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ASSESSING THE ACCURACY OF HISTORICAL MAPS OF CITIES: METHODS AND PROBLEMS

Object of the study

Representing the city as a two-dimensional abstraction took a long time to become an established practice and it was only at the end of the 18th century that it overtook other forms of cartographic representation[1].

Leon Battista Alberti's work, the *Descriptio Urbis Romae,* is undoubtedly a milestone in scientific urban surveying. It was done around 1450[2] and integrated into the theoretical and descriptive part by his *Ludi matematici*[3]. It consisted of a list of coordinates of prominent city features, calculated in a local coordinate system. Alberti did not illustrate graphically his survey. The *Pianta di Roma secondo le misure di Leon Battista Alberti* we owe firstly to Professor Alessandro Capannari, who carried out a reconstruction on behalf of Domenico Gnolli[4] in 1884

[1] The history of urban representation is examined in great detail in various works. Among the most notable Italian works we find: L. Nuti, *Ritratti di città, visione e memoria tra Medioevo e Settecento*, Venezia, Marsiglio, 1996; M. Docci-D. Maestri, *Storia del rilevamento architettonico e urbano*, Roma-Bari, Laterza, 1993.

[2] L. Vagnetti, *La "Descriptio Urbis Romae" di L.B. Alberti*, «Quaderno dell'Istituto di Elementi di Architettura e Rilievo dei Monumenti dell'Università di Genova», I, Ottobre 1968, pp. 25-79.

[3] In *Ludi matematici* Alberti describes the urban surveying system and the tools for carrying it out by proposing a series of measurements by forward intersection (definition of the triangle by measuring one side and two contiguous angles), using as station points significant places in the territory. Cfr: M. Docci-D. Maestri, *Storia del rilevamento architettonico e urbano*, cit., p. 101.

[4] On the 19th century version of the map of Rome drawn from the *Descriptio Urbis Romae,* the reader is referred to: A.P. Frutaz, *Le Piante di Roma*, Roma, Istituto di Studi Romani, 1962, 2 voll., I, tav. 151, pp. 127-128; F. Furlan, *In margine all'edizione degli Ex ludis rerum mathematicarum: Ossia osservazioni e note per l'edizione di un testo scientifico e delle sue figure***,** «Revue d'histoire des sciences» 59-2, 2006, pp. 197–217: pp. 208-209, see in particular note 15.

and then to Luigi Vagnetti, who in 1968 proposed a map of the places indicated by Alberti, comparing the original coordinates with the coordinates gathered from the cartography of the IGM (fig. 1)[5].

Alberti's method consists in defining the coordinates of points that are visible from the extremes of a measured baseline, placed, as Alberti himself suggests, on the tops of two towers. Although it is sufficient for surveying the characteristic points such as obelisks, columns, towers, bell towers and domes, it cannot be used for surveying the detailed urban, since the intersections of the road axes would be difficult to see.

If we leave aside the problems linked to the accuracy of the instruments used for surveying and concentrate on the procedures described by Alberti, we can still assert that applying his method would allow us to carry out a main triangulation and thus to construct a reference grid for the emerging parts of the city, to be completed with detailed surveys made by working on ground level. This procedure is no different from how a ground survey of an urban area would be carried out today.

The map of Imola drawn by Leonardo Da Vinci in 1502 is generally recognized as the first example of a complete ichnographic representation of an urban space to be made scientifically. The surveying method for traverses can be traced back to Alberti's. Ground measures were probably taken by means of an instrument with a dioptre for sighting and a compass for calculating the angles of the various alignments[6].

The evolution of ichnographic representation has gone hand in hand with the evolution of direct and indirect measuring systems and the development of treatise writing on the subject. A fundamental turning point in the procedures of topographic surveying came with the introduction of the *tavoletta pretoriana* or plane table. This tool linked the definition of the angular directions with

[5] According to Alberti the points to be surveyed are inflection points on the Aurelian walls, and important buildings or buildings easily visible from the main station, namely the Capitol. The reconstruction of the map drawn from these points is in L. VAGNETTI, *La "Descriptio Urbis Romae" di L.B. Alberti*, cit; further details can be found in: ID., *Lo studio di Roma negli scritti albertiani*, «Convegno internazionale indetto nel V centenario di Leon Battista Alberti», Roma, Accademia Nazionale dei Lincei, 1974, pp. 73-110.

[6] Cf: M. DOCCI-D. MAESTRI, *Storia del rilevamento architettonico e urbano*, cit., pp. 105-106; L. VAGNETTI, *Mieux vaut voir que courir 1*, in *Cartes et figures de la Terre*, Paris, Centre Georges Pompidou, 1980, pp. 242-247: 247; F. MANCINI, *Urbanistica rinascimentale a Imola da Girolamo Riario a Leonardo Da Vinci (1474-1502)*, 2 voll., Imola, Grafiche Galeati, 1979. See also: B. GAUTHIEZ, *Historie del la cartographie des villes aux XVI-XVIII siècles*, «Città e Storia», 2, 2006, pp. 359-376: p. 360. For a detailed bibliography of studies on the Pianta di Imola, please refer to: D. FRIEDMAN, *La pianta di Imola di Leonardo, 1502*, in *Rappresentare la città. Topografie urbane nell'Italia di antico regime*, M. Folin (ed.), Reggio Emilia, Diabasis, 2010, pp. 121-144.

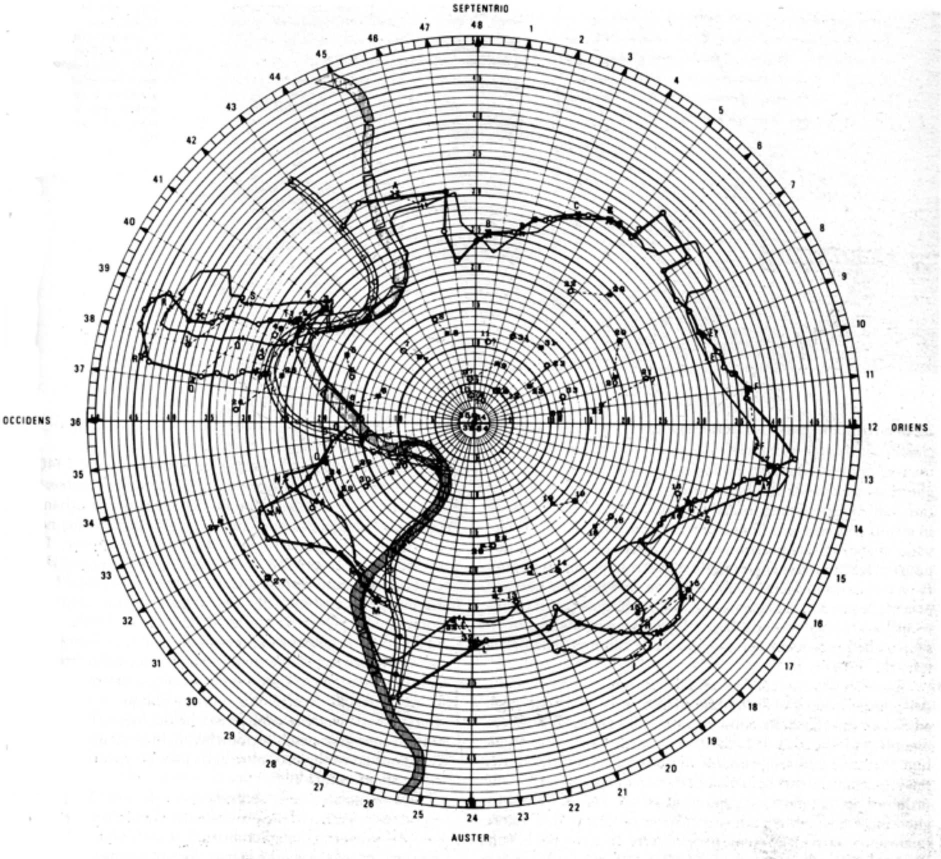


Fig. 1 - Plan of the *Descriptio Urbis Romae*, drawn by L. Vagnetti based on the writings of L.B. Alberti. The map by Alberti is drawn with darker lines while an equivalent representation from 1950, drawn from the IGM plan on a scale of 1:50.000, is drawn with a lighter colour.

their immediate graphic transposition, thus obviating the problem of drawing the angles and greatly accelerating survey times. It took its name in Italian from its inventor, the German mathematician and astronomer Johannes Praetorius (Richter) (1537-1616), and consists of a wooden board joined to a base supported by a tripod: the base makes levelling possible as it has three levelling screws and it can be rotated thanks to a spherical joint linking it to the tripod. There is a sighting or telescopic dioptre and a compass on the board. Directions are identified using the dioptre and traced with a ruler, and thus it is possible to make surveys by irradiation and by intersection, which are worked out graphically on a sheet of drawing paper placed on the board[7]. It was used for drawing

[7] Cf: A.M. Ceneri, *L'uso dello strumento geometrico detto la tavoletta pretoriana*, Bologna, Lelio d. Volpe, 1728; G. A. Alberti, *Istruzioni pratiche per l'ingegnero civile: o sia perito agrimensore, e perito d'acque*, Savioni, Venezia, 1748; A. Righini, *Trattato di Topografia*, Torino, S. Franco, 1860. On the surveying methods at the beginning of the modern period see: D. Stroffolino, *La città misurata.*

small and medium scale maps since it made it possible to make detailed drawings but it was gradually replaced from the end of the 18th century by tools that were able to make more accurate measurements of angular directions and distances, such as theodolites and tachymeters, and then substituted for good during the 20th century by aerophotogrammetric surveys.

