$\underline{2\eta \quad \Sigma\epsilon\iota\rho\acute{a} \ A\sigma\kappa\acute{\eta}\sigma\epsilon\omega\nu}$

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$\alpha = AM_{mod}10 + 1 = 6 + 1 = 7$

2.1) $x_1(t) = \sin(20\alpha\pi t)\dfrac{\cos(300\alpha t)}{\pi t} = \sin(140\pi t)\dfrac{\cos(300\pi t)}{\pi t}$
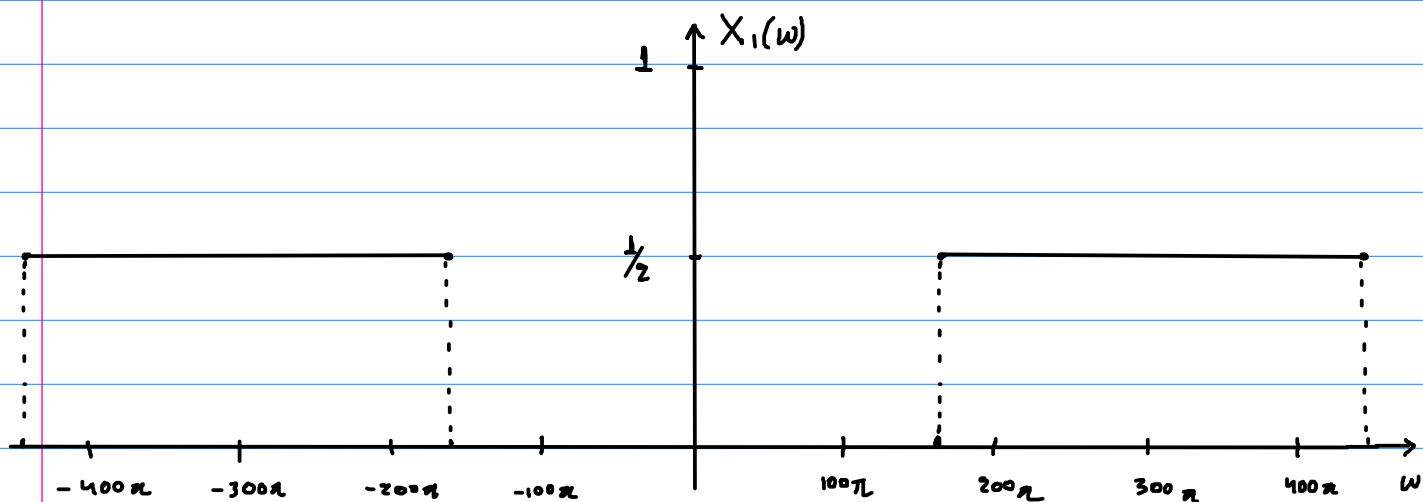
$x_2(t) = 2\dfrac{\sin(500\alpha t)}{\pi t}$

$x_1(t) = 2\sin(140\pi t)\cos(300\alpha t) \cdot \dfrac{1}{2\pi t} =$

$= \left[\sin\left(140\pi t + 300\alpha t\right) + \sin\left(140\alpha t - 300\alpha t\right)\right] \cdot \dfrac{1}{2\pi t} =$

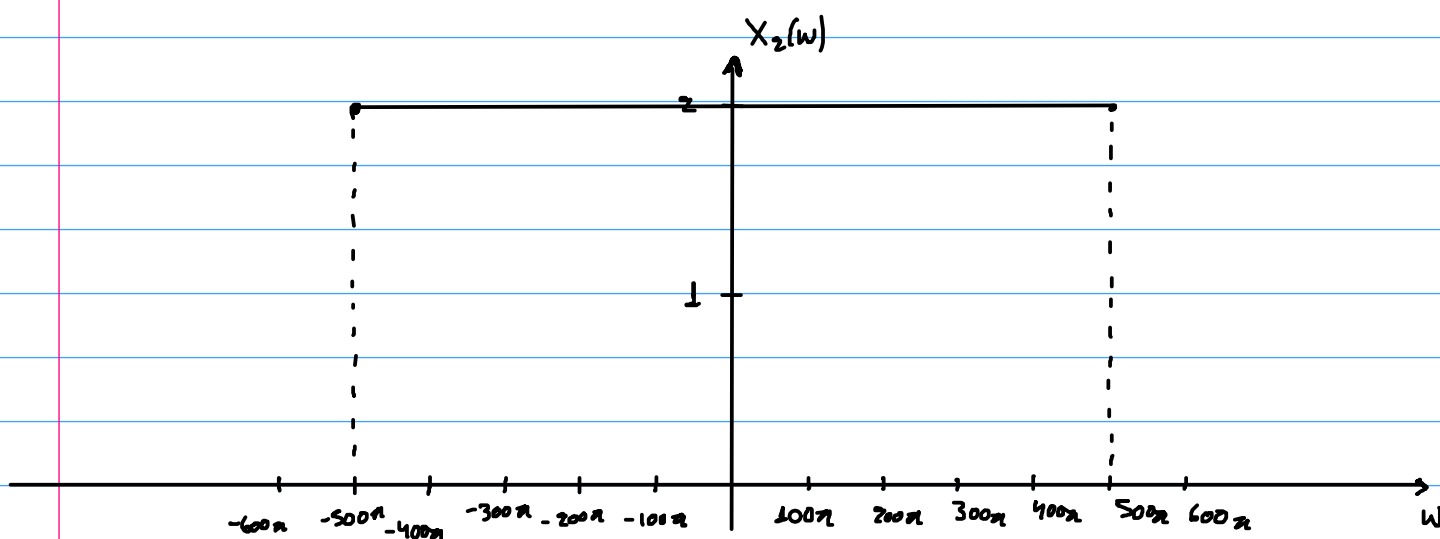$= \dfrac{\sin 440\pi t}{2\pi t} - \dfrac{\sin 160\pi t}{2\pi t}$

$X_1(\omega) = F[x_1(t)] = \dfrac{1}{2}F\left[\dfrac{\sin 440\alpha t}{\pi t}\right] - \dfrac{1}{2}F\left[\dfrac{\sin 160\alpha t}{\pi t}\right]$

$X_1(\omega) = \begin{cases} 0, & \omega \in (-\infty, -440\pi) \cup (-160\pi, 160\pi) \cup (440\alpha, +\infty) \\[4pt] \dfrac{1}{2}, & \omega \in (-440\alpha, -160\alpha) \cup (160\alpha, 440\alpha) \end{cases}$

$$X_2(\omega) = F[x_2(t)] = 2F\left[\frac{\sin(500\pi t)}{\pi t}\right] =$$

$$= \begin{cases} 0, & \omega \in (-\infty, -500\pi) \cup (500\pi, +\infty) \\ 2, & -500\pi < \omega < 500\pi \end{cases}$$
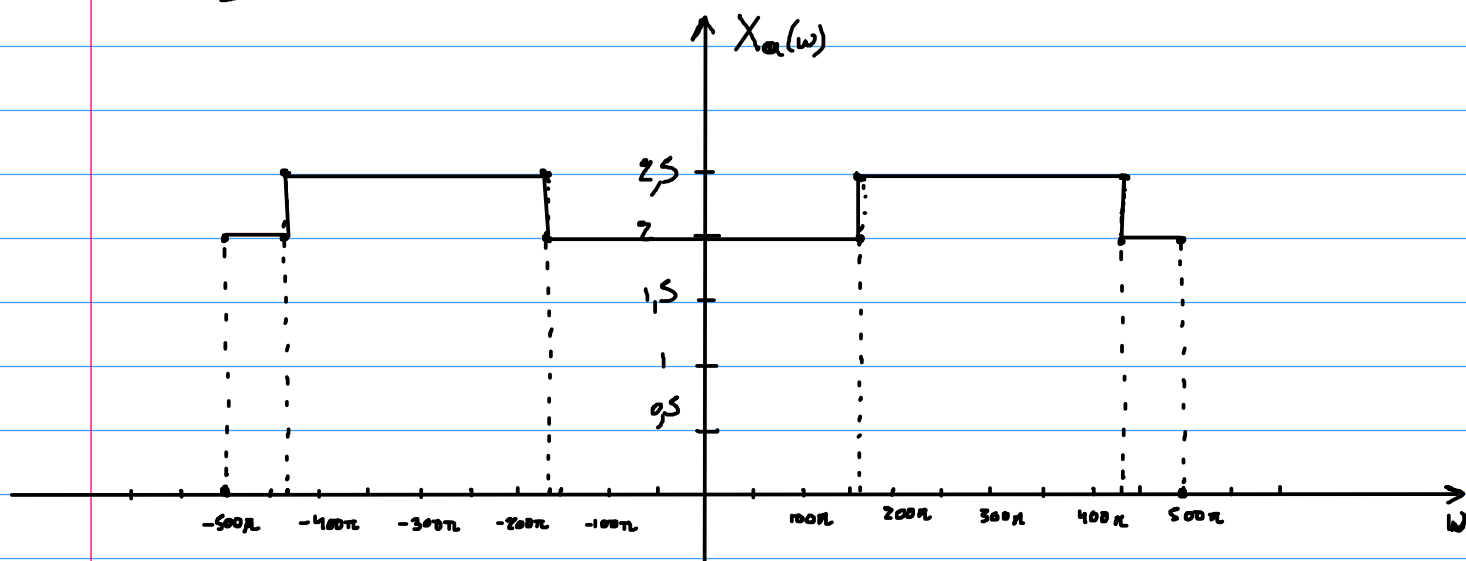
a) $x_1(t) + x_2(t)$

