FIN2704/X
Week 3

Time value of money

Present value: value today (at t=0)

- **Discounting** some future amount

$$PV = \frac{FV}{(1+i)^n}$$

Future value: value sometime in the future (at t>0)

- **Compounding** some present amount

$$FV = PV(1+i)^n$$

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PV and FV

PV is inversely related to future value <u>on the effect of interest (i) and number of periods (n)</u>

(referring to slide 27)

<u>Interest rate (i)</u>

What is the present value?

$PV = FV/(1+i)^n = 100/(1+i)^1$

i=0% → PV = $100
i=5% → PV = $95.24
i=10% → PV = $90.91

What is the future value?

$FV = PV*(1+i)^n = 100(1+i)^1$

i=0% → FV = $100
i=5% → FV = $105
i=10% → FV = $110

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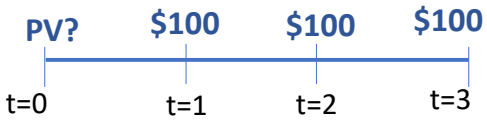
PV and FV (cont.)

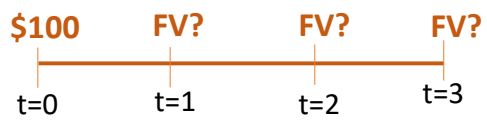
Number of periods (n)



What is the present value?
(Assume i=5%)

$PV = FV/(1+i)^n = 100/(1.05)^n$

n=1 → PV = $95.24
n=2 → PV = $90.70
n=3 → PV = $86.38



What is the future value?
(Assume i=5%)

$FV = PV*(1+i)^n = 100(1.05)^n$

n=1 → FV = $105
n=2 → FV = $110.25
n=3 → FV = $115.76

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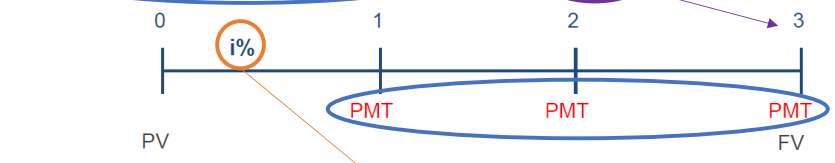
Annuity and perpetuity

Annuity:

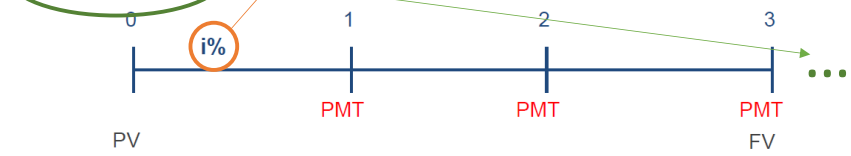
- Steady periodic payments for a finite number of periods



Period rate, matching the periodicity of the payments

Perpetuity:

- Steady periodic payments for infinite number of periods **(forever)**



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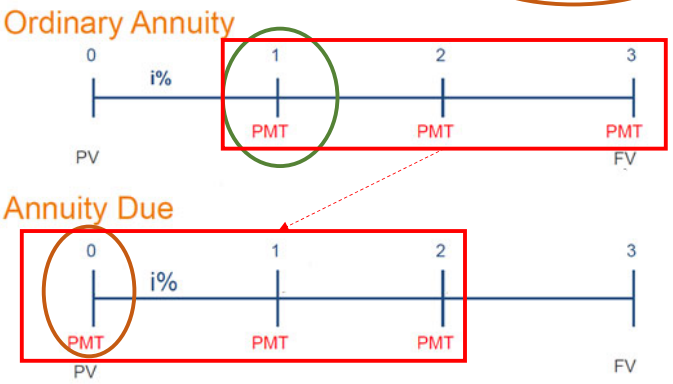
Ordinary annuity and annuity due

Ordinary annuity

- First cash flow occurs one period from now

Annuity due

- First cash flow occurs immediately



PV Annuity Due =
PV Annuity * (1+i)

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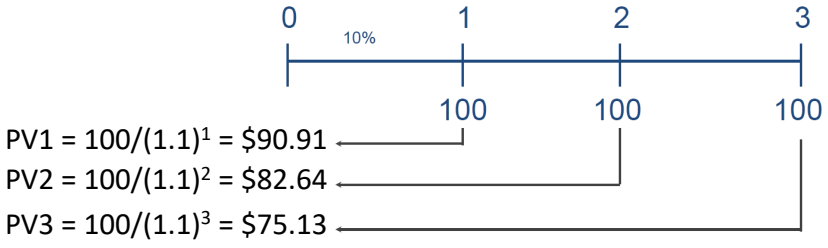
PV of ordinary annuity

What is the present value of a 3-year ordinary annuity with periodic payment of $100 and period rate of 10%?



