2η Σειρά Ασκήσεων

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$a = AM \bmod 10 + 1 = 6 + 1 = 7$

2.1) $x_1(t) = \sin(20a\pi t)\frac{\cos(300\pi t)}{\pi t} = \sin(140\pi t)\frac{\cos(300\pi t)}{\pi t}$

$x_2(t) = 2\frac{\sin(500\pi t)}{\pi t}$

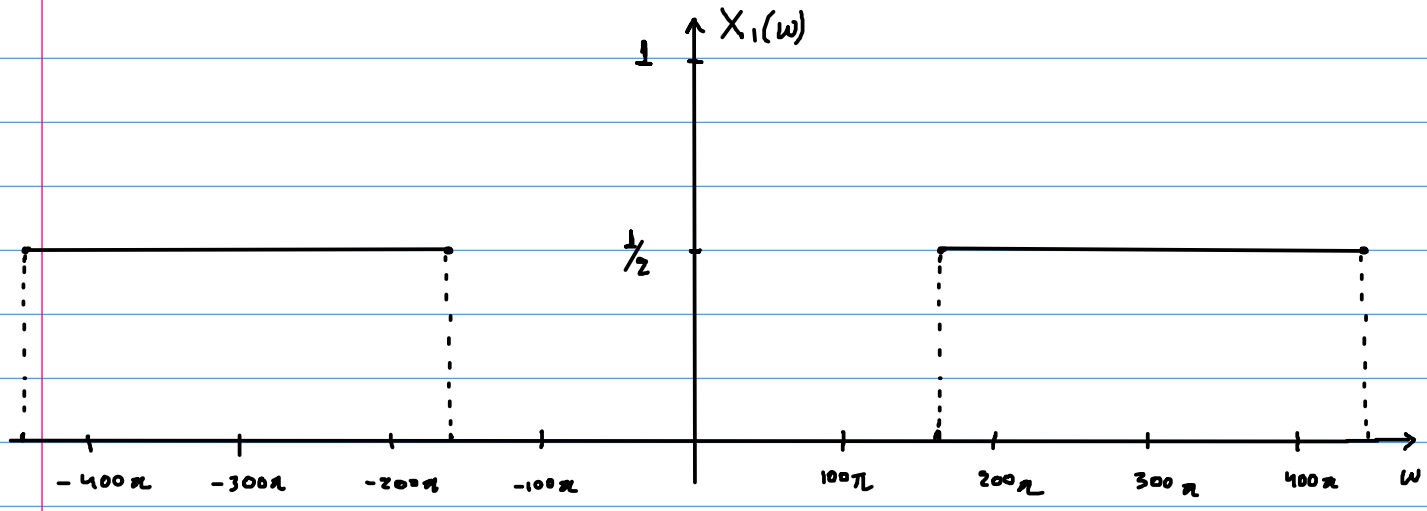
$x_1(t) = 2\sin(140\pi t)\cos(300\pi t)\cdot\frac{1}{2\pi t} =$

$= \left[\sin(140\pi t + 300\pi t) + \sin(140\pi t - 300\pi t)\right]\cdot\frac{1}{2\pi t} =$

$= \frac{\sin 440\pi t}{2\pi t} - \frac{\sin 160\pi t}{2\pi t}$

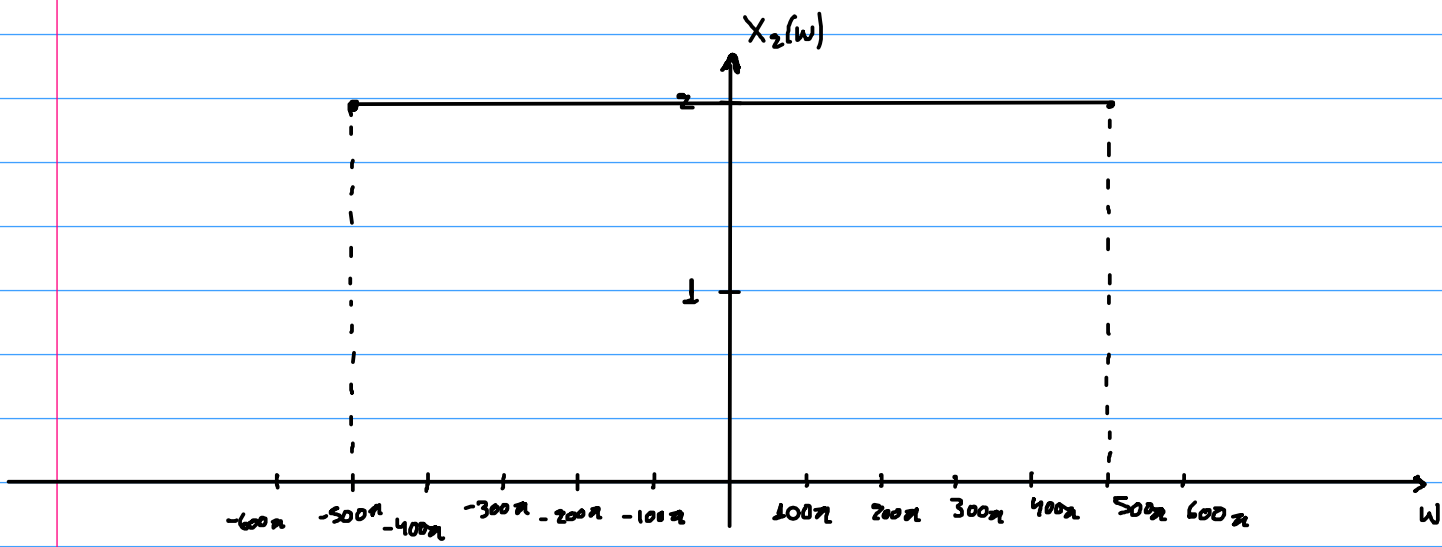
$X_1(\omega) = F[x_1(t)] = \frac{1}{2}F\left[\frac{\sin 440\pi t}{\pi t}\right] - \frac{1}{2}F\left[\frac{\sin 160\pi t}{\pi t}\right]$

$$X_1(\omega) = \begin{cases} 0, & \omega \in (-\infty, -440\pi) \cup (-160\pi, 160\pi) \cup (440\pi, +\infty) \\ \frac{1}{2}, & \omega \in (-440\pi, -160\pi) \cup (160\pi, 440\pi) \end{cases}$$



$$X_2(w) = F[x_2(t)] = 2F\left[\frac{\sin(500\pi t)}{\pi t}\right] =$$

$$= \begin{cases} 0, & w \in (-\infty, -500\pi) \cup (500\pi, +\infty) \\ 2, & -500\pi < w < 500\pi \end{cases}$$