During the 18th century urban cartography experienced a crucial stage of development at international level due to the significant improvement of topographic survey techniques[8]. The geometric accuracy was claimed by the progress in the scientific culture but also by the necessity for a more effective and balanced system of taxation[9]. A significant step forward towards the standardization of cartographic representations was done at the end of the century, with the introduction of the metric system, and the graphical representation of the orography as contour lines[10].

The 18th century gave us one of the most striking examples of an ichnographic representation of a city: the *Nuova Pianta di Roma* (New map of Rome) published in 1748 by G.B. Nolli[11].

Tecniche e strumenti di rilevamento nei trattati a stampa del Cinquecento, Roma, Salerno, 1990.

[8] On the 18th urban cartography see: B. MARIN, *Le grandi planimetrie urbane stampate in Europa nel XVIII secolo. Produzioni e utilizzi*, in *Roma nel Settecento, Immagini e realtà di una capitale attraverso la pianta di G.B. Nolli*, C.M. Travaglini-K. Lelo (eds.), 2 voll., Roma, CROMA-Università degli studi Roma Tre and Edilstampa, I, 2013, pp. 175-206; J. BOUTIER, *La cartographie urbaine à l'époque des Lumières*, in *De l'esprit des villes. Nancy et l'Europe urbaine au siècle des Lumières, 1720-1770*, A. Gady-J.M. Pérouse de Montclos (eds.), Musée des Beaux-Arts de Nancy, 7 maggio-22 agosto 2005, Versailles, Arthlys, pp. 130-141. On the cartographic representations of Rome, see M. BEVILACQUA, *Piante e vedute di Roma dall'Umanesimo all'Illuminismo*, in *Imago Urbis Romae. L'immagine di Roma in età moderna* (catalogue of the exbition, Rome, 11 february-15 may 2005), Cesare de Seta (ed.), Milano, Electa, 2005, pp. 93-103.

[9] Surveying techniques in the eighteenth century, with reference to the cartographic work of Rizzi Zannoni, are analyzed in detail by Ilario Principe: G.A. RIZZI ZANNONI, *Atlante geografico del Regno di Napoli,* I. Principe (ed.), Soveria Mannelli, Rubbettino, 1994, pp. 13-43. Cf. also: V. VALERIO, *L'Italia nei manoscritti dell'Officina Topografica*, Napoli, Istituto Italiano per gli studi Filosofici, 1985; V. VALERIO, *Società uomini e istituzioni cartografiche nel Mezzogiorno d'Italia*, Firenze, IGM, 1993; M. IULIANO, *Rizzi Zannoni e la «Gran Carta di Roma»* in *Roma nel Settecento,* cit., pp. 251-256.

[10] K. LELO-C.M. TRAVAGLINI, *Dalla «Nuova Pianta» del Nolli al Catasto Pio-Gregoriano: l'immagine di Roma all'epoca del* Grand Tour, «Città e Storia», I, 2006, 2, pp. 431-456.

[11] Original title of the work: «Nuova Pianta / di / Roma / data in luce / da / Gianbattista Nolli / l'anno / M DCC XLVIII». On the importance/influence of this map, both in the Italian and international cartographic production of the second half of the 18th century, see: C. FACCIOLI, *Giovanni Battista Nolli (1701- 1756) e la sua gran "Pianta di Roma" del 1748*, «Studi Romani», XIV, 1966, 2, pp. 415-442; ID., *Formazione romana di un incisore comasco del Settecento*, «L'Urbe», XXI, 1968, 3, pp. 22-28; I. INSOLERA, *Roma. Immagini e realtà dal X al XX secolo,* Roma- Bari, Laterza, 1980;

The *Nuova Pianta* is the second ichnographic representation of Rome after the Leonardo Bufalini's Plan of 1551[12]. If we look at its precision and its metric accuracy, we can assert that this map is the first geometrically correct bi-dimensional cartography of the urban fabric, produced through the meticulous application of the most advanced surveying techniques of the time. These techniques, are analysed in detail in this essay, because we believe that this cartographic work can be used as an example of best practice for assessing the accuracy of historical city maps.

The *Nuova Pianta* by G.B. Nolli consists of a volume obtained from the printing of 36 engraved copper matrices. This work, designed to be bound, contains: the *Pianta Grande* (big map), composed of 12 sheets and their annexed indexes, the *Pianta Piccola* (small map), a reduced one-sheet version of the big map and a revisited version of the map of Rome in 1551 by Leonardo Bufalini.

G.B. Nolli illustrates the use of the *tavoletta pretoriana* in detail in the *Pianta Grande*. The map's surrounding details include a rich variety of figures, buildings and inscriptions. Of particular interest are the putti engraved on the lower right-hand portion of the map, shown busily at work surveying the city; the putto in the foreground is using the *tavoletta pretoriana* equipped with an alidade and a compass, while the putti in the background are using rods to measure distances

G. Spagnesi, *L'immagine di Roma barocca da Sisto V a Clemente XII: la pianta di G.B. Nolli del 1748*, in *Immagini del Barocco. Bernini e la cultura del Seicento*, M. Fagiolo-G. Spagnesi (eds.), Roma, Istituto della Enciclopedia Italiana, 1982, pp. 145-156; A. Ceen, *Rome 1748. The Pianta grande di Roma of Gian Battista Nolli in facsimile,* New York, Aronson, 1984; Id., *Piranesi and Nolli: Imago Urbis Romae*, in *Piranesi: Rome recorded […]*, [catalogue of the exbition, New York-Roma, 1990], Roma, Accademia Americana, 1990, pp. 17-22; E. Guidoni, *L'urbanistica di Roma tra miti e progetti*, Roma-Bari, Laterza, 1990; S. Borsi, *Roma di Benedetto XIV. La Pianta di Giovan Battista Nolli*, 1748, Roma, Officina Edizioni, 1993; M. Bevilacqua, *Roma nel secolo dei Lumi. Architettura erudizione scienza nella Pianta di G.B. Nolli «celebre geometra»*, Napoli, Electa Napoli, 1998; P. Micalizzi, *Roma nel Settecento. Le trasformazioni della città tra magnificenza barocca e regola neoclassica*, in *Roma nel XVIII secolo*, P. Micalizzi (ed.), 2 voll., I, Roma, Kappa, 2003, pp. 7-82; *Nolli Vasi Piranesi. Immagine di Roma Antica e Moderna. Rappresentare e conoscere la metropoli dei Lumi,* [catalogue of the exbition, Rome, 2004-2005], M. Bevilacqua (ed.), Roma, Artemide, 2004; M. Fagiolo, *L'immagine di Roma moderna,* in *Nolli, Vasi, Piranesi*, cit., pp. 11-17; *Piante di Roma dal Rinascimento ai Catasti*, M. Bevilacqua-M. Fagiolo (eds.), Roma, Artemide, 2012; K. Lelo, *Nolli e la visione della città: rigore ed estetica nella rappresentazione dello spazio urbano*, in *Roma nel Settecento,* cit., pp. 3-42.

[12] Cf: I. Insolera, *Roma: immagini e realtà dal X al XX secolo,* Bari, Laterza, 1980, pp. 112-122; G. Aragozzini-M. Nocca, *Le piante di Roma dal Cinquecento all'Ottocento* Roma, Dino Audino, 1993, pp. 9–33; *Roma, disegno e immagine della Città Eterna: le piante di Roma dal II secolo d.Cr. ai giorni nostri,* B. Tellini Santoni-A. Manodori (eds.), Roma, De Luca, 1994, pp. 18–19; P. Schlapobersky-D. Friedman, *Leonardo Bufalini's orthogonal Roma (1551)*, «Thresholds» 28, 2005, pp. 10–16; G. Maier, *Mapping Past and Present: Leonardo Bufalini's Plan of Rome (1551),* «Imago Mundi», 59, 1, 2007, pp. 1-23.

a

b

Fig. 2 - G.B. Nolli, 1748. Details of the *Pianta Grande*. Illustration of the topographic survey. a) putti illustrating the use of a compass; b) putto drawing on the *tavoletta pretoriana* and putti measuring distances on the ground using rods.

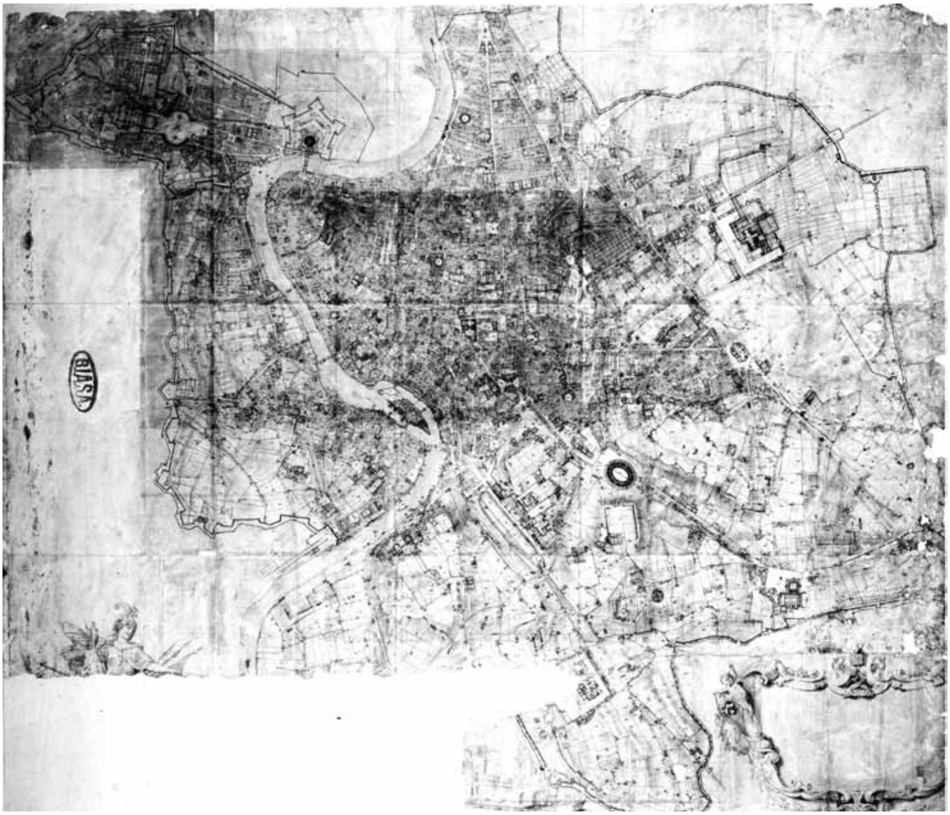


Fig. 3 - G.B. Nolli, 1736-1744 ca. Preparatory drawing for the *Nuova Pianta di Roma* (Rome, Library of Archaeology and Art History).

on the ground. The two putti to the right of the scale bar are checking the orientation of the map by looking at a compass (fig. 2).

