NANYANG TECHNOLOGICAL UNIVERSITY SINGAPORE

CZ3005 Artificial Intelligence

Week 12a – Fuzzy Logic

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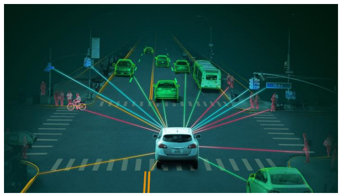
Learning Goals

Understanding the:

- Basic definitions and terminology
- Set-theoretic operations
- Membership Function (MF) formulation
  - MFs parameterization
  - Linguistic modifier/hedges

Example: Safe Autonomous Vehicles

- Autonomous Cars implement Duty of Care
  - an individual should exercise “reasonable care” while performing acts that could harm others
- “On a Formal Model of Safe and Scalable Self-driving Cars”, by Shalev-Swartz, Shammah, and Shashua, arXiv 1708.06374
  - Responsibility Sensitive Safety – mathematical safety assurance
  - System design that adheres to the mathematical model



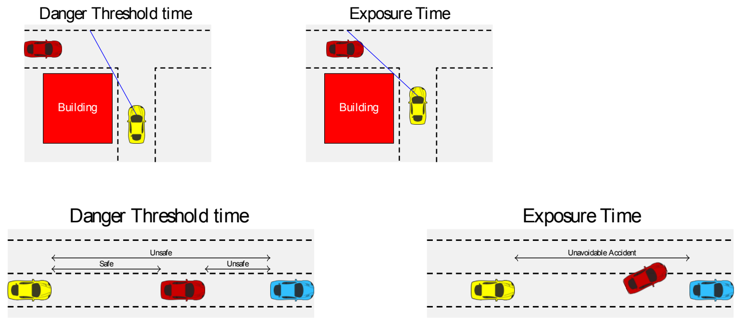
Example: Safe Autonomous Vehicles

- Responsibility Sensitive Safety (RSS)
  - Do not hit someone from behind
  - Do not cut-in recklessly
  - Right-of-way is given, not taken
  - Be careful of areas with limited visibility
  - If you can avoid an accident without causing another one, you must do so

Example: Safe Autonomous Vehicles

- Responsibility Sensitive Safety (RSS)
  - Do not hit someone from behind
    - Even if not your fault?
  - Do not cut-in recklessly
  - Right-of-way is given, not taken
    - How to resolve polite deadlocks?
  - Be careful of areas with limited visibility
  - If you can avoid an accident without causing another one, you must do so
    - Emergency breaking can cause whiplash

Example: Safe Autonomous Vehicles



"On a Formal Model of Safe and Scalable Self-driving Cars",
by Shalev-Swartz, Shammah, and Shashua, arXiv 1708.06374

Example: Safe Autonomous Vehicles

- Use of Semantic Action Space
  - Not "drive for 5.33 kilometers, then reduce speed at the rate of 1 m/s^2"
  - Slow down as you approach red light to stop at the line.
    - IF approach red light, THEN slow down and stop

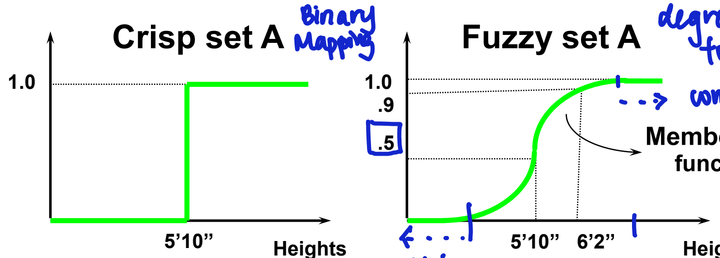
"On a Formal Model of Safe and Scalable Self-driving Cars",
by Shalev-Swartz, Shammah, and Shashua, arXiv 1708.06374

Idea: Fuzzy logic ⇒ mapping function from crisp domain (Good for machies) to more fuzzy Rules (i.e slow down a little bit)

