

Decision Trees

MIRI / MVA - Practice #7 Report

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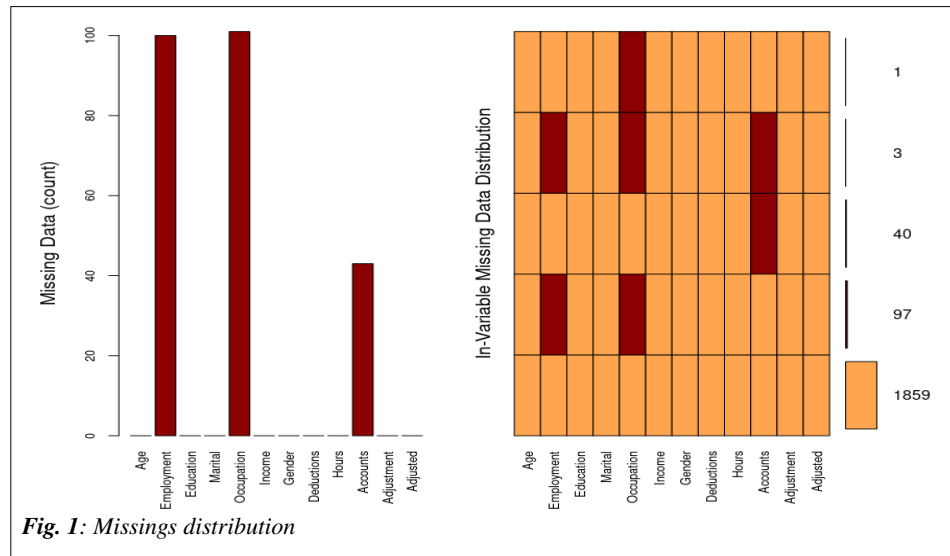
Delivery: before 2018.05.27 – 23:55

The audit data-set is a simplified financial audit data-set for modelling productive and non-productive audits of a person's financial statement. The dataset is used to illustrate binary classification. A productive audit is one which identifies errors or inaccuracies in the information provided by a client. A non-productive audit is usually an audit which found all supplied information to be in order. The target variable is identified as Adjusted.

1. Import the Audit.xlsx file and convert it to the csv format

Using the R package `xlsx`, we import the `xlsx` formatted file, convert it to `csv` format using column ID as row names, before saving it to disk as a `csv` file. Subsequent calls to the data set, are made by directly loading the `csv` file in memory.

The data-set contains 2000 observations, and 11 variables beside the boolean target/dependent/response variable “Adjusted”. 1859 observations have no missing value. 141 have missings distributed according to Figure 1.



We do not perform imputations as the R package `rpart()` uses surrogate variables efficiently to handle missings.

2. Decide which predictors you will use. Pre-process the corresponding variables as needed.

We define 10 predictors and 2 supplementary variables as follows:

- 6 active categorical variables: "Employment", "Education", "Marital", "Occupation", "Gender", "Accounts"
- 4 active continuous variables: "Age", "Income", "Deductions", "Hours"
- 1 supplementary continuous variable: "Adjustment", which illustrates a productive audit and `Adjusted=1`.
- 1 supplementary categorical (yes/no or 0/1) variable: "Adjusted", which is our target/dependent variable

We pre-process the continuous variables “Age”, “Income”, “Hours” to transform them in as many categorical variables. Care is taken so each variable’s resulting bins (modalities) exhibit similar counts. “Deductions” remains a continuous variable as its discretization would yield very lopsided bins’ frequencies, between two basic modalities: “With” (4.15%) and “Without” (95.85%). See Figure 2. Table 1 (to the right) summarizes the discretization.

