NANYANG TECHNOLOGICAL UNIVERSITY SINGAPORE

SC3000/CZ3005 Artificial Intelligence

Constraint satisfaction and adversarial search

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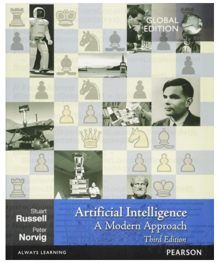
Lesson Outline

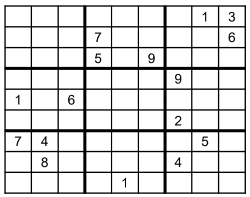
- Constraint Satisfaction
- Adversarial search (Game Playing)



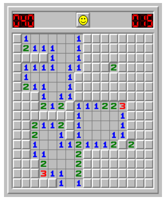
Constraint Satisfaction Problem (CSP)

Goal: discover some state that satisfies a given set of constraints

Example: Sudoku



Example: Minesweeper



Examples: Real-world CSPs

- Assignment problems
  - e.g. who teaches what class
- Timetabling problems
  - e.g. which class is offered when and where?
- Hardware configuration
- Transportation scheduling
- Factory scheduling
- Floor-planning

CSP

State

- defined by variables V_i with values from domain D_i

Example: 8-queens

- Variables: locations of each of the eight queens
- Values: squares on the board

Goal test

- a set of constraints specifying allowable combinations of values for subsets of variables

Example: 8-queens

- Goal test: No two queens in the same row, column or diagonal

Example: Cryptarithmetic Puzzle

```
    S E N D
+   M O R E
-----------
  M O N E Y
```

M=1

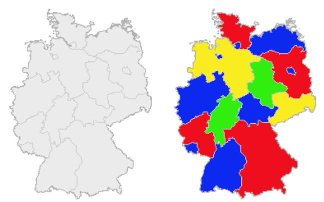
Starting from left you can solve the problem more easily: domain knowledge

- Variables: D, E, M, N, O, R, S, Y
- Domains: {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}
- Constraints
  - $Y = D + E$ *or* $Y = D + E - 10$, etc
  - D ≠ E, D ≠ M, D ≠ N, etc.
  - M ≠ 0, S ≠ 0 (unary constraints: concern the value of a single variable)

Example: Map Colouring

NP hard

Colour a map so that no adjacent parts have the same colour



- Variables: Countries *Ci*
- Domains: {*Red, Blue, Green*}
- Contraints: C1 ≠ C2, C1 ≠ C5, etc.
  - binary constraints

Some Definitions

- A state of the problem is defined by an **assignment** of values to some or all of the variables.
- An assignment that does not violate any constraints is called a **consistent** or **legal** assignment.
- A **solution** to a CSP is an assignment with every variable given a value (**complete**) and the assignment satisfies all the constraints.

Applying Standard Search

- States: defined by the values assigned so far
- Initial state: all variables unassigned
- Actions: assign a value to an unassigned variable
- Goal test: all variables assigned, no constraints violated

Applying Standard Search

Question: How to represent constraints?

Answer: Explicitly (e.g., D ≠ E)

Example

- Row the 1st queen occupies: $V_1 \in \{1, 2, 3, 4, 5, 6, 7, 8\}$ (similarly, for V_2)
- No-attack constraint for V_1 and V_2: { <1, 3> , <1, 4>, <1, 5>, … , <2, 4>, <2, 5>, …}

