

# Re-exam for: DAT038, TDA417, DAT525 (and DAT037, TDA416)

## Datastrukturer och algoritmer

Tuesday, 2022-03-15, 14:00–18:00, Johanneberg

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**Teachers** Peter Ljunglöf, 0766–075561  
Nick Smallbone, 0707–183062  
  
will visit around 15:00 and 16:30

**Allowed aids** None

**Exam review** After the exams have been graded, they are available for review in the student office on floor 4 in the EDIT building.  
If you want to discuss the grading, please come to the exam review:

Friday, 25 March, 15–17, in room 6128 (EDIT building 6th floor)

In that case, you should leave the exam in the student office until after the review. We will bring all exams to the review meeting.

**Notes** Write your anonymous code (not your name) on every page.  
You may answer in English or Swedish.  
Excessively complicated answers might be rejected.  
Write legibly – we need to be able to read your answer!  
You can write explanations on the question sheet or on a separate paper.

Grade	Sections with $\geq 1$ correct answer	Sections with both answers correct
3	5	—
4	5	2
5	6	4

**Good luck!**

Write your anonymous code (*not* your name):

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## Section 1: Complexity

### Question 1A

What is the asymptotic complexity of the following code in terms of  $n$ ?

Write your answer in O-notation. Be as exact and simple as possible. Justify that the complexity of the program has this order of growth.

```
M = new red-black map
for i from 1 to n:
    M.put(i, 0)
    for j from 1 to i*i:
        M.put(i, M.get(i)+j)
```

Answer: \_\_\_\_\_

Justification:

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Points for question (to be filled by the grader):

Write your anonymous code (*not* your name):

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### Question 1B

This question is about the order of growth of a mathematical expression. Consider the following expression in  $x$ ,  $y$  and  $z$ :

$$3xy \log_2(x) + (z + 1)^2 + xyz$$

- a. Suppose that  $y$  and  $z$  are constants, but the value of  $x$  can vary.  
What is the order of growth in terms of  $x$ ?
- b. Now suppose that  $x$  is a constant, but the values of  $y$  and  $z$  can vary.  
What is the order of growth in terms of  $y$  and  $z$  together?

Write your answers in O-notation, as simply and exactly as possible. Explain your reasoning, e.g., by showing the steps you took to simplify the function to an order of growth.

Answer (a): \_\_\_\_\_

Answer (b): \_\_\_\_\_

Explanation:

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Points for question (to be filled by the grader):

## Section 2: Using data types and data structures

### Question 2A

Pair each of the following four abstract data types (ADTs) with a use case for which it is well-suited. Note that each ADT goes with exactly one use case, and each use case goes with exactly one ADT.

**A:** stack

**P:** a queue for an emergency room at a hospital, where the sickest patients should be treated first

**B:** queue

**Q:** a database of people, where you can find information about a person by giving their personnummer

**C:** map

**R:** testing if a string contains matching parentheses and brackets  
e.g. correct: ( [] ( ( [] ) ( ) ) [ ( ) ] ), incorrect: [ ( ) )

**D:** priority queue

**S:** a waiting list for a course – when someone drops out of the course, the student who signed up earliest takes their place

You can e.g. draw lines between each ADT and its use case.

For each ADT, say which operations the ADT supports that are important for the use case:

**A.**

**B.**

**C.**

**D.**

## Question 2B

In this question, you will show how to implement a data structure you may not have seen before, called a *bidirectional map*. A bidirectional map is a map which supports two kinds of lookup: given a key, you can find the corresponding value, and given a value, you can find the corresponding key. All entries (key-value pairs) in a bidirectional map must have a different key (like in an ordinary map) and also a *different value*.

A bidirectional map supports the following operations from maps:

- `put (k, v)` : add the entry  $k \mapsto v$  to the map.  
Any existing item with key ***k*** **or value *v*** is removed.
- `get (k)` : if the map contains an entry  $k \mapsto v$ , return  $v$ .

plus the following *reverse lookup* operation:

- `reverseGet (v)` : if the map contains an entry  $k \mapsto v$ , return  $k$ .

The following table shows an example sequence of operations.

