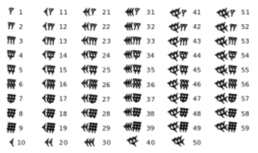
Data Structures & Algorithms 1

Topic 10 – Bit Manipulation

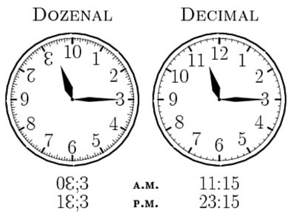
Positional number system

- The Hindu-Arabic numeral system, which is base 10, is the most commonly used system in the world
- The system evolved in India around 300BC, with zero identified about 1,000 years later, popularized by Fibonacci and spreading across Europe by around 1400
- The ancient Babylonians used a base 60 system
- The Mayans used a base 20 system



Positional number system

- There have been arguments for a base 12 system



Face off: Telling the time using a dozenal clock is simpler, once you get used to the two new digits. This reads "elv dit three", and "onequa elv dit three".

Positional number system

- In base 10 1004 means
  - 4 units
  - 0 10s
  - 0 10^2s
  - 1 10^3s = 1,004 (in base 10 – obviously!)
- In base 12 1004 means
  - 4 units
  - 0 12s
  - 0 12^2s
  - 1 12^3s = 1,732 (in base 10)

Converting base

- To convert a number in base 10 to any other base, we need to figure out how many units of each power it has
- E.g. convert 1004 from base 10 to base 12
- Find how many 12^3s it has: 0
- Find out how many 12^2s it has: 6 with remainder 140
- Find out how many 12s it has: 11 with remainder 8
- Find out how many units it has: 8
- So the answer is 6-11-8 or 6elv8

1004 % 12 = 8	1004 – 8 = 996	996 / 12 = 83
83 % 12 = 11	83 – 11 = 72	72 / 12 = 6
6 % 12 = 6	6 – 6 = 0	0 / 12 = 0

Converting base

- In mathematics and computing, **hexadecimal** (also base 16, or hex) is a positional numeral system with a radix, or base, of 16. It uses sixteen distinct symbols, most often the symbols 0–9 to represent values zero to nine, and A, B, C, D, E, F (or alternatively a, b, c, d, e, f) to represent values ten to fifteen
- Convert 9A3 to decimal base 10
- 9A3 =
- $9 * 16^2$ = 2,304
- $10 * 16^1$ = 160
- $3 * 16^0$ = 3
- Total = 2,304 + 160 + 3 = 2,467

Bit Manipulation

- Why does Bender freak out when he sees this number in the mirror?

Bit representation

- In Java an `int` is represented as 32 bits
- Starting from the right, each bit represents increasing powers of 2
- The leftmost bit is special. It is negative, and represents -2^31, which is -2,147,483,647
- The effective range is therefore from 2,147,483,646 to -2,147,483,647
- There are several operators for directly manipulating the bit representation

Bit representation

This is negative – if the value of this bit is 1 it counts as -2^31

2^{31}	2^{30}	2^{29}	2^{28}	2^{27}	2^{26}	2^{25}	2^{24}	2^{23}	2^{22}	2^{21}	2^{0}	2^{19}	2^{18}	2^{17}	2^{16}
1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0

2^{15}	2^{14}	2^{13}	2^{12}	2^{11}	2^{10}	2^{9}	2^{8}	2^{7}	2^{6}	2^{5}	2^{4}	2^{3}	2^{2}	2^{1}	2^{0}
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

- The `int` value of this 32 bit string is -2^31 + 2^30 + 2^29 which is -536,870,912

Big Endian vs Little Endian

- Little Endian is used by Java, also PowerPC, ARM, iPhone, Xbox 360 and PS3 and encodes the bytes in this order:

2^{31}	2^{30}	2^{29}	2^{28}	2^{27}	2^{26}	2^{25}	2^{24}	2^{23}	2^{22}	2^{21}	2^{0}	2^{19}	2^{18}	2^{17}	2^{16}
1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0

2^{15}	2^{14}	2^{13}	2^{12}	2^{11}	2^{10}	2^{9}	2^{8}	2^{7}	2^{6}	2^{5}	2^{4}	2^{3}	2^{2}	2^{1}	2^{0}
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

- Big Endian is used by Intel (C / C++ depends on the system) and encodes bytes in this order:

2^{7}	2^{6}	2^{5}	2^{4}	2^{3}	2^{2}	2^{1}	2^{0}	2^{15}	2^{14}	2^{13}	2^{12}	2^{11}	2^{10}	2^{9}	2^{8}
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

2^{23}	2^{22}	2^{21}	2^{0}	2^{19}	2^{18}	2^{17}	2^{16}	2^{31}	2^{30}	2^{29}	2^{28}	2^{27}	2^{26}	2^{25}	2^{24}
0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0

Picking out a byte

- Say you want to pick out the second byte in a 32-bit integer, you can do it as follows

```
int num = 3534464;  //we want byte 2 of this
int filter = Integer.parseInt
     ("00000000111111110000000000000000",2);
int secondbyte = num & filter;
```

- The bits in the position of a 0 are lost, the bits in the position of a 1 are preserved

Bit representation

- 0 is 00000000 00000000 00000000 00000000
- -1 is 11111111 11111111 11111111 11111111
- -2 is 11111111 11111111 11111111 11111110
- 65535 is 00000000 00000000 11111111 11111111
- -65535 is 11111111 11111111 00000000 00000001

Bitwise operators

Operator	Name	Description
&	bitwise AND	The bits in the result are set to 1 where the corresponding bits in the two operands are both 1, the other bits are set to 0
|	bitwise inclusive OR	The bits in the result are set to 1 where at least one of the corresponding bits in the two operands is 1, the other bits are set to 0
^	bitwise exclusive OR	The bits in the result are set to 1 where the corresponding bits in the two operands are different, the other bits (where the corresponding bits in the two operands are the same) are set to 0
<<	left shift	Shifts the bits of the first operand left by the number of bits specified by the second operand; fill from the right with 0.
>>	signed right shift	Shifts the bits of the first operand right by the number of bits specified by the second operand. If the first operand is negative, 1s are filled in from the left; otherwise, 0s are filled in from the left.
>>>	unsigned right shift	Shifts the bits of the first operand right by the number of bits specified by the second operand; 0s are filled in from the left.
~	bitwise complement	All 0 bits are set to 1, and all 1 bits are set to 0.

Bitwise AND operator (&)

Bit 1	Bit 2	**Bit 1 & Bit 2**
0	0	0
1	0	0
0	1	0
1	1	1

- Take the two numbers, convert into bit form, follow the above rules to combine them, and then convert back to an integer value
- `int result = 7 & 3;`
  - 7 = ...111
  - 3 = ...011
  - 7 & 3 = ...011 = 3

Bitwise OR operator (|)

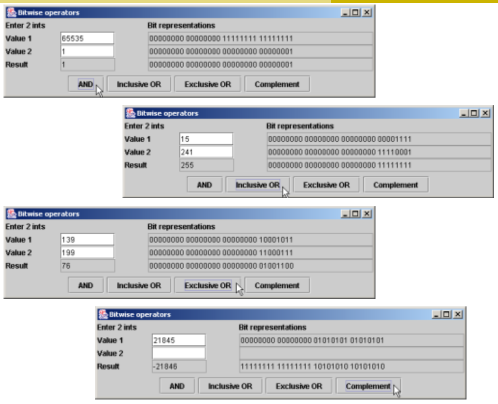
Bit 1	Bit 2	**Bit 1 | Bit 2**
0	0	0
1	0	1
0	1	1
1	1	1

- Take the two numbers, convert into bit form, follow the above rules to combine them, and then convert back to an integer value
- `int result = 11 | 8;`
  - 11 = ...1011
  - 8 = ...1000
  - 11 | 8 = ...1011 = 11

Bitwise XOR operator (^)

Bit 1	Bit 2	**Bit 1 & Bit 2**
0	0	0
1	0	1
0	1	1
1	1	0

- Take the two numbers, convert into bit form, follow the above rules to combine them, and then convert back to an integer value
- `int result = 15 ^ 6;`
  - 15 = ...1111
  - 6 = ...0110
  - 15 ^ 6 = ...1001 = 9



Enter 2 ints

		Bit representations
Value 1	65535	00000000 00000000 11111111 11111111
Value 2	1	00000000 00000000 00000000 00000001
Result	1	00000000 00000000 00000000 00000001