$X_a(\omega) = F[x_1(t) + x_2(t)] = F[x_1(t)] + F[x_2(t)] =$

$= X_1(\omega) + X_2(\omega) =$

$$\begin{cases} 0, & \omega \in (-\infty, -500\pi) \cup (500\pi, +\infty) \\ 2, & \omega \in (-500\pi, -440\pi) \cup (-160\pi, 160\pi) \cup (440\pi, 500\pi) \\ \dfrac{5}{2}, & \omega \in (-440\pi, -160\pi) \cup (160\pi, 440\pi) \end{cases}$$

$\omega = 1000\pi \Rightarrow \omega_s > 2\omega \Rightarrow \dfrac{2\pi}{T_s} > 2\omega \Rightarrow T_s < \dfrac{\pi}{\omega} = \dfrac{\pi}{1000\pi} = 10^{-3} \, s$

b) $x_6(t) = x_1^3(t) = x_1(t) \cdot x_1^2(t)$

$x_1^2(t) \Rightarrow F[x_1^2(t)] = X_1(\omega) * X_1(\omega)$,

$W_{x^2} = W_{x_1} + W_{x_1} = 880\pi + 880\pi = 1760\pi$

$X_6(\omega) = X_1(\omega) * F[x_1^2(t)]$,

$$W = W_{x_1} + W_{x_2} = 880\pi + 1760\pi = 2640\pi$$

$$W_s > 2W \Rightarrow \frac{2\pi}{T_s} > 2W \Rightarrow T_s < \frac{2640\pi}{\pi} = 2640s$$

$\gamma)$ $x_\gamma(t) = x_1(t) + x_2(t)$

$X_\gamma(w) = X_1(w) \cdot X_2(w)$

$W = \min\{w_1, w_2\} = \{880\pi, 1000\pi\} = 880\pi$

$$W_s > 2W \Rightarrow \frac{2\pi}{T_s} > 2 \cdot 880\pi \Rightarrow T_s < 880s$$

$\delta)$ $x_\delta(t) = x_1(t) \cdot [x_2(t)]^2 + [x_1(t)]^2 * x_2(t)$

$$X_\delta(w) = F\left[x_1(t) \cdot [x_2(t)]^2\right] + F\left[[x_1(t)]^2 * x_2(t)\right]$$

$$W = \max\left\{w_1 + 2w_2, \min\{2w_1, w_2\}\right\} =$$

$$= \max\left\{880\pi + 2000\pi, \min\{1760\pi, 1000\pi\}\right\} =$$

$$= \max\left\{2880\pi, 1000\pi\right\} = 2880\pi$$

$$W_s > 2W \Rightarrow \frac{2\pi}{T_s} > 2W \Rightarrow T_s < 2880s$$

2.3) $W_S = (2 + 7 \mod 3) W_M = (2 + 1) W_M = 3 W_M$

a) $y_d[n] = \dfrac{x[n-1] - 2x[n] + x[n+1]}{4} = \dfrac{1}{4} x[n-1] - \dfrac{1}{2} x[n] + \dfrac{1}{4} x[n+1]$

$$Y_d[\Omega] = \dfrac{1}{4} e^{-j\Omega} X_d[\Omega] - \dfrac{1}{2} X_d[\Omega] + \dfrac{1}{4} e^{j\Omega} X_d[\Omega] \Rightarrow$$

$$\dfrac{Y_d[\Omega]}{X_d[\Omega]} = \dfrac{1}{2}\left(\dfrac{e^{-j\Omega} + e^{j\Omega}}{2} - 1 \right) = \dfrac{1}{2}\left(\cos\Omega - 1 \right)$$

$$H_d[\Omega] = \dfrac{1}{2}\left(\cos\Omega - 1 \right)$$

$$h_d[n] = F^{-1}[H_d[\Omega]] = \dfrac{1}{4} F^{-1}\left[e^{-j\Omega} + e^{j\Omega} \right] - \dfrac{1}{2} F^{-1}[1] =$$

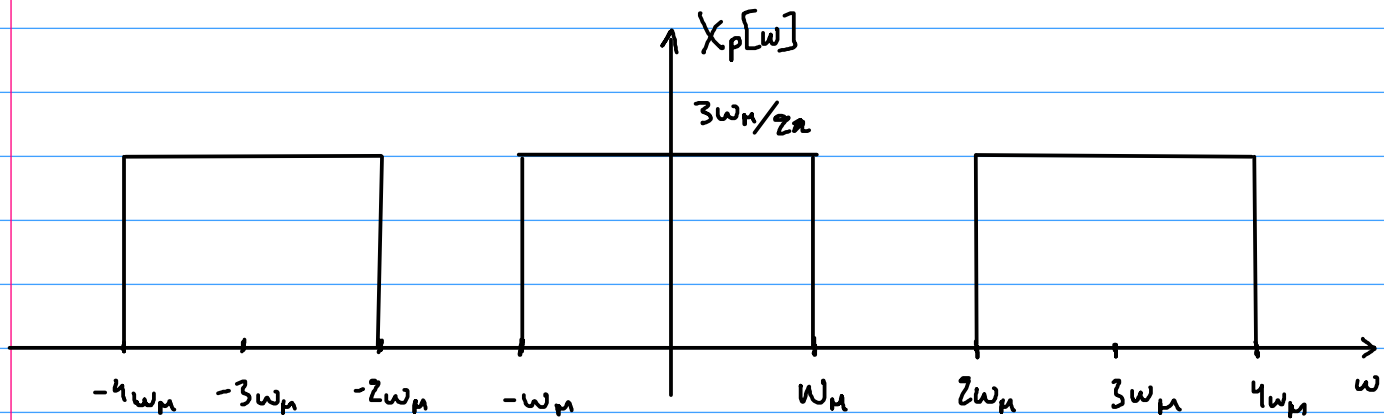$$= \dfrac{1}{4} \delta[n-1] + \dfrac{1}{4} \delta[n+1] - \dfrac{1}{2} \delta[n] =$$

$$= \dfrac{\delta[n-1] - 2\delta[n] + \delta[n+1]}{4}$$

b) $x_c(t) = \dfrac{\sin(W_M t)}{\pi t}$, $\quad W_S = \dfrac{2a}{T_S} \Rightarrow T_S = \dfrac{2a}{W_S} = \dfrac{2a}{3W_M}$
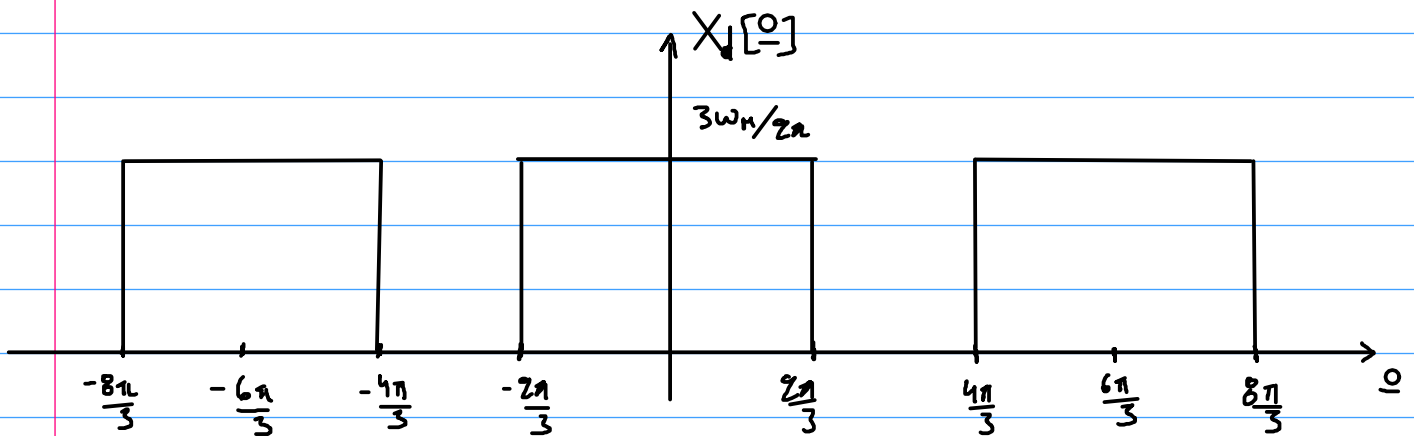
$$x_p(t) = x_c(t) \cdot P(t) = \dfrac{\sin(W_M t)}{\pi t} \sum_n \delta(t - nT)$$

