Geometric Shapes: 3D Curves

Module 3

Lecture 3

We have learnt that

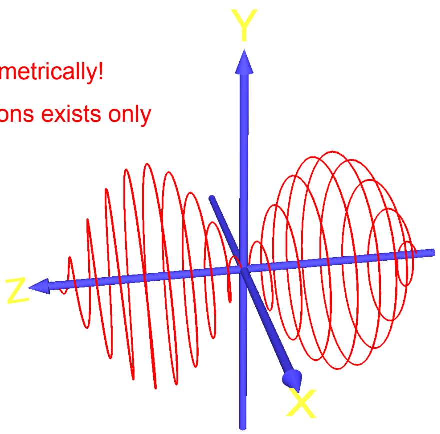
- We think of a 2D curve as a point moving with one degree of freedom (forward and backward)
- 2D curves can be defined analytically by
  - **Implicit functions**
    $f(x,y)=0$ – Slow for rendering
  - **Explicit functions**
    $y=f(x)$ *or* $x=f(y)$ – Fast but axes dependent
  - **Parametric functions**
    One parameter only
    $x=x(t),\quad y=y(t)\quad t=[t_1, t_2]$ – Fast and axes independent
- Curves are usually interpolated by polylines – connected segments

3D Curves

Can be only defined parametrically!

Explicit and implicit functions exists only for plane curves.

$$x = f_x(\tau)$$
$$y = f_y(\tau)$$
$$z = f_z(\tau)$$
$$\tau = [\tau_1, \tau_2]$$



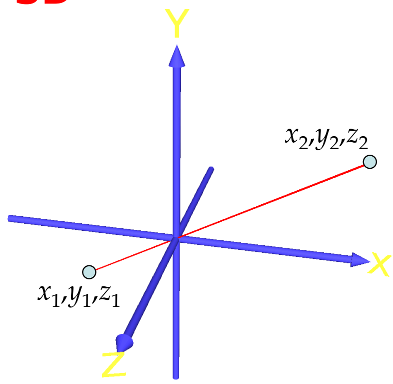
Still one parameter !

Straight Line in 3D

$$x = x_1 + \tau(x_2 - x_1)$$
$$y = y_1 + \tau(y_2 - y_1)$$
$$z = z_1 + \tau(z_2 - z_1)$$
$\tau = [0,1]$ Segment

$\tau = [0,\infty)$ Ray

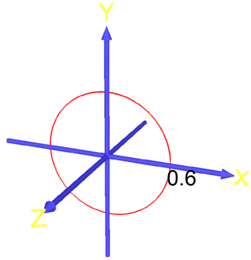
$\tau = (-\infty,\infty)$ Straight line



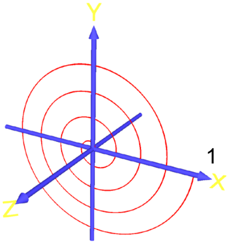
No explicit and implicit representation !

Experimenting with Parametric Curves

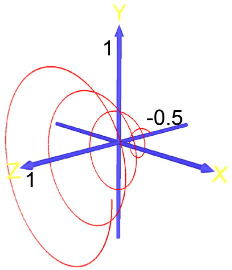
$x = 0.6 \cdot cos(u \cdot 2\pi)$
$y = 0.6 \cdot sin(u \cdot 2\pi)$
$z = 0$
$0 \le u \le 1$



$x = \mathbf{u} \cdot cos(u \cdot \mathbf{8}\pi)$
$y = \mathbf{u} \cdot sin(u \cdot \mathbf{8}\pi)$
$z = 0$
$0 \le u \le 1$

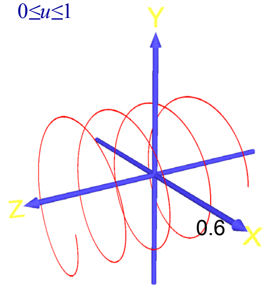


$x = u \cdot cos(u \cdot 8\pi)$
$y = u \cdot sin(u \cdot 8\pi)$
$z = \mathbf{-0.5+1.5 \cdot u}$
$0 \le u \le 1$

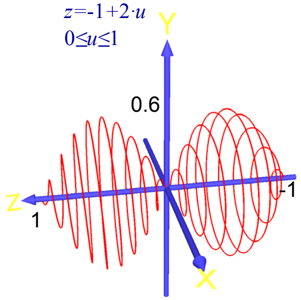


Experimenting with Parametric Curves

$x = \mathbf{0.6} \cdot cos(u \cdot 8\pi)$
$y = \mathbf{0.6} \cdot sin(u \cdot 8\pi)$
$z = -0.5+1.5 \cdot u$
$0 \le u \le 1$



$x = 0.6|\mathbf{sin(u*2\pi)}| \cdot cos(u \cdot 36\pi)$
$y = 0.6|\mathbf{sin(u*2\pi)}| \cdot sin(u \cdot 36\pi)$
$z = -1+2 \cdot u$
$0 \le u \le 1$



Curves. Summary

- 2D and 3D. A point moving with one degree of freedom (forward and backward)
- Polylines – interpolation by connected straight line segments
- 2D:
  - **Explicit**
    y=f(x) or x=f(y) – axes dependent, no arcs and segments
  - **Implicit**
    f(x,y)=0 – no arcs and segments
  - **Parametric**
    One parameter only. Any curve, even with self-intersections. The concept of a moving point as a function of the parameter. Curve interpolation.
    $x=x(t), \quad y=y(t) \qquad t=[t_1, t_2]$
- 3D:
  - **Parametric**
    One parameter only. Any curve, even with self-intersections. The concept of a moving point as a function of the parameter. Curve interpolation.
    $x=x(t), \quad y=y(t), \quad z=z(t) \qquad t=[t_1, t_2]$