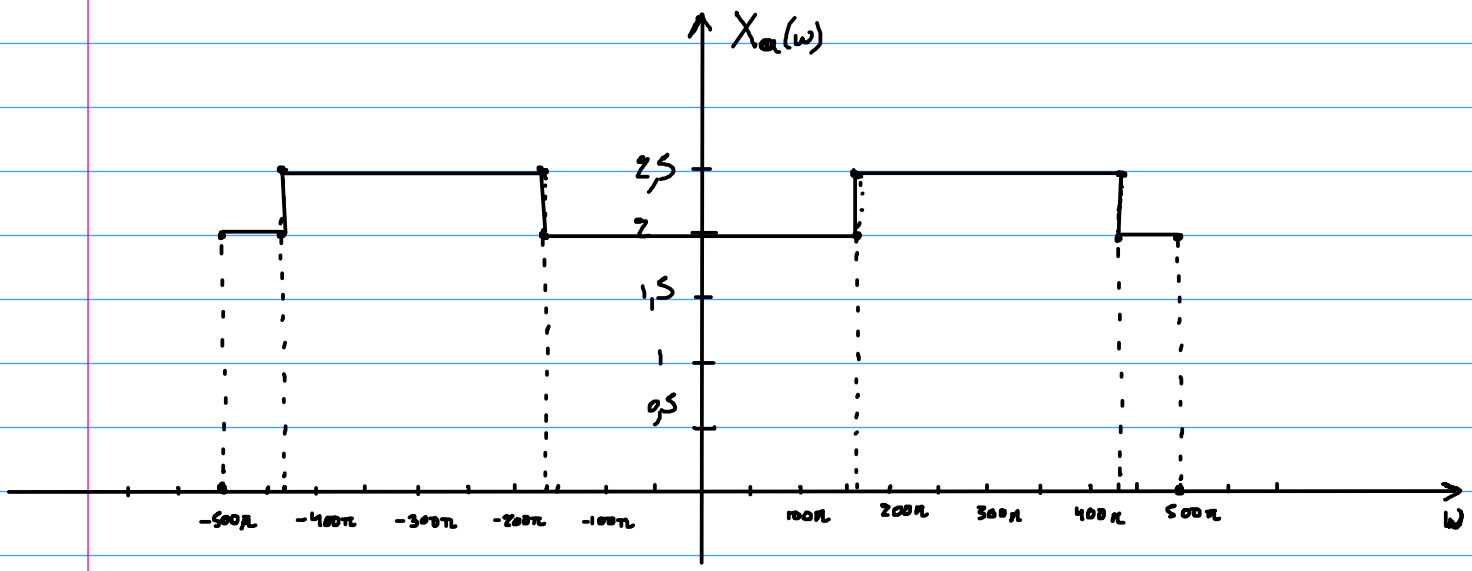


a) $x_1(t) + x_2(t)$

$X_a(w) = F[x_1(t) + x_2(t)] = F[x_1(t)] + F[x_2(t)] =$

$= X_1(w) + X_2(w) =$

$$\begin{cases} 0, & w \in (-\infty, -500\pi) \cup (500\pi, +\infty) \\ 2, & w \in (-500\pi, -440\pi) \cup (-160\pi, 160\pi) \cup (440\pi, 500\pi) \\ \frac{5}{2}, & w \in (-440\pi, -160\pi) \cup (160\pi, 440\pi) \end{cases}$$



$w = 1000\pi \Rightarrow w_s \geq 2w \Rightarrow \frac{2\pi}{T_s} \geq 2w \Rightarrow T_s < \frac{\pi}{w} = \frac{\pi}{1000\pi} = 10^{-3}\,s$

b) $x_b(t) = x_1^3(t) = x_1(t) \cdot x_1^2(t)$

$x_1^2(t) \Rightarrow F[x_1^2(t)] = X_1(w) * X_1(w)$,

$w_{x_1^2} = w_{x_1} + w_{x_1} = 880\pi + 880\pi = 1760\pi$

$X_b(w) = X_1(w) * F[x_1^2(t)]$,

$$w = w_{x_1} + w_{x_2} = 880\pi + 1760\pi = 2640\pi$$

$$w_s > 2w \Rightarrow \frac{2\pi}{T_s} > 2w \Rightarrow T_s < \frac{2640\pi}{\pi} = 2640\,s$$

γ) $x_\gamma(t) = x_1(t) * x_2(t)$

$X_\gamma(w) = X_1(w) \cdot X_2(w)$

$w = \min\{w_1, w_2\} = \{880\pi, 1000\pi\} = 880\pi$

$$w_s > 2w \Rightarrow \frac{2\pi}{T_s} > 2 \cdot 880\pi \Rightarrow T_s < 880\,s$$

δ) $x_\delta(t) = x_1(t) \cdot [x_2(t)]^2 + [x_1(t)]^2 * x_2(t)$

$X_\delta(w) = F[x_1(t) \cdot [x_2(t)]^2] + F[[x_1(t)]^2 * x_2(t)]$

$w = \max\{w_1 + 2w_2, \min\{2w_1, w_2\}\} =$

$= \max\{880\pi + 2000\pi, \min\{1760\pi, 1000\pi\}\} =$

$= \max\{2880\pi, 1000\pi\} = 2880\pi$

$$w_s > 2w \Rightarrow \frac{2\pi}{T_s} > 2w \Rightarrow T_s < 2880\,s$$

2.3) $\omega_s = (2+7 \bmod 3)\,\omega_M = (2+1)\omega_M = 3\omega_M$

a) $y_d[n] = \frac{x[n-1] - 2x[n] + x[n+1]}{4} = \frac{1}{4}x[n-1] - \frac{1}{2}x[n] + \frac{1}{4}x[n+1]$

$$Y_d[\Omega] = \frac{1}{4}e^{-j\Omega}X_d[\Omega] - \frac{1}{2}X_d[\Omega] + \frac{1}{4}e^{j\Omega}X_d[\Omega] \Rightarrow$$

$$\frac{Y_d[\Omega]}{X_d[\Omega]} = \frac{1}{2}\left(\frac{e^{-j\Omega} + e^{j\Omega}}{2} - 1\right) = \frac{1}{2}(\cos\Omega - 1)$$

