.7.0.1. none

Where the length of each individual is exactly 1 for all ages, in which case the `none` length-weight relationship must also be used.

### 4.7.0.2. von\_bertalanffy

$$\bar{s}(\text{age}) = L_{\infty} (1 - \exp(-k(\text{age} - t_0))) \quad (4.32)$$

### 4.7.0.3. schnute

$$\bar{s}(\text{age}) = \begin{cases} \left[ y_1^b + (y_2^b - y_1^b) \frac{1 - \exp(-a(\text{age} - \tau_1))}{1 - \exp(-a(\tau_2 - \tau_1))} \right]^{1/b}, & \text{if } a \neq 0 \text{ and } b \neq 0 \\ y_1 \exp \left[ \ln(y_2/y_1) \frac{1 - \exp(-a(\text{age} - \tau_1))}{1 - \exp(-a(\tau_2 - \tau_1))} \right], & \text{if } a \neq 0 \text{ and } b = 0 \\ \left[ y_1^b + (y_2^b - y_1^b) \frac{\text{age} - \tau_1}{\tau_2 - \tau_1} \right]^{1/b}, & \text{if } a = 0 \text{ and } b \neq 0 \\ y_1 \exp \left[ \ln(y_2/y_1) \frac{\text{age} - \tau_1}{\tau_2 - \tau_1} \right], & \text{if } a = 0 \text{ and } b = 0 \end{cases} \quad (4.33)$$

Note, the von Bertalanffy curve is parameterised by  $L_{\infty}$ ,  $k$ , and  $t_0$ ; the Schnute curve (Schnute, 1981) by  $y_1$  and  $y_2$ , which are the mean lengths at reference ages  $\tau_1$  and  $\tau_2$ , and  $a$  and  $b$  (when  $b = 1$ , this reduces to the von Bertalanffy with  $k = a$ ).

**4.7.0.4. data**

There is an option for users to input empirical length at age by year, this is an alternative to going through a age length growth model such as the von Bertalanffy and Schnute model. CASAL2 will do a lot of interpolations of missing years for the user and across time steps. There is a condition that the measurements of length at age throughout the model years occur in the same time step.

**4.8. Length-weight relationship**

There are two length-weight relationships available in CASAL2. The first is the 'naive' — which the relationship where the weight of an individual, regardless of length, is always 1. The second is the 'basic' relationship, which is the standard cubic function of length.

**4.8.0.5. none**

$$\text{mean weight} = 1 \quad (4.34)$$

**4.8.0.6. basic**

The mean weight  $\hat{w}_a$  of an individual at age  $a$  is,

$$\hat{w}_a = a \hat{l}_a^b \quad (4.35)$$

where  $\hat{l}_a$  is the mean length at age  $a$ . Note that if a distribution of length-at-age is specified, then the mean weight is calculated over the distribution of lengths,

$$\hat{w}_a = (a \hat{l}_a^b) (1 + cv^2)^{\frac{b(b-1)}{2}} \quad (4.36)$$

where the  $cv$  is the coefficient of variation (c.v.) of lengths-at-age. This adjustment is exact for lognormal distributions, and a close approximation for normal distributions if the c.v. is not large (Bull et al., 2012). For users comparing CASAL with CASAL2, there is a small difference between the two programs. CASAL only adjusted the c.v.s by `length` when c.v.s are used in distribution calculations (length based selectivities, length based processes and observations), and is not done in the above correction.

Be careful about the scale of  $a$  — this can easily be specified incorrectly. If the catch is in tonnes and the growth curve in centimetres, then  $a$  should be on the right scale to convert a length in centimetres to a weight in tonnes. Note that there are reports available that can be used to help check that the units specified are plausible (see Section 7).

```
@length_weight length_weight
type basic
units tonnes
a 0.00000123
b 3.132
```

**4.9. Age-weight relationship**

CASAL2 also allows users to input direct weight at age measurements. This is different to the method above as it uses empirical data to evaluate weight at age, rather than calculating it via the growth functions (age->length->weight).

This class represents the weight at age for categories at a point in time. They can be used in weight based derived quantities, processes and observations. An example of applying this in functionality in CASAL2 model.

```
type Data
units tonnes
table data
year 1 2 3 4 5 6 7 8 9 10
1986 0.134 0.686 1.639 2.719 3.649 4.901 6.329 6.591 7.238 7.491
1987 0.132 0.724 1.534 2.829 4.092 4.853 5.705 6.143 7.179 8.089
1988 0.122 0.641 1.533 2.641 3.796 5.054 5.652 6.356 6.95 8.857
1989 0.137 0.722 1.606 2.416 3.629 5.027 5.561 6.35 6.933 7.217
1990 0.138 0.773 1.645 2.74 3.711 4.506 5.684 6.929 7.424 7.479
end_table
```

If weight is purely defined by the weight at age functionality, you can choose not to specify the age length block in the @categories block.

```
@categories
format stock
names Stock
```

if a weight/biomass based derived quantity, process or observation has a age\_weight\_label subcommand then it can use the @age\_weight class to calculate mean weight at age.

#### 4.10. Weightless model

To model abundance (i.e., not convert the population to weight), the @length\_weight argument is turned off by specifying the keyword none in the @age\_length block, e.g.,

```
@age_length age_size
type schnute
...
length_weight none
```

In this case any "biomass" generated by CASAL2 will actually be abundance, and care should be taken with interpretation of the output when using this setting.

#### 4.11. Maturity, in models without maturing in the partition

If maturity is not a character of the partition it can easily be derived at an instance in time using selectivities. Applying a maturity selectivity to the partition allows CASAL2 to use mature elements in processes, derive mature biomass estimates (using derived quantities), and report the mature partition as an output.

#### 4.12. Selectivities

A selectivity is a function that can have a different value for each age class. Selectivities are used throughout CASAL2 to interpret observations (Section 5) or to modify the effects of processes on each age class (Section 4). CASAL2 implements a number of different parametric forms, including

logistic, knife edge, and double normal selectivities. Selectivities are defined in their own command block (@selectivity), where the unique label of the selectivity is used by observations and processes to identify which selectivity to apply.

Selectivities are indexed by age, with indices from `min_age` to `max_age`. For example, for a logistic age-based selectivity with 50% selected at age 5 and 95% selected at age 7, would be defined by the `type=logistic` with parameters  $a_{50} = 5$  and  $a_{95} = (7 - 5) = 2$ . The value of the selectivity at age  $x = 7$  is 0.95, and the value at age  $x = 3$  is 0.05. Note, while selectivities can be length based, use with caution as more testing is needed for this functionality.

The function values for some choices of parameters, for some selectivities, can result in a computer numeric overflow error (i.e., the number calculated from parameter values is either too large or too small to be represented in computer memory). CASAL2 implements range checks on some parameters to test for a possible numeric overflow error before attempting to calculate function values. For example, the logistic selectivity is implemented such that if  $(a_{50} - x)/a_{95} > 5$  then the value of the selectivity at  $x = 0$ , i.e., for  $a_{50} = 5$ ,  $a_{95} = 0.1$ , then the value of the selectivity at  $x = 1$ , without range checking would be  $7.1 \times 10^{-52}$ . With range checking, that value is 0 (as  $(a_{50} - x)/a_{95} = 40 > 5$ ).

The available selectivities are;

- Constant
- Knife-edge
- All values
- All values bounded
- Increasing
- Logistic
- Inverse logistic
- Logistic producing
- Double normal
- Double exponential

The available selectivities are described below.

#### 4.12.1. constant

$$f(x) = C \tag{4.37}$$

The constant selectivity has the estimable parameter C.

#### 4.12.2. knife\_edge

$$f(x) = \begin{cases} 0, & \text{if } x < E \\ \alpha, & \text{if } x \geq E \end{cases} \tag{4.38}$$

The knife-edge ogive has the estimable parameter E and a scaling parameter  $\alpha$ , where the default value of  $\alpha = 1$ .

**4.12.3. all\_values**

$$f(x) = V_x \quad (4.39)$$

The all-values selectivity has estimable parameters  $V_{low}, V_{low+1} \dots V_{high}$ . Here, you need to provide the selectivity value for each age class.

**4.12.4. all\_values\_bounded**

$$f(x) = \begin{cases} 0, & \text{if } x < L \\ V_x, & \text{if } L \leq x \leq H \\ V_H, & \text{if } x > H \end{cases} \quad (4.40)$$

The all-values-bounded selectivity has non-estimable parameters  $L$  and  $H$ . The estimable parameters are  $V_L, V_{L+1} \dots V_H$ . Here, you need to provide an selectivity value for each age class from  $L \dots H$ .

**4.12.5. increasing**

$$f(x) = \begin{cases} 0, & \text{if } x < L \\ f(x-1) + \pi_x(\alpha - f(x-1)), & \text{if } L \leq x \leq H \\ f(\alpha), & \text{if } x \geq H \end{cases} \quad (4.41)$$

The increasing ogive has non-estimable parameters  $L$  and  $H$ . The estimable parameters are  $\pi_L, \pi_{L+1} \dots \pi_H$  (but if these are estimated, they should always be constrained to be between 0 and 1).  $\alpha$  is a scaling parameter, with default value of  $\alpha = 1$ . Note that the increasing ogive is similar to the all-values-bounded ogive, but is constrained to be non-decreasing.

**4.12.6. logistic**

$$f(x) = \alpha / [1 + 19^{(a_{50}-x)/a_{t095}}] \quad (4.42)$$

The logistic selectivity has estimable parameters  $a_{50}$  and  $a_{t095}$ .  $\alpha$  is a scaling parameter, with default value of  $\alpha = 1$ . The logistic selectivity takes values  $0.5\alpha$  at  $x = a_{50}$  and  $0.95\alpha$  at  $x = a_{50} + a_{t095}$ .

**4.12.7. inverse\_logistic**

$$f(x) = \alpha - \alpha / [1 + 19^{(a_{50}-x)/a_{t095}}] \quad (4.43)$$

The inverse logistic selectivity has estimable parameters  $a_{50}$  and  $a_{t095}$ .  $\alpha$  is a scaling parameter, with default value of  $\alpha = 1$ . The logistic selectivity takes values  $0.5\alpha$  at  $x = a_{50}$  and  $0.95\alpha$  at  $x = a_{50} - a_{t095}$ .

**4.12.8. logistic\_producing**

$$f(x) = \begin{cases} 0, & \text{if } x < L \\ \lambda(L), & \text{if } x = L \\ (\lambda(x) - \lambda(x-1)) / (1 - \lambda(x-1)), & \text{if } L < x < H \\ 1, & \text{if } x \geq H \end{cases} \quad (4.44)$$

The logistic-producing selectivity has the non-estimable parameters  $L$  and  $H$ , and has estimable parameters  $a_{50}$  and  $a_{t095}$ .  $\alpha$  is a scaling parameter, with default value of  $\alpha = 1$ . For category transitions,  $f(x)$  represents the proportion moving, not the proportion that have moved. This selectivity was designed for use in an age-based model to model maturity. In such a model, a logistic-producing maturation selectivity will (in the absence of other influences) make the proportions mature follow a logistic curve with parameters  $a_{50}$ ,  $a_{t095}$ .

#### 4.12.9. double\_normal

$$f(x) = \begin{cases} \alpha 2^{-[(x-\mu)/\sigma_L]^2}, & \text{if } x \leq \mu \\ \alpha 2^{-[(x-\mu)/\sigma_R]^2}, & \text{if } x \geq \mu \end{cases} \quad (4.45)$$

The double-normal selectivity has estimable parameters  $a_1$ ,  $s_L$ , and  $s_R$ .  $\alpha$  is a scaling parameter, with default value of  $\alpha = 1$ . It has values  $\alpha$  at  $x = a_1$ , and  $0.5\alpha$  at  $x = a_1 - s_L$  and  $x = a_1 + s_R$ .

#### 4.12.10. double\_exponential

$$f(x) = \begin{cases} \alpha y_0 (y_1/y_0)^{(x-x_0)/(x_1-x_0)}, & \text{if } x \leq x_0 \\ \alpha y_0 (y_2/y_0)^{(x-x_0)/(x_2-x_0)}, & \text{if } x > x_0 \end{cases} \quad (4.46)$$

The double-exponential selectivity has non-estimable parameters  $x_1$  and  $x_2$ , and estimable parameters  $x_0$ ,  $y_0$ ,  $y_1$ , and  $y_2$ .  $\alpha$  is a scaling parameter, with default value of  $\alpha = 1$ . It can be ‘U-shaped’. Bounds for  $x_0$  must be such that  $x_1 < x_0 < x_2$ . With  $\alpha = 1$ , the selectivity passes through the points  $(x_1, y_1)$ ,  $(x_0, y_0)$ , and  $(x_2, y_2)$ . If both  $y_1$  and  $y_2$  are greater than  $y_0$  the selectivity is ‘U-shaped’ with minimum at  $(x_0, y_0)$ .



**Figure 4.3: Examples of the functional forms of selectivities available in CASAL2.**

Selectivities `all_values` and `all_values_bounded` can be addressed in additional priors using the following syntax,

```
@selectivity maturity
type all_values
v 0.001 0.1 0.2 0.3 0.4 0.3 0.2 0.1

## encourage ages 3-8 to be smooth.
@additional_prior smooth_maturity
type vector_smooth
parameter selectivity[maturity].values{3:8}
```

### 4.13. Projections

This section lists all the projections classes available, their functionality and an example of the syntax.

#### 4.13.1. constant

A parameter can either be fixed during all projection years or specified individually for each projection year. This is a deterministic assumption, where the parameter is assumed to be known without error during projection years.

```
@project Future_ycs
type constant
parameter process[Recruitment].yces_values
years 2012:2016
values 1 2 1 2 0.5
multiplier 1
```

#### 4.13.2. empirical\_resampling

Parameters that have time components associated with them can be re-sampled uniformly with replacement over a range of years and used as the projected years' values. The year range which users must specify are between `start_year` and `final_year`

```
@project Future_ycs
type empirical_sampling
parameter process[Recruitment].yces_values
years 2012:2016
start_year 1988
final_year 2008
multiplier 1
```

#### 4.13.3. lognormal

The parameters are originally drawn from a Gaussian distribution in log space and exponentiated out to form the lognormal distribution,

$$X_p = e^{\varepsilon_p - 0.5\sigma^2} \quad (4.47)$$

where  $\varepsilon_p \stackrel{iid}{\sim} N(\mu, \sigma)$  and  $X_p$  is the projected value for parameter  $X$ , and  $\mu, \sigma$  is the mean and standard deviation on the log scale. An example of applying this process is if we wanted to draw future year class parameters from a lognormal distribution with mean 1 and standard deviation 0.8, we would define the syntax as,

```
@project Future_ycs
type lognormal
parameter process[Recruitment].yces_values
years 2012:2016
mean 0
sigma 0.8
multiplier 1
```



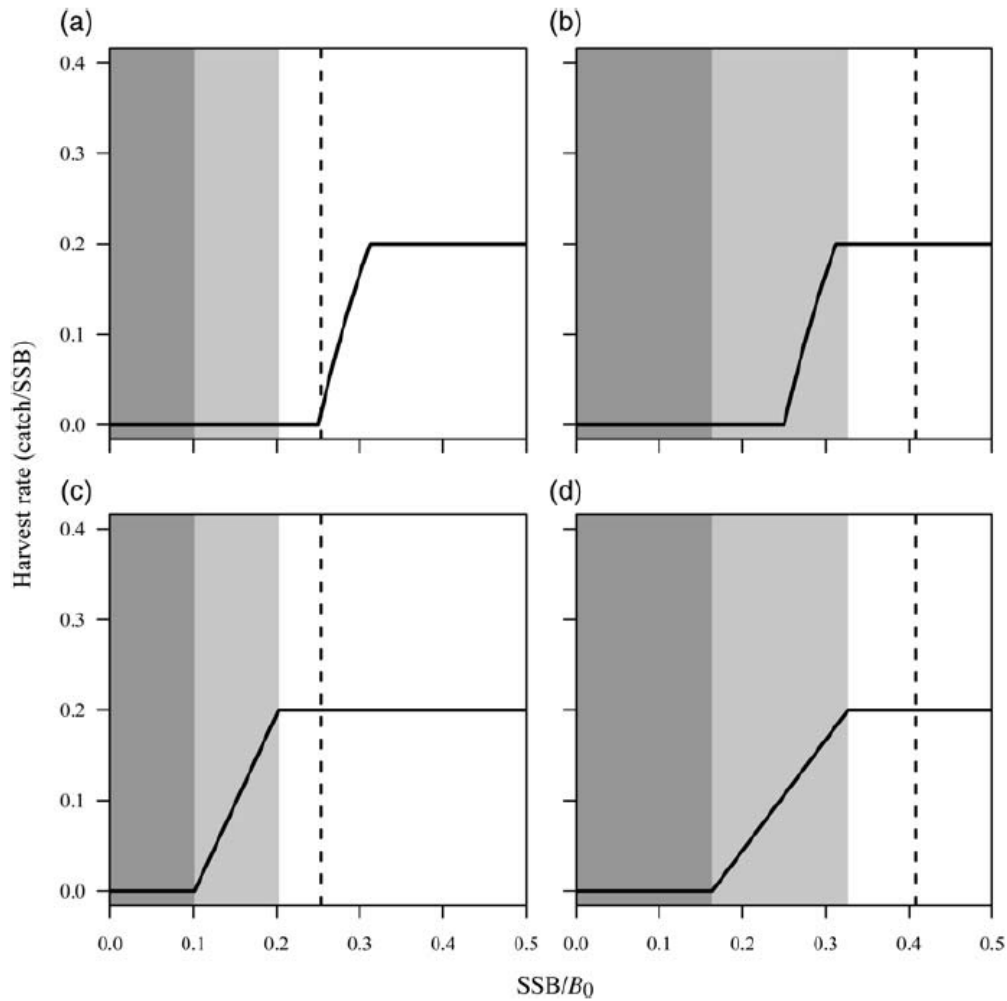
#### 4.13.4. lognormal\_empirical

This class applies a lognormal draw as in the `LogNormal` class but it allows the user to specify a year range which is re-sampled uniformly without replacement. These re-sampled values are then used to calculate the standard deviation of the distribution. Then equation (4.47) is used to generate future values with user defined  $\mu$  and empirically calculated  $\sigma$ ,

```
@project Future_ycs
type lognormal_empirical
parameter process[Recruitment].ycs_values
years 2012:2016
mean 0
start_year 1988
final_year 2008
multiplier 1
```

#### 4.13.5. user\_defined

This class allows the use of the equation parser to define the future values of a parameter during projection mode. This was originally set-up to apply harvest control rules (i.e. apply a management action such as changing the TACC based on the current or previous state). In fisheries, this would be setting the catch based on an exploitation rate multiplied by the vulnerable biomass, where the exploitation rate is based on some rule as shown in figure 4.4.



**Figure 4.4: Examples of control rules based on current stock status.**

```
@project HCR_2015
type user_defined
parameter process[Instantaneous_Mortality].method_Sub_Ant_F
years 2015
equation if(derived_quantity[SSB].values{2014} / process[Recruitment].b0 <= 0.1, 0.0,
if(derived_quantity[SSB].values{2014} / process[Recruitment].b0 > 0.1 &&
derived_quantity[SSB].values{2014} / process[Recruitment].b0 < 0.2,
derived_quantity[SSB].values{2014} * derived_quantity[SSB].values{2014}
/ process[Recruitment].b0,
derived_quantity[SSB].values{2014} * 0.2))
```

The syntax of the equation parser can be messy and care should be taken when writing it. In words the above equation is, if  $%B_{2014} \leq 0.1$  then set next years catch to 0.0, else if  $%B_{2014} > 0.1$  &  $%B_{2014} \leq 0.2$  then set next years catch equal to  $%B_{2014} \times SSB_{2014}$ , else set next years catch to  $0.2SSB_{2014}$ .

#### 4.13.6.

Projections! Catches are unique in that they are known inputs in a table format. To project

catches that are in a table use this example projection block,

```
# fishing process
@process Fishing
type mortality_instantaneous_retained
m 0.17*6 #0.17 #testing at old values
time_step_ratio 1
selectivities One*6 #for age based M
categories *
table catches
year FishingLine FishingPot Recreation
1900 0 0 0
1901 13.2 0 22.9
1902 26.4 0 23.5
1903 39.6 0 24
end_table

# projection block
@project future_catch
type constant
parameter process[Fishing].method_fishingpot
years 2020:2029
values 4000
```

This follows the syntax, `block_type[block_label].method_fishinglabel`. **note** the fishing label which is defined in the table needs to be lower case form in the `@projection` block.

#### 4.14. Time Varying Parameters

CASAL2 has the functionality to vary any parameter annually between the start and final year of a model run. This can be for blocks of years or specific years. For years that are not specified the parameter will default to the input, or if in an iterative state such as estimation mode, the value being trialled at that iteration. Method types for time varying a parameter are; `constant`, `random_walk`, `exogenous`, `linear`, `annual_shift`, `random_draw`. This allows users to let a parameter be known in a year, be the result of a deterministic equation, or stochastic. **Note** stochastic time varying was added for simulation purposes. It has not been tested in an estimation context. To implement hierarchical I have estimated the prior values with hyper-priors. If you were to try an implement a hierarchical model using the time varying functionality, firstly you could only do this at MCMC estimation because an MPD we would not be doing that integral which is required to obtain unbiased estimates. In an MCMC context you would be assuming a Gibbs sampler. That is every draw is from a conditional distribution and so every draw is a candidate value.

When allowing removals to have annual varying catchabilities, selectivities and other more realistic model components, simulated observations more closely model real data and associated conclusions become more useful. Another driver for implementing time-varying parameters was allowing mean or location parameters of selectivities to change between years based on an explanatory variable. An example of this is in the New Zealand Hoki fishery where we allow the  $\mu$  and  $a_{50}$  parameters to shift depending on when the fishing season occurs. Descriptive analysis showed that when fishing was earlier relative to other years smaller fish were caught and vice versa. This can be shown in the CASAL2/Examples/2stock directory, implemented at line: 382 in the `population.csl2` file.

#### 4.14.1. constant

Allows a parameter to have an alternative value during certain years, which can be estimated.

```
@time_varying q_time_var
type constant
parameter catchability[survey_q].q
years 1975:1988
values 0.001
```

Users can also estimate these year values using the following syntax, caution is that you shouldn't estimate the actual parameter and its time varying counterpart, as the time varying value will overwrite the actual parameter making the first value unidentifiable and an impossible optimization process.

```
@estimate q_time_var
type uniform
parameter time_varying[q_time_var].values(1975:1976)
lower_bound 1e-6 1e-6
upper_bound 2 2
```

**Caution required:** the actual parameter and its time varying counterpart should not both be estimated, as the time varying value will overwrite the actual parameter making the first value unidentifiable and an impossible optimization process.

#### 4.14.2. random\_walk

A random deviate is added into the last value drawn from a standard normal distribution. This has an estimable parameter  $\sigma_p$  for each time varying parameter  $p$ . For reproducible modelling, it is highly recommended that users set the seed (see Section 3.4) when using stochastic functionality like this, otherwise reproducing models becomes almost impossible.

```
@time_varying q_time_var
type random_walk
parameter catchability[survey_q].q
distribution normal
mean 0
sigma 3
```

If the parameter specified in the @time\_varying block is associated with an @estimate block then the parameter is constrained to stay within the lower and upper bounds of the @estimate block. **WARNING**, if the parameter does not have an associated @estimate block then there is no safe guard for a random deviate to put the parameter in a space where the model fails, i.e generates NA or INF values. To avoid this, we recommended an @estimate block is specified even though the parameter is not actually being estimated, see example syntax, below. A constraint whilst using this functionality is that a parameter cannot be less than 0.0, if it is CASAL2 sets it equal to 0.01.

```
@estimate survey_q_est
type uniform
parameter catchability[survey_q].q
lower_bound 1e-6
upper_bound 10
```

This will insure the random walk time varying process will set the any new candidate within the lower and upper bound of the @estimate block.

#### 4.14.3. annual\_shift

A parameter generated in year  $y$  ( $\theta'_y$ ) depends on the value specified by the user ( $\theta_y$ ) along with three coefficients  $a, b$  and  $c$  as follows,

$$\bar{\theta}_y = \frac{\sum_y^Y \theta_y}{Y} \quad (4.48)$$

$$\theta'_y = a\bar{\theta}_y + b\bar{\theta}_y^2 + c\bar{\theta}_y^3 \quad (4.49)$$

#### 4.14.4. exogenous

Parameters are shifted based on an exogenous variable, an example of this is an exploitation selectivity parameters that may vary between years based on known changes in exploitation behaviour such as season, start time, and average depth of exploitation.

$$\delta_y = a(E_y - \bar{E}) \quad (4.50)$$

$$\theta'_y = \theta_y + \delta_y \quad (4.51)$$

where  $\delta_y$  is the shift or deviation in parameter  $\theta_y$  in year  $y$  to generate the new parameter value in year  $y$  ( $\theta'_y$ ).  $a$  is an estimable shift parameter,  $E$  is the exogenous variable and  $E_y$  is the value of this variable in year  $y$ . For more information readers can see Francis et al. (2003).

### 4.15. Equation Parser

CASAL2 has the ability to use an equation parser, this is currently implemented in Projections (section 4.13), Derived quantities (section 4.6) and Reports (section 7). Examples of syntax for implementing the equation parser follow from here. For a more detailed look at the parser see <https://github.com/nickgammon/parser/blob/master/parser.cpp>

```
equation process[Recruitment].r0 * (2-1)
```

and can apply routine mathematical functions such as log, exp, cos, sin, tan, e.g.,

```
equation sqrt(process[Recruitment].r0)
```

apply exponents,

```
equation pow(2, 3)
```

evaluate the absolute of an equation using the abs() syntax,

```
equation abs(sqrt(process[Recruitment].r0) * 1.33)
```

use if else statements

```
equation if(process[Recruitment].r0 > 23, 44, 55)
## if R0 is greater than 23 return 44 else return 55
```

if else statements can also be linked, but the syntax becomes a little messy, e.g.,

```
equation if(process[Recruitment].r0 > 23, 44,
            if(process[Recruitment].r0 > 10, 55, 66))
## if R0 is greater than 23 return 44 else if R0 less than 23 but greater than 10 return 55,
else R0 must be less than 10 return 66
```

Only singletons can be reference, so an equation cannot be applied to vector parameters, e.g. `process[Recruit].yrs_values{1974:1980}` can't be referenced. To see which parameters can be included in an equation parser go to the syntax section (Section 12). Any subcommand that has a type `estimable` could, in theory, be addressed in an equation parser.

With the equation parser it is difficult to catch all user configuration errors, we cannot check whether a parameter that exists in the system has been populated when the user requires it. For example, the wrong year could easily be misspecified in the case of next years (2015) removals to be based on the this years (2014) state of the population,

```
parameter process[removals].catch
year 2015
equation derived_quantity[percent_b0].values{2020}
```

The above would be an acceptable equation but obviously will cause nonsensical results, because you are asking for a value in 2020 when you are in 2015. This is just a caution, for although the equation parser adds a great deal of flexibility, users should be careful because it is easy to misspecify models in this manner.

---

## 5. The estimation section

### 5.1. Role of the estimation section

The role of the estimation section is to define the tasks carried out by CASAL2:

1. Define the objective function (see Section 5.2)
2. Define the parameters to be estimated (see Section 5.3)
3. Calculate a point estimate, i.e., the maximum posterior density estimate (MPD) (see Section 5.4)
4. Calculate a posterior profile selected parameters, i.e., find, for each of a series of values of a parameter, allowing the other estimated parameters to vary, the minimum value of the objective function (see Section 5.5)
5. Generate an MCMC sample from the posterior distribution (see Section 5.6)
6. Calculate the approximate covariance matrix of the parameters as the inverse of the minimizer's approximation to the Hessian, and the corresponding correlation matrix (see Section 5.4)

The estimation section defines the objective function, parameters of the model, and the method of estimation (point estimates, Bayesian posteriors, profiles, etc.). The objective function is based on a goodness-of-fit measure of the model to observations, priors and penalties. See the observation section for a description of the observations, likelihoods, priors and penalties.

