SYSTEMATIC PREDICATE ABSTRACTION USING DEEP LEARNING

A PREPRINT

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ABSTRACT

Systematic Predicate Abstraction using Deep Learning

***K*eywords** First keyword · Second keyword · More

1 Introduction

Systematic Predicate Abstraction using Deep Learning

Reduce CEGAR iterations.

Summarize the pipline:

2 Background

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2.1 Abstraction Based Model Checking

Model checking is a technique for program verification. It builds a transition system based on variables' states in all control locations and to check if there is a path to the error states. The error states are built by specifications given by users. But, programs with infinite states are ubiquitous. Therefore, we need to find a finite transition system (abstract transition system $\widehat{M}$) to represent the infinite transition system M, such that if there is no path to the error state in $\widehat{M}$, M has no error as well, and if there is a path to the error state in $\widehat{M}$, we can use this concrete error path to check if there is a error in M.

The major challenge in this idea is to find a appropriate $\widehat{M}$. CEGAR [1] is a state-of-art framework to find $\widehat{M}$ iteratively. CEGAR framework can be described 1. It starts with a initial abstract transition system $\widehat{M}$. The model checker can check if there is a path to the error state. If there is no such path, the the system M is safe, otherwise give the path as a counterexample to check the feasibility. Checking feasibility means to use this counterexample as input to M If M return a error, this counterexample is feasible and M is not safe. If there is no error returned, this counterexample is infeasible which means $\widehat{M}$ does not represent M. Therefore, we need a new abstract transition system $\widehat{M}'$ obtained by a refinement process.

$\widehat{M}$ is built by a set of predicates $Pred$. To update $\widehat{M}$ to $\widehat{M}'$ means to update $Pred$ to a new predicate set $Pred'$ by adding new predicates. The new predicates are constructed by splitting the counterexample path to two parts A and B

*All contributions are considered equal.

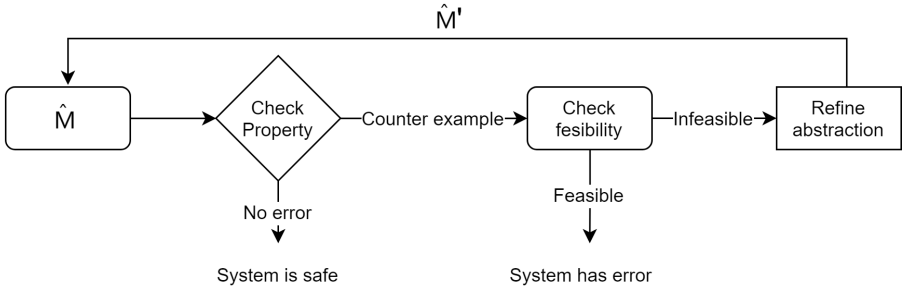

Figure 1: CEGAR framework

and find the Interpolants I of A and B, such that I consists of common elements in A and B, $A \Rightarrow I$, and $I \Rightarrow \neg B$. I is the new predicates to be added to $Pred$.

2.2 Abstract Interpolation

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3 Problem Overview

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4 Program representation

Eldarica transforms SMT or C format programs to intermediate horn clause format and the semantics are preserved [cite]. By using horn clauses as the learning inputs, we don't need to care about the input languages but focus on solving the logic problems. We have an example in Figure 2a and 2b to explain how the horn clauses capture semantics of the program.

In Figure 2a, we have a simple while loop C program which assumes variable x and y equal to external input Integer n and $n \geq 0$. The assertion is $y == 0$. The corresponding Horn clauses are described in Figure 2b. Every line is consists of $Head :- Body, [constraint]$. For example, in line 0, $Head$ is $inv\_main4(x, y)$ which means in control location $while(x! = 0)$ in the main function, there are two variables x and y. $Body$ is empty here because this is the initial state of the program. The $constraint$ is $n \geq 0 \wedge x = n \wedge n = 0 \wedge y = n$ which comes from the $assume(x == n \&\& y == n \&\& n >= 0)$ in the C program. Line 0 means in control location $while(x! = 0)$, there are two variables x, and y, which satisfy the $constraint : n \geq 0 \wedge x = n \wedge n = 0 \wedge y = n$. In line 1, The body is not empty, this means that there is a transition from body to head and this transition satisfy the $constraint$. Line 1 means from control location $while(x! = 0)$ to next iteration, it must satisfy that, $x! = 0$ and in the new iteration, the variables $arg1$ and $arg2$ in the control location $while(x! = 0)$ must equal to $x - 1$ and $y - 1$, (values of x, y come from last iteration). Line 2 transforms the semantic of assertion $y == 0$ to horn clauses. Line 2 means that from control location $while(x! = 0)$ to false state, in the control location $while(x! = 0)$, the variable x is 0 and y satisfy $y! = 0$.

```
0  extern int n;
1  void main(){
2      int x,y;
3      assume(x==n&&y==n&&n>=0);
4      while(x!=0){
5          x--;
6          y--;
7      }
8      assert(y==0);
9  }
```