Implicitly: use a function to test for constraint satisfaction

$[g(x)-0] \leftarrow$ penalty

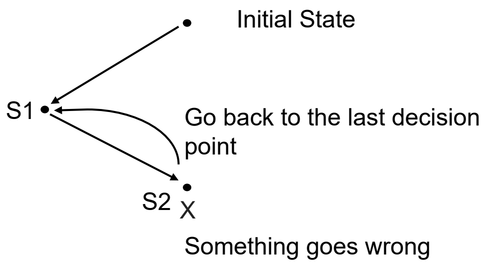
$\min f(x) + g(x)$

s.t. $g(x) \leq 0$

Language multiplier → essentially same

Backtracking Search

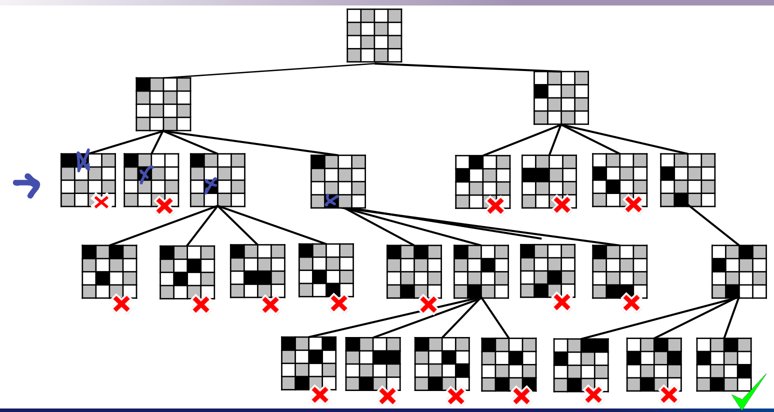
Backtracking search: Do not waste time searching when constraints have already been violated



- Before generating successors, check for constraint violations
- If yes, backtrack to try something else

uninformed search

Example (4-Queens)



Heuristics for CSPs

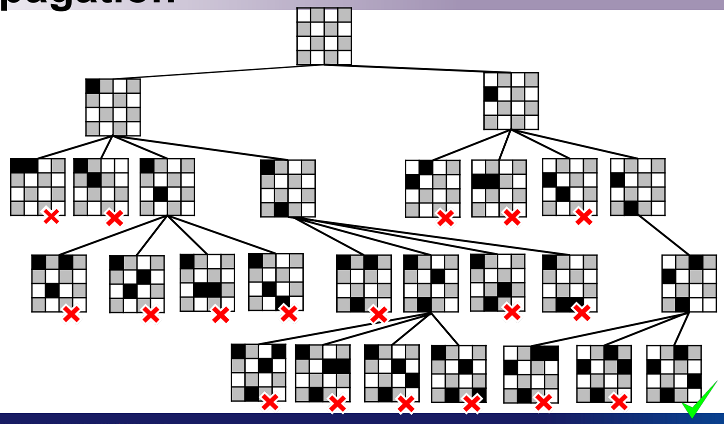
Plain backtracking is an uninformed algorithm!!

More intelligent search that takes into consideration

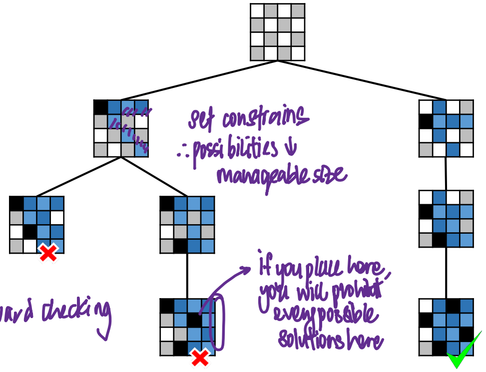
- Which variable to assign next
- What order of the values to try for each variable
- Implications of current variable assignments for the other unassigned variables
    - forward checking and **constraint propagation**

Constraint propagation: propagating the implications of a constraint on one variable onto other variables

Example (4-Queens) without Constraint Propagation



Search Tree of 4-Queens with Constraint Propagation



set constrains ∴ possibilities ↓ manageable size

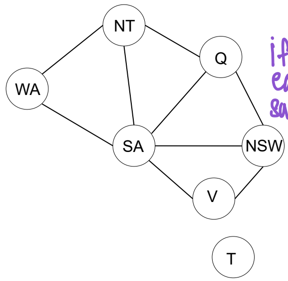
order might matter too!