Enter 2 ints

		Bit representations
Value 1	15	00000000 00000000 00000000 00001111
Value 2	241	00000000 00000000 00000000 11110001
Result	255	00000000 00000000 00000000 11111111

Enter 2 ints

		Bit representations
Value 1	139	00000000 00000000 00000000 10001011
Value 2	199	00000000 00000000 00000000 11000111
Result	76	00000000 00000000 00000000 01001100

Enter 2 ints

		Bit representations
Value 1	21845	00000000 00000000 01010101 01010101
Value 2		
Result	-21846	11111111 11111111 10101010 10101010

Bitwise left shift (<<)

- Take the number, convert into bit form, shift the bits to the left and fill in the spaces on the right with 0s
- `int result = 15 << 3;`
  - 15 = 00000000 00000000 00000000 00001111
  - 15 << 3 = 00000000 00000000 00000000 01111000
  - 15 << 3 = 120 (15 *2^3)
- `int result = 116 << 5;`
  - 116 = 00000000 00000000 00000000 01110100
  - 116 << 5 = 00000000 00000000 00001110 10000000
  - 116 << 5 = 3,712 (116 *2^5)

Bitwise signed right shift (>>)

- Take the number, convert into bit form, shift the bits to the right and fill in the spaces on the right with 1s if it's a negative number, 0s otherwise
- `int result = 116 >> 3;`
  - 116 = 00000000 00000000 00000000 01110100
  - 116 >>3 = 00000000 00000000 00000000 00001110
  - 116 >>3 = 14 which is (around 116 / 2^3)
- `int result = -116 >> 3;`
  - -116 = 11111111 11111111 11111111 10001100
  - -116 >>3 = 11111111 11111111 11111111 11110001
  - -116 >>3 = -15 which is (around -116 / 2^3)

Bitwise unsigned right shift (>>>)

- Take the number, convert into bit form, shift the bits to the right and fill in the spaces with 0s no matter what
- `int result = 116 >>> 3;`
  - 116 = 00000000 00000000 00000000 01110100
  - 116 >>>3 = 00000000 00000000 00000000 00001110
  - 116 >>>3 = 14 which is (around 116 / 2^3)
- `int result = -116 >> 3;`
  - -116 = 11111111 11111111 11111111 10001100
  - -116 >>>3 = 00011111 11111111 11111111 11110001
  - -116 >>>3 = 536,870,897

Bitwise complement (~)

- Take the number, convert into bit form, flip every 1 to a 0 and vice versa
- This operation on n is the same as (n*-1) - 1
- `int result = ~116;`
  - 116 = 00000000 00000000 00000000 01110100
  - ~116 = 11111111 11111111 11111111 10001011
  - ~116 = -117

Interview question

- Explain what the following code does:
  ((n & (n-1) == 0)
- What does it mean if A & B == 0?
- It means that A and B never have a 1 bit in the same place
- So if n & (n-1) == 0, then n and n-1 never share a 1

Interview question

- What does n-1 look like (as compared with n)?
- Try doing subtraction by hand in base 2

	1	1	0	1	0	1	1	0	0	0
-										1
=	1	1	0	1	0	1	0	1	1	1

- When you subtract 1 from a number you look at the least significant bit
- If it's a 1 you change it to a 0 and you're done
- If it's a zero, you must borrow from a larger bit, so you go to increasingly larger bits, changing each from a zero to a 1, until you find a 1
- You flip that to a 0 and you're done

Interview question

- So what does n & (n – 1) indicate?
- n and (n – 1) must have no 1s in common
- Therefore all the Xs below must be zeroes

	X	X	X	X	X	X	1	0	0	0
-										1
=	X	X	X	X	X	X	0	1	1	1

- So the number n must look like this: 00001000
- n is therefore a power of two
- ((n & (n-1)) == 0) checks if n is a power of 2

Interview question

- Subtract two numbers without using minus
- Bitwise complement gives you the negative version of a number - 1

```
• int first = 10;
• int second = 3;
• result = first + ~second + 1;
```

Interview question

- Add two numbers together without using +, - , * or /

```
public int addition(int a, int b) {
   if(b==0) {
      return a;
   } else {
      sum = a^b;
      carry = (a&b)<<1;
      return addition(sum,carry);
   }
}
```