2η Σειρά Ασκήσεων

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$a = AM \bmod 10 + 1 = 6 + 1 = 7$

2.1) $x_1(t) = \sin(20a\pi t)\dfrac{\cos(300\pi t)}{\pi t} = \sin(140\pi t)\dfrac{\cos(300\pi t)}{\pi t}$

$x_2(t) = 2\dfrac{\sin(500\pi t)}{\pi t}$

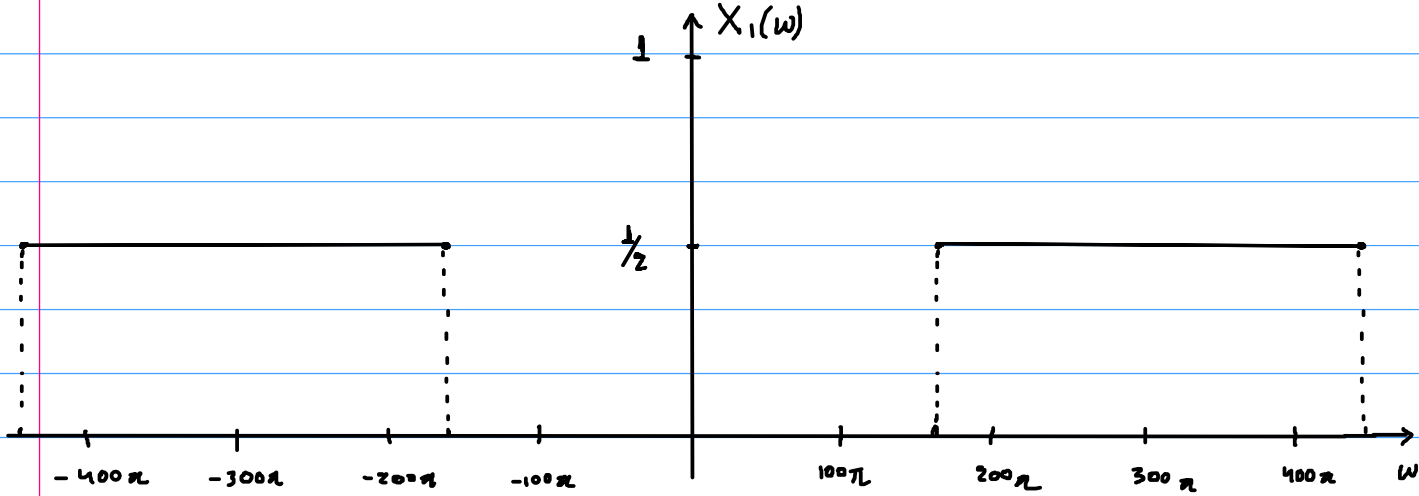
$$x_1(t) = 2\sin(140\pi t)\cos(300\pi t)\cdot\frac{1}{2\pi t} =$$

$$= \left[\sin\left(140\pi t + 300\pi t\right) + \sin\left(140\pi t - 300\pi t\right)\right]\cdot\frac{1}{2\pi t} =$$

$$= \frac{\sin 440\pi t}{2\pi t} - \frac{\sin 160\pi t}{2\pi t}$$

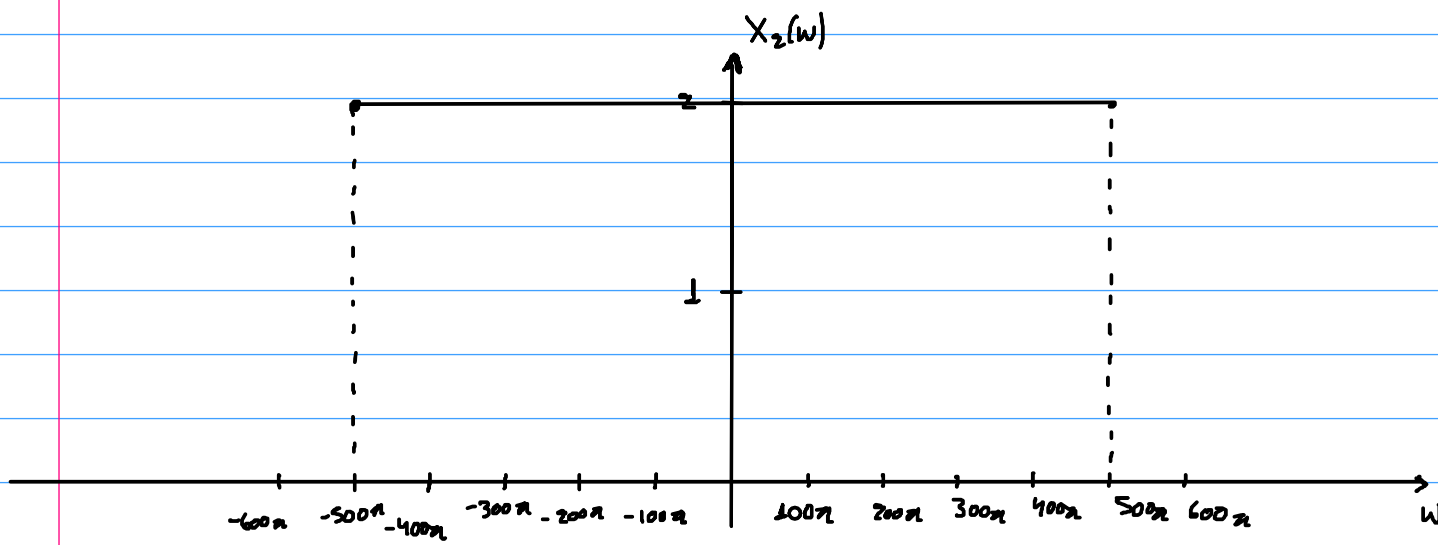
$$X_1(\omega) = F[x_1(t)] = \frac{1}{2}F\left[\frac{\sin 440\pi t}{\pi t}\right] - \frac{1}{2}F\left[\frac{\sin 160\pi t}{\pi t}\right]$$

$$X_1(\omega) = \begin{cases} 0, & \omega \in (-\infty, -440\pi) \cup (-160\pi, 160\pi) \cup (440\pi, +\infty) \\ \frac{1}{2}, & \omega \in (-440\pi, -160\pi) \cup (160\pi, 440\pi) \end{cases}$$



$$X_2(w) = F[x_2(t)] = 2F\left[\frac{\sin(500\pi t)}{\pi t}\right] =$$

$$= \begin{cases} 0, & w \in (-\infty, -500\pi) \cup (500\pi, +\infty) \\ 2, & -500\pi < w < 500\pi \end{cases}$$