if you place here, you will prohibit every possible solutions here

forward checking

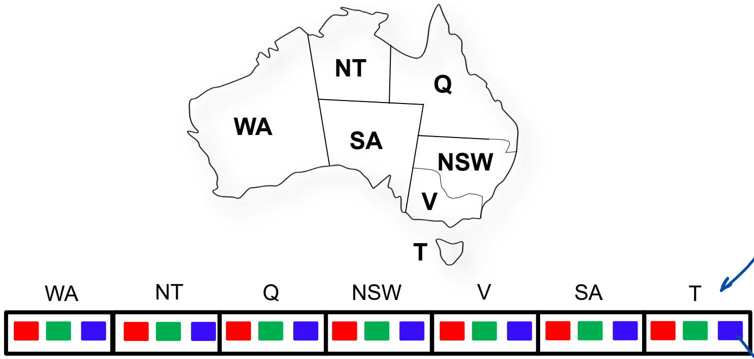
Example (Map Colouring)

Goal: No two same colour for neighbours



if there's an edge, should not use same colors for two states

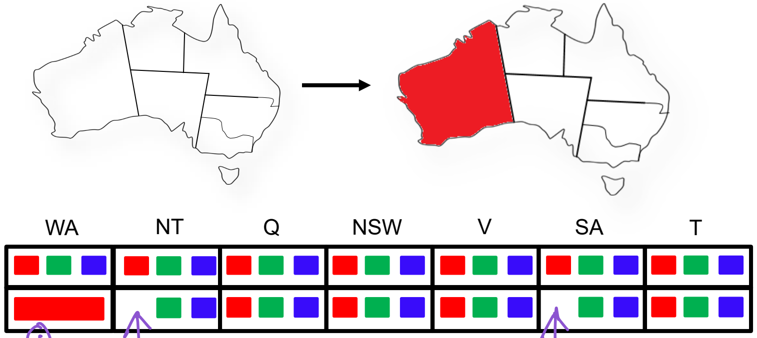
Example (Map Colouring)...



every state is available

domain is 3 colours

Example (Map Colouring)...



1

Red is removed

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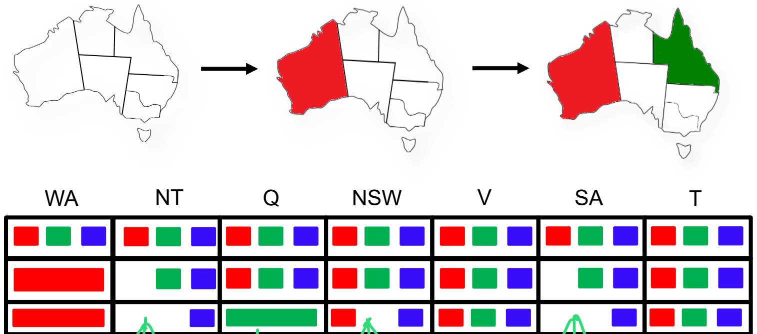
Example (Map Colouring)...



2

Removed.

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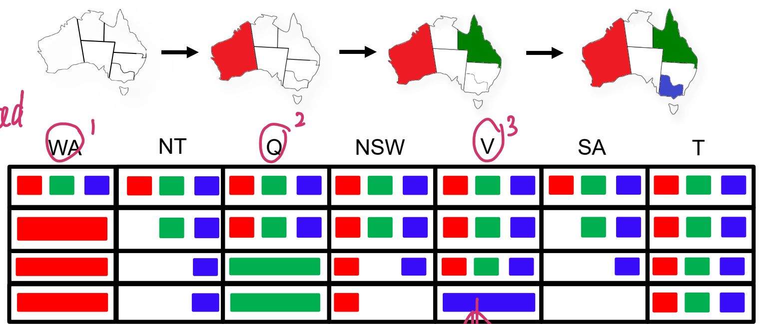
Example (Map Colouring)...

Should we have started elsewhere



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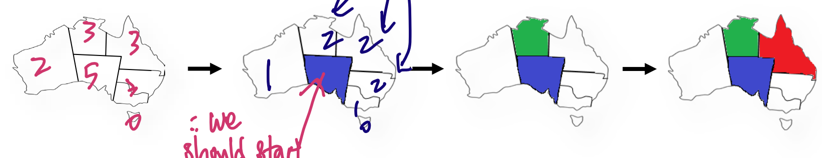
IDEA: Identify the most constraining variable