Fuzzy Sets

- Sets with fuzzy boundaries

A = Set of tall people



Binary Mapping

degree of membership to work

completly belongs to set of tall people

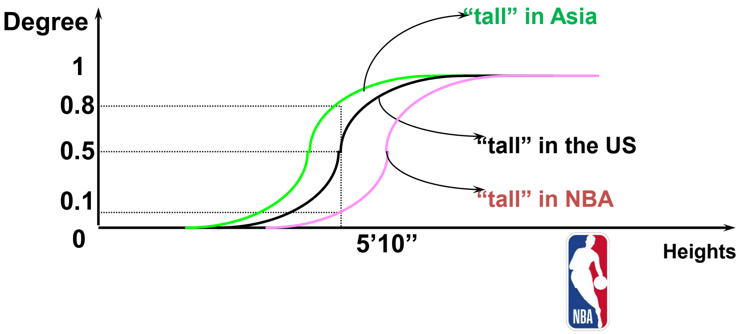
ie as long as > 5'10 you are tall person

Not tall

in real world this range is still considered tall person.

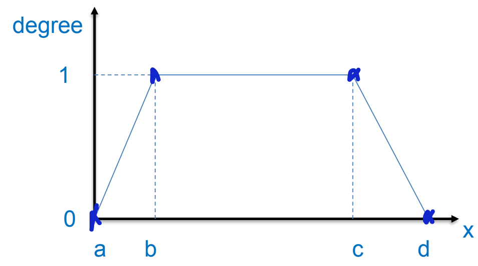
Fuzzy Membership Function

- Characteristics of a fuzzy MF:
  - Subjective measures between 0 and 1
  - Not probability functions



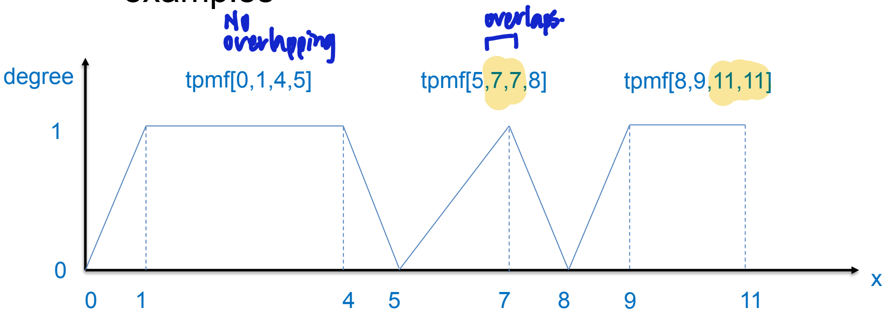
Fuzzy Membership Function

- Trapezoidal MF:
  - *tpmf[a,b,c,d]*
  - where
    - degree=0 at x=a
    - degree=1 at x=b
    - degree=1 at x=c
    - degree=0 at x=d



Fuzzy Membership Function

- Trapezoidal MF:
  - examples



Fuzzy Sets

Formal definition:

A fuzzy set *A* in *X* is expressed as a set of ordered pairs:

$$A = \{(\overbrace{x}^{\text{Crisp input}}, \underbrace{\mu_A(x)}_{\text{Membership function (MF)}}) | x \in X\}$$

Fuzzy set

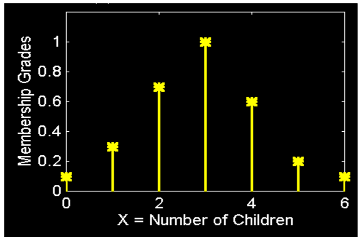
Universe or universe of discourse

A fuzzy set is totally characterized by a membership function (MF).

Fuzzy Sets – Discrete Universes

- Fuzzy set C = "desirable city to live in"
  X = {SF, Boston, LA} (discrete and non-ordered)
  C = {(SF, 0.9), (Boston, 0.8), (LA, 0.6)}

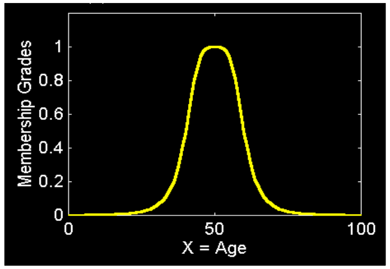
- Fuzzy set A = "sensible number of children to have"
  X = {0, 1, 2, 3, 4, 5, 6} (discrete universe)
  A = {(0, .1), (1, .3), (2, .7), (3, 1), (4, .6), (5, .2), (6, .1)}