### 5.2. The objective function

In Bayesian estimation, the objective function is a negative log-posterior,

$$Objective(p) = -\sum_i \log [L(\mathbf{p}|O_i)] - \log [\pi(\mathbf{p})] \quad (5.1)$$

where  $\pi$  is the joint prior density of the parameters  $p$ .

The contribution to the objective function from the likelihoods are defined in Section 6.2. In addition to likelihoods, priors (see Section 5.7) and penalties (see Section 5.8) are components of the objective function. Note that if the priors are specified as uniform, then the prior contribution is zero and the estimation problem turns into penalised-likelihood and not Bayesian.

Penalties can be used to ensure that the exploitation rate constraints on mortality events (i.e., fisheries) are not breached (otherwise there is nothing to prevent the model from having abundances so low that the recorded mortalities could not have been taken), penalties on category transitions (to ensure there are enough individuals to move), and possibly penalties to encourage estimated values to be similar or smooth, etc. Equation 5.1 can mathematically reduce to a penalised likelihood equation if all priors are assumed to be uniform. This is because uniform priors have a zero contribution to the objective function so Equation 5.1 reduces to likelihoods plus penalties.

### 5.3. Specifying the parameters to be estimated

The parameters that will be estimated (estimables) are defined using `@estimate` commands (see Section 9). An `@estimate` command-block looks like,

```
@estimate male.m
parameter process[NaturalMortality].m{male}
lower_bound 0.1
upper_bound 0.4
type uniform
```

See Section 3.5.5 for instructions on how to generate the parameter name. At least one parameter is to be estimated if doing an estimation `-e`, profile `-p`, or MCMC `-m` run. Initial values for the parameters to be estimated will still need to be provided, and these are used as the starting values for the minimiser. However, these may be overwritten if you provide a set of alternative starting values (i.e., using `casal2 -i`, see Section 3.4).

All parameters are estimated within bounds. For each parameter to be estimated, you need to specify the bounds and the prior (`type`) (Section 5.7). Note that the bounds and prior for each parameter refer to the values of the parameters, not the actual values resulting from the application of the parameter to an equation. Bounds should be carefully chosen as they effect the space in which the minimisers search over. Some minimisers convert lower and upper bound into a minimisation space (for example `-1,1` space for the numerical differences algorithm). If estimating only some elements of a vector, either define each element of the vector to be estimated (see 3.5.5) or fix the others by setting the bounds equal.

## 5.4. Point estimation

Point estimation is invoked with `casal2 -e`. Mathematically, it is an attempt to find a minimum of the objective function. CASAL2 has multiple algorithms for solving (minimising) the optimisation problem. There are three non auto differential minimisers: numerical differences, differential evolution minimiser (`de_solver`), and the `dlib` minimiser. There are also three auto differential (AD) minimisers: ADOL-C, CPPAD, and BETADIFF. For references see Section 1.8. AD minimisers are recommended for complex models as they are on average much faster and tend to find a better minimum when exploring a complex objective surface.

Recently, with the number of parameters growing in these models, an important input parameter on most minimisers is the `tolerance` parameter. This is the stopping rule that minimisers use to define when they have found a 'solution' (remember there is no guarantee that a solution is the global solution, such is the world we live in). Try alternative starting values, this is easily done with the `-i parameter_file.txt` in CASAL2. We recommend, when estimating any model with CASAL2 that you try smaller values for the `tolerance` parameter. We have found for some models that if you make it say 0.00000002 that the solution is quite a bit different than when using the default (0.002). This is not ideal model behaviour and that more investigative work may be required to determine what parameters are causing the behaviour. An aside note: this will also effect your covariance matrix — with different tolerances and searches you may end up with a different approximate Hessian matrix which is inverted to solve for the covariance matrix. We tested the difference this might make on MCMC results (because the covariance is incorporated into the proposal distribution). However, MCMC runs with varying tolerances and have not been found, so far, to effect the posterior distribution of the MCMC.

### 5.4.1. The numerical differences minimiser

The minimiser has three kinds of (non-error) exit status, depending on the minimiser:

1. Successful convergence (suggests you have found a local minimum, at least).



2. Convergence failure (you have not reached a local minimum, though you may deem yourself to be ‘close enough’ at your own risk).
3. Convergence unclear (the minimiser halted but was unable to determine if convergence occurred. You may be at a local minimum, although you should check by restarting the minimiser at the final values of the estimated parameters).

You can choose the maximum number of quasi-Newton iterations and objective function evaluations allotted to the minimiser. If it exceeds either limit, it exits with a convergence failure. We recommend large numbers of evaluations and iterations (at least the defaults of 300 and 1000) unless you successfully reach convergence with less. You can also specify an alternative starting point of the minimiser using `casal2 -i`.

We want to stress that the minimisers are local optimisation algorithms trying to solve a global optimisation problem. What this means is that, even if you get a ‘successful convergence’ message, your solution may be only a local minimum, not a global one. To diagnose this problem, try doing multiple runs from different starting points and comparing the results, or doing profiles of one or more key parameters and seeing if any of the profiled estimates finds a better optimum than the original point estimate.

The approximate covariance matrix of the estimated parameters can be calculated as the inverse of the minimiser’s approximation to the Hessian, and the corresponding correlation matrix is also calculated. Be aware that

- the Hessian approximation develops over many minimiser steps, so if the minimiser has only run for a small number of iterations the covariance matrix can be a very poor approximation; and
- the inverse Hessian is not a good approximation to the covariance matrix of the estimated parameters, and may not be useful to construct, for example, confidence intervals.

Also note that if an estimated parameter has equal lower and upper bounds, it will have entries of ‘0’ in the covariance matrix and NaN or -1.#IND (depending on the operating system) in the correlation matrix.

```
@minimiser numerical_diff
type numerical_differences
tolerance 1e-6
iterations 2500
evaluations 4000
```

### 5.4.2. The differential evolution minimiser

The differential evolution minimiser is a simple population based, stochastic function minimizer, but is claimed to be quite powerful in solving minimisation problems. It is a method of mathematical optimization of multi-dimensional functions and belongs to the class of evolution strategy optimizers. Initially, the procedure randomly generates and evaluates a number of solution vectors (the population size), each with  $p$  parameters. Then, for each generation (iteration), the algorithm creates a candidate solution for each existing solution by random mutation and uniform crossover. The random mutation generates a new solution by multiplying the difference between two randomly selected solution vectors by some scale factor, then adding the result to a third vector. Then an element-wise crossover takes place with probability  $P_{cr}$ , to generate a potential candidate solution. If this is better than the initial solution vector, it replaces it, otherwise the original

solution is retained. The algorithm is terminated after either a predefined number of generations (`max_generations`) or when the maximum difference between the scaled individual parameters from the candidate solutions from all populations is less than some predefined amount `tolerance`.

The differential evolution minimiser can be good at finding global minimums in surfaces that may have local minima. However, the speed of the minimiser, and the ability to find a good minima depend on the number of initial ‘populations’. Some authors recommend that the number of populations be set at about  $10 * p$ , where  $p$  is the number of free parameters. However, depending on your problem, you may find that you may need more, or that less will suffice.

We note that there is no proof of convergence for the differential evolution solver, but several papers have found it to be an efficient method of solving multidimensional problems. Our (limited) experience suggests that it can often find a better minima and may be faster or longer (depending on the actual model specification) at finding a solution when compared with the numerical differences minimiser. Comparisons with auto-differentiation minimisers or other more sophisticated algorithms have not been made.

```
@minimiser DE_solver
type de_solver
tolerance 1e-6
iterations 2500
evaluations 4000
```

#### 5.4.3. Betadiff minimiser

An auto-differentiable minimiser for non-linear models, This is the minimiser from the original CASAL package, based on ADOL-C.

```
@minimiser beta_diff
type beta_diff
tolerance 1e-6
iterations 2500
evaluations 4000
```

#### 5.4.4. ADOL-C minimiser

An auto-differentiable minimiser for non-linear models.

```
@minimiser ADOLC
type adolc
step_size 1e-6
iterations 2500
evaluations 4000
tolerance 1e-6
```

#### 5.4.5. CPPAD minimiser

An auto-differentiable minimiser for non-linear models using the mumps solver, see [https://www.coin-or.org/CppAD/Doc/ipopt\\_solve.htm](https://www.coin-or.org/CppAD/Doc/ipopt_solve.htm) for more information about this solver.

```
@minimiser CPPAD
type cppad
```

We have found this solver to be by far the quickest solver for models that have a reasonably well defined solution, i.e., there is 'good' information in the data to identify all the parameters. Now you may be thinking...shouldn't this be the case for all minimisers? Short answer is yes, but the other minimisers are quicker than `cppad` to tell you there is not a reasonable solution.

#### 5.4.6. Dlib minimiser

Non auto-diff minimiser

```
@minimiser Dlib
type dlib
tolerance 1e-6
iterations 2500
evaluations 4000
```

### 5.5. Posterior profiles

If profiles are requested `casal2 -p`, CASAL2 will first calculate a point estimate. For each scalar parameter or, in the case of vectors or selectivities, the element of the parameter to be profiled, CASAL2 will fix its value at a sequence of  $n$  evenly spaced numbers (*step*) between a specified lower and upper bounds  $l$  and  $u$ , and calculate a point estimate at each value.

By default  $step = 10$ , and  $(l, u) = (\text{lower bound on parameter plus } (range/(2n)), \text{upper bound on parameter less } (range/(2n)))$ . Each minimisation starts at the final parameter values from the previous resulting value of the parameter being profiled. CASAL2 will report the objective function for each parameter value. Note that an initial point estimate should be compared with the profile, not least to check that none of the other points along the profile have a better objective function value than the initial 'minimum'.

You specify which parameters are to be profiled, and optionally the number of steps, lower bound, and upper bound for each. In the case of vector parameters, you will also need to specify the element of the vector being profiled.

You can also supply the initial starting point for the estimation using `casal2 -i file` — this may improve the minimiser performance for the profiles.

If you get an implausible profile, it may be a result of not using enough iterations in the minimiser or a poor choice of minimiser control variables (e.g., the minimiser tolerance). It also may be useful to try both if the minimisers in CASAL2 and compare the results.

### 5.6. Bayesian estimation

CASAL2 can use a Monte Carlo Markov Chain (MCMC) to generate a sample from the posterior distribution of the estimated parameters `casal2 -m` and output the sampled values to a file (optionally keeping only every  $n$ th set of values).

As CASAL2 has no post-processing capabilities. CASAL2 cannot produce MCMC convergence diagnostics (use a package such as BOA) or plot/summarize the posterior distributions of the output quantities (for example, use a general-purpose statistical or spreadsheet package such as S-Plus, **R**, or Microsoft Excel).

Bayesian methodology and MCMC are both large and complex topic. See Gelman et al. (1995) and Gilks et al. (1994) for details of both Bayesian analysis and MCMC methods. In addition, see Punt

& Hilborn (2001) for an introduction to quantitative fish stock assessment using Bayesian methods. This section briefly describes the MCMC algorithms used in CASAL2. See Section 9.3 for a description of the sequence of CASAL2 commands used in a full Bayesian analysis.

CASAL2 uses a straightforward implementation of the Metropolis-Hastings algorithm (Gelman et al., 1995, Gilks et al., 1994). The Metropolis-Hastings algorithm attempts to draw a sample from a Bayesian posterior distribution, and calculates the posterior density  $\pi$ , scaled by an unknown constant. The algorithm generates a ‘chain’ or sequence of values. Typically the beginning of the chain is discarded and every  $N$ th element of the remainder is taken as the posterior sample. The chain is produced by taking an initial point  $x_0$  and repeatedly applying the following rule, where  $x_i$  is the current point:

- Draw a candidate step  $s$  from a proposal distribution  $J$ , which should be symmetric i.e.,  $J(-s) = J(s)$
- Calculate  $r = \min(\pi(x_i + s)/\pi(x_i), 1)$
- Let  $x_{i+1} = x_i + s$  with probability  $r$ , or  $x_i$  with probability  $1 - r$

An initial point estimate is produced before the chain starts, which is done so as to calculate the approximate covariance matrix of the estimated parameters (as the inverse Hessian), and may also be used as the starting point of the chain.

The user can specify the starting point of the point estimate minimiser using `casal2 -i`. Don’t start it too close to the actual estimate (either by using `casal2 -i`, or by changing the initial parameter values in input configuration file) as it takes a few iterations to form a reasonable approximation to the Hessian.

There is currently two options for the starting point of the Markov Chain:

- Start from the point estimate; and
- Restart a chain given a covariance matrix and sand a previous starting point.

The chain moves in natural space, i.e., no transformations are applied to the estimated parameters. The default proposal distribution is a multivariate  $t$  centred on the current point, with covariance matrix equal to a matrix based on the approximate covariance produced by the minimiser, times some stepsize factor. The following steps define the initial covariance matrix of the proposal distribution:

- The covariance matrix is taken as the inverse of the approximate Hessian from the quasi-Newton minimiser.
- The covariance matrix is modified so as to decrease all correlations greater than `@mcmc.max_correlation` down to `@mcmc.max_correlation`, and similarly to increase all correlations less than `-@mcmc.max_correlation` up to `-@mcmc.max_correlation` (the `@mcmc.max_correlation` parameter defaults to 0.8). This should help to avoid getting ‘stuck’ in a lower-dimensional subspace.
- The covariance matrix is then modified either by,
  - `@mcmc.adjustment_method=covariance`: that if the variance of the  $i$ th parameter is non-zero and less than `@mcmc.min_difference` times the difference between the parameters’ lower and upper bound, then the variance is changed, without changing the associated correlations, to  $k = \min\_diff(upper\_bound_i - lower\_bound_i)$ . This is done

by setting

$$\text{Cov}(i, j)' = \text{sqrt}(k) \text{Cov}(i, j) / \text{sd}(i)$$

for  $i \neq j$ , and  $\text{var}(i)' = k$

- `@mcmc.adjustment_method=correlation`: that if the variance of the  $i$ th parameter is non-zero and less than `@mcmc.min_difference` times the difference between the parameters' lower and upper bound, then its variance is changed to  $k = \text{min\_diff}(\text{upper\_bound}_i - \text{lower\_bound}_i)$ . This differs from (i) above in that the effect of this option is that it also modifies the resulting correlations between the  $i$ th parameter and all other parameters.

This allows each estimated parameter to move in the MCMC even if its variance is very small according to the inverse Hessian. In both cases, the `@mcmc.min_difference` parameter defaults to 0.0001.

- The `@mcmc.stepsize` (a scalar factor applied to the covariance matrix to improve the acceptance probability) is chosen by the user. The default is  $2.4d^{-0.5}$  where  $d$  is the number of estimated parameters, as recommended by Gelman et al. (Gelman et al., 1995). However, you may find that a smaller value may often be better.

The proposal distribution can also change adaptively during the chain, using two different mechanisms. Both are offered as means of improving the convergence properties of the chain. It is important to note that any adaptive behaviour must finish before the end of the burn-in period, i.e., the proposal distribution must be finalised before the kept portion of the chain starts. The adaptive mechanisms are as follows:

1. You can request that the stepsize change adaptively at one or more sample numbers (See next paragraph for details on the stepsize adaptation methods)
2. You can request that the entire covariance matrix change adaptively at one or more sample numbers. At each adaptation, the covariance matrix is replaced with an empirical covariance, derived from the MCMC chain. The idea here is that an empirical covariance is a better approximation to the proposal distribution than the inverse of the hessian matrix, and can improve convergence and mixing of your chain.

The two methods that you can choose to adapt the step size are `double_half` or `ratio`, this is done through the input parameter `adapt_stepsize_method`. The `double_half` method is used in CASAL and (See Gelman et al. (Gelman et al., 1995) for justification). The algorithm for `double_half` is, at each adaptation, the stepsize is doubled if the acceptance rate since the last adaptation is more than 0.5, or halved if the acceptance rate is less than 0.2. The `ratio` is taken from SPM. It adapts the current step size by, the acceptance rate since the last adaptation multiplied by 4.1667 to reach an acceptance rate of  $\approx 0.24$  see Sherlock and Roberts (2009) for justification on that acceptance rate.

The stepsize parameter is now on a completely different scale, and must be reset. It is set to a user-specified value (which may or may not be the same as the initial stepsize). We recommend that some of the stepsize adaptations are set to occur after this, so that the stepsize can be readjusted to an appropriate value which gives good acceptance probabilities with the new matrix.

All modified versions of the covariance matrix are printed to the standard output, but only the initial covariance matrix (inverse Hessian) is saved to the objectives file. The number of covariance modifications by each iteration is recorded as a column on the objectives file.

The variance-covariance matrix of this sub-sample of chain is calculated. As above, correlations greater than `@mcmc.max_correlation` are reduced to `@mcmc.max_correlation`, correlations less than `@mcmc.max_correlation` are increased to `@mcmc.max_correlation`, and very small non-zero variances are increased (`@mcmc.covariance_adjustment` and `@mcmc.min_difference`). The result is the new variance-covariance matrix of the proposal distribution.

The procedure used to choose the sample of points is as follows. First, all points on the chain so far are taken. All points in an initial user-specified period are discarded. The assumption is that the chain will have started moving during this period - if this is incorrect and the chain has still not moved by the end of this period, it is a fatal error and CASAL2 stops. The remaining set of points must contain at least some user-specified number of transitions - if this is incorrect and the chain has not moved this often, it is again a fatal error. If this test is passed, the set of points is systematically sub-sampled down to 1000 points (it must be at least this long to start with).

The probability of acceptance for each jump is 0 if it would move out of the bounds, or 1 if it improves the posterior, or (new posterior/old posterior) otherwise. You can specify how often the position of the chain is recorded using the `keep` parameter. For example, with `keep 10`, only every 10th sample is recorded.

You have the option to specify that some of the estimated parameters are fixed during the MCMC. If the chain starts at the point estimate or at a random location, these fixed parameters are set to their values at the point estimate.

If you specify the start of the chain using `casal2 -i`, these fixed parameters are set to the values in the file.

Restarting an MCMC chain, in the case where computers get turned off and the MCMC execution was halted. This allows the ability to restart it from where it finished,

```
casal2 -m --resume --objective-file Objective_file_name --sample-file Sample_file_name
```

where `Objective_file_name` is the file name containing the objective report and `Sample_file_name` is the file name containing the sample report from a MCMC chain.

The posterior sample can be used for (projections (Section 4.4.5)) or simulations (Section 6.6) with the values supplied using `casal2 -i file`.

A multivariate t distribution is used as an alternative to the multivariate normal proposal distribution. If you request multivariate t proposals, you may want to change the degrees of freedom from the default of 4. As the degrees of freedom decrease, the t distribution becomes more heavy tailed. This may lead to better convergence properties. Note the default is the multivariate t.

Given a posterior (sub)sample, CASAL2 can calculate a list of output quantities for each sample point (see Section 7 specifically tabular report). These quantities can be dumped into a file (using `casal2 -r -tabular`) and read into an external software package where the posterior distributions can be plotted and/or summarised.

The posterior sample can also be used for projections (Section 4.4.5). The advantage of this is that the parameter uncertainty, as expressed in your posterior distribution, can be included into the risk estimates.

## 5.7. Priors

In a Bayesian analysis, you need to give a prior for every parameter that is being estimated. There are no default priors.

Note that when some of these priors are parameterised in terms of mean, c.v., and standard deviation, these refer to the parameters of the distribution before bounds are applied. The moments of the prior after the bounds are applied may differ.

CASAL2 has the following priors (expressed in terms of their contribution to the objective function):

1. Uniform

$$-\log(\pi(p)) = 0 \quad (5.2)$$

2. Uniform-log (i.e.,  $\log(p) \sim \text{uniform}$ )

$$-\log(\pi(p)) = \log(p) \quad (5.3)$$

3. Normal with mean  $\mu$  and c.v.  $c$

$$-\log(\pi(p)) = 0.5 \left( \frac{p - \mu}{c\mu} \right)^2 \quad (5.4)$$

4. Normal with mean  $\mu$  and standard deviation  $\sigma$

$$-\log(\pi(p)) = 0.5 \left( \frac{p - \mu}{\sigma} \right)^2 \quad (5.5)$$

5. Lognormal with mean  $\mu$  and c.v.  $c$

$$-\log(\pi(p)) = \log(p) + 0.5 \left( \frac{\log(p/\mu)}{s} + \frac{s}{2} \right)^2 \quad (5.6)$$

where  $s$  is the standard deviation of  $\log(p)$  and  $s = \sqrt{\log(1 + c^2)}$ .

6. Beta with mean  $\mu$  and standard deviation  $\sigma$ , and range parameters  $A$  and  $B$

$$-\log(\pi(p)) = (1 - m) \log(p - A) + (1 - n) \log(B - p) \quad (5.7)$$

where  $v = \frac{\mu - A}{B - A}$ , and  $\tau = \frac{(\mu - A)(B - \mu)}{\sigma^2} - 1$  and then  $\mu = \tau v$  and  $n = \tau(1 - v)$ . Note that the beta prior is undefined when  $\tau \leq 0$ .

Vectors of parameters can be independently (but not necessarily identically) distributed according to any of the above forms, in which case the joint negative-log-prior for the vector is the sum of the negative-log-priors of the components. Values of each parameter need to be specified for each element of the vector. Example of syntax to define the estimation of a parameter and the prior assumed follows;

```
## uniform-log example estimate
@estimate B0
type uniform_log # this command "type" defines the prior type.
parameter process[Recruitment].b0 # "Recruitment" is the label of your process
upper_bound 20000
lower_bound 1000

## Lognormal YCS estimation
@estimate year_class_strengths_1990_1995
type lognormal
parameter process[Recruitment].yces_values{1990:1995}
#yces_year 1990 1991 1992 1993 1994 1995
mu 1 1 1 1 1 1
cv 0.9 0.9 0.9 0.9 0.9 0.9
lower_bound 0.01 0.01 0.01 0.01 0.01 0.01
upper_bound 9 9 9 9 9 9
```

### 5.8. Penalties

Penalties are associated with processes and can be used to discourage parameter values or model outputs that are nonsensical, by adding a penalty to the objective function. For example, parameter estimates that do not allow a known mortality event to remove enough individuals from the population can be discouraged within an event mortality process. CASAL2 requires penalty functions for processes that remove or shift a *number* of individuals between categories or from the partition. For CASAL users many of the penalties that were available in CASAL have been moved to be additional priors, see Section 5.9.

For most penalties, you need to specify a multiplier, and the objective function is increased by this multiplier times the penalty value as described below. In some cases you will need to make the multiplier quite large to prohibit some model behaviour.

Currently, the penalties for the processes `@process[label].type=event_mortality`, `@process[label].type=mortality_instantaneous`, `@process[label].type=tag_by_length`, `@process[label].type=tag_by_length` and `@process[label].type=category_transition` are the only penalties implemented.

For these processes, two types of penalty can be defined, natural scale (the default) and log scale. Both of these types add a penalty value of the squared difference between the observed value (i.e., the actual number of individuals to be removed in an event mortality process or the actual number of individuals to shift in a category transition process), and the number that were moved (if less than or equal), times the penalty multiplier.

The natural scale penalty just uses at the squared difference on a natural scale, while the log scale penalty uses the squared difference of the logged values. An example of applying a penalty,

```
@process Mortality
type mortality_instantaneous
penalty CatchMustBeTaken

# define the penalty in an @penalty block
@penalty CatchMustBeTaken
type process
log_scale True
multiplier 10000
```



Penalties are added to the objective function in the following ways;

$$Penalty = (X_1 - X_2)^2 \quad (5.8)$$

or if `log_scale` true

$$Penalty = (\log(X_1) - \log(X_2))^2 \quad (5.9)$$

Where  $X_1$  could be a known catch and  $X_2$  is the model catch under a given set of parameter values. These are usually applied in situations where you have known numbers or weight. Another obvious example is tagging, we know for a fact that we tagged  $N$  fish this year so don't allow your model to apply less than that because that is nonsensical.

## 5.9. Additional Priors

Additional priors can be thought of as the inverse of penalties and for CASAL users most of the legacy `@penalty` blocks have now been migrated as `@additional_prior` blocks. They constrain or encourage parameters in user defined spaces. The types of additional priors available in CASAL2 are `vector_smoothing`, `vector_averaging`, `uniform_log`, `lognormal`, `element_difference` and `Beta`, which are defined as,

### 1. `vector_averaging`

Applied to a vector parameter. Sum of squares of  $r$ th differences, optionally on a log scale. This encourages the vector to be like a polynomial of degree  $(r-1)$ . Note, a range of the vector to be smoothed can be specified (and if not, the smoother is applied to the entire vector), but this must be specified by an index of the vector and must be between 1 and the length of the vector, inclusive.

### 2. `vector_smoothing`

Applied to a vector parameter. Square of  $(\text{mean}(\text{vector})-k)$ , or of  $(\text{mean}(\log(\text{vector}))-l)$ , or of  $(\log(\text{mean}(\text{vector})/m))$ . Encourages the vector to average arithmetically to  $k$  or  $m$ , or geometrically to  $\exp(l)$ . Typically used for YCS with  $k=1$  or  $m=1$  or  $l=0$ , to encourage the YCS to centre on 1. Optionally, you can choose to exclude indices outside a given set of bounds.

### 3. `lognormal` with mean $\mu$ and c.v. $c$

$$-\log(\pi(p)) = \log(p) + 0.5 \left( \frac{\log(p/\mu)}{s} + \frac{s}{2} \right)^2 \quad (5.10)$$

### 4. `uniform_log`

$$-\log(\pi(p)) = \log(p) \quad (5.11)$$

## 5. element\_difference

$$-\log(\pi(p_1, p_2)) = \sum_{i=1}^n (p_{1,i} - p_{2,i})^2 \quad (5.12)$$

6. Beta Beta with mean  $\mu$  and standard deviation  $\sigma$ , and range parameters  $A$  and  $B$ , for parameter value =  $p$

$$-\log(\pi(p)) = (1 - m) \log(p - A) + (1 - n) \log(B - p) \quad (5.13)$$

where  $v = \frac{\mu - A}{B - A}$ , and  $\tau = \frac{(\mu - A)(B - \mu)}{\sigma^2} - 1$  and then  $m = \tau v$  and  $n = \tau(1 - v)$ . Note that the beta prior is undefined when  $\tau \leq 0$ .

Methods available for the type `vector_average` are l, k, m. For a target vector parameter  $\mathbf{X}$  and desired average  $k$ , the contribution to the objective score is.

- method k

$$-\log(\pi(p)) = (\bar{X} - k)^2$$

- method l

$$-\log(\pi(p)) = (\overline{\ln(X)} - k)^2$$

- method m

$$-\log(\pi(p)) = (\ln(\bar{X}) - k)^2$$

where  $\overline{\ln(X)}$  is the mean of the logged values.

There are a range of parameters and derived values that users can apply additional priors to. Here are a list of non-estimated (all parameters that can be estimated can have an additional prior attached to them) parameters that you can apply additional priors on. This should be a useful guide for users who are trying to apply the equivalent old CASAL penalties to their updated CASAL2 models.

- `selectivity[Selectivity_label].values{3:8}`. This applies a selectivity to the actual selectivity value by age (in this case for ages 3-8). This is only available for certain types of selectivities (`all_values`, `all_values_bounded`, `double_exponential`). See the Hoki stock assessment for an example of applying additional priors on selectivities.
- `catchability[Catchability_label].q` this is only for catchabilities that are of type nuisance. Because nuisance  $q$ 's are not free parameters to replicate legacy CASAL models with `@estimate` blocks in nuisance  $q$ 's we now apply additional priors. Note, if legacy models applied uniform priors this has a null effect and you can ignore functionality when converting to CASAL2 models.

## 5.10. Estimate Transformations

CASAL2 has multiple methods to transform a parameter, with some methods developed for legacy purposes and others are more recent ideas. All transformations are implemented for the same reason

— to try and achieve 'better' model optimisation. It is no surprise that complex population models can have highly correlated parameters so transforming them to be orthogonal or to be in a different space is a way of trying to remove correlations, and to allow the minimiser to find a 'global' minimum quicker. To read more about transformations and get a better understanding of why they are used we refer you to Gilks et al. (1995), specifically chapter 6.