(a) An input example: C program

```
0  inv_main4(x, y) :- ,[n >= 0 & x = n & 0 = n & y =
       n].
1  inv_main4(arg1, arg2) :- inv_main4(x, y), [x != 0
       & x + -1 = arg1 & y + -1 = arg2 & n = 0].
2  false :- inv_main4(0, y), [y != 0].
```

(b) Horn clauses for C program

Figure 2

Table 1: Nodes and hyperedges in graph

Graph elements	Name	Elements	Inputs	Outputs
V_{CL}	Control location node	$CL_{Initial}$, CL_{Main_k}, CL_{false}	he_{CL}	he_{CF}
V_{Vri}	Variable node	Arg_k(argument),FV_k (free variable)	v_{OP}, v_{Vri}, v_C,he_{DF}	he_{DF}
V_C	Constant node	Constants	-	v_{OP}, v_{Arg}, v_{FV}, he_{DF}, he_{CF}
V_{Op}	Operator node	Operators	v_{Arg}, v_{FV}, v_C, he_{DF}	v_{Arg}, v_{Op}, v_{FV}, he_{DF}, he_{CF}
V_{Pred}	Predicate node	Template types	v_{OP}, v_{Vri}, v_C	v_{CL}
HE_{CF}	Guarded control flow hyperedge	-	(v_{OP} or v_C, v_{CL})	v_{CL}
HE_{DF}	Guarded data flow hyperedge	-	(v_{OP} or v_C, v_{Arg} or v_{FV})	v_{Arg}

Table 2: Edges in graph

Graph elements	Name	Start from	End at
E_{CFI}	Control flow in hyperedge	v_{CL}	he_{CF}
E_{CFO}	Control flow out hyperedge	he_{CF}	v_{CL}
E_{DFI}	Data flow in hyperedge	v_{OP}, v_{Arg}, v_{FV}, v_C	he_{DF}
E_{DFO}	Data flow out hyperedge	he_{DF}	v_{OP}, v_{Arg}
E_{CD}	Condition edge	v_C, v_{OP}	(he_{DF}, he_{CF})
E_{Arg}	Argument edge	v_{Arg}	v_{CL}

How to transform program semantics to horn clauses and why it can be transformed in this way can be found in [cite]. Our main purpose is to use this horn clauses format as input of deep learning structure to select better templates to reduce the number of iterations in CEGAR loop.

Text streamed Horn clauses as inputs are not enough to make multilayer perceptron understand the semantics because it is hard to use text-level embedding to capture the structural information included in the horn clauses. Therefore, we represent horn clauses by a graph which contains both control flow and data flow information, then we embed the graph to represent the original program.

The graph in Figure represents horn clauses in Figure 2b.

Formally, the graph consists of four categories of nodes, two categories of hyperedges, and six categories of edges. The graph is defined as $G = (V_{CL}, V_{Vri}, V_C, V_{Op}, HE_{CF}, HE_{DF}, E_{CFI}, E_{CFO}, E_{DFI}, E_{DFO}, E_{CD}, E_{Arg}, V_{Pred})$, in which $V_{CL} = \{cl_{Initial}, cl_{Main_k}, cl_{false}\}$ is control location node set, $V_{Arg} = \{Arg_k\}$ is argument set, V_C is constant value set, $V_{Op} = \{+, -, ==, ...\}$ is operator set,$V_{FV} = \{v_k\}$ is free variables set. k is a positive integer. HE_{CF} is a set of Guarded control flow hyper edges. it is used between two control locations. it takes two inputs, a control location $cl_{Initial}$ or cl_{Mian_k} and a boolean value from operator. It has a control flow out edge e_{CFO} (one element from E_{CFO}) connected to next control location node. Similarly, HE_{DF} is a set of Guarded data flow hyper edges. It guards the data flow from $Body$ to $Head$. It takes two inputs, a value from argument Arg_k or free variable v_k and a boolean value and has a data flow out edge e_{DFO} (one element from E_{DFO}) connected to next operator or argument node. E_{CFI}, E_{CFO} are control flow in and control flow out edges, they are only used when there are connections between nodes and guarded control flow hyperedges. similarly, E_{DFI}, E_{DFO} are data flow in and out edges, they are only corresponding to guarded data flow hyperedges. E_{CD} is condition edge. It connects boolean operator to the hyperedges to send boolean values to the hyperedges. E_{Arg} is argument edge. It connects arguments to corresponding control location node. We use lower case to represent one elements in the set. For example v_{CL} means one control location node from V_{CL}. V_{Pred} is predicate node set. It connects the predicate's AST tree and the predicate's location.