Most Constraining Variable

Example: map colouring

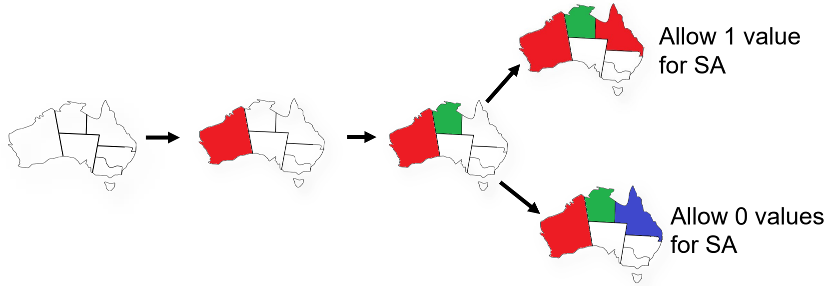
choose one of these.



∴ we should start here!

To reduce the branching factor on future choices by selecting the variable that is involved in the largest number of constraints on unassigned variables.

Least Constraining Value

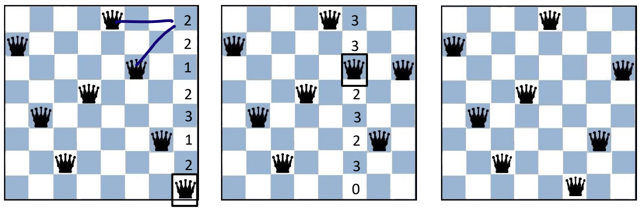


Choose the value that leaves maximum flexibility for subsequent variable assignments

Min-Conflicts Heuristic (8-queens)

- A local heuristic search method for solving CSPs
- Given an initial assignment, selects a variable in the scope of a violated constraint and assigns it to the value that minimises the number of violated constraints

No garantee



Games as Search Problems

Abstraction

- Ideal representation of real world problems
  - e.g. board games, chess, go, etc. as an abstraction of war games
  - Perfect information, i.e. fully observable
- Accurate formulation: state space representation

Uncertainty

- Account for the existence of hostile agents (players)
  - Other agents acting so as to diminish the agent's well-being
  - Uncertainty (about other agents' actions):
    - not due to the effect of non-deterministic actions
    - not due to randomness

    → Contingency problem

Games as Search Problems...

Complexity

- Games are abstract but not simple
  - e.g. chess: average branching factor = 35, game length > 50
  - → complexity = 35^{50}(only 10^{40} for legal moves)

- Games are usually time limited
  - Complete search (for the optimal solution) not possible
  - → uncertainty on actions desirability
  - Search efficiency is crucial

Types of Games

	Deterministic	**Chance**
Perfect information	Chess, Checkers, Go, Othello	Backgammon, Monopoly
Imperfect information		Bridge, Poker, Scrabble, Nuclear war

Perfect information

- each player has complete information about his opponent's position and about the choices available to him

Game as a Search Problem

- Initial state: initial board configuration and indication of who makes the first move
- Operators: legal moves
- Terminal test: determines when the game is over
  - states where the game has ended: terminal states
- Utility function (payoff function): returns a numeric score to quantify the outcome of a game

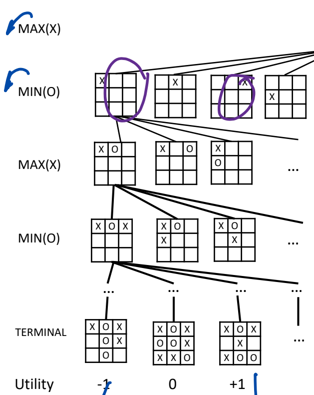
Example: Chess

Win (+1), loss(-1) or draw (0)

Game Tree for Tic-Tac-Toe

you max your credits / utility

opponent: wants to min your credits / max his credits



→ same states.

How should we prune the tree?

→ Reduce # of states (9 → 3)

- The utility value of the terminal state is from the point of view of MAX
- MAX uses the search tree to determine the best move

→ symmetric states

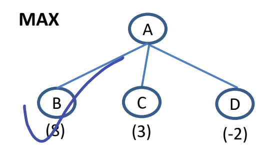
opponents win!

-1 because your utility is minimized by me.