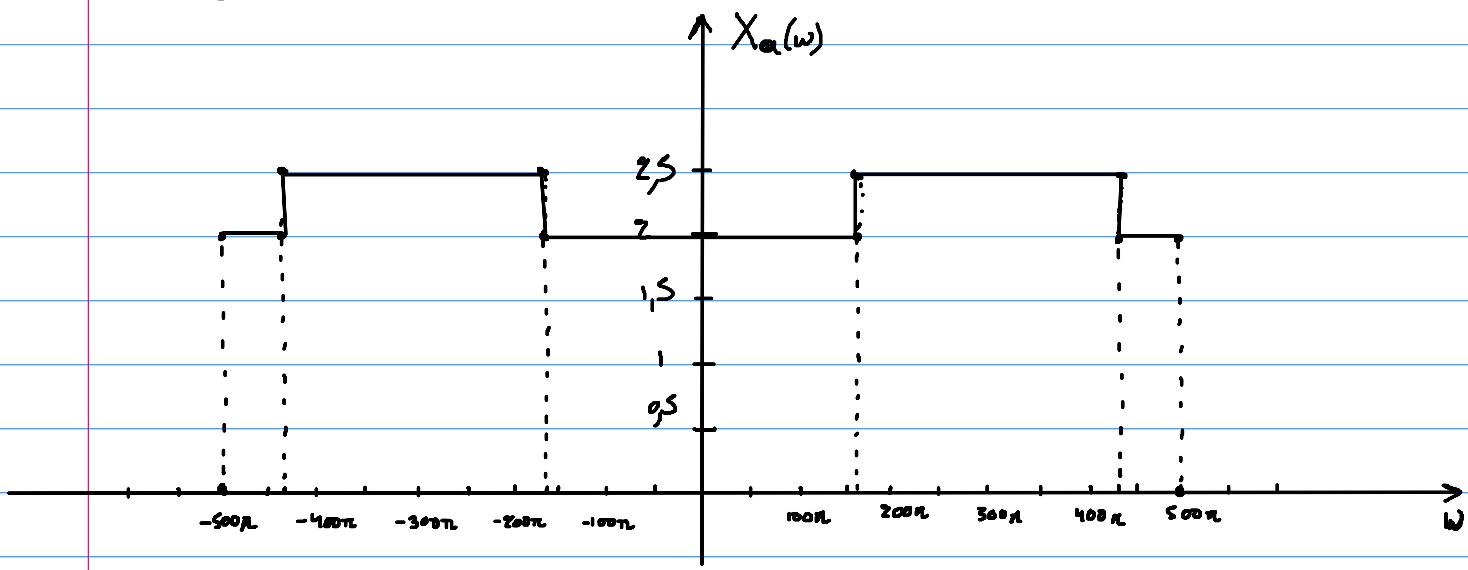


a) $x_1(t) + x_2(t)$

$$X_a(\omega) = F[x_1(t) + x_2(t)] = F[x_1(t)] + F[x_2(t)] =$$

$$= X_1(\omega) + X_2(\omega) =$$

$$\begin{cases} 0, & \omega \in (-\infty, -500\pi) \cup (500\pi, +\infty) \\ 2, & \omega \in (-500\pi, -440\pi) \cup (-160\pi, 160\pi) \cup (440\pi, 500\pi) \\ \frac{5}{2}, & \omega \in (-440\pi, -160\pi) \cup (160\pi, 440\pi) \end{cases}$$



$$\omega = 1000\pi \Rightarrow \omega_s > 2\omega \Rightarrow \frac{\cancel{2}\pi}{T_s} > \cancel{2}\omega \Rightarrow T_s < \frac{\pi}{\omega} = \frac{\pi}{1000\pi} = 10^{-3}\,s$$

b) $x_b(t) = x_1^3(t) = x_1(t) \cdot x_1^2(t)$

$$x_1^2(t) \Rightarrow F[x_1^2(t)] = X_1(\omega) * X_1(\omega),$$

$$\omega_{x_1^2} = \omega_{x_1} + \omega_{x_1} = 880\pi + 880\pi = 1760\pi$$

$$X_b(\omega) = X_1(\omega) * F[x_1^2(t)],$$

$$W = W_{x_1} + W_{x_1^2} = 880\pi + 1760\pi = 2640\pi$$

$$\omega_s > 2W \Rightarrow \frac{\cancel{2}\cancel{\pi}}{T_s} > \cancel{2}W \Rightarrow T_s < \frac{2640\cancel{\pi}}{\cancel{\pi}} = 2640\,s$$

γ) $x_\gamma(t) = x_1(t) * x_2(t)$

$X_\gamma(\omega) = X_1(\omega) \cdot X_2(\omega)$

$W = \min\{W_1, W_2\} = \{880\pi, 1000\pi\} = 880\pi$

$$\omega_s > 2W \Rightarrow \frac{\cancel{2}\cancel{\pi}}{T_s} > \cancel{2}\cdot 880\cancel{\pi} \Rightarrow T_s < 880\,s$$

δ) $x_\delta(t) = x_1(t) \cdot [x_2(t)]^2 + [x_1(t)]^2 * x_2(t)$

$$X_\delta(\omega) = F\left[x_1(t) \cdot [x_2(t)]^2\right] + F\left[[x_1(t)]^2 * x_2(t)\right]$$

$$W = \max\{W_1 + 2W_2, \min\{2W_1, W_2\}\} =$$

$$= \max\{880\pi + 2000\pi, \min\{1760\pi, 1000\pi\}\} =$$

$$= \max\{2880\pi, 1000\pi\} = 2880\pi$$

$$\omega_s > 2W \Rightarrow \frac{\cancel{2}\pi}{T_s} > \cancel{2}W \Rightarrow T_s < 2880\,s$$

2.3) $\omega_s = (2 + 7 \bmod 3)\omega_M = (2+1)\omega_M = 3\omega_M$

a) $y_d[n] = \dfrac{x[n-1] - 2x[n] + x[n+1]}{4} = \dfrac{1}{4}x[n-1] - \dfrac{1}{2}x[n] + \dfrac{1}{4}x[n+1]$

$$Y_d[\Omega] = \frac{1}{4}e^{-j\Omega}X_d[\Omega] - \frac{1}{2}X_d[\Omega] + \frac{1}{4}e^{j\Omega}X_d[\Omega] \Rightarrow$$

$$\frac{Y_d[\Omega]}{X_d[\Omega]} = \frac{1}{2}\left(\frac{e^{-j\Omega} + e^{j\Omega}}{2} - 1\right) = \frac{1}{2}\left(\cos\Omega - 1\right)$$

$$H_d[\Omega] = \frac{1}{2}\left(\cos\Omega - 1\right)$$

$$h_d[n] = F^{-1}\left[H_d[\Omega]\right] = \frac{1}{4}F^{-1}\left[e^{-j\Omega} + e^{j\Omega}\right] - \frac{1}{2}F^{-1}[1] =$$

$$= \frac{1}{4}\delta[n-1] + \frac{1}{4}\delta[n+1] - \frac{1}{2}\delta[n] =$$

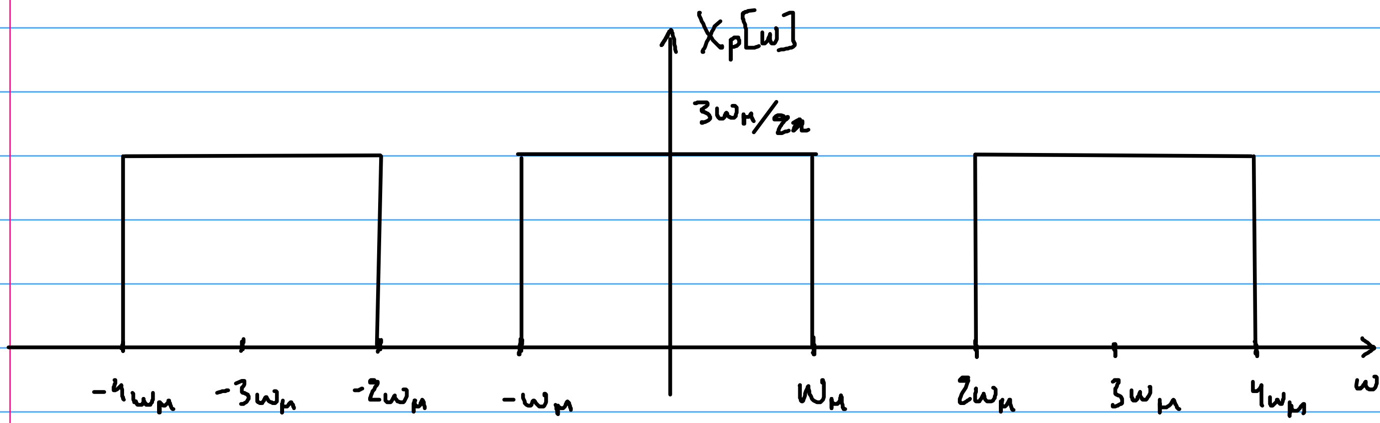
$$= \frac{\delta[n-1] - 2\delta[n] + \delta[n+1]}{4}$$

b) $x_c(t) = \dfrac{\sin(\omega_M t)}{\pi t}$, $\omega_s = \dfrac{2\pi}{T_s} \Rightarrow T_s = \dfrac{2\pi}{\omega_s} = \dfrac{2\pi}{3\omega_M}$