The summary of G's nodes and hyperedges are shown in Table 1 and edges are shown in 2 respectively.

Graph construction. The graph is constructed by parsing the horn clauses line by line. The $Head$ contains a control location and arguments. Argument nodes (represented as circles) and the control location nodes (represented as

rectangular) are connected by the $Argumentedges$ (dotted edges). $Body$ has same structure with $Head$. If $Head$ and $Body$ have the same control location, then they share the control location and arguments.

The $Constraint$ contains the constraints to make the transition from $Body$ to $Head$ satisfiable. We parse it to two information, data flow AST and constraint AST. Data flow AST takes $Body'$ arguments, constant value, or free variables in $constraint$ as inputs, and go through some arithmetic operator to the root as output (a value) eventually to an $Head'$ argument. The constraint AST takes $Body$' arguments, free variables in $constraint$ as the inputs. The root of the constraint AST tree is a boolean operator which can output a boolean value to all guarded control and data flow hyperedges from $Body$ to $Head$. If there is no $constraint$, the boolean value inputs to the hyperedges are set to true.

We can represent the transition by the definition of graph:

$Head(CL_{Main_m}, Arg_{head}) : -Body(CL_{Main_n}, Arg_{body}), [Constraint(Arg_{head}, Arg_{body}, V_C, V_{Op}, V_{FV})]$, where m and n are positive integer numbers. They can be equal or not equal. Arg_{head} and Arg_{body} are the set of arguments in head and body respectively. If $m == n$, $Arg_{head} == Arg_{body}$.

To associate $Head$ and $Body$, we represent control flow information first by connecting the control location from $Body$ to $Head$, and between the control locations, there is a guarded control flow hyperedge which takes $Body$' control location and the boolean value from AST tree constructed from $constraint$ as inputs and output the control location information to $Head$'s control location. Then, we build data flow between $Head$ and $Body$. Every argument in $Head$ is guarded by a guarded data flow hyper edge which takes $Body$' arguments, data flow AST tree root, or free variables and the boolean value from $constraint$'s AST tree as inputs. In summary, all the control and data flows from $Body$ to $Head$ are guarded by a hyperedge, and one of the inputs of the hyperedges is the boolean value from the root of constraint AST tree constructed by the predicates in $constraint$.

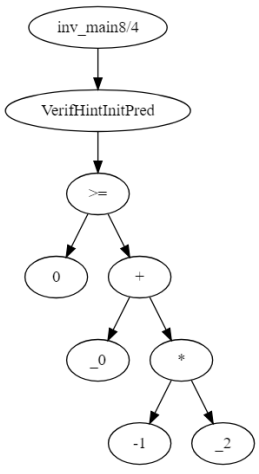
Graph Embedding.

4.1 Templates Representation

Each template contain location, category, and predicate information. One example template is "inv_main8:VerifHintInitPred(((_0 + -1 * _2) >= 0))" in which "inv_main8" is its control location,"VerifHintInitPred" is its category (The meaning of the categories can be found in appendix), and "(((_0 + -1 * _2) >= 0))" is the predicate. _0 and _2 are canonical encoding of variable names in source code. For example, _0 is x and _2 is y. We can combine the three components into one graph. The root node contains the location information ("inv_main8") as its node attribute. The only child of the root contains the category information ("VerifHintInitPred"). The predicate ("(((_0 + -1 * _2) >= 0))") can be represent as a binary tree. We connect the binary tree's root to the category node to connect all parts together. One example is shown in Figure 3b. We use graphviz to manipulate the graph. The corresponding graphviz text representation of the template is shown in 3a

```
0  digraph dag {
1  0 [label="inv_main8/4"];
2  1 [label="
       VerifHintInitPred"];
3  2 [label=">="];
4  3 [label="0"];
5  4 [label="+"];
6  5 [label="_0"];
7  6 [label="*"];
8  7 [label="-1"];
9  8 [label="_2"];
10 0->1
11 1->2
12 2->4
13 2->3
14 4->6
15 4->5
16 6->8
17 6->7
```

(a) Template graph example in graphviz format



(b) Template graph example

Figure 3: Graph representation for an example template "inv_main8:VerifHintInitPred(((_0 + -1 * _2) >= 0))"

