Tutorial 4 : Risk and Return II

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Systematic and Unsystematic Risk

- When you diversify across assets that are not perfectly correlated, the portfolio's risk is less than the weighted sum of the risks of the individual securities in the portfolio.

- The risk that can be diversified away during the portfolio construction process is called the asset's **unsystematic risk** (also called unique, diversifiable, or firm-specific risk). Since the market portfolio contains all risky assets, it must represent the ultimate in diversification. All the risk that can be diversified away must be gone.

- The risk that is left cannot be diversified away, since there is nothing left to add to the portfolio. This risk that remains is called the **systematic risk** (also called non-diversifiable risk or market risk).

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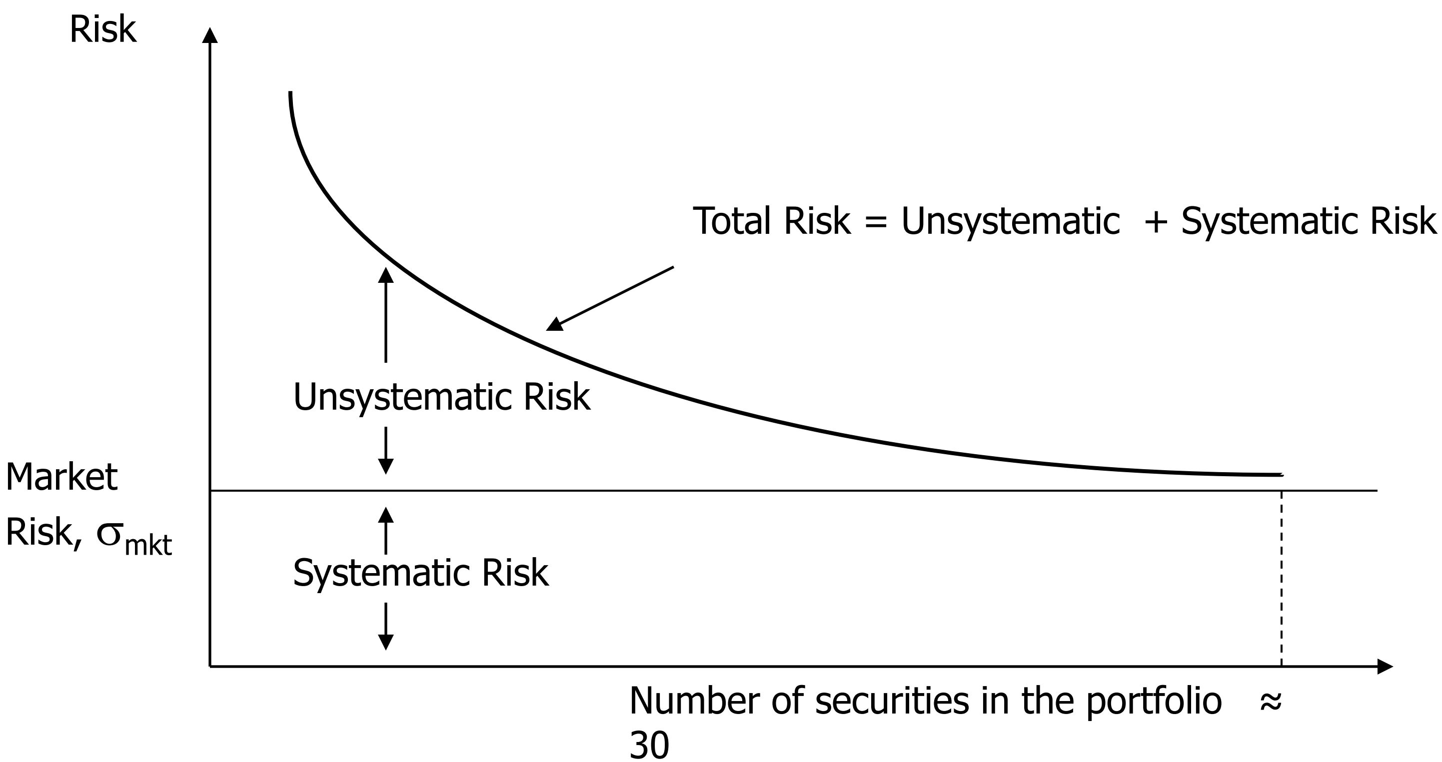
- The concept of systematic risk applies to individual securities as well as to portfolios. This risk is caused by general factors such as changes in inflation rates, interest rates, and market sentiments that affect all investment types albeit by varying degrees.



