
Emulating Satellite Tangential Scale Distortion

1 Problem statement

Remote sensing systems, provide an instantaneous “snapshot” view of the Earth from directly overhead. Each detector essentially takes a “snapshot” of each ground resolution cell [1].

However, a distortion occurs due to the rotation of the scanning optics. As the sensor scans across each line, the distance from the sensor to the ground increases further away from the centre of the swath. Although the scanning mirror rotates at a constant speed, the relative motion of the sensor is faster (relative to the ground) and scans a larger area as it moves closer to the edges. This effect results in the compression of image features at points away from the nadir and is called **tangential scale distortion** [1].

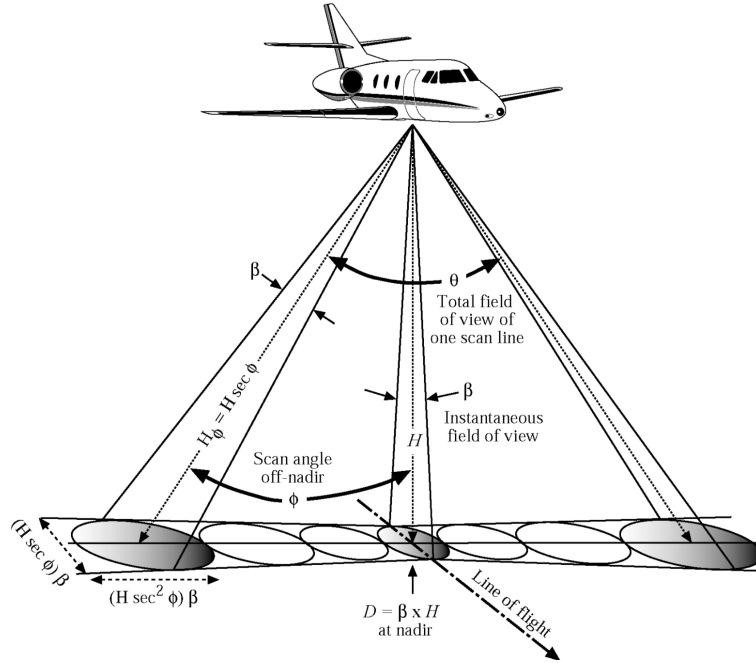


Figure 1: The ground resolution cell size along a single across-track scan. Figure from [2]

To reflect this deformation when generating synthetic imagery samples, we propose to apply an unidimensional sigmoid application to coordinates along a virtual scanning line. Explicitely, let σ_λ be :

$$\sigma_\lambda : \begin{cases} \mathbb{R} & \longrightarrow [0, 1] \\ x & \longmapsto \frac{1}{1+e^{-\lambda x}} \end{cases}, \quad \lambda > 0$$

Let now $\ell > 0$ denote the image size along the deformation line and the center position $c = \frac{\ell}{2}$ corresponding to a virtual nadir. To reflect invariance at nadir and compression at edges, we apply along the swath line $[0, \ell]$ the application $x \in [0, \ell] \longmapsto \ell \sigma_\lambda(x - c)$.

In practice nearest method can be used for shifted pixels assignation and interpolation to cope with missing positions.

