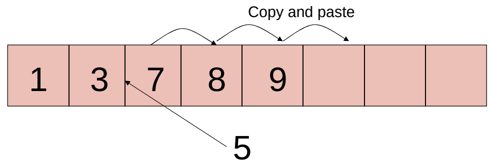
Data Structures & Algorithms 1

Topic 8 – Linked Lists

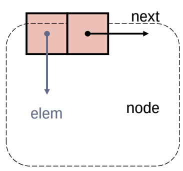
Why are arrays limited?

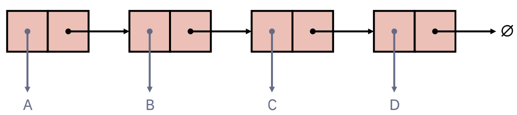
- Every data structure we have considered thus far has involved the use of arrays
- The main problem with arrays is that in order to move an item from one slot to another it has to be copied, pasted and deleted
- This is very time consuming – can we come up with an alternative that avoids this process?



Linked List

- A linked list is an abstract data structure consisting of a sequence of links
- Each link (also called a node) stores
  - data
  - link to the next node





Advantages

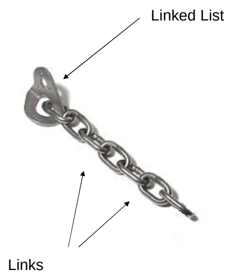
- Why are linked lists better than arrays?
- Arrays waste space because they aren't always full
- When they get full it is not easy to extend them
- If you want to insert a new element in a particular slot you have to copy all the items that need to be moved
- Linked lists avoid all of these problems because they can adapt their ordering by changing the items they point to – no memory is wasted and extra links can easily be added

Array

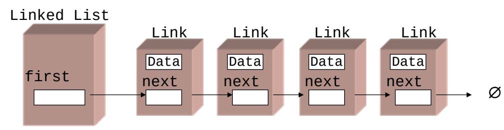
Linked List

Structure

- One special type of class called Linked List stores the first or anchor link for all the subsequent Link objects
- A Linked List object is instantiated to point to the start of the list and multiple Link objects are created for each link in the list
- All contain references to the next link in the list



Structure



- The Linked List class stores a reference to the first link
- The Link class stores a bit of data and a reference to the next link

Java Implementation

```
class LinkedList {
  private Link first;

  public LinkedList( ){
       first=null;
  }

  public boolean isEmpty( ){
       return (first==null);
  }
  ……
}
```

- Linked list only contains a reference to the first link
- This is originally set to null

Java Implementation

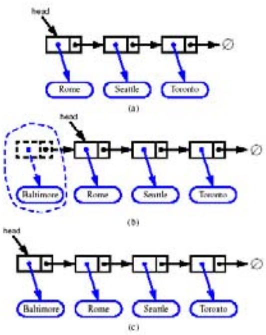
```
public class Link{

        public int data;
        public Link next;

        public Link(int datain) // constructor
        {
                 data = datain;    // initialize data
                                   // 'next' is automatically set to null
         }
}
```

- Note that although the Link object contains another Link (next), this is only a reference to the next Link
- This is the same for any object, the label (e.g. next) is only a reference and does not represent the actual object itself
- `next   = new Link( )` : next acts as a reference to this object which is created somewhere else in memory

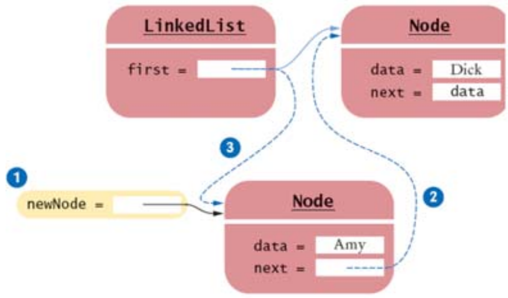
Inserting at the Head

- The head is the first link
- The tail is the last link
- When inserting or removing links we always take care not to break the chain