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- Academic studies have shown that as you increase the number of stocks in a portfolio, the portfolio’s risk falls toward the level of market risk.
- One study showed that it only took about 12 to 18 stocks in a portfolio to achieve 90 percent of the maximum diversification possible.
- Another study indicated it took 30 securities. Whatever the number, it is significantly less than all the securities.

Systematic Risk Is Relevant in Portfolios

- In the previous section on the Markowitz portfolio model, it was noted that the relevant risk to consider when adding a security to large portfolio is **its average covariance with all other assets in the portfolio**.

- Combining this concept with the idea that the only relevant portfolio is the market portfolio, results in the conclusion that the relevant risk measure for an individual risky asset is **its covariance with the market portfolio**, also known as its systematic risk.

- Since unsystematic risk can be diversified away, and therefore, unsystematic risk is not relevant to investors. The only relevant risk is systematic risk. Therefore, investors should be compensated for accepting risk that cannot be diversified away i.e. the systematic risk and not the unsystematic risk.

Capital Asset Pricing Model (CAPM)

- The CAPM states that in perfect markets, all unsystematic risk can be eliminated through diversification by forming portfolios of two or more securities.

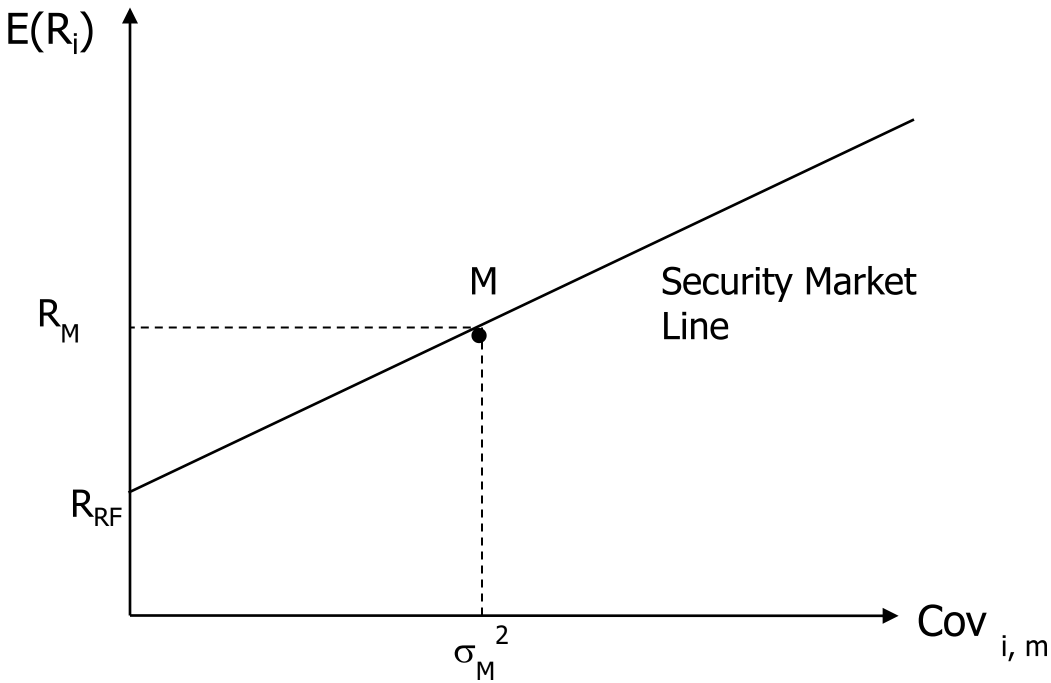
- A perfect market is one where there is no trading friction, that is, there are no transaction costs, information is costless, and all assets are divisible. These are also conditions for an efficient market.