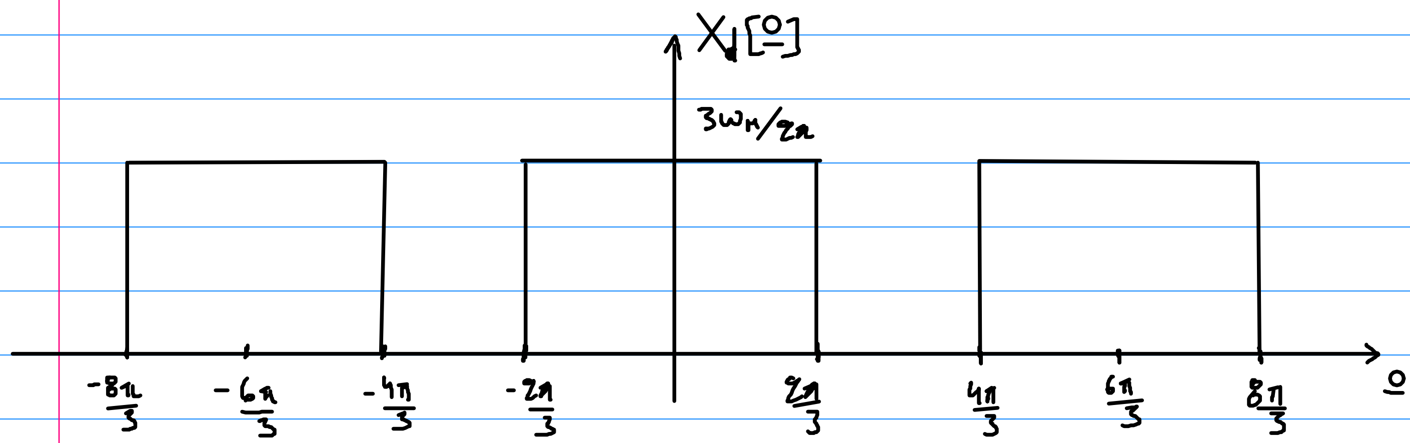
$$x_p(t) = x_c(t) \cdot p(t) = \frac{\sin(\omega_M t)}{\pi t} \sum_n \delta(t - nT)$$

$$X_p[\omega] = X_c(\omega) * F\left[\sum_n \delta(t - nT_s)\right] = X_c * \left[\frac{1}{T_s}\sum_{m=-\infty}^{+\infty}\delta(\omega - m\omega_s)\right] =$$

$$= \frac{1}{T_s}\sum_{m=-\infty}^{+\infty} X_c(\omega - m\omega_s) = \frac{3\omega_M}{2\pi}\sum_{m=-\infty}^{+\infty} X_c(\omega - m\omega_s)$$

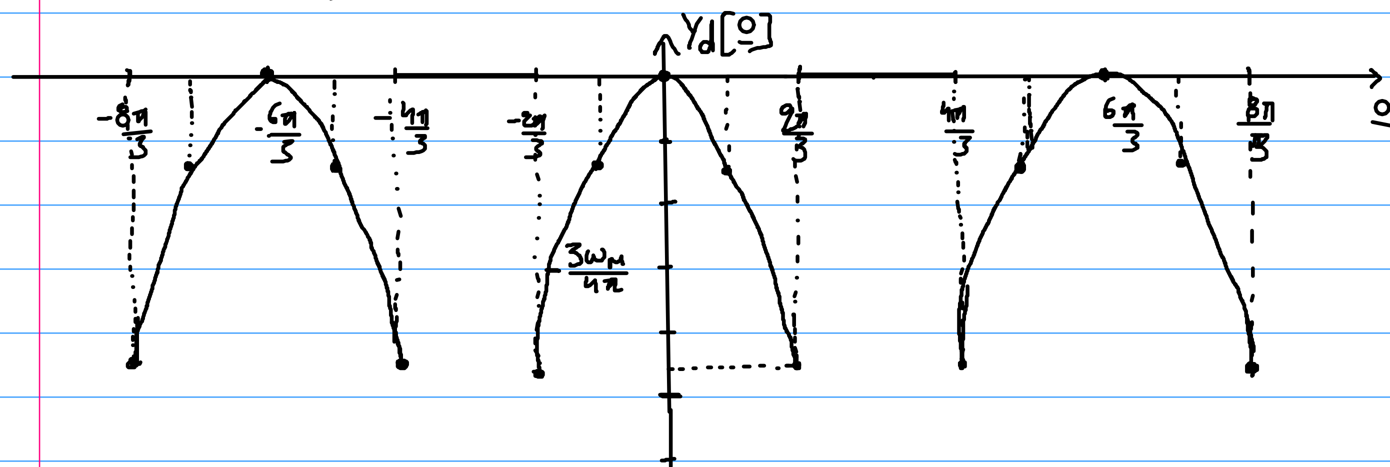


$$X_d(\Omega) = X_p\left(\frac{\Omega}{T_s}\right) = X_p\left(\frac{3\omega_M}{2\pi}\cdot\Omega\right)$$

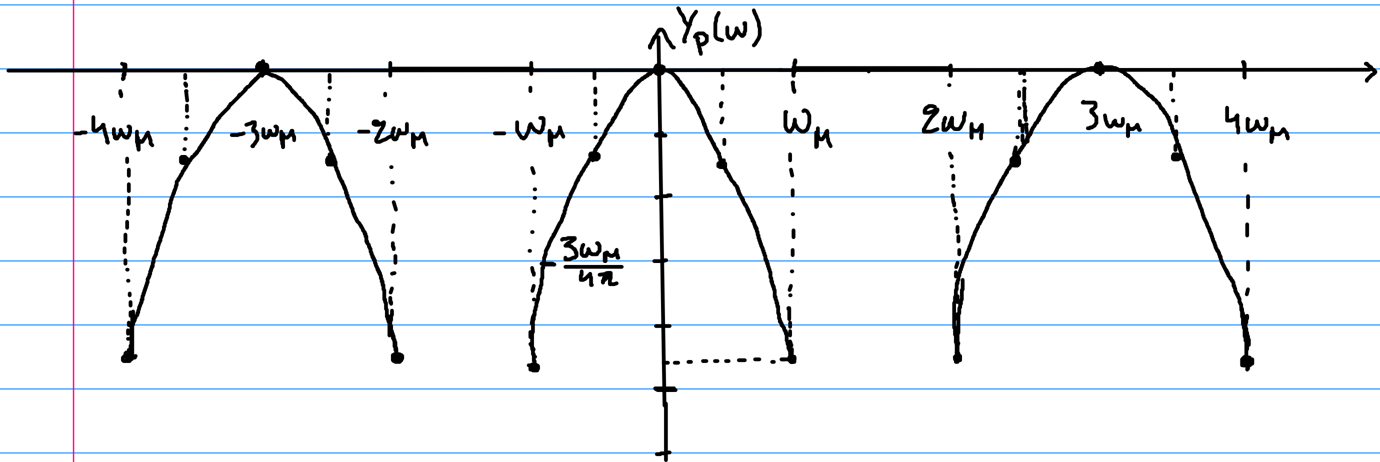


$$Y_d[\Omega] = \frac{1}{4}e^{j\Omega}X_d[\Omega] - \frac{1}{2}X_d[\Omega] + \frac{1}{4}e^{-j\Omega}X_d[\Omega] =$$

$$= \frac{X_d[\Omega]}{2}\left(\frac{e^{-j\Omega}+e^{j\Omega}}{2} - 1\right) = \frac{X_d[\Omega]}{2}(\cos\Omega - 1)$$