Operation	Result
create new map	Map is {}
<code>put (1, "hello")</code>	Map is $\{1 \mapsto \text{"hello"}\}$
<code>put (2, "world")</code>	Map is $\{1 \mapsto \text{"hello"}, 2 \mapsto \text{"world"}\}$
<code>get (1)</code>	Returns "hello"
<code>put (3, "hello")</code>	Map is $\{2 \mapsto \text{"world"}, 3 \mapsto \text{"hello"}\}$ . Notice that the item $1 \mapsto \text{"hello"}$ is replaced by $3 \mapsto \text{"hello"}$ since it is not allowed for two entries to have the same value.
<code>reverseGet ("world")</code>	Returns 2

We can implement a bidirectional map using *two* maps:

- `forward` is a map from keys to values.  
In the example above, at the end, it contains  $2 \mapsto \text{"world"}$  and  $3 \mapsto \text{"hello"}$ .
- `backward` is a map from *values* to *keys*.  
In the example above, it contains  $\text{"world"} \mapsto 2$  and  $\text{"hello"} \mapsto 3$ .

The data structure has the following invariant: `forward` contains the entry  $k \mapsto v$  if and only if `backward` contains the entry  $v \mapsto k$ . In other words, the implementation must make sure that `forward` and `backward` are always “mirror images” of one another.

On the next page is an incomplete pseudocode implementation of bidirectional maps. The `get` and `reverseGet` operations have already been implemented. Your job is to implement `put`! Make sure that your code works on the previous example — there is an important detail which is easy to miss.

Write your anonymous code (*not* your name):

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In this code, forward and backward are implemented as AVL trees. You can use all of the standard map operations on forward and backward, such as get, put, containsKey and remove.

```
class BidirectionalMap[Key, Value]:  
  // This class implements a bidirectional map where the keys are  
  // objects of class 'Key' and the values are objects of class 'Value'.  
  
  // We implement the maps 'forward' and 'backward' as AVL trees,  
  // but only remember that they are maps.  
  forward: Map[Key, Value] = empty AVL map  
  backward: Map[Value, Key] = empty AVL map  
  
  // Look up the value for a given key.  
  // Note on syntax: this declares that 'get' is a method, taking one  
  // parameter 'key' of type 'Key', returning an object of type 'Value'.  
  get(key: Key) -> Value:  
    return forward.get(key)  
  
  // Look up the key for a given value.  
  reverseGet(value: Value) -> Key:  
    return backward.get(value)  
  
  // Add an entry to the bidirectional map.  
  // This overwrites any existing entry with that key or value.  
  put(key: Key, value: Value):  
    // Your code goes here! Write your answer below,  
    // or on a separate piece of paper.  
    // You can use Java or Python syntax, or pseudo-code,  
    // as long as it is clear enough.
```

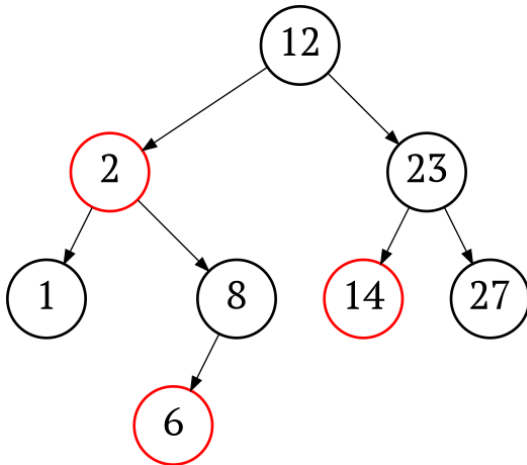
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Points for question (to be filled by the grader):

## Section 3: Search trees

In this section, the key ordering of search trees is the natural ordering of integers.

### Question 3A

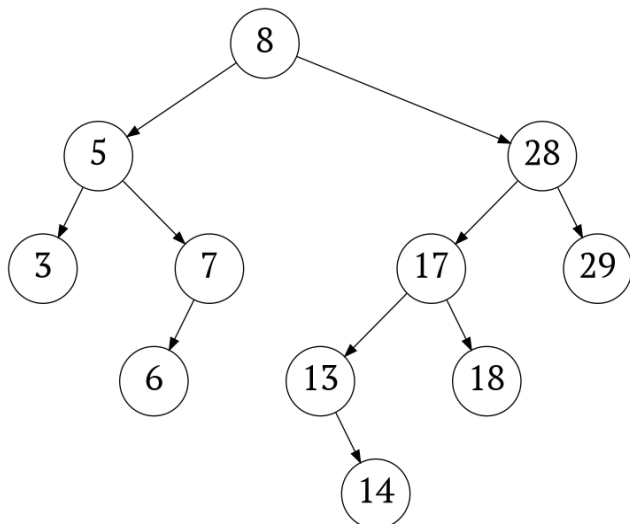


Is this a red-black tree?