Fuzzy Sets – Continuous Universes

- Fuzzy set B = "about 50 years old"
  X = Set of positive real numbers (continuous)
  B = {(x, μ_B(x)) | x in X}

$$\mu_B(x) = \frac{1}{1 + \left(\frac{x-50}{10}\right)^2}$$



Alternative Notation

A fuzzy set A can be alternatively denoted as follows:

X is discrete

$$A = \sum_{x_i \in X} \mu_A(x_i) / x_i$$

X = {0, 1, 2, 3, 4, 5, 6} (discrete universe)
A = {(0, .1), (1, .3), (2, .7), (3, 1), (4, .6), (5, .2), (6, .1)}

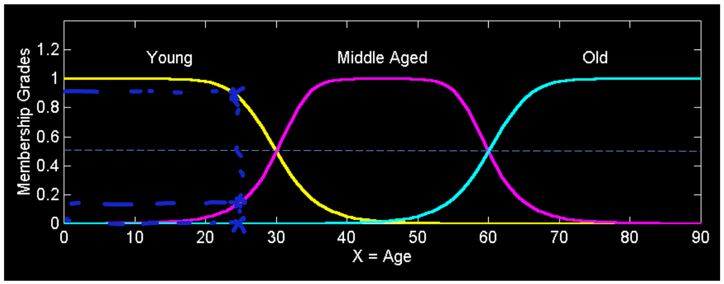
X is continuous

$$A = \int_X \mu_A(x) / x$$

Note that Σ and integral signs stand for the union of membership grades;
"/" stands for a marker and does not imply division.

Fuzzy Partition

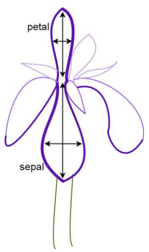
- Fuzzy partitions formed by the linguistic values "young", "middle aged", and "old":



