ST2131 Cheatsheet
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Chapter 01 - Combinatorial Analysis

Multinomial Theorem

Let n be a non-negative integer, then

$$(x_1 + x_2 + \ldots + x_r)^n = \sum_{n_1+\ldots+n_r=n} \binom{n}{n_1, n_2, \ldots, n_r} x_1^{r_1} x_2^{r_2} \ldots x_r^{n_r}$$

where $\binom{n}{n_1,n_2,\ldots,n_r}$ is the Multinomial Coefficient, and it satisfies

$$\binom{n}{n_1, n_2, \ldots, n_r} = \binom{n}{n_1}\binom{n-n_1}{n_2} \ldots \binom{n-n_1-\ldots-n_{r-1}}{n_r} = \frac{n!}{n_1!n_2!\ldots n_r!}$$

Some Combinatorial Identities

For all non-negative integers m, n, k and $k \le n$,

- $k\binom{n}{k} = (n-k+1)\binom{n}{k-1} = n\binom{n-1}{k-1}$ (*AY20/21Sem2 Tut1Qn7*)
- $\sum_{k=1}^{n} k\binom{n}{k} = n2^{n-1}$ (*AY20/21Sem2 Tut1Qn8*)
- $\binom{n+m}{k} = \binom{n}{0}\binom{m}{k} + \binom{n}{1}\binom{m}{k-1} + \ldots + \binom{n}{r}\binom{m}{0}$ (*AY20/21Sem2 Tut1Qn9*)
- $\binom{2n}{n} = \sum_{k=0}^{n} \binom{n}{k}^2$ (*AY20/21Sem2 Tut1Qn10*)

Chapter 02 - Axioms of Probability

Inclusion/Exclusion Principle (Prop. 2.6)

Let $E_1, E_2, \ldots, E_n$ be any n events, then

$$\begin{aligned} P(E_1 \cup E_2 \cup \ldots E_n) = &\sum_{i=1}^{n} P(E_i) - \sum_{1 \le i_1 < i_2 \le n} P(E_{i_1} \cap E_{i_2}) + \ldots \\ &+ (-1)^{r+1} \sum_{1 \le i_1 < \ldots < i_r \le n} P(E_{i_1} \cap \ldots \cap E_{i_r}) \\ &+ \ldots + (-1)^{n+1} P(E_1 \cap \ldots \cap E_n) \end{aligned}$$

Generalised Bonferroni's Inequality

(*AY20/21Sem2 Tut2Qn16*) Let $E_1, E_2, \ldots, E_n$ be any n events, then

$$P(E_1 E_2 \ldots E_n) \ge P(E_1) + \ldots + P(E_n) - (n-1)$$

When $n = 2$,

$$P(E_1 E_2) \ge P(E_1) + P(E_2) - 1$$