Characterized by an exceptionally high level of detail, the *Pianta Grande* is expressed in Roman palms of architecture, corresponding to a scale of 1:2.910 in the metric system[13].

The transfer of the mapping work onto printed paper was carried out by means of a trans position onto 12 copper matrices measuring circa 470x700 mm ca., of the so-called *disegno preparatorio* (preparatory drawing), which is a priceless cartographic document. It measures 1660x1895 mm, is drawn by hand in pencil and sepia ink and shows the survey of the urban fabric within the Aurelian

[13] The Roman palm in architecture (palmo romano d'architettura) is equal to 0,223422 m. According to this conversion, the equivalent scale in the metric system of the map is 1:2.910. On the scale of the Nolli map see also M. BEVILACQUA, *Roma nel secolo dei lumi*, cit., p. 81, note 26; K. LELO, *Nolli e la visione della città*, cit., p. 18, note 9.

walls. The surveying operations began in 1736, the year in which Nolli and his team began to draw the outlines of the blocks on the drawing, inside which they inserted subsequently, in a reduced scale, the plans of the religious buildings surveyed separately[14]. The part to be reproduced on each of the matrices is directly indicated on the original drawing by thin pencil lines. The preparatory drawing seems to be almost entirely without overlaps, second thoughts or corrections. It is kept in the Library of the Institute of Archaeology and Art History in Rome where it is preserved under glass: there are some blackened areas and some damage to the bottom part (fig. 3).

In order to assess the distortions of the preparatory drawing, a preliminary investigation was carried out, in which well-known places which have not changed over time were compared on the historical maps and on today's maps. As well as being used to rectify the historical maps, the so-called *ground control points* (GCPs) may be utilized to establish a "correspondence" with the present-day cartography without needing to go as far as making a georeference[15].

The free software MapAnalyst makes possible a rapid estimate of the planimetric precision of the historical cartography, and generates various types of visualization that show its distortions[16].

The distortion grid shows the deformations and the rotation angle of the map. If the *historical cartography* is oriented towards the geographical North Pole and has no distortions, the grid will be regular and not rotated.

The vectors of displacement graphically illustrate the accuracy of each pair of points. A vector connects the position of a point with its transformed counterpart. Each vector line starts at a point on the analyzed map and ends at the position where the point would be if the analyzed map were as accurate as the modern reference map. This endpoint results from a similar transformation between the two sets of points.

In order to evaluate the planimetric precision of the preparatory drawing, Helmert's 4 parameter[17] transformation was used on 256 points uniformly dis-

[14] M. Bevilacqua, *Roma nel secolo dei lumi*, cit., p. 76.

[15] F. Guerra, *2W: new technologies for the georeferenced visualization of historic cartography*, «International archives of photogrammetry and remote sensing», 33, Part B5, 2000, pp. 339-345: 341; C. Boutoura-E. Livieratos, *Some fundamentals for the study of the geometry of early maps by comparative methods*, «*e*-Perimetron», 1-1, 2006, pp. 60-70: 62-69.

[16] www.mapanalyst.org. See also: B. Jenny-A. Weber-L. Hurni, *Visualizing the Planimetric Accuracy of Historical Maps with MapAnalyst,* «Cartographica», 42-1, 2007, pp. 89-94.

[17] Helmert's transformation is a flat rototranslation with a variation in the scale factor (also called a *similarity transformation*) to 4 parameters: translation of X and Y, rotation and variation in the scale factor. It does not introduce deformations of the shape and geometries of the map,

Fig. 4 - Vectors of displacement in the preparatory drawing. The grid step is 200 m. For representation purposes, the true length of the vectors is multiplied by a scale factor of 10.

tributed on the map and recognised in today's cartography. The points were mainly positioned at the corners of buildings and, when the conditions of comparability were favourable, along the perimeter of the Aurelian walls (fig. 4).

As can be seen in figure 4, the slight distortions of the grid, which are hard to see at first glance, are mainly in the densely built-up area on the bend of the Tiber, just north of the Tiber Island. The rest of the grid appears to be regular. It is rotated by 13 degrees, which confirms the orientation of the preparatory drawing towards the North Magnetic Pole.

As far as the accuracy of the positioning of the control points is concerned, it can be affirmed that the minor deviations are found in the area of the Tridente in the city centre, while the major ones belong to points situated on the Aurelian walls and generally in undeveloped peripheral areas. The deviations in question, however, have absolute values that vary between 0,46 and 20,20 m, with an average of 4,17 m, which may be considered as acceptable, given the uncertainties of the instruments used at that time and also bearing in mind the fact that the preparatory drawing shows clear distortions, gaps and alterations in the paper support.

The large scale survey by Nolli restores a cartographic document of exceptional quality to us not only in terms of the details of the buildings surveyed, but also and above all because of its geometrical precision. This map represents a fertile research subject for the study of the metric content and the testing of georeferencing techniques on historical cartography.

Survey method: hypothesis

In his work Nolli does not really observe Rome from any specific point of view, as was done by almost all of his predecessors, using the slopes of the Janiculum Hill as a privileged observation spot. He walks through streets, alleys, gardens or vineyards, measuring angles and distances homogeneously, and transferring everything that can be measured to the preparatory drawing: buildings, fountains, ruins, walls, streets, gardens, embankments, shores and so on, with an incredible level of detail. The use of the tavoletta pretoriana or plane table, illustrated in the *Pianta Grande* through the gestures of the putti, highlights how his accurate surveys were obtained locally, sheet after sheet, by positioning the instrument at the station points.

and for this reason it is the most suitable transformation for assessing the distortions of historical cartography. Cf. QGIS TEAM, *Documentazione QGis 2.2*, http://docs.qgis.org/2.2/it/docs/user_manual/, 2014. On the use of this transformation in georeferencing historical maps see: V. BAIOCCHI-K. LELO-M.V. MILONE-M. MORMILE, *Accuracy of different georeferencing strategies on historical maps of Rome*, «Geographia Technica», 1, 2013, pp.10-16.

Having been made this way, these surveys could have been put together like a mosaic, based only the recognition of the corresponding points on adjacent sheets. This procedure, though easy to carry out, would however have put the final result at risk of accumulating and spreading both accidental and gross errors.

There are no recorded discoveries/confirmations of all the various phases of his topographical surveying work. Nevertheless, previous studies have confirmed the hypothesis of a trigonometric measuring procedure, undertaken in order to avoid an accumulation of metric errors in the map-making[18]. According to Bevilaqua:

> having been warned that using the tavoletta pretoriana by itself, despite the "myth" of its astonishing precision, would not have made possible, …, the construction of a map with lower margins of approximation, Nolli had to continue anyway,…, with a careful review of the measurements of the perimeter and of the area of the city using triangulation methods, whose vertices could be fixed at the four obelisks erected by Sixtus V and the two spiral columns, and the base of which could perhaps be constructed from one of the main straight road axes[19].

The proven metric accuracy of map[20] suggests that there must have been a geodetic phase before the details were surveyed. It is impossible for the various sheets or "tavole" not to have been combined by means of a network of measurement points which must all have been visible at the same time from one or more prominent points and intervisible among themselves on the ground (in pairs).

One hypothesis on this subject could be that the survey was made at two different times: first there would have been a precise survey just of the coordinates of some significant points (the tops of obelisks, columns, towers and domes) that may also have all been visible at the same time from a higher point – similar to Alberti's method, which used the Capitol as his key point in his *Descriptio Urbis Romae* – and then the detailed sheets were drawn using the tavoletta pretoriana. These sheets would have been oriented towards the geographical North Pole and (at least some) would have contained the traces of the significant points. The "tavole" thus made, could then have been transferred to the preparatory drawing

[18] See G. SPAGNESI, *L'immagine di Roma barocca da Sisto V a Clemente XII,* cit., pp. 145-156. An extended description of the preparatory drawing can be found in M. BEVILACQUA, *Roma nel secolo dei lumi*, cit., pp. 65-82.

[19] *Ivi*, p. 76.

[20] Cf: V. BAIOCCHI-K. LELO, *Georeferenziazione di cartografie storiche in ambiente GIS e loro verifica mediante rilievi GPS*, in "Atti della 5ª Conferenza nazionale ASITA" (Rimini, 2001); V. BAIOCCHI-K. LELO, *Confronto di cartografie storiche con cartografie attuali per l'area del centro storico di Roma*, in "Atti della 6ª Conferenza nazionale ASITA", (Perugia, 2002).