5 Data Collection

For a single instance of data extracting from manual[2] or simple heuristics[3], the inputs are a list of templates $T_0, T_1, ..., T_n$ and a program P. The output is a list of templates with 0 or 1 labels $T_0^l, T_1^l, ..., T_n^l, l = 0$ or 1. A concrete example for an output is $T_0^1 = (\text{"}inv\_main8 : VerifHintInitPred(((\_0 + -1 * \_2) >= 0))\text{"}, 1)$

Algorithm 1: Templates extracting process

Result: $T_0^l, T_1^l, ..., T_n^l$
initialization: CurrentTemplateList = $\{T_0, T_1, ..., T_n\}$;
Solvalbility=CEGAR(CurrentTemplateList,HornClauses);
if *Solvalbility == True* **then**
  while *CurrentTemplateList is not empty* **do**
    CurrentTemplateList = CurrentTemplateList $-\{T_k\}(0 \leqslant k \leqslant n)$;
    Solvalbility=CEGAR(CurrentTemplateList,HornClauses);
    if *Solvalbility == True* **then**
      RedunantTemplateList=RedunantTemplateList $\cup\ T_k^0$;
    else
      CriticalTemplateList=CriticalTemplateList $\cup\ T_k^1$;
    end
    Templates=RedunantTemplateList $\cup$ CriticalTemplateList;
  end
else
  Cannot solve within timeout, no template extracted;
end

Algorithm 2: Predicates extracting process

Result: $T_0^l, T_1^l, ..., T_n^l$
initialization: CurrentTemplateList = $\{T_0, T_1, ..., T_n\}$;
Solvalbility,CurrentPredicateList=CEGAR(CurrentTemplateList,HornClauses);
if *Solvalbility == True* **then**
  while *CurrentPredicateList is not empty* **do**
    CurrentPredicateList = CurrentPredicateList $-\{P_k\}(0 \leqslant k \leqslant n)$;
    CurrentTemplateList=transform(CurrentPredicateList)
    Solvalbility=CEGAR(CurrentTemplateList,HornClauses);
    if *Solvalbility == True* **then**
      RedunantPredicateList=RedunantPredicateList $\cup\ P_k^0$;
    else
      CriticalPredicateList=CriticalPredicateList $\cup\ P_k^1$;
    end
  end
  Templates=transform(RedunantPredicateList $\cup$ CriticalPredicateList);
else
  Cannot solve within timeout, no template extracted;
end

The final goal is to have proper predicates that can represent the abstract transition system. The templates are heuristics to generate the predicates in each CEGAR iteration. There are additional predicates added to solve the program in every CEGAR iterationl but the templates are given and fixed before the CEGAR iteration.

Extracting data with variable timeout: First, confirm the solvability (i.e. the program can be solved within 60 seconds with the abstraction heuristic). Second, record the solving time with and without the abstraction heuristic. If Eldarica takes less solving time with abstraction heuristic, pass this solving time as timeout to Eldarica to label the templates used in the program.

1. chc-comp benchmarks: 40/1216. 32 programs (204 templates)for training and 8 (47 templates) for testing. 8/8 solved by read templates. Read templates time consumption/original templates consumption = 40.25/40.86 (in seconds)

2. sv-comp smt benchmarks: 15/5911. 12 programs (254 templates)for training and 3 (74 templates) for testing. 3/3 solved by read templates. Read templates time consumption/original templates consumption = 28.07/27.28 (in seconds)
3. sv-comp c benchmarks: 41/555. 32 programs (4635 templates)for training and 9 (350 templates) for testing. 8/9 solved by read templates. Read templates time consumption/original templates consumption = 34.04/53.46 (in seconds)

96 training programs. 76 programs for training, 20 programs for testing. 18/20 solved by read templates. Read templates time consumption/original templates consumption = 140.67/126.84 (in seconds)

Extracting data on larger benchmarks, and use these three benchmarks only for testing.

We try to use balanced and imbalanced data set to train the neural network.

5.1 Argument labelling

After we separate templates according to their usefulness, for every argument in a location, we count the occurrence in useful templates in that location to be argument's score which is the training label.