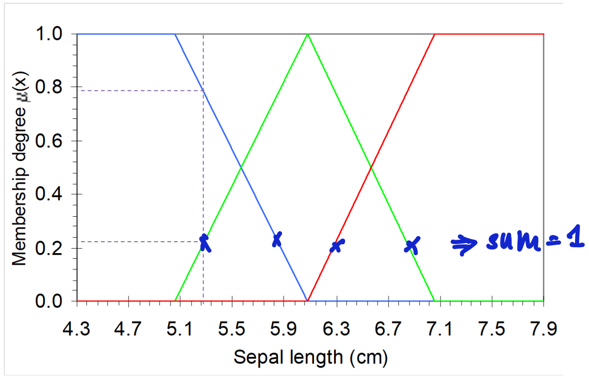
Pseudo Fuzzy Partition

A fuzzy space is a pseudo fuzzy partition when

The sum of the MF values at any *x* is always 1



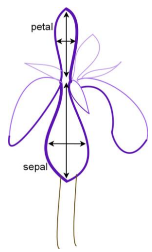
Iris flower



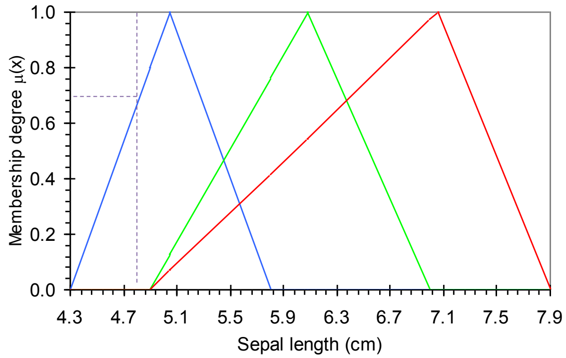
Non-Pseudo Fuzzy Partition

A fuzzy space is a non-pseudo fuzzy partition when

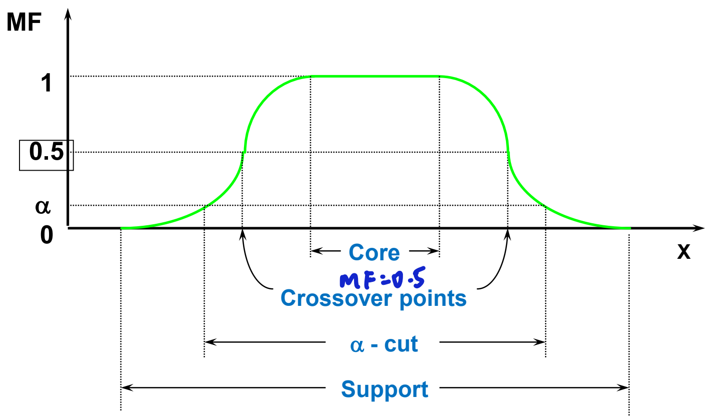
The sum of the MF values at any *x* is NOT always 1



Iris flower



MF Terminology

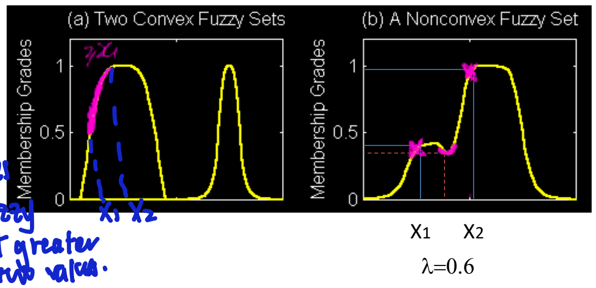


Convexity of Fuzzy Sets

A fuzzy set *A* is convex if for any λ within [0, 1]:

$$\mu_A(\lambda x_1 + (1-\lambda)x_2) \geq \min(\mu_A(x_1), \mu_A(x_2))$$

convex:
Any two values on x-axis, any two values between the two values should correspond to fuzzy membership value that greater than min value btw two values.



Alternatively, *A* is convex if all its α–*cuts* are convex.

Set-Theoretic Operations

- Subset:

A is subset of B
Any value in B appears in A

$$A \subseteq B \Leftrightarrow \mu_A \le \mu_B$$

- Complement:

$$\bar{A} = X - A \Leftrightarrow \mu_{\bar{A}}(x) = 1 - \mu_A(x)$$

- Union: (OR, max())

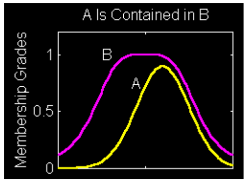
$$C = A \cup B \Leftrightarrow \mu_c(x) = \max(\mu_A(x), \mu_B(x)) = \mu_A(x) \vee \mu_B(x)$$

- Intersection: (AND, min())

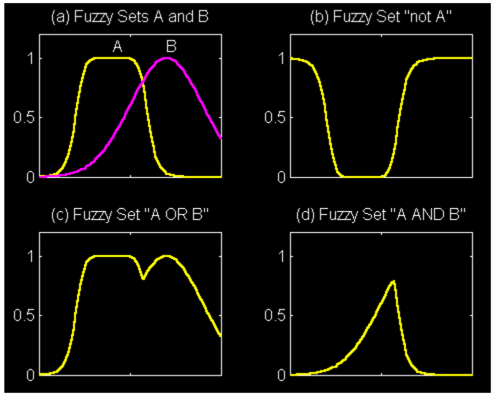
$$C = A \cap B \Leftrightarrow \mu_c(x) = \min(\mu_A(x), \mu_B(x)) = \mu_A(x) \wedge \mu_B(x)$$