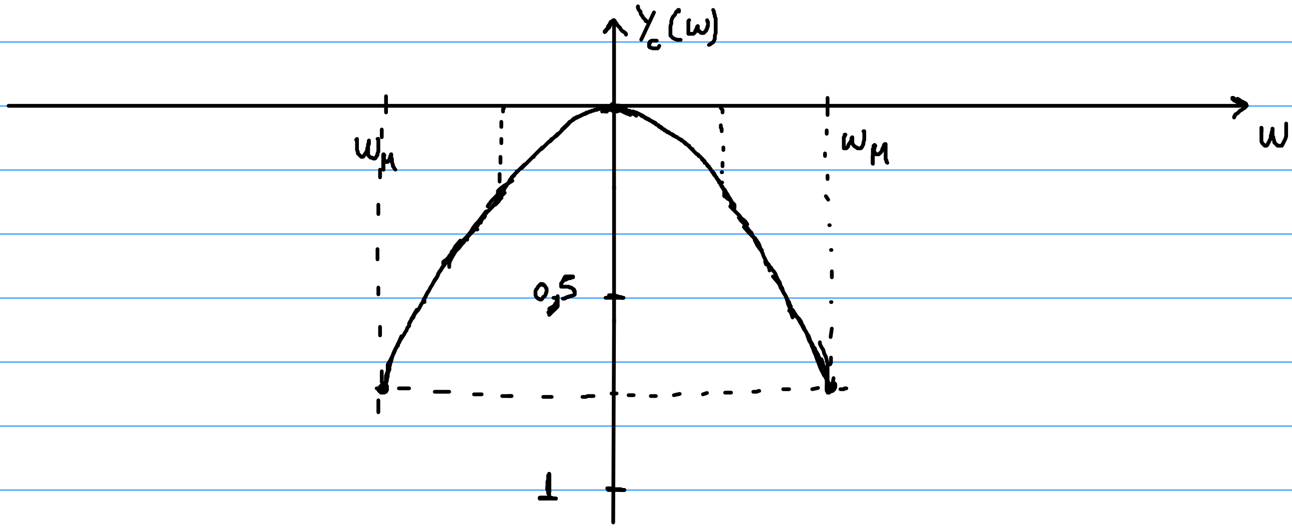


$$Y_d[\Omega] = Y_p\left(\frac{\Omega}{T_s}\right) \Rightarrow Y_p(\omega) = Y_d[\omega T_s] = Y_d\left[\omega \cdot \frac{2\pi}{3\omega_M}\right]$$



$$H_r(\omega) = \begin{cases} T & , |\omega| < \frac{\omega_s}{2} = \frac{3\omega_M}{2} \\ 0 & , |\omega| > \frac{\omega_s}{2} = \frac{3\omega_M}{2} \end{cases}$$

$$Y_c(\omega) = Y_p(\omega) \cdot H_r(\omega) = \begin{cases} \frac{2\pi}{3\omega_M} \cdot Y_p(\omega) & , |\omega| < \frac{3\omega_M}{2} \\ 0 & , |\omega| > \frac{3\omega_M}{2} \end{cases}$$



γ) $$H_c(\omega) = \frac{Y_c(\omega)}{X_c(\omega)} = \frac{Y_p(\omega) \cdot H_r(\omega)}{X_c(\omega)} = \frac{Y_d(\omega T_s) H_r(\omega)}{X_c(\omega)} =$$

$$= \frac{X_d(\omega T_s) H_d(\omega T_s) H_r(\omega)}{X_c(\omega)} = \frac{X_p(\omega) H_d(\omega T_s) H_r(\omega)}{X_c(\omega)} = \frac{H_d(\omega T_s) \cdot \cancel{X_c(\omega)}}{\cancel{X_c(\omega)}} = H_d(\omega T_s)$$

Επομένως $H_c(\omega) = \begin{cases} H_d(\omega T_s) &, |\omega| \leq \frac{\pi}{T_s} = \frac{\pi}{\frac{2\pi}{3\omega_M}} = \frac{3\omega_M}{2} \\ 0 &, |\omega| > \frac{3\omega_M}{2} \end{cases} \Rightarrow$

$$H_c(\omega) = \begin{cases} \frac{1}{2}\left(\cos\left(\frac{2\pi}{3}\cdot\frac{\omega}{\omega_M}\right) - 1\right) &, |\omega| \leq \frac{3\omega_M}{2} \\ 0 &, |\omega| > \frac{3\omega_M}{2} \end{cases}$$

2.4) a) $x_1[n] = 7^n \sin[\pi(n+1)/3]\cos[\pi(n+1)/6]u[n] =$

$$= \frac{7^n}{2}\cdot\left(\sin\frac{\pi}{2}(n+1) + \sin\frac{\pi}{6}(n+1)\right)u[n] =$$

$$= \frac{7^n}{2}\left[\frac{e^{j\frac{\pi}{2}(n+1)} - e^{-j\frac{\pi}{2}(n+1)}}{2j} + \frac{e^{j\frac{\pi}{6}(n+1)} - e^{-j\frac{\pi}{6}(n+1)}}{2j}\right]u[n] =$$

$$= \frac{7^n}{4}\left(\frac{\cancel{j}e^{j\frac{\pi}{2}n} + \cancel{j}e^{-j\frac{\pi}{2}n}}{\cancel{j}} + \frac{\left(\frac{\sqrt{3}}{2}+j\frac{1}{2}\right)e^{j\frac{\pi}{6}n} - \left(\frac{\sqrt{3}}{2} - j\frac{1}{2}\right)e^{-j\frac{\pi}{6}n}}{j}\right)u[n] =$$

$$= \frac{7^n}{4}\left(e^{j\frac{\pi}{2}n} + e^{-j\frac{\pi}{2}n} + \frac{\sqrt{3}}{2j}\left(e^{j\frac{\pi}{6}n} - e^{-j\frac{\pi}{6}n}\right) + \frac{1}{2}\left(e^{j\frac{\pi}{6}n} + e^{-j\frac{\pi}{6}n}\right)\right)u[n]$$

$$= \frac{7^n}{4}\left(2\cos\frac{\pi}{2}n + \sqrt{3}\sin\frac{\pi}{6}n + \cos\frac{\pi}{6}n\right)u[n] =$$

$$= \frac{1}{2}7^n\cos\left(\frac{\pi}{2}n\right)u[n] + \frac{\sqrt{3}}{4}7^n\sin\left(\frac{\pi}{6}n\right)u[n] + \frac{1}{4}7^n\cos\left(\frac{\pi}{6}n\right)u[n]$$

$$X_1(z) = \frac{1}{2}\cdot\frac{1-[7\cos\frac{\pi}{2}]z^{-1}}{1-2[7\cos\frac{\pi}{2}]z^{-1} + 7^2 z^{-2}} + \frac{\sqrt{3}}{4}\cdot\frac{[7\sin\frac{\pi}{6}]z^{-1}}{1-[2\cdot 7\cos\frac{\pi}{6}]z^{-1} + {}^2 z^{-2}} +$$

$$+\frac{1}{4}\cdot\frac{1-\left[7\cos\frac{\pi}{6}\right]z^{-1}}{1-\left[2\cdot 7\cos\frac{\pi}{6}\right]z^{-1}+7^2z^{-2}}=\frac{1}{2}\cdot\frac{1}{1+\left(\frac{7}{z}\right)^2}+\frac{\sqrt{3}}{\cancel{4}2}\cdot\frac{\frac{7}{\cancel{2}z}}{1-7\sqrt{3}z^{-1}+\left(\frac{7}{z}\right)^2}+$$