$$X_p[W] = X_c(w) * F\left[\sum_n \delta(t - nT_S) \right] = X_c * \left[\dfrac{1}{T_S} \sum_{m=-\infty}^{+\infty} \delta(w - mw_s) \right] =$$
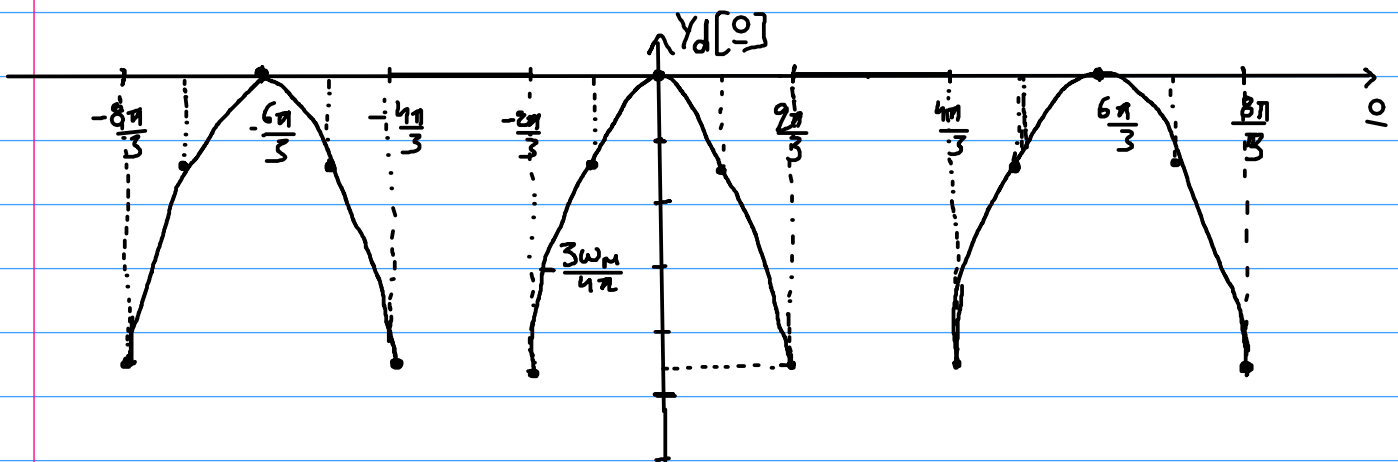
$$= \dfrac{1}{T_S} \sum_{m=-\infty}^{+\infty} X_c(w - mw_s) = \dfrac{3W_M}{2a} \sum_{m=-\infty}^{+\infty} X_c(w - mw_s)$$

$$X_d\left(\underline{O}\right) = X_p\left(\frac{O}{T_s}\right) = X_p\left(\frac{3w_m}{2\pi} \cdot \underline{O}\right)$$

$$Y_d[\underline{O}] = \frac{1}{4} e^{j\underline{O}} X_d[\underline{O}] - \frac{1}{2} X_d[\underline{O}] + \frac{1}{4} e^{-j\underline{O}} X_d[\underline{O}] =$$

$$= \frac{X_d[\underline{O}]}{2}\left(\frac{e^{-j\underline{O}} + e^{j\underline{O}}}{2} - 1\right) = \frac{X_d[\underline{O}]}{2}\left(\cos\underline{O} - 1\right)$$

$$Y_d[\underline{0}] = Y_P\left(\frac{0}{T_s}\right) \Rightarrow Y_P(\omega) = Y_d[\omega T_s] = Y_d\left[\omega \cdot \frac{2\pi}{3\omega_M}\right]$$
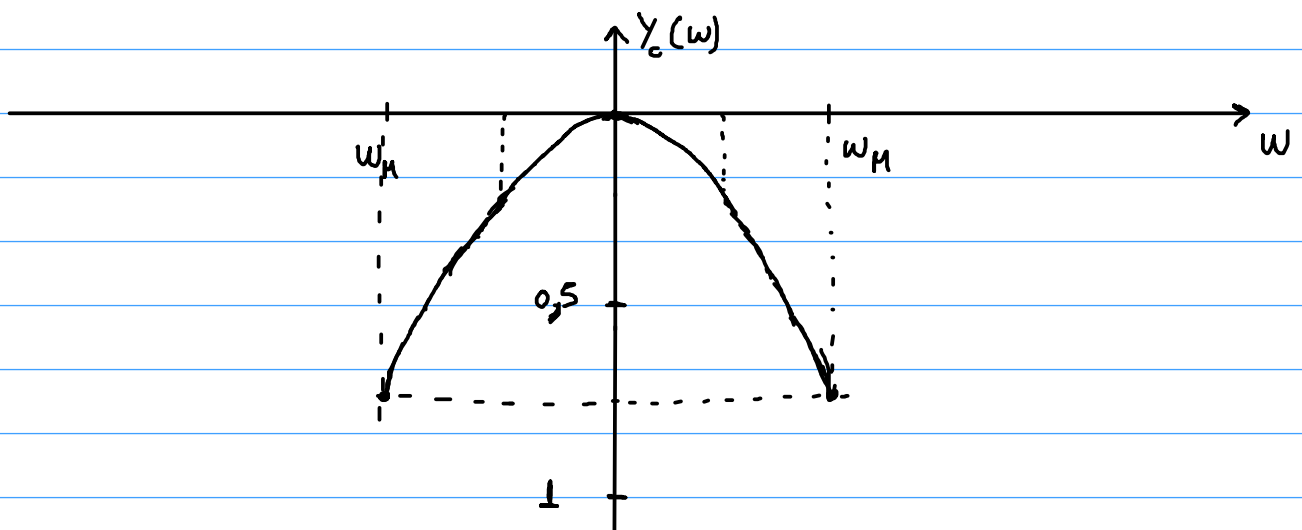

$$H_r(\omega) = \begin{cases} T & , \ |\omega| < \frac{\omega_s}{2} = \frac{3\omega_M}{2} \\ 0 & , \ |\omega| > \frac{\omega_s}{2} = \frac{3\omega_M}{2} \end{cases}$$

$$Y_c(\omega) = Y_P(\omega) \cdot H_r(\omega) = \begin{cases} \frac{2\pi}{3\omega_M} \cdot Y_P(\omega) & , \ |\omega| < \frac{3\omega_M}{2} \\ 0 & , \ |\omega| > \frac{3\omega_M}{2} \end{cases}$$

$\gamma)$
$$H_c(\omega) = \frac{Y_c(\omega)}{X_c(\omega)} = \frac{Y_P(\omega) \cdot H_r(\omega)}{X_c(\omega)} = \frac{Y_d(\omega T_s) H_r(\omega)}{X_c(\omega)} =$$

$$= \frac{X_d(\omega T_s) H_d(\omega T_s) H_r(\omega)}{X_c(\omega)} = \frac{X_P(\omega) H_d(\omega T_s) H_r(\omega)}{X_c(\omega)} = \frac{H_d(\omega T_s) \cdot X_c(\omega)}{X_c(\omega)} = H_d(\omega T_s)$$

Επομένως $\quad H_c(\omega) = \begin{cases} H_d(\omega T_s) & , |\omega| \le \dfrac{\pi}{T_s} = \dfrac{\pi}{\frac{2\pi}{3\omega_M}} = \dfrac{3\omega_M}{2} \\ \\ 0 & , |\omega| > \dfrac{3\omega_M}{2} \end{cases} \Rightarrow$

$$H_c(\omega) = \begin{cases} \dfrac{1}{2}\left(\cos\left(\dfrac{2\pi}{3}\cdot\dfrac{\omega}{\omega_M}\right) - 1\right) & , |\omega| \le \dfrac{3\omega_M}{2} \\ \\ 0 & , |\omega| > \dfrac{3\omega_M}{2} \end{cases}$$

2.4) a) $X_1[n] = 7^n \sin[\pi(n+1)/3]\cos[\pi(n+1)/6]u[n] =$

$= \dfrac{7^n}{2}\cdot\left(\sin\dfrac{\pi}{2}(n+1) + \sin\dfrac{\pi}{6}(n+1)\right)u[n] =$

$= \dfrac{7^n}{2}\left[\dfrac{e^{j\frac{\pi}{2}(n+1)} - e^{-j\frac{\pi}{2}(n+1)}}{2j} + \dfrac{e^{j\frac{\pi}{6}(n+1)} - e^{-j\frac{\pi}{6}(n+1)}}{2j}\right]u[n] =$