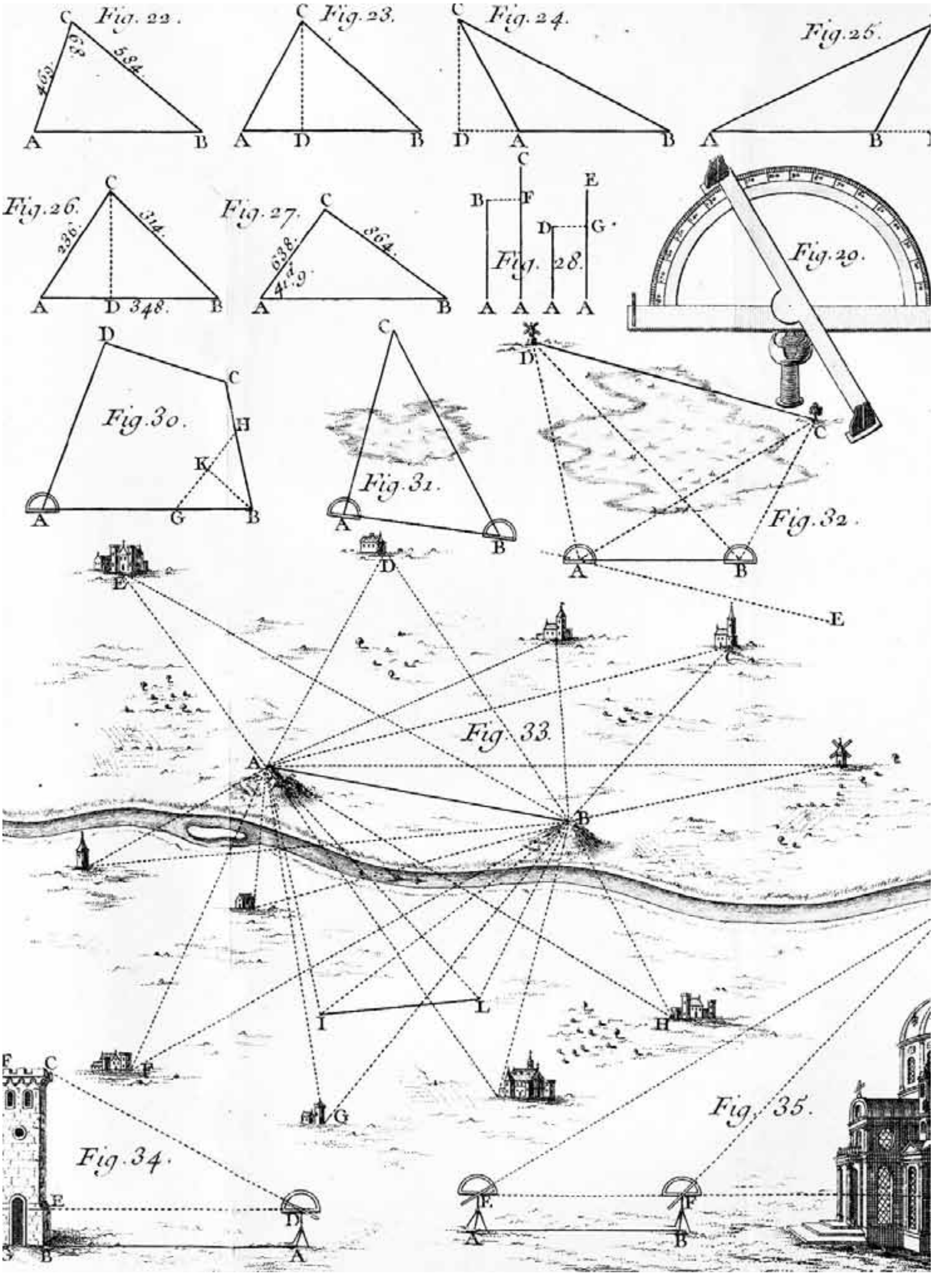


Fig. 5 - Illustration taken from: A. Deparcieux, *Nouveau traités de trigonométrie rectiligne et sphérique,* Paris, 1741.

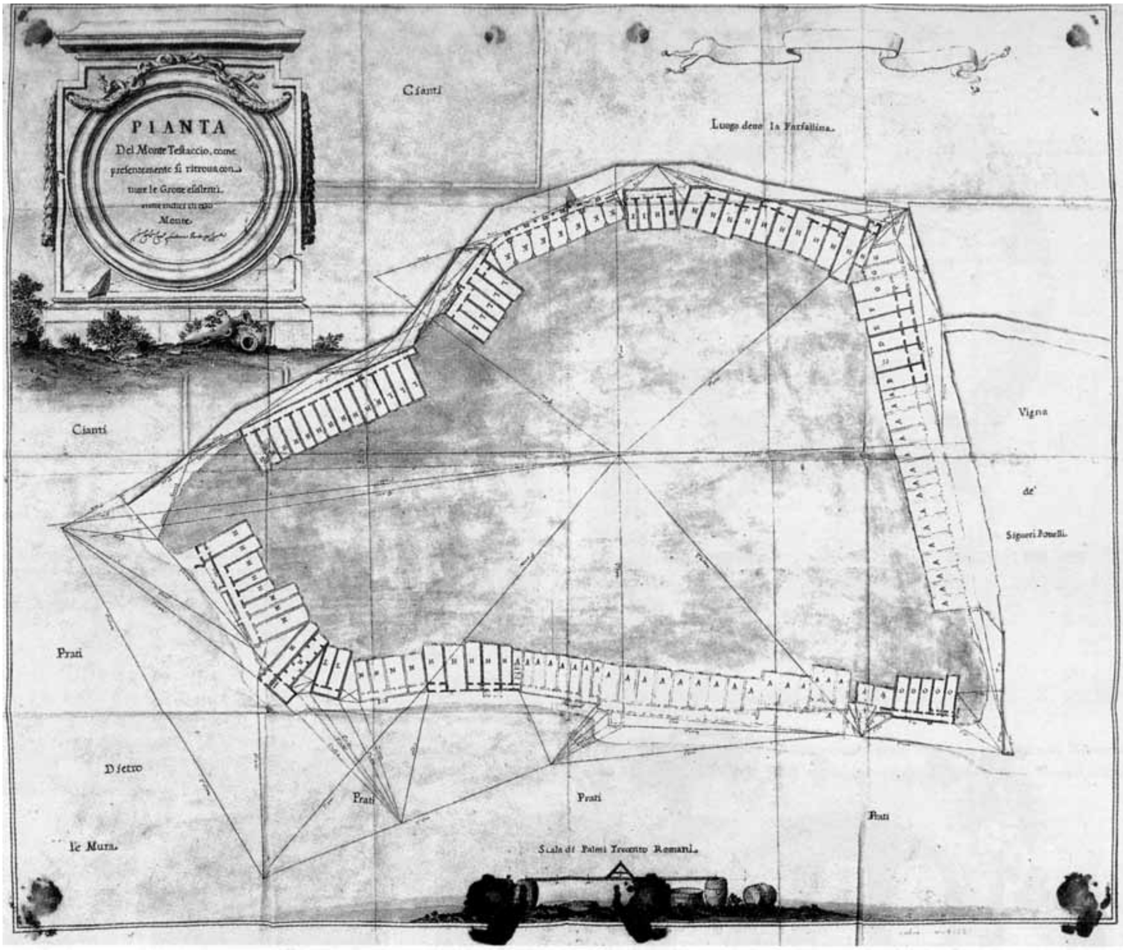


Fig. 6 - C. Fontana and V. Giordano, 1695. *La pianta del Monte Testaccio, come presentemente si trova con tutte le grotte esistenti nelle radici di esso*. (Plan of Monte Testaccio as it is today with all the caves at its roots) (Roma, Archivio Storico Capitolino, Cred. 10. T. 22, c. 57).

which probably already contained the positions of the significant points and also the orientation of the meridian. This would have made it possible to "georeference" the individual sheets (partial surveys) by referring them to a visible point and orienting them towards the North Magnetic Pole. This procedure would have left out of the georeferencing procedure sheets with no significant points, but they could still have been easily placed together with the rest of the mosaic, that had already been oriented.