Set-Theoretic Operations

$1 - \mu_A(x)$

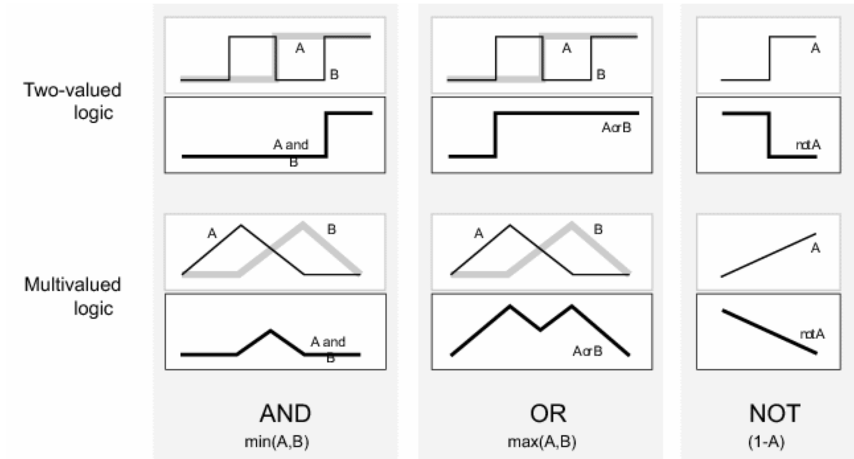


$\mu_A \le \mu_B$



$\max(\mu_A(x), \mu_B(x))$ $\min(\mu_A(x), \mu_B(x))$

Fuzzy Logical Operation



Thank you!

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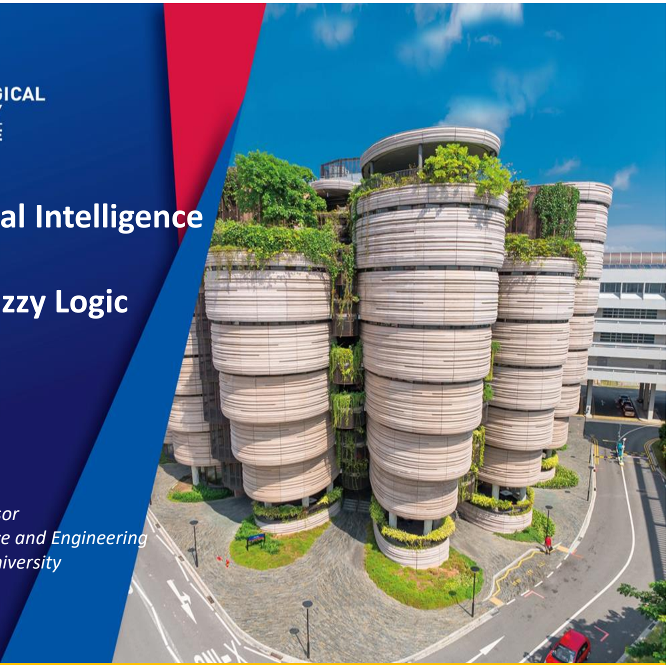
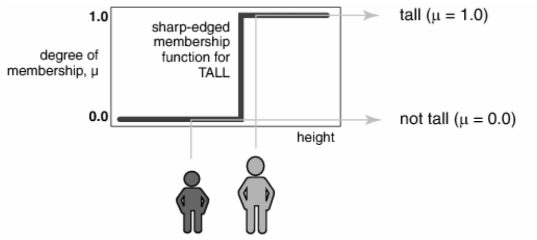
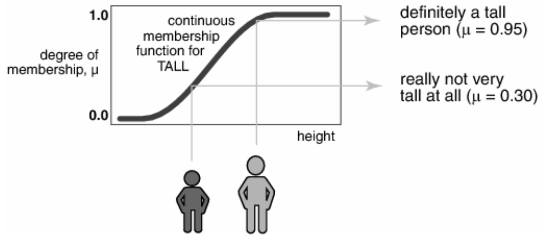


Recap – Fuzzy Membership Function



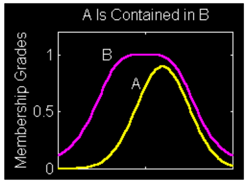
A person who is 180 cm is tall

A person who is 179 cm is **NOT** tall!



A person who is 180 cm is tall **to a higher degree than** another person who is 179 cm

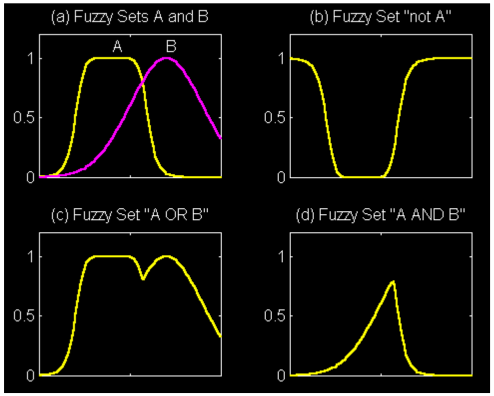
Recap – Fuzzy Set-Theoretic Operators



$\mu_A \le \mu_B$

Subset

$1-\mu_A(x)$ **Complement**



$\max(\mu_A(x), \mu_B(x))$

Union

$\min(\mu_A(x), \mu_B(x))$

Intersection

Learning Goals

Understanding the:

