Datastrukturer exam, 2022-01-13: solutions

1A.

A.	push: O(n)	pop: O(1)
B.	push: O(log n)	pop: O(log n)
C.	push: O(1)	pop: O(1)

1B. O(n)

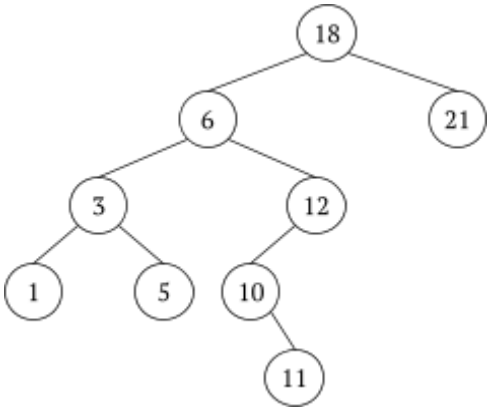
$O(n + n/2 + n/4 + n/8 + \ldots) = O(n \cdot (1 + ½ + ¼ + \ldots)) = O(n \cdot 2) = O(n)$

2A. Red-black tree / AA tree

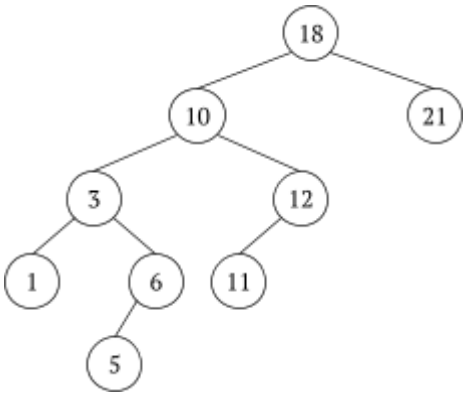
2B.

- Minimum: the root
- Maximum: the root's left and right children (= root.left and root.right)
    - [Optional: also the root, if the heap has size 1.]

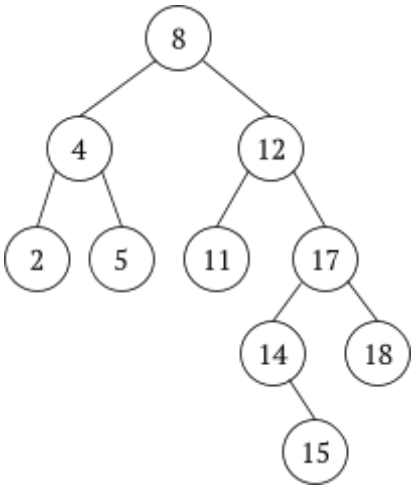
3A. Alternative 1:



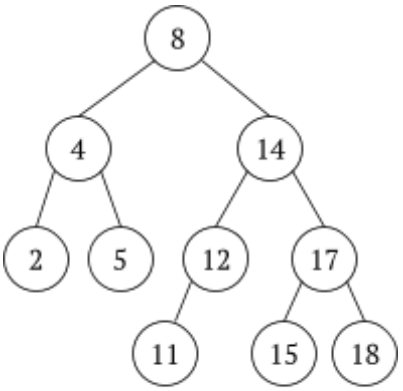
Alternative 2:



3B. After insertion:



After rebalancing:



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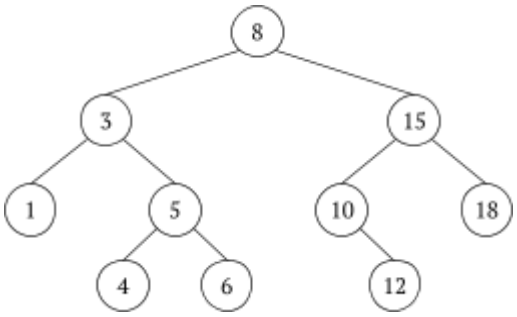
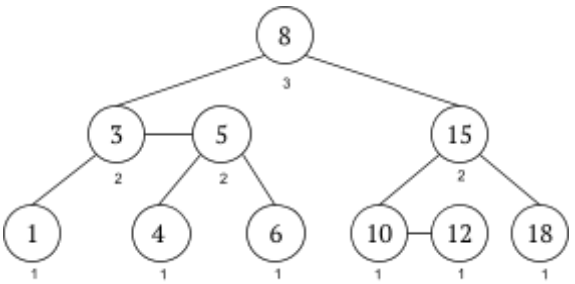
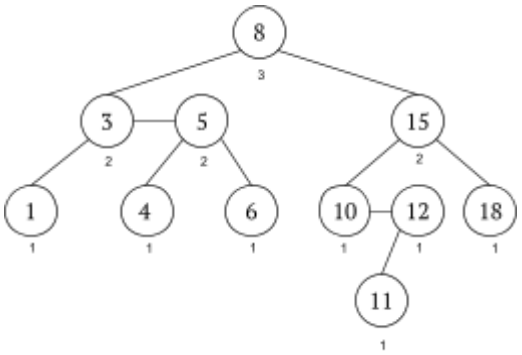
3B AAtree