PV of ordinary annuity = $248.69

- Using formula: $PV\ of\ Annuity = PMT * \frac{1}{r} * (1 - \frac{1}{(1+r)^n})$

PV of ordinary annuity = 100 * 1/0.1*(1-(1/1.1^3)) = 248.69

- Using financial calculator:

2nd FV (clear TVM); N=3; I/Y = 10; PMT = 100; cpt PV (-248.69)

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Lottery example

(Referring to slide 50)

Suppose you win a $10 million lottery prize. The money is paid in equal annual end-of-year installments of $333,333.33 over 30 years. If the appropriate discount rate is 5%, how much is the sweepstakes actually worth today?

- You are not going to receive the $10m prize immediately in full (lump sum)
- Ordinary annuity
- Periodicity of payment: annually

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APR & EAR

Annual Percentage Rate (APR)

- Annual rate that is quoted by law **without compounding** considered
- Period rate:

  APR = Period rate * the number of periods per year

Effective Annual Rate (EAR)

- Actual rate paid (or received) after taking into consideration any **compounding** that may occur during the year
- Period rate:

  EAR = [(1 + Period rate)^the number of periods per year] - 1

Example:

The annual percentage rate of a saving account is 12%, compounding monthly. What is the effective annual rate of this saving account?

APR = 12% → period rate (i.e., monthly rate) = 1%

Number of periods per year = 12

EAR = (1+0.01)^12 - 1 = 12.68%

Another APR & EAR example

The annual percentage rate of a saving account is 12%, compounding annually. What is the effective annual rate of this saving account?

APR = 12% → period rate (i.e., annual rate) = 12%

Number of periods per year = 1

EAR = (1+0.12)^1 - 1= 12%

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Car loan example

(Referring to slide 67)

Supposed you need a car loan of $100,000 from a bank. The bank charges an APR of 6% for a 5-year loan with monthly payments.

- This is an annuity. Why?
  - Identical amount of payment each period
- Periodicity of the payment: monthly
  - Periodic rate = monthly rate = I/Y = 0.5%
- PV of the loan: the principal of the loan that you receive now
  - $100,000 (=PV) → Positive because it is a cash inflow from your perspective
- Number of periods = how many payments are you going to be making
  - N= 5*12 = 60

How much is your monthly car loan payment?

Annuity formula: $100{,}000 = PMT / 0.005 * (1 - (1/(1+0.005)^{60}))$

Financial calculator: 2nd FV (clear TVM); N=60; I/Y=0.5; PV=100000; cpt PMT (=1933.28)

What is the EAR:

EAR formula: EAR = (1+0.005)^12 - 1 = 6.2%

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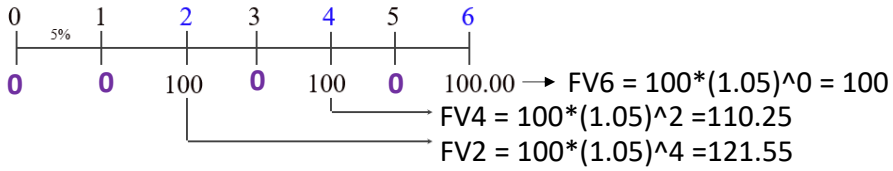
Example on slides 81 - 84

What's the value at the end of Year 3 of the following CF stream if the quoted interest rate is *10%, compounded semiannually?*

- Quoted interest rate: APR
- Periodicity of compounding: semiannually

Method 1:



- Periodicity of payments: semiannually
  - No cashflows in period 0, 1, 3, 5
- Period rate (i.e., semiannual rate) = 10%/2 = 5%

Future value = 100 + 110.26 + 121.55 = $331.80

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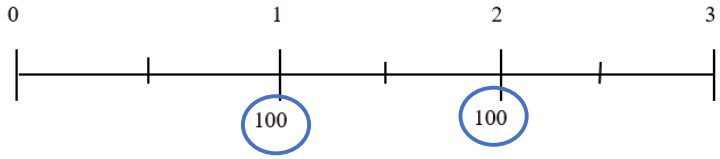
Example on slides 81 – 84 (cont.)