There are two main methods available in CASAL2, `transform_with_jacobian` and `prior_applies_to_transform`. When using Transform-with-Jacobian the user defines priors on parameters in natural/model space (business as usual priors) but when we pass the parameter to the minimiser it gets transformed and a Jacobian is added to the objective function to account for the transformation. The second method is when users can specify bounds and prior parameters on the parameters in transformed space. Note that you cannot specify both `prior_applies_to_transform` and `transform_with_jacobian` true, CASAL2 should gracefully tell you this.

There are two ways users can apply estimate transformations. The first is within the `@estimate` block, this is for univariate (simple) transformations only. For complicated transformations you will have to specify a `@estimate_transformation` block to describe the transformation. Examples of these two implementations,

```
## simple transformation
@estimate log_R0
type lognormal
transformation log
parameter process[Recruitment].r0
transform_with_jacobian true
mu 442413
cv 0.2
lower_bound 3000
upper_bound 24154953

## Complicated
@estimate R0
type lognormal
parameter process[Recruitment].r0
mu 442413
cv 0.2
lower_bound 3000
upper_bound 24154953

@estiamte_transformation Log_R0
type log
estimate_label log_R0
transform_with_jacobian true
```

### Transform with Jacobian

The support of a random variable  $X$  with density  $p_X(x)$  is that subset of values for which it has non-zero density,

$$\text{supp}(X) = \{x | p_X(x) > 0\} \quad (5.14)$$

If  $f$  is a transformation function defined on the support of  $X$ , then  $Y = f(X)$  is a new random

variable (transformed variable). This section shows the available transformations in CASAL2 and the probability density function of  $Y$ .

Suppose  $X$  is one dimensional and  $f: \text{supp}(X) \rightarrow \mathbf{R}$  is a one-to-one, monotonic function with a differentiable inverse  $f^{-1}$ . Then the density of  $Y$  is given by

$$p_Y(y) = p_X(f^{-1}(y)) \left| \frac{\partial}{\partial y} f^{-1}(y) \right| \quad (5.15)$$

where  $\left| \frac{\partial}{\partial y} f^{-1}(y) \right|$  is the Jacobian term. The Jacobian measures how the scale of the transformed variable changes with respect to the underlying variable. This can be expanded to the multivariate case where the Jacobian becomes a matrix of partial derivatives, see later for some example of multivariate cases. In equation 5.15 the term  $p_X(f^{-1}(y)) = p_X(X)$  and in a Bayesian context is the prior of the untransformed variable/parameter. **Note:** if this functionality is in use be careful interpreting the covariance matrix as this will be related to the transformed variable not the variable space that you may be thinking it is in e.g. if you are estimating natural mortality ( $M$ ) as  $Y = M/2$  the covariance matrix will be described for  $Y$ .

```
@estimate log_R0
type lognormal
transformation log
parameter process[Recruitment].r0
transform_with_jacobian true
mu 442413
cv 0.2
lower_bound 3000
upper_bound 24154953
```

### Transform without Jacobian but prior defined in transformed space

This is where users define the priors in transformed space (this class of transformations will contain functionality that was implemented in the original CASAL). If  $f()$  is a transformation function defined on the support of  $X$ , then  $Y = f(X)$  is a new random variable (transformed variable). In this class users specify *a priori* information with regard to  $p_Y(y)$  and  $X$  can be thought of as a derived quantity. An example of this syntax,

```
@estimate log_R0
type lognormal
parameter process[Recruitment].r0
prior_applies_to_transform true
mu 13
cv 0.5
lower_bound 8
upper_bound 17
```

### Transformation types

- log natural logarithm transformation
  - is\_simple = true
  - jacobian defined = true

$$Y = \ln(X)$$

$$\left| \frac{\partial}{\partial y} f^{-1}(y) \right| = X^{-1}$$

- inverse

$$\text{is\_simple} = \text{true}$$

$$\text{jacobian defined} = \text{true}$$

$$Y = X^{-1}$$

$$\left| \frac{\partial}{\partial y} f^{-1}(y) \right| = -X^{-2}$$

- sqrt Square Root transformation

$$\text{is\_simple} = \text{true}$$

$$\text{jacobian defined} = \text{true}$$

$$Y = \sqrt{X}$$

$$\left| \frac{\partial}{\partial y} f^{-1}(y) \right| = -X^{-1.5}$$

- average\_difference Take two parameters  $\theta_1$  and  $\theta_2$  and transform to  $Y_1$  and  $Y_2$ , where  $Y_1$  is the average of the original parameters and  $Y_2$  is the difference between the mean and each parameter.

$$\text{is\_simple} = \text{false}$$

$$\text{jacobian defined} = \text{false}$$

$$Y_1 = \frac{\theta_1 + \theta_2}{2}$$

$$Y_2 = (Y_1 - \theta_2)2$$

Restore transformations

$$\theta_1 = Y_1 + 0.5Y_2$$

$$\theta_2 = \theta_1 - 0.5Y_2$$

$$\left| \frac{\partial}{\partial y} f^{-1}(y) \right| \text{ Hasn't been assessed (i.e it could exist)}$$

- log\_sum Take two parameters  $\theta_1$  and  $\theta_2$  and transform to  $Y_1$  and  $Y_2$ , where  $Y_1$  is the natural logarithm of the sum of  $\theta_1$  and  $\theta_2$ .  $Y_2$  describes the proportion of the sum with respect to  $\theta_1$

$$\text{is\_simple} = \text{false}$$

$$\text{jacobian defined} = \text{false}$$

$$Y_1 = \ln(\theta_1 + \theta_2)$$

$$Y_2 = \theta_1 / (\theta_1 + \theta_2)$$

Restore transformations

$$\theta_1 = \exp(Y_1)Y_2$$

$$\theta_2 = \exp(Y_1)(1 - Y_2)$$

$$\left| \frac{\partial}{\partial y} f^{-1}(y) \right| \text{ Hasn't been assessed (i.e it could exist)}$$

- orthogonal Take two parameters  $\theta_1$  and  $\theta_2$  and transform to  $Y_1$  and  $Y_2$ , where  $Y_1$  is the multiplication of  $\theta_1$  and  $\theta_2$ .  $Y_2$  is the division of  $\theta_1$  and  $\theta_2$

$$\text{is\_simple} = \text{false}$$

$$\text{jacobian defined} = \text{true}$$

$$Y_1 = \theta_1 \theta_2$$

$$Y_2 = \theta_1 / \theta_2$$

Restore transformations

$$\theta_1 = \sqrt{Y_1 Y_2}$$

$$\theta_2 = \sqrt{Y_1 / Y_2}$$

$$\left| \frac{\partial}{\partial y} f^{-1}(y) \right| = 2Y_2$$

- **SumToOne** Take two parameters  $\theta_1$  and  $\theta_2$  that have the constraint  $\sum_{i=1}^2 \theta_i$  and estimate only  $\theta_1$  given  $\theta_2 = 1 - \theta_1$   
is\_simple = false  
jacobian defined = false

---

## 6. The observation section

### 6.1. Observations

The objective function is based on the goodness-of-fit of the model to the supplied observational data. Observations are typically supplied at an instance in time, over a group of aggregated categories. Most observations are formed from time, i.e., data which were recorded for one or more years, in the same format each year. Examples of time series data types include relative abundance indices, commercial catch length frequencies, and survey numbers-at-age.

Definitions for each type of observation are described below, including how the observed values should be formatted, how CASAL2 calculates the expected values, and the likelihoods that are available for each type of observation.

There are two main types of observations available in CASAL2. The first are observations that are associated with a **mortality block** and, secondly, observations that are associated with a specific process. These can be distinguished by the `type` subcommand. If an observation type begins with `process` it is an observation that is associated with a process. If a type does **not** begin with `process` it is associated with the mortality block of the defined time step. For example, the observation type `process_abundance` is a process based observation, whereas `process_abundance` abundance is an observation that is associated with a mortality block.

Process specific observations can also be broken into two types. **Specific process observations** are observations that are associated to a specific process (e.g. `process_proportions_migrating`), and **general process observations** are observations that can be associated with any process (e.g. `process_proportions_at_age`). These tiers of observations have been separated in different sections as to reduce the confusion.

#### 6.1.1. Mortality block associated observations

All observations within this class are calculated in a similar fashion. That is, an expectation is calculated at the beginning of the mortality block and at the end of the mortality block. CASAL2 then uses a linear interpolation to approximate an expectation part way through a mortality block using the subcommand `time_step_proportion`. This could be useful if a survey occurs part-way through an exploitation phase, e.g when modelling a fish population this may be part-way through a fishing season. Each observation in this class will evaluate different expectations of the partition (explained in the following descriptions). A list of observation `types` available with this class of observations are:

- `abundance`
- `biomass`
- `proportions_at_age`
- `proportions_at_length`
- `proportions_by_category`
- `tag_recapture_by_length`
- `tag_recapture_by_age`

**Abundance or biomass observations**

Abundance (or biomass) observations are observations of either a relative or absolute number (or biomass) of individuals from a set of categories after applying a selectivity. The observation classes are the same, except that a biomass observation will use the biomass as the observed (and expected) value (calculated from mean weight of individuals within each age and category) while an abundance observation is just the number of individuals.

Each observation is for a given year and time-step, for some selected age classes of the population (i.e., for a range of ages multiplied by a selectivity), for aggregated categories. Further, you need to provide the label of the catchability coefficient  $q$ , which can either be estimated or fixed. For absolute abundance or absolute biomass observations, define a catchability where  $q = 1$ .

The observations can be supplied for any set of categories. For example, for a model with the two categories *male* and *female*, we might supply an observation of the total abundance/biomass (male + female) or just male abundance/biomass. The subcommand `categories` defines the categories used to aggregate the abundance/biomass. In addition, each category must have an associated selectivity, defined by `selectivities`. For example,

```
categories male
selectivities male-selectivity
```

defines an observation for males after applying the selectivity `male-selectivity`. CASAL2 then expects that there will be a single observation supplied. The expected values for the observations will be the expected abundance (or biomass) of males, after applying the selectivities, at the year and time-step specified.

CASAL2 calculates the expected values by summing over the defined ages (via the age range and selectivity) and categories at both the beginning and end of a mortality block. You can prompt CASAL2 to approximate the expectation part way through the mortality block using the `time_step_proportion`. The default value CASAL2 uses is 0.5, which does linear interpolation between the start and end abundance (or biomass) from the mortality block.

For an abundance observation the expectation is calculated as follows,

$$E_{i,1} = \sum_{c=1}^C \sum_{a=1}^A S_{a,c} N_{a,c,i,1} \quad (6.1)$$

$$E_{i,2} = \sum_{c=1}^C \sum_{a=1}^A S_a N_{a,c,i,2} \quad (6.2)$$

Where  $E_{i,1}$  is the expectation at the beginning of time step and  $E_{i,2}$  is the expectation at the end of the time-step.  $S_a$  is the selectivity for age  $a$  and category  $c$ . If there is no mortality related to this observation then  $E_i$  which is used in the likelihood contribution is  $E_{i,1}$ . If this was a biomass observation we would replace  $N_{a,c,i,1}$  in Equation (6.1) and (6.2) with  $N_{a,c,i,1} \bar{w}_{a,c}$ , where  $\bar{w}_{a,c}$  is the mean weight of category  $c$  at age  $a$ . If the user wishes to apply 100% mortality then  $E_i = E_{i,2}$ . For applying quantities of mortality between these values ( $M_i$ ), CASAL2 does the following linear interpolation.

$$E_i = |E_{i,1} - E_{i,2}| M_i \quad (6.3)$$



```
@observation MyAbundance
type abundance
years 1999
...
categories male
obs 1000
...
```

Or, for an observation aggregated over multiple categories,

```
@observation MyAbundance
type abundance
years 1990 1991
...
categories male+female
table obs
1990 1000
1991 1200
end_table
...
```

Note that, to define a biomass observation instead of an abundance observation, use

```
@observation MyBiomass
type biomass
...
```

### Proportions-at-age

Proportions-at-age observations are observations of the relative number of individuals at age, via some selectivity.

The observation is supplied for a given year and time-step, for some selected age classes of the population (i.e., for a range of ages multiplied by a selectivity), for categories aggregated over a set of spatial cells. Note that the categories defined in the observations must have an associated selectivity, defined by `selectivities`.

The age range must be ages defined in the partition (i.e., between `@model.min_age` and `@model.max_age` inclusive), but the upper end of the age range can optionally be a plus group — which must be either the same or less than the plus group defined for the partition.

Proportions-at-age observations can be supplied as;

1. a set of proportions for a single category,
2. a set of proportions for multiple categories, or
3. a set of proportions across aggregated categories.

The method of evaluating expectations are the same for all three of these sceneries. We will describe how you define these different scenarios and the expected dimensions of observation and error inputs that CASAL2 expects for each respective scenario with examples.

Like all types of observations that are associated with the mortality block, CASAL2 will evaluate the numbers at age before the mortality block (after taking into account a selectivity that the user defines) and after for the specified time step of the observation. CASAL2 will generate expectations from

the partition part way through the mortality block using the subcommand `time_step_proportion`. This approximation is a linear interpolation of the numbers at age over the mortality block.

Once the interpolation is evaluated CASAL2 will apply ageing error if the user has specified it. CASAL2 finally converts numbers at age to proportions at age by dividing all numbers in an age bin by the total and sending that to the likelihood to be evaluated.

Defining an observation for a single category is the simplest, and is used to model a set of proportions of a single category by age class. For example, to specify that the observations are of the proportions of male within each age class, then the subcommand `categories` for the `@observation[label].type=proportion_by_age` command is,

```
categories male
```

CASAL2 then expects that there will be a single vector of proportions supplied, with one proportion for each age class within the defined age range, and that these proportions sum to one.

For example, if the age range was 3 to 10, then 8 proportions should be supplied (one proportion for each of the the ages 3, 4, 5, 6, 7, 8, 9, and 10). The expected values will be the expected proportions of males within each of these age classes (after ignoring any males aged less than 3 or older than 10), after applying a selectivity at the year and time-step specified. The supplied vector of proportions (i.e., in this example, the 8 proportions) must sum to one, which is evaluated with a default tolerance of 0.001.

```
@observation MyProportions
type proportions_at_age
...
categories male
min_age 3
max_age 9
years 1990
table obs
1990 0.01 0.09 0.20 0.20 0.35 0.10 0.05
end_table
...
```

Defining an observation for multiple categories extends on the single category implementation. It is used to model a set of proportions over several categories by age class. For example, to specify that the observations are of the proportions of male or females within each age class, then the subcommand `categories` for the `@observation[label].type=proportion_by_age` command is,

```
categories male female
```

CASAL2 then expects that there will be a single vector of proportions supplied, with one proportion for each category and age class combination, and that these proportions sum to one across all ages and categories.

For example, if there were two categories and the age range was 3 to 10, then 16 proportions should be supplied (one proportion for each of the the ages 3, 4, 5, 6, 7, 8, 9, and 10, for each category male and female). The expected values will be the expected proportions of males and within each of these age classes (after ignoring those aged less than 3 or older than 10), after applying a selectivity

at the year and time-step specified. The supplied vector of proportions (i.e., in this example, the 16 proportions) must sum to one, which is evaluated with a default tolerance of 0.001.

For example,

```
@observation MyProportions
type proportions_at_age
...
categories male female
min_age 1
max_age 5
years 1990 1991
table obs
1990 0.01 0.05 0.10 0.20 0.20 0.01 0.05 0.15 0.20 0.03
1991 0.02 0.06 0.10 0.21 0.18 0.02 0.03 0.17 0.20 0.01
end_table
...
```

Defining an observation across aggregated categories allows categories to be aggregated before the proportions are calculated. It is used to model a set of proportions from several categories that have been combined by age class. To indicate that two (or more) categories are to be aggregated, separate them with a '+' symbol. For example, to specify that the observations are of the proportions of male and females combined within each age class, then the subcommand `categories` for the `@observation[label].type=proportion_by_age` command is,

```
categories male + female
```

CASAL2 then expects that there will be a single vector of proportions supplied, with one proportion for each age class, and that these proportions sum to one.

For example, if there were two categories and the age range was 3 to 10, then 8 proportions should be supplied (one proportion for each of the the ages 3, 4, 5, 6, 7, 8, 9, and 10, for the sum of males and females within each age class). The expected values will be the expected proportions of males + females within each of these age classes (after ignoring those aged less than 3 or older than 10), after applying a selectivity at the year and time-step specified. The supplied vector of proportions (i.e., in this example, the 16 proportions) must sum to one, which is evaluated with a default tolerance of 0.001.

For example,

```
@observation MyProportions
type proportions_at_age
...
years 1990 1991
categories male+female
min_age 1
max_age 5
table obs
1990 0.02 0.13 0.25 0.30 0.30
1991 0.02 0.06 0.18 0.35 0.39
end_table
...
```

The later form can then be extended to include multiple categories, or multiple aggregated categories. For example, to describe proportions for the three groups: immature males, mature

males, and all females (immature and mature females added together) for ages 1–4, a total of 12 proportions are required

```
@observation MyProportions
type proportions_at_age
...
categories male_immature male_mature female_immature+female_mature
min_age 1
max_age 4
years 1990
table obs
year 1990 0.05 0.15 0.15 0.05 0.02 0.03 0.08 0.04 0.05 0.15 0.15 0.08
end_table
...
```

### Proportions-at-length

Functionality regarding defining combinations of categories and aggregated categories directly translates over from proportions at age to proportions at length. The difference is the observation is over length bins instead of age-classes. CASAL2 calculates expectations of numbers at length by converting numbers at age to numbers by length by using the age-length relationship and distribution specified for the category specified in the @age\_length block. Instead of supplying a minimum and maximum age users must supply a vector of length bins. If there is no plus group, i.e., length\_plus=false, then CASAL2 expects a vector of proportions for each year that is  $n - 1$ , where  $n$  is the number of lengths supplied. If length\_plus=true then CASAL2 expects a vector of proportions for each year that is  $n$ . The last proportion represents the numbers from the last length bin to the maximum length the age-length relationship allows.

```
@observation Observed_Length_frequency_ChathamRise
type process_removals_by_length
years 1991 1992
likelihood multinomial
time_step Summer
fishery EastChathamRise
process instant_mort
categories male
length_plus false
length_bins 0 20 40 60 80 110
table obs
1991 0.2 0.25 0.15 0.2 0.2
1992 0.12 0.25 0.28 0.25 0.1
end_table
table error_values
1991 25
1992 37
end_table
```

### Proportions-by-category observations

Proportions-by-category observations are observations of either the relative number of individuals between categories within age classes, or relative biomass between categories within age classes.

The observation is supplied for a given year and time-step, for some selected age classes of the population (i.e., for a range of ages multiplied by a selectivity).

The age range must be ages defined in the partition (i.e., between `@model.min_age` and `@model.max_age` inclusive), but the upper end of the age range can optionally be a plus group — which may or may not be the same as the plus group defined for the partition.

Proportions-by-category observations can be supplied for any set of categories as a proportion of themselves and any set of additional categories. For example, for a model with the two categories *male* and *female*, we might supply observations of the proportions of males in the population at each age class. The subcommand `categories` defines the categories for the numerator in the calculation of the proportion, and the subcommand `categories2` supplies the additional categories to be used in the denominator of the calculation. In addition, each category must have an associated selectivity, defined by `selectivities` for the numerator categories and `selectivities2` for the additional categories used in the denominator, e.g.,

```
categories male
categories2 female
selectivities male-selectivity
selectivities2 female-selectivity
```

defines that the proportion of males in each age class as a proportion of males + females. CASAL2 then expects that there will be a vector of proportions supplied, with one proportion for each age class within the defined age range, i.e., if the age range was 3 to 10, then 8 proportions should be supplied (one proportion for each of the ages 3, 4, 5, 6, 7, 8, 9, and 10). The expected values will be the expected proportions of male to male + female within each of these age classes, after applying the selectivities at the year and time-step specified.

The observations must be supplied using all or some of the values defined by a categorical layer. CASAL2 calculates the expected values by summing over the ages (via the age range and selectivity) and categories for those spatial cells where the categorical layer has the same value as defined for each vector of observations i.e.,

```
@observation MyProportions
type proportions_by_category
years 1990 1991
...
categories male
categories2 female
min_age 1
max_age 5
table obs
1990 0.01 0.05 0.10 0.20 0.20
1991 0.02 0.06 0.10 0.21 0.18
end_table
...
```

### Tag Recapture by length

Tag data is primarily used to estimate the population abundance of fish. In some models, this estimation can only be made outside the model and the result is used as an estimate of abundance in the model. But in CASAL2 the tagging data can, alternatively, be fitted within the model.

Before adding a tag-recapture time series, you will need to define a tag-release process (Section 4.5.5). Tagging events list the labels of the tags which are modelled, and define the events where fish are tagged (i.e., CASAL2 moves fish into the section of the partition

corresponding to a specific tag).

The observations are divided into two parts: (i) the number of fish that were scanned, and (ii) the number of tags that were recaptured. Each can be specified by categories, or for combinations of categories. The precise content of the scanned and recaptured observations depends on the sampling method, and the available options are:

1. age: both scanned and recaptured are vectors containing numbers-at-age. Only available in an age-based model. The selectivity ogive is redundant and cannot be supplied.
2. size: both scanned and recaptured are vectors containing numbers-at-size. Can be used in either an age- or size-based model. The selectivity ogive is redundant and cannot be supplied.

When defining the tag-recapture time series, you also need to specify:

- the time step,
- the years (unlike a tag-release process, the tag-recapture observations can occur over several years),
- the probability that each scanned tagged fish is detected as tagged (may be less than 1 if the observers are not infallible). The expected number of tags detected is calculated by multiplying this number by the number of tagged fish in the sample,
- the tagged category or categories (Make up the recaptures),
- the categories scanned (All the fish sampled for tags),
- A selectivity used in the recapture process,
- the size classes if the observations are size-based in an age-based model.

An example of a tag recapture observation applied in CASAL2

```
## For the following partition
@categories
format sex.area.tag
names male.Areal.2011,notag female.Areal.2011,notag

@observation Tag_2011_Areal_recap_2012 ## individuals tagged in 2011 and recaptured in 2012
## in Areal
type tag_recapture_by_length
categories format=*.Areal.*+ ## scanned categories in Areal
tagged_categories *.Areal.2011+ ## male and female tagged categories
detection 0.85 ## detection probability
likelihood binomial ## likelihood choice
selectivities One ## label of selectivity for tagged
tagged_selectivities One ## label of selectivity for scanned
years 2012 ## years to apply observation
time_step step2 ## time_step to apply observation
time_step_proportion 0.5 ## proportion of mortality applied before observation is calculated

table scanned
2012 281271 41360 30239 12234
end_table

table recaptured
2012 15 20 12 2
end_table
```

```
delta 1e-11 ## robustification value
dispersion 6.3 ## dispersion factor
```

The observed ( $O_{y,l}$ ) and expected ( $E_{y,l}$ ) values in year  $y$  and length  $l$  of this observation are calculated as followed;

$$O_{y,l} = \frac{R_{y,l}}{S_{y,l}} \quad (6.4)$$

where  $R_{y,l}$  is the recaptures in year  $y$  at length  $l$  and  $S_{y,l}$  are the scanned values, supplied by the user.

$$E_{y,l} = d \frac{\tilde{N}_{y,l,t} + (\tilde{N}_{y,l,t+1} - \tilde{N}_{y,l,t}) \times p}{N_{y,l,t} + (N_{y,l,t+1} - N_{y,l,t}) \times p} \quad (6.5)$$

where  $\tilde{N}_{y,l,t}$  is an element in the tagged categories at the beginning of time step  $t$  and  $\tilde{N}_{y,l,t+1}$  is an element in the tagged categories at the end of time step  $t$ .  $N_{y,l,t}$  is the sum of the categories that were vulnerable to sampling when the observation occurred.  $p$  is the proportion of the time step that the observation was taken,  $d$  is the detection probability. For cases where there are multiple tagged categories and multiple categories that were vulnerable to sampling.

$$\tilde{N}_{y,l,t} = \sum_{j=1}^J N_{y,l,t,j} \quad (6.6)$$

where  $j = \{1, 2, 3, \dots, J\}$  are all the tagged categories, the same method is applied to the vulnerable categories to get  $N_{y,l,t}$ . Remember that the tagged categories should be defined in the vulnerable categories. If you think about an extreme case where we tag every individual in the population this would be divide by zero. So to constrain the expectation to be between 0-1, we need the numerator to be in the denominator.

The tag-recapture likelihood (binomial) is specified below as it is a modified version of the more general binomial. Note that this likelihood does not have any user-set precision parameters such as  $N$  or  $c.v.$  (though there are user-specified robustification and dispersion parameters available). Note that factorials are calculated using the log-gamma function, to allow for non-integer arguments where necessary (and avoid overflow errors).

### 6.1.2. General process observations

A list of types that are associated with this set of observations:

- process\_abundance
- process\_biomass
- process\_proportions\_at\_age
- process\_proportions\_at\_length
- process\_proportions\_by\_category

These observations have the same expectations as the mortality block versions described in Section 4.4.2. With the exception that instead of wrapping a mortality block they can wrap any process type available in CASAL2.

### 6.1.3. Specific process observations

A list of types that are associated with this set of observations are:

- `process_removals_by_age`
- `process_removals_by_age_retained`
- `process_removals_by_age_retained_total`
- `process_removals_by_length`
- `process_removals_by_length_retained`
- `process_removals_by_length_retained_total`
- `process_proportions_migrating`

#### Process removals by age

Removals at age observations are observations of the relative number of individuals at age, partway through a process of type `mortality_instantaneous`. This observation is exclusively associated with the process of type `mortality_instantaneous`, and will error out if associated with any other process type.

The observation is supplied for a given year and time-step, for some selected age classes of the population (i.e., for a range of ages multiplied by a selectivity that is associated with the process).

The age range must be ages defined in the partition (i.e., between `@model.min_age` and `@model.max_age` inclusive), but the upper end of the age range can optionally be a plus group — which must be either the same or less than the plus group defined for the partition.

The expectations from this observation are generated whilst the process is being executed. The expectation of numbers at age  $a$  for category  $c$  from exploitation method  $m$  ( $E[N_{a,c,m}]$ ) are defined as,

$$E[N_{a,c,m}] = N_{a,c} U_{a,m} S_{a,c,m} 0.5 M_{a,c} \quad (6.7)$$

where,  $N_{a,c}$  are the numbers at age in category  $c$  before the process is executed,  $U_{a,m}$  is the exploitation rate for age  $a$  from method  $m$ .  $S_{a,c,m}$  is the selectivity and  $M$  is the natural mortality. These are all relevant to the time step which the user defines.

The observation class then acquires the variable  $E[N_{a,c,m}]$  and applies ageing error if the user has specified it. Then it amalgamates the observations by method and category depending on how the user specifies the observation, before converting numbers at age to proportions and sending them to the likelihood to be evaluated.

Likelihoods that are available for this observation class are the multinomial, dirichlet and the lognormal. See Section 6.2 for information on the respected likelihood.



### Process removals by age retained

Observations of retained and total catches by age are permitted, using the labels `process_removals_by_age_retained` and `process_removals_by_age_retained_total` respectively. Examples of two such observations are given below, with the associated process `Instantaneous_Mortality_Retained` having the form of the example in Section 4.5.3.4. First, for retained catch:

```
@observation potFishAFtotal #test syntax get catch AF out
type process_removals_by_age_retained_total
mortality_instantaneous_process Instantaneous_Mortality_Retained
method_of_removal FishingPot
years 2005
time_step 1
categories male
### ageing_error Normal_ageing
min_age 3
max_age 15
plus_group True
table obs
2005 0.0002814574 0.0095351205 0.1661896098 0.2701718827 0.2214454177 0.1661869474 0.1107930285 0.
end_table
table error_values
2005 651
end_table
likelihood multinomial
delta 1e-11
```

and similarly, for total catch:

```
@observation potFishAFretained #test syntax --> fits to discards not catch
type process_removals_by_age_retained
mortality_instantaneous_process Instantaneous_Mortality_Retained
method_of_removal FishingPot
years 2005
time_step 1
categories male
# ageing_error Normal_ageing
min_age 3
max_age 15
plus_group True
table obs
2005 1.650990e-10 7.566419e-07 1.771126e-03 1.962050e-01 3.192775e-01 2.413644e-01 1.609208e-01 8.
end_table
table error_values
2005 651
end_table
likelihood multinomial
delta 1e-11
```

### Process removals by length

Removals by length observations are observations of the relative number of individuals at length, partway through a process of type `mortality_instantaneous`. This observation is exclusively associated with the process of type `mortality_instantaneous`, and will error

out if associated with any other process type.