Age	$]-\text{inf}, 27]$	$]27, 38]$	$]38, 50]$	$]50, \text{inf}[$
modality	Prime	Middle	Mature	Senior
bin counts	498	558	537	407
Income	$]-\text{inf}, 34.5\text{k}]$	$]34.5\text{k}, 60\text{k}]$	$]60\text{k}, 115\text{k}]$	$]115\text{k}, \text{inf}[$
modality	Low	Medium	High	Obscene
bin counts	501	502	503	494
Hours	$]-\text{inf}, 30]$	$]30, 40]$	$]40, \text{inf}[$	
modality	Part	Reduced	Full	
bin counts	344	1063	593	

Table 1: Discretization of active continuous variables "Age", "Income", "Hours"

3. Select 1/3 of the data at the end of the data set, as test data.

The training data-set consists of 1333 observations, while the test data-set consists of 667 observations.

4. Build the decision tree to predict var “Adjusted” using training data. Determine cutoff value for optimal decision making.

The fully grown tree for the training dataset using 10 Cross-Validation (CV) replicas, and a complexity parameter of 10^{-3} is shown on Figure 3.

Figure 4 represents the CV normalized error mean (tree cost $R(T)$) and the whole data set based training error as a function of tree size (or α value). The complexity parameter table used to build Figure 4 is available in Appendix B.

The red horizontal dashed line represents the minimum tree impurity (MTI) level, and the red dotted line above it MTI + 1

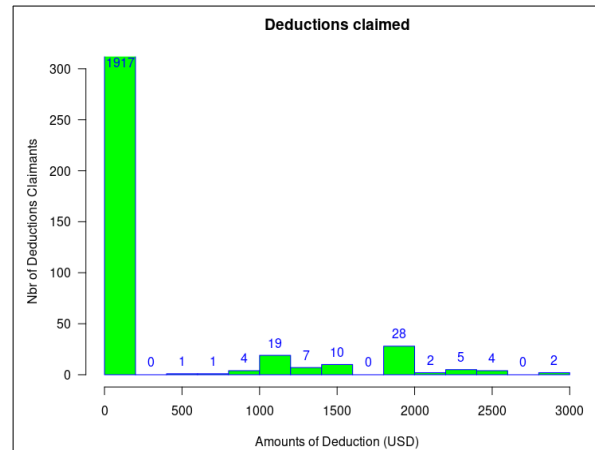


Fig. 2: Histogram of deductions claimed (Numbers in blue are counts for each bucket.)