$$+\frac{1}{4}\cdot\frac{1-\frac{\sqrt{3}}{2z}}{1-\frac{7\sqrt{3}}{z}+\left(\frac{7}{z}\right)^2}=\frac{z^2}{2(z^2+7^2)}+\frac{\sqrt{3}}{2}\cdot\frac{7z}{z^2-7\sqrt{3}z+7^2}+\frac{1}{4}\cdot\frac{\frac{2z-7\sqrt{3}}{2\cancel{z}}}{\frac{z^2-7\sqrt{3}z+7^2}{z^{\cancel{2}}}}=$$

$$=\frac{z^2}{2(z^2+7^2)}+\frac{\sqrt{3}}{2}\cdot\frac{7z}{z^2-z\sqrt{3}+7^2}+\frac{1}{2}\cdot\frac{z(2z-7\sqrt{3})}{z^2-7z\sqrt{3}+7^2}=$$

$$=\frac{z^2}{2(z^2+7^2)}+\frac{\cancel{7z\sqrt{3}}+\cancel{2}z^2-\cancel{7z\sqrt{3}}}{\cancel{2}(z^2-7z\sqrt{3}+7^2)}=z^2\cdot\frac{z^2+7^2-7z\sqrt{3}+2z^2+2\cdot 7^2}{2(z^2+7^2)(z^2-7z\sqrt{3}+7^2)}=$$

$$=z^2\cdot\frac{3z^2-7z\sqrt{3}+3\cdot 7^2}{2(z^2+7^2)(z^2-7z\sqrt{3}+7^2)}=z^2\cdot\frac{3z^2-7\sqrt{3}z+147}{2(z^2+49)(z^2-7\sqrt{3}z+49)},\quad |z|>7$$

b) $x_2[n]=n7^{-|n|}=n7^{-n}u[n]+n7^{n}u[-n]$

$$X_2(z)=Z[x_2[n]]=Z[n7^{-n}u[n]]+Z[n7^{n}u[-n]]=$$

$$=Z\left[n\left(\tfrac{1}{7}\right)^n u[n]\right]-Z\Big[\underbrace{-n\left(\tfrac{1}{7}\right)^{-n}u[-n]}_{\text{Χρονική αντιστροφή}}\Big]=$$

$$=\frac{\frac{1}{7}z^{-1}}{\left(1-\frac{1}{7}z^{-1}\right)^2}-\frac{\frac{1}{7}(z^{-1})^{-1}}{\left(1-\frac{1}{7}(z^{-1})^{-1}\right)^2}=\frac{\frac{1}{7z}}{\left(1-\frac{1}{7z}\right)^2}-\frac{\frac{z}{7}}{\left(1-\frac{z}{7}\right)^2}=$$

$$=\frac{\frac{1}{\cancel{7z}}}{\frac{(7z-1)^2}{(7z)^2}}-\frac{\frac{z}{\cancel{7}}}{\frac{(7-z)^2}{7^2}}=\frac{7z}{(7z-1)^2}-\frac{7z}{(7-z)^2}=$$

$$=7z\cdot\frac{(7-z-7z+1)(7-z+7z-1)}{(7-z)^2(7z-1)^2}=7z\cdot\frac{(7+1)(1-z)(7-1)(1+z)}{(7-z)^2(7z-1)^2}=7z\,\frac{48(1-z^2)}{(7-z)^2(7z-1)^2}=$$

$$= \frac{336 z(1-z^2)}{(7-z)^2(7z-1)^2}, |z| > \frac{1}{7}, \quad \left|\frac{1}{z}\right| > \frac{1}{7} \Rightarrow |z| < 7$$

Άρα $\frac{1}{7} < |z| < 7$

γ) $$x_3[n] = \frac{n}{7^n} \sin\left[\frac{\pi n}{3} + \frac{\pi}{4}\right] u[n] =$$

$$= \frac{n}{7^n}\left[\sin\left(\frac{\pi n}{3}\right)\cos\left(\frac{\pi}{4}\right) + \cos\left(\frac{\pi n}{3}\right)\sin\left(\frac{\pi}{4}\right)\right] u[n] =$$

$$= \frac{n}{7^n}\left[\frac{\sqrt{2}}{2}\sin\left(\frac{\pi n}{3}\right) + \frac{\sqrt{2}}{2}\cos\left(\frac{\pi n}{3}\right)\right] u[n] =$$

$$= \frac{\sqrt{2}}{2}\left[\frac{n}{7^n}\left(\sin\left(\frac{\pi n}{3}\right) + \cos\left(\frac{\pi n}{3}\right)\right)\right] u[n] =$$

$$= \frac{\sqrt{2}}{2}\left[n(7^{-1})^n \sin\left(\frac{\pi n}{3}\right) u[n] + n(7^{-1})^n \cos\left(\frac{\pi n}{3}\right) u[n]\right]$$

$$X_3(z) = \frac{\sqrt{2}}{2} Z\left[n(7^{-1})^n \sin\frac{\pi n}{3} \cdot u[n] + n(7^{-1})^n \cos\left(\frac{\pi n}{3}\right) u[n]\right] =$$

$$= \frac{\sqrt{2}}{2}\left[-z\frac{d Z\left[(7^{-1})^n \sin\frac{\pi n}{3} u[n]\right]}{dz} - z\frac{d Z\left[(7^{-1})^n \cos\left(\frac{\pi n}{3}\right) u[n]\right]}{dz}\right] =$$

$$= \frac{\sqrt{2}}{2}\left[-z \cdot \frac{d}{dz}\left(\frac{7^{-1}\sin\frac{\pi}{3} z^{-1}}{1 - 2\cdot 7^{-1}\cos\frac{\pi}{3} z^{-1} + 7^{-2}z^{-2}}\right) - z\frac{d}{dz}\left(\frac{1 - 7^{-1}\cos\frac{\pi}{3} z^{-1}}{1 - 2\cdot 7^{-1}\cos\frac{\pi}{3} z^{-1} + 7^{-2}z^{-2}}\right)\right] =$$

$$= \frac{\sqrt{2}}{2}\left[-z\frac{d}{dz}\left(\frac{\frac{\sqrt{3}}{2\cdot 7\cdot z}}{1 - \frac{1}{7z} + \frac{1}{7^2 z^2}}\right) - z\frac{d}{dz}\left(\frac{1 - \frac{1}{2\cdot 7\cdot z}}{1 - \frac{1}{7\cdot z} + \frac{1}{7^2\cdot z^2}}\right)\right] =$$

$$= \frac{\sqrt{2}}{2}\left[-z\frac{d}{dz}\left(\frac{\sqrt{3}}{2} \cdot \frac{7z}{49z^2 - 7z + 1}\right) - z\frac{d}{dz}\left(\frac{\frac{14z-1}{2\cdot 7z}}{\frac{7^2z^2 - 7z + 1}{7^2z^2}}\right)\right] =$$

$$= \frac{\sqrt{2}}{2}\left[-z\frac{d}{dz}\left(\frac{\sqrt{3}}{2}\cdot\frac{7z}{49z^2-7z+1}\right)-z\frac{d}{dz}\left(\frac{1}{2}\cdot\frac{7z(14z-1)}{49z^2-7z+1}\right)\right]=$$

$$= -\frac{7\sqrt{2}}{4}\left[\sqrt{3}\,z\cdot\frac{49z^2-7z+1-z(98z-7)}{(49z^2-7z+1)^2}\right. +$$

$$\left. + z\cdot\frac{(28z-1)(49z^2-7z+1)-(14z^2-1)(98z-7)}{(49z^2-7z+1)^2}\right] =$$

$$= -\frac{7\sqrt{2}}{4}\cdot z\left[\sqrt{3}\,\frac{-49z^2+1}{(49z^2-7z+1)^2}\right. +$$

$$\left. + \frac{\cancel{4\cdot 7^3 z^3}-4\cdot 7^2 z^2+4\cdot 7z-7^2z^2+7z-1-\cancel{4\cdot 7^3 z^3}+2\cdot 7^2 z^2+2\cdot 7^2 z-7}{(49z^2-7z+1)}\right] =$$

$$= -\frac{7\sqrt{2}}{4}z\cdot\frac{-49\sqrt{3}z^2+\sqrt{3}-147z^2+133z-8}{(49z^2-7z+1)} =$$

$$= \frac{7\sqrt{2}}{4}\cdot\frac{156z^3-133z^2+(8-\sqrt{3})z}{(49z^2-7z+1)}, \quad |z|>\frac{1}{7}$$