$$H_d[\Omega] = \frac{1}{2}(\cos\Omega - 1)$$

$$h_d[n] = F^{-1}[H_d[\Omega]] = \frac{1}{4}F^{-1}[e^{-j\Omega} + e^{j\Omega}] - \frac{1}{2}F^{-1}[1] =$$

$$= \frac{1}{4}\delta[n-1] + \frac{1}{4}\delta[n+1] - \frac{1}{2}\delta[n] =$$

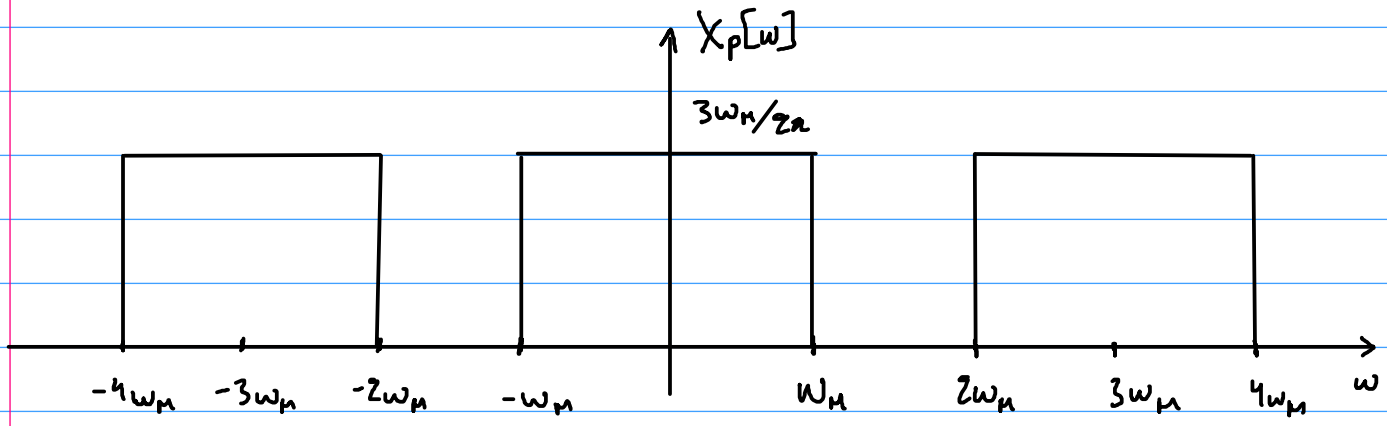
$$= \frac{\delta[n-1] - 2\delta[n] + \delta[n+1]}{4}$$

b) $x_c(t) = \frac{\sin(\omega_M t)}{\pi t}$, $\omega_s = \frac{2\pi}{T_s} \Rightarrow T_s = \frac{2\pi}{\omega_s} = \frac{2\pi}{3\omega_M}$

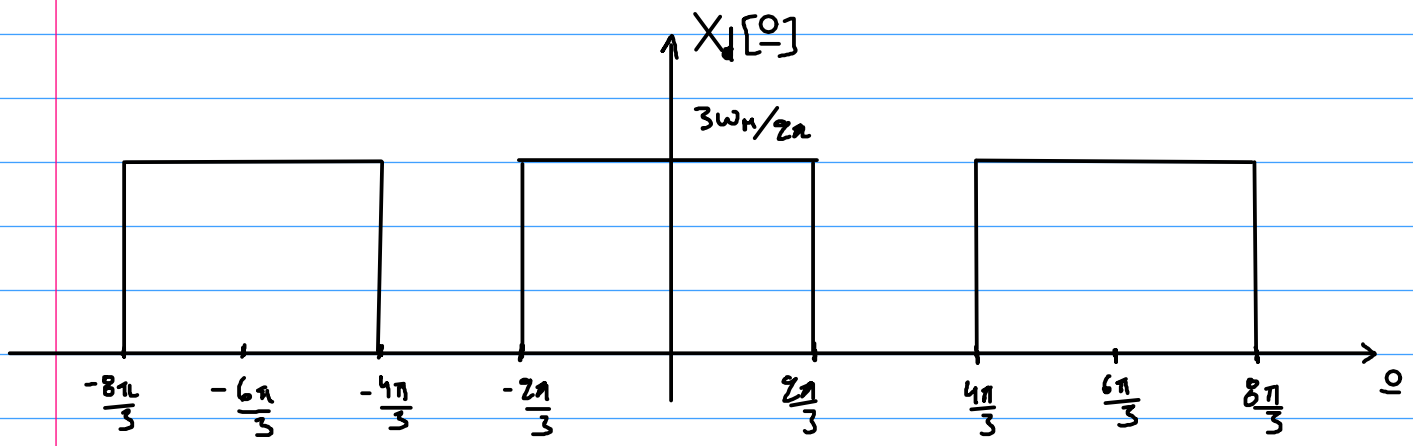
$$x_p(t) = x_c(t) \cdot p(t) = \frac{\sin(\omega_M t)}{\pi t}\sum_n \delta(t - nT)$$

$$X_p[\omega] = X_c(\omega) * F\left[\sum_n \delta(t - nT_s)\right] = X_c * \left[\frac{1}{T_s}\sum_{m=-\infty}^{+\infty}\delta(\omega - m\omega_s)\right] =$$

$$= \frac{1}{T_s}\sum_{m=-\infty}^{+\infty} X_c(\omega - m\omega_s) = \frac{3\omega_M}{2\pi}\sum_{m=-\infty}^{+\infty} X_c(\omega - m\omega_s)$$