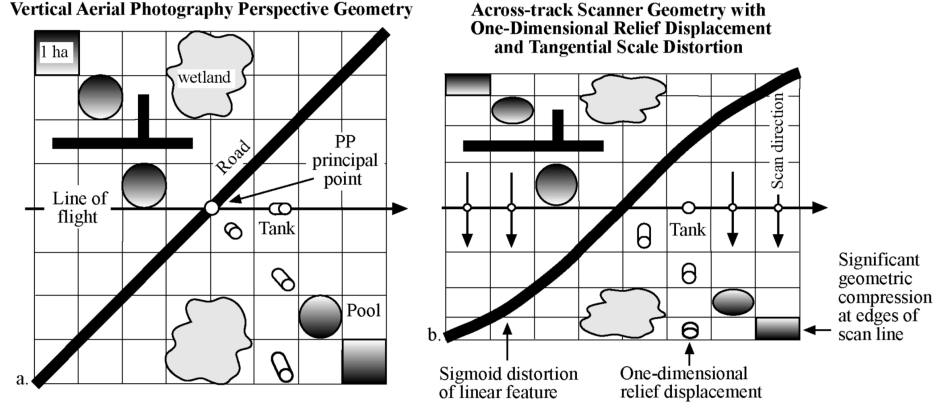


Figure 2: One-dimensional relief displacement is introduced in both directions away from nadir for each sweep of the across-track mirror. Linear features trending across the terrain are often recorded with s-shaped or sigmoid curvature characteristics due to tangential scale distortion and image compression. Figure from [2]

Tangential scale distortion does not however expand image representation and only acts as a compression of the edges. Mathematically, it means that $x \mapsto \ell\sigma_\lambda(x - c)$ should be a contractive¹ mapping. We find that setting $\lambda \leq \frac{4}{\ell}$ allows to satisfy this condition, proof is provided at the end.

This value sets an upper bound in the choice of λ which can be used to model high-resolution satellites. For lower-resolution images generating larger deformation, lower values can be chosen.

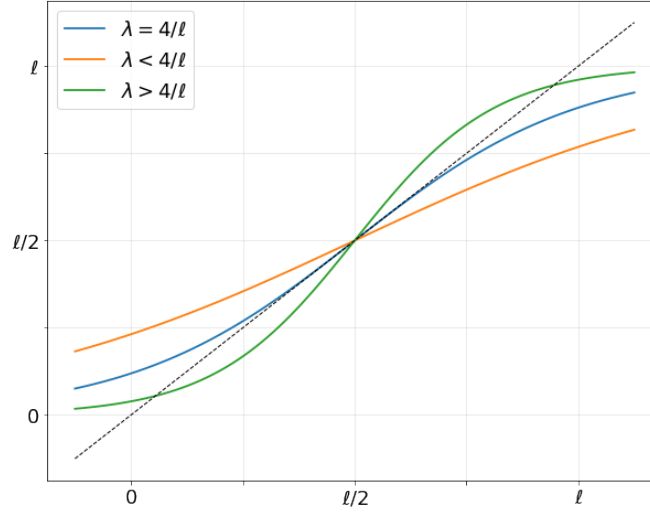


Figure 3: Plot of $\ell\sigma_\lambda(\cdot - c)$ for various choices of λ . In green, we observe the curve to grow faster than the identity, hence provoking interval expansion rather than compression.

¹actually 1-Lipschitzian, contractive is stricter

2 Implementation

We approximate this transformation by a piecewise affine deformation of the image based on scikit-image [3] `PiecewiseAffineTransform` class.

Our implementation comes as a transformer class `TangentialScaleDistortion` initialized by :

- `image.size` (tuple[int]) : width and height of the input image
- `mesh.size` (tuple[int]) : number of anchors points in width and height
- `axis` (int) : distortion axis (height:0 or width:1)
- `growth_rate` (float) : λ parameter, if none specified taken as $4/\ell$

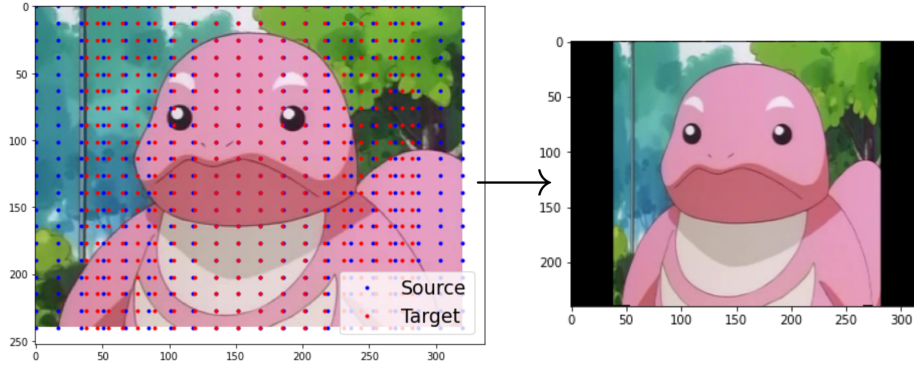


Figure 4: *Left* : Original image with piecewise affine transform anchor points. Source anchor points (blue) are mapped to target anchor points (red). We observe compression only affects edges as center source and target anchors overlap. *Right* : Resulting deformed image