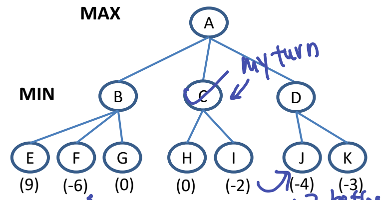
you win

What Search Strategy?

One-play



Two-play



From my perspective

opponent -9

if I choose B opponent chooses F

-2 better than -4

Minimax Search Strategy

Search strategy
- Find a sequence of moves that leads to a terminal state (goal)

Minimax search strategy
- Maximise one's own utility and minimise the opponent's
  - Assumption is that the opponent does the same

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Minimax Search Strategy

3-step process

1. Generate the entire game tree down to terminal states
2. Calculate utility
   a) Assess the utility of each terminal state
   b) Determine the best utility of the parents of the terminal state (bottom up fashion)
   c) Repeat the process for their parents until the root is reached
3. Select the best move (i.e. the move with the highest utility value)

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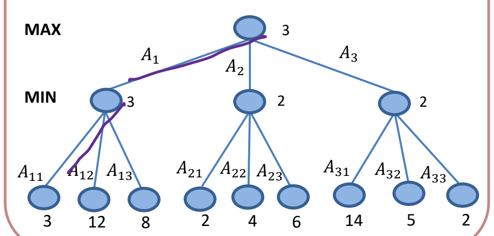
Perfect Decisions by Minimax Algorithm

Perfect decisions: no time limit is imposed
- generate the complete search tree

Two players: MAX and MIN
- Choose move with best achievable payoff against best play
- MAX tries to max the utility, assuming that MIN will try to min it

Example



*Remember whose turn it is.

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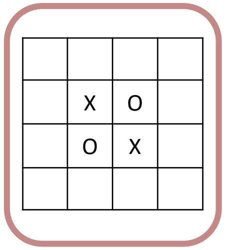
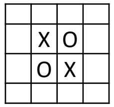
Othello 4



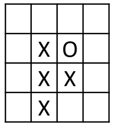
- A player can place a new piece in a position if there exists at least one straight (horizontal, vertical, or diagonal) occupied line between the new piece and another piece of the same kind, with one or more contiguous pieces from the opponent player between them
- After placing the new piece, the pieces from the opponent player will be captured and become the pieces from the same Player
- The player with the most pieces on the board wins

`X' plays first

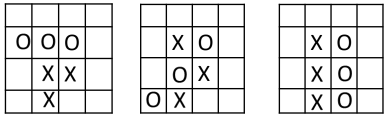
X considers the game now



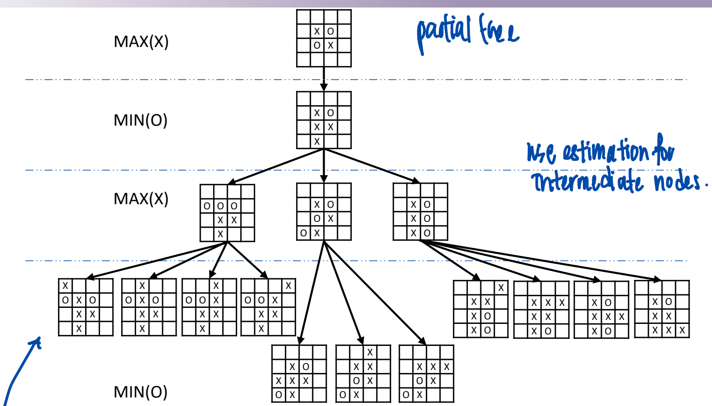
O considers the game now



X considers the game now



Game Tree Othello 4



partial tree

use estimation for intermediate nodes.

this is not the terminal nodes.