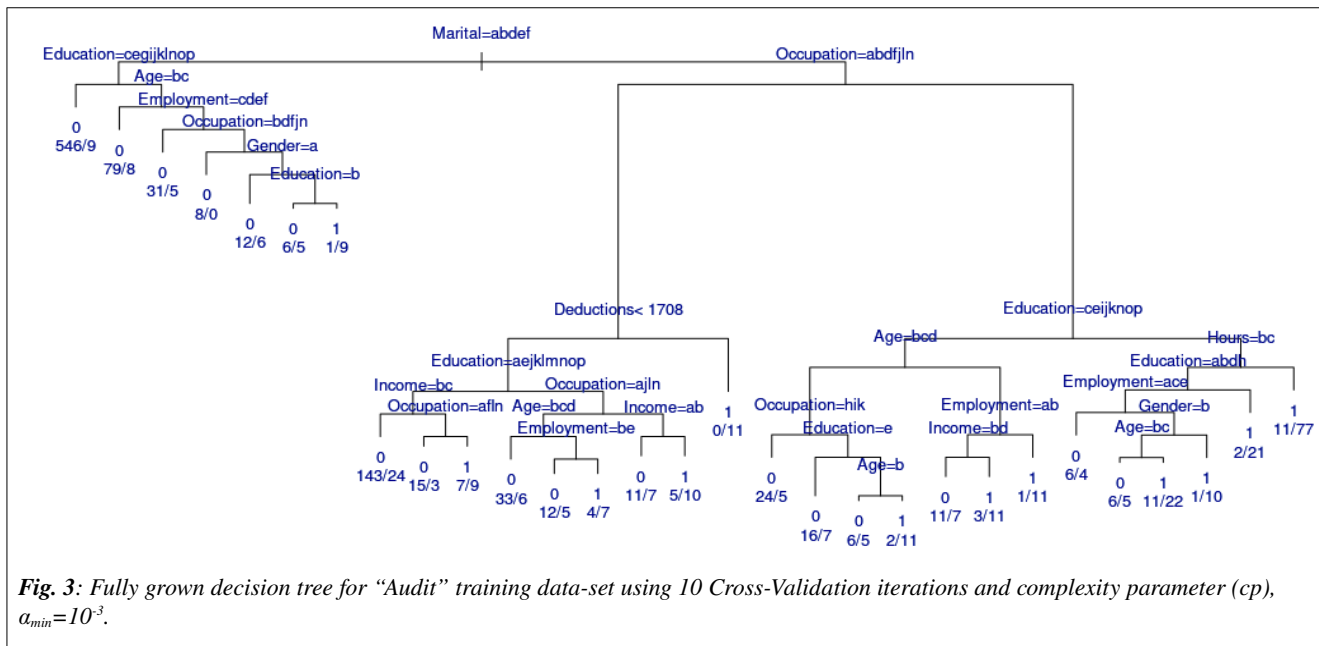
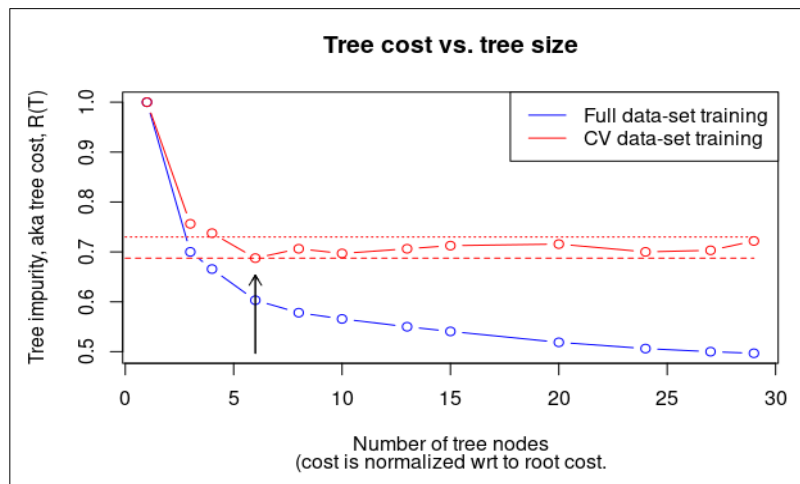
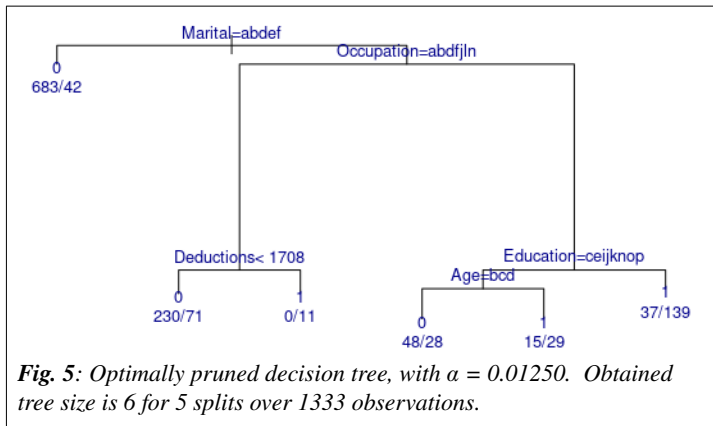


Fig. 3: Fully grown decision tree for “Audit” training data-set using 10 Cross-Validation iterations and complexity parameter (cp), $\alpha_{min}=10^{-3}$.

standard error, calculated over the CV errors for value of the complexity parameter, α , in the closed interval $[0.001, 0.150]$. The optimum value of α is obtained by inspection. It corresponds to the first value of CV tree cost smaller than MTI + 1 standard error, when scanning normalized mean values of tree impurity starting at root (tree size = 1 for $\alpha = 0.150$): $\alpha_{opt} = 0.01250$.