- Linguistic modifier/hedges
- Fuzzy Rule Based System
  - Fuzzy Rule
  - Fuzzy Inference
  - Defuzzification

Linguistic Hedge - Modifiers

- Linguistic Hedge/Modifiers are operations that modify the meaning of a term – fuzzy label (fuzzy set).
  - "*very Tall*", the word *very* modifies *"TALL"* which is a fuzzy set.

- Other modifiers are:
  - "more or less" (morl), "possibly", and "definitely"

Linguistic Hedge – Modifiers

- very **a** = $\mathbf{a}^2$
- more or less (morl) **a** = $\mathbf{a}^{0.5}$
- extremely **a** = $\mathbf{a}^3$
- slightly **a** = $\mathbf{a}^{0.333}$
- somewhat **a** = morl **a** and not slightly **a**

e.g.,

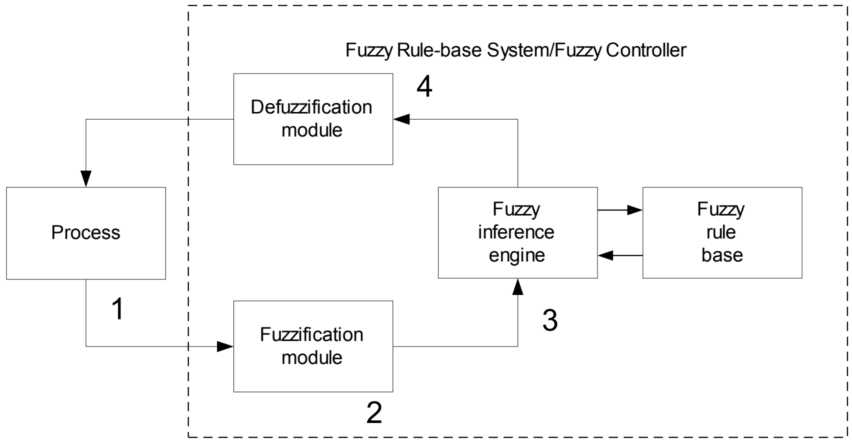
young = [1/0, 0.6/20, 0.1/40, 0.0/60, 0.0/80]

very young = young^2

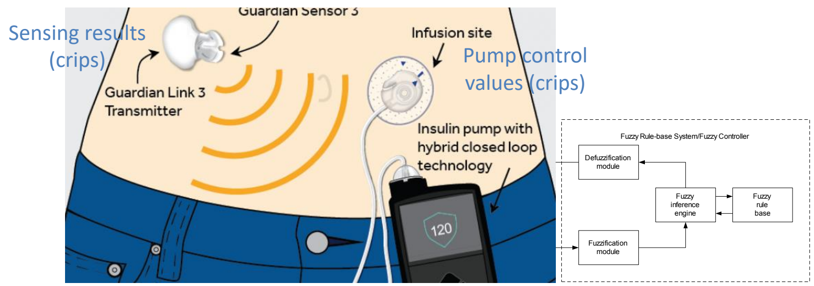
= [1/0, 0.36/20, 0.01/40, 0.0/60, 0.0/80]

lower than young set

Fuzzy Rule-Based (FRB) Systems



Example FRB Continuous Insulin Pump



Steps of an FRB System

1. **Fuzzify inputs**: Resolve all fuzzy statements in the antecedent to a degree of membership between 0 and 1
2. **Apply Fuzzy Operators**: If there are multiple parts to the antecedent, apply fuzzy logic operators (AND, OR, etc.) and resolve the antecedent to a single number between 0 and 1.
3. **Apply Implication Method**: Use the degree of support for the entire rule to shape the output fuzzy set. The consequent of a fuzzy rule assigns an entire fuzzy set to the output. This fuzzy set is represented by a membership function that is chosen to indicate the qualities of the consequent.
4. **Aggregation** of the consequents across all rules (if there are multiple rules).
5. **Defuzzification**

Fuzzy Rules

A single fuzzy if – then rule:

If *x* is *A* then *y* is *B*

A and *B* are linguistic values defined by fuzzy sets on the range of discourse for fuzzy variables *x* and *y*.