$= \dfrac{7^n}{4}\left(\dfrac{je^{j\frac{\pi}{2}n} + je^{-j\frac{\pi}{2}n}}{j} + \dfrac{\left(\frac{\sqrt3}{2}+j\frac{1}{2}\right)e^{j\frac{\pi}{6}n} - \left(\frac{\sqrt3}{2}-j\frac{1}{2}\right)e^{-j\frac{\pi}{6}n}}{j}\right)u[n] =$

$= \dfrac{7^n}{4}\left(e^{j\frac{\pi}{2}n} + e^{-j\frac{\pi}{2}n} + \dfrac{\sqrt3}{2j}\left(e^{j\frac{\pi}{6}n} - e^{-j\frac{\pi}{6}n}\right) + \dfrac{1}{2}\left(e^{j\frac{\pi}{6}n} + e^{-j\frac{\pi}{6}n}\right)\right)u[n]$

$= \dfrac{7^n}{4}\left(2\cos\dfrac{\pi}{2}n + \sqrt3\sin\dfrac{\pi}{6}n + \cos\dfrac{\pi}{6}n\right)u[n] =$

$= \dfrac{1}{2}7^n\cos\left(\dfrac{\pi}{2}n\right)u[n] + \dfrac{\sqrt3}{4}7^n\sin\left(\dfrac{\pi}{6}n\right)u[n] + \dfrac{1}{4}7^n\cos\left(\dfrac{\pi}{6}n\right)u[n]$

$X_1(z) = \dfrac{1}{2}\cdot\dfrac{1 - \left[7\cos\frac{\pi}{2}\right]z^{-1}}{1 - 2\left[7\cos\frac{\pi}{2}\right]z^{-1} + 7^2z^{-2}} + \dfrac{\sqrt3}{4}\cdot\dfrac{\left[7\sin\frac{\pi}{6}\right]z^{-1}}{1 - \left[2\cdot7\cos\frac{\pi}{6}\right]z^{-1} + 7^2z^{-2}} +$

$$+\frac{1}{4}\cdot\frac{1-\left[7\cos\frac{\pi}{6}\right]z^{-1}}{1-\left[2\cdot7\cos\frac{\pi}{6}\right]z^{-1}+7^2z^{-2}}=\frac{1}{2}\cdot\frac{1}{1+\left(\frac{7}{z}\right)^2}+\frac{\sqrt{3}}{4z}\cdot\frac{\frac{7}{2z}}{1-7\sqrt{3}\,z^{-1}+\left(\frac{7}{z}\right)^2}+$$

$$+\frac{1}{4}\cdot\frac{1-\frac{\sqrt{3}}{2z}}{1-\frac{7\sqrt{3}}{z}+\left(\frac{7}{z}\right)^2}=\frac{z^2}{2(z^2+7^2)}+\frac{\sqrt{3}}{2}\cdot\frac{7z}{z^2-7\sqrt{3}z+7^2}+\frac{1}{4}\cdot\frac{\frac{2z-7\sqrt{3}}{2z}}{\frac{z^2-7\sqrt{3}z+7^2}{z^2}}=$$

$$=\frac{z^2}{2(z^2+7^2)}+\frac{\sqrt{3}}{2}\cdot\frac{7z}{z^2-z\sqrt{3}+7^2}+\frac{1}{2}\cdot\frac{z(2z-7\sqrt{3})}{z^2-7z\sqrt{3}+7^2}=$$

$$=\frac{z^2}{2(z^2+7^2)}+\frac{7z\sqrt{3}+2z^2-7z\sqrt{3}}{2(z^2-7z\sqrt{3}+7^2)}=z^2\cdot\frac{z^2+7^2-7z\sqrt{3}+2z^2+2\cdot7^2}{2(z^2+7^2)(z^2-7z\sqrt{3}+7^2)}=$$

$$=z^2\cdot\frac{3z^2-7\cdot z\sqrt{3}+3\cdot7^2}{2(z^2+7)(z^2-7z\sqrt{3}+7^2)}=z^2\cdot\frac{3z^2-7\sqrt{3}z+147}{2(z^2+49)(z^2-7\sqrt{3}z+49)}\,,\quad |z|>7$$

b) $x_2[n]=n7^{-|n|}=n7^{-n}u[n]+n7^nu[-n]$

$$X_2(z)=Z[x_2[n]]=Z[n7^{-n}u[n]]+Z[n7^nu[-n]]=$$

$$=Z\left[n\left(\frac{1}{7}\right)^nu[n]\right]-Z\left[-n\left(\frac{1}{7}\right)^{-n}u[-n]\right]=$$

$$\underbrace{\qquad\qquad}_{\text{Χρονική αναόζροφή}}$$

$$=\frac{\frac{1}{7}\cdot z^{-1}}{\left(1-\frac{1}{7}z^{-1}\right)^2}-\frac{\frac{1}{7}(z^{-1})^{-1}}{\left(1-\frac{1}{7}(z^{-1})^{-1}\right)^2}=\frac{\frac{1}{7z}}{\left(1-\frac{1}{7z}\right)^2}-\frac{\frac{z}{7}}{\left(1-\frac{z}{7}\right)^2}=$$

$$=\frac{\frac{1}{7z}}{\frac{(7z-1)^2}{(7z)^2}}-\frac{\frac{z}{7}}{\frac{(7-z)^2}{7^2}}=\frac{7z}{(7z-1)^2}-\frac{7z}{(7-z)^2}=$$

$$=7z\cdot\frac{(7-z-7z+1)(7-z+7z-1)}{(7-z)^2(7z-1)^2}=7z\cdot\frac{(7+1)(1-z)(7-1)(1+z)}{(7-z)^2(7z-1)^2}=7z\cdot\frac{48(1-z^2)}{(7-z)^2(7z-1)^2}=$$

$$= \frac{336 z (1-z^2)}{(7-z)^2 (7z-1)^2}, \; |z| > \frac{1}{7}, \; \left|\frac{1}{z}\right| > \frac{1}{7} \Rightarrow |z| < 7$$

Apa $\frac{1}{7} < |z| < 7$

γ) $x_3[n] = \frac{n}{7^n} \sin\left[\frac{\pi n}{3} + \frac{\pi}{4}\right] u[n] =$

$$= \frac{n}{7^n}\left[\sin\left(\frac{\pi n}{3}\right)\cos\left(\frac{\pi}{4}\right) + \cos\left(\frac{\pi n}{3}\right)\sin\left(\frac{\pi}{4}\right)\right] u[n] =$$

$$= \frac{n}{7^n}\left[\frac{\sqrt{2}}{2}\sin\left(\frac{\pi n}{3}\right) + \frac{\sqrt{2}}{2}\cos\left(\frac{\pi n}{3}\right)\right] u[n] =$$

$$= \frac{\sqrt{2}}{2}\left[\frac{n}{7^n}\left(\sin\left(\frac{\pi n}{3}\right) + \cos\left(\frac{\pi n}{3}\right)\right)\right] u[n] =$$

$$= \frac{\sqrt{2}}{2}\left[n(7^{-1})^n \sin\left(\frac{\pi n}{3}\right) u[n] + n(7^{-1})^n \cos\left(\frac{\pi n}{3}\right) u[n]\right]$$

$$X_3(z) = \frac{\sqrt{2}}{2} Z\left[n(7^{-1})^n \sin\frac{\pi n}{3} \cdot u[n] + n(7^{-1})^n \cos\left(\frac{\pi n}{3}\right) u[n]\right] =$$

$$= \frac{\sqrt{2}}{2}\left[-z\frac{d Z\left[(7^{-1})^n \sin\frac{\pi n}{3} u[n]\right]}{dz} - z\frac{d Z\left[(7^{-1})^n \cos\left(\frac{\pi n}{3}\right) u[n]\right]}{dz}\right] =$$

$$= \frac{\sqrt{2}}{2}\left[-z\cdot\frac{d}{dz}\left(\frac{7^{-1}\sin\frac{\pi}{3} z^{-1}}{1 - 2\cdot 7^{-1}\cos\frac{\pi}{3} z^{-1} + 7^{-2}z^{-2}}\right) - z\frac{d}{dz}\left(\frac{1 - 7^{-1}\cos\frac{\pi}{3} z^{-1}}{1 - 2\cdot 7^{-1}\cos\frac{\pi}{3} z^{-1} + 7^{-2}z^{-2}}\right)\right] =$$

$$= \frac{\sqrt{2}}{2}\left[-z\frac{d}{dz}\left(\frac{\frac{\sqrt{3}}{2\cdot 7\cdot z}}{1 - \frac{1}{7z} + \frac{1}{7^2z^2}}\right) - z\frac{d}{dz}\left(\frac{1 - \frac{1}{2\cdot 7\cdot z}}{1 - \frac{1}{7z} + \frac{1}{7^2\cdot z^2}}\right)\right] =$$