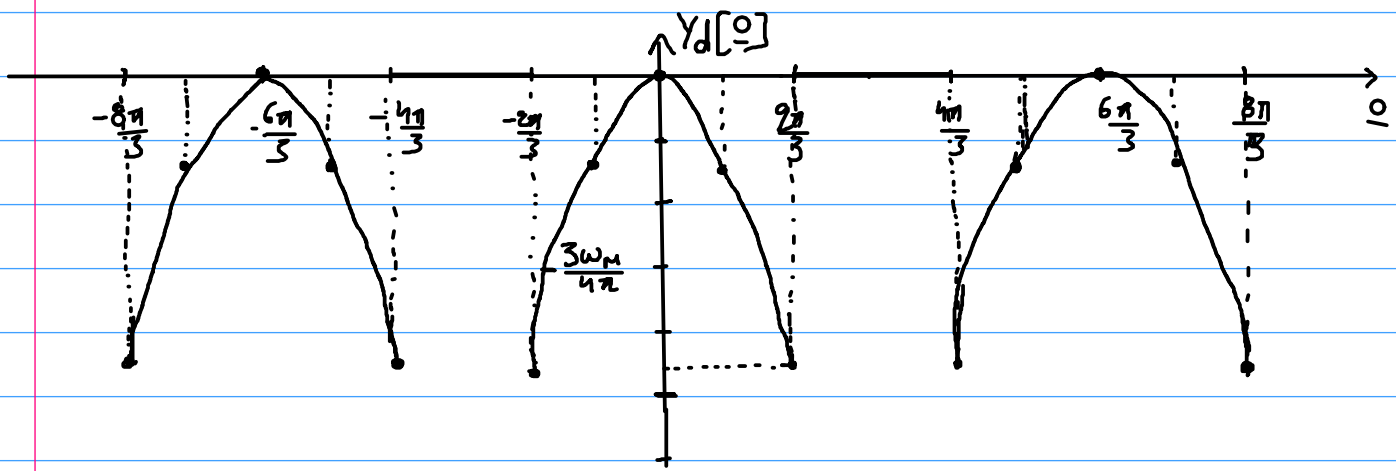


$$X_d(\underline{\omega}) = X_p\left(\frac{\underline{\omega}}{T_s}\right) = X_p\left(\frac{3\omega_M}{2\pi}\cdot\underline{\omega}\right)$$

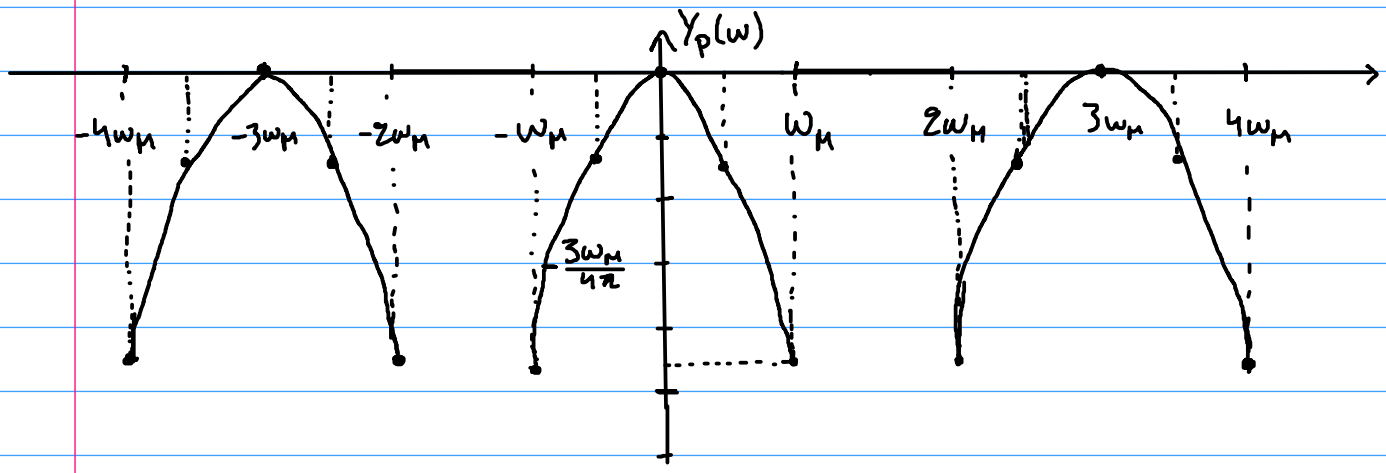


$$y_d[\underline{\omega}] = \frac{1}{4}e^{j\underline{\omega}}X_d[\underline{\omega}] - \frac{1}{2}X_d[\underline{\omega}] + \frac{1}{4}e^{-j\underline{\omega}}X_d[\underline{\omega}] =$$

$$= \frac{X_d[\underline{\omega}]}{2}\left(\frac{e^{-j\underline{\omega}} + e^{j\underline{\omega}}}{2} - 1\right) = \frac{X_d[\underline{\omega}]}{2}(\cos\underline{\omega} - 1)$$

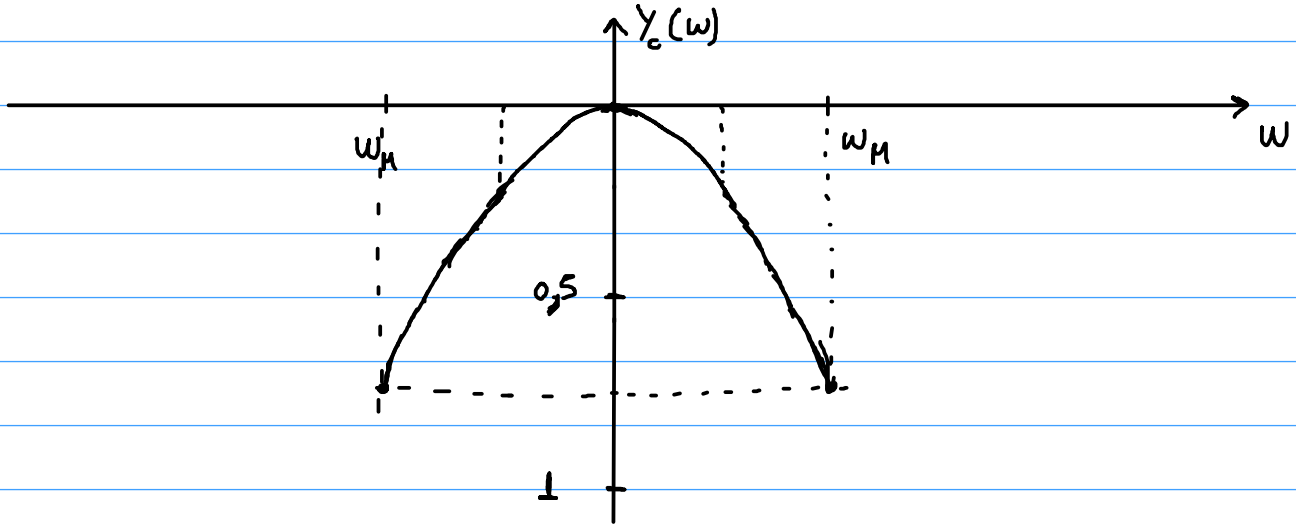


$$Y_d[n] = Y_p\left(\frac{n}{T_s}\right) \Rightarrow Y_p(w) = Y_d[wT_s] = Y_d\left[w \cdot \frac{2\pi}{3w_M}\right]$$



$$H_r(w) = \begin{cases} T, & |w| < \frac{w_s}{2} = \frac{3w_M}{2} \\ 0, & |w| > \frac{w_s}{2} = \frac{3w_M}{2} \end{cases}$$

$$Y_c(w) = Y_p(w) \cdot H_r(w) = \begin{cases} \frac{2\pi}{3w_M} \cdot Y_p(w), & |w| < \frac{3w_M}{2} \\ 0, & |w| > \frac{3w_M}{2} \end{cases}$$