References

- [1] Geometric distortion in imagery. <https://www.nrcan.gc.ca/maps-tools-publications/satellite-imagery-air-photos/remote-sensing-tutorials/satellites-sensors/geometric-distortion-imagery/9401>.
- [2] A. Bhattacharya. Correcting rs system detector error geometric correction. <http://www.csre.iitb.ac.in/~avikb/GNR401/DIP/>.
- [3] Stéfan van der Walt, Johannes L. Schönberger, Juan Nunez-Iglesias, François Boulogne, Joshua D. Warner, Neil Yager, Emmanuelle Gouillart, Tony Yu, and the scikit-image contributors. scikit-image: image processing in Python. *PeerJ*, 2014.

Proof. Let $\ell > 0$ denote the image size along the deformation line and the center position $c = \frac{\ell}{2}$. In accordance with the previous notations, λ must allow to satisfy :

$$\forall x \leq c, \ell \sigma_{\lambda}(x - c) \geq x \quad (1)$$

$$\forall x \geq c, \ell \sigma_{\lambda}(x - c) \leq x \quad (2)$$

$$(1) \Rightarrow \forall x \in]0, c[\quad x - c \geq \sigma_\lambda^{-1} \left(\frac{x}{\ell} \right) \quad (3)$$

$$\Rightarrow \forall x \in]0, c[\quad \lambda \leq \sigma_{x-c}^{-1} \left(\frac{x}{\ell} \right) \quad (4)$$

where $\sigma_\lambda^{-1} = x \in]0, 1[\mapsto \frac{1}{\lambda} \log \left(\frac{x}{1-x} \right)$.

Similarly,

$$(2) \Rightarrow \forall x \in]c, \ell[\quad x - c \leq \sigma_\lambda^{-1} \left(\frac{x}{\ell} \right) \quad (5)$$

$$\Rightarrow \forall x \in]c, \ell[\quad \lambda \leq \sigma_{x-c}^{-1} \left(\frac{x}{\ell} \right) \quad (6)$$

Hence $\forall x \in]0, \ell[\setminus \{c\}$ we have

$$\lambda \leq \frac{1}{x-c} \log \left(\frac{x/\ell}{1-x/\ell} \right) = \underbrace{\frac{1}{x-c} \log \left(\frac{x}{\ell-x} \right)}_{\psi_\ell(x)}$$

ψ_ℓ decreases on $]0, c[$ and increases on $]c, \ell[$. It thus admits a lower bound in c , i.e.

$$\liminf_{]0, \ell[\setminus \{c\}} \psi_\ell = \lim_{x \rightarrow c} \psi_\ell(x)$$

To compute this limit, we observe that :

$$\begin{aligned} \log \left(\frac{x}{\ell-x} \right) &= \log(x) - \log(\ell-x) \\ &= \log(c + (x-c)) - \log(c - (x-c)) \\ &\underset{x \rightarrow c}{=} \frac{x-c}{c} + \frac{x-c}{c} + o(x-c) \\ &\underset{x \rightarrow c}{=} \frac{4}{\ell} (x-c) + o(x-c) \end{aligned}$$

Which entails :

$$\psi_\ell(x) \underset{x \rightarrow c}{=} \frac{1}{x-c} \left[\frac{4}{\ell} (x-c) + o(x-c) \right] \underset{x \rightarrow c}{=} \frac{4}{\ell} + o(1) \Rightarrow \lim_{x \rightarrow c} \psi_\ell(x) = \frac{4}{\ell}$$

Reversely, we verify that choosing $\lambda \leq \frac{4}{\ell}$ allows to verify (1) and (2), hence making $x \mapsto \ell \sigma_\lambda(x-c)$ contractive. \square