Fig.4: Data-sets’ training error without (blue) and with (red) cross validation. The red dashed line represents the smallest value of CV training error while the red dotted line above it represents the same + 1 standard error. The black arrow point to the optimum number of nodes for post-pruning.





This optimum complexity parameter value allows us to post-prune the decision tree at the 5th node split. The result is shown in Figure 5 along with corresponding split rules in Appendix C. Rules obtained are self-explanatory and usefully complement the tree of Fig.5.

We notice only one small pure node (for a productive audit, i.e. Adjusted =1) in the optimal decision tree, at rule 13 and 5th node split. The leaf corresponds to only 11/13.33 \approx 1% of the population.

Not all predictors are shown in the tree: at each node split, only the best predictor is kept at any given node.

5. Plot the importance of variables in the prediction.

Figure 6 shows variables' importance. Most important are:

- Marital, Income and Occupation.

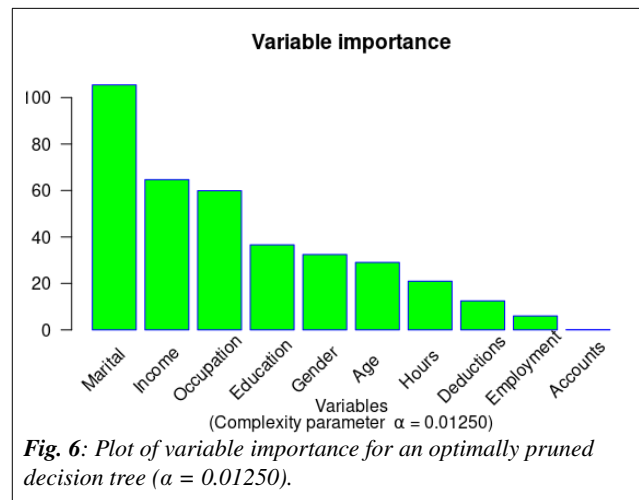
A second group of intermediate importance is made of:

- Education, Gender, and Age.

Least important variables are:

- Hours, Deductions, Employment, and Accounts.

The variable Accounts is the least important. As it is not a predictor of variable Adjusted, it would be appropriate to remove it from the decision-tree analysis, with no adverse effect on the analysis' outcome.



6. Compute the accuracy, precision, recall and AUC from the test data.

Using the training model constructed with cross-validation (denoted `AuditTree_optimum` in the adjoined R script), a classification for the test data can be predicted. On that basis we cross-tabulate observed and predicted classes with associated statistics. In doing so we consider that a positive outcome is a constructive audit, characterized by Adjusted = 1. Table 2 summarizes predicted classification results in the form of a Confusion Matrix.

Out of n=667 observations in the reference test sample, 143 are positives (P, constructive audits) and 524 negatives (N, non-constructive audits).

$$\text{Accuracy} = 1 - \frac{FN + FP}{n} = 82.46\%$$

$$\text{Precision} = \frac{1}{2} \left[\frac{TP}{TP + FP} + \frac{TN}{TN + FN} \right] = 74.51\%$$

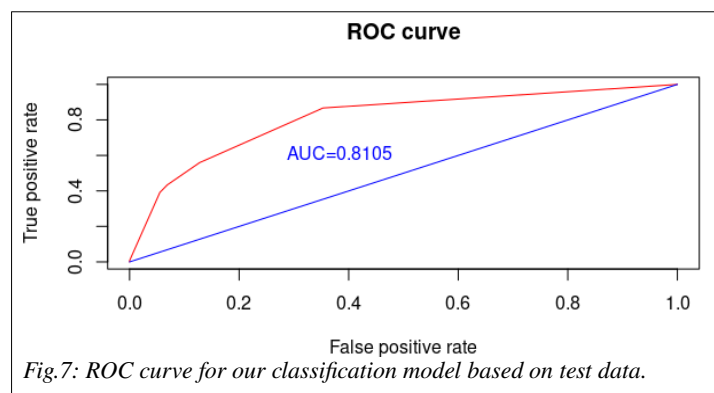
$$\text{Sensitivity} = \text{True Positive Rate} = \frac{TP}{P} = 43.36\%$$

$$AUC = 0.8105$$

AUC illustrates the ability of a binary classifier to discriminate between true and false positives. Values of $AUC > 0.7$ are considered good