- Under perfect markets conditions, since all unsystematic risks can be diversified away, and there is no cost to diversification, no one will be motivated to pay for (or attach any premium to) unsystematic risk. Investors will only be willing to pay for (or attach a premium) to the systematic risk which cannot be diversified away.

- Therefore, the CAPM maintains that the only risk that remains in an effectively diversified portfolio is the systematic risk. This systematic risk is termed beta (symbol β).

Security Market Line (SML)

- The security market line is a visual representation of the relationship between risk and the expected or required rate of return on an asset. Because the relevant risk measure for an individual risky asset is its covariance with the market portfolio ($Cov_{i,M}$), the following risk-return relationship can be drawn.



$$E(R_i) = R_{RF} + \frac{R_M - R_{RF}}{\sigma_M^2} (Cov_{i, m})$$

$$= R_{RF} + \frac{Cov_{i, m}}{\sigma_M^2} (R_M - R_{RF})$$

If $\beta_i = Cov_{i, m} / \sigma_M^2$, then

$$\boxed{E(R_i) = R_{RF} + \beta_i (R_M - R_{RF})}$$

Beta (β)

- Beta is standardized measure of an asset's systematic risk, in that it relates an asset's covariance with the market to the variance of the market portfolio.

- The beta for market portfolio is 1.0. An asset with a beta greater than 1.0 has higher systematic risk than the market portfolio – meaning it is more volatile than the overall market portfolio. The reverse is true if an asset's beta is less than 1.0.

$$\beta_i = Cov_{i,\,m} / \sigma_M^2$$

- The SML can be redrawn to use beta as the relevant risk measure as shown in the next slide.