The observation is supplied for a given year and time-step, for some selected age classes of the population (i.e., for a range of ages multiplied by a selectivity that is associated with the process).

The expectations from this observation are generated whilst the process is being executed. The expectation of numbers at age  $a$  for category  $c$  from exploitation method  $m$  ( $E[N_{a,c,m}]$ ) are defined as,

$$E[N_{a,c,m}] = N_{a,c} U_{a,m} S_{a,c,m} 0.5 M_{a,c} \quad (6.8)$$

where,  $N_{a,c}$  are the numbers at age in category  $c$  before the process is executed,  $U_{a,m}$  is the exploitation rate for age  $a$  from method  $m$ .  $S_{a,c,m}$  is the selectivity and  $M$  is the natural mortality. These are all relevant to the time step which the user defines.

The observation class acquires the variable  $E[N_{a,c,m}]$  from the process and applies the age-length relationship specified in the model. This converts numbers at age to numbers at age and length, where CASAL2 then converts to numbers at length. Then it amalgamates the observations by method and category depending on how the user specifies the observation, before converting numbers at age to proportions and sending them to the likelihood to be evaluated.

```
@observation observation_fishery_LF
type process_removals_by_length
...
years 1993 1994 1995
method_of_removal FishingEast
mortality_instantaneous_process instant_mort
length_plus false
length_bins 0 20 40 60 80 110
delta 1e-5
table obs
1993    0.0    0.05    0.05    0.10    0.80
1994    0.05   0.1    0.05    0.05    0.75
1995    0.3    0.4    0.2    0.05    0.05
end_table

table error_values
1993 31
1994 34
1995 22
end_table
```

Likelihoods that are available for this observation are the multinomial, dirichlet and the lognormal. See Section 6.2 for information on the respected likelihood.

### Process removals by age retained

Observations of retained and total catches by length are permitted, using the labels `process_removals_by_length_retained` and `process_removals_by_length_retained_total` respectively. Examples of two such observations are given below, with the associated process `Instantaneous_Mortality_Retained` having the form of the example in Section 4.5.3.4. First, for retained catch:

```

@observation potFishLFtotal #test syntax get catch LF out
type process_removals_by_length_retained_total
mortality_instantaneous_process Instantaneous_Mortality_Retained
method_of_removal FishingPot
years 2005
time_step 1
categories male
length_bins 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 # for LF in catch
length_plus False
table obs
2005 0.05344612 0.06432242 0.07357780 0.08050385 0.08473451 0.08619620 0.08502982 0.08152921 0.076
end_table
table error_values
2005 651
end_table
likelihood multinomial
delta 1e-11

```

and similarly, for total catch:

```

@observation potFishLFretained #test syntax get retained LF out
type process_removals_by_length_retained
mortality_instantaneous_process Instantaneous_Mortality_Retained
method_of_removal FishingPot
years 2005
time_step 1
categories male
length_bins 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 # for LF in catch
length_plus False
table obs
2005 0.02462879 0.03536036 0.04759163 0.06025858 0.07205340 0.08169356 0.08817806 0.09095124 0.089
end_table
table error_values
2005 651
end_table
likelihood multinomial
delta 1e-11

```

### Proportions migrating

This observation is of the proportion migrating from one area to another. This observation is exclusively associated with the process type `transition_category`, and will error out when trying to associate with any other process type. This observation is used to inform migration rates in migration processes. This observation class is used in the Hoki stock assessment see Francis et al. (2003) for more information on how these observations are collected and the situation you would use it. This observation calculates an expectation  $E_a$  of proportions for each age class  $a$  that have migrated, by evaluating the following,

$$E_a = \frac{N_a - N'_a}{N_a} \quad (6.9)$$

where,  $N_a$  are the numbers of individuals in age  $a$  before the migration process occurs and  $N'_a$  is the number of individuals after the migration process occurs. The likelihoods that are allowed for this observation are the lognormal, multinomial and dirichlet.

An extract of the Hoki stock assessment is as follows,

```
@observation pspawn_1993
type process_proportions_migrating
years 1993
time_step step4
process Wspmg ## migration process that the observation is associated with
age_plus true
min_age 4
max_age 9
likelihood lognormal
categories male.west+female.west ## Categories to evaluate the prportion for
ageing_error Normal_offset ## label for an @ageing_error block
table obs
#age    4    5    6    7    8    9
1993 0.64 0.58 0.65 0.66 0.71 0.60
end_table

table error_values
## if lognormal these are c.v.'s
1993 0.25
end_table
```

## 6.2. Likelihoods

### 6.2.1. Likelihoods for proportions-at-age observations

CASAL2 implements three likelihoods for proportions-at-age observations, the multinomial likelihood, dirichlet, and the lognormal likelihood.

#### The multinomial likelihood

For the observed proportions at age  $O_i$  for age classes  $i$ , with sample size  $N$ , and the expected proportions at the same age classes  $E_i$ , the negative log-likelihood is defined as;

$$-\log(L) = -\log(N!) + \sum_i \log((NO_i)!) - NO_i \log(Z(E_i, \delta)) \quad (6.10)$$

where  $\sum_i O_i = 1$  and  $\sum_i E_i = 1$ .  $Z(\theta, \delta)$  is a robustifying function to prevent division by zero errors, with parameter  $\delta > 0$ .  $Z(\theta, \delta)$  is defined as,

$$Z(\theta, \delta) = \begin{cases} \theta, & \text{where } \theta \geq r \\ \delta / (2 - \theta / \delta), & \text{otherwise} \end{cases} \quad (6.11)$$

The default value of  $\delta$  is  $1 \times 10^{-11}$ .

### The dirichlet likelihood

For the observed proportions at age  $O_i$  for age classes  $i$ , with sample size  $N$ , and the expected proportions at the same age classes  $E_i$ , the negative log-likelihood is defined as;

$$-\log(L) = -\log(\Gamma(\sum_i \alpha_i)) + \sum_i \log(\Gamma(\alpha_i)) - \sum_i (\alpha_i - 1) \log(Z(O_i, \delta)) \quad (6.12)$$

where  $\alpha_i = Z(NE_i, \delta)$ ,  $\sum_i O_i = 1$ , and  $\sum_i E_i = 1$ .  $Z(\theta, \delta)$  is a robustifying function to prevent division by zero errors, with parameter  $\delta > 0$ .  $Z(\theta, \delta)$  is defined as,

$$Z(\theta, \delta) = \begin{cases} \theta, & \text{where } \theta \geq r \\ \delta / (2 - \theta/\delta), & \text{otherwise} \end{cases} \quad (6.13)$$

The default value of  $\delta$  is  $1 \times 10^{-11}$ .

### The lognormal likelihood

For the observed proportions at age  $O_i$  for age classes  $i$ , with c.v.  $c_i$ , and the expected proportions at the same age classes  $E_i$ , the negative log-likelihood is defined as;

$$-\log(L) = \sum_i \left( \log(\sigma_i) + 0.5 \left( \frac{\log(O_i/Z(E_i, \delta))}{\sigma_i} + 0.5\sigma_i \right)^2 \right) \quad (6.14)$$

where

$$\sigma_i = \sqrt{\log(1 + c_i^2)} \quad (6.15)$$

and the  $c_i$ 's are the c.v.s for each age class  $i$ , and  $Z(\theta, \delta)$  is a robustifying function to prevent division by zero errors, with parameter  $\delta > 0$ .  $Z(\theta, \delta)$  is defined as,

$$Z(\theta, \delta) = \begin{cases} \theta, & \text{where } \theta \geq r \\ \delta / (2 - \theta/\delta), & \text{otherwise} \end{cases} \quad (6.16)$$

The default value of  $\delta$  is  $1 \times 10^{-11}$ .

#### 6.2.2. Likelihoods for abundance and biomass observations

Abundance and biomass observations are expected as an annual time series in CASAL2, where they select the same categories over that time series. The parameters and inputs needed to use this observation class are: a observation  $O_i$ , c.v.  $c_i$ , catchability coefficient  $q$ , where  $i$  indexed the year. CASAL2 calculates an expectation  $E_i$  and scales it by  $q$  before comparing it to  $O_i$ . This means that the value chosen for  $q$  will determine whether the observation is relative ( $q \neq 1$ ) or absolute  $q = 1$ . Before we describe each of the likelihoods we will discuss the methods available to handle  $q$ 's:

1. The  $q$ 's can be treated as "nuisance" parameters. For each set of values of the free parameters, the model uses the values of the  $q$ 's which minimise the objective function. These optimal  $q$ 's are calculated algebraically (see Section 6.4). If one of the  $q$ 's falls outside the bounds specified by the user, it is set equal to the closest bound. This approach reduces the size of the parameter vector and hence should improve the performance of the estimation method. However, it is not correct when calculating a sample from the posterior in a Bayesian analysis (except asymptotically, see Walters and Ludwig (1994)) and we offer the following alternative;
2. The  $q$ 's can be treated as ordinary free parameters.

For both options, it is necessary to evaluate the contribution of  $O_i$  to the negative loglikelihood for a given value of  $q$ . Each observation  $O_i$  varies about  $qE_i$  – express the variability of  $O_i$  in terms of its c.v.  $c_i$  (or in one case, its standard deviation  $s_i$ ). Here are the likelihoods, which are expressed on the objective-function scale of  $-\log(L)$ :

### The lognormal likelihood

The negative log likelihood for a the lognormal is as follows,

$$-\log(L) = \sum_i \left( \log(\sigma_i) + 0.5 \left( \frac{\log(O_i/qZ(E_i, \delta))}{\sigma_i} + 0.5\sigma_i \right)^2 \right) \quad (6.17)$$

where

$$\sigma_i = \sqrt{\log(1 + c_i^2)} \quad (6.18)$$

and  $Z(\theta, \delta)$  is a robustifying function to prevent division by zero errors, with parameter  $\delta > 0$ .  $Z(\theta, \delta)$  is defined as,

This reflects the distributional assumptions that  $O_i$  has the lognormal distribution, that the mean of  $O_i$  is  $qE_i$  and the c.v. of  $O_i$  is  $c_i$ .

$$Z(\theta, \delta) = \begin{cases} \theta, & \text{where } \theta \geq r \\ \delta/(2 - \theta/\delta), & \text{otherwise} \end{cases} \quad (6.19)$$

The default value of  $\delta$  is  $1 \times 10^{-11}$ .

### The normal likelihood

For observations  $O_i$ , c.v.  $c_i$ , and expected values  $qE_i$ , the negative log-likelihood is defined as;

$$-\log(L) = \sum_i \left( \log(c_i E_i) + 0.5 \left( \frac{O_i - E_i}{Z(c_i E_i, \delta)} \right)^2 \right) \quad (6.20)$$

and  $Z(\theta, \delta)$  is a robustifying function to prevent division by zero errors, with parameter  $\delta > 0$ .  $Z(\theta, \delta)$  is defined as,

$$Z(\theta, \delta) = \begin{cases} \theta, & \text{where } \theta \geq r \\ \delta / (2 - \theta / \delta), & \text{otherwise} \end{cases} \quad (6.21)$$

The default value of  $\delta$  is  $1 \times 10^{-11}$ .

This reflects the distributional assumptions that  $O_i$  has the normal distribution, that the mean of  $O_i$  is  $qE_i$  and the c.v. of  $O_i$  is  $c_i$ .

### 6.2.3. Likelihoods for tag recapture by age and length observations

#### The binomial likelihood

Designed for situations where the size frequencies or age frequencies of the recaptured tagged fish and of the scanned fish are known. Available in both age or size based models.

Here we define the likelihood as a binomial, but based on sizes, rather than ages,

$$\begin{aligned} -\log(L)' = & -\sum_i [\log(n_i!) - \log((n_i - m_i)!) - \log((m_i)!) + m_i \log\left(Z\left(\frac{M_i}{N_i}, \delta\right)\right) \\ & + (n_i - m_i) \log\left(Z\left(1 - \frac{M_i}{N_i}, \delta\right)\right)] \end{aligned} \quad (6.22)$$

where

$n_i$  = number of fish at size or age  $i$  that were scanned

$m_i$  = number of fish at size or age  $i$  that were recaptured

$N_i$  = number of fish at size or age  $i$  in the available population (tagged and untagged)

$M_i$  = number of fish at size or age  $i$  in the available population that have the tag after a detection probability  $p_d$  has been applied,  $M_i = M'_i p_d$ , where  $M'_i$  is the expected available population that have the tag.

where  $Z(x, \delta)$  is a robustifying function with parameter  $r > 0$  (to prevent division by zero errors), defined as

$$Z(x, \delta) = \begin{cases} x & \text{where } x \geq \delta \\ \frac{\delta}{(2 - x/\delta)} & \text{otherwise} \end{cases}$$

Finally if a dispersion parameter ( $\tau$ ) is described in the observation then the final negative log likelihood  $-\log(L)$  contribution is,

$$-\log(L) = -\log(L)' / \tau$$

### 6.2.4. Likelihoods for proportions-by-category observations

CASAL2 implements two likelihoods for proportions-by-category observations, the binomial likelihood, and the normal approximation to the binomial (binomial-approx).

### The binomial likelihood

For observed proportions  $O_i$  for age class  $i$ , where  $E_i$  are the expected proportions for age class  $i$ , and  $N_i$  is the effective sample size for age class  $i$ , then the negative log-likelihood is defined as;

$$-\log(L) = -\sum_i [\log(N_i!) - \log((N_i(1 - O_i))!) - \log((N_i O_i)!) + N_i O_i \log(Z(E_i, \delta)) + N_i(1 - O_i) \log(Z(1 - E_i, \delta))] \quad (6.23)$$

where  $Z(\theta, \delta)$  is a robustifying function to prevent division by zero errors, with parameter  $\delta > 0$ .  $Z(\theta, \delta)$  is defined as,

$$Z(\theta, \delta) = \begin{cases} \theta, & \text{where } \theta \geq r \\ \delta / (2 - \theta / \delta), & \text{otherwise} \end{cases} \quad (6.24)$$

The default value of  $\delta$  is  $1 \times 10^{-11}$ .

### The normal approximation to the binomial likelihood

For observed proportions  $O_i$  for age class  $i$ , where  $E_i$  are the expected proportions for age class  $i$ , and  $N_i$  is the effective sample size for age class  $i$ , then the negative log-likelihood is defined as;

$$-\log(L) = \sum_i \log\left(\sqrt{Z(E_i, \delta) Z(1 - E_i, \delta) / N_i}\right) + \frac{1}{2} \left( \frac{O_i - E_i}{\sqrt{Z(E_i, \delta) Z(1 - E_i, \delta) / N_i}} \right)^2 \quad (6.25)$$

where  $Z(\theta, \delta)$  is a robustifying function to prevent division by zero errors, with parameter  $\delta > 0$ .  $Z(\theta, \delta)$  is defined as,

$$Z(\theta, \delta) = \begin{cases} \theta, & \text{where } \theta \geq r \\ \delta / (2 - \theta / \delta), & \text{otherwise} \end{cases} \quad (6.26)$$

The default value of  $\delta$  is  $1 \times 10^{-11}$ .

### 6.3. Process error

Additional ‘process error’ can be defined for each set of observations. Additional process error has the effect of increasing the observation error in the data, and hence of decreasing the relative weight given to the data in the fitting process.

For observations where the likelihood is parameterised by the c.v., you can specify the process error for a given set of observations as a c.v., in which case all the c.v.s  $c_i$  are changed to

$$c'_i = \sqrt{c_i^2 + c_{process\_error}^2} \quad (6.27)$$



Note that  $c_{process\_error} \geq 0$ , and that  $c_{process\_error} = 0$  is equivalent to no process error.

Similarly, if the likelihood is parameterised by the effective sample size  $N$ ,

$$N'_i = \frac{1}{1/N_i + 1/N_{process\_error}} \quad (6.28)$$

Note that this requires that  $N_{process\_error} > 0$ , but we allow the special case of  $N_{process\_error} = 0$ , and define  $N_{process\_error} = 0$  as no process error (i.e., defined to be equivalent to  $N_{process\_error} = \infty$ ).

For both the c.v. and  $N$  process errors, the process error has more effect on small errors than on large ones. Be clear that a large value for the  $N$  process error means a small process error.

#### 6.4. Calculating nuisance $q$ 's

This section describes the theory used to calculate nuisance (analytical) catchability coefficients  $q$ s (see Section 6.2.2). From the user's point of view, the essence is that you can use nuisance  $q$ s in the following situations:

1. With maximum likelihood.
2. With Bayesian estimation, providing that you provide an additional prior on the  $q$  is one of the following:
  - none (default)
  - Uniform-log
  - Lognormal with observations distributed lognormal, robustified lognormal

Table 6.1 displays the scenarios when the nuisance catchability can be used for a Bayesian analysis.

**Table 6.1: Equations used to calculate nuisance  $q$ 's. (\*=no analytic solution found.)**

Distribution of observations	Maximum Likelihood	None	Uniform-log	Normal	lognormal
Normal	(6.29)	(6.29)	(6.31)	*	*
Lognormal	(6.32)	(6.32)	(6.36)	*	(6.37)

Note that  $q$ s are calculated for robustified lognormal likelihoods as if they were ordinary lognormal likelihoods.

The equations and their derivations follow. Let  $\sigma_i = \sqrt{\log(1 + c_i^2)}$  throughout, and let  $n$  be the number of observations in the time series. The case of multiple time series sharing the same  $q$ , and the modifications required for the assumption of curvature, are addressed at the end of this subsection.

First, consider maximum likelihood estimation. When the  $(O_i)$  are assumed to be normally distributed,

$$-\log(L) = \sum_i \log(c_i q E_i) + 0.5 \sum_i \left( \frac{O_i - q E_i}{c_i q E_i} \right)^2 \quad (6.29)$$

The value of  $q$  which minimises the objective function is found by solving for  $q$  under the following condition,  $\partial/\partial q(-\log(L)) = 0$

$$\frac{\partial}{\partial q}(-\log(L)) = \frac{n}{q} + \frac{1}{q^2} \sum_i \frac{O_i}{c_i^2 E_i} - \frac{1}{q^3} \sum_i \left( \frac{O_i}{c_i E_i} \right)^2 \quad (6.30)$$

hence

$$\hat{q} = \frac{-S_1 + \sqrt{S_1^2 + 4nS_2}}{2n} \quad (6.31)$$

where  $S_1 = \sum_i (O_i/c_i^2 E_i)$  and  $S_2 = \sum_i (O_i/c_i E_i)^2$

When the  $(O_i)$  are assumed to be lognormally distributed,

$$-\log(L) = \sum_i \log(\sigma_i) + 0.5 \sum_i \left( \frac{\log(O_i) - \log(qE_i) + 0.5\sigma_i^2}{\sigma_i} \right)^2 \quad (6.32)$$

$$\frac{\partial}{\partial q}(-\log(L)) = \frac{-1}{q} \sum_i \left( \frac{\log(O_i/E_i) - \log(q) + 0.5\sigma_i^2}{\sigma_i^2} \right) \quad (6.33)$$

$$\hat{q} = \exp \frac{0.5n + S_3}{S_4} \quad (6.34)$$

where  $S_3 = \sum_i (\log(O_i/E_i)/\sigma_i^2)$  and  $S_4 = \sum_i (1/\sigma_i^2)$

Next consider Bayesian estimation, where we must also specify a prior for  $q$ .

The effects of the prior on the equations are to replace likelihood  $L$  by posterior  $P$  throughout, to add  $-\log(\pi(q))$  to the equation for  $-\log(P)$  and  $\partial/\partial q(-\log(-\pi(q)))$  to the equation for  $\partial/\partial q(-\log(P))$

This last term is 0 for a uniform prior on  $q$ ,  $1/q$  for a log-uniform prior, and  $\frac{1}{q} \left( 1.5 + \frac{\log(q) - \log(\mu_q)}{\sigma_q^2} \right)$  for a lognormal prior,

where  $\mu_q$  and  $c_q$  are the mean and c.v of the prior on  $q$  and  $\sigma_q = \sqrt{\log(1 + c_q^2)}$ . Clearly, if the prior is uniform, the equation for  $\hat{q}$  is the same as the maximum likelihood estimation.

When the  $(O_i)$  are assumed to be normally distributed and the prior is log-uniform equation (6.31) becomes,

$$\hat{q} = \frac{-S_1 + \sqrt{S_1^2 + 4(n+1)S_2}}{2(n+1)} \quad (6.35)$$

but we cannot solve for  $\hat{q}$  with either a normal or lognormal prior.

When the  $O_i$  are assumed to be lognormally distributed and the prior is log-uniform, equation (6.34) becomes

$$\hat{q} = \exp \frac{0.5n - 1 + S_3}{S_4} \quad (6.36)$$

and if the prior is lognormal,

$$\hat{q} = \exp \frac{0.5n - 1.5 + \log(\mu_q)/\sigma_q^2 + S_3}{S_4 + 1/\sigma_q^2} \quad (6.37)$$

but it is not possible to solve for  $\hat{q}$  with a normal prior. An example of specifying the syntax and an equivalent additional prior see below

```
@catchability chatTANq
type nuisance
upper_bound 0.6
lower_bound 0.0001

@additional_prior chatTANq_prior
type lognormal
parameter catchabilityp[chatTANq].q
mu 0.3
cv 0.2
```

## 6.5. Ageing error

CASAL2 can apply ageing error to expected age frequency generated by the model. The ageing error is applied as a misclassification matrix, which has the effect of 'smearing' the expected age frequencies. This is mimicking the error involved in identifying the age of individuals. For example fish species are aged by reading the ear bones (otoliths) which can be quite difficult depending on the species. These are used in calculating the fits to the observed values, and hence the contribution to the total objective function.

Ageing error is optional, and if it is used, it may be omitted for any individual time series. Different ageing error models may be applied for different observation commands. See Section 7.14 for reporting the misclassification matrix at the end of model run.

The ageing error models implemented are,

1. None: The default model is to apply no ageing error.
2. Off by one: Proportion  $p_1$  of individuals of each age  $a$  are misclassified as age  $a - 1$  and proportion  $p_2$  are misclassified as age  $a + 1$ . Individuals of age  $a < k$  are not misclassified. If there is no plus group in the population model, then proportion  $p_2$  of the oldest age class will 'fall off the edge' and disappear.
3. Normal: Individuals of age  $a$  are classified as ages which are normally distributed with mean  $a$  and constant c.v.  $c$ . As above, if there is no plus group in the population model, some individuals of the older age classes may disappear. If  $c$  is high enough, some of the younger age classes may 'fall off the other edge'. Individuals of age  $a < k$  are not misclassified.

Note that the expected values (fits) reported by CASAL2 for observations with ageing error will have had the ageing error applied.

## 6.6. Simulating observations

CASAL2 can generate simulated observations for a given model with given parameter values using `casal2 -s 1` (To simulate one set of simulated observations). Simulated observations are randomly distributed values, generated according to the error assumptions defined for each observation, around fits calculated from one or more sets of the 'true' parameter values. Simulating from a set of parameters can be used to generate observations from an operating model or as a form of parametric bootstrap.

The procedure CASAL2 uses for simulating observations is to first run using the 'true' parameter values and generate the expected values. Then, if a set of observations uses ageing error, ageing error is applied. Finally a random value for each observed value is generated based on (i) the expected values, (ii) the type of likelihood specified, and (iii) the variability parameters (e.g., `error_value` and `process_error`).

Methods for generating the random error, and hence simulated values, depend on the specific likelihood type of each observation.

1. Normal likelihood parameterised by c.v.: Let  $E_i$  be the fitted value for observation  $i$ , and  $c_i$  be the corresponding c.v. (adjusted by the process error if applicable). Each simulated observation value  $S_i$  is generated as an independent normal deviate with mean  $E_i$  and standard deviation  $E_i c_i$ .
2. Log-normal likelihood: Let  $E_i$  be the fitted value for observation  $i$  and  $c_i$  be the corresponding c.v. (adjusted by the process error if applicable). Each simulated observation value  $S_i$  is generated as an independent lognormal deviate with mean and standard deviation (on the natural scale, not the log-scale) of  $E_i$  and  $E_i c_i$  respectively. The robustification parameter  $\delta$  is ignored.
3. Multinomial likelihood: Let  $E_i$  be the fitted value for observation  $i$ , for  $i$  between 1 and  $n$ , and let  $N$  be the sample size (adjusted by process error if applicable, and then rounded up to the next whole number). The robustification parameter  $\delta$  is ignored. Then,
  - a) A sample of  $N$  values from 1 to  $n$  is generated using the multinomial distribution, using sample probabilities proportional to the values of  $E_i$ .
  - b) Each simulated observation value  $S_i$  is calculated as the proportion of the  $N$  sampled values equalling  $i$
  - c) The simulated observation values  $S_i$  are then rescaled so that their sum is equal to 1
4. Binomial and the normal approximation to the binomial likelihoods: Let  $E_i$  be the fitted value for observation  $i$ , for  $i$  between 1 and  $n$ , and  $N_i$  the corresponding equivalent sample size (adjusted by process error if applicable, and then rounded up to the next whole number). The robustification parameter  $\delta$  is ignored. Then,
  - a) A sample of  $N_i$  independent binary variates is generated, equalling 1 with probability  $E_i$
  - b) The simulated observation value  $S_i$  is calculated as the sum of these binary variates divided by  $N_i$

**An important note when simulating:** CASAL2 will **not** automatically report simulated observations when users undertake a `casal2 -s 1` run, you must write an explicit report using the `simulated_observation` report (`@report[label].type=observation`). See Section 7 for more information on how to write this report.

## 6.7. Pseudo-observations

CASAL2 can generate expected values for observations without them contributing to the total objective function. These are called pseudo-observations, and can be used to either generate the expected values from CASAL2 for reporting or diagnostic purposes. To define an observation as a pseudo-observation, use the command `@observation[label].likelihood=none`. Any observation type can be used as a pseudo-observation. CASAL2 can also generate simulated observations from pseudo-observations. Note that;

- Output will only be generated if a report command `@report[label].type=observation` is specified.
- The observed values should be supplied (even if they are ‘dummy’ observation). These will be processed by CASAL2 as if they were actual observation values, and must conform to the validations carried out for the other types of likelihood.
- The subcommands `likelihood`, `obs`, `error_value` and `process_error` have no effect when generating the expected values for the pseudo-observation.
- When simulating observations, CASAL2 needs the subcommand `simulation_likelihood` to tell it what sort of likelihood to use. In this case, the `obs`, `error_value` and `process_error` are used to determine the appropriate terms to use for the likelihood when simulating.

## 6.8. Residuals

CASAL2 will only print the usual residual (i.e. observed less fitted) using the report type `@report.type=observation`. For an observation  $O$  and  $F$  the corresponding fit ( $=qE$  for relative observations), then

- Residuals =  $O - F$

Pearson and Normalised residuals can be generated using CASAL2 **R** package with-in the **R** environment. For specific R functions see Section 15. The definitions used in the calculations are as follows,

1. *Pearson residuals* attempt to express the residual relative to the variability of the observation, and are defined as  $(O-F)/\text{std.dev.}(O)$ , where  $\text{std.dev.}(O)$  is calculated as
  - $F \times cv$  for normal, lognormal, robustified lognormal, and normal-log error distributions.
  - $s$  for normal-by-standard deviation error distributions.
  - $\sqrt{\frac{Z(F,r)(1-Z(F,r))}{N}}$  for multinomial or binomial likelihoods.
  - $\sqrt{\frac{(F+r)(1-F+r)}{N}}$  for binomial-approx likelihood likelihoods.
2. *Normalised residuals* to express the residual on a standard normal scale, and are defined as:
  - Equal to the Pearson residuals for normal error distributions.
  - $(\log(O/F)+0.5\sigma^2)/\sigma$  for lognormal (including robustified lognormal) error distributions, where  $\sigma = \sqrt{\log(1+cv^2)}$ .
  - $\log(O/F)/\sigma$  for normal-log error distributions, again with  $\sigma = \sqrt{\log(1+cv^2)}$ .
  - And are otherwise undefined.

where  $Z(F, r)$  is the robustifying term on  $F$  (fit or expectation of the observation). This robustifying is described earlier in the likelihood section.