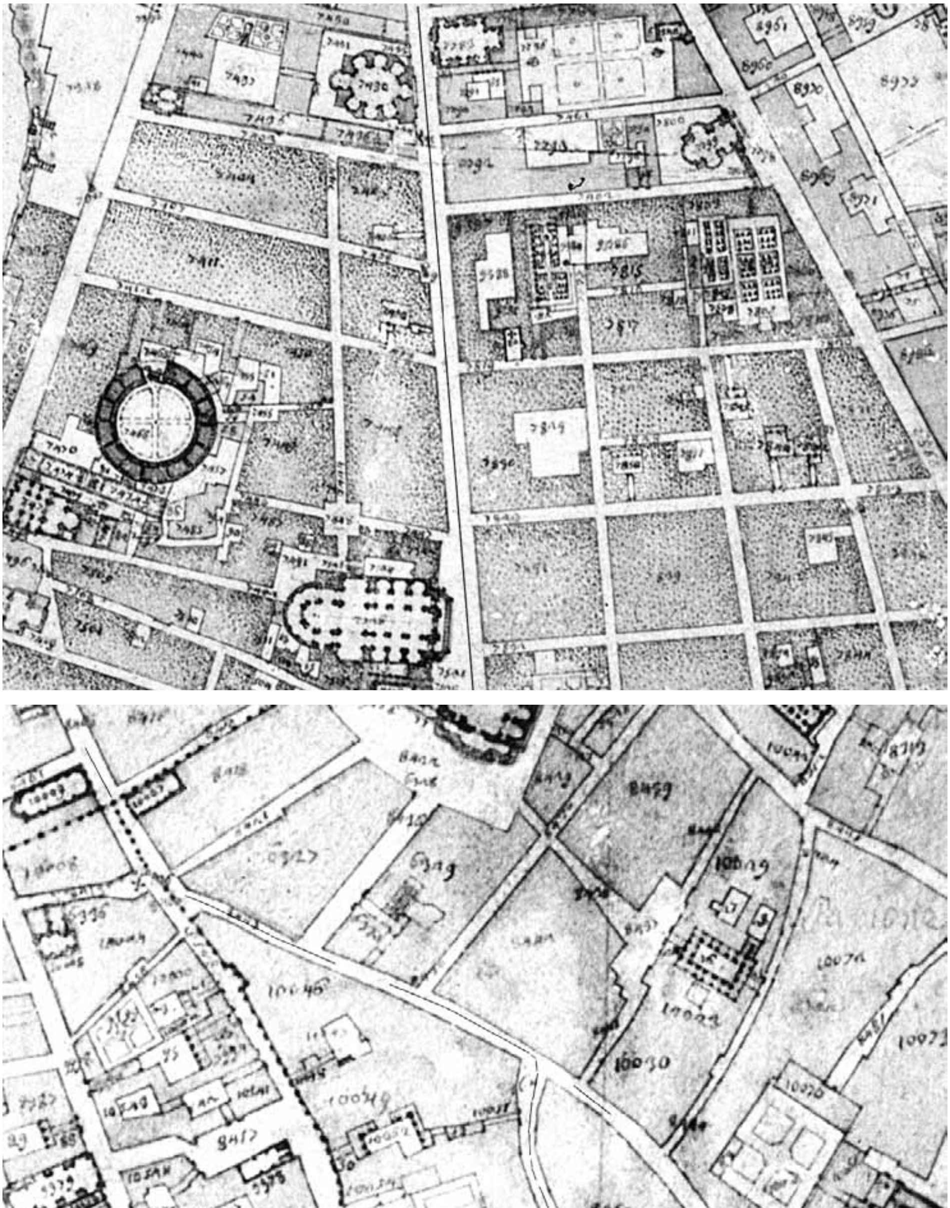
This methodology, similar in concept to the orientation of single pictures for aerophotogrammetric surveys which is still used today, would have had the huge advantage of avoiding the propagation of errors and their distribution over all the map, thus obtaining an end result that was overall more accurate and more geometrically.

It is curious, however, to observe that Nolli did not seem to think necessary to leave any documentation[21] of this preparatory geodetic work, although it was clearly laborious, at least as far as the calculations were concerned. And yet in that historical period it was typical to find illustrated treatises on trigonometry and gnomonics[22], while there are also splendid examples of representations of trilaterizations together with the cartography (figs. 5 and 6).

There are no traces in the preparatory drawing that can definitely be attributed to geodetic measuring procedures. There are several lines in pencil that are difficult to interpret, such as, for example, the centre lines of some roads (fig. 7), or some grids, which were presumably used for the construction of repeated details (fig. 8). Among these marks, it's worth mentioning the long, straight line traced to correspond with the axis of a part of Via del Corso. The line starts at the north end of the road, where there are the so-called "twin churches" in Piazza del Popolo (n. 484: S. Maria dei Miracoli and n. 404: S. Maria di Monte Santo), and continues uninterrupted for 1.500 metres, as far as what was then Piazza di Sciarra, where via del Corso (n. 302) widens just south of Piazza Colonna. It might be risky to hypothesize that this line was used as the measured base for the geodetic measuring procedures, but with reference to the analysis of the accuracy of the positioning in the preparatory drawing illustrated in the previous paragraph, it should be pointed out that the urban sector crossed by

[21] The detailed study of Nolli's work, carried out by Mario Bevilaqua, pointed out the almost complete lack of archive material complementary to the preparatory drawing. The only thing noted by Nolli was to do with the measurements taken along the perimeter of the walls. Cf. M. Bevilacqua, *Roma nel secolo dei lumi*, cit., p. 76 and p. 82, note 75.

[22] The treatises on cartographic surveying in the 18th and 19th centuries constitutes an interesting current of research which could be developed further. The topic is beyond the scope of this brief article. Just as an example we refer to: A. Deparcieux, *Nouveau traités de trigonométrie rectiligne et sphérique,* Paris, Guerin, 1741.



a

b

Fig. 7 – Detail from the preparatory drawing with pencil traces on the centre lines of: a) Via del Corso; b) Strada del Pellegrino and Vicolo de' Cappellari. It is difficult to make out the light pencil marks, given the reduced scale and their reproduction in black and white, and this is why the line has been digitally enhanced.

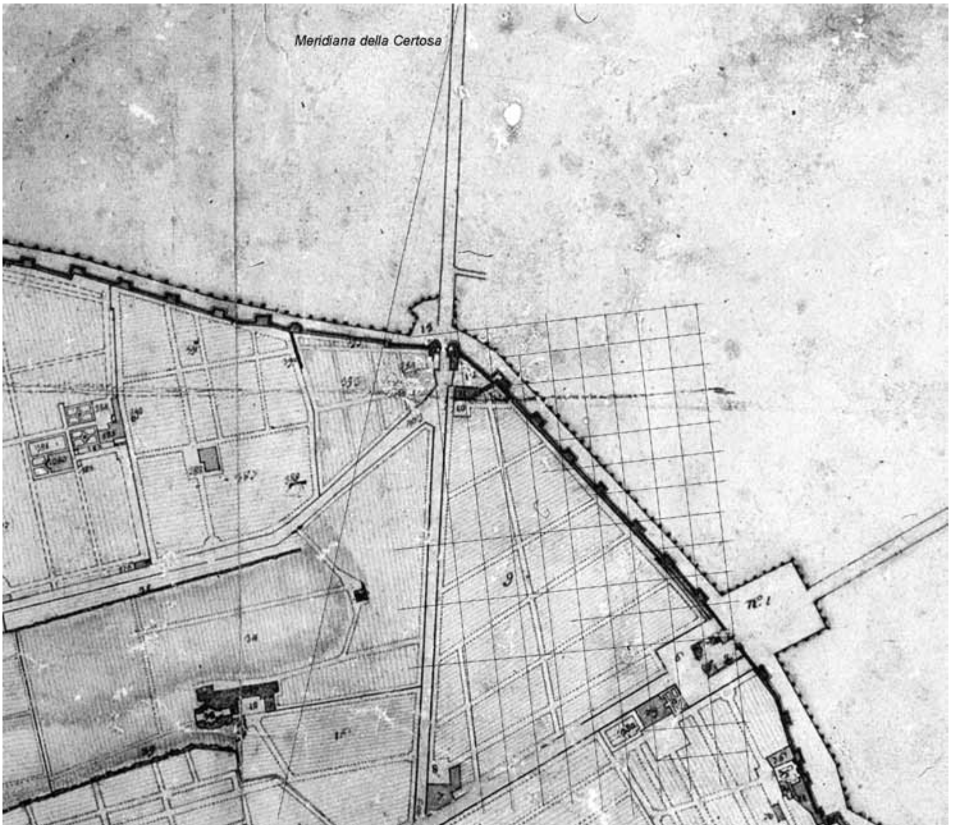


Fig. 8 - Detail from the preparatory drawing with pencil lines of a grid – apparently used for drawing a part of the Aurelian walls – and the Meridiana della Certosa visible from outside Porta Salaria. It is difficult to see these light pencil lines, given the reduced scale and their reproduction in black and white and this is why the lines have been digitally enhanced and the writing is in a different place compared to the original.

Via del Corso is the most accurate one when compared with the rest of the city within the walls. In actual fact, there seems to be a positive relation between the accuracy of the positioning of the GCPs (fig. 4) and their distance from the line traced on Via del Corso (graph 1). The correlation index between these two variables is equal to 0,76.

There are also a lot of holes made by a compass on the preparatory drawing. The presence of these marks usually indicates the complex operations for reducing the scale and the subsequent addition to the preparatory drawing of the plans of churches, buildings and antiquities to the drawing, all done on an architectural scale. However there are also a lot of compass holes external to the buildings, either in high points or where the road junctions are, and some of these are lightly