$$= \frac{\sqrt{2}}{2}\left[-z\frac{d}{dz}\left(\frac{\sqrt{3}}{2}\cdot\frac{7z}{49z^2 - 7z + 1}\right) - z\frac{d}{dz}\left(\frac{\frac{14z-1}{2\cdot 7z}}{\frac{7^2z^2 - 7z + 1}{7^2z^2}}\right)\right] =$$

$$= \frac{\sqrt{2}}{2}\left[-z\frac{d}{dz}\left(\frac{\sqrt{3}}{2}\cdot\frac{7z}{49z^2-7z+1}\right) - z\frac{d}{dz}\left(\frac{1}{2}\cdot\frac{7z(14z-1)}{49z^2-7z+1}\right)\right] =$$

$$= -\frac{7\sqrt{2}}{4}\left[\sqrt{3}\,z\cdot\frac{49z^2-7z+1 -z(98z-7)}{(49z^2-7z+1)^2} \right. +$$

$$\left. + z\cdot\frac{(28z-1)(49z^2-7z+1)-(14z^2-1)(98z-7)}{(49z^2-7z+1)^2}\right] =$$

$$= -\frac{7\sqrt{2}}{4}\cdot z\left[\sqrt{3}\,\frac{-49z^2 +1}{(49z^2-7z+1)^2} \right. +$$

$$\left. + \frac{4\cdot7^3 z^3-4\cdot7^2 z^2+4\cdot7z-7^2z^2+7z-1 - 4\cdot7^3 z^3 +2\cdot7^2 z^2+2\cdot7^2 z -7}{(49z^2-7z+1)}\right] =$$

$$= -\frac{7\sqrt{2}}{4}z\cdot\frac{-49\sqrt{3}z^2+\sqrt{3} -147z^2+133z-8}{(49z^2-7z+1)} =$$

$$= \frac{7\sqrt{2}}{4}\cdot\frac{156z^3-133z^2 +(8-\sqrt{3})z}{(49z^2-7z+1)} \quad,\ |z|>\frac{1}{7}$$

б)
$$X_4[n] = \begin{cases} 7^n &,\ n<0 \Rightarrow n\leq -1 \\ \left(\frac{1}{2}\right)^n &,\ n=0,2,4,\ldots \\ \left(\frac{1}{3}\right)^n &,\ n=1,3,5,\ldots \end{cases}$$

$$X_4[n] = 7^n\, u[-n-1] + \left(\frac{1}{2}\right)^n\frac{\cos(n\pi)+1}{2}\,u[n] + \left(\frac{1}{3}\right)^n\left[\frac{\cos(n\pi)+1}{2} - \cos(n\pi)\right]u[n] =$$

$$= 7^n\, u[-n-1] + \left(\frac{1}{2}\right)^n\cdot\frac{\cos(n\pi)+1}{2}\,u[n] + \left(\frac{1}{3}\right)^n\cdot\frac{1-\cos(n\pi)}{2}\,u[n] =$$

$$= -\left(-7^n u[-n-1]\right) + \left(\frac{1}{2}\right)^n \frac{\cos(n\pi) H}{2} u[n] + \left(\frac{1}{3}\right)^n \cdot \frac{1-\cos(n\pi)}{2} u[n] =$$

$$= -\left(-7^n u[-n-1]\right) + \frac{1}{2}\cdot\left(\frac{1}{2}\right)^n \cos(n\pi) u[n] + \frac{1}{2}\left(\frac{1}{2}\right)^n u[n] +$$

$$+ \frac{1}{2}\left(\frac{1}{3}\right)^n u[n] - \frac{1}{2}\left(\frac{1}{3}\right)^n \cos(n\pi) u[n]$$

$$X_4(z) = -Z\left[-7^n u[-n-1]\right] + \frac{1}{2}Z\left[\left(\frac{1}{2}\right)^n \cos(\pi n) u[n]\right] + \frac{1}{2}Z\left[\left(\frac{1}{2}\right)^n u[n]\right] +$$

$$+ \frac{1}{2}Z\left[\left(\frac{1}{3}\right)^n u[n]\right] - \frac{1}{2}Z\left[\left(\frac{1}{3}\right)^n \cos(\pi n) u[n]\right] =$$

$$= -\frac{1}{1-7z^{-1}} + \frac{1}{2}\cdot\frac{1-\frac{1}{2}\cos\pi \cdot z^{-1}}{1-2\cdot\frac{1}{2}\cos\pi \cdot z^{-1} +\frac{1}{4}z^{-2}} + \frac{1}{2}\cdot\frac{1}{1-\frac{1}{2}z^{-1}} + \frac{1}{2}\cdot\frac{1}{1-\frac{1}{3}z^{-1}} -$$

$$-\frac{1}{2}\cdot\frac{1-\frac{1}{3}\cos\pi z^{-1}}{1-2\cdot\frac{1}{3}\cos\pi z^{-1}+\frac{1}{9}z^{-2}} = -\frac{z}{z-7} + \frac{1}{2}\cdot\frac{\frac{2z+1}{2z}}{1+\frac{1}{2}+\frac{1}{4z^2}} + \frac{1}{2}\cdot\frac{2z}{2z-1} +$$

$$+ \frac{1}{2}\cdot\frac{3z}{3z-1} - \frac{1}{2}\cdot\frac{\frac{3z+1}{3z}}{1+\frac{2}{3z}+\frac{1}{9z^2}} =$$

$$= \frac{z}{7-z} + \frac{z(2z+1)}{4z^2+4z+1} + \frac{z}{2z-1} + \frac{1}{2}\cdot\frac{3z}{3z-1} - \frac{1}{2}\cdot\frac{3z(3z+1)}{9z^2+6z+1} =$$

$$= \frac{z}{7-z} + \frac{z}{2z+1} + \frac{z}{2z-1} + \frac{1}{2}\left(\frac{3z}{3z-1} - \frac{3z}{3z+1}\right) =$$

$$= \frac{z}{7-z} + z\cdot\frac{2z-1+2z+1}{4z^2-1} + \frac{3z}{2}\left(\frac{3z+1-3z+1}{9z^2-1}\right) =$$

$$= \frac{z}{7-z} + \frac{4z^2}{4z^2-1} + \frac{3z}{9z^2-1} \quad, \quad |z|<7 \text{ και } |z|>\frac{1}{2} \text{ και } |z|>\frac{1}{3} \Rightarrow$$

$$\boxed{\frac{1}{2}<|z|<7}$$

2.5) a) $X_1(z) = \dfrac{z-3}{z^2-2z+2}$, $x_1[n]$ αιτιατό

$X_1(z) = \dfrac{z-3}{z^2-2\frac{\sqrt{2}\cdot\sqrt{2}}{2}z+(\sqrt{2})^2}$

$\Omega_0 = \dfrac{\pi}{4}$, $r=\sqrt{2}$ $\qquad x[n] = (\sqrt{2})^n \cos\left(\dfrac{\pi}{4}n\right)u[n]$

$X(z) = \dfrac{1-\sqrt{2}\cdot\frac{\sqrt{2}}{2}\cdot z^{-1}}{1-2\cdot\sqrt{2}\cdot\frac{\sqrt{2}}{2}z^{-1}+(\sqrt{2})^2\cdot z^{-2}} = \dfrac{\frac{z-1}{z}}{\frac{z^2-2z+2}{z^2}} = \dfrac{z^2-z}{z^2-2z+2} =$

$= \dfrac{z^2-2z+2+z-2}{z^2-2z+2} = 1 + \dfrac{z-2}{z^2-2z+2}$

$x[n] = (\sqrt{2})^n \sin\left(\dfrac{\pi}{4}n\right)u[n]$

$X(z) = \dfrac{\sqrt{2}\cdot\frac{\sqrt{2}}{2}z^{-1}}{1-2\cdot\sqrt{2}\cdot\frac{\sqrt{2}}{2}z^{-1}+2z^{-2}} = \dfrac{z^{-1}}{\frac{z^2-2z+2}{z^2}} = \dfrac{z}{z^2-2z+2}$

$\dfrac{z-3}{z^2-2z+2} = a\left(1+\dfrac{z-2}{z^2-2z+2}\right) + b\cdot\dfrac{z}{z^2-2z+2} + \gamma \Rightarrow$

$z-3 = az^2-2az+2a+az-2a + bz + \gamma z^2-2\gamma z+2\gamma \Rightarrow$
$z-3 = (a+\gamma)z^2 + (b-a-2\gamma)z + 2\gamma$

$a+\gamma=0 \Rightarrow$, $2\gamma=-3 \Rightarrow$, $b-a-2\gamma=1 \Rightarrow$
$a=-\gamma=\dfrac{3}{2}$ $\qquad \gamma=-\dfrac{3}{2}$ $\qquad b=2\gamma+a+1 = -3+\dfrac{3}{2}+1=-\dfrac{1}{2}$