Imperfect Decisions

Go: Huge search game tree
- difficult to generate complete game tree

For chess, branching factor ≈ 35, each player typically makes 50 moves → for the complete game tree, need to examine 35^{100} positions

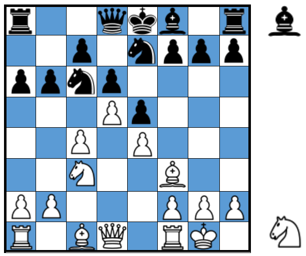
Time/space requirements → complete game tree search is intractable → impractical to make perfect decisions

Modifications to minimax algorithm

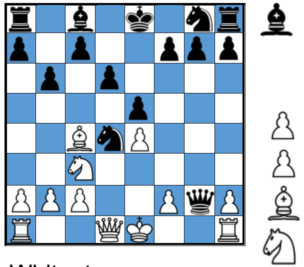
1. replace utility function by an estimated desirability of the position
   - Evaluation function
2. partial tree search
   - E.g., depth limit
   - Replace terminal test by a cut-off test

Evaluation Functions

Returns an estimate of the expected utility of the game from a given position



Black: to move
White: slightly better



White: to move
Black: winning

Evaluation Functions...

Requirements

- Computation is efficient
- Agrees with utility function on terminal states
- Accurately reflects the chances of winning

Trade off between accuracy and efficiency

Example (Chess)

Define features

- e.g. , (number of white queens) – (number of black queens)

Use a weighted sum of these features

$$Eval(s) = w_1 f_1(s) + w_2 f_2(s) + \cdots w_n f_n(s)$$

- Need to learn the weight

heuristic
no one knows the truth

- for AlphaGo Eval
can be a deep neural
Network

Evaluation Functions for Othello 4

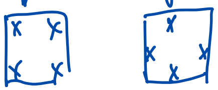
- A corner of the board is one of the most important positions. A piece at the corner can help capture other pieces from the opponent player
- A square at the border is also more important than any position in the middle of the board

Heuristics for `X' is proposed as follows:

- For any non-terminal game state, the evaluation function is computed as

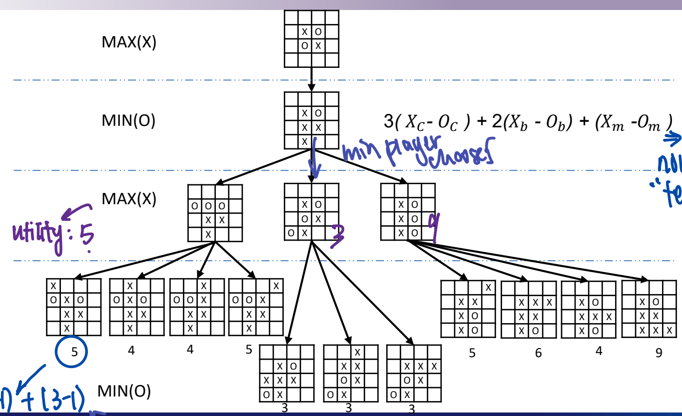
$$3(X_C - O_C) + 2(X_b - O_b) + (X_m - O_m)$$

where X_C is the number of X's at corners,
X_b is the number of X's at the border (excluding corners) ,
X_m is the number of X's in the middle of the grid,
O_C , O_b and O_m are the number of O's at the corners, the border and the middle of the board



mediocre
spots.

Evaluation Functions for Othello 4 ...



$3(X_C - O_C) + 2(X_b - O_b) + (X_m - O_m)$ → Helps us to assume non-terminal states to "terminal states"

min player chooses

utility: 5

3

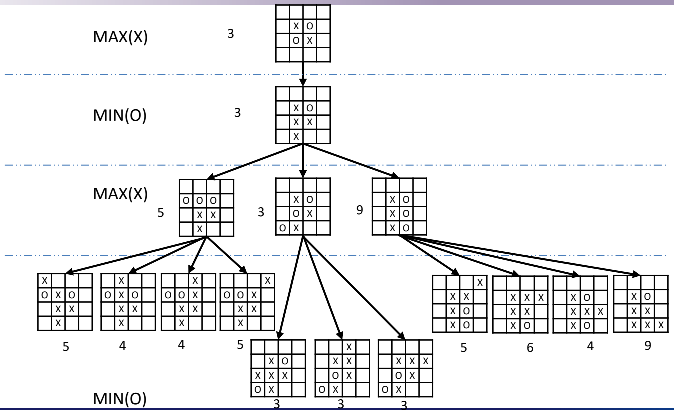
4

X_C O_C

$3(1-0)+2(1-1)+(3-1)$
$=5$

X_b O_b X_m O_m

Minimax Search for Othello 4



When do you stop

- use depth limit / mem
...
- use quiescence search s find a settlepoint

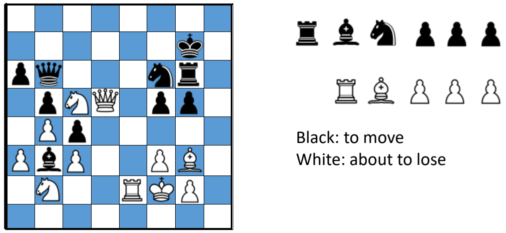
Quiescent Search (heuristic)