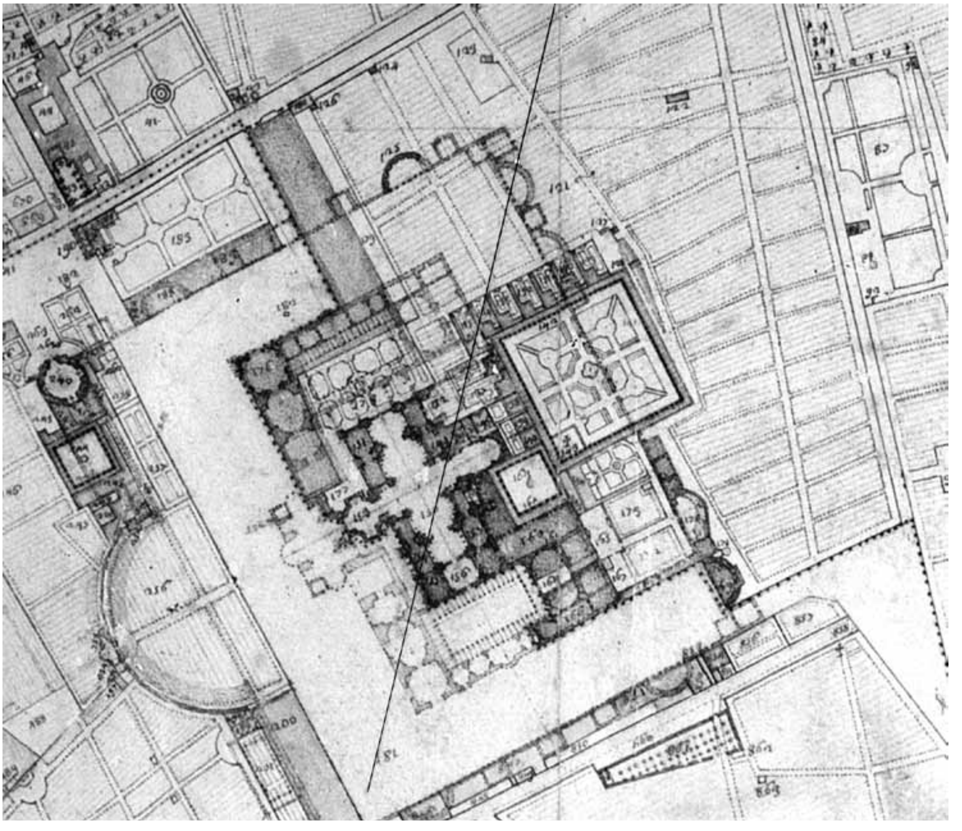
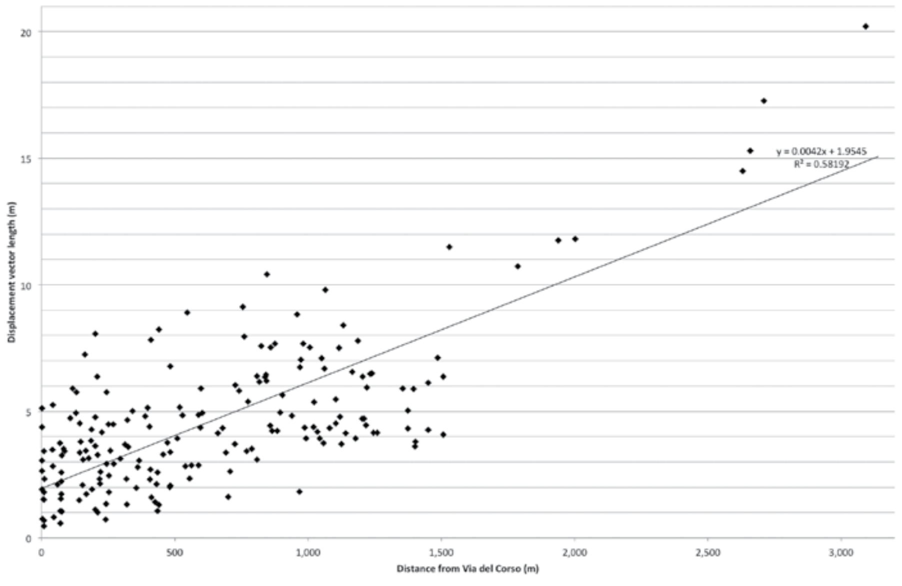


Fig. 9 - Detail from the preparatory drawing with the pencil line of the Meridiana della Certosa which crosses the floor of the church of S. Maria degli Angeli. It is difficult to make out the light pencil line, given the reduced scale and its reproduction in black and white, and this is why the line has been digitally enhanced.

circled in pencil. The drawing is sometimes so damaged around these points that some restoration work has been necessary in order to close these holes. Specialized investigations are needed to try and establish whether these points could have been used, at least partly, as "station points" for Nolli's geodetic measuring operations. When studying the conditions of intervisibilty on a reduced number of cases[23], the impression given is that the points in question were used to make partial surveys.

[23] The visual analysis of the preparatory drawing was anything but easy. This huge map is currently preserved in a glass case in the Sala della Crociera, which is attached to the Biblioteca dell'Archeologie e Storia dell'Arte. This case is slightly tilted to make it easier to see the map, but it cannot be opened or moved in any way. For this reason it is impossible to inspect the centre of the drawing in any detail.



Graph 1 - Relationship between the dispersion vectors' length value (fig. 4) and the distance of the GCPs from the line drawn on the axis of Via del Corso (fig. 7a).

It is however reasonable to suppose that, starting from these marks it is possible to reconstruct the planimetric course of some of these networks.

On observing the drawing against the light, some erasure marks made by a rubber can be seen, which led us to speculate that pre-existing traces of triangulation were removed to make room for the insertion of minutely detailed surveys, something which was undoubtedly very important for Nolli.

The orientation of the map is the only certain geodetic detail included in the preparatory drawing. The use of a compass clearly suggested the orientation towards the magnetic rather than the geographical North Pole, which would have been more accurate. The geographical North Pole is also shown on the preparatory drawing, the reference for which is the «Meridiana della Certosa», a line traced in 1702 on the marble floor of the church of Santa Maria degli Angeli for defining precisely the equinoxes and thus Easter days[24]. The line is drawn in pencil and is

[24] The tracing of the meridian on the floor of the church Santa Maria degli Angeli was done by Francesco Bianchini, a renowned scientist, historian and antiquary of the eighteenth century, best known for his extensive astronomical studies. M. BEVILACQUA, *Nolli Piranesi Vasi. Percorsi e incontri nella città del Settecento*, in *Nolli Vasi Piranesi. Immagine di Roma Antica e Moderna*, Mario Bevilaqua (ed.), Artemide Edizioni, Roma, 2004, p. 24.

clearly visible as it goes across the whole drawing from north to south. In the northern part, just beyond the walls at porta Salaria, the words «Meridiana della Certosa» (figs. 8 and 9) can just be made out. There are lines parallel to the meridian drawn in various parts of the preparatory drawing. All these lines, after the georefencing, have been rotated by about 2 degrees clockwise and this rotation is due to the convergence of the meridian[25]. The magnetic North Pole has a vertical pencil line as its reference, traced to the west of the boundary walls in Trastevere, with the words «Meridiana della Bussola» (fig. 10). The wind rose, which is missing from the preparatory drawing, was etched directly onto the matrices of the *Pianta Grande,* and indicates both the magnetic and the geographical North Pole. After the georeferencing this last feature shows a rotation of about 1,6 degrees clockwise[26].

Survey accuracy: evaluation

The city survey carried out by Nolli using tried and tested methods and instruments between the end of the 17th and the beginning of the 18th century produced the *Pianta Grande*, which is of undoubted quality as far as geometrical accuracy is concerned. With reference to the 18th and19th cartographic production on Rome, it is certainly the map that guarantees the best terms of comparison with present-day cartographies. The GIS system was used to subject the map to procedures of calibration, rectification, georefencing and for transforming the projection in order to enable a comparison to be made between historical cartographies and with present-day ones, by allowing the effects of deformation of the paper supports and those due to the various systems of representation and surveying[27] to be reduced.

As is well-known, historical cartographies either have no geographical refer-

[25] Value estimated after georeferencing using the coordinates UTM-WGS84 of the GCPs. The value γ of the convergence between the geographic and cartographic Norths (of the division into squares) in the IGM cartography IGM is 1°43', or about 1,73 degrees.

[26] Cf: K. LELO, *Nolli e la visione della città*, cit., p. 4.

[27] CROMA (Centro di ateneo per lo studio di Roma) – Università degli studi Roma Tre, undertook the georeferencing operations for some 18th and 19th century Roman cartographies as part of the "Atlas of modern and contemporary Rome" project. On the guidelines, methods and analytical processes designed for the project, refer to: K. LELO, *Storia urbana e sistemi informativi geografici* in *I territori di Roma. Storie, popolazioni, geografie*, R. Morelli-E. Sonnino-C.M. Travaglini, Roma, Università degli Studi di Roma La Sapienza-Tor Vergata-Roma Tre, 2002; K. LELO-C.M. TRAVAGLINI (eds.), *The GIS-based Historical Atlas of Rome*, in "XX International CIPA Symposium International Cooperation to save the World's Cultural Heritage" (Torino, 2005); K. LELO-C.M. TRAVAGLINI, *Computerization and Management of Archive Sources for the Study of Urban Cultural Heritage*, in "XXI International CIPA Symposium International Cooperation to save the World's Cultural Heritage" (Athens, 2007); K. LELO-C. M. TRAVAGLINI, *Dalla «Nuova Pianta» del Nolli al Catasto urbano Pio-Gregoriano: l'immagine di Roma all'epoca del Grand Tour*, «Città e Storia» I, 2006, 2, pp. 431-456.