Λόγω γραμμικότητας :

$$\text{Apa} \quad x_1[n] = \frac{3}{2}(\sqrt{2})^n \cos\left(\frac{\pi}{4}n\right)u[n] - \frac{1}{2}(\sqrt{2})^n \sin\left(\frac{\pi}{4}n\right)u[n] - \frac{3}{2}\delta[n]$$

б) $X_2(z) = \dfrac{z + 2z^{-1}}{z^2 + 4}$, $|z| > 2$

$$X_2(z) = \frac{z^{-1} + 2z^{-3}}{1 + 4z^{-2}} = \frac{z^{-1}}{1 + 4z^{-2}} + \frac{2z^{-3}}{1 + 4z^{-2}} =$$

$$= \frac{1}{2} \cdot \frac{2z^{-1}}{1 + 4z^{-2}} + \frac{2z^{-1}}{1 + 4z^{-2}} \cdot z^{-2} \qquad , \quad \frac{O}{\underline{\quad}}_o = \frac{\pi}{2} \ , \ r = 2 \ , \ m = 2$$

$$X_2[n] = \frac{1}{2} \cdot 2^n \sin\left(\frac{\pi}{2}n\right)u[n] + 2^n \sin\left(\frac{\pi}{2}n\right)u[n] \ast \delta[n-2] =$$

$$= \frac{1}{2} \cdot 2^n \sin\left(\frac{\pi}{2}n\right)u[n] + 2^{n-2} \cdot \sin\left(\frac{\pi}{2}(n-2)\right)u[n-2] =$$

$$= 2^{n-1} \sin\left(\frac{\pi}{2}n\right)u[n] - 2^{n-2} \sin\left(\frac{\pi}{2}n\right)u[n-2] =$$

$$= 2^{n-1} \sin\left(\frac{\pi}{2}n\right)\left[u[n] - \frac{u[n-2]}{2}\right] =$$

$$= 2^{n-1} \sin\left(\frac{\pi}{2}n\right) \cdot \frac{2u[n] - u[n-2]}{2} =$$

$$= 2^{n-2} \sin\left(\frac{\pi}{2}n\right)\left(2u[n] - u[n-2]\right)$$

γ) $X_3(z) = \dfrac{7z^2 + 4z}{z^3 + z^2 - 4z - 4}$, $|z| < 1$

$$X_3(z) = \frac{7z^2 + 4z}{z^2(z+1) - 4(z+1)} = \frac{7z^2 + 4z}{(z+1)(z-2)(z+2)}$$

$$\frac{7z^2+4z}{(z+1)(z+2)(z-2)} = \frac{A}{z+1} + \frac{B}{z+2} + \frac{\Gamma}{z-2} \Rightarrow$$

$$7z^2+4z = A(z^2-4) + B(z+1)(z-2) + \Gamma(z+1)(z+2) \Rightarrow$$
$$7z^2+4z = Az^2 - 4A + Bz^2 - Bz - 2B + \Gamma z^2 + 3\Gamma z + 2\Gamma \Rightarrow$$
$$7z^2+4z = (A+B+\Gamma)z^2 + (3\Gamma-B)z + 2\Gamma - 4A - 2B$$

$$A+B+\Gamma=0 \Rightarrow A+2-3A+2-A=0 \Rightarrow \boxed{A=4/3}$$
$$3\Gamma-B=4 \Rightarrow B=3\Gamma-4 \Rightarrow B=6A+3B-4 \Rightarrow 2B=4-6A \Rightarrow$$
$$2\Gamma-4A-2B=0 \Rightarrow \Gamma=2A+B=2A+2-3A=2-A \qquad B=2-3A$$

$$\boxed{\begin{array}{l} B=2-4=-2 \\ \Gamma=\dfrac{2}{3} \end{array}}$$

$$X_3(z) = \frac{4}{3} \cdot \frac{1}{z+1} - 2 \cdot \frac{1}{z+2} + \frac{2}{3} \cdot \frac{1}{z-2} =$$

$$= \frac{4}{3} \cdot \frac{z^{-1}}{1+z^{-1}} - 2 \cdot \frac{z^{-1}}{1+2z^{-1}} + \frac{2}{3} \cdot \frac{z^{-1}}{1-2z^{-1}} =$$

$$= \frac{4}{3}\left(1 - \frac{1}{1+z^{-1}}\right) - \left(1 - \frac{1}{1+2z^{-1}}\right) + \frac{1}{3}\left(-1 + \frac{1}{1-2z^{-1}}\right) =$$

$$= \frac{4}{3} - 1 - \frac{1}{3} - \frac{4}{3} \cdot \frac{1}{1+z^{-1}} + \frac{1}{1+2z^{-1}} + \frac{1}{3} \cdot \frac{1}{1-2z^{-1}}$$

$$X_3[n] = -\frac{4}{3}\left(-(-1)^n u[-n-1]\right) - (-2)^n u[-n-1] + \frac{1}{3} 2^n u[-n-1] =$$

$$= \frac{4(-1)^n - 3(-2)^n + 2^n}{3} \cdot u[-n-1]$$

Περιοχή, $|z| < |-1| = 1$ και $|z| < |-2| = 2$ και $|z| < |2| = 2$

Άρα $|z| < 1$

δ) $X_4(z) = \dfrac{z^3(5z+1)}{(z+1)(z^2-2z-3)}$, $1 < |z| < 3$

$X_4(z) = \dfrac{5z^4 + z^3}{z^3 - z^2 - 5z - 3}$

$$\begin{array}{r|l}
5z^4 + z^3 + 0z^2 + 0z + 0 & z^3 - z^2 - 5z - 3 \\
\underline{-5z^4 + 5z^3 + 25z^2 + 15z} & 5z + 6 \\
6z^3 + 25z^2 + 15z & \\
\underline{-6z^3 + 6z^2 + 30z + 18} & \\
31z^2 + 45z + 18 &
\end{array}$$

$X_4(z) = \dfrac{(5z+6)(z^3-z^2-5z-3) + 31z^2 + 45z + 18}{z^3 - z^2 - 5z - 3} = 5z + 6 + \dfrac{31z^2 + 45z + 18}{(z+1)^2(z-3)}$

$\dfrac{31z^2 + 45z + 18}{(z+1)^2(z-3)} = \dfrac{A}{z+1} + \dfrac{B}{(z+1)^2} + \dfrac{\Gamma}{z-3} \Rightarrow$

$31z^2 + 45z + 18 = Az^2 - 2Az - 3A + Bz - 3B + \Gamma z^2 + 2\Gamma z + \Gamma \Rightarrow$

$31z^2 + 45z + 18 = (A + \Gamma)z^2 + (B + 2\Gamma - 2A)z - 3A - 3B + \Gamma$

$A + \Gamma = 31 \Rightarrow \Gamma = 31 - A$

$B + 2(\Gamma - A) = 45 \Rightarrow B + 2(31 - 2A) = 45 \Rightarrow 4A = B + 62 - 45 = B + 17$ $\Big\}\Rightarrow$

$\Gamma - 3(A + B) = 18 \Rightarrow 31 - A - 3A - 3B = 18 \Rightarrow 4A = -3B + 13$

$B + 17 = -3B + 13 \Rightarrow 4B = -4 \Rightarrow \boxed{B = -1}$

$A = \dfrac{-1 + 17}{4} = \dfrac{16}{4} = \boxed{4}$

$\Gamma = 31 - 4 = \boxed{27}$

$$X_4(z) = 5z + 6 + \frac{4}{z+1} - \frac{1}{(z+1)^2} + \frac{27}{z-3} =$$

$$= 5z^{-(-1)} + 6 + \frac{4}{z-(-1)} + \frac{-1}{(z-(-1))^2} + 27 \cdot \frac{z^{-1}}{1-3z^{-1}} =$$

$$= 5z^{-(-1)} + 6 + \frac{4}{z-(-1)} + \frac{-1}{(z-(-1))^2} + 9 \cdot \left(-1 + \frac{1}{1-3z^{-1}}\right) =$$

$$= 5z^{-(-1)} - 3 + \frac{4}{z-(-1)} + \frac{-1}{(z-(-1))^2} + 9 \cdot \frac{1}{1-3z^{-1}}$$

$$x_4[n] = 5\delta[n+1] - 3\delta[n] + 4(-1)^{n-1} u[n-1] + (n-1)(-1)^{n-1} u[n-1] -$$

$$- 9 \cdot 3^n u[-n-1] =$$

$$= 5\delta[n+1] - 3\delta[n] + (n+3)(-1)^{n-1} u[n-1] - 3^{n+2} u[-n-1]$$

Περιοχη $\begin{array}{l} |z| > |-1| = 1 \\ |z| < |3| = 3 \end{array}\Bigg\} \Rightarrow 1 < |z| < 3$