1. Create a new link
2. Have new link point to old first link
3. Update Linked List to point to new link



Adding at the Head



Adding a Node to the Head of a Linked List

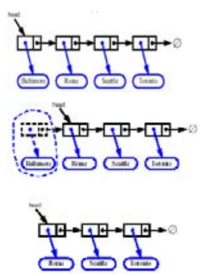
Insert at Head

```
public void insertHead (int number) {
  Link newLink = new Link(number);
  newLink.next = first;
  first = newLink;
}
```

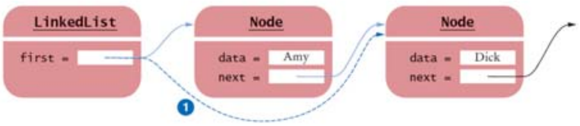
- This insertHead( ) method in LinkedList does the following
  - takes in a number
  - creates a new link with that piece of data
  - sets that link to point to the old first link
  - sets the first link to point to the new link

Deleting at the Head

- Update head to point to next node in the list
- Garbage collector will now reclaim the former first node



Removing the First Node from a Linked List



Removing the First Node from a Linked List

Delete at Head

```
public Link deleteHead () {
  Link temp =first;
  first = first.next;
  return temp;
}
```

- This method in LinkedList does the following
  - Backs up the first link
  - Gets the link after the first link
  - Puts the old first link equal to that
  - Returns the link that has been 'bypassed'

Traversing a linked list

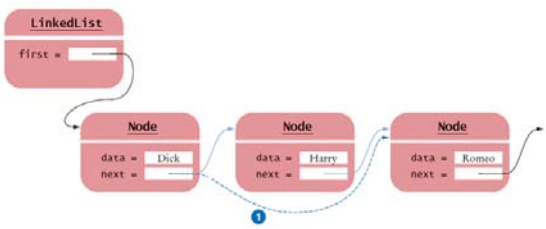
- With arrays, each item occupies a particular slot, and we can index any slot we like
- With a linked list, you have to start at the beginning of the chain and work your way along in order to find an item
- Binary search not possible – big disadvantage of linked lists

```
public void display {
  Link current = first; // start with first link
  while(current!=null){
      current.displayLink();  //print out the link
      current=current.next;
            //keep going until you come to the end
    }
}
```

Finding a link

- What if we want to find or delete one particular node?
- The algorithm works like this:
  - Take in the value to be found or deleted
  - Start at the beginning of the list
  - Keep moving down the links until we find the correct one
  - All the while keep tracking the current link and the previous link (so we can join them up when required)
  - Now update the references to bypass the link to be deleted

Removing a Node From the Middle of a Linked List



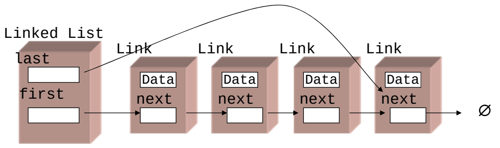
Removing a Node From the Middle of a Linked List

Removing a Node From the Middle of a Linked List

```
public Link delete(int key) {         // delete link with given key
     Link current = first;              // search for link
     Link previous = first;            //put these equal to first Link
     while(current.data != key){
        if(current.next == null){
           return null;                 // didn't find it
        }else {
           previous = current;          // go to next link
           current = current.next;
        }
     }                                // found it
     if(current == first){                // if first link,
        first = first.next;             //    change first
     }else{                              // otherwise,
        previous.next = current.next;   //    bypass it
     }
     return current;
}                                   NOTE: Assumes list is not empty
```

Double-Ended Linked Lists

- A double-ended list is similar to an ordinary linked list with one additional feature
- It as a reference to the last link as well as the first
- This allows a new link to be inserted or deleted at the end as well as the beginning
- Handy for implementing a queue (items arrive one end and leave at the other)