γ) $$H_c(w) = \frac{Y_c(w)}{X_c(w)} = \frac{Y_p(w) \cdot H_r(w)}{X_c(w)} = \frac{Y_d(wT_s) H_r(w)}{X_c(w)} =$$

$$= \frac{X_d(wT_s) H_d(wT_s) H_r(w)}{X_c(w)} = \frac{X_p(w) H_d(wT_s) H_r(w)}{X_c(w)} = \frac{H_d(wT_s) \cdot \cancel{X_c(w)}}{\cancel{X_c(w)}} = H_d(wT_s)$$

Επομένως $H_c(\omega) = \begin{cases} H_d(\omega T_s) , & |\omega| \le \frac{\pi}{T_s} = \frac{\pi}{\frac{2\pi}{3\omega_M}} = \frac{3\omega_M}{2} \\ 0 , & |\omega| > \frac{3\omega_M}{2} \end{cases} \Rightarrow$

$$H_c(\omega) = \begin{cases} \frac{1}{2}\left(\cos\left(\frac{2\pi}{3}\cdot\frac{\omega}{\omega_M}\right) - 1\right) , & |\omega| \le \frac{3\omega_M}{2} \\ 0 , & |\omega| > \frac{3\omega_M}{2} \end{cases}$$

2.4) a) $x_1[n] = a^n \sin[\pi(n+1)/3]\cos[\pi(n+1)/6]u[n] =$

$= \frac{a^n}{2}\cdot\left(\sin\frac{\pi}{2}(n+1) + \sin\frac{\pi}{6}(n+1)\right)u[n] =$

$= \frac{a^n}{2}\left[\frac{e^{j\frac{\pi}{2}(n+1)} - e^{-j\frac{\pi}{2}(n+1)}}{2j} + \frac{e^{j\frac{\pi}{6}(n+1)} - e^{-j\frac{\pi}{6}(n+1)}}{2j}\right]u[n] =$

$= \frac{a^n}{4}\left(\frac{je^{j\frac{\pi}{2}n} + je^{-j\frac{\pi}{2}n}}{j} + \frac{\left(\frac{\sqrt{3}}{2} + j\frac{1}{2}\right)e^{j\frac{\pi}{6}n} - \left(\frac{\sqrt{3}}{2} - j\frac{1}{2}\right)e^{-j\frac{\pi}{6}n}}{j}\right)u[n] =$

$= \frac{a^n}{4}\left(e^{j\frac{\pi}{2}n} + e^{-j\frac{\pi}{2}n} + \frac{\sqrt{3}}{2j}\left(e^{j\frac{\pi}{6}n} - e^{-j\frac{\pi}{6}n}\right) + \frac{1}{2}\left(e^{j\frac{\pi}{6}n} + e^{-j\frac{\pi}{6}n}\right)\right)u[n]$

$= \frac{a^n}{4}\left(2\cos\frac{\pi}{2}n + \sqrt{3}\sin\frac{\pi}{6}n + \cos\frac{\pi}{6}n\right)u[n] =$

$= \frac{1}{2}a^n\cos\left(\frac{\pi}{2}n\right)u[n] + \frac{\sqrt{3}}{4}a^n\sin\left(\frac{\pi}{6}n\right)u[n] + \frac{1}{4}a^n\cos\left(\frac{\pi}{6}n\right)u[n]$

$$X_1(z) = \frac{1}{2}\cdot\frac{1 - [a\cos\frac{\pi}{2}]z^{-1}}{1 - 2[a\cos\frac{\pi}{2}]z^{-1} + a^2z^{-2}} + \frac{\sqrt{3}}{4}\cdot\frac{[a\sin\frac{\pi}{6}]z^{-1}}{1 - [2a\cos\frac{\pi}{6}]z^{-1} + a^2z^{-2}} +$$

$$+\frac{1}{4}\cdot\frac{1-\left[a\cos\frac{\pi}{6}\right]z^{-1}}{1-\left[2a\cos\frac{\pi}{6}\right]z^{-1}+a^2z^{-2}}=\frac{1}{2}\cdot\frac{1}{1+\left(\frac{a}{z}\right)^2}+\frac{\sqrt{3}}{4z}\cdot\frac{\frac{a}{2z}}{1-a\sqrt{3}z^{-1}+\left(\frac{a}{z}\right)^2}+$$

$$+\frac{1}{4}\cdot\frac{1-\frac{a\sqrt{3}}{2z}}{1-\frac{a\sqrt{3}}{z}+\left(\frac{a}{z}\right)^2}=\frac{z^2}{2(z^2+a^2)}+\frac{\sqrt{3}}{2}\cdot\frac{az}{z^2-a\sqrt{3}z+a^2}+\frac{1}{4}\cdot\frac{\frac{2z-a\sqrt{3}}{2z}}{\frac{z^2-a\sqrt{3}z+a^2}{z^2}}=$$

$$=\frac{z^2}{2(z^2+a^2)}+\frac{\sqrt{3}}{2}\cdot\frac{az}{z^2-az\sqrt{3}+a^2}+\frac{1}{2}\cdot\frac{z(2z-a\sqrt{3})}{z^2-az\sqrt{3}+a^2}=$$

$$=\frac{z^2}{2(z^2+a^2)}+\frac{az\sqrt{3}+2z^2-az\sqrt{3}}{2(z^2-az\sqrt{3}+a^2)}=z^2\cdot\frac{z^2+a^2-az\sqrt{3}+2z^2+2a^2}{2(z^2+a^2)(z^2-az\sqrt{3}+a^2)}=$$

$$=z^2\cdot\frac{3z^2-az\sqrt{3}+3a^2}{2(z^2+a^2)(z^2-az\sqrt{3}+a^2)},\ |z|>|a|$$