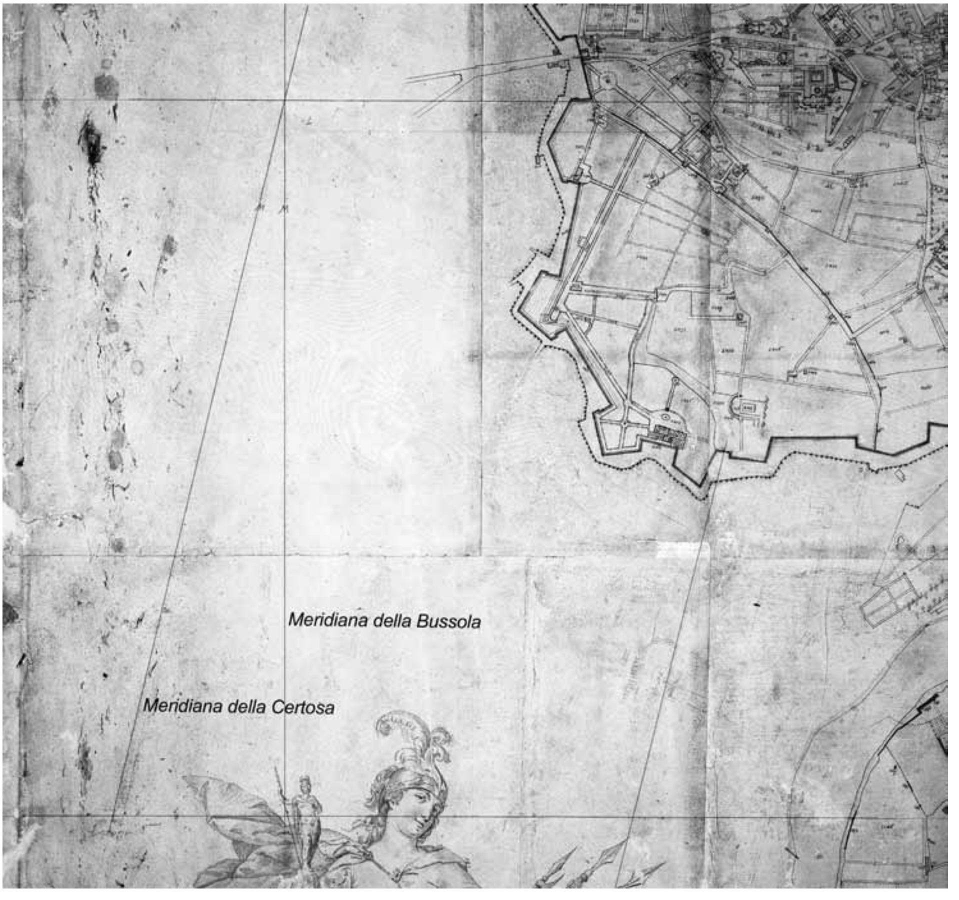


Fig. 10 - Detail from the preparatory drawing with the pencil lines of the Meridiana della Certosa and of the Meridiana della Bussola which cross in the southwest part of the map outside the ramparts in Trastevere. It is difficult to see these light pencil lines, given the reduced scale and their reproduction in black and white and this is why the lines have been digitally enhanced and the writings are in a different place compared to the original.

ence system or they have one which is different from those used nowadays. When the state of their geometric accuracy makes it possible, these documents can be georeferenced by calculating the coordinates of well-identified cartographic elements. The fastest way to determine these coordinates is by retrieving them from current cartographies on which the same elements are visible. The procedure of 'tying' a historical cartography to a modern one permits the local estimation of the error and guarantees control over the georeferencing accuracy, which is necessary for the eventual redefinition of the control points.

As far as the *Pianta Grande* is concerned, in addition to the historical survey errors that, nevertheless exist, the georeferencing accuracy was also affected by the fact that official contemporary cartographies of the study area were on a smaller scale when compared to the historical map in the early 2000s, when the project for georeferencing the historical maps of Rome started, while the large scale of Nolli's requires at least a decimetric accuracy of the GCPs (ground control points). Moreover, different cases of the wrong restitution of modern cartography were identified[28].

For these reasons a differential GPS survey was planned for the purpose of 'positioning' the historical maps in an absolute reference system. In the first stage, the localization of 'correctly measurable' points on the historical cartography was carried out. The most suitable points are those incorporated within buildings (for example building corners). Less suitable ones are easily recognizable points such as fountains or obelisks that may have been subject to modifications or re-arrangements. Regrettably, building corners coordinates are difficult to measure unless we can reach the top so as to guarantee a good GPS cover. This involves an increase in logistic access problems.

The GPS network, whose objective was to provide ground control points for georeferencing various 18th and 19th century historical cartographical sources, was established as a result of two measuring campaigns undertaken between 2000 and 2001. The survey was designed to provide a homogeneous coverage of the historic centre. The search for accessible points that could be identified for certain on the historical cartographies needed to be carried out very carefully since, apart from the univocity of the position, there also needed to be sufficient GPS coverage. 31 points were acquired all together: 20 in the first and 11 in the second measuring phase. The network of points was surveyed in rapid-static mode with baseline length limited to ten kilometres. Their planimetric accuracy is assessable in a few centimetres while the altimetric accuracy, which is usually a bit lower, was not taken into consideration since the height is not required by the georeferencing software[29]. 25 of the points acquired during the two measuring campaigns were identified for certain on Nolli's map and as a result were then used for georeferencing the twelve sheets of the *Pianta Grande*.

[28] An initial attempt to georeference a historical map of Rome – the 1866 Census Map – using as a reference the coordinates of the regional technical map of Latium from 1990 on a scale of 1:10.000, which was the only official map available at that time, produced controversial results: the discrepancies found could not always be traced back to the inaccuracy of the historcal map. Cf: V. BAIOCCHI-K. LELO, *Georeferenziazione di cartografie storiche in ambiente GIS e loro verifica mediante rilievi GPS*, in "Atti della 5ª Conferenza nazionale ASITA" (Rimini, 2001).

[29] Height is not necessary for georeferencing and rectifying cartographies, which already have a flat projection; it may be needed for the orthorectification of historical aerial images but obviously has no place in this study.

The *Pianta Grande* obtained by printing from the matrices maintains the exactness of the survey, although it was subjected to limits of precision that we may define as being "technical", since they are associated with the process of engraving on the copper which leads to generalization levels that are higher compared to the preparatory drawing (the thickness of the line engraved on the copper is greater than that drawn in pencil on the original document). Another reason for the reduction in accuracy is the subdivision into different sheets: there are deformations along the joining lines on the tables that can be attributed to the redrawing and engraving work.

The copies of the twelve sheets of Nolli's[30] *Pianta Grande*, used for the acquisition in a raster format, showed anisotropic deformations which meant it was necessary to do a preventive calibration of the images so as to be able to mosaic them correctly.

The real accuracy that can be obtained by the georeferencing procedure was calculated by using some of the most common georeferencing algorithms which use GCPs (ground control points).

In the field of geographical information systems, the error that is still present after the georeferencing of a raster file is generally only reported in terms of the residuals on the points used for calculating the georeferencing itself. As a matter of fact, most software, both commercial and open source, gives this value as the only index of quality for the georeferencing carried out.

Although the results are correlated with the overall reliability of the coordinates, they may also contain an overestimate of the accuracy of the georeferenced cartography. This is due to the fact that the calculation of the transformation parameters is carried out using the well-known least squares estimator, which in the case of georeferencing tends to minimize the square of the deviations (from which the name is derived) on the points given as a reference, i.e. the above-mentioned GCPs[31].

[30] In order to acquire the *Pianta Grande* in digital format reproductions in phototypies were used by the Biblioteca Apostolica Vaticana. Cf. F. Ehrle S.I., *Roma al tempo di Benedetto XIV. La pianta di Giambattista Nolli del 1748 riprodotta da una copia vaticana*, Città del Vaticano, Biblioteca Apostolica Vaticana, 1932. This choice was made bearing two aspects in mind: firstly, copies do not show the deformations of the paper supports caused by the binding of the tables, which are found in most of the original examples. Secondly, to avoid damaging the originals, given the need to obtain a scan with very high resolution, which in the past could only be achieved by using a rolling scan instrument. The reproduction, made by Ehrle, is on a different scale compared to the original. The size of the sheet is ca.56x36 cm which led to a reduction of the map to a scale of ca. 1:3.600.

[31] G. Molnar, *Making a georeferenced mosaic of historical map series using constrained polynomial fit*, «Acta Geodaetica et Geophysica Hungarica», 45, 2010, 1, pp. 24-30.

The average of the residuals on the GCPs alone may provide an estimate of the precision of the model understood as the mutual coherence of the GCPs: this means that even points with a gross error but an error which is the same for each one (for example a translation) would be coherent and accurate.

If we define accuracy as "the proximity of the results obtained to the real values", it is easy to understand that it will be the GCPs that give a higher agreement with the real values while what is needed for a correct calculation of the accuracy of the historical maps is the deviation that can be observed on average on all the points on the map, not only on the "best" ones.

In order to verify the accuracy that can be obtained using georeferencing, we believed the correct thing to do was to use points extracted from the present-day map on a scale of 1:2.000 as control points. This type of cartographic work should guarantee a level of accuracy of around 40 cm.

The estimate of the residuals on the points not used for calculating the georeferencing parameters, and therefore independent from the process, is only possible with some GIS software, among which the QGis. We therefore decided to collimate the points obtained from the GPS survey and from the present-day map using the QGis "georeferencer" module. The GPS points were inserted as GCPs in order to estimate the various transformations whereas the points from present-day cartography were inserted only as verification points, which remain independent of the least squares estimation and which we will call check points (CPs) just as in photogrammetry.

In order to confirm the hypothesis that Nolli used a network of geodetic measurements made up of significant points in the city, on which the partial surveys were the "georeferenced", we decided to evaluate the accuracy of the *Pianta Grande* by using two different CPs networks. The first network consists of points from the present-day map, mainly building corners, univocally recognizable on the historical map and uniformly distributed over the city within the walls. In contrast, the second one consists of "potential" significant points that Nolli might have been used for his triangulation work. These are obelisks, columns, towers and bell towers, easily visible from most of the city and easily sighted at ground level. The hypothesized network of points is partially superimposed on the one used by Alberti for his *Descriptio Urbis Romae*.

It is expected that the level of accuracy of the historical map measured on the network of the presumed significant points of the city is higher compared to that measured on the network of points chosen randomly.

Results and discussion

The results of the georeferencing, obtained by using 25 GCPs and 32 CPs chosen from the cartography and uniformly distributed over the city within the walls, were calculated using all the georeferencing models provided by the QGis software, that is to say *polynomial*[32] from the first to the third order, *spline*[33] and *projective*[34]. The most basic transformations, such as that of Helmert, have not been taken into consideration here since they do not permit the elimination of the effects of the distortion suffered by the paper support of the historical cartographies[35].

The best results in terms of accuracy were obtained using a third-order polynomial transformation (P3) and a projective transformation. The numerical values are very close, but we decided to use the P3 transformation because it is documented better in the literature (tab. 1)[36].

The results of the P3 transformation were later recalculated using 32 points from present-day maps as CPs, which were chosen from the presumed network of significant points as described in the previous paragraph. As was expected, the results shown in Table 2 display a significant improvement in the level of measured accuracy.