---

## 7. The report section

The report section specifies the printouts and other output from the model. CASAL2 does not, in general, produce any output unless specified by a valid `@report` block.

### 7.1. Report command block format

Reports from CASAL2 can be defined to print partition and states objects at a particular point in time, observation summaries, estimated and derived parameter values, and objective function values.

```
@report observation_age ## label of report
type observation ## Type of report
observation age_1990 ## label corresponding to an @observation report, shown below

@observation age_1990
type proportion_at_age
year 1990
plus_group
etc ...
```

### 7.2. Report block output format

Reports from CASAL2 have a standard style (with one exception, the `output_parameters` report, see below). The standard style is that reports are prefixed with an asterisk followed by a user-defined label and type of report in brackets (e.g., `*label (type)`), with the report ending with the line `*end`. For example,

```
*My_report (type)
...
*end
```

This report block output format should make it easier for other software packages to read and process CASAL2 output. The `extract` functions in the **R** CASAL2 package use this information to identify and read CASAL2 output.

The `output_parameters` report does not print either a header or `*end` at the end of the report block. This is because the `output_parameters` report is designed to provide a single line vector of the estimated parameter values, or multiple lines for more than one set, which can be read by CASAL2 with the command `casal2 -i`. This is a specialised report for the `casal2 -o filename` command.

For estimated values in standard output use the `type=estimate_value` report.

Reports can be defined in a `@report` command block but may not be output, e.g., a report to print the partition for a year and/or timestep that does not exist, or reporting the covariance matrix when not estimation run mode.

Certain reports are associated with certain CASAL2 run modes. These reports are ignored by CASAL2 and the program will not generate any output for these reports, although they must still conform to CASAL2 syntax requirements.

Not all reports will be generated in all run modes. Some reports are only available in some run modes. For example, when simulating, only the simulation reports will be output.

### 7.3. Print the partition at the end of an initialisation

This report prints the partition following the initialisation phase, which includes the numbers of individuals in each age class and category in the partition. This report will print out in run modes `-r`, `-e`, `-f`.

### 7.4. Print the partition

This report prints the numbers of individuals in each age class and category in the partition for each given year or given years and timestep. This report is evaluated at the end of the timestep in the given year(s). This report will print out in run modes `-r`, `-e`, `-f`.

### 7.5. Print the age length and length weight values

This report prints the length and weight value for each age class and category in the partition for each given year or given years and timestep. This report is evaluated at the end of the timestep in the given year(s). This report will print out in run modes `-r`, `-e`, `-f`.

```
@report length_weight_at_age
type partition_mean_weight
time_step step2
years 1900:2013
```

### 7.6. Print a process summary

Depending on the process, different summaries are produced. These reports typically detail the type of process, its parameters and other options, and any associated details. This report will print out in run modes `-r`, `-e`, `-f`.

### 7.7. Print derived quantities

This report prints the description of the derived quantity, and the values of the derived quantity as recorded in the model state, for each year of the model, and for all years in the initialisation phase. This report will print out in run modes `-r`, `-e`, `-f`.

### 7.8. Print the estimated parameters

This report prints a summary of the estimated parameters using the type `estimate_summary`, including the parameter name, lower and upper bounds, the label of the prior, and its value. This report will print out in run modes `-r`, `-e`.



### 7.9. Print the estimated parameters in a vector format

This report prints the estimated parameter values out as a vector. The `estimate_values` report prints the name of the parameter, followed by the value for that run. This report will print out in run modes `-r`, `-e`.

### 7.10. Print the objective function

This report prints the total objective function value, the value of all observation likelihood components, the values of all priors, and the value of any penalties that have been incurred. If an individual model run does not incur a penalty, then the penalty will not be reported. This report will print out in run modes `-r`, `-e`, `-f`.

### 7.11. Print the covariance matrix

This report prints the Hessian and covariance matrices if in estimation run mode and if the covariance has been requested by `@minimiser[label].covariance=true`.

### 7.12. Print observations, fits, and residuals

This report prints, for each category or combination of categories, the expected values, residuals (observed – expected), the error value, process error, the total error (i.e., the error value as modified by any additional process error), and the contribution to the total objective function of that individual datum in the observation.

Constants in the likelihood components are often ignored in the objective function score of individual observation values. Hence, the total score from an observation equals the contribution of the objective function scores from each individual observation value plus a constant term (if applicable). In likelihood components without a constant term, the total score from an observation will equal the contribution of the objective function scores from each individual observation value.

If CASAL2 is in simulation run mode, then the contribution to the objective function of each observation is reported as zero.

```
@report Tan_at_age_obs
type observation
observation TAN_AT_AGE
```

### 7.13. Print simulated observations

This report prints a complete set of observation values in the form specified by `@report[label].type=observation`, with observed values replaced by randomly generated simulated values. The output is in a form suitable for use within a CASAL2 input configuration file, reproducing the command and subcommands from the input configuration file. This report will print out in run mode `-s`.

**7.14. Print the ageing error misclassification matrix**

This report prints the ageing error misclassification matrix used to offset observations within during model the model fitting procedure.

**7.15. Print selectivities**

This report prints the values of a selectivity for each age in the partition, for a given year and at then end of a given timestep.

**7.16. Print the random number seed**

This report prints the random number seed used by CASAL2 to initialise the generated random number sequence. Additional runs which use the same random number seed and the same model will produce identical outputs.

**7.17. Print the results of an MCMC**

This report prints the MCMC samples, objective function values, and proposal covariance matrix following an MCMC. This report will print out in run mode `-m`.

**7.18. Print the MCMC samples as they are calculated**

This report prints the MCMC samples for each new *i*th sample as they are calculated while doing an MCMC. The output file will be appended with each new sample as it is calculated by CASAL2. This report will print out in run mode `-m`.

**7.19. Print the MCMC objective function values as they are calculated**

This report prints the MCMC objective function values, along with the proposal covariance matrix, for each new *i*th sample as they are calculated while doing an MCMC. The output file will be appended with each new set of objective function values as it is calculated by CASAL2. This report will print out in run mode `-m`.

**7.20. Print time varying parameters**

This report prints all `@time_varying` blocks with the values and years in which they were specified. This report will print out in run modes `-r`, `-e`, `-m`.

```
@report time_varying_parameters
type time_varying
```

**7.21. Tabular reporting format**

An alternative reporting framework to the standard output is the tabular reporting format. Tabular reporting is used with multi-line `-i` input files (like the MCMC sample or `-o` outputs). Tabular reports will print out a row that will correspond with each row of the `-i` input files.

Tabular reporting is specified using the `--tabular` argument (`casal2 -r --tabular -i file_name`).

Derived quantities, processes, observations, and `estimate_values` are the only report types that can be output with this format. For each input file the output will begin with the names of each column followed by a multi-line report ending with the `*end` syntax.

These tables can be read with **R** using the `CASAL2` package. An example usage is reading in files of MCMC posterior values of derived quantities, which can then be plotted. This command is the same as running `casal -v` in `CASAL`.



---

## 8. Population command and subcommand syntax

### 8.1. Model structure

**@model** *label*     Define an object of type *model*

**start\_year**     Define the first year of the model, immediately following initialisation

Type: non-negative integer

Default: No Default

Value: R, Defines the first year of the model,  $\geq 1$ , e.g. 1990,

**final\_year**     Define the final year of the model, excluding years in the projection period

Type: non-negative integer

Default: No Default

Value: Defines the last year of the model, i.e., the model is run from start\_year to final\_year

**min\_age**     Minimum age of individuals in the population

Type: non-negative integer

Default: 0

Value: R,  $0 \leq \text{age}_{\min} \leq \text{age}_{\max}$ ,

**max\_age**     Maximum age of individuals in the population

Type: non-negative integer

Default: 0

Value: R,  $0 \leq \text{age}_{\min} \leq \text{age}_{\max}$ ,

**age\_plus**     Define the oldest age or extra length midpoint ,plus group size, as a plus group

Type: boolean

Default: false

Value: true, false

**initialisation\_phases**     Define the labels of the phases of the initialisation

Type: string vector

Default: true

Value: R, A list of valid labels defined by @initialisation\_phase,

**time\_steps**     Define the labels of the time steps, in the order that they are applied, to form the annual cycle

Type: string vector

Default: No Default

Value: R, A list of valid labels defined by @time\_step,

**projection\_final\_year**     Define the final year of the model in projection mode

Type: non-negative integer

Default: 0

Value: R, Defines the last year of the projection period, i.e., the projection period runs from final\_year+1 to projection\_final\_year. For the default, 0, no projections are run.,

`length_bins`      The minimum length in each length bin

Type: non-negative integer vector

Default: true

Value:  $R, 0 \leq \text{length}_{\min} \leq \text{length}_{\max}$ ,

`length_plus`      Specify whether there is a length plus group or not

Type: boolean

Default: true

Value: true, false

`length_plus_group`      Mean length of length plus group

Type: non-negative integer

Default: 0

Value:  $R, \text{length}_{\max}, \text{length\_plus\_group}$ ,

`base_weight_units`      Define the units for the base weight. This will be the default unit of any weight input parameters

Type: string

Default: tonnes

Value: grams, kgs or tonnes

Allowed Values: grams, tonnes, kgs

## 8.2. Initialisation

**@initialisation\_phase** *label*      Define an object of type *initialisation\_phase*

*label*      The label of the initialisation phase

Type: string

Default: No Default

*type*      The type of initialisation

Type: string

Default: iterative

### 8.2.1. @initialisation\_\_phase[label].type=cinitial

*categories*      The list of categories for the Cinitial initialisation

Type: string vector

Default: No Default

**8.2.2. @initialisation\_\_phase[label].type=derived**

`insert_processes` Additional processes not defined in the annual cycle, that are to be inserted into this initialisation phase

Type: string vector

Default: true

`exclude_processes` Processes in the annual cycle to be excluded from this initialisation phase

Type: string vector

Default: true

`casal_initialisation_switch` Run an extra annual cycle to evaluate equilibrium SSB's. Warning - if true, this may not correctly evaluate the equilibrium state. Use true if attempting to replicate a legacy CASAL model

Type: boolean

Default: false

**8.2.3. @initialisation\_\_phase[label].type=iterative**

`years` The number of iterations, years, over which to execute this initialisation phase

Type: non-negative integer

Default: No Default

`insert_processes` ,years, over which to execute this initialisation phase

Type: string vector

Default: true

`exclude_processes` Processes in the annual cycle to be excluded from this initialisation phase

Type: string vector

Default: true

`convergence_years` The iteration, year, when the test for convergence, lambda, is evaluated

Type: non-negative integer vector

Default: true

`lambda` The maximum value of the absolute sum of differences, lambda, between the partition at year-1 and year that indicates successful convergence

Type: constant

Default: 0.0

**8.2.4. @initialisation\_\_phase[label].type=state\_category\_by\_age**

`categories` The list of categories for the category state initialisation

Type: string vector

Default: No Default

`min_age`      The minimum age of values supplied in the definition of the category state  
Type: non-negative integer  
Default: No Default

`max_age`      The minimum age of values supplied in the definition of the category state  
Type: non-negative integer  
Default: No Default

### 8.3. Categories

**@categories** *label*      Define an object of type *categories*

`format`      The format that the category names adhere too  
Type: string  
Default: No Default

`names`      The names of the categories to be used in the model  
Type: string vector  
Default: No Default

`years`      The years that individual categories will be active for. This overrides the model values  
Type: string vector  
Default: true

`age_lengths`      R, The labels of `age_length` objects that are assigned to categories,  
Type: string vector  
Default: true

`length_weight`      R, The labels of the `length_weight` objects that are assigned to categories,  
Type: string vector  
Default: true

`age_weight`      R, The labels of the `age_weight` objects that are assigned to categories,  
Type: string vector  
Default: true

### 8.4. Time-steps

**@time\_step** *label*      Define an object of type *time\_step*

`label`      The label of the timestep  
Type: string  
Default: No Default



`processes`      The labels of the processes for this time step in the order that they occur  
Type: string vector  
Default: No Default

## 8.5. Processes

**@process** *label*      Define an object of type *process*

`label`      The label of the process  
Type: string  
Default: No Default

`type`      The type of process  
Type: string  
Default: ""

### 8.5.1. @process[label].type=ageing

`categories`      The labels of the categories  
Type: string vector  
Default: No Default

### 8.5.2. @process[label].type=growth\_basic

`categories`      The labels of the categories  
Type: string vector  
Default: No Default

`number_of_growth_episodes`      Number of growth episodes per year  
Type: non-negative integer  
Default: No Default

`growth_time_steps`      Time step in which each growth episode occurs  
Type: string vector  
Default: No Default

`cv`      c.v. for the growth model  
Type: constant  
Default: 0.0  
Lower Bound: 0.0 (inclusive)

`sigma_min`      Lower bound on sigma for the growth model  
Type: constant  
Default: 0.0

### 8.5.3. `@process[label].type=maturation`

`from`      List of categories to mature from

    Type: string vector

    Default: No Default

`to`        List of categories to mature too

    Type: string vector

    Default: No Default

`selectivities`      List of selectivities to use for maturation

    Type: string vector

    Default: No Default

`years`      The years to be associated with rates

    Type: non-negative integer vector

    Default: No Default

`rates`      The rates to mature for each year

    Type: estimable vector

    Default: No Default

### 8.5.4. `@process[label].type=mortality_constant_rate`

`categories`      List of categories labels

    Type: string vector

    Default: No Default

`m`        Mortality rates

    Type: estimable

    Default: No Default

    Lower Bound: 0.0 (inclusive)

`time_step_ratio`      Time step ratios for the mortality rates

    Type: constant vector

    Default: true

    Lower Bound: 0.0 (inclusive)

    Upper Bound: 1.0 (inclusive)

**8.5.5. @process[label].type=mortality\_event**

categories      Categories

Type: string vector

Default: No Default

years      Years in which to apply the mortality process

Type: non-negative integer vector

Default: No Default

catches      The number of removals ,catches, to apply for each year

Type: estimable vector

Default: No Default

u\_max      Maximum exploitation rate , $U_{max}$ ,

Type: estimable

Default: 0.99

selectivities      List of selectivities

Type: string vector

Default: No Default

penalty      The label of the penalty to apply if the total number of removals cannot be taken

Type: string

Default: ""

**8.5.6. @process[label].type=mortality\_event\_biomass**

categories      Category labels

Type: string vector

Default: No Default

selectivities      The labels of the selectivities for each of the categories

Type: string vector

Default: No Default

years      Years in which to apply the mortality process

Type: non-negative integer vector

Default: No Default

catches      The biomass of removals ,catches, to apply for each year

Type: estimable vector

Default: No Default

u\_max      Maximum exploitation rate , $U_{max}$ ,

Type: estimable  
Default: 0.99

penalty      The label of the penalty to apply if the total biomass of removals cannot be taken  
Type: string  
Default: ""

### 8.5.7. `@process[label].type=mortality_holling_rate`

prey\_categories      Prey Categories labels  
Type: string vector  
Default: No Default

predator\_categories      Predator Categories labels  
Type: string vector  
Default: No Default

is\_abundance      Is vulnerable amount of prey and predator an abundance [true] or biomass [false]  
Type: boolean  
Default: true

a      parameter a  
Type: estimable  
Default: No Default  
Lower Bound: 0.0 (inclusive)

b      parameter b  
Type: estimable  
Default: No Default  
Lower Bound: 0.0 (inclusive)

x      This parameter controls the type of functional form, Holling function type 2 , $x=2$ , or 3 , $x=3$ , or generalised ,Michaelis Menten,  $x=1$ ,  
Type: estimable  
Default: No Default  
Lower Bound: 1.0 (inclusive)

u\_max      Maximum exploitation rate , $U_{max}$ ,  
Type: constant  
Default: No Default  
Lower Bound: 0.0 (inclusive)  
Upper Bound: 1.0 (inclusive)

prey\_selectivities      Selectivities for prey categories

Type: string vector  
Default: true

predator\_selectivities      Selectivities for predator categories  
Type: string vector  
Default: true

penalty      Label of penalty to be applied  
Type: string  
Default: ""

years      Years in which to apply the mortality process  
Type: non-negative integer vector  
Default: No Default

#### **8.5.8. @process[label].type=mortality\_initialisation\_event**

categories      Categories  
Type: string vector  
Default: No Default

catch      The number of removals ,catches, to apply for each year  
Type: estimable  
Default: No Default

u\_max      Maximum exploitation rate , $U_{max}$ ,  
Type: estimable  
Default: 0.99

selectivities      List of selectivities  
Type: string vector  
Default: No Default

penalty      The label of the penalty to apply if the total number of removals cannot be taken  
Type: string  
Default: ""

#### **8.5.9. @process[label].type=mortality\_initialisation\_event\_biomass**

categories      Categories  
Type: string vector  
Default: No Default

catch      The number of removals ,catches, to apply for each year

Type: estimable  
Default: No Default

`u_max`      Maximum exploitation rate  $U_{max}$ ,  
Type: estimable  
Default: 0.99

`selectivities`      List of selectivities  
Type: string vector  
Default: No Default

`penalty`      The label of the penalty to apply if the total number of removals cannot be taken  
Type: string  
Default: ""

#### **8.5.10. `@process[label].type=mortality_instantaneous`**

`categories`      Categories for instantaneous mortality  
Type: string vector  
Default: No Default

`m`      Natural mortality rates for each category  
Type: estimable  
Default: No Default  
Lower Bound: 0.0 (inclusive)

`time_step_ratio`      Time step ratios for natural mortality  
Type: constant vector  
Default: true  
Lower Bound: 0.0 (inclusive)  
Upper Bound: 1.0 (inclusive)

`selectivities`      The selectivities to apply on the categories for natural mortality  
Type: string vector  
Default: No Default

#### **8.5.11. `@process[label].type=mortality_instantaneous_retained`**

`categories`      Categories for instantaneous mortality  
Type: string vector  
Default: No Default

`m`      Natural mortality rates for each category

Type: estimable  
Default: No Default  
Lower Bound: 0.0 (inclusive)

time\_step\_ratio      Time step ratios for natural mortality  
Type: constant vector  
Default: true  
Lower Bound: 0.0 (inclusive)  
Upper Bound: 1.0 (inclusive)

selectivities      The selectivities to apply on the categories for natural mortality  
Type: string vector  
Default: No Default

### **8.5.12. @process[label].type=mortality\_prey\_suitability**

prey\_categories      Prey Categories labels  
Type: string vector  
Default: No Default

predator\_categories      Predator Categories labels  
Type: string vector  
Default: No Default

consumption\_rate      Predator consumption rate  
Type: estimable  
Default: No Default  
Lower Bound: 0.0 (inclusive)  
Upper Bound: 1.0 (inclusive)

electivities      Prey Electivities  
Type: estimable vector  
Default: No Default  
Lower Bound: 0.0 (inclusive)  
Upper Bound: 1.0 (inclusive)

u\_max      Umax  
Type: constant  
Default: No Default  
Lower Bound: 0.0 (inclusive)  
Upper Bound: 1.0 (inclusive)

prey\_selectivities      Selectivities for prey categories  
Type: string vector  
Default: No Default

`predator_selectivities`      Selectivities for predator categories  
Type: string vector  
Default: No Default

`penalty`      Label of penalty to be applied  
Type: string  
Default: ""

`years`      Year that process occurs  
Type: non-negative integer vector  
Default: No Default

### **8.5.13. @process[label].type=recruitment\_beverton\_holt**

`categories`      Category labels  
Type: string vector  
Default: No Default

`r0`      R0  
Type: estimable  
Default: false

`b0`      B0  
Type: estimable  
Default: false

`proportions`      Proportions  
Type: estimable  
Default: No Default

`age`      Age to recruit at  
Type: non-negative integer  
Default: true

`ssb_offset`      Spawning biomass year offset  
Type: non-negative integer  
Default: true

`steepness`      Steepness  
Type: estimable  
Default: 1.0

`ssb`      SSB Label ,derived quantity,  
Type: string  
Default: No Default



`b0_initialisation_phase`      Initialisation phase Label that b0 is from  
Type: string  
Default: ""

`yces_values`      YCS Values  
Type: estimable vector  
Default: No Default

`yces_years`      Recruitment years. A vector of years that relates to the year of the spawning event that created this cohort  
Type: non-negative integer vector  
Default: false

`standardise_yces_years`      Years that are included for year class standardisation  
Type: non-negative integer vector  
Default: true

#### **8.5.14. `@process[label].type=recruitment_beverton_holt_with_deviations`**

`categories`      Category labels  
Type: string vector  
Default: No Default

`r0`      R0  
Type: estimable  
Default: false

`b0`      B0  
Type: estimable  
Default: false

`proportions`      Proportions  
Type: estimable  
Default: No Default

`age`      Age to recruit at  
Type: non-negative integer  
Default: true

`ssb_offset`      Spawning biomass year offset  
Type: non-negative integer  
Default: true

steepness      Steepness

Type: estimable

Default: 1.0

ssb      SSB Label ,derived quantity,

Type: string

Default: No Default

sigma\_r      Sigma r

Type: estimable

Default: No Default

b\_max      Max bias adjustment

Type: estimable

Default: 0.85

Lower Bound: 0.0 (inclusive)

Upper Bound: 1.0 (inclusive)

last\_year\_with\_no\_bias      Last year with no bias adjustment

Type: non-negative integer

Default: false

first\_year\_with\_bias      First year with full bias adjustment

Type: non-negative integer

Default: false

last\_year\_with\_bias      Last year with full bias adjustment

Type: non-negative integer

Default: false

first\_recent\_year\_with\_no\_bias      First recent year with no bias adjustment

Type: non-negative integer

Default: false

b0\_initialisation\_phase      Initialisation phase Label that b0 is from

Type: string

Default: ""

deviation\_values      Recruitment deviation values

Type: estimable vector

Default: No Default

deviation\_years      Recruitment years. A vector of years that relates to the year of the spawning event that created this cohort

Type: non-negative integer vector

Default: false

**8.5.15. @process[label].type=recruitment\_constant**

categories      Categories

Type: string vector

Default: No Default

proportions      Proportions

Type: estimable

Default: true

length\_bins      The length bins recruits are uniformly distributed over, when recruitment occurs

Type: non-negative integer vector

Default: No Default

r0      R0

Type: estimable

Default: No Default

Lower Bound: 0.0 (exclusive)

**8.5.16. @process[label].type=survival\_constant\_rate**

categories      List of categories

Type: string vector

Default: No Default

s      Survival rates

Type: estimable

Default: No Default

time\_step\_ratio      Time step ratios for S

Type: constant vector

Default: true

selectivities      Selectivity label

Type: string vector

Default: No Default

**8.5.17. @process[label].type=tag\_by\_age**

from      Categories to transition from

Type: string vector

Default: No Default

to      Categories to transition to

Type: string vector

Default: No Default

min\_age      Minimum age to transition

Type: non-negative integer

Default: No Default

max\_age      Maximum age to transition

Type: non-negative integer

Default: No Default

penalty      Penalty label

Type: string

Default: ""

u\_max      U Max

Type: constant

Default: 0.99

years      Years to execute the transition in

Type: non-negative integer vector

Default: No Default

initial\_mortality

Type: constant

Default: 0

initial\_mortality\_selectivity

Type: string

Default: ""

loss\_rate

Type: constant vector

Default: No Default

loss\_rate\_selectivities

Type: string vector

Default: true

selectivities

Type: string vector

Default: No Default

n

Type: constant vector

Default: true

### 8.5.18. @process[label].type=tag\_by\_length

from Categories to transition from

Type: string vector

Default: No Default

to ategories to transition to

Type: string vector

Default: No Default

penalty Penalty label

Type: string

Default: ""

u\_max U Max

Type: constant

Default: 0.99

years Years to execute the transition in

Type: non-negative integer vector

Default: No Default

initial\_mortality

Type: constant

Default: 0

initial\_mortality\_selectivity

Type: string

Default: ""

selectivities

Type: string vector

Default: No Default

n

Type: constant vector

Default: true

### 8.5.19. @process[label].type=tag\_loss

categories      List of categories

    Type: string vector

    Default: No Default

tag\_loss\_rate      Tag Loss rates

    Type: constant vector

    Default: No Default

time\_step\_ratio      Time step ratios for Tag Loss

    Type: constant vector

    Default: true

tag\_loss\_type      Type of tag loss

    Type: string

    Default: No Default

selectivities      Selectivities

    Type: string vector

    Default: No Default

year      The year the first tagging release process was executed

    Type: non-negative integer

    Default: No Default

### 8.5.20. @process[label].type=transition\_category

from      From

    Type: string vector

    Default: No Default

to      To

    Type: string vector

    Default: No Default

proportions      Proportions

    Type: estimable

    Default: No Default

selectivities      Selectivity names

    Type: string vector

    Default: No Default

**8.5.21. @process[label].type=transition\_category\_by\_age**

from      Categories to transition from

Type: string vector

Default: No Default

to        Categories to transition to

Type: string vector

Default: No Default

min\_age    Minimum age to transition

Type: non-negative integer

Default: No Default

max\_age    Maximum age to transition

Type: non-negative integer

Default: No Default

penalty    Penalty label

Type: string

Default: ""

u\_max      U Max

Type: constant

Default: 0.99

years      Years to execute the transition in

Type: non-negative integer vector

Default: No Default

**8.6. Time varying parameters**

**@time\_varying** *label*    Define an object of type *time\_varying*

label      The time-varying label

Type: string

Default: No Default

type        The time-varying type

Type: string

Default: ""

years      Years in which to vary the values

Type: non-negative integer vector

Default: No Default

`parameter`      The name of the parameter to time vary  
Type: string  
Default: No Default

### 8.7. Derived quantities

**@derived\_quantity** *label*      Define an object of type *derived\_quantity*

`label`      Label of the derived quantity  
Type: string  
Default: No Default

`type`      Type of derived quantity  
Type: string  
Default: No Default

`time_step`      The time step in which to calculate the derived quantity after  
Type: string  
Default: No Default

`categories`      The list of categories to use when calculating the derived quantity  
Type: string vector  
Default: No Default

`selectivities`      A list of one selectivity  
Type: string vector  
Default: No Default

`time_step_proportion`      Proportion through the mortality block of the time step when  
calculated  
Type: constant  
Default: 0.5  
Lower Bound: 0.0 (inclusive)  
Upper Bound: 1.0 (inclusive)

`time_step_proportion_method`      Method for interpolating for the proportion through the  
mortality block  
Type: string  
Default: weighted\_sum  
Allowed Values: weighted\_sum, weighted\_product

`values`  
Type: Addressable vector  
Default: No Default



**8.7.1. @derived\_\_quantity[label].type=abundance**

**8.7.2. @derived\_\_quantity[label].type=biomass**

## 8.8. Age-length relationship

**@age\_length** *label* Define an object of type *age\_length*

*label* Label of the age length relationship

Type: string

Default: No Default

*type* Type of age length relationship

Type: string

Default: No Default

*time\_step\_proportions* the fraction of the year applied in each time step that is added to the age for the purposes of evaluating the length, i.e., a value of 0.5 for a time step will evaluate the length of individuals at age+0.5 in that time step

Type: constant vector

Default: true

Lower Bound: 0.0 (inclusive)

Upper Bound: 1.0 (inclusive)

*distribution* The assumed distribution for the growth curve

Type: string

Default: normal

*cv\_first* CV for the first age class

Type: estimable

Default: 0.0

Lower Bound: 0.0 (inclusive)

*cv\_last* CV for last age class

Type: estimable

Default: 0.0

Lower Bound: 0.0 (inclusive)

*casal\_switch* If true, use the ,less accurate, equation for the cumulative normal function as was used in the legacy version of CASAL.