2.7) $x[n] = \delta[n] + 2\delta[n-2]$

a) $X[k] = \sum\limits_{n=0}^{5} x[n]e^{-\frac{j2\pi}{6}\cdot kn} =$

$= \sum\limits_{n=0}^{4} e^{-j\frac{2\pi}{6}kn}\left(\delta[n] + 2\delta[n-2]\right) =$

$= 1 + 0 + 2e^{-j\frac{4\pi}{6}k} + 0+0+0 = 1 + 2e^{-j\frac{2\pi}{3}k}$

$X[0] = 1 + 2 = 3$

$X[1] = 1 + 2e^{-j\frac{2\pi}{3}} = 1 + 2\left(-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right) = -j\sqrt{3}$

$X[2] = 1 + 2e^{-j\frac{4\pi}{3}} = 1 + 2\left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right) = j\sqrt{3}$

$X[3] = 1 + 2e^{-j2\pi} = 1 + 2 = 3$

$X[4] = 1 + 2e^{-j\frac{8\pi}{3}} = 1 + 2e^{-j\frac{2\pi}{3}} = -j\sqrt{3}$

$X[5] = 1 + 2e^{-j\frac{10\pi}{3}} = 1 + 2e^{-j\frac{4\pi}{3}} = j\sqrt{3}$

b) $Z[k] = e^{j\frac{L2\pi}{6}}\cdot X[k] = e^{j\frac{k2\pi}{6}}\left[1 + 2e^{-j\frac{4\pi}{6}k}\right] =$

$= e^{j\frac{2\pi k}{6}} + 2e^{-j\frac{2\pi k}{6}}$

$z[n] = \frac{1}{N}\sum\limits_{k=0}^{N-1} Z[k]e^{j\frac{2\pi}{N}kn} =$

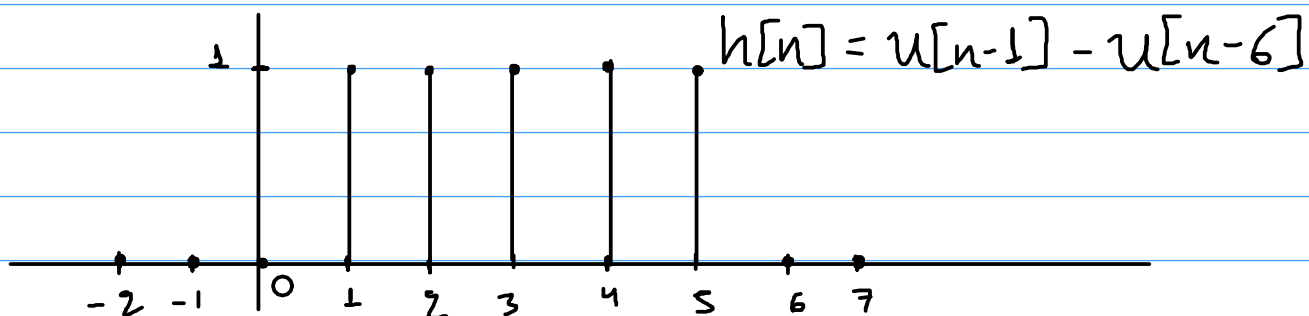$= \frac{1}{6}\sum\limits_{k=0}^{5}\left[e^{j\frac{2\pi k}{6}} + 2e^{-j\frac{2\pi k}{6}}\right]e^{j\frac{2\pi k}{6}n} =$

$$= \frac{1}{6} \sum_{k=0}^{5} \left[e^{j\frac{\pi k}{3}} + 2e^{-j\frac{\pi k}{3}} \right] e^{j\frac{\pi k}{3}n} =$$

$$= \frac{1}{6} \left[3 + \left(e^{j\frac{\pi}{3}} + 2e^{-j\frac{\pi}{3}} \right) e^{j\frac{\pi}{3}n} + \left(e^{j\frac{2\pi}{3}} + 2e^{-j\frac{2\pi}{3}} \right) e^{j\frac{2\pi}{3}n} + \right.$$

$$\left. + \left(e^{j\pi} + 2e^{-j\pi} \right) e^{j\pi n} + \left(e^{j\frac{4\pi}{3}} + 2e^{-j\frac{4\pi}{3}} \right) e^{j\frac{4\pi}{3}n} + \left(e^{j\frac{5\pi}{3}} + 2e^{-j\frac{5\pi}{3}} \right) e^{j\frac{5\pi}{3}n} \right] =$$

$$= \frac{1}{6} \left[3 + \left(\frac{1}{2} + j\frac{\sqrt{3}}{2} + 2\cdot\frac{1}{2} - 2\cdot j\frac{\sqrt{3}}{2} \right) e^{j\frac{\pi}{3}n} + \left(-\frac{1}{2} + j\frac{\sqrt{3}}{2} - 2\cdot\frac{1}{2} - j2\cdot\frac{\sqrt{3}}{2} \right) e^{j\frac{2\pi}{3}n} + \right.$$

$$\left. + \left(-1 - 2 \right)(-1)^n + \left(-\frac{1}{2} - j\frac{\sqrt{3}}{2} - 2\cdot\frac{1}{2} + j\frac{\sqrt{3}}{2}\cdot 2 \right) e^{j\frac{4\pi}{3}n} + \left(\frac{1}{2} - j\frac{\sqrt{3}}{2} + 2\cdot\frac{1}{2} + 2j\frac{\sqrt{3}}{2} \right) e^{j\frac{5\pi}{3}n} \right] =$$

$$= \frac{1}{6} \left[3\left(1 + (-1)^{n+1}\right) + \left(\frac{3}{2} - j\frac{\sqrt{3}}{2} \right) e^{j\frac{\pi}{3}n} - \left(\frac{3}{2} + j\frac{\sqrt{3}}{2} \right) e^{j\frac{2\pi}{3}n} + \left(\frac{3}{2} - j\frac{\sqrt{3}}{2} \right)(-1)^{n+1} e^{j\frac{\pi}{3}n} + \right.$$

$$\left. - \left(\frac{3}{2} + j\frac{\sqrt{3}}{2} \right)(-1)^{n+1} e^{j\frac{2\pi}{3}n} \right] = \frac{1}{6} \left[3\left(1 + (-1)^{n+1}\right) + \left(\frac{3}{2} - j\frac{\sqrt{3}}{2} \right) e^{j\frac{\pi}{3}n}\left(1 + (-1)^n\right) - \right.$$

$$\left. - \left(\frac{3}{2} + j\frac{\sqrt{3}}{2} \right) e^{j\frac{2\pi}{3}n}\left(1 + (-1)^{n+1}\right) \right] = \frac{\left(1 + (-1)^{n+1}\right)}{6} \left[3 + \left(\frac{3}{2} - j\frac{\sqrt{3}}{2} \right) e^{j\frac{\pi}{3}n} - \left(\frac{3}{2} + j\frac{\sqrt{3}}{2} \right) e^{j\frac{2\pi}{3}n} \right]$$

δ) $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad h[n] = u[n-1] - u[n-6]$

$$h[n] = \delta[n-1] + \delta[n-2] + \delta[n-3] + \delta[n-4] + \delta[n-5]$$

$$H[k] = \sum_{n=0}^{N-1} h[n] e^{-\frac{2j\pi}{N}kn} =$$

$$= \sum_{n=0}^{5} \left(\delta[n-1] + \delta[n-2] + \delta[n-3] + \delta[n-4] + \delta[n-5] \right) e^{-\frac{j\pi}{3}kn} =$$

$$= e^{-j\frac{\pi k}{3}} + e^{-j\frac{2\pi}{3}k} + e^{-j\pi k} + e^{-j\frac{4\pi}{3}k} + e^{-j\frac{5\pi}{3}k} =$$

$H[0] = 5$, $H[1] = e^{-j\frac{\pi}{3}} + e^{j\frac{\pi}{3}} + e^{-j\frac{2\pi}{3}} + e^{j\frac{2\pi}{3}} + e^{-j\pi} = 1 - 1 - 1 = -1$

$H[2] = e^{-j\frac{2\pi}{3}} - e^{-j\frac{\pi}{3}} + e^{-j2\pi} + e^{-j\frac{2\pi}{3}} - e^{-j\frac{\pi}{3}} = 2(e^{-j\frac{2\pi}{3}} - e^{-j\frac{\pi}{3}}) + 1 = 2(-\frac{1}{2} - j\frac{\sqrt{3}}{2} - \frac{1}{2} + j\frac{\sqrt{3}}{2}) + 1 = -1$

$H[3] = e^{-j\pi} + e^{-j2\pi} + e^{-j3\pi} + e^{-j4\pi} + e^{-j5\pi} = -1 + 1 - 1 + 1 - 1 = -1$