δ)

$$x_4[n] = \begin{cases} 7^n, & n<0 \Rightarrow n\leq -1 \\ \left(\frac{1}{2}\right)^n, & n=0,2,4,\dots \\ \left(\frac{1}{3}\right)^n, & n=1,3,5,\dots \end{cases}$$

$$x_4[n] = 7^n u[-n-1] + \left(\frac{1}{2}\right)^n \frac{\cos(n\pi)+1}{2} + \left(\frac{1}{3}\right)^n\left[\frac{\cos(n\pi)+1}{2}-\cos(n\pi)\right] =$$

$$= 7^n u[-n-1] + \left(\frac{1}{2}\right)^n\cdot\frac{\cos(n\pi)+1}{2} + \left(\frac{1}{3}\right)^n\cdot\frac{1-\cos(n\pi)}{2} =$$

$$= -\left(-7^n u[-n-1]\right) + \left(\frac{1}{2}\right)^n \frac{\cos(n\pi)+1}{2} u[n] + \left(\frac{1}{3}\right)^n \cdot \frac{1-\cos(n\pi)}{2} u[n] =$$

$$= -\left(-7^n u[-n-1]\right) + \frac{1}{2}\cdot\left(\frac{1}{2}\right)^n \cos(n\pi) u[n] + \frac{1}{2}\left(\frac{1}{2}\right)^n u[n] +$$

$$+ \frac{1}{2}\left(\frac{1}{3}\right)^n u[n] - \frac{1}{2}\left(\frac{1}{3}\right)^n \cos(n\pi) u[n]$$

$$X_4(z) = -Z\left[-7^n u[-n-1]\right] + \frac{1}{2} Z\left[\left(\frac{1}{2}\right)^n \cos(\pi n) u[n]\right] + \frac{1}{2} Z\left[\left(\frac{1}{2}\right)^n u[n]\right] +$$

$$+ \frac{1}{2} Z\left[\left(\frac{1}{3}\right)^n u[n]\right] - \frac{1}{2} Z\left[\left(\frac{1}{3}\right)^n \cos(\pi n) u[n]\right] =$$

$$= -\frac{1}{1-7z^{-1}} + \frac{1}{2}\cdot \frac{1-\frac{1}{2}\cos\pi\cdot z^{-1}}{1-2\cdot\frac{1}{2}\cos\pi\cdot z^{-1}+\frac{1}{4}z^{-2}} + \frac{1}{2}\cdot\frac{1}{1-\frac{1}{2}z^{-1}} + \frac{1}{2}\cdot\frac{1}{1-\frac{1}{3}z^{-1}} -$$

$$-\frac{1}{2}\cdot\frac{1-\frac{1}{3}\cos\pi z^{-1}}{1-2\cdot\frac{1}{3}\cos\pi z^{-1}+\frac{1}{9}z^{-2}} = -\frac{z}{z-7} + \frac{1}{2}\cdot\frac{\frac{2z+1}{2z}}{1+\frac{1}{z}+\frac{1}{4z^2}} + \frac{1}{\cancel{2}}\cdot\frac{\cancel{2}z}{2z-1} +$$

$$+\frac{1}{2}\cdot\frac{3z}{3z-1} - \frac{1}{2}\cdot\frac{\frac{3z+1}{3z}}{1+\frac{2}{3z}+\frac{1}{9z^2}} =$$

$$= \frac{z}{7-z} + \frac{z(2z+1)}{4z^2+4z+1} + \frac{z}{2z-1} + \frac{1}{2}\cdot\frac{3z}{3z-1} - \frac{1}{2}\cdot\frac{3z(3z+1)}{9z^2+6z+1} =$$

$$= \frac{z}{7-z} + \frac{z}{2z+1} + \frac{z}{2z-1} + \frac{1}{2}\left(\frac{3z}{3z-1} - \frac{3z}{3z+1}\right) =$$

$$= \frac{z}{7-z} + z\cdot\frac{2z\cancel{-1}+2z\cancel{+1}}{4z^2-1} + \frac{3z}{\cancel{2}}\left(\frac{\cancel{3z}+1-\cancel{3z}+1}{9z^2-1}\right) =$$

$$= \frac{z}{7-z} + \frac{4z^2}{4z^2-1} + \frac{3z}{9z^2-1}, \quad |z|<7 \text{ και } |z|>\frac{1}{2} \text{ και } |z|>\frac{1}{3} \Rightarrow$$

$$\boxed{\frac{1}{2}<|z|<7}$$

2.7) $x[n] = \delta[n] + 2\delta[n-2]$

a) $X[k] = \sum_{n=0}^{5} x[n] e^{-\frac{j2\pi}{6} \cdot kn} =$

$= \sum_{n=0}^{4} e^{-j\frac{2\pi}{6}kn} \left(\delta[n] + 2\delta[n-2]\right) =$

$= 1 + 0 + 2e^{-j\frac{4\pi}{6}k} + 0 + 0 + 0 = 1 + 2e^{-j\frac{2\pi}{3}k}$

$X[0] = 1 + 2 = 3$

$X[1] = 1 + 2e^{-j\frac{2\pi}{3}} = 1 + 2\left(-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right) = -j\sqrt{3}$

$X[2] = 1 + 2e^{-j\frac{4\pi}{3}} = 1 + 2\left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right) = j\sqrt{3}$

$X[3] = 1 + 2e^{-j2\pi} = 1 + 2 = 3$

$X[4] = 1 + 2e^{-j\frac{8\pi}{3}} = 1 + 2e^{-j\frac{2\pi}{3}} = -j\sqrt{3}$

$X[5] = 1 + 2e^{-j\frac{10\pi}{3}} = 1 + 2e^{-j\frac{4\pi}{3}} = j\sqrt{3}$

b) $Z[k] = e^{jk\frac{2\pi}{6}} \cdot X[k] = e^{jk\frac{2\pi}{6}} \left[1 + 2e^{-j\frac{4\pi}{6}k}\right] =$