6 Training Model

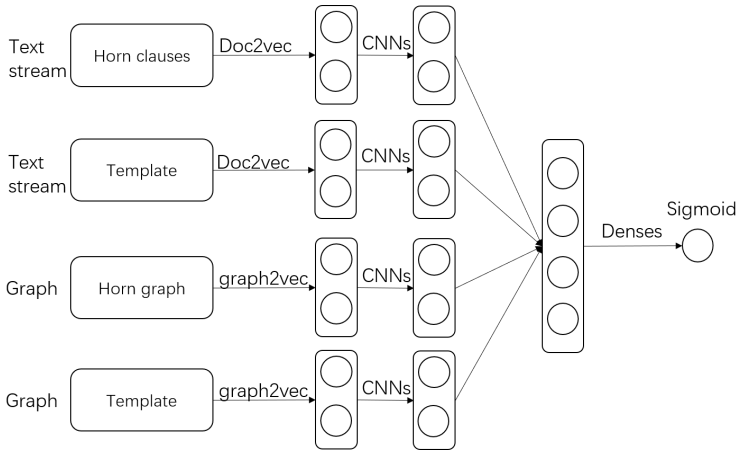


Figure 4: Neural network structure

6.1 Program Embedding

6.1.1 Text Level

Doc2Vec [4] embed different length of sentences to fixed vector. Program text streams are embedded into 100 dimensions. Template text streams are embedded to 20 dimensions.

6.1.2 Graph Level

Graph2Vec[5]. WL relabeling process [6] to generate random subgraphs (can be seen as sentences). Then use Doc2Vec embed subgraphs to fixed vector. Program graphs (horn graphs) are embedded into 100 dimensions. Templates are embedded to 20 dimensions.

7 Experiments

The predicted result is a score for a template. We add two rank modes to decide which templates will be used for solving a particular program. For example, we have 10 templates, their scores ranged from 0 to 1. The first rank mode can specify a threshold for the scores. If the threshold is 0.4, all templates that have scores larger than 0.4 will be used to solve the program, and the left templates are discarded. The second rank mode can specify a hierarchical number (N)

Table 3: Benchmarks

Benchmarks	File type	Total file	Solved	sat	unsat	Unsolved		
chc-comp-smt	.smt2	1216	421	339	82	795		
sv-comp-smt	.smt2	5911	4730	4516	214	1181		
sv-comp-c	.c	555	323	262	61	231		
In total	-	-	-	-	-	-	-	-

which decides to use top N ranked templates by score. If we specify the hierarchical number (N) be 5, then the top 5 ranked templates by score will be used to solve the program, and the left 5 templates will be discarded.

7.1 Ablation studies

Table 4: Imbalanced Predicates with control flow and template graph

Benchmarks	Training program	Testing program	Training predicates	Testing predicates	Solved programs	Total time consumption (predicted:abstract)
chc-comp-smt	130	33	969	225	33/33	63.92 : 63.72 (s)
sv-comp-smt	784	196	5907	1406	196/196	514.60 : 511.186
sv-comp-c	116	29	4148	847	20/29	578.33 : 56.74
In total	1030	258	10588	2914	250/258	1119.73 : 629.82

Table 5: Imbalanced Templates with control flow and template graph

Benchmarks	Training program	Testing program	Training predicates	Testing predicates	Solved programs	Time consumption (predicted/abstract)
chc-comp-smt	-	-	-	-	-	- (s)
sv-comp-smt	-	-	-	-	-	-
sv-comp-c	-	-	-	-	-	-
In total	-	-	-	-		

CEGAR iterations:

Solvability:

Benchmarks	Total file	with template graph	with horn graph	with control flow graph	with control flow and template graphs	with horn and template graphs
chc-comp-smt	1216	-	-			
sv-comp-smt	5911	-	-			
sv-comp-c	555	-	-			

Time consumption:

Benchmarks	Total file	with template graph	with horn graph	with control flow graph	with control flow and template graphs	with horn and template graphs
chc-comp-smt	1216	-	-			
sv-comp-smt	5911	-	-			
sv-comp-c	555	-	-			

8 Related work

Text level learning for improving ATP (formal verification?) [7]

Guiding formal method's search process.

Program graph representation.

Graph neural networks. End-to-end graph embedding. Message passing, convolutional graph neural network. Define a the first state of nodes. Update node representations by edges type and the connected node. Iteration is intuitively the filters of CNNs. The edges, hyper-edges, messages should capture the characteristics or relations between nodes. To deal with arbitrary length of information (different number of nodes), aggregation or abstraction then aggregation.

AST embedding: code2vec [8]. TBCNN [9].Learning to represent programs with graphs[10].

FormulaNet [11]. GAT [12].

Unsupervised graph embedding Before combining with deep learning, graph embedding works in the context of dimensionality reduction. The typical techniques are principle component analysis (PCA) and multidimensional scaling (MDS) DeepWalk maps the graph's nodes to sentence's words and uses word embedding technique skip-gram to perform the node embedding. along with skip-gram word embedding later on been extended to Doc2Vec, DeepWalk node embedding is extended to Graph2Vec.

9 Discussion and Future work

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