$H[4] = -e^{-j\frac{\pi}{3}} + e^{-j\frac{2\pi}{3}} + e^{-j4\pi} - e^{-j\frac{\pi}{3}} + e^{-j\frac{2\pi}{3}} = 2(e^{-j\frac{2\pi}{3}} - e^{-j\frac{\pi}{3}}) + 1 = -1$

$H[5] = e^{j\frac{2\pi}{3}} + e^{j\frac{\pi}{3}} + e^{-j5\pi} + e^{-j\frac{2\pi}{3}} + e^{-j\frac{\pi}{3}} = 2\cdot\frac{1}{2} - 2\cdot\frac{1}{2} - 1 = -1$

$$F[k] = X[k] \cdot H[k]$$

$F[0] = 3 \cdot 5 = 15$ $F[3] = 3 \cdot (-1) = -3$

$F[1] = -j\sqrt{3} \cdot (-1) = j\sqrt{3}$ $F[4] = -j\sqrt{3} \cdot (-1) = j\sqrt{3}$

$F[2] = j\sqrt{3} \cdot (-1) = -j\sqrt{3}$ $F[5] = j\sqrt{3} \cdot (-1) = -j\sqrt{3}$

$$f[n] = \frac{1}{N} \sum_{k=0}^{N-1} F[k] e^{j\frac{2\pi}{N}kn} = \frac{1}{6} \sum_{k=0}^{5} F[k] e^{j\frac{\pi}{3}kn} =$$

$$= \frac{1}{6}\left[15 + j\sqrt{3} e^{j\frac{\pi}{3}n} - j\sqrt{3} e^{j\frac{2\pi}{3}n} - 3e^{j\pi n} + j\sqrt{3} e^{j\frac{4\pi}{3}n} - j\sqrt{3} e^{j\frac{5\pi}{3}n} \right] =$$

$$= \frac{1}{6}\left[15 + 3(-1)^{n+1} + j\sqrt{3}\left(e^{j\frac{\pi}{3}n} + e^{j\pi n} e^{j\frac{\pi}{3}n} - e^{j\frac{2\pi}{3}n} - e^{j\pi n} e^{j\frac{2\pi}{3}n} \right) \right] =$$

$$= \frac{1}{6}\left[15 + 3(-1)^{n+1} + j\sqrt{3}\left(e^{j\frac{\pi}{3}n}(1 + (-1)^n) - e^{j\frac{2\pi}{3}n}(1 + (-1)^n) \right) \right] =$$

$$= \frac{1}{6}\left[3(5 + (-1)^{n+1}) + j\sqrt{3}(1 + (-1)^n)(e^{j\frac{\pi}{3}n} - e^{j\frac{2\pi}{3}n}) \right]$$

$$f[0] = \frac{1}{6}\left[3 \cdot 4 + j2\sqrt{3} \cdot (1 - 1) \right] = 2$$

$$f[1] = \frac{1}{6}\left[3 \cdot 6 + j\sqrt{3} \cdot 0 \right] = 3$$

$$f[2] = \frac{1}{6}\left[3\cdot4 + j2\sqrt{3}\left(e^{j\frac{2\pi}{3}} - e^{j\frac{4\pi}{3}}\right)\right] = 2 + j\frac{\sqrt{3}}{3}\left(-\frac{1}{2} + j\frac{\sqrt{3}}{2} + \frac{1}{2} + j\frac{\sqrt{3}}{2}\right) =$$

$$= 2 - \frac{\sqrt{3}\cdot\sqrt{3}}{3} = 1$$

$$f[3] = \frac{1}{6}\left[3\cdot6 + j\sqrt{3}\cdot0\right] = 3$$

$$f[4] = \frac{1}{6}\left[3\cdot4 + j2\sqrt{3}\left(e^{j\frac{4\pi}{3}} - e^{j\frac{8\pi}{3}}\right)\right] = 2 + j\frac{\sqrt{3}}{3}\left(-\frac{1}{2} - j\frac{\sqrt{3}}{2} + \frac{1}{2} - j\frac{\sqrt{3}}{2}\right) =$$

$$= 2 + \frac{\sqrt{3}\cdot\sqrt{3}}{3} = 3$$

$$f[5] = \frac{1}{6}\left[3\cdot6 + j\sqrt{3}\cdot0\right] = 3$$

б) $G[k] = X[k]H[k] \Rightarrow g[n] = x[n] \circledast h[n]$

$$g[n] = x[n]\circledast h[n] = \sum_{m=0}^{N-1} x[m]\, h\big([n-m]\big)_N =$$

$$= \sum_{m=0}^{7} x[m]\, h\big([n-m]\big)_8 =$$

$$h[n] = \begin{cases} 1, & n = 1,2,3,4,5 \\ 0, & \text{иначе} \end{cases}$$

$$g[0] = \sum_{m=0}^{7} x[m]h[-m] = 0$$

$$g[1] = \sum_{m=0}^{7} x[m]h[1-m] = x[0] = 1$$

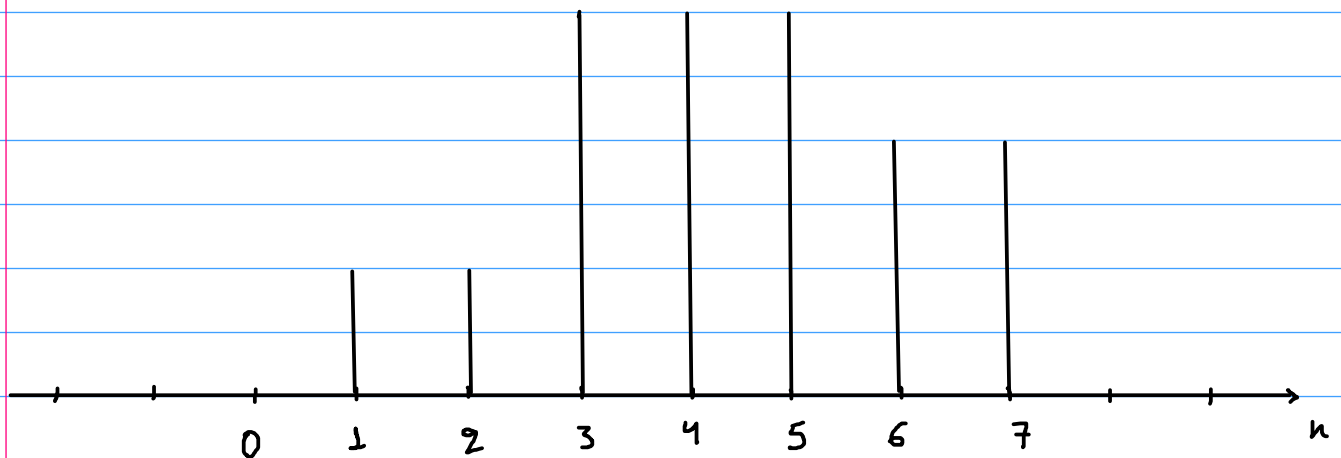$$g[2] = \sum_{m=0}^{7} x[m]h[2-m] = x[0] + x[1] = 1 + 0 = 1$$

$$g[3] = \sum_{m=0}^{7} x[m]h[3-m] = x[0] + x[1] + x[2] = 1 + 0 + 2 = 3$$

$$g[4] = \sum_{m=0}^{7} x[m]h[4-m] = X[0]+X[1]+X[2]+X[3] = 1+0+2+0 = 3$$

$$g[5] = \sum_{m=0}^{7} x[m]h[5-m] = X[0]+\cancel{X[1]}+X[2]+\cancel{X[3]}+\cancel{X[4]} = 3$$

$$g[6] = \sum_{m=0}^{7} x[m]h[6-m] = \cancel{X[1]}+X[2]+\cancel{X[3]}+\cancel{X[4]}+\cancel{X[5]} = 2$$

$$g[7] = \sum_{m=0}^{7} x[m]h[7-m] = X[2]+\cancel{X[3]}+\cancel{X[4]}+\cancel{X[5]}+\cancel{X[6]} = 2$$

$\varepsilon)$ $y[n] = x[n] * h[n] = \left(\delta[n] + 2\delta[n-2]\right) * \left(u[n-1] - u[n-6]\right) =$

$= \delta[n]*u[n-1] - \delta[n]*u[n-6] + 2\delta[n-2]u[n-1] - 2\delta[n-2]u[n-6] =$

$= u[n-1] - u[n-6] + 2u[n-3] - 2u[n-8] =$
$= \delta[n-1]+\delta[n-2]+\delta[n-3]+\delta[n-4]+\delta[n-5] + 2\big(\delta[n-3]+\delta[n-4]+$
$+\delta[n-5]+\delta[n-6]+\delta[n-7]\big) =$
$= \delta[n-1]+\delta[n-2]+3\delta[n-3]+3\delta[n-4]+3\delta[n-5]+2\delta[n-6]+2\delta[n-7]$

$y[n] \equiv g[n] \neq f[n]$