$= e^{j\frac{2\pi k}{6}} + 2e^{-j\frac{2\pi k}{6}}$

$z[n] = \frac{1}{N} \sum_{k=0}^{N-1} Z[k] e^{j\frac{2\pi}{N}kn} =$

$= \frac{1}{6} \sum_{k=0}^{5} \left[e^{j\frac{2\pi k}{6}} + 2e^{-j\frac{2\pi k}{6}}\right] e^{j\frac{2\pi k}{6}n} =$

$$= \frac{1}{6}\sum_{k=0}^{5}\left[e^{j\frac{\pi k}{3}} + 2e^{-j\frac{\pi k}{3}}\right]e^{j\frac{\pi k}{3}n} =$$

$$= \frac{1}{6}\Big[3 + \left(e^{j\frac{\pi}{3}} + 2e^{-j\frac{\pi}{3}}\right)e^{j\frac{\pi}{3}n} + \left(e^{j\frac{2\pi}{3}} + 2e^{-j\frac{2\pi}{3}}\right)e^{j\frac{2\pi}{3}n} +$$

$$+ \left(e^{j\pi} + 2e^{-j\pi}\right)e^{j\pi n} + \left(e^{j\frac{4\pi}{3}} + 2e^{-j\frac{4\pi}{3}}\right)e^{j\frac{4\pi}{3}n} + \left(e^{j\frac{5\pi}{3}} + 2e^{-j\frac{5\pi}{3}}\right)e^{j\frac{5\pi}{3}n}\Big] =$$

$$= \frac{1}{6}\Big[3 + \left(\frac{1}{2} + j\frac{\sqrt{3}}{2} + 2\cdot\frac{1}{2} - 2\cdot j\frac{\sqrt{3}}{2}\right)e^{j\frac{\pi}{3}n} + \left(-\frac{1}{2} + j\frac{\sqrt{3}}{2} - 2\cdot\frac{1}{2} - j2\cdot\frac{\sqrt{3}}{2}\right)e^{j\frac{2\pi}{3}n} +$$

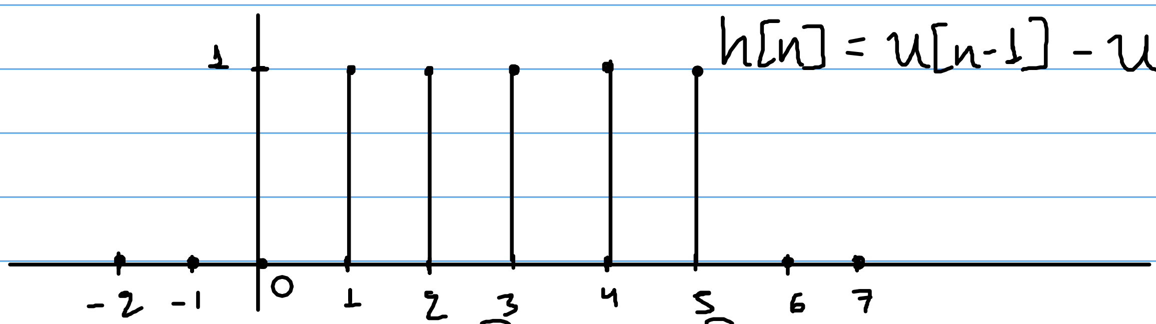
$$+ (-1-2)(-1)^n + \left(-\frac{1}{2} - j\frac{\sqrt{3}}{2} - 2\cdot\frac{1}{2} + j\frac{\sqrt{3}}{2}\cdot 2\right)e^{j\frac{4\pi}{3}n} + \left(\frac{1}{2} - j\frac{\sqrt{3}}{2} + 2\cdot\frac{1}{2} + 2j\frac{\sqrt{3}}{2}\right)e^{j\frac{5\pi}{3}n}\Big] =$$

$$= \frac{1}{6}\Big[3\left(1+(-1)^{n+1}\right) + \left(\frac{3}{2} - j\frac{\sqrt{3}}{2}\right)e^{j\frac{\pi}{3}n} - \left(\frac{3}{2} + j\frac{\sqrt{3}}{2}\right)e^{j\frac{2\pi}{3}n} + \left(\frac{3}{2} - j\frac{\sqrt{3}}{2}\right)(-1)^{n+1}e^{j\frac{\pi}{3}n} +$$

$$- \left(\frac{3}{2} + j\frac{\sqrt{3}}{2}\right)(-1)^{n+1}e^{j\frac{2\pi}{3}n}\Big] = \frac{1}{6}\Big[3\left(1+(-1)^{n+1}\right) + \left(\frac{3}{2} - j\frac{\sqrt{3}}{2}\right)e^{j\frac{\pi}{3}n}\left(1+(-1)^n\right) -$$

$$- \left(\frac{3}{2} + j\frac{\sqrt{3}}{2}\right)e^{j\frac{2\pi}{3}n}\left(1+(-1)^{n+1}\right)\Big] = \frac{\left(1+(-1)^{n+1}\right)}{6}\Big[3 + \left(\frac{3}{2} - j\frac{\sqrt{3}}{2}\right)e^{j\frac{\pi}{3}n} - \left(\frac{3}{2} + j\frac{\sqrt{3}}{2}\right)e^{j\frac{2\pi}{3}n}\Big]$$

γ)



$h[n] = u[n-1] - u[n-6]$

$$h[n] = \delta[n-1] + \delta[n-2] + \delta[n-3] + \delta[n-4] + \delta[n-5]$$

$$H[k] = \sum_{n=0}^{N-1} h[n]e^{-\frac{2j\pi}{N}kn} =$$

$$= \sum_{n=0}^{5}\left(\delta[n-1] + \delta[n-2] + \delta[n-3] + \delta[n-4] + \delta[n-5]\right)e^{-\frac{j\pi}{3}kn} =$$

$$= e^{-j\frac{\pi k}{3}} + e^{-j\frac{2\pi}{3}k} + e^{-j\pi k} + e^{-j\frac{4\pi}{3}k} + e^{-j\frac{5\pi}{3}k} =$$

$$H[0] = 5 \quad , \quad H[1] = e^{-j\frac{\pi}{3}} + e^{j\frac{\pi}{3}} + e^{-j\frac{2\pi}{3}} + e^{j\frac{2\pi}{3}} + e^{-j\pi} = 1 - 1 - 1 = -1$$

$$H[2] = e^{-j\frac{2\pi}{3}} - e^{-j\frac{\pi}{3}} + e^{-j2\pi} + e^{-j\frac{2\pi}{3}} - e^{-j\frac{\pi}{3}} = 2\left(e^{-j\frac{2\pi}{3}} - e^{-j\frac{\pi}{3}}\right) + 1 = 2\left(-\frac{1}{2} - j\frac{\cancel{\sqrt{3}}}{2} - \frac{1}{2} + j\frac{\cancel{\sqrt{3}}}{2}\right) + 1 = -1$$

$$H[3] = e^{-j\pi} + e^{-j2\pi} + e^{-j3\pi} + e^{-j4\pi} + e^{-j5\pi} = \cancel{-1} + \cancel{1} - \cancel{1} + \cancel{1} - 1 = -1$$

$$H[4] = -e^{-j\frac{\pi}{3}} + e^{-j\frac{2\pi}{3}} + e^{-j4\pi} - e^{-j\frac{\pi}{3}} + e^{-j\frac{2\pi}{3}} = 2\left(e^{-j\frac{2\pi}{3}} - e^{-j\frac{\pi}{3}}\right) + 1 = -1$$

$$H[5] = e^{j\frac{2\pi}{3}} + e^{j\frac{\pi}{3}} + e^{-j5\pi} + e^{-j\frac{2\pi}{3}} + {}^{-j\frac{\pi}{3}} = \cancel{2\cdot\frac{1}{2}} - \cancel{2\cdot\frac{1}{2}} - 1 = -1$$

$$F[k] = X[k] \cdot H[k]$$

$$F[0] = 3 \cdot 5 = 15 \qquad F[3] = 3 \cdot (-1) = -3$$

$$F[1] = -j\sqrt{3} \cdot (-1) = j\sqrt{3} \qquad F[4] = -j\sqrt{3} \cdot (-1) = j\sqrt{3}$$

$$F[2] = j\sqrt{3} \cdot (-1) = -j\sqrt{3} \qquad F[5] = j\sqrt{3} \cdot (-1) = -j\sqrt{3}$$

$$f[n] = \frac{1}{N}\sum_{k=0}^{N-1} F[k] e^{j\frac{2\pi}{N}kn} = \frac{1}{6}\sum_{k=0}^{5} F[k] e^{j\frac{\pi}{3}kn} =$$