☐ Yes.

☐ No.

If not, point out a concrete problem (be specific):



Is this an AVL tree?

☐ Yes.

☐ No.

If not, point out a concrete problem (be specific):

Write your anonymous code (*not* your name):

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### Question 3B

Find a 2-3 tree with the following properties:

- Its keys are the integers 1–8 except 4. In particular, each integer 1, 2, 3, 5, 6, 7, 8 appears exactly once in the tree.
- Inserting the key 4 will increase the height by one.

Draw the tree before inserting 4:

Draw the resulting tree after inserting 4:

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Points for question (to be filled by the grader):



## Section 4: Sorting

### Question 4A

Let `arr` be an array of  $n$  integers. The following algorithm is supposed to build a sorted version `sorted` of the array `arr`. Unfortunately, your cat has eaten the delicious parts of the algorithm:

```
sorted = new array of size n
for i from 0 to n:
    minIndex = i
    for j from i+1 to n:
        if _____ <= _____:
            minIndex = j
    sorted[i] = _____
    arr[minIndex] = arr[i]
```

(Clarification: A loop `for i from a to b` iterates over the values  $a, a+1, \dots, b-1$ .)

Repair the algorithm by filling in the missing expressions.

## Question 4B

In this question, we consider sorting a list of items by some kind of *key*. For example, we may want to sort a list of strings by their length, so that shorter strings come first. More formally, we have a function *key* whose values we can compare, and we sort the list so that  $x$  comes before  $y$  if  $\text{key}(x) < \text{key}(y)$ . In this example,  $\text{key}(x)$  would be the length of the string  $x$ .

A key-based sorting algorithm is *stable* if any two objects with the same key have the same relative order before and after sorting. That is, if  $x$  and  $y$  have the same key, and  $x$  comes before  $y$  in the input list, then  $x$  comes before  $y$  in the sorted output list. For example, consider the following list of strings:

*big, cats, eat, icecream, some, tuesdays*

If we sort these strings by their length, using a stable sorting algorithm, the result will be:

*big, eat, cats, some, icecream, tuesdays*

Notice that *big* and *eat* both have length 3, and *big* comes before *eat* both before and after sorting. Similarly, *cats* comes before *some*, and *icecream* comes before *tuesdays*. The strings are sorted in order of length, but strings with the same length have the same relative order as they did originally.

The following version of insertion sort takes an array `arr` of size  $n$  and sorts it by `key`:

```
for i from 0 to n:
    j = i
    while j > 0 and key(arr[j]) <= key(arr[j-1]):
        swap arr[j] and arr[j-1]
        j = j-1
```

Unfortunately, it is not stable.

Your tasks:

- The list “*big cats...*” above is an example for which stability is violated. Show how the resulting list looks after applying the sorting algorithm above.
- Change a single line in the pseudocode so that the algorithm becomes stable.
- Justify why your improved version is stable.

**(there is more space to answer on the next page)**

Write your anonymous code (*not* your name):

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**(more space to answer Question 4B)**

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Points for question (to be filled by the grader):

## Section 5: Stacks, queues, priority queue, hash tables

### Question 5A

Exactly one of the following arrays is **not** a binary min-heap.

**A**

1	2	1	3	2	2	1	4	3	3
---	---	---	---	---	---	---	---	---	---

**B**

1	2	3	4	8	5	6	6	7	7
---	---	---	---	---	---	---	---	---	---

**C**

0	3	2	5	4	8	9	7	6	5
---	---	---	---	---	---	---	---	---	---

Which one is it? Point out a concrete problem with it (be specific).

Write your anonymous code (*not* your name):

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### Question 5B

We consider an open addressing hash table with linear probing and modular compression. The maximum load factor is 75%, and when the table becomes overloaded, its array size grows by 50%.

Initially, the hash table looks like this:

0	1	2	3	4	5	6	7	8	9
18	39		13	63				28	9

Insert the following elements into the table, in this order: 30, 33, 14. Use the numbers themselves as hash values (modulo the array size to get the array index).

Draw the hash table *after each individual insertion*. If the hash table needs to be resized, draw it *both before and after* resizing. (This means that you should draw at least 3 hash tables below.)