Type: boolean

Default: false

*by\_length* Specifies if the linear interpolation of CV's is a linear function of mean length at age. Default is just by age

Type: boolean

Default: true

### 8.8.1. @age\_\_length[label].type=data

external\_gaps

Type: string

Default: mean

Allowed Values: mean, nearest\_neighbour

internal\_gaps

Type: string

Default: mean

Allowed Values: mean, nearest\_neighbour, interpolate

length\_weight      The label from an associated length-weight block

Type: string

Default: No Default

time\_step\_measurements\_were\_made      Time step label for which size-at-age data are provided

Type: string

Default: No Default

### 8.8.2. @age\_\_length[label].type=none

### 8.8.3. @age\_\_length[label].type=schnute

y1      Define the y1 parameter of the Schnute relationship

Type: estimable

Default: No Default

y2      Define the y2 parameter of the Schnute relationship

Type: estimable

Default: No Default

tau1      Define the  $\tau_1$  parameter of the Schnute relationship

Type: estimable

Default: No Default

tau2      Define the  $\tau_2$  parameter of the Schnute relationship

Type: estimable

Default: No Default

a      Define the  $a$  parameter of the Schnute relationship

Type: estimable

Default: No Default

Lower Bound: 0.0 (inclusive)

**b** Define the  $b$  parameter of the Schnute relationship

Type: estimable

Default: No Default

Lower Bound: 0.0 (exclusive)

**length\_weight** Define the label of the associated length-weight relationship

Type: string

Default: No Default

#### 8.8.4. @age\_\_length[label].type=von\_bertalanffy

**linf** Define the  $L_{infinity}$  parameter of the von Bertalanffy relationship

Type: estimable

Default: No Default

Lower Bound: 0.0 (inclusive)

**k** Define the  $k$  parameter of the von Bertalanffy relationship

Type: estimable

Default: No Default

Lower Bound: 0.0 (inclusive)

**t0** Define the  $t_0$  parameter of the von Bertalanffy relationship

Type: estimable

Default: No Default

**length\_weight** Define the label of the associated length-weight relationship

Type: string

Default: No Default

### 8.9. Length-weight

**@length\_weight label** Define an object of type *length\_weight*

**label** The label of the length-weight relationship

Type: string

Default: No Default

**type** The type of the length-weight relationship

Type: string

Default: No Default

### 8.10. Selectivities

**@selectivity label** Define an object of type *selectivity*

`label`      The label for this selectivity

    Type: string

    Default: No Default

`type`      The type of selectivity

    Type: string

    Default: No Default

`length_based`      Is the selectivity length based

    Type: boolean

    Default: false

`intervals`      Number of quantiles to evaluate a length based selectivity over the age length distribution

    Type: non-negative integer

    Default: 5

`partition_type`      The type of partition this selectivity will support, Defaults to same as the model

    Type: string

    Default: model

    Allowed Values: model, age, length, hybrid

`values`

    Type: Addressable vector

    Default: No Default

`length_values`

    Type: Addressable vector

    Default: No Default

## 9. Estimation command and subcommand syntax

### 9.1. Estimation methods

**@estimate** *label*      Define an object of type *estimate*

`label`      The label of the estimate

    Type: string

    Default: ""

`type`      The prior type for the estimate

    Type: string

    Default: No Default

<code>parameter</code>	The name of the parameter to estimate in the model Type: string Default: No Default
<code>lower_bound</code>	The lower bound for the parameter Type: constant Default: No Default
<code>upper_bound</code>	The upper bound for the parameter Type: constant Default: No Default
<code>same</code>	List of parameters that are constrained to have the same value as this parameter Type: string vector Default: ""
<code>estimation_phase</code>	The first estimation phase to allow this to be estimated Type: non-negative integer Default: 1
<code>mcmc</code>	Indicates if this parameter is estimated at the point estimate but fixed during MCMC estimation run Type: boolean Default: false
<code>transformation</code>	Type of simple transformation to apply to estimate Type: string Default: ""
<code>transform_with_jacobian</code>	Apply jacobian during transformation Type: boolean Default: false
<code>prior_applies_to_transform</code>	Does the prior apply to the transformed parameter? a legacy switch, see Manual for more information Type: boolean Default: false

## 9.2. Point estimation

<code>@minimiser</code>	<code>label</code> Define an object of type <i>minimiser</i>
<code>label</code>	The minimiser label Type: string Default: No Default

`type`      The type of minimiser to use

Type: string

Default: No Default

`active`      Indicates if this minimiser is active

Type: boolean

Default: false

`covariance`      Indicates if a covariance matrix should be generated

Type: boolean

Default: true

### 9.3. Markov chain Monte Carlo (MCMC)

**@mcmc** *label*      Define an object of type *mcmc*

`label`      The label of the MCMC

Type: string

Default: No Default

`type`      The type of MCMC

Type: string

Default: ""

`length`      The number of iterations in for the MCMC chain

Type: non-negative integer

Default: No Default

`active`      Indicates if this is the active MCMC algorithm

Type: boolean

Default: true

`print_default_reports`      Indicates if the output prints the default reports

Type: boolean

Default: true

`step_size`      Initial stepsize ,as a multiplier of the approximate covariance matrix,

Type: constant

Default: 0.02

### 9.4. Profiles

**@profile** *label*      Define an object of type *profile*

`label`      Label

Type: string  
Default: ""

`steps`      The number of steps to take between the lower and upper bound  
Type: non-negative integer  
Default: No Default

`lower_bound`      The lower bounds  
Type: constant  
Default: No Default

`upper_bound`      The upper bounds  
Type: constant  
Default: No Default

`parameter`      The system parameter to profile  
Type: string  
Default: No Default

`same`      A Parameter that are constrained to have the same value as the parameter being profiled  
Type: string  
Default: ""

## 9.5. Defining catchability constants

**@catchability** *label*      Define an object of type *catchability*

`label`      Label of the catchability  
Type: string  
Default: No Default

`type`      Type of catchability  
Type: string  
Default: No Default

## 9.6. Defining penalties

**@penalty** *label*      Define an object of type *penalty*

`label`      The label of the penalty  
Type: string  
Default: No Default

`type`      The type of penalty

Type: string  
Default: No Default

## 9.7. Defining priors on parameter ratios, differences, and means

**@additional\_prior** *label* Define an object of type *additional\_prior*

*parameter* Name of the parameter to generate additional prior on  
Type: string  
Default: No Default

*label* Label for the additional prior  
Type: string  
Default: No Default

*type* Type of additional prior  
Type: string  
Default: No Default

## 10. Observation command and subcommand syntax

### 10.1. Observation types

The observation types available are:

- Observations of proportions of individuals by age class
- Observations of proportions of individuals by category and age class
- Relative and absolute abundance observations
- Relative and absolute biomass observations

Each type of observation requires a set of subcommands and arguments specific to that process.

**@observation** *label* Define an object of type *observation*

*label* Label  
Type: string  
Default: No Default

*type* Type of observation  
Type: string  
Default: No Default

*likelihood* Type of likelihood to use  
Type: string  
Default: No Default



categories      Category labels to use

Type: string vector

Default: true

delta      Robustification value ,delta, for the likelihood

Type: constant

Default: DELTA

Lower Bound: 0.0 (inclusive)

simulation\_likelihood      Simulation likelihood to use

Type: string

Default: ""

likelihood\_multiplier      Likelihood score multiplier

Type: constant

Default: double

error\_value\_multiplier      Error value multiplier for likelihood

Type: constant

Default: double

#### **10.1.1. @observation[label].type=abundance**

selectivities      Labels of the selectivities

Type: string vector

Default: true

time\_step      The label of time-step that the observation occurs in

Type: string

Default: No Default

#### **10.1.2. @observation[label].type=biomass**

catchability      The time-step of the observation

Type: string

Default: No Default

time\_step      The label of time-step that the observation occurs in

Type: string

Default: No Default

obs      The observed values

Type: string vector

Default: No Default

`years`      The years of the observed values

Type: non-negative integer vector

Default: No Default

`error_value`      The error values of the observed values ,note the units depend on the likelihood,

Type: constant vector

Default: No Default

`selectivities`      Labels of the selectivities

Type: string vector

Default: true

`process_error`      Value for process error

Type: estimable

Default: 0.0

`age_weight_labels`      R,The labels for the `@age_weight` block which corresponds to each category, if you want to use that weight calculation method for biomass calculations,

Type: string vector

Default: ""

### 10.1.3. `@observation[label].type=process_removals_by_age`

`min_age`      Minimum age

Type: non-negative integer

Default: No Default

`max_age`      Maximum age

Type: non-negative integer

Default: No Default

`plus_group`      Use age plus group

Type: boolean

Default: true

`time_step`      The label of time-step that the observation occurs in

Type: string vector

Default: No Default

`tolerance`      Tolerance

Type: constant

Default: double

years      Years for which there are observations  
Type: non-negative integer vector  
Default: No Default

process\_errors      Label of process error to use  
Type: estimable vector  
Default: true

ageing\_error      Label of ageing error to use  
Type: string  
Default: ""

method\_of\_removal      Label of observed method of removals  
Type: string vector  
Default: ""

mortality\_instantaneous\_process      The label of the mortality instantaneous process for the observation  
Type: string  
Default: No Default

#### **10.1.4. @observation[label].type=process\_removals\_by\_age\_retained**

min\_age      Minimum age  
Type: non-negative integer  
Default: No Default

max\_age      Maximum age  
Type: non-negative integer  
Default: No Default

plus\_group      Use age plus group  
Type: boolean  
Default: true

time\_step      The label of time-step that the observation occurs in  
Type: string vector  
Default: No Default

tolerance      Tolerance  
Type: constant  
Default: double

years      Years for which there are observations

Type: non-negative integer vector

Default: No Default

`process_errors`      Label of process error to use

Type: estimable vector

Default: true

`ageing_error`      Label of ageing error to use

Type: string

Default: ""

`method_of_removal`      Label of observed method of removals

Type: string vector

Default: ""

`mortality_instantaneous_process`      The label of the mortality instantaneous process for the observation

Type: string

Default: No Default

### **10.1.5. `@observation[label].type=process_removals_by_age_retained_total`**

`min_age`      Minimum age

Type: non-negative integer

Default: No Default

`max_age`      Maximum age

Type: non-negative integer

Default: No Default

`plus_group`      Use age plus group

Type: boolean

Default: true

`time_step`      The label of time-step that the observation occurs in

Type: string vector

Default: No Default

`tolerance`      Tolerance

Type: constant

Default: double

`years`      Years for which there are observations

Type: non-negative integer vector

Default: No Default

`process_errors`      Label of process error to use  
Type: estimable vector  
Default: true

`ageing_error`      Label of ageing error to use  
Type: string  
Default: ""

`method_of_removal`      Label of observed method of removals  
Type: string vector  
Default: ""

`mortality_instantaneous_process`      The label of the mortality instantaneous process for the observation  
Type: string  
Default: No Default

#### **10.1.6. @observation[label].type=process\_removals\_by\_length**

`length_bins`      Length bins  
Type: constant vector  
Default: No Default

`time_step`      Time step to execute in  
Type: string  
Default: No Default

`length_plus`      Is the last bin a plus group  
Type: boolean  
Default: true

`tolerance`      Tolerance for rescaling proportions  
Type: constant  
Default: double

`years`      Years for which there are observations  
Type: non-negative integer vector  
Default: No Default

`process_errors`      the value of process error  
Type: estimable vector  
Default: true

method\_of\_removal      Label of observed method of removals  
 Type: string  
 Default: ""

mortality\_instantaneous\_process      The label of the mortality instantaneous process for the observation  
 Type: string  
 Default: No Default

### 10.1.7. **@observation[label].type=process\_removals\_by\_length\_retained**

length\_bins      Length bins  
 Type: constant vector  
 Default: No Default

time\_step      Time step to execute in  
 Type: string  
 Default: No Default

length\_plus      Is the last bin a plus group  
 Type: boolean  
 Default: true

tolerance      Tolerance for rescaling proportions  
 Type: constant  
 Default: double

years      Years for which there are observations  
 Type: non-negative integer vector  
 Default: No Default

process\_errors      the value of process error  
 Type: estimable vector  
 Default: true

method\_of\_removal      Label of observed method of removals  
 Type: string  
 Default: ""

mortality\_instantaneous\_process      The label of the mortality instantaneous process for the observation  
 Type: string  
 Default: No Default

**10.1.8. @observation[label].type=process\_removals\_by\_length\_retained\_total**

length\_bins      Length bins

Type: constant vector

Default: No Default

time\_step      Time step to execute in

Type: string

Default: No Default

length\_plus      Is the last bin a plus group

Type: boolean

Default: true

tolerance      Tolerance for rescaling proportions

Type: constant

Default: double

years      Years for which there are observations

Type: non-negative integer vector

Default: No Default

process\_errors      the value of process error

Type: estimable vector

Default: true

method\_of\_removal      Label of observed method of removals

Type: string

Default: ""

mortality\_instantaneous\_process      The label of the mortality instantaneous process for the observation

Type: string

Default: No Default

**10.1.9. @observation[label].type=proportions\_at\_age**

min\_age      Minimum age

Type: non-negative integer

Default: No Default

max\_age      Maximum age

Type: non-negative integer

Default: No Default

plus\_group      Use age plus group  
Type: boolean  
Default: true

time\_step      The label of time-step that the observation occurs in  
Type: string  
Default: No Default

tolerance      Tolerance on the constraint, that for each year the sum of proportions in each age must equal one e.g. tolerance = 0.1 then  $1 - \text{Sum,Proportions}$ , can be as great as 0.1  
Type: constant  
Default: double

years          The years of the observed values  
Type: non-negative integer vector  
Default: No Default

selectivities   Labels of the selectivities  
Type: string vector  
Default: true

process\_errors   Process error  
Type: constant vector  
Default: true

ageing\_error   Label of ageing error to use  
Type: string  
Default: ""

#### 10.1.10. @observation[label].type=proportions\_at\_length

time\_step      The label of time-step that the observation occurs in  
Type: string  
Default: No Default

tolerance      Tolerance for rescaling proportions  
Type: constant  
Default: double

years          Years for which there are observations  
Type: non-negative integer vector  
Default: No Default

selectivities   The labels of the selectivities



Type: string vector  
Default: true

process\_errors      Process error  
Type: constant vector  
Default: true

### 10.1.11. @observation[label].type=proportions\_by\_category

min\_age      Minimum age  
Type: non-negative integer  
Default: No Default

max\_age      Maximum age  
Type: non-negative integer  
Default: No Default

time\_step      The label of time-step that the observation occurs in  
Type: string  
Default: No Default

plus\_group      Use age plus group  
Type: boolean  
Default: true

years      Years for which there are observations  
Type: non-negative integer vector  
Default: No Default

selectivities      The labels of the selectivities  
Type: string vector  
Default: true

categories2      Target Categories  
Type: string vector  
Default: No Default

selectivities2      Target Selectivities  
Type: string vector  
Default: No Default

**10.1.12. @observation[label].type=proportions\_mature\_by\_age**

min\_age      Minimum age

Type: non-negative integer

Default: No Default

max\_age      Maximum age

Type: non-negative integer

Default: No Default

time\_step    The label of time-step that the observation occurs in

Type: string

Default: No Default

plus\_group   Use age plus group

Type: boolean

Default: true

years        Years for which there are observations

Type: non-negative integer vector

Default: No Default

ageing\_error   Label of ageing error to use

Type: string

Default: ""

total\_categories   All category labels that were vulnerable to sampling at the time of this observation ,not including the categories already given,

Type: string vector

Default: true

time\_step\_proportion   Proportion through the mortality block of the time step when the observation is evaluated

Type: constant

Default: double

**10.1.13. @observation[label].type=proportions\_migrating**

min\_age      Minimum age

Type: non-negative integer

Default: No Default

max\_age      Maximum age

Type: non-negative integer

Default: No Default

`time_step`      The label of time-step that the observation occurs in  
Type: string  
Default: No Default

`plus_group`      Use age plus group  
Type: boolean  
Default: true

`years`      Years for which there are observations  
Type: non-negative integer vector  
Default: No Default

`process_errors`      Process error  
Type: constant vector  
Default: true

`ageing_error`      Label of ageing error to use  
Type: string  
Default: ""

`process`      Process label  
Type: string  
Default: No Default

#### **10.1.14. `@observation[label].type=tag_recapture_by_age`**

`min_age`      Minimum age  
Type: non-negative integer  
Default: No Default

`max_age`      Maximum age  
Type: non-negative integer  
Default: No Default

`plus_group`      Use age plus group  
Type: boolean  
Default: true

`years`      Years for which there are observations  
Type: non-negative integer vector  
Default: No Default

`categories2`      The available categories in the partition

Type: string vector  
 Default: No Default

`selectivities`      The labels of the selectivities  
 Type: string vector  
 Default: true

`time_step`      The label of time-step that the observation occurs in  
 Type: string  
 Default: No Default

`selectivities2`      The categories of tagged individuals for the observation  
 Type: string vector  
 Default: No Default

`detection`      Probability of detecting a recaptured individual  
 Type: constant  
 Default: No Default

`time_step_proportion`      Proportion through the mortality block of the time step when the observation is evaluated  
 Type: constant  
 Default: double

#### **10.1.15. `@observation[label].type=tag_recapture_by_length`**

`years`      Years for which there are observations  
 Type: non-negative integer vector  
 Default: No Default

`length_bins`      Length bins  
 Type: non-negative integer vector  
 Default: true

`length_plus`      Is the last bin a plus group  
 Type: boolean  
 Default: model

`selectivities`      The labels of the selectivities used for untagged categories  
 Type: string vector  
 Default: true

`tagged_selectivities`      The labels of the tag category selectivities  
 Type: string vector  
 Default: No Default

`detection`      Probability of detecting a recaptured individual  
Type: constant  
Default: No Default

`dispersion`      Over-dispersion parameter  $\phi$ ,  
Type: constant  
Default: double

`time_step_proportion`      Proportion through the mortality block of the time step when the observation is evaluated  
Type: constant  
Default: double

## 10.2. Likelihoods

**@likelihood** *label*      Define an object of type *likelihood*

## 10.3. Defining ageing error

The methods for including ageing error into estimation with observations are:

- None
- Data
- Normal
- Off-by-one

Each type of ageing error requires a set of subcommands and arguments specific to its type.

**@ageing\_error** *label*      Define an object of type *ageing\_error*

`label`      Label of the ageing error  
Type: string  
Default: No Default

`type`      Type of ageing error  
Type: string  
Default: No Default

**10.3.1. @ageing\_\_error[label].type=data**

**10.3.2. @ageing\_\_error[label].type=none**

**10.3.3. @ageing\_\_error[label].type=normal**

cv CV of the misclassification matrix

Type: estimable

Default: No Default

Lower Bound: 0.0 (inclusive)

k k defines the minimum age of individuals which can be misclassified, e.g., individuals of age less than k have no ageing error

Type: non-negative integer

Default: 0u

**10.3.4. @ageing\_\_error[label].type=off\_by\_one**

p1 proportion misclassified as one year younger, e.g., the proportion of age 3 individuals that were misclassified as age 2

Type: estimable

Default: No Default

Lower Bound: 0.0 (inclusive)

Upper Bound: 1.0 (inclusive)

p2 proportion misclassified as one year older, e.g., the proportion of age 3 individuals that were misclassified as age 4

Type: estimable

Default: No Default

Lower Bound: 0.0 (inclusive)

Upper Bound: 1.0 (inclusive)

k The minimum age of fish which can be misclassified, i.e., fish of age less than k are assumed to be correctly classified

Type: non-negative integer

Default: 0u

Lower Bound: 0.0 (inclusive)

Upper Bound: 1.0 (inclusive)

## 11. Report command and subcommand syntax

### 11.1. Report commands and subcommands

**@report label** Define an object of type *report*

label The label for the report

Type: string  
Default: No Default

type      The type of report  
Type: string  
Default: No Default

file\_name      The File Name if you want this report to be in a separate file  
Type: string  
Default: ""

write\_mode      The write mode  
Type: string  
Default: overwrite  
Allowed Values: overwrite, append, incremental\_suffix

#### **11.1.1. @report [label] .type=age\_length**

time\_step      Time Step label  
Type: string  
Default: ""

years      Years  
Type: non-negative integer vector  
Default: true

age\_length  
Type: string  
Default: No Default

category  
Type: string  
Default: No Default

#### **11.1.2. @report [label] .type=ageing\_error\_matrix**

ageing\_error      Ageing Error label  
Type: string  
Default: No Default

**11.1.3. @report[label].type=initialisation\_partition\_mean\_weight****11.1.4. @report[label].type=partition**

time\_step      Time Step label  
Type: string  
Default: ""

years          Years  
Type: non-negative integer vector  
Default: true

**11.1.5. @report[label].type=partition\_biomass**

time\_step      Time Step label  
Type: string  
Default: ""

years          Years  
Type: non-negative integer vector  
Default: true

**11.1.6. @report[label].type=partition\_mean\_weight**

time\_step      Time Step label  
Type: string  
Default: ""

years          Years  
Type: non-negative integer vector  
Default: true

**11.1.7. @report[label].type=partition\_year\_cross\_age\_matrix****12. Including commands from other files**

**@include** *file*      Include an external file

*file*      The name of the external file to include

Type: string

Default: No default

Value: A valid external file

Condition: The file name must be enclosed in double quotes

Example: @include "my\_file.csl2"

Note: @include does not denote the end of the previous command block as is the case for all other



---

commands



---

### 13. Tips for setting up Casal2 model based on an existing CASAL model

For many users that are about to embark on the CASAL2 journey, firstly good luck, but secondly, most of you will be coming from a functioning CASAL model. This section focuses on transitioning from CASAL to CASAL2.

There are a range of expected reasons why CASAL2 will provide (report) different values when comparing model output to CASAL models. There are also reasons why values will differ that are not so obvious such as, reasons caused from using different compilers on different machines where over/underflow might occur. It is thought/assumed that the latter inconspicuous reasons are insignificant (or at least should be), and the 'overall' behaviour when it comes to estimation will be the same between CASAL and CASAL2. Reasons why you can expect different values reported between CASAL and CASAL2 that I have discovered so far are;

- Report rounding. There are setting with respect to `std::out` in CASAL that set significant figures for writing to files. So if things look truncated, there might be a very simple reason for this.
- Priors on parameters that are turned off with `upper_bound = lower_bound`. In both programs you can turn off the estimation of parameters by setting the bounds equal. CASAL will evaluate the prior value and add this to the objective function, you don't need to worry as this contribution is a constant so will not effect parameter inference. It may however confuse you when comparing output between the two models.
- Default values... This one seems obvious but there are a lot of switches in these programs, and even subtle things like the `delta` in CASAL2 or `r` parameter in CASAL for robustifying likelihoods can catch you out.
- order of processes. CASAL has a predefined sequence in which it executes processes with in a time step, where as CASAL2 is completely user defined.
- Length based process/observations. CASAL2 has updated the normal distribution cdf calculation (its approximated no closed form solution) with better approximations.
- Age observations currently Casal2 doesn't have the `sum_to_one` subcommand, where as CASAL does this behind the scenes. Check that this is false if you want to truly compare
- Tag Penalties. CASAL applies a penalty as sum of squares on total tagged fish in an a 'tagging episode' from the model compared to observed number tagged fish. CASAL2 applies a penalty on the transition rate by length. If you ask to apply a tags in a length bin that doesn't have fish e.g. asking to tag 2 fish of length 60-61cm when there is 0 will flag a penalty. Unsure the consequence of this during estimation.

Many of the switches between CASAL and CASAL2 are pretty similar but if there is any confusion you should go to the syntax section of this document (Sections 8). So it should be easy to get a model up and running between the two programs. One tip I have is never do an estimation run (`casal2 -e`) until you have convinced yourself that the programs give the same (keep in mind the points above) results with a **range** of parameter values using the deterministic run commands (`casal2 -r`).

The first thing you should look/investigate at when setting up a comparison between CASAL2 and CASAL is focus on the stock dynamics outputs which I call the process dynamics model (i.e. ignore observations). This is your initial age-structure, SSB's and the like. If these components differ between programs then your observations will certainly be different and thus, if you blindly did an estimation you would almost certainly get different results and possibly conclude there is a bug or something.

There are few links that you can make with certain stock outputs that will point you in a direction of processes that are misspecified. Any difference between proportions in the initial age-structure (assuming an equilibrium state) is due to  $M$  (natural mortality). For difference in absolute initial age-structure (defined as  $R_0$  in the recruitment process) will be due to growth (@age\_length or @length\_weight). Most of our models are  $B_0$  initialised so  $R_0$  is a back calculation through the growth curve.

If you have successfully got the initial age-structure between the programs the same, then you can move on to focusing on derived quantities such as SSB's. Difference in these will generally be caused by how fishing and recruitment processes are configured. Look at things like which year class values are standardised, and choice of selectivities etc.

Once you are happy that the process dynamic model is doing the same between the two programs. I reiterate, do this with a few different set of parameter values (I suggest by using the `-i` functionality). Then you can move on to investigating the observation model. Things you want to pull out and examine are expected values between CASAL and CASAL2 assuming you have input the correct observations the difference in objective function will come from model expectations and likelihood configurations, this is where subcommands such as robustification and default values will annoy you.

Once you are satisfied that the process model and observation models are the same between CASAL and CASAL2 you can unleash an estimation run (`casal2 -e`). Now I would love to say on the first attempt everything will work out and both CASAL and CASAL2 will minimise to the same values, but from my experience they wont. If this happens to you, what I suggest you do is get the parameter values from CASAL and do a deterministic run with CASAL2 using CASAL estimated parameter values (`casal2 -r -i CASAL_mpd_pars.txt`). Then once again look at the process dynamic model, once you are satisfied inspect the observation model and see if you can identify the culprit.

The next question is how close do the model estimates and outputs have to be, before we can conclusively say the models are identical? This is an ongoing decision historically we have used subjective qualitative measures to decide whether the models are doing the same thing. A recorded comparison for the hake stock assessment can be found at Appendix B in Horn (2017).

---

## 14. Syntax conventions, examples and niceties

### 14.1. Input File Specification

The file format used for CASAL2 is based on the formats used for CASAL and SPM. It's a standard text file that contains definitions organised into blocks.

Without exception, every object specified in a configuration file is part of a block. At the top level blocks have a one-to-one relationships with components in the system.

Example:

```
@block1 label
parameter value
parameter value_1 value 2
```

```
@block2 label
parameter value
table table_name
column_1 column_2
data_1 data_2
data_3 data_4
end_table
```

Some general notes about writing configuration files:

1. Whitespace can be used freely. Tabs and spaces are both accepted
2. A block ends only at the beginning of a new block or end of final configuration file
3. You can include another configuration file from anywhere
4. Included files are placed inline, so you can continue a block in a new file
5. The configuration files support inline declarations of objects

#### 14.1.1. Keywords And Reserved Characters

In order to allow efficient creation of input files CASAL2's file format contains special keywords and characters that cannot be used for labels etc.

##### @Block Definitions

Every new block in the configuration file must start with a block definition character. The reserved character for this is the @character

Example:

```
@block1 <label>
type <type>

@block2 <label>
type <type>
```

**'type' Keyword**

The 'type' keyword is used for declaring the sub-type of a defined block. Any block object that has multiple sub-types will use the type keyword.

Example:

```
@block1 <label>
type <sub_type>

@block2 <label>
type <sub_type>
```

**# (Single-Line Comment)**

Comments are supported in the configuration file in either single-line (to end-of-line) or multi-line

Example:

```
@block <label>
type <sub_type> #Descriptive comment
#parameter <value_1> â€” This whole line is commented out
parameter <value_1> #<value_2>(value_2 is commented out)
```

**/\* \*/ (Multi-Line Comment)**

Multiple line comments are supported by surrounding the comments in /\* and \*/

Example:

```
@block <label>
type <sub_type>
parameter <value_1>
parameter <value_1> <value_2>

\*
Do not load this process
@block <label>
type <sub_type>
parameter <value_1>
parameter <value_1> <value_2>
*\
```

**{ } (Indexing Parameters)**

Users can reference individual elements of a map using the { } syntax, for example when estimating ycs\_values you may only want to estimate a block of YCS not all of them say between 1975 and 2012. Example:

```
@estimate YCS
parameter process[Recruitment].ycs_values{1975:2012}
type uniform
lower_bound
upper_bound
```

**':' (Range Specifier)**

The range specifier allows you to specify a range of values at once instead of having to input them manually. Ranges can be either incremental or decremental.