$$= \frac{1}{6}\left[15 + j\sqrt{3}e^{j\frac{\pi}{3}n} - j\sqrt{3}e^{j\frac{2\pi}{3}n} - 3e^{j\pi n} + j\sqrt{3}e^{j\frac{4\pi}{3}n} - j\sqrt{3}e^{j\frac{5\pi}{3}n}\right] =$$

$$= \frac{1}{6}\left[15 + 3(-1)^{n+1} + j\sqrt{3}\left(e^{j\frac{\pi}{3}n} + e^{j\pi n}e^{j\frac{\pi}{3}n} - e^{j\frac{2\pi}{3}n} - e^{j\pi n}e^{j\frac{2\pi}{3}n}\right)\right] =$$

$$= \frac{1}{6}\left[15 + 3(-1)^{n+1} + j\sqrt{3}\left(e^{j\frac{\pi}{3}n}\left(1 + (-1)^n\right) - e^{j\frac{2\pi}{3}n}\left(1 + (-1)^n\right)\right)\right] =$$

$$= \frac{1}{6}\left[3\left(5 + (-1)^{n+1}\right) + j\sqrt{3}\left(1 + (-1)^n\right)\left(e^{j\frac{\pi}{3}n} - e^{j\frac{2\pi}{3}n}\right)\right]$$

$$f[0] = \frac{1}{6}\left[3 \cdot 4 + j2\sqrt{3} \cdot (\cancel{1} - \cancel{1})\right] = 2$$

$$f[1] = \frac{1}{6}\left[3 \cdot 6 + j\sqrt{3} \cdot 0\right] = 3$$

$$f[2] = \frac{1}{6}\left[3\cdot 4 + j2\sqrt{3}\left(e^{j\frac{2\pi}{3}} - e^{j\frac{4\pi}{3}}\right)\right] = 2 + j\frac{\sqrt{3}}{3}\left(\cancel{-\frac{1}{2}} + j\frac{\sqrt{3}}{2} \cancel{+\frac{1}{2}} + j\frac{\sqrt{3}}{2}\right) =$$

$$= 2 - \frac{\sqrt{3}\cdot\sqrt{3}}{3} = 1$$

$$f[3] = \frac{1}{6}\left[3\cdot 6 + j\sqrt{3}\cdot 0\right] = 3$$

$$f[4] = \frac{1}{6}\left[3\cdot 4 + j2\sqrt{3}\left(e^{j\frac{4\pi}{3}} - e^{j\frac{8\pi}{3}}\right)\right] = 2 + j\frac{\sqrt{3}}{3}\left(\cancel{-\frac{1}{2}} - j\frac{\sqrt{3}}{2} \cancel{+\frac{1}{2}} - j\frac{\sqrt{3}}{2}\right) =$$

$$= 2 + \frac{\sqrt{3}\cdot\sqrt{3}}{3} = 3$$

$$f[5] = \frac{1}{6}\left[3\cdot 6 + j\sqrt{3}\cdot 0\right] = 3$$

δ) $G[k] = X[k]H[k] \Rightarrow g[n] = x[n] \circledast h[n]$

$$g[n] = x[n] \circledast h[n] = \sum_{m=0}^{N-1} x[m]\, h\big([n-m]\big)_N =$$

$$= \sum_{m=0}^{7} x[m]\, h\big([n-m]\big)_8 =$$

$$h[n] = \begin{cases} 1, & n = 1,2,3,4,5 \\ 0, & \text{αλλού} \end{cases}$$

$$g[0] = \sum_{m=0}^{7} x[m]h[-m] = 0$$

$$g[1] = \sum_{m=0}^{7} x[m]h[1-m] = x[0] = 1$$

$$g[2] = \sum_{m=0}^{7} x[m]h[2-m] = x[0] + x[1] = 1 + 0 = 1$$

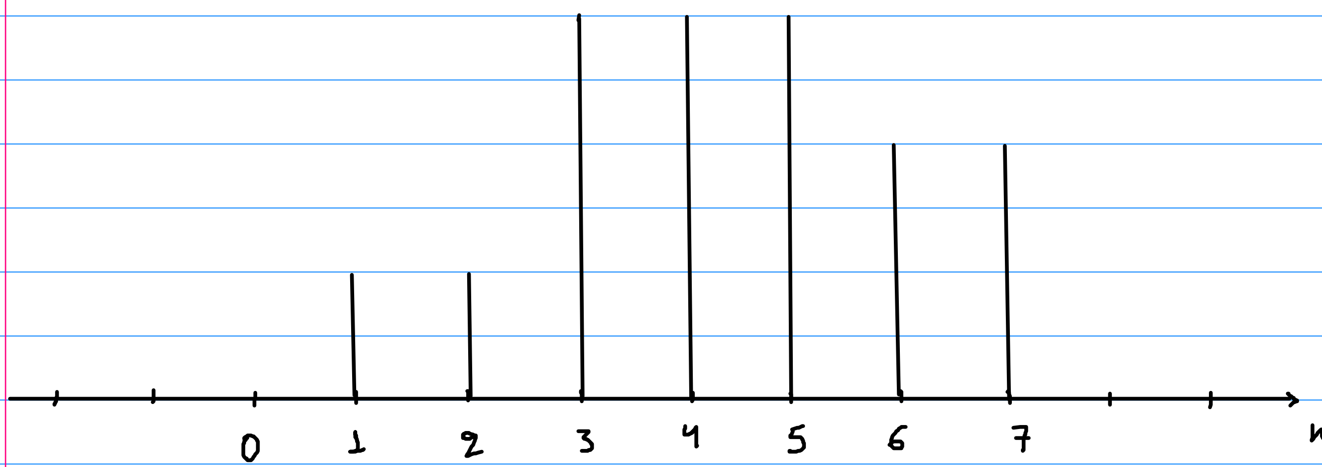
$$g[3] = \sum_{m=0}^{7} x[m]h[3-m] = x[0] + x[1] + x[2] = 1 + 0 + 2 = 3$$

$$g[4] = \sum_{m=0}^{7} x[m]h[4-m] = X[0]+X[1]+X[2]+X[3] = 1+0+2+0 = 3$$

$$g[5] = \sum_{m=0}^{7} x[m]h[5-m] = X[0]+\cancel{X[1]}+X[2]+\cancel{X[3]}+\cancel{X[4]} = 3$$

$$g[6] = \sum_{m=0}^{7} x[m]h[6-m] = \cancel{X[1]}+X[2]+\cancel{X[3]}+\cancel{X[4]}+\cancel{X[5]} = 2$$

$$g[7] = \sum_{m=0}^{7} x[m]h[7-m] = X[2]+\cancel{X[3]}+\cancel{X[4]}+X[5]+\cancel{X[6]} = 2$$



ε) $y[n] = x[n]*h[n] = \left(\delta[n]+2\delta[n-2]\right)*\left(u[n-1]-u[n-6]\right) =$

$= \delta[n]*u[n-1] - \delta[n]*u[n-6] + 2\delta[n-2]u[n-1] - 2\delta[n-2]u[n-6] =$

$= u[n-1] - u[n-6] + 2u[n-3] - 2u[n-8] =$

$= \delta[n-1]+\delta[n-2]+\delta[n-3]+\delta[n-4]+\delta[n-5]+2\big(\delta[n-3]+\delta[n-4]+$
$+\delta[n-5]+\delta[n-6]+\delta[n-7]\big) =$

$= \delta[n-1]+\delta[n-2]+3\delta[n-3]+3\delta[n-4]+3\delta[n-5]+2\delta[n-6]+2\delta[n-7]$

$y[n] \equiv g[n] \neq f[n]$