β) $x_2[n]=na^{-|n|}=na^{-n}u[n]+na^{n}u[-n]$

$$X_2(z)=\mathcal{Z}[x_2[n]]=\mathcal{Z}[na^{-n}u[n]]+\mathcal{Z}[na^{n}u[-n]]=$$

$$=\mathcal{Z}\left[n\left(\frac{1}{a}\right)^n u[n]\right]-\underbrace{\mathcal{Z}\left[-n\left(\frac{1}{a}\right)^{-n}u[-n]\right]}_{\text{χρειάζεται αντιστροφή}}=$$

$$=\frac{\frac{1}{a}\cdot z^{-1}}{\left(1-\frac{1}{a}z^{-1}\right)^2}-\frac{\frac{1}{a}(z^{-1})^{-1}}{\left(1-\frac{1}{a}(z^{-1})^{-1}\right)^2}=\frac{\frac{1}{az}}{\left(1-\frac{1}{az}\right)^2}-\frac{\frac{z}{a}}{\left(1-\frac{z}{a}\right)^2}=$$

$$=\frac{\frac{1}{az}}{\frac{(az-1)^2}{(az)^2}}-\frac{\frac{z}{a}}{\frac{(a-z)^2}{a^2}}=\frac{az}{(az-1)^2}-\frac{az}{(a-z)^2}=$$

$$=az\cdot\frac{(a-z-az+1)(a-z+az-1)}{(a-z)^2(az-1)^2}=az\cdot\frac{(a+1)(1-z)(a-1)(1+z)}{(a-z)^2(az-1)^2}=az\frac{(a^2-1)(1-z^2)}{(a-z)^2(az-1)^2}$$

$|z| > \left|\frac{1}{a}\right|$, $\left|\frac{1}{z}\right| > \left|\frac{1}{a}\right| \Rightarrow |z| < |a|$

$|a|^{-1} < |z| < |a|$

2.7) $x[n] = \delta[n] + 2\delta[n-2]$

a) $X[k] = \sum_{n=0}^{5} x[n] e^{-\frac{j2\pi}{6} \cdot kn} =$

$= \sum_{n=0}^{4} e^{-j\frac{2\pi}{6}kn} \left(\delta[n] + 2\delta[n-2]\right) =$

$= 1 + 0 + 2e^{-j\frac{4\pi}{6}k} + 0+0+0 = 1 + 2e^{-j\frac{2\pi}{3}k}$

$X[0] = 1 + 2 = 3$
$X[1] = 1 + 2e^{-j\frac{2\pi}{3}} = 1 + 2\left(-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right) = -j\sqrt{3}$
$X[2] = 1 + 2e^{-j\frac{4\pi}{3}} = 1 + 2\left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right) = j\sqrt{3}$
$X[3] = 1 + 2e^{-j2\pi} = 1 + 2 = 3$
$X[4] = 1 + 2e^{-j\frac{8\pi}{3}} = 1 + 2e^{-j\frac{2\pi}{3}} = -j\sqrt{3}$
$X[5] = 1 + 2e^{-j\frac{10\pi}{3}} = 1 + 2e^{-j\frac{4\pi}{3}} = j\sqrt{3}$

b) $Z[k] = e^{jk\frac{2\pi}{6}} \cdot X[k] = e^{jk\frac{2\pi}{6}} \left[1 + 2e^{-j\frac{4\pi}{6}k}\right] =$

$= e^{j\frac{2\pi k}{6}} + 2e^{-j\frac{2\pi k}{6}}$

$z[n] = \frac{1}{N} \sum_{k=0}^{N-1} Z[k] e^{j\frac{2\pi}{N}kn} =$

$= \frac{1}{6} \sum_{k=0}^{5} \left[e^{j\frac{2\pi k}{6}} + 2e^{-j\frac{2\pi k}{6}}\right] e^{j\frac{2\pi k}{6}n} =$

$$= \frac{1}{6} \sum_{k=0}^{5} \left[e^{j\frac{\pi k}{3}} + 2e^{-j\frac{\pi k}{3}} \right] e^{j\frac{\pi k}{3} n} =$$

$$= \frac{1}{6} \left[3 + \left(e^{j\frac{\pi}{3}} + 2e^{-j\frac{\pi}{3}} \right) e^{j\frac{\pi}{3}n} + \left(e^{j\frac{2\pi}{3}} + 2e^{-j\frac{2\pi}{3}} \right) e^{j\frac{2\pi}{3}n} + \right.$$

$$\left. + \left(e^{j\pi} + 2e^{-j\pi} \right) e^{j\pi n} + \left(e^{j\frac{4\pi}{3}} + 2e^{-j\frac{4\pi}{3}} \right) e^{j\frac{4\pi}{3}n} + \left(e^{j\frac{5\pi}{3}} + 2e^{-j\frac{5\pi}{3}} \right) e^{j\frac{5\pi}{3}n} \right] =$$

$$= \frac{1}{6} \left[3 + \left(\frac{1}{2} + j\frac{\sqrt{3}}{2} + 2\cdot\frac{1}{2} - 2\cdot j\frac{\sqrt{3}}{2} \right) e^{j\frac{\pi}{3}n} + \left(-\frac{1}{2} + j\frac{\sqrt{3}}{2} - 2\cdot\frac{1}{2} - j2\cdot\frac{\sqrt{3}}{2} \right) e^{j\frac{2\pi}{3}n} + \right.$$

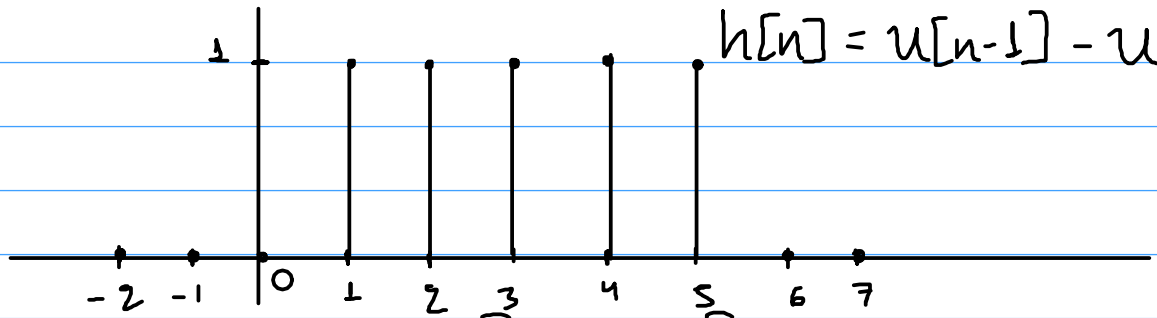
$$\left. + (-1-2)(-1)^n + \left(-\frac{1}{2} - j\frac{\sqrt{3}}{2} - 2\cdot\frac{1}{2} + j\frac{\sqrt{3}}{2}\cdot 2 \right) e^{j\frac{4\pi}{3}n} + \left(\frac{1}{2} - j\frac{\sqrt{3}}{2} + 2\cdot\frac{1}{2} + 2j\frac{\sqrt{3}}{2} \right) e^{j\frac{5\pi}{3}n} \right] =$$