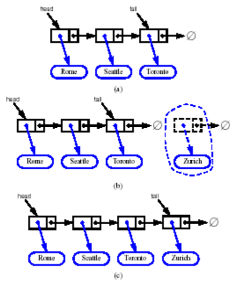


Double-Ended Linked Lists

- A double ended Linked List class will have the following variables
  - first
  - last
- It will have the following methods
  - insertFirst(int data)
  - insertLast(int data)
  - deleteFirst( )
  - deleteLast( ) (requires a doubly-linked list)
- In these methods, the variables first and last are updated accordingly

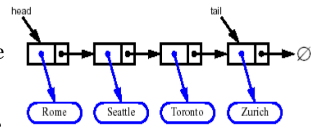
Inserting at the Tail

- Create a new link
- Have new link point to null
- Have old last link point to new node
- Update tail of Linked List object to point to new node



Removing at the Tail

- Removing at the tail of a singly-linked double-ended list is not efficient!
- There is no constant-time way to update the tail to point to the previous node
- We need to use double links (that point to both next and previous link)



Linked-List Efficiency

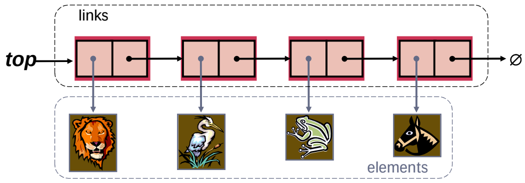
- Insertion and deletion at the beginning of a link list are very fast → O(1)
- Finding, deleting or inserting in a particular location requires traveling through an average of half the links → O(n)
- However, nothing needs to be copied, so it is a faster O(n) than array insert – only references are updated
- Major advantage is that no memory is wasted
- Vectors in Java are expandable arrays but these usually expand more than they need to

Abstraction

- It doesn't matter if we implement the stack or queue using an array or linked list
- In object-oriented programming an Abstract Data Type (ADT) is considered without regard to its implementation
- We only want to know how to use the methods, not how they carry out their tasks
- The ADT specification is called the *interface* (e.g. in a stack these would be push( ) and pop( )

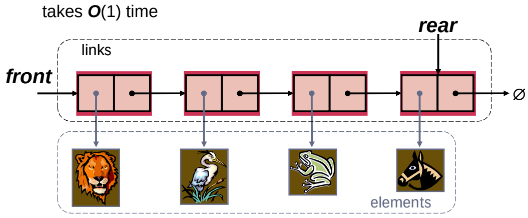
Stack with a Singly Linked List

- We can implement a stack with a singly linked list
- The top element is stored at the first node of the list
- The space used is ***O***(***n***) and each operation of the Stack ADT takes ***O***(1) time



Queue with a Singly Linked List

- We can implement a queue with a double ended linked list
  - The front element is stored at the first node
  - The rear element is stored at the last node
- The space used is ***O***(***n***) and each operation of the Queue ADT takes ***O***(1) time

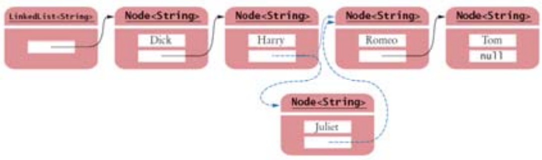


Sorted Lists

- Remember the priority queue – this required the items to be sorted
- In order to keep a linked list sorted, we have to insert at particular locations
- Only way to find the correct location is to search through the list – can't use binary search
- When you find the correct location (i.e. when you find a value bigger than the one you want to insert) update the pointers of the relevant links

Insertion

- After finding the correct location for the item we must update the pointers
  - The new element must be set to point to the next element in the linked list
  - The previous element must be updated to point to the new element



Insertion to ordered list

```
public void insertOrdered(Link newlink){  // insert (in order)

      Link previous = null;              // start at first
      Link current = first;              // until end of list

      while(current != null && newlink.data > current.data)
      {                                  // while key > current
         previous = current;
         current = current.next;         // go to next item
      }

      if(previous==null){                // at beginning of list
         first = newlink;                // first --> newlink
      }else{                             // not at beginning
         previous.next = newlink;        // old prev --> newlink
         newlink.next = current;        // newlink --> old current
      }
}                                       // end insert()
```