**Note:** A simple way of calculating modulo N is to repeatedly subtract N until you get a value below N.

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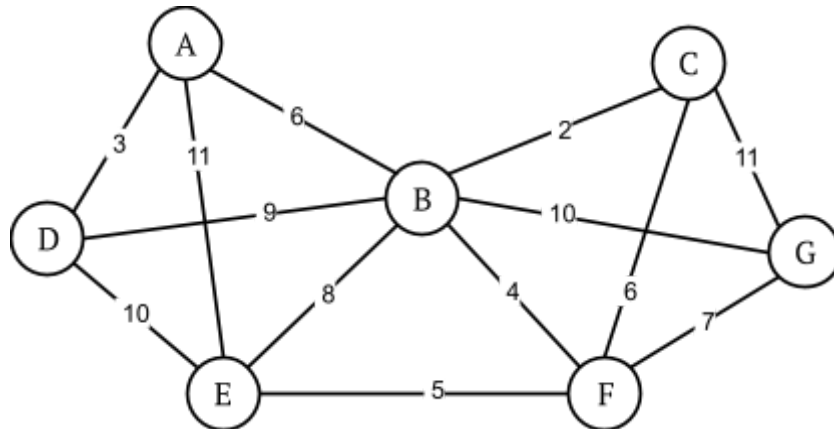
Points for question (to be filled by the grader):

Write your anonymous code (*not* your name):

## Section 6: Graphs

### Question 6A

Perform Kruskal's algorithm on the following weighted undirected graph.



List the edges of the minimum spanning tree in the order they are added:

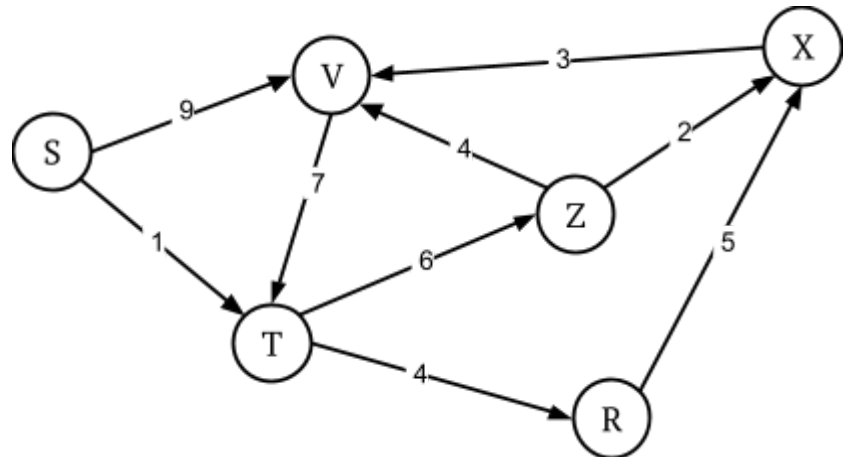

**Note:** The table has more rows than needed.

Points for question (to be filled by the grader):

Write your anonymous code (*not* your name):

### Question 6B

Here is a weighted directed graph:



And here is a trace of a run of Dijkstra's algorithm (UCS) on the graph, starting at node S:

Removed node	Added node(s)	Visited nodes	Priority queue after adding node(s)
—	S	—	S:0
S	T, V	S	T:1, V:9
T	R, Z	S, T	R:4, Z:6, V:9
R	X	R, S, T	X:5, Z:6, V:9
X	V	R, S, T, X	V:3, Z:6, V:9
V	T	R, S, T, V, X	Z:6, T:7, V:9
Z	V, X	R, S, T, V, X, Z	X:2, V:4, T:7, V:9
X	—	R, S, T, V, X, Z	V:4, T:7, V:9
V	—	R, S, T, V, X, Z	T:7, V:9
T	—	R, S, T, V, X, Z	V:9
V	—	R, S, T, V, X, Z	—

However, the person implementing the algorithm made some errors. Because of this, the above trace does not produce the correct shortest path tree.

Points for question (to be filled by the grader):

Write your anonymous code (*not* your name):

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**Question 6B (continued)**

a. Where in the trace does the algorithm start going wrong?

b. What is the problem with the implementation?

c. Show the trace for a corrected implementation:

Removed node	Added node(s)	Visited nodes	Priority queue after adding node(s)

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Points for question (to be filled by the grader):