$$= \frac{1}{6} \left[3\left(1+(-1)^{n+1}\right) + \left(\frac{3}{2} - j\frac{\sqrt{3}}{2} \right) e^{j\frac{\pi}{3}n} - \left(\frac{3}{2} + j\frac{\sqrt{3}}{2} \right) e^{j\frac{2\pi}{3}n} + \left(\frac{3}{2} - j\frac{\sqrt{3}}{2} \right) (-1)^{n+1} e^{j\frac{\pi}{3}n} + \right.$$

$$\left. - \left(\frac{3}{2} + j\frac{\sqrt{3}}{2} \right) (-1)^{n+1} e^{j\frac{2\pi}{3}n} \right] = \frac{1}{6} \left[3\left(1+(-1)^{n+1}\right) + \left(\frac{3}{2} - j\frac{\sqrt{3}}{2} \right) e^{j\frac{\pi}{3}n} \left(1+(-1)^n\right) - \right.$$

$$\left. - \left(\frac{3}{2} + j\frac{\sqrt{3}}{2} \right) e^{j\frac{2\pi}{3}n} \left(1+(-1)^{n+1}\right) \right] = \frac{\left(1+(-1)^{n+1}\right)}{6} \left[3 + \left(\frac{3}{2} - j\frac{\sqrt{3}}{2} \right) e^{j\frac{\pi}{3}n} - \left(\frac{3}{2} + j\frac{\sqrt{3}}{2} \right) e^{j\frac{2\pi}{3}n} \right]$$

γ)



$$h[n] = u[n-1] - u[n-6]$$

$$h[n] = \delta[n-1] + \delta[n-2] + \delta[n-3] + \delta[n-4] + \delta[n-5]$$

$$H[k] = \sum_{n=0}^{N-1} h[n] e^{-\frac{2j\pi}{N}kn} =$$

$$= \sum_{n=0}^{5} \left(\delta[n-1] + \delta[n-2] + \delta[n-3] + \delta[n-4] + \delta[n-5] \right) e^{\frac{-j\pi}{3}kn} =$$

$$= e^{-j\frac{\pi}{3}k} + e^{-j\frac{2\pi}{3}k} + e^{-j\pi k} + e^{-j\frac{4\pi}{3}k} + e^{-j\frac{5\pi}{3}k} =$$

$H[0] = 5$, $H[1] = e^{-j\frac{\pi}{3}} + e^{j\frac{\pi}{3}} + e^{-j\frac{2\pi}{3}} + e^{j\frac{2\pi}{3}} + e^{-j\pi} = 1 - 1 - 1 = -1$

$H[2] = e^{-j\frac{2\pi}{3}} - e^{j\frac{\pi}{3}} + e^{-j2\pi} + e^{-j\frac{2\pi}{3}} - e^{-j\frac{\pi}{3}} = 2\left(e^{-j\frac{2\pi}{3}} - e^{-j\frac{\pi}{3}}\right) + 1 = 2\left(-\frac{1}{2} - j\frac{\sqrt{3}}{2} - \frac{1}{2} + j\frac{\sqrt{3}}{2}\right) + 1 = -1$

$H[3] = e^{-j\pi} + e^{-j2\pi} + e^{-j3\pi} + e^{-j4\pi} + e^{-j5\pi} = -1 + 1 - 1 + 1 - 1 = -1$

$H[4] = -e^{-j\frac{\pi}{3}} + e^{-j\frac{2\pi}{3}} + e^{-j4\pi} - e^{-j\frac{\pi}{3}} + e^{-j\frac{2\pi}{3}} = 2\left(e^{-j\frac{2\pi}{3}} - e^{-j\frac{\pi}{3}}\right) + 1 = -1$

$H[5] = e^{j\frac{2\pi}{3}} + e^{j\frac{\pi}{3}} + e^{-j5\pi} + e^{-j\frac{2\pi}{3}} + e^{-j\frac{\pi}{3}} = \frac{2\cdot 1}{2} - \frac{2\cdot 1}{2} - 1 = -1$

$F[k] = X[k] \cdot H[k]$

$F[0] = 3 \cdot 5 = 15$ $\qquad$ $F[3] = 3 \cdot (-1) = -3$

$F[1] = -j\sqrt{3} \cdot (-1) = j\sqrt{3}$ $\qquad$ $F[4] = -j\sqrt{3} \cdot (-1) = j\sqrt{3}$

$F[2] = j\sqrt{3} \cdot (-1) = -j\sqrt{3}$ $\qquad$ $F[5] = j\sqrt{3} \cdot (-1) = -j\sqrt{3}$

$$f[n] = \frac{1}{N}\sum_{k=0}^{N-1} F[k] e^{j\frac{2\pi}{N}kn} = \frac{1}{6}\sum_{k=0}^{5} F[k] e^{j\frac{\pi}{3}kn} =$$

$$= \frac{1}{6}\left[15 + j\sqrt{3}e^{j\frac{\pi}{3}n} - j\sqrt{3}e^{j\frac{2\pi}{3}n} - 3e^{j\pi n} + j\sqrt{3}e^{j\frac{4\pi}{3}n} - j\sqrt{3}e^{j\frac{5\pi}{3}n}\right] =$$

$$= \frac{1}{6}\left[15 + 3(-1)^{n+1} + j\sqrt{3}\left(e^{j\frac{\pi}{3}n} + e^{j\pi n}e^{j\frac{\pi}{3}n} - e^{j\frac{2\pi}{3}n} - e^{j\pi n}e^{j\frac{2\pi}{3}n}\right)\right] =$$

$$= \frac{1}{6}\left[15 + 3(-1)^{n+1} + j\sqrt{3}\left(e^{j\frac{\pi}{3}n}\left(1 + (-1)^n\right) - e^{j\frac{2\pi}{3}n}\left(1 + (-1)^n\right)\right)\right] =$$

$$= \frac{1}{6}\left[3\left(5 + (-1)^{n+1}\right) + j\sqrt{3}\left(1 + (-1)^n\right)\left(e^{j\frac{\pi}{3}n} - e^{j\frac{2\pi}{3}n}\right)\right]$$