Efficiency of Sorted Linked Lists

- Insertion and deletion of items in the sorted linked lists requires O(n) comparisons (n/2 on average) because we have to step through half of the list
- The minimum (or max) can be found or deleted in O(1) time (will be at the top of the sorted linked list)
- If an application frequently accesses the minimum item and fast insertion isn't critical then this is a good choice

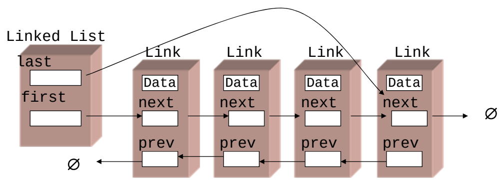
Using linked lists for array sorting

- Take the items from your unsorted array
- Put them into a sorted linked list one by one
- You can then put them back in the array and it will be sorted
- Actually turns out to be significantly more efficient than insertion sort because fewer copies are necessary!
- Still $O(n^2)$ comparisons, however each item is only copied twice
  - Once from the array to the linked list
  - Once back from the linked list into the array
- 2n copies is much better than the usual $O(n^2)$ copies using Insertion Sort

Doubly Linked Lists

- Removing an element at the tail of a singly linked list is not easy since
  - We can't move backwards!!!
- Indeed, there are many times we need to know the predecessor as well as the successor of a particular node
- A linked list where you can traverse it forwards or backwards is a doubly linked list (references going both ways)

Double-Ended Doubly Linked Lists



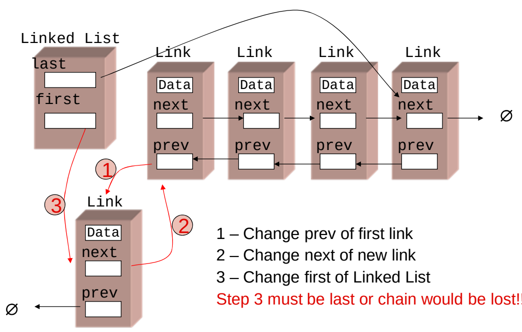
Double-Ended Doubly Linked Lists

- The specification for the link class now looks like this:

```
public class Link{
  public int data;
  public Link next;
  public Link previous;
}
```

- Now every time we insert or delete a link, we must deal with four links instead of two
  - Update the two attachments between the previous link and the new one
  - Update the two attachments between the next link and the new one
- Now we can move backwards or forwards through the list

Insertion at beginning

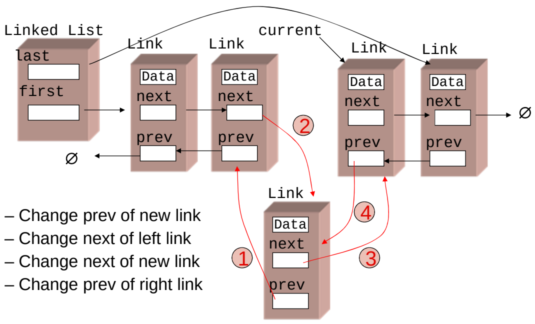


1 – Change prev of first link
2 – Change next of new link
3 – Change first of Linked List
Step 3 must be last or chain would be lost!!!

Java Implementation

```
public void insertHead(long data) {   // insert at head

     Link newLink = new Link(data);   // make new link

     if( isEmpty() ){                // if empty list,
        last = newLink;              // newLink <-- last

    }else{
        first.previous = newLink;    // newLink <-- old first
     }

     newLink.next = first;           // newLink --> old first
     first = newLink;                // first --> newLink
 }
```