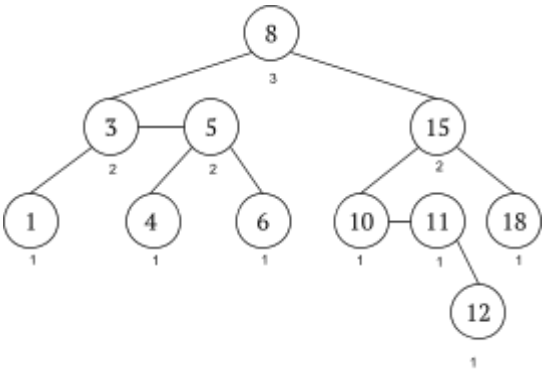




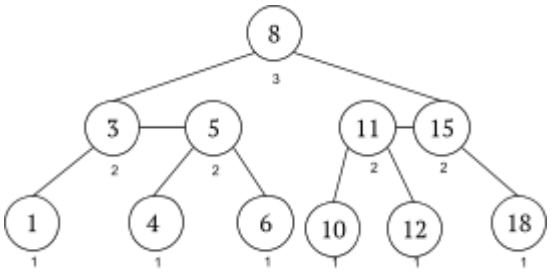
Insert 11:



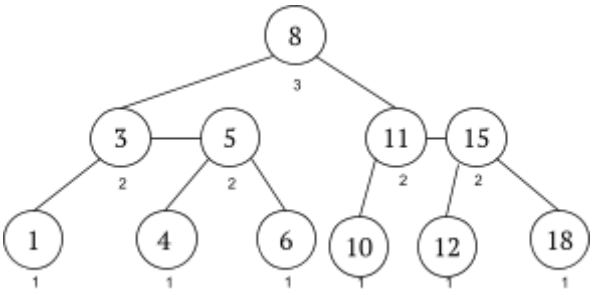
Skew:



Split:



Skew:



Datastrukturer exam, 2022-01-13: solutions

4A. (The ~~overcrossed~~ cells can also be empty)

0	1	2	3	4	5	6	7	8	9	10	11
mat							~~*the*~~	~~*cat*~~	~~*sat*~~	*on*	*the*

↑ rear

↑ front

4B. The array can also be shifted (so that the min element is in position 1)

Initial heap:

0	1	2	3	4	5	6	7	8	9	10	11
2	*4*	*6*	*10*	*13*	*9*	*7*	*20*	*15*	*17*		

After removeMin:

0	1	2	3	4	5	6	7	8	9	10	11
4	*10*	*6*	*15*	*13*	*9*	*7*	*20*	*17*			

Datastrukturer exam, 2022-01-13: solutions

5A. Your task is to sort the following array: `[13 7 98 45 97 30 71 9]`

```
[13] + [7] -> [7 13]                          13<7
[98] + [45] -> [45 98]                        98<45

[7 13] + [45 98] -> [7 13 45 98]              7<45, 13<45

[97] + [30] -> [30 97]                        97<30
[71] + [9] -> [9 71]                          71<9

[30 97] + [9 71] -> [9 30 71 97]              30<9, 30<71, 97<71

[7 13 45 98] + [9 30 71 97] -> [7 9 13 30 45 71 97 98]
                    7<9, 13<9, 13<30, 45<30, 45<71, 98<71, 98<97
```

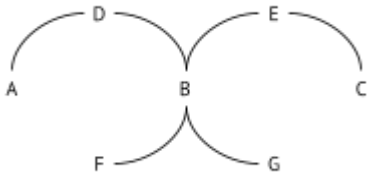
Step	First	Second	Step	First	Second
1	13	7	**9**	97	71
2	98	45	**10**	7	9
3	7	45	**11**	13	9
4	13	45	**12**	13	30
5	97	30	**13**	45	30
6	71	9	**14**	45	71
7	30	9	**15**	98	71
8	30	71	**16**	98	97

5B.

- Worst case: 15 = 5+4+3+2+1 comparisons
- Best case: 8 = 5+2+1 comparisons

Datastrukturer exam, 2022-01-13: solutions

6A. Option 3:



6B.

Removed	Added	Priority queue after adding	Comments
–	A	A:0	
A	D B	D:1 B:12	
D	B	B:10 (B:12)	Ok to remove B:12 from PQ
B	G F	(B:12) G:13 F:16	
(B)	–	G:13 F:16	Not necessary
G	C E	F:16 C:18 E:20	
F	(A)	C:18 E:20 (A:27)	Not necessary – A already visited
C	–	E:20 (A:27)	
E	(C) (B)	(C:22) (B:24) (A:27)	Not necessary – B, C already visited
(C)	–	(B:24) (A:27)	Not necessary
(B)	–	(A:27)	Not necessary
(A)	–	–	Not necessary

Final SPT from A (the total costs are optional):