$f[0] = \frac{1}{6}\left[3 \cdot 4 + j2\sqrt{3} \cdot (1 - 1)\right] = 2$

$f[1] = \frac{1}{6}\left[3 \cdot 6 + j\sqrt{3} \cdot 0\right] = 3$

$$f[2] = \frac{1}{6}\left[3\cdot 4 + j2\sqrt{3}\left(e^{j\frac{2\pi}{3}} - e^{j\frac{4\pi}{3}}\right)\right] = 2 + j\frac{\sqrt{3}}{3}\left(-\tfrac{1}{2} + j\tfrac{\sqrt{3}}{2} + \tfrac{1}{2} + j\tfrac{\sqrt{3}}{2}\right) =$$

$$= 2 - \frac{\sqrt{3}\cdot\sqrt{3}}{3} = 1$$

$$f[3] = \frac{1}{6}\left[3\cdot 6 + j\sqrt{3}\cdot 0\right] = 3$$

$$f[4] = \frac{1}{6}\left[3\cdot 4 + j2\sqrt{3}\left(e^{j\frac{4\pi}{3}} - e^{j\frac{8\pi}{3}}\right)\right] = 2 + j\frac{\sqrt{3}}{3}\left(-\tfrac{1}{2} - j\tfrac{\sqrt{3}}{2} + \tfrac{1}{2} - j\tfrac{\sqrt{3}}{2}\right) =$$

$$= 2 + \frac{\sqrt{3}\cdot\sqrt{3}}{3} = 3$$

$$f[5] = \frac{1}{6}\left[3\cdot 6 + j\sqrt{3}\cdot 0\right] = 3$$

δ) $G_2[k] = X[k]H[k] \Rightarrow g[n] = x[n] \circledast h[n]$

$$g[n] = x[n] \circledast h[n] = \sum_{m=0}^{N-1} x[m]\, h\left([n-m]\right)_N =$$

$$= \sum_{m=0}^{7} x[m]\, h\left([n-m]\right)_8 =$$

$$h[n] = \begin{cases} 1, & n = 1,2,3,4,5 \\ 0, & \text{αλλού} \end{cases}$$

$$g[0] = \sum_{m=0}^{7} x[m]\, h[-m] = 0$$

$$g[1] = \sum_{m=0}^{7} x[m]\, h[1-m] = x[0] = 1$$

$$g[2] = \sum_{m=0}^{7} x[m]\, h[2-m] = x[0] + x[1] = 1 + 0 = 1$$

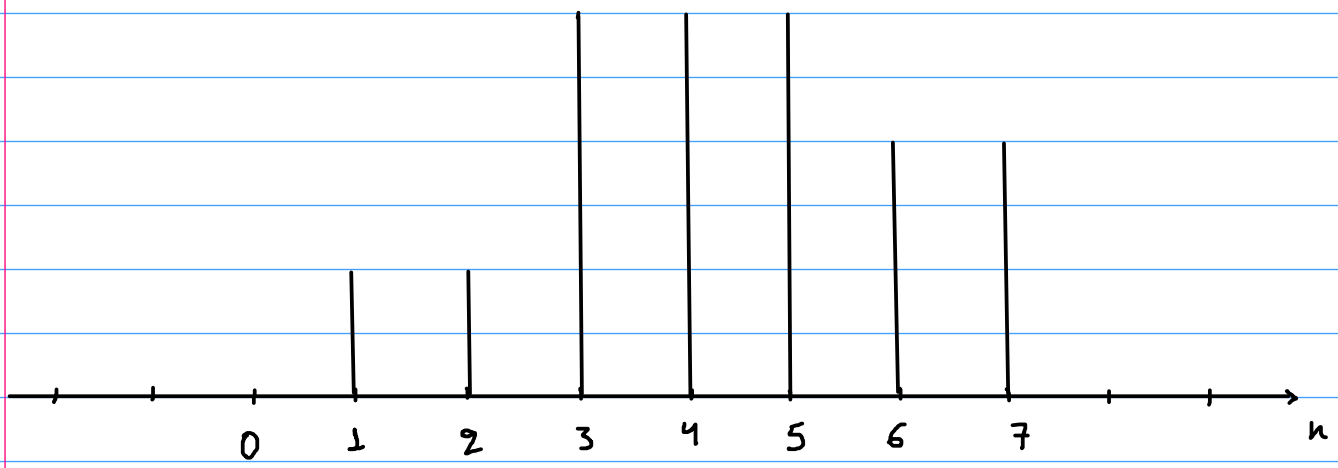
$$g[3] = \sum_{m=0}^{7} x[m]\, h[3-m] = x[0] + x[1] + x[2] = 1 + 0 + 2 = 3$$

$$g[4] = \sum_{m=0}^{7} x[m]h[4-m] = x[0]+x[1]+x[2]+x[3] = 1+0+2+0 = 3$$

$$g[5] = \sum_{m=0}^{7} x[m]h[5-m] = x[0]+x[1]+x[2]+x[3]+x[4] = 3$$

$$g[6] = \sum_{m=0}^{7} x[m]h[6-m] = x[1]+x[2]+x[3]+x[4]+x[5] = 2$$

$$g[7] = \sum_{m=0}^{7} x[m]h[7-m] = x[2]+x[3]+x[4]+x[5]+x[6] = 2$$



ε) $y[n] = x[n] * h[n] = (\delta[n] + 2\delta[n-2]) * (u[n-1] - u[n-6]) =$

$= \delta[n]*u[n-1] - \delta[n]*u[n-6] + 2\delta[n-2]u[n-1] - 2\delta[n-2]u[n-6] =$

$= u[n-1] - u[n-6] + 2u[n-3] - 2u[n-8] =$

$= \delta[n-1] + \delta[n-2] + \delta[n-3] + \delta[n-4] + \delta[n-5] + 2(\delta[n-3] + \delta[n-4] +$
$+ \delta[n-5] + \delta[n-6] + \delta[n-7]) =$

$= \delta[n-1] + \delta[n-2] + 3\delta[n-3] + 3\delta[n-4] + 3\delta[n-5] + 2\delta[n-6] + 2\delta[n-7]$

$y[n] \equiv g[n] \neq f[n]$