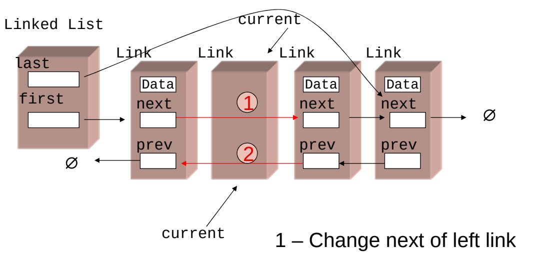
Insertion in order



1 – Change prev of new link
2 – Change next of left link
3 – Change next of new link
4 – Change prev of right link

```
public void insertOrdered(long data){      //inserts data in order

      Link current = first;                // start at beginning
      while(current!=null && data > current.data) {
         current = current.next;           // move to next link

      }

      Link newLink = new Link(data);       // make new link

      if(current==first) {                 // if insertion at head
         insertHead(data);

      }else if(current==last){             //if insertion at tail
         insertTail(data);

     } else {                             // somewhere in middle
          newLink.previous = current.previous;    // step 1
          current.previous.next = newLink;        // step 2
          newLink.next = current;                 // step 3
         current.previous = newLink;             // step 4
      }
}
```

Deletion



1 – Change next of left link
2 – Change prev of right link

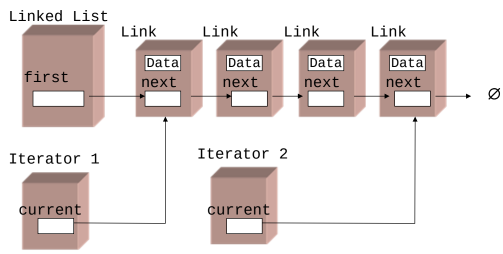
```
public Link delete (long key) {       // delete item with given key

      Link current = first;            // start at beginning

      while(current.data != key){      // until match is found
         current = current.next;       // move to next link
      }

       if(current == null){
          return null;                   // didn't find it
       }

       if(current==first){               // found it; first item?
          first = current.next;          // first --> old next
      } else {                           // not first
                                          // old previous --> old next
          current.previous.next = current.next;
      }

       if(current==last){                 // last item?
          last = current.previous;        // old previous <-- last
       }else{                             // not last item
                                          // old previous <-- old next
          current.next.previous = current.previous;
      }
       return current;                   // return value
}
```

Iterators

- In order to find a link we started at the beginning and searched all the way through
- This doesn't give the user any control over the items passed over
- Suppose you want to traverse a list and perform some operation on certain links
  - Increase the wages of all employees being paid the minimum without affecting anyone else's
  - Delete all customers in a database who haven't ordered anything for the past six months
- This is easy in an array because every element has a fixed index
- We need a method that can step from link to link performing an action on each one

List Iterator

- Iterators always point to some link in the list
- They are associated with the list but are not part of it



Iterator Class

- Objects containing references to items in data structures, used to traverse these structures, are commonly called iterators
- The main advantage of using an object is that we can create as many references as we want

```
class ListIterator(){
  private Link current;
  …
}
```

- It easiest to get the linked list class to create the iterator so we add a getIterator( ) method to it

```
LinkedList theList = new LinkedList();
ListIterator iter1 = theList.getIterator();
Link aLink = iter1.getCurrent();  //access link at
  iterator
iter1.nextLink();                     // move iterator to
  next Link
```

Additional Iterator Features

- We've seen that's it's handy to store a reference to the previous link
- If our linked list isn't doubly-linked then the iterator should store both current and previous so that it can delete links
- It is also handy for the iterator to store a reference to the linked list class so it can access the first element of the list

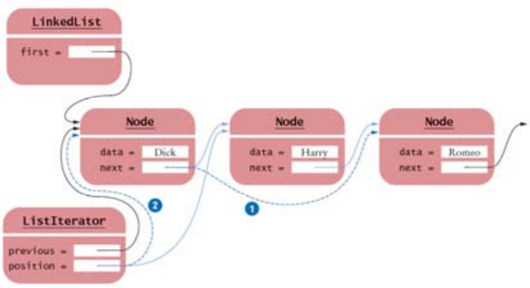
Iterator Code

```
class ListIterator
   {
   private Link current;          // current link
   private Link previous;         // previous link
   private LinkList ourList;      // our linked list

   public ListIterator(LinkList list) // constructor
      {
      ourList = list;
      reset();
      }

   public void reset()            // start at 'first'
      {
      current = ourList.getFirst();
      previous = null;
      }

   public void nextLink()
     {
     previous = current; //set previous to this
     current = current.next;      //set this to next
     }
```