The evaluation function should only be applied to quiescent positions

– positions that are not likely to have large variations in evaluation in the near future

Example (Chess)

Positions in which favorable captures can be made

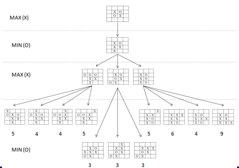


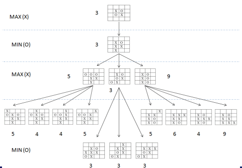
Black: to move
White: about to lose

Expansion of non-quiescent positions until quiescent positions are reached

AlphaGo: Key Ideas

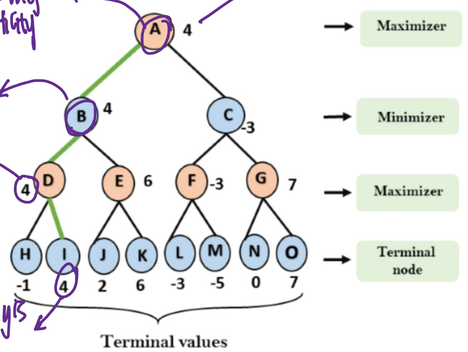
- Objective: Reduce search space without sacrificing quality
- Key Idea 1: Take advantage of human top players' data
  - Deep learning
- Key Idea 2: Self-play
  - Reinforcement learning
- Key Idea 3: Looking ahead
  - Monte Carlo tree search
  - We learned Minimax search with evaluation functions





3)

A Toy Min-Max Example



max my utility

this value is useless

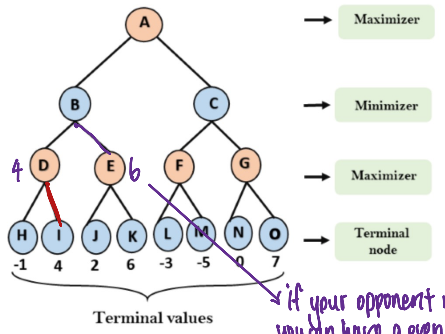
opponent: min my utility

if opponent comes here, my future=4

my utility is 4

this is how both of us would play

A Toy Min-Max Example



4

6

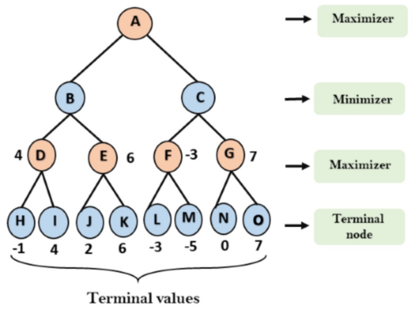
Assume you have the whole search tree

Step 1:
Expand the whole game tree.
Initialize each node to –Inf for max
+Inf for min

if your opponent makes a stupid move, you can have a even better future/utility

if you over estimate your opponent your actual utility might be lower.

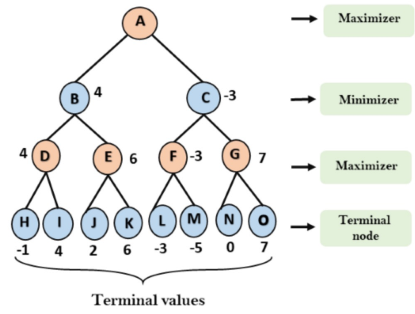
A Toy Min-Max Example



Step 2:
Update the value from btm to top

e.g., for node D:
We want to win→max the value (-1, 4)

A Toy Min-Max Example



Step 2:
Update the value from btm to top

e.g., for node D:
I want to win→max the value (-1, 4)

e.g., for node B:
You want me to lose → min the value (4, 6)

A Toy Min-Max Example