Example:

```
@process my_recruitment_process
type constant_recruitment
years_to_run 1999:2009 #With range specifier

@process my_mortality_process
type natural_mortality
years_to_run 2000 2001 2002 2003 2004 2005 2006 2007 #Without range specifier
```

**',' (List Specifier)**

When a parameter supports multiple values in a single entry you can use the list specifier to supply multiple values as a single parameter.

Example:

```
@categories
format sex.stage
names male,female.immature,mature #With list specifier

@categories
format sex.stage
names male.immature male.mature female.immature female.mature #Without list specifier
```

**'table' and 'end\_table' Keyword**

The table keyword is used to define a table of information used as a parameter. The line following the table declaration must contain a list of columns to be used. Following lines are rows of the table. Each row must have the same number of values as the number of columns specified. The table definition must end with the 'end\_table' keyword on it's own line. The first row of a table will be the name of the columns if required.

Example:

```
@block <label>
type <sub_type>
parameter <value_1>
table <table_label>
<column_1> <column_2> <column_n>
<row1_value1> <row1_value2> <row1_valueN>
<row2_value1> <row2_value2> <row2_valueN>
end_table
```

**[ ] (Inline Declarations)**

When an object takes the label of a target object as a parameter this can be replaced with an inline declaration. An inline declaration is a complete declaration of an object one line. This is designed to allow the configuration writer to simplify the configuration writing process.

Example:

```
#With inline declaration with label specified for time step
```

```

@model
time_steps step_one=[type=iterative; processes=recruitment ageing]

#With inline declaration with default label (model.1)
@model
time_steps [type=iterative; processes=recruitment ageing]

#Without inline declaration
@model
time_steps step_one

@time_step step_one
processes recruitment ageing

```

## Categories

The CASAL2 model is essentially a 2-dimensional model. The model partition is: Categories x Ages/Lengths.

Each category supports the ability to have a different range of ages/lengths and accessibility during different time periods.

Because each category is quite complicated, the syntax for defining categories has been structured to allow complex definitions using a simple short-hand structure.

The "format" parameter allows you to tell the model the structure of the category labels. By using a "." (period) character between each segment we can utilise this later in the model to do short-hand lookups of categories.

The "names" parameter is a list of the category names. The syntax of these names will need to match the "format" parameter so CASAL2 can organise and search on them. Using the "list specifier" and range characters we can shorten this parameter significantly.

Example:

```

@categories
format sex.stage.tag
names male.immature.notag male.immature.2001 male.mature.notag male.mature.2001

names male.immature #Invalid: No tag information
names female #Invalid: no stage of tag information
names female.immature.notag.1 #Invalid: Extra format segment not defined

names male,female.immature,mature.notag,2001:2005 #OK!
#Without short-hand. You'd have to write:
names male.immature.notag male.immature.2001 male.immature.2002 male.immature.2003 male.immature.2004

```

When we have specific data for a year in a category we don't want the model to process this category during other years (or the initialisation stages). We can define a list of years where each category will be available, this will override the default of all years in the model. Any category where you overwrite the default will no longer be accessible in the initialisation phases.

Examples:

```

@model
start_year 1998
final_year 2010

```



```
@categories
format sex.stage.tag
names male,female.immature,mature.notag,2001:2005 #OK!
years tag=2001=1999:2003 tag=2005=2003:2007
# Categories with the tag value 'IJ2001' will be available during years 1999, 2000, 2001, 2002 and 2003
# Categories with the tag value 'IJ2005' will be available during the years 2003, 2004, 2005, 2006, 2007
```

## 14.2. More examples of shorthand syntax and use of CASAL2's reserved and key characters

### Categories

CASAL2 allows many user defined categories so shorthand syntax has been added to aid in the readability of complex configuration scripts and partition structures. For example when defining categories you can use a comma for shortening lists of categories. The following syntax is how we would specify the categories the long way.

```
@categories
format sex.stage
names male.immature male.mature female.immature female.mature
```

for the exact same partition structure but specified in a shorter way users could define the categories as, (note the use of the list character ','),

```
@categories
format sex.stage
names male,female.immature,mature
```

CASAL2 asks for categories in processes and observations so that it can apply the right model dynamics to the right elements of the partition. For the same reason as defining categories shorthand syntax aids in readability and input management. An example of a process where categories need to be supplied as an input command is in ageing,

```
# 1. The standard way
@ageing my_ageing
categories male.immature male.mature female.immature female.mature

# 2. The 1st short-hand way
@ageing my_ageing
categories male,female.immature,mature

# 3. Wild Card (all categories)
@ageing my_ageing
categories *

# 4. The 2nd short-hand way
@ageing my_ageing
categories sex=male sex=female
```

Sometimes in observations we want to amalgamate categories together for example if we had a biomass estimate of the population that was made up of both males and females in the population you can specify this using the + special character, for example

```
@observation CPUE
type biomass
catchability Fishq
time_step one
categories male+female
selectivities FishSel
likelihood lognormal
years 1992:2001
time_step_proportion 1.0
obs 1.50 1.10 0.93 1.33 1.53 0.90 0.68 0.75 0.57 1.23
error_value 0.35
```

Another helpful short cut using the amalgamation symbol + is if your observation wants to compare to the total combined population you can use the following format.

```
@observation CPUE
type biomass
catchability Fishq
time_step one
categories *+
selectivities FishSel
likelihood lognormal
years 1992:2001
time_step_proportion 1.0
obs 1.50 1.10 0.93 1.33 1.53 0.90 0.68 0.75 0.57 1.23
error_value 0.35
```

if male and female are the only categories in your population, then this is the same syntax as the observation just above it.

Shorthand syntax can be useful when applying processes to a select group of categories from the partition, for example. If we wanted to apply a spawning migration to the mature categories in the partition and the partition was defined by the categories below,

```
@categories
format area.maturity.tag
names north.immature.notag,2011 north.mature.notag,2011 south.immature.notag,2011
south.mature.notag,2011
```

If we wanted to migrate a portion of the mature population from the southern area to the northern are you could use the following syntax,

```
@process spawn_migration
type transition_category
from format=south.mature.*
to format=north.mature.*
proportions 1.0
selectivities One
```

## Parameters

CASAL2 also allows parameters that are of type vector or map to be referenced and estimated partially. An example of a parameter that is type vector is `yces_values` in a recruitment process. Let say a recruitment block was specified as follows,

```
@process WestRecruitment
type recruitment_beverton_holt
r0 400000
years
ycs_values 1 1 1 1 1 1 1 1
ycs_years 1975:1983
An alternative specification to the sequence of values you can use an astrix to
shorthand repeating integers e.g.

ycs_values 1*8

steepness 0.9
age 1
```

Lets say we wanted to only estimate the last four years of the parameter `process[WestRecruitment].ycs_values`. This can be done as specified in the following `@estimate` block,

```
@estimate
parameter process[WestRecruitment].ycs_values{1979:1983}
type uniform
lower_bound 0.1 0.1 0.1 0.1
upper_bound 10 10 10 10
```

Note the first element of a vector is indexed by 1. This syntax can be applied to parameters that are of type `map` as well, for information on what type a parameter is see the syntax section. An example of a parameter that is of type `map` is `@time_varying[label].type=constant`. For the following `@time_varying` block,

```
@time_varying q_step1
type constant
parameter catchability[Fishq].q
years 1992 1993 1994 1995
value 0.2 0.2 0.2 0.2
```

In this example a user may want to estimate only one element of the map (say 1992), but force all other years to be the same as the one estimate. This can be done in an estimate block as follows,

```
@estimate
parameter time_varying[q_step1].value{1992}
same time_varying[q_step1].value{1993:1995}
type uniform
lower_bound 0.1 0.1 0.1 0.1
upper_bound 10 10 10 10
```

### In line declaration

In line declarations can help shorten models by passing `@` blocks, for example

```
@observation chatCPUE
type biomass
catchability [q=6.52606e-005]
time_step one
```

```

categories male+female
selectivities chatFselMale chatFselFemale
likelihood lognormal
years 1992:2001
time_step_proportion 1.0
obs 1.50 1.10 0.93 1.33 1.53 0.90 0.68 0.75 0.57 1.23
error_value 0.35

@estimate
parameter catchability[chatTANbiomass.one].q
type uniform_log
lower_bound 1e-2
upper_bound 1
In line declaration tips

```

In the above code we are defining and estimating catchability without explicitly creating an `@catchability` block.

When you do an inline declaration the new object will be created with the name of the creator's `label.index` where `index` will be the word if it's one-nine and the number if it's 10+, for example,

```

@mortality halfm
selectivities [type=constant; c=1]

would create
@selectivity halfm.one

```

if there were 10 categories all with there own selectivity the 10<sup>th</sup> selectivity would be labelled,

```
@selectivity halfm.10
```

### 14.3. Processes

Processes are special in how they can be defined, all throughout this document we have been referring to specifying a process as follows,

```

@process Recruitment
type recruitment_beverton_holt

```

However for convenience and for file clarity you could equally specify this block as follows,

```

@recruitment Recruitment
type beverton_holt

```

The trick is that you can replace the keyword `process` with the first word of the process type, in the example above this is the `recruitment` this can be away of creating more reader friendly/lay term configuration scripts. More examples follow;

```

@mortality Fishing_and_M
type instantaneous

@transition Migration
type category

```

## 14.4. An example of a simple model

This example implements a very simple single species and area model, with recruitment, maturation, natural and fishing mortality, and an annual age increment. The population structure has ages 1 – 30<sup>+</sup> with a single category.

CASAL2 default file to search for in your current working directory is `casal2.csl2`. In this example, `casal2.csl2` specifies all the files necessary to run your CASAL2 model from your current working directory. This is done using the `!include` command as follows.

```
!include "population.csl2"
!include "reports.csl2"
!include "Observation.csl2"
!include "estimation.csl2"
```

Breaking up a CASAL2 model into sections is recommended, as it aids in readability and error checking. `population.csl2` contains the population information. The model runs from 1975-2012 and is initialised over a 120 year period prior to 1975, which applies the following processes,

1. A Beverton-Holt recruitment process, recruiting a constant number of individuals to the first age class (i.e.,  $age = 1$ ).
2. A constant mortality process representing natural mortality( $M$ ). This process is repeated in all three time steps, so that each with its own time step proportion of  $M$  applied.
3. An ageing process, where all individuals are aged by one year, and with a plus group accumulator age class at  $age = 30$ .

Following initialisation, the model runs from the years 1975 to 2012 iterating through two time-steps. The first time-step applies processes of recruitment, and  $\frac{1}{2}M_1 + F + \frac{1}{2}M_1$  processes, where  $M_1$  is the proportion of  $M$  applied in the first time step. The exploitation process (fishing) is applied in the years 1975–2012. Catches are defined in the catches table and attribute information on each fishery such as selectivity and time-step they are implemented are in the fisheries table in the `@process` block.

The second time-step applies an age increment and the remaining natural mortality.

The first 28 lines of the main section of the `population.csl2` are,

```
#THE MODEL constraints
@model
start_year 1975
final_year 2012
min_age 1
max_age 30
age_plus true
base_weight_units tonnes
initialisation_phases Equilibrium_state
time_steps Sep_Feb Mar_May Jun_Aug ##
length_bins 10 20 30 40 50 60 70 80 90 100 110 120 130 140 150 160 170 180 190 200 210 220 230
            240 250

#CATEGORIES
@categories
format stock ## Single sex and area population
names HAK4
age_lengths age_size

@initialisation_phase Equilibrium_state
type derived
```

```
## Define the processes in the Annual Cycle
## This is a list of labels that correspond to a process
@time_step Sep_Feb
processes Recruitment Instantaneous_Mortality

@time_step Mar_May
processes Instantaneous_Mortality
```

To carry out a run of the model (to verify that the model runs without any syntax errors), use the command `casal2 -r`. Note that as CASAL2 looks for a file named `casal2.txt` by default, we can override this. Hypothetically speaking if our model was all written in `Mymodel.txt` we could call it using the `-c` command like `casal2 -r -c Mymodel.txt`.

To run an estimation, and hence estimate the parameters defined in the file `estimation.csl2` (the catchability constant  $q$ , recruitment  $R_0$ , and the selectivity parameters  $a_{50}$  and  $a_{t095}$ ), use `casal2 -e`. Here, we have piped the output to `estimate.log` using the command `casal2 -e > estimate.log`, reports the user defined reports `reports.csl2` from the final iteration of the estimation, and successful convergence printed to screen,

```
Total elapsed time: 1 second
Completed
```

The main part of the output from the estimation run is summarised in the file `estimate.log`, and the final MPD parameter values can be piped out as a separate report, in this case named `paramaters.out`, using the command `casal2 -e -o paramaters.out > estimate.log`.

A profile on the  $R_0$  parameter can also be run, using `casal2 -p > profile.log`. See the examples folder for the example of the output.

---

## 15. Post processing output using R

**R** is the main program used to process and visualise output from a CASAL2 model. **R** is free and can be downloaded from <https://cran.r-project.org/>. Once you have installed **R** you can load the `casal2` **R** package for investigating model runs.

In the downloaded bundle there is a **R**-package (`casal2_1.0.tar.gz`) that reads CASAL2 output into **R**. The CASAL2 package has multiple functionalities, it can import CASAL2 output into **R** as a list. It also has plotting and summarising functions for CASAL2 reports.

There are three types of output that CASAL2 will produce and these depend on what type of analysis you are doing. These outputs are Standard output, MCMC output, Derived Quantity outputs. Standard output are the typical reports that are produced in most CASAL2 runmodes (with the exception of `-s` and `-m`). This output can be split into further categories where you can have a single parameter run (`casal2 -r`) or multiparameter run e.g. `casal2 -r -i many_pars.out` or running in projection mode (`-f 1`). Both of the standard output's can be read into **R** using the `extract.mpd()` function. The second type of output is generated when doing an MCMC analysis (`casal2 -m`), which automatically generates two files `mcmc_objective.out` and `mcmc_sampeles.out`. These reports will be summarised to conclude convergence or chain behaviour, it will also be used to view marginal posteriors and quantify key model parameter uncertainty. The last output type is Derive Quantity outputs or sometimes referred to as tabular output. Usually this is generated after an MCMC is done, and the user wants to see the marginal posteriors for quantities that aren't explicit estimates. For example in fisheries a commonly reported quantity is the spawning stock biomass from the initial year to the final year. These aren't estimated parameters and so you would need to run the following command to get posteriors for these derived quantities (`casal2 -r -i mcmc_samples.out -tabular > Tabular_report.out`). This can then be read into an **R** environment using the `extract.tabular()` function.

CASAL2's reported output is written so that each `@report` will start with a `'*` and end with `*end'`. Users can use this format as the basis to construct **R** or other functions that read CASAL2 output to identify and read individual reports for post-processing.

These extract functions differ by how the expected output is structured and they each create a different `casal2` object in **R** that when run with `summary`, `plot` functions will generate different plots. To query an object from an `extract()` function just type `class(object)`. The following is a list of available **R** functions with their descriptions that are available in the `casal2` **R** package.

- `extract.mpd()` - import CASAL2 default output into **R** as a list
- `extract.mcmc()` - import CASAL2 mcmc output into **R** as a list
- `extract.tabular()` - import CASAL2 tabular output into **R** as a list
- `extract.parameters()` - import CASAL2 parameter files into **R** as a list
- `generate.starting.pars()` - Reads in a file that contains your `@estimate` blocks and generates 'N' starting values to test 'global convergence'. Pretty handy little bugger I think, encouraged to use.
- `burn.in.tabular()` - takes a `casal2TAB` object and cuts off the first 'n' rows.
- `plot.derived_quantities()` - plot derived quantities
- `plot.selectivities()` - plot selectivities
- `plot.ycs()` - plot true ycs strengths
- `plot.pressure()` - plot fishing pressures

- `summary()` - summarise a model run
- `extract.csl2.file` - read CASAL2 .csl2 (configuration) file into **R** as a list
- `write.csl2.file` - write CASAL2 .csl2 (configuration) file from **R**
- `ReadSimulatedData` - read CASAL2 output into **R** from a `casal2 -s` run
- `Method.TA1.8()` return a weighting factor for compositional data. See Francis (2011) for more detail.
- `apply.dataweighting.to.csl2()` - takes a configuration that contains `@observation` blocks, it applies a weighting factor to a compositional data set and generates a new .csl2 file, with modified effective sample size.

specific settings for these functions can be queried once you have loaded the CASAL2 R library into your current R session via `(library(casal2))`, using the standard **R** help syntax `?` (i.e. `?param.profile()`). Most of the help files should have example data that you can replicate to help understand with syntax.

### Standard diagnostics and plots for model output

An aspect of CASAL2 that we have added to aid in usability is an **R** package that has functions that can summarise and diagnose a model with ease. Stock synthesis is exceptional for having a well tested and used **R** package, this our attempt of trying to follow suit.

```
plot.derived_quantity()
```

Generally when comparing model outputs we are either comparing different parameters for the same model structure (Option 1) or comparing outputs from multiple model structures (Option 2). This function in theory should work for both of these.

```
plot.selectivities() plot.pressure() plot.fit() plot.ycs()
```

### DataWeighting

An important component of stock assessment is dealing with data conflicts through the use of dataweighting. There are a range of methods for this for a great summary see Francis (2011). The function implemented in the CASAL2 R-library is `TA1.8`, which can be invoked using `Method.TA1.8()`. We have tried to automate this process for users by adding an assist function named `apply.dataweighting.to.csl2`. This function allows users to automatically apply a weighting factor to a specific compositional `@observation` block, and generate a new .csl2 file to be re-estimated.

```
library(casal2)

## read in a reported output from a casal2 -e run
## in a command prompt run 'casal2 -e > estimate.log'
## make sure there is @report block for the observation of interest.

Mpd = extract.mpd(estimate.log)
## calculate weighting factor from Francis method
WeightingFactor = Method.TA1.8(model = Mpd, observation_labels = "chatTANage")
## Apply weighting factor to your Observation.csl2 file
apply.dataweighting.to.csl2(weighting_factor = WeightingFactor, Observation_csl2_file = "
  Observations.csl2", Observation_label = "chatTANage", Observation_out_filename = "
  Observation.csl2.0")
## this will generate a new Observation.csl.0 file with the re-weighted effective sample size.
```



---

Another handy piece of code automates the dataweighting process, as follows

```
library(casal2)
## setwd to your working environment, to your Casal2 files and output
setwd()
MPD = extract.mpd(file = "estimate.log")
Factor = Method.TA1.8(MPD, observation_labels = c("ObserverProportionsAtAge"))
## Before you start this loop, I suggest making a copy of your Observation.csl2 e.g.
   Observation_orig.csl2, so that if something goes wrong, you can come back to it.
while(abs(Factor - 1) > 0.01) {
  cmd = "betadiff & casal2 -e > estimate.log 2> log.out"
  shell(cmd);
  new_mpd = extract.mpd(file = "estimate.log")
  Factor = Method.TA1.8(new_mpd, observation_labels = c("ObserverProportionsAtAge"))
  apply.dataweighting.to.csl2(weighting_factor = Factor, Observation_csl2_file = "Observation.
    csl2", Observation_out_filename = "Observation.csl2", Observation_label = c("
    ObserverProportionsAtAge"))
  print(Factor)
}
```

## Trouble Shooting R package

This section documents some issues with solutions that we have come across when using the R package that may come in handy for future users.

if you get the following error when using one of the extract functions

```
Read 1 item
Warning messages:
1: In scan(filename, what = "", sep = "\n", fileEncoding = fileEncoding) :
  embedded nul(s) found in input
2: In extract.mpd(file = "results.txt", fileEncoding = "") :
  File is empty, no reports found
```

We have resolved the abover error by trying an alternative UTF formatting using the fileEncoding parameter e.g,

```
library(casal2)
MyOutput = extract.mpd(file = "Estimate.log", path = getwd(), , fileEncoding = "UTF-16LE")
```



---

## 16. Troubleshooting

This section is to aid users in debugging models. If you cannot resolve an issue using these guidelines then please contact the development team. To report an issue please follow the format described in Section 16.2.1.

Most user errors should be well documented and CASAL2 should produce informative error messages. There are runtime options that users can enable to attempt to resolve or at least isolate an error or bug, including different levels of logging.

### 16.1. Logging

CASAL2's internal logging system can be invoked at the command line with argument `-loglevel` followed by one of these options: `trace`, `finest`, `fine`, `medium`.

An example of logging with trace level output:

- On Windows: `casal2 -r -loglevel trace > output.log 2> log.out`
- On Linux: `casal2 -r -loglevel trace > output.log 2&> log.out`

This argument will output CASAL2's reports to the file "output.log", and the "2>" or "2&>" syntax will print the error logged information to the file "log.out". You should be able to see where CASAL2 is exiting by going to the end of the "log.out" file.

The optimal level of logging will depend on what run mode you are using and the granularity of information that you would like to see. There is an ordering in the options, with `medium` being the most coarse, and `trace` being the finest level, with `fine` and `finest` in between. We suggest that if you are running CASAL2 in an iterative state such as for estimation (`casal2 -e`) or MCMC you use `medium` level. This is because the logging can print a lot of information for a single model run, so an estimation which could comprise thousands of model runs can produce very large text files with the finer logging option specified. For a single iteration run such as `casal2 -r` each of the logging options can be useful during different phases of model development.

You can see how CASAL2 creates these reports by looking in the ".cpp" files in the Observation or Processes source code subdirectories and see code such as in `Model/Model.cpp`,

```
LOG_FINE() « "Model: State change to Execute";
```

### 16.2. Reporting errors

If you find a bug or error in CASAL2, please submit an issue in the GitHub repository at <https://github.com/NIWAFisheriesModelling/CASAL2/issues>.

Please follow the guidelines below so that the bug or error can be reproduced. It is helpful to be as detailed and specific as possible when describing the observed behavior as well as the expected behaviour.

#### 16.2.1. Guidelines for reporting an error with CASAL2

1. Ensure you are using the most recent version of CASAL2, as the bug or error you are having may have already been resolved.

2. Provide the version of CASAL2 you are using, e.g., "CASAL2 v2019-12-11 (rev. 69b27ff)". The version is output by CASAL2 with the command `casal2 -v`.
3. Provide the operating system you are using, e.g., "IBM-PC Intel CPU with Microsoft Windows 10 Enterprise".
4. Provide a brief description of the problem, e.g., "a segmentation fault was produced".
5. If the problem is reproducible, please describe in detail the steps required to cause it, and include the CASAL2 configuration files, other input files, and any output files generated. Specify the *exact* command line arguments that were used, e.g., "Using the command `casal2 -e -q` produced a segmentation fault. The input configuration files are attached."
6. If the problem is not reproducible (it happened only once, or occasionally for no apparent reason), please describe in detail the circumstances in which it occurred and the behaviour observed, e.g., "CASAL2 crashed, but I have not been able to reproduce the issue. It seemed to be related to a local network crash but I cannot be sure."
7. If the problem produced any error messages, please give the *exact* text displayed, e.g., "segmentation fault (core dumped)".
8. Attach all relevant input and output files so that the problem can be reproduced; these files can be compressed into a single file e.g., a zip file, and uploaded to GitHub.

---

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## 18. Acknowledgements

We thank the developers of CASAL (Bull et al., 2012) for their ideas that led to the development of CASAL2. The CASAL2 logo was designed by Ian Doonan and Erika Mackay (NIWA).

Much of the structure of CASAL2, equations, and documentation in this manual draw heavily on similar components of the fisheries population model CASAL (Bull et al., 2012) and the spatial model SPM (Dunn et al., 2015). We thank the authors of CASAL and SPM for their permission to use their work as the basis for parts of CASAL2 and allow the use of the definitions, concepts, and documentation.

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## 21. Quick reference

**@additional\_prior** *label* Define an object of type *additional\_prior*

parameter Name of the parameter to generate additional prior on

label Label for the additional prior

type Type of additional prior

**@ageing\_error** *label* Define an object of type *ageing\_error*

label Label of the ageing error

type Type of ageing error

**@ageing\_\_error[label].type=data**

**@ageing\_\_error[label].type=none**

**@ageing\_\_error[label].type=normal**

cv CV of the misclassification matrix

k k defines the minimum age of individuals which can be misclassified, e.g., individuals of age less than k have no ageing error

**@ageing\_\_error[label].type=off\_by\_one**

p1 proportion misclassified as one year younger, e.g., the proportion of age 3 individuals that were misclassified as age 2

p2 proportion misclassified as one year older, e.g., the proportion of age 3 individuals that were misclassified as age 4

k The minimum age of fish which can be misclassified, i.e., fish of age less than k are assumed to be correctly classified

**@age\_length** *label* Define an object of type *age\_length*

label Label of the age length relationship

type Type of age length relationship

time\_step\_proportions the fraction of the year applied in each time step that is added to the age for the purposes of evaluating the length, i.e., a value of 0.5 for a time step will evaluate the length of individuals at age+0.5 in that time step

distribution The assumed distribution for the growth curve

cv\_first CV for the first age class

cv\_last CV for last age class

casal\_switch If true, use the ,less accurate, equation for the cumulative normal function as was used in the legacy version of CASAL.

by\_length Specifies if the linear interpolation of CV's is a linear function of mean length at age. Default is just by age

**@age\_\_length[label].type=data**

external\_gaps

internal\_gaps

length\_weight The label from an associated length-weight block

time\_step\_measurements\_were\_made Time step label for which size-at-age data are provided

**@age\_\_length[label].type=none**

**@age\_\_length[label].type=schnute**

y1     Define the  $y_1$  parameter of the Schnute relationship  
y2     Define the  $y_2$  parameter of the Schnute relationship  
tau1    Define the  $\tau_1$  parameter of the Schnute relationship  
tau2    Define the  $\tau_2$  parameter of the Schnute relationship  
a     Define the  $a$  parameter of the Schnute relationship  
b     Define the  $b$  parameter of the Schnute relationship  
length\_weight     Define the label of the associated length-weight relationship

**@age\_\_length[label].type=von\_bertalanffy**

linf    Define the  $L_{infinity}$  parameter of the von Bertalanffy relationship  
k     Define the  $k$  parameter of the von Bertalanffy relationship  
t0     Define the  $t_0$  parameter of the von Bertalanffy relationship  
length\_weight     Define the label of the associated length-weight relationship  
**@catchability** *label*     Define an object of type *catchability*  
label    Label of the catchability  
type    Type of catchability  
**@categories** *label*     Define an object of type *categories*  
format    The format that the category names adhere too  
names    The names of the categories to be used in the model  
years    The years that individual categories will be active for. This overrides the model values  
age\_lengths    R, The labels of age\_length objects that are assigned to categories,  
length\_weight    R, The labels of the length\_weight objects that are assigned to categories,  
age\_weight    R, The labels of the age\_weight objects that are assigned to categories,  
**@derived\_quantity** *label*     Define an object of type *derived\_quantity*  
label    Label of the derived quantity  
type    Type of derived quantity  
time\_step    The time step in which to calculate the derived quantity after  
categories    The list of categories to use when calculating the derived quantity  
selectivities    A list of one selectivity  
time\_step\_proportion    Proportion through the mortality block of the time step when  
calculated  
time\_step\_proportion\_method    Method for interpolating for the proportion through the  
mortality block  
values

**@derived\_\_quantity[label].type=abundance****@derived\_\_quantity[label].type=biomass****@estimate** *label*     Define an object of type *estimate*

label    The label of the estimate

---

type      The prior type for the estimate  
 parameter      The name of the parameter to estimate in the model  
 lower\_bound      The lower bound for the parameter  
 upper\_bound      The upper bound for the parameter  
 same      List of parameters that are constrained to have the same value as this parameter  
 estimation\_phase      The first estimation phase to allow this to be estimated  
 mcmc      Indicates if this parameter is estimated at the point estimate but fixed during MCMC estimation run  
 transformation      Type of simple transformation to apply to estimate  
 transform\_with\_jacobian      Apply jacobian during transformation  
 prior\_applies\_to\_transform      Does the prior apply to the transformed parameter? a legacy switch, see Manual for more information  
**@estimate\_transformation** *label*      Define an object of type *estimate\_transformation*  
 label      Label for the transformation block  
 type      Type of transformation  
 transform\_with\_jacobian      Apply jacobian during transformation  
**@initialisation\_phase** *label*      Define an object of type *initialisation\_phase*  
 label      The label of the initialisation phase  
 type      The type of initialisation