Iterator Methods

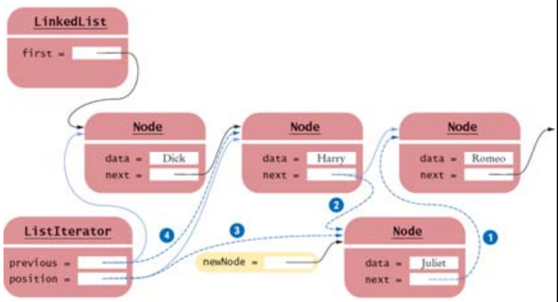
- Additional methods can make the iterator a flexible and powerful class
- All operations that involve iterating through the list are more naturally performed by the iterator
  - reset() – Sets the iterator to the start of the list
  - nextLink() – moves the iterator to the next link
  - getCurrent() – returns the link at the iterator
  - atEnd() – returns true if iterator is at end of list
  - insertAfter() – inserts a new link after the iterator
  - insertBefore() – inserts a new link before the iterator
  - deleteCurrent() – deletes the link at the iterator

Removing a Link using the Iterator



Removing a Link From the Middle of a Linked List

Adding a Link Using an Iterator



Adding a Link to the Middle of a Linked List

Where does the Iterator point?

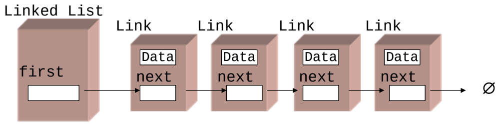
- When you delete an item, should the iterator point to the next item, previous item, or back to the beginning of the list?
- Keeping it in the vicinity is usually convenient
- However, can't set it to the previous item because there's no way to reset the iterator's previous field to the one before that (unless it's doubly linked)
- Move it to the next item, and back to the start if you've just deleted the last item

The atEnd() Method

- Does this return true when the iterator points to the last valid link or when the iterator points *past* the last link?
- If it points past the last link, then you can't do anything with that link
- You also can't back up an interator in a singly linked-list (e.g. you couldn't search for the last link and then delete it)
- The better approach is to make sure that the iterator always points to a valid link – should return true when it points to the last valid link (has to always check if next is null)

Stacks using Linked Lists

- Implement a stack structure for reversing a word using a single-ended singly-linked list
- We need a single-ended singly-linked list where we can insert and delete at the head



Link

- First, create your link

```
class Link{

   public char data;               // data item
   public Link next;                 // next link in list

   public Link(char data){       // constructor
      this.data = data;              // initialize data
   }
}
```

Linked List

- Now, make your list

```
public class LinkedList {

   private Link first;            // ref to first link

  public LinkedList(){             // constructor
     first = null;               // no links on list yet
   }

  public boolean isEmpty(){       // true if list is empty

         return (first==null);
   }
    ……

   …add in the insertHead() and removeHead() methods…
```

Methods

- We need insertHead( ) and deleteHead( )

```
public void insertHead(char data){   // make new link
     Link newLink = new Link(data);
     newLink.next = first;       // newLink --> old first
     first = newLink;            // first --> newLink
}

public Link removeHead() {      // delete first item
                               // (assumes list not empty)
     Link temp = first;         // save reference to link
     first = first.next;        // delete it: first-->old
 next
     return temp;               // return deleted link
}
```

Stack

- Wrap it all up in a stack - abstraction

```
public class Stack{

   private LinkedList list;

   public Stack(){               // constructor
      list = new LinkedList();
   }

  public void push(char data){    // put item on top of stack
     list.insertHead(data);
   }

   public char pop(){          // take item from top of stack
     return list.removeHead().data;
   }
```