		Prediction	
		0	1
Reference (test sample)	0	488 TN	36 FP
	1	81 FN	62 TP

Table 2: Confusion matrix for data set "audit" (667 observations); TN = "True Negative", FN = "False Negative", FP = "False Positive", TP = "True Positive".



7. Perform a Random Forest on test data

α	Nbr of tree nodes	Learn error (no CV)	CV-learn error mean	CV-learn error std-dev
0.150	1	1.0000	1.0000	0.04873
0.034	3	0.7000	0.7563	0.04398
0.031	4	0.6656	0.7375	0.04355
0.013	6	0.6031	0.6875	0.04235
0.006	8	0.5781	0.7063	0.04281
0.005	10	0.5656	0.6969	0.04258
0.005	13	0.5500	0.7063	0.04281
0.004	15	0.5406	0.7125	0.04296
0.003	20	0.5188	0.7156	0.04304
0.002	24	0.5062	0.7000	0.04266
0.002	27	0.5000	0.7031	0.04274
0.001	29	0.4969	0.7219	0.04318

Appendix A: Data-set's variables' dictionary

Rule number: 13 [Adjusted=1 cover=11 (11/1333=1%) prob=1.00]
Marital= Married
Occupation= Cleaner, Clerical, Farming, Machinist, Repair, Service, Transport
Deductions >= 1708

Rule number: 15 [Adjusted=1 cover=176 (176/1333=13%) prob=0.79]
Marital= Married
Occupation= Executive, Professional, Protective, Sales, Support
Education= Associate, Bachelor, Doctorate, Master, Professional

Rule number: 29 [Adjusted=1 cover=44 (44/1333=3%) prob=0.66]
Marital= Married
Occupation= Executive, Professional, Protective, Sales, Support
Education= College, HSgrad, Vocational, Yr10, Yr11, Yr5t6, Yr7t8, Yr9
Age= Mature

Rule number: 28 [Adjusted=0 cover=76 (76/1333=6%) prob=0.37]
Marital= Married
Occupation= Executive, Professional, Protective, Sales, Support
Education= College, HSgrad, Vocational, Yr10, Yr11, Yr5t6, Yr7t8, Yr9
Age= Middle, Prime, Senior

Rule number: 12 [Adjusted=0 cover=301 (301/1333=23%) prob=0.24]
Marital= Married
Occupation= Cleaner, Clerical, Farming, Machinist, Repair, Service, Transport
Deductions < 1708

Rule number: 2 [Adjusted=0 cover=725 (725/1333=54%) prob=0.06]
Marital= Absent, Divorced, Married-spouse-absent, Unmarried, Widowed

ID	Unique identifier for the person's being audited.
Age	Age of the person being audited.
Employment	Type of employment.
Education	Highest level of education.
Marital	Current marital status.
Occupation	Type of occupation.
Income	Amount of income declared.
Gender	Person's gender.
Deductions	Total amount of expenses that a person claims in their financial statement.
Hours	Average number of hours worked per week.
Accounts	Country in which the person has most of their money banked.
Adjustment	Monetary amount of any adjustment to the person's financial claims as a result of a productive audit. This variable is thus a measure of the size of the risk associated with the person.
Adjusted	Boolean; indicates non-productive (0) and productive (1) audits.

Appendix B: Complex parameter, α , table for the decision tree

Appendix C: Rules derived from the optimal decision tree ($\alpha = 0.01250$, rounded to 0.013 in Appendix B)