**@initialisation\_\_phase[label].type=cinitial**

categories      The list of categories for the Cinitial initialisation

**@initialisation\_\_phase[label].type=derived**

insert\_processes      Additional processes not defined in the annual cycle, that are to be inserted into this initialisation phase  
 exclude\_processes      Processes in the annual cycle to be excluded from this initialisation phase  
 casal\_initialisation\_switch      Run an extra annual cycle to evaluate equilibrium SSB's. Warning - if true, this may not correctly evaluate the equilibrium state. Use true if attempting to replicate a legacy CASAL model

**@initialisation\_\_phase[label].type=iterative**

years      The number of iterations ,years, over which to execute this initialisation phase  
 insert\_processes      ,years, over which to execute this initialisation phase  
 exclude\_processes      Processes in the annual cycle to be excluded from this initialisation phase  
 convergence\_years      The iteration ,year, when the test for convergence ,lambda, is evaluated  
 lambda      The maximum value of the absolute sum of differences ,lambda, between the partition at year-1 and year that indicates successful convergence

**@initialisation\_\_phase[label].type=state\_category\_by\_age**

categories      The list of categories for the category state initialisation  
 min\_age      The minimum age of values supplied in the definition of the category state  
 max\_age      The minimum age of values supplied in the definition of the category state  
**@length\_weight** *label*      Define an object of type *length\_weight*  
 label      The label of the length-weight relationship

`type`     The type of the length-weight relationship  
**@likelihood** *label*     Define an object of type *likelihood*  
**@mcmc** *label*     Define an object of type *mcmc*  
`label`     The label of the MCMC  
`type`     The type of MCMC  
`length`     The number of iterations in for the MCMC chain  
`active`     Indicates if this is the active MCMC algorithm  
`print_default_reports`     Indicates if the output prints the default reports  
`step_size`     Initial stepsize ,as a multiplier of the approximate covariance matrix,  
**@minimiser** *label*     Define an object of type *minimiser*  
`label`     The minimiser label  
`type`     The type of minimiser to use  
`active`     Indicates if this minimiser is active  
`covariance`     Indicates if a covariance matrix should be generated  
**@model** *label*     Define an object of type *model*  
`start_year`     Define the first year of the model, immediately following initialisation  
`final_year`     Define the final year of the model, excluding years in the projection period  
`min_age`     Minimum age of individuals in the population  
`max_age`     Maximum age of individuals in the population  
`age_plus`     Define the oldest age or extra length midpoint ,plus group size, as a plus group  
`initialisation_phases`     Define the labels of the phases of the initialisation  
`time_steps`     Define the labels of the time steps, in the order that they are applied, to form the annual cycle  
`projection_final_year`     Define the final year of the model in projection mode  
`length_bins`     The minimum length in each length bin  
`length_plus`     Specify whether there is a length plus group or not  
`length_plus_group`     Mean length of length plus group  
`base_weight_units`     Define the units for the base weight. This will be the default unit of any weight input parameters  
**@observation** *label*     Define an object of type *observation*  
`label`     Label  
`type`     Type of observation  
`likelihood`     Type of likelihood to use  
`categories`     Category labels to use  
`delta`     Robustification value ,delta, for the likelihood  
`simulation_likelihood`     Simulation likelihood to use  
`likelihood_multiplier`     Likelihood score multiplier  
`error_value_multiplier`     Error value multiplier for likelihood  
  
**@observation[label].type=abundance**  
`selectivities`     Labels of the selectivities  
`time_step`     The label of time-step that the observation occurs in  
  
**@observation[label].type=biomass**  
`catchability`     The time-step of the observation

---

time\_step    The label of time-step that the observation occurs in  
obs        The observed values  
years       The years of the observed values  
error\_value    The error values of the observed values ,note the units depend on the likelihood,  
selectivities    Labels of the selectivities  
process\_error    Value for process error  
age\_weight\_labels    R,The labels for the @age\_weight block which corresponds to each category, if you want to use that weight calculation method for biomass calculations,

**@observation[label].type=process\_removals\_by\_age**

min\_age    Minimum age  
max\_age    Maximum age  
plus\_group    Use age plus group  
time\_step    The label of time-step that the observation occurs in  
tolerance    Tolerance  
years       Years for which there are observations  
process\_errors    Label of process error to use  
ageing\_error    Label of ageing error to use  
method\_of\_removal    Label of observed method of removals  
mortality\_instantaneous\_process    The label of the mortality instantaneous process for the observation

**@observation[label].type=process\_removals\_by\_age\_retained**

min\_age    Minimum age  
max\_age    Maximum age  
plus\_group    Use age plus group  
time\_step    The label of time-step that the observation occurs in  
tolerance    Tolerance  
years       Years for which there are observations  
process\_errors    Label of process error to use  
ageing\_error    Label of ageing error to use  
method\_of\_removal    Label of observed method of removals  
mortality\_instantaneous\_process    The label of the mortality instantaneous process for the observation

**@observation[label].type=process\_removals\_by\_age\_retained\_total**

min\_age    Minimum age  
max\_age    Maximum age  
plus\_group    Use age plus group  
time\_step    The label of time-step that the observation occurs in  
tolerance    Tolerance  
years       Years for which there are observations  
process\_errors    Label of process error to use  
ageing\_error    Label of ageing error to use  
method\_of\_removal    Label of observed method of removals  
mortality\_instantaneous\_process    The label of the mortality instantaneous process for the observation

**@observation[label].type=process\_removals\_by\_length**

length\_bins      Length bins  
time\_step      Time step to execute in  
length\_plus      Is the last bin a plus group  
tolerance      Tolerance for rescaling proportions  
years      Years for which there are observations  
process\_errors      the value of process error  
method\_of\_removal      Label of observed method of removals  
mortality\_instantaneous\_process      The label of the mortality instantaneous process for the observation

**@observation[label].type=process\_removals\_by\_length\_retained**

length\_bins      Length bins  
time\_step      Time step to execute in  
length\_plus      Is the last bin a plus group  
tolerance      Tolerance for rescaling proportions  
years      Years for which there are observations  
process\_errors      the value of process error  
method\_of\_removal      Label of observed method of removals  
mortality\_instantaneous\_process      The label of the mortality instantaneous process for the observation

**@observation[label].type=process\_removals\_by\_length\_retained\_total**

length\_bins      Length bins  
time\_step      Time step to execute in  
length\_plus      Is the last bin a plus group  
tolerance      Tolerance for rescaling proportions  
years      Years for which there are observations  
process\_errors      the value of process error  
method\_of\_removal      Label of observed method of removals  
mortality\_instantaneous\_process      The label of the mortality instantaneous process for the observation

**@observation[label].type=proportions\_at\_age**

min\_age      Minimum age  
max\_age      Maximum age  
plus\_group      Use age plus group  
time\_step      The label of time-step that the observation occurs in  
tolerance      Tolerance on the constraint, that for each year the sum of proportions in each age must equal one e.g. tolerance = 0.1 then  $1 - \sum \text{Proportions}$ , can be as great as 0.1  
years      The years of the observed values  
selectivities      Labels of the selectivities  
process\_errors      Process error  
ageing\_error      Label of ageing error to use



---

**@observation[label].type=proportions\_at\_length**

time\_step     The label of time-step that the observation occurs in  
tolerance     Tolerance for rescaling proportions  
years        Years for which there are observations  
selectivities   The labels of the selectivities  
process\_errors   Process error

**@observation[label].type=proportions\_by\_category**

min\_age     Minimum age  
max\_age     Maximum age  
time\_step     The label of time-step that the observation occurs in  
plus\_group     Use age plus group  
years        Years for which there are observations  
selectivities   The labels of the selectivities  
categories2     Target Categories  
selectivities2   Target Selectivities

**@observation[label].type=proportions\_mature\_by\_age**

min\_age     Minimum age  
max\_age     Maximum age  
time\_step     The label of time-step that the observation occurs in  
plus\_group     Use age plus group  
years        Years for which there are observations  
ageing\_error   Label of ageing error to use  
total\_categories   All category labels that were vulnerable to sampling at the time of this  
observation ,not including the categories already given,  
time\_step\_proportion   Proportion through the mortality block of the time step when the  
observation is evaluated

**@observation[label].type=proportions\_migrating**

min\_age     Minimum age  
max\_age     Maximum age  
time\_step     The label of time-step that the observation occurs in  
plus\_group     Use age plus group  
years        Years for which there are observations  
process\_errors   Process error  
ageing\_error   Label of ageing error to use  
process     Process label

**@observation[label].type=tag\_recapture\_by\_age**

min\_age     Minimum age

max\_age      Maximum age  
plus\_group    Use age plus group  
years        Years for which there are observations  
categories2    The available categories in the partition  
selectivities    The labels of the selectivities  
time\_step     The label of time-step that the observation occurs in  
selectivities2    The categories of tagged individuals for the observation  
detection     Probability of detecting a recaptured individual  
time\_step\_proportion    Proportion through the mortality block of the time step when the observation is evaluated

**@observation[label].type=tag\_recapture\_by\_length**

years        Years for which there are observations  
length\_bins    Length bins  
length\_plus    Is the last bin a plus group  
selectivities    The labels of the selectivities used for untagged categories  
tagged\_selectivities    The labels of the tag category selectivities  
detection     Probability of detecting a recaptured individual  
dispersion    Over-dispersion parameter  $\phi$ ,  
time\_step\_proportion    Proportion through the mortality block of the time step when the observation is evaluated

**@penalty label**    Define an object of type *penalty*

label        The label of the penalty  
type        The type of penalty  
**@process label**    Define an object of type *process*  
label        The label of the process  
type        The type of process

**@process[label].type=ageing**

categories    The labels of the categories

**@process[label].type=growth\_basic**

categories    The labels of the categories  
number\_of\_growth\_episodes    Number of growth episodes per year  
growth\_time\_steps    Time step in which each growth episode occurs  
cv        c.v. for the growth model  
sigma\_min    Lower bound on sigma for the growth model

**@process[label].type=maturation**

from        List of categories to mature from  
to        List of categories to mature too  
selectivities    List of selectivities to use for maturation  
years        The years to be associated with rates  
rates        The rates to mature for each year

---

**@process[label].type=mortality\_constant\_rate**

categories     List of categories labels  
m     Mortality rates  
time\_step\_ratio     Time step ratios for the mortality rates

**@process[label].type=mortality\_event**

categories     Categories  
years     Years in which to apply the mortality process  
catches     The number of removals ,catches, to apply for each year  
u\_max     Maximum exploitation rate , $U_{max}$ ,  
selectivities     List of selectivities  
penalty     The label of the penalty to apply if the total number of removals cannot be taken

**@process[label].type=mortality\_event\_biomass**

categories     Category labels  
selectivities     The labels of the selectivities for each of the categories  
years     Years in which to apply the mortality process  
catches     The biomass of removals ,catches, to apply for each year  
u\_max     Maximum exploitation rate , $U_{max}$ ,  
penalty     The label of the penalty to apply if the total biomass of removals cannot be taken

**@process[label].type=mortality\_holling\_rate**

prey\_categories     Prey Categories labels  
predator\_categories     Predator Categories labels  
is\_abundance     Is vulnerable amount of prey and predator an abundance [true] or biomass [false]  
a     parameter a  
b     parameter b  
x     This parameter controls the type of functional form, Holling function type 2 , $x=2$ , or 3 , $x=3$ , or generalised ,Michaelis Menten,  $x=1$ ,  
u\_max     Maximum exploitation rate , $U_{max}$ ,  
prey\_selectivities     Selectivities for prey categories  
predator\_selectivities     Selectivities for predator categories  
penalty     Label of penalty to be applied  
years     Years in which to apply the mortality process

**@process[label].type=mortality\_initialisation\_event**

categories     Categories  
catch     The number of removals ,catches, to apply for each year  
u\_max     Maximum exploitation rate , $U_{max}$ ,  
selectivities     List of selectivities  
penalty     The label of the penalty to apply if the total number of removals cannot be taken

**@process[label].type=mortality\_initialisation\_event\_biomass**

categories     Categories

catch      The number of removals ,catches, to apply for each year  
u\_max      Maximum exploitation rate , $U_{max}$ ,  
selectivities      List of selectivities  
penalty      The label of the penalty to apply if the total number of removals cannot be taken

**@process[label].type=mortality\_instantaneous**

categories      Categories for instantaneous mortality  
m      Natural mortality rates for each category  
time\_step\_ratio      Time step ratios for natural mortality  
selectivities      The selectivities to apply on the categories for natural mortality

**@process[label].type=mortality\_instantaneous\_retained**

categories      Categories for instantaneous mortality  
m      Natural mortality rates for each category  
time\_step\_ratio      Time step ratios for natural mortality  
selectivities      The selectivities to apply on the categories for natural mortality

**@process[label].type=mortality\_prey\_suitability**

prey\_categories      Prey Categories labels  
predator\_categories      Predator Categories labels  
consumption\_rate      Predator consumption rate  
electivities      Prey Electivities  
u\_max       $U_{max}$   
prey\_selectivities      Selectivities for prey categories  
predator\_selectivities      Selectivities for predator categories  
penalty      Label of penalty to be applied  
years      Year that process occurs

**@process[label].type=recruitment\_beverton\_holt**

categories      Category labels  
r0       $R_0$   
b0       $B_0$   
proportions      Proportions  
age      Age to recruit at  
ssb\_offset      Spawning biomass year offset  
steepness      Steepness  
ssb      SSB Label ,derived quantity,  
b0\_initialisation\_phase      Initialisation phase Label that b0 is from  
ycs\_values      YCS Values  
ycs\_years      Recruitment years. A vector of years that relates to the year of the spawning event that created this cohort  
standardise\_ycs\_years      Years that are included for year class standardisation

**@process[label].type=recruitment\_beverton\_holt\_with\_deviations**

categories      Category labels

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r0     R0  
 b0     B0  
 proportions     Proportions  
 age     Age to recruit at  
 ssb\_offset     Spawning biomass year offset  
 steepness     Steepness  
 ssb     SSB Label ,derived quantity,  
 sigma\_r     Sigma r  
 b\_max     Max bias adjustment  
 last\_year\_with\_no\_bias     Last year with no bias adjustment  
 first\_year\_with\_bias     First year with full bias adjustment  
 last\_year\_with\_bias     Last year with full bias adjustment  
 first\_recent\_year\_with\_no\_bias     First recent year with no bias adjustment  
 b0\_initialisation\_phase     Initialisation phase Label that b0 is from  
 deviation\_values     Recruitment deviation values  
 deviation\_years     Recruitment years. A vector of years that relates to the year of the spawning event that created this cohort

**@process[label].type=recruitment\_constant**

categories     Categories  
 proportions     Proportions  
 length\_bins     The length bins recruits are uniformly distributed over, when recruitment occurs  
 r0     R0

**@process[label].type=survival\_constant\_rate**

categories     List of categories  
 s     Survival rates  
 time\_step\_ratio     Time step ratios for S  
 selectivities     Selectivity label

**@process[label].type=tag\_by\_age**

from     Categories to transition from  
 to     Categories to transition to  
 min\_age     Minimum age to transition  
 max\_age     Maximum age to transition  
 penalty     Penalty label  
 u\_max     U Max  
 years     Years to execute the transition in  
 initial\_mortality  
 initial\_mortality\_selectivity  
 loss\_rate  
 loss\_rate\_selectivities  
 selectivities  
 n

**@process[label].type=tag\_by\_length**

from     Categories to transition from

to      categories to transition to  
penalty      Penalty label  
u\_max      U Max  
years      Years to execute the transition in  
initial\_mortality  
initial\_mortality\_selectivity  
selectivities  
n

**@process[label].type=tag\_loss**

categories      List of categories  
tag\_loss\_rate      Tag Loss rates  
time\_step\_ratio      Time step ratios for Tag Loss  
tag\_loss\_type      Type of tag loss  
selectivities      Selectivities  
year      The year the first tagging release process was executed

**@process[label].type=transition\_category**

from      From  
to      To  
proportions      Proportions  
selectivities      Selectivity names

**@process[label].type=transition\_category\_by\_age**

from      Categories to transition from  
to      Categories to transition to  
min\_age      Minimum age to transition  
max\_age      Maximum age to transition  
penalty      Penalty label  
u\_max      U Max  
years      Years to execute the transition in

**@profile label**      Define an object of type *profile*

label      Label  
steps      The number of steps to take between the lower and upper bound  
lower\_bound      The lower bounds  
upper\_bound      The upper bounds  
parameter      The system parameter to profile  
same      A Parameter that are constrained to have the same value as the parameter being profiled

**@project label**      Define an object of type *project*

label      Label  
type      Type  
years      Years to recalculate the values  
parameter      Parameter to project  
multiplier      Multiplier that is applied to the projected value

**@report label**      Define an object of type *report*

label      The label for the report

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type      The type of report  
file\_name      The File Name if you want this report to be in a separate file  
write\_mode      The write mode

**@report[label].type=age\_length**

time\_step      Time Step label  
years      Years  
age\_length  
category

**@report[label].type=ageing\_error\_matrix**

ageing\_error      Ageing Error label

**@report[label].type=initialisation\_partition\_mean\_weight**

**@report[label].type=partition**

time\_step      Time Step label  
years      Years

**@report[label].type=partition\_biomass**

time\_step      Time Step label  
years      Years

**@report[label].type=partition\_mean\_weight**

time\_step      Time Step label  
years      Years

**@report[label].type=partition\_year\_cross\_age\_matrix**

**@selectivity label**      Define an object of type *selectivity*

label      The label for this selectivity  
type      The type of selectivity  
length\_based      Is the selectivity length based  
intervals      Number of quantiles to evaluate a length based selectivity over the age length  
distribution  
partition\_type      The type of partition this selectivity will support, Defaults to same as the  
model  
values  
length\_values

**@simulate label**      Define an object of type *simulate*

label      Label  
type      Type  
years      Years to recalculate the values  
parameter      Parameter to Simulate  
**@time\_step label**      Define an object of type *time\_step*  
label      The label of the timestep

processes     The labels of the processes for this time step in the order that they occur  
**@time\_varying** *label*     Define an object of type *time\_varying*  
label     The time-varying label  
type     The time-varying type  
years     Years in which to vary the values  
parameter     The name of the parameter to time vary



# Appendices

## A. Investigating two options for YCS prior distribution formulations

There are two common ways of parameterising the log-Normal prior distribution of year class strength (YCS) when fitting models in fisheries.

Let  $YCS_y$  represent the YCS for year  $y$ . The two parameterisations used are:

1. Option 1:  $YCS_y \sim LN(\mu, \sigma_R^2)$ , with  $\mu$  chosen so that  $E(YCS_y) = 1$ .
2. Option 2:  $YCS_y = e^{\varepsilon_y - \frac{1}{2}\sigma_R^2}$ , where  $\varepsilon_y \sim N(0, \sigma_R^2)$ .

To check whether the two representations are equivalent, we will determine, in each case the density function of  $YCS_y$  on the log-scale.

Note that, in general, if  $Y \sim LN(\mu, \sigma_R^2)$  (ie random variable  $Y$  follows a log-Normal distribution with parameters  $\mu$  and  $\sigma_R^2$ ), then the expectation,  $E(Y)$ , and variance,  $Var(Y)$ , of  $Y$  are given by

$$E(Y) = e^{\mu + \frac{1}{2}\sigma_R^2},$$

and

$$Var(Y) = \left[ e^{\sigma_R^2} - 1 \right] e^{2\mu + \sigma_R^2}.$$

The log-Normal distribution can be expressed on the log scale as follows:

$$\log Y \sim \text{Normal}(\mu, \sigma_R^2).$$

**Option 1:**  $YCS_y \sim LN(\mu, \sigma_R^2)$ , with  $E(YCS_y) = 1$

Setting  $E(YCS_y) = 1$  implies

$$\begin{aligned} e^{\mu + \frac{1}{2}\sigma_R^2} &= 1 \\ \Rightarrow \mu + \frac{1}{2}\sigma_R^2 &= \log 1 \\ \Rightarrow \mu &= -\frac{1}{2}\sigma_R^2 \end{aligned} \tag{A.1}$$

and

$$\begin{aligned} Var(YCS_y) &= \left[ e^{\sigma_R^2} - 1 \right] e^{2\mu + \sigma_R^2} \\ &= \left[ e^{\sigma_R^2} - 1 \right] e^{2(-\frac{1}{2}\sigma_R^2) + \sigma_R^2} \\ &= \left[ e^{\sigma_R^2} - 1 \right] e^0 \\ &= e^{\sigma_R^2} - 1. \end{aligned} \tag{A.2}$$

So, on the log scale we have:

$$\log YCS_y \sim N\left(-\frac{1}{2}\sigma_R^2, \sigma_R^2\right).$$

**Option 2:**  $YCS_y = e^{\varepsilon_y - \frac{1}{2}\sigma_R^2}$ , **where**  $\varepsilon_y \sim N(0, \sigma_R^2)$

In this case,  $YCS_y = e^{\varepsilon_y - \frac{1}{2}\sigma_R^2}$  implies

$$\log YCS_y = \varepsilon_y - \frac{1}{2}\sigma_R^2$$

and

$$E(\log YCS_y) = E\left(\varepsilon_y - \frac{1}{2}\sigma_R^2\right) = -\frac{1}{2}\sigma_R^2,$$

since  $E(\varepsilon_y) = 0$ . We also have:

$$\begin{aligned} \text{Var}(\log(YCS_y)) &= \text{Var}\left(\varepsilon_y - \frac{1}{2}\sigma_R^2\right) \\ &= \text{Var}(\varepsilon_y) \\ &= \sigma_R^2 \end{aligned} \tag{A.3}$$

Therefore

$$\log YCS_y \sim N\left(-\frac{1}{2}\sigma_R^2, \sigma_R^2\right).$$

Therefore the two parameterisations result in exactly the same distribution for  $YCS_y$  values and should give the same results **if expressed correctly** in MCMC algorithms.

To illustrate that these two distributions are exactly the same, we first use simulations to show that we get the same  $YCS_y$  values when generating sequences from these two formulations. One is generated directly from the log-Normal distribution, while the other is obtained by transforming a Normal random variable.

### Investigating prior specification

Given the two different representations, the question is how should their prior distribution contributions to the negative log-posterior be specified?

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### Prior based on Option 1

For Option 1, this is straight-forward. The  $YCS_y$ 's are generated from a log-Normal distribution, so the contribution to the log posterior is based on the log-Normal density function. That is, if we let  $Y = YCS_y$ , then the density function of  $Y$  is given by

$$f(y) = \frac{1}{y\sigma_R\sqrt{2\pi}} e^{-\frac{1}{2\sigma_R^2}(\log y - \mu)^2}.$$

Since  $\mu = -\frac{1}{2}\sigma_R^2$  as shown in equation A.1, we have

$$-\log f(y) = \log y + \log \sigma_R + \frac{1}{2} \log 2\pi + \frac{1}{2\sigma_R^2} \left( \log y - \left(-\frac{1}{2}\sigma_R^2\right) \right)^2. \quad (\text{A.4})$$

### Prior based on Option 2

For Option 2, I will look at two ways used to specify the prior and say which one is correct.

#### Prior 2 - Normal distribution for $\epsilon_y$ .

In this approach, we have  $YCS_y = e^{\epsilon_y - \frac{1}{2}\sigma_R^2}$ , where  $\epsilon_y \sim N(0, \sigma_R^2)$ .

What is sometimes done is to then express the contribution to the negative log-posterior using  $-\log f(\epsilon_y)$ ,

where

$$f(\epsilon_y) = \frac{1}{\sigma_R\sqrt{2\pi}} e^{-\frac{1}{2\sigma_R^2}\epsilon_y^2},$$

and therefore

$$-\log f(\epsilon_y) = \log \sigma_R + \frac{1}{2} \log 2\pi + \frac{1}{2\sigma_R^2}\epsilon_y^2. \quad (\text{A.5})$$

However, this contribution is based on the density function for  $\epsilon_y$  and not for  $Y$ . Such an approach is incorrect. The two contributions are different as seen in equations A.4 and A.5, and as shown below.

So what does this mean in practice?

The overall model is based on  $YCS_y$ , rather than  $\epsilon_y$ . Using a negative log-posterior based on  $f(\epsilon_y)$  gives incorrect weights to the  $YCS_y$  values in the model, meaning that in MCMC steps, acceptance probabilities will be incorrect. When using the specification based on  $\epsilon_y$ , the correct approach is to use variable transformation methods to obtain probability densities for the  $YCS_y$  values. These density values based on  $f(Y)$  are the ones to use in the negative log-posterior.

### Prior 3 - Variable transformation to obtain $f(Y)$ based on $f(\epsilon_y)$

Given  $Y = e^{\epsilon_y - \frac{1}{2}\sigma_R^2}$ , where  $\epsilon_y \sim N(0, \sigma_R^2)$ , we need to find  $g(y)$ , the distribution of the transformed variable  $YCS_y$ .

Variable transformation theory tells us that:

$$g(y) = f(s(y)) \left| \frac{ds(y)}{dy} \right|,$$

where  $s(y) = \epsilon_y(y)$  is the result of the conversion from  $Y$  to  $\epsilon_y$ , and  $\left| \frac{ds(y)}{dy} \right|$  is the Jacobian of the transformation.

We find  $s(y)$  by expressing  $\epsilon_y$  as a function of  $y$ :

$$\begin{aligned} y &= e^{\epsilon_y - \frac{1}{2}\sigma_R^2} \\ \Rightarrow \log y &= \epsilon_y - \frac{1}{2}\sigma_R^2 \\ \Rightarrow \epsilon_y &= \log y + \frac{1}{2}\sigma_R^2. \end{aligned} \tag{A.6}$$

Therefore

$$\frac{ds(y)}{dy} = \frac{d\epsilon_y(y)}{dy} = \frac{d}{dy} \left( \log y + \frac{1}{2}\sigma_R^2 \right) = \frac{1}{y}.$$

Then

$$\begin{aligned} g(y) &= \frac{1}{\sigma_R \sqrt{2\pi}} e^{-\frac{1}{2\sigma_R^2} [\log y + \frac{1}{2}\sigma_R^2]^2} \cdot \left| \frac{1}{y} \right| \\ &= \frac{1}{y\sigma_R \sqrt{2\pi}} \exp \left\{ -\frac{[\log y - (-\frac{1}{2}\sigma_R^2)]^2}{2\sigma_R^2} \right\}. \end{aligned} \tag{A.7}$$

This is the density function of a logNormal distribution with parameters  $\mu = -\frac{1}{2}\sigma_R^2$  and  $\sigma_R^2$ , that is

$$Y \sim LN \left( -\frac{1}{2}\sigma_R^2, \sigma_R^2 \right),$$

and from equation A.7 we see that

$$-\log g(y) = \log y + \log \sigma_R + \frac{1}{2} \log 2\pi + \frac{1}{2\sigma_R^2} \left( \log y - \left( -\frac{1}{2}\sigma_R^2 \right) \right)^2. \tag{A.8}$$

which is exactly the same expression as that for  $-\log f(y)$  in equation A.4.

---

Therefore if used correctly, Option 2 parameterisation results in the same contribution,  $-\log g(y)$ , to the negative log posterior as Option 1 ( $-\log f(y)$ ).

Note that  $-\log f(y) = -\log f(\epsilon_y)$  only for  $Y = 1$ .

I'll carry out a brief simulation exercise to illustrate the point. I will use the density functions derived above, rather than existing R functions and calculate  $-\log f(y) = -\log g(y)$  and  $-\log f(\epsilon_y)$  for a sequence of  $YCS_y$  values.

The simulation results indicate that  $-\log f(y) = -\log g(y) = -\log f(\epsilon_y)$  only for  $Y = 1$ . For other values of  $Y$ , the size of the difference between  $-\log f(y)$  and  $-\log f(\epsilon_y)$  is given by  $|\log y|$  and does not depend on  $\sigma_R^2$ . Therefore, the differences increase in size as the  $YCS$  value diverges further from 1.

What is the implication of this?

Incorrect use of the prior based on  $\epsilon_y$  (ie using  $-\log f(\epsilon_y)$  in place of  $-\log g(y)$ ) results in prior contributions to the negative log-posterior that are lower or higher by  $\log YCS_y$  than what they should be.