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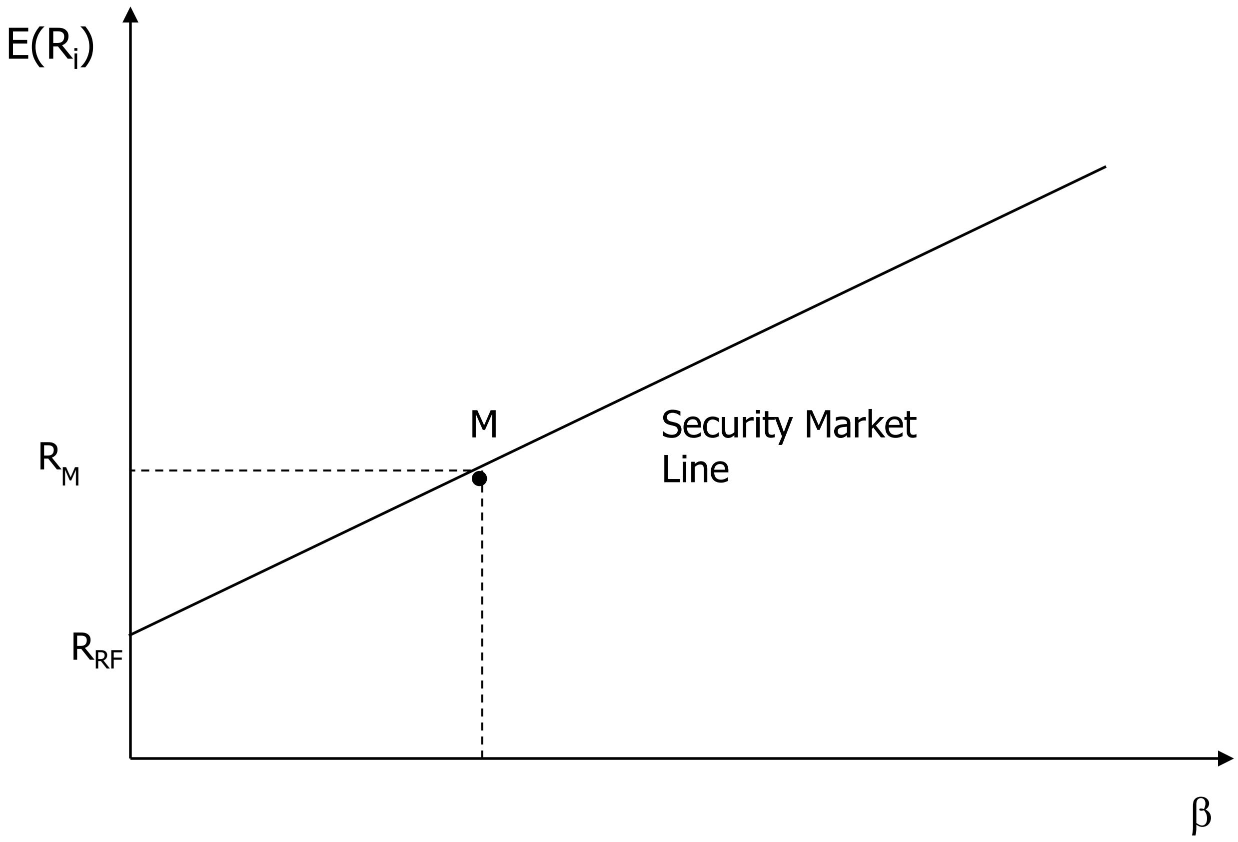
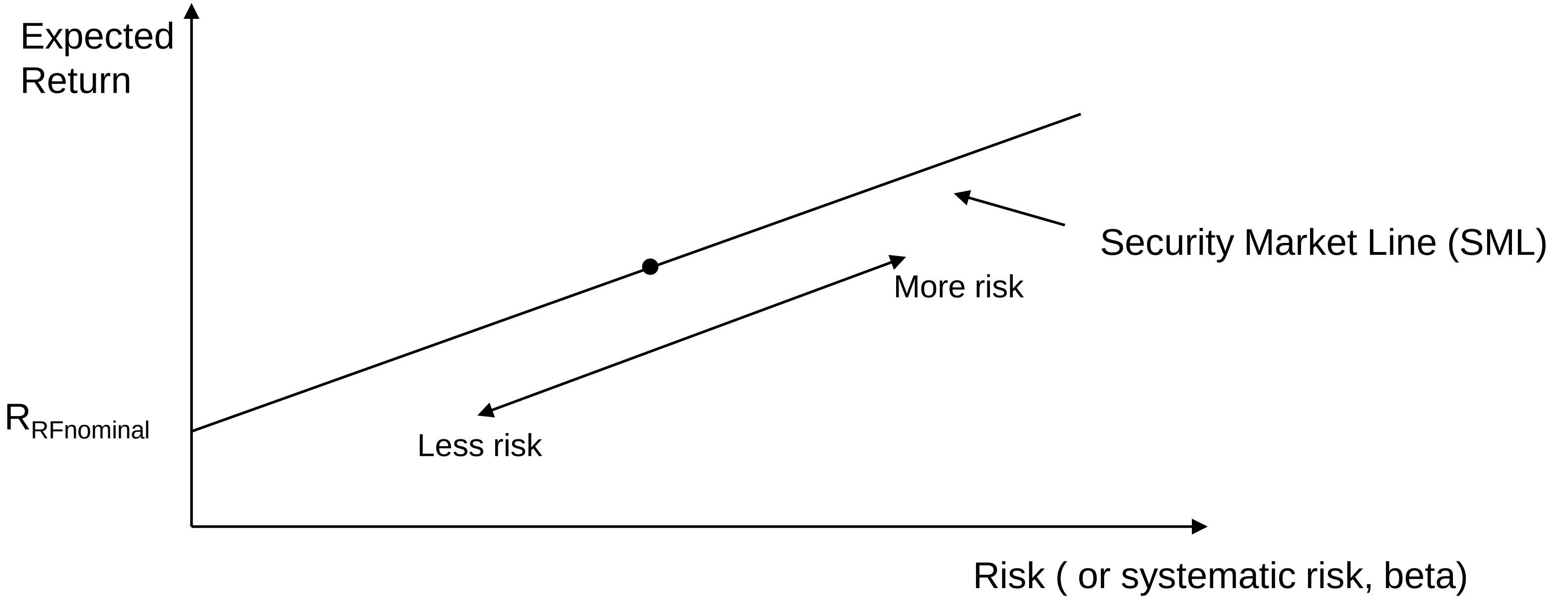


Security Market Line (SML)

- There are various risk/return combinations at any one point in time.
- Modern portfolio theory asserts that, as more risk is assumed, the expected return demanded by investors will increase proportionately.