E.g.,

Real world concept

If *service* is *good* then *tips* is *average*

x *A* *y* *B*

Fuzzy set; linguistic label

fuzzy antecedent — fuzzy conclusion

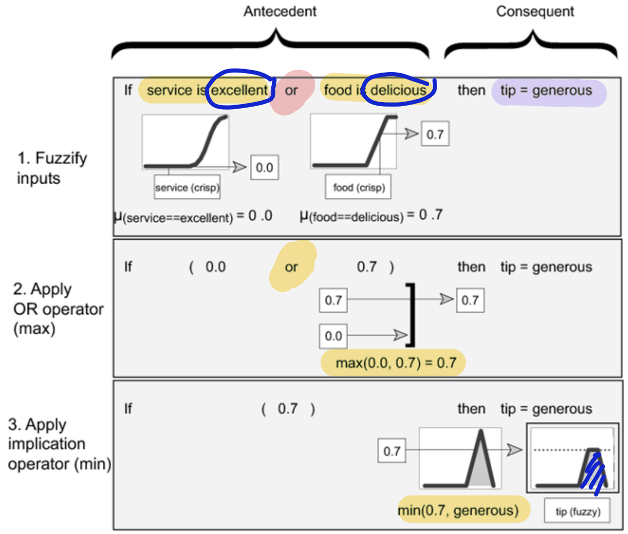
Fuzzy Inference

Interpreting a single fuzzy if-then rule:

- Evaluating antecedent
  - *fuzzifying input* (fuzzy function application)
  - *applying necessary fuzzy operators*
- Applying the result to the consequent
  - *known as fuzzy implication:*

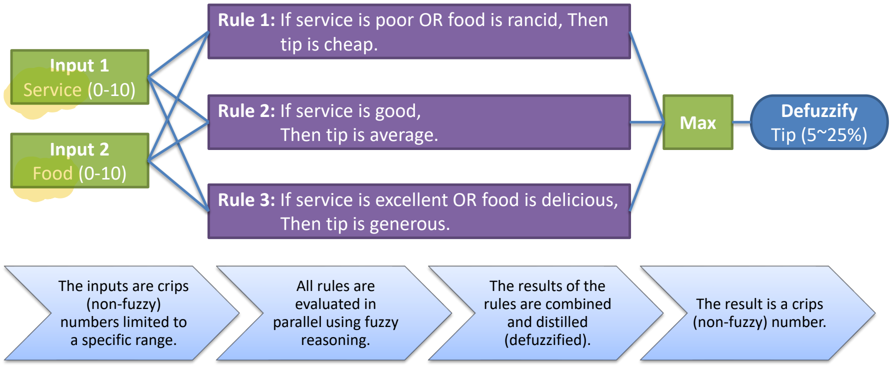
min(fuzzy antecedent, fuzzy consequent)

Fuzzy Inference – Example



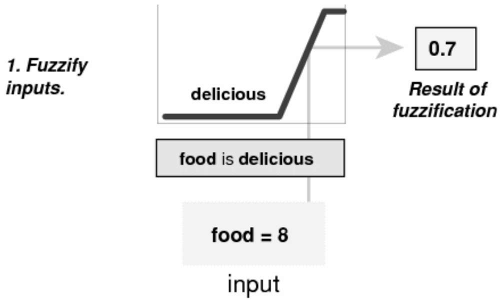
Fuzzy Inference with Multiple Fuzzy Rules

Consider the example of the service for a dinner for two



Step 1: Fuzzification

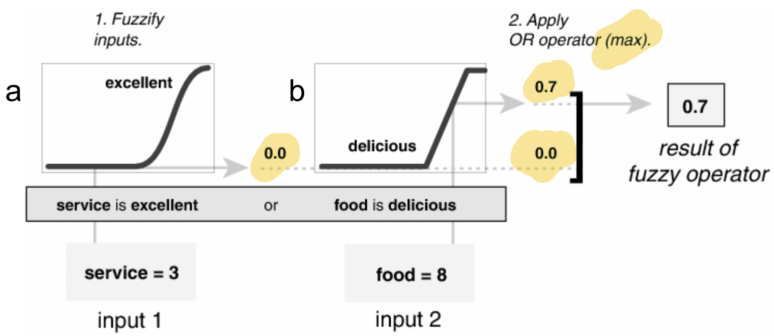
- Fuzzification of the input amounts to either a table lookup or a function evaluation.