Despite there being metric errors greater than the errors of graphicism conventionally accepted nowadays, we can affirm that such errors are perfectly acceptable, given that the instruments used in the 18th century, though conceptually advanced, did not allow for high margins of precision.

The high level of accuracy of this cartographic product is a proven fact which is demonstrated by a further test whose aim is to measure the variations in scale of the *Pianta Grande*. The scale ratio 1:2.910, quoted in the bibliographical sources is calculated based on the correspondence of the Roman palm in the metric system, the palm being the original unit of measure.

[32] This consists of a rigid rototranslation with a variation in the isotropic scale. Cf: QGis Team, *Documentazione QGis 2.2*, http://docs.qgis.org/2.2/it/docs/user_manual/_2014.

[33] The Polynomial algorithms 1-3 are among the most widely used algorithms introduced to match source and destination ground control points. The most widely used polynomial algorithm is the second-order polynomial transformation, which allows some curvature. First-order polynomial transformation (affine) preserves collinearity and allows scaling, translation and rotation only, *ivi*.

[34] The transformation algorithm Thin Plate Spline (TPS) is a recent method of georeferencing, which makes it possible to insert localized distortions into the data. This algorithm is particularly useful for georeferencing images of poor quality, *ibidem*.

[35] V. Baiocchi-K. Lelo-M.V. Milone-M. Mormile, *Accuracy of different georeferencing strategies on historical maps of Rome*, cit., pp.10-16.

[36] M.A. Brovelli-M. Minghini, *Georeferencing old maps: a polynomial-based approach for Como historical cadastres,* «e-Perimetron», 2012, 7, 3, pp. 97-110.

This test includes the most precise measurement possible of segments of various lengths placed in various parts of the map, which are representative parts of the urban fabric that have remained unchanged over time, followed by the calculation of the scale of each segment and a comparison with the real measure.

In order to bypass the problem of the predictable deformations of the paper support, which are found both in the preparatory drawing and in the various printed versions of the *Nuova Pianta*, we decided to take the measurements on the matrices, since a hard support is less subject to deformations. The chalcographic matrices of the twelve sheets of Nolli's *Pianta Grande*, made of steel-clad copper and measuring ca 470x700 mm, are preserved in excellent condition in Rome, in the Chalcography of the National Institute of Graphics[37]. We were able to examine and measure the segments easily and then the measurements were reported to the GIS system in order to calculate the lengths. The mapping of the segments with their respective scales of representation was followed by a procedure for interpolating the scale values over the whole surface occupied by the city and by the generation of isolines that join points that have the same scale. The result illustrated in figure 11 shows variations that range from scale values of 1:2.910 to scale values of 1:2.820, with a variability of 3%.

It is interesting to notice how the perfect coincidence between the local scale and the scale declared by the map occurs in the area of the Tridente which has also shown higher levels of accuracy in the positioning of GCPs than the rest of the city. The scale is therefore reported correctly in the northern sector of the map, and becomes less correct towards the southern and south-western quadrants (fig. 11).

The analysis of the variations of scale merits further study in light of the results obtained from the detailed analysis of the metric contents of the map carried out for this paper. The collection of measurements, carried out on the matrices several years ago during the initial phase of this project, should be completed in order to include parts of the city that were excluded, especially the segments along the walls and in the peripheral area towards the south-eastern quadrant.

As a conclusion, we can assert that the use of the *tavoletta pretoriana*, allowing for fast and precise on-site drawings, coupled with the triangulation of prominent city features such as obelisks, towers and domes, enabled Nolli to achieve the accuracy for which his plan became famous all over Europe.

The intuitions regarding the method of geodetic surveying, which was certainly used though not documented by Nolli, seem to be supported in this phase of our work by the results showing the map's metric accuracy. These results are so encouraging that we firmly believe that the research requires further development.

[37] Istituto Nazionale per la Grafica – Calcografia, inv. 1785/1-16.

Table 1 - Accuracy of the historical cartography georeferenced using a third-order Polynomial transformation, compared to the GCPs and CPs chosen randomly from present-day cartography.

N.	GCP (pixels)	GCP (meters)
1	16,88	2,19
2	21,04	2,74
3	21,06	2,74
4	73,42	9,54
5	11,21	1,46
6	19,61	2,55
7	42,44	5,52
8	17,93	2,33
9	7,72	1,00
10	15,81	2,05
11	5,98	0,78
12	34,17	4,44
13	25,67	3,34
14	33,86	4,40
15	12,34	1,60
16	15,00	1,95
17	15,13	1,97
18	12,06	1,57
19	18,36	2,39
20	24,19	3,15
21	14,52	1,89
22	5,40	0,70
23	11,37	1,48
24	39,97	5,20
25	2,70	0,35
	Mean of absolute values	*2,69*
	St. dev. of absolute values	*1,92*
	Minimum value	*0,35*
	Maximum value	*9,54*

N.	CP (pixels)	CP (meters)
1	26,18	3,40
2	17,57	2,28
3	8,66	1,13
4	11,83	1,54
5	20,22	2,63
6	20,43	2,66
7	11,88	1,54
8	22,30	2,90
9	6,49	0,84
10	23,50	3,05
11	27,75	3,61
12	8,21	1,07
13	15,87	2,06
14	40,33	5,24
15	19,69	2,56
16	26,59	3,46
17	34,22	4,45
18	25,78	3,35
19	38,35	4,99
20	29,07	3,78
21	36,57	4,75
22	11,32	1,47
23	15,55	2,02
24	16,96	2,20
25	39,68	5,16
26	48,91	6,36
27	22,25	2,89
28	12,11	1,57
29	46,43	6,04
30	33,54	4,36
31	50,34	6,54
32	10,86	1,41
	Mean of absolute values	*3,45*
	St. dev. of absolute values	*1,65*
	Minimum value	*0,84*
	Maximum value	*6,54*

Table 2 - Accuracy of the historical cartography, georeferenced using a third-order Polynomial transformation, compared to presumed significant points of the 18th century city, identified on present-day cartography.

N.	CP (pixels)	CP (meters)	N. Nolli	Description	Rione	***Descriptio Urbis***
1	11,18	1,45	1319	Castel S. Angelo	Borgo	sì
2	26,06	3,39	917	S. Maria d'Araceli	Campitelli	sì
3	21,95	2,85	920	Campidoglio	Campitelli	sì
4	35,56	4,62	404	S. Maria di Monte Santo	Campomarzo	
5	9,64	1,25	484	S. Maria dei Miracoli	Campomarzo	
6	11,68	1,52	310	Colonna di Marco Aurelio	Colonna	sì
7	9,84	1,28	1	S. Stefano Rotondo	Monti	sì
8	12,92	1,68	21	S. Croce in Gerusalemme	Monti	sì
9	21,66	2,82	49	S. Maria Maggiore	Monti	sì
10	17,40	2,26	64	S. Pietro in Vincoli	Monti	sì
11	30,79	4,00	97	SS. Martina e Luca	Monti	
12	30,45	3,96	113	Colonna Traiana	Monti	sì
13	30,83	4,01	164	S. Lorenzo in Pane e Perna	Monti	sì
14	31,95	4,15	178	S. Vitale	Monti	
15	19,51	2,54	195	S. Bernardo	Monti	
16	21,74	2,83	203	S. Maria degli Angeli	Monti	sì
17	22,78	2,96	608	S. Agnese	Parione	
18	35,06	4,56	610	Torre Millina	Parione	sì
19	39,87	5,18	656	S. Maria in Vallicella	Parione	
20	32,36	4,21	837	S. Maria ad Martyres gia' Pantheon	Pigna	sì
21	33,84	4,40	902	Gesu'	Pigna	
22	28,54	3,71	535	SSmo. Salvatore in Lauro	Ponte	
23	21,06	2,74	549	S. Giovanni dei Fiorentini	Ponte	
24	13,71	1,78	1059	S. Prisca	Ripa	sì
25	17,21	2,24	775	S. Andrea della Valle	S. Eustachio	
26	24,83	3,23	800	S. Ivo nella Sapienza	S. Eustachio	
27	17,50	2,27	1143	S. Grisogono	Trastevere	sì
28	31,94	4,15	1165	S. Maria in Trastevere	Trastevere	sì
29	14,83	1,93	1186	S. Pietro in Montorio	Trastevere	sì
30	6,46	0,84	211	S. Nicola di Tolentino	Trevi	
31	22,34	2,90	272	Nome di Maria	Trevi	
32	14,75	1,92	274	S. Maria di Loreto	Trevi	
Mean of absolute values		*2,97*				
St. dev. of absolute values		*1,15*				
Minimum value		*0,84*				
Maximum value		*5,18*				

Fig. 11 - Variations in the scale of the New map. Interpolation carried out on some measurements taken on the matrices.

As a starting point we believe it is useful to carry out an in-depth study on the network of significant points of the 18th century city. One possible line of research could deal with an analysis of the intervisibility between the presumed significant points. This would involve complex and lengthy work for reconstructing the urban morphology since it would be necessary to retrace, acquire and analyse a great deal of historical data (land altimetry, heights of buildings and lost heritage buildings) from various sources. This work would allow us to identify the *real* significant points on historical maps, or rather those points that can be seen at the same time from one or more prominent points and that are intervisible at ground level.

Despite not having all the information necessary for reconstructing a complete and precise morphology of the city in the 18th century thus the network of significant points, it could be possible to try and reconstruct some parts of the network, traces of which could be searched for in the preparatory drawing. As mentioned previously, some assumed station points can be distinguished in the preparatory drawing which, if they have remained unchanged (or almost), can be used as a base from which to measure again the significant points present in the 18th century and still visible today. This work would also involve complex and laborious operations, which in turn would require economic resources and also access to locations that are not always accessible.

Valerio Baiocchi - Keti Lelo