Method 2:



- Can use annuity formula because, the payments are steady ($100) & periodic (annually)
- Periodicity of the payments: annually
- APR is 10%, compounding semiannually
  - The periodicity of the payments and the interest compounding DO NOT MATCH
    - Can't use semiannual rate because it does not match the periodicity of the annual payments
- **Need period rate that matches the periodicity of the payments (i.e., need annual interest rate, taking into account the semiannual compounding)**

  Period rate that matches the periodicity of the payment
  = annual rate = (1 + 0.05)^2 – 1 = 10.25%

Plug into the annuity formula:

FV = $100 * [(1 / 0.1025) * ((1.1025)^3-1)] = $331.80

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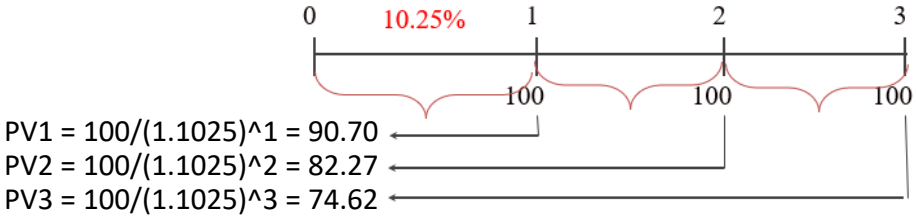
Example on slides 81 – 84 (cont.)

Method 3:

Similar setup as method 2

- Periodicity of payments: annual
- Periodicity of interest compounding: semiannual
- As such, need to find annual interest after taking into account compounding



PV = 90.70 + 82.27 + 74.62 = 247.59

FV at t=3 = 247.59 * (1.1025) ^ 3 = 331.80

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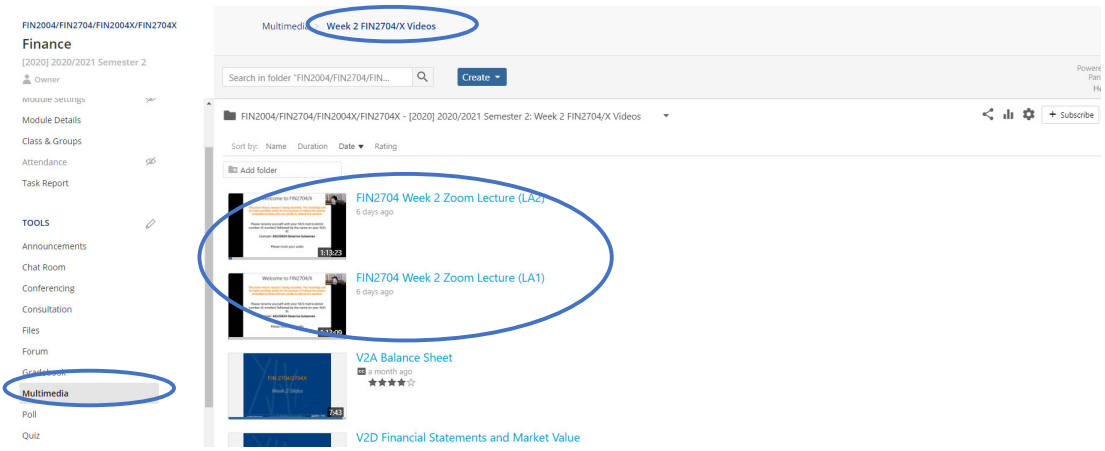
Negative interest rates

- Rarely charged to retail investors
- Although the savings interest rate may be negative, the BORROWING interest rates are unlikely to be negative.

Class management

- Recordings



- Financial calculators
- Past weeks topics

Other types of loans (not amortized loans)

Example on how to calculate the payments are on slides 71-72, 73-74, 75-77

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Week 3
List of topics

Note:

You are responsible for all materials covered in the pre-recorded videos posted on LumiNUS, unless they are marked "not examinable". This list only serves to help you in your revisions.

Week 3 topics

Time value of money

A dollar paid today is worth more than a dollar paid tomorrow

- Present value
- Future value

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Week 3 topics (cont.)

- Principal
- Interest (also referred to as discount rate, cost of capital, opportunity cost of capital, required return)
    - Simple interest
    - Compound interest
- Annuity & annuity due
- Perpetuity
- Multiple cash flows
    - Use timeline

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Week 3 topics (Cont.)

- Annual Percentage Rate (APR)
  - Period rate
- Effective Annual Rate (EAR)

- Different types of loan (independent study)

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FIN2704/X
Week 2

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Week 2 topics

- CFFA (review Week 2 Zoom lecture recordings)

Cash Flow From Assets (CFFA^) =

Operating Cash Flow (OCF) → Cashflow from a firm's day-to-day operation

– Net Capital Spending (NCS)

– Changes in NOWC (Net Operating Working Capital) → Investments needed for the day-to-day operation

- Dupont Identity
  - Read textbook pages 69 – 71

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