- In this manner, each input is fuzzified over all the qualifying membership functions required by the rules.

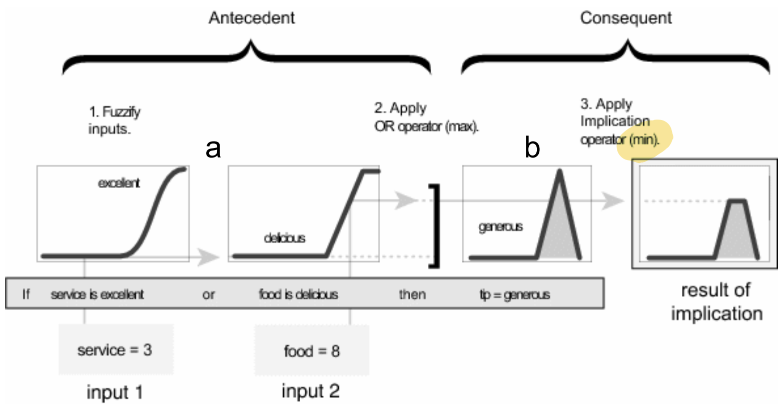
Step 2: Applying Fuzzy Operators

OR - max(a, b), AND - min(a, b):



Step 3: Applying Implication Method

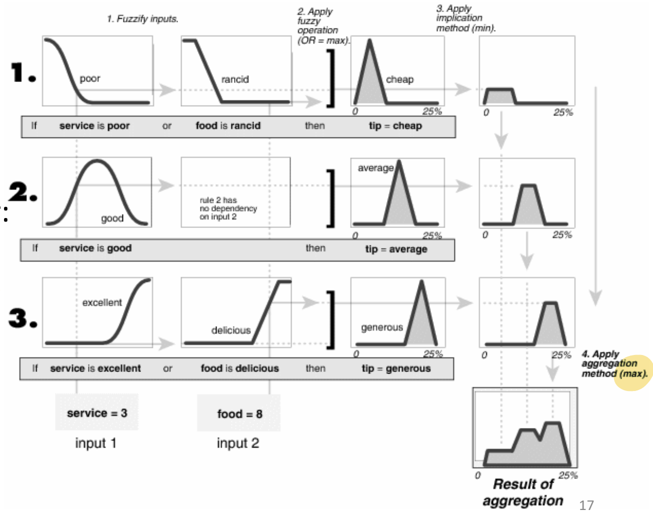
Implication (i.e., then) - min(a, b):



Step 4: Aggregating All Outputs (3 Rules)

Combining the outputs of all fuzzy rules into a single fuzzy set through the union operator:

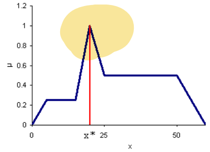
max(R1, R2, R3)



Step 5: Defuzzification

Types of defuzzification method:

- Max-membership $\sup_x(\mu(X))$ defuzzification method



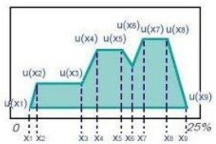
- Centroid Defuzzification Technique

$$x^* = \frac{\sum_{i=1}^{n} x_i\,\mu(x_i)}{\sum_{i=1}^{n} \mu(x_i)}$$

Step 5: Defuzzification

- The input for the defuzzification process is a fuzzy set (the aggregate output fuzzy set)
- The output is a single number.

5. Defuzzify the aggregate output (centroid)



Fuzzy membership value

$$x^* = \frac{\sum_{i=1}^{9} x_i\mu(x_i)}{\sum_{i=1}^{9} \mu(x_i)} = 16.7$$

Result of defuzzification

- The final tip to pay for a rating of service=3 and food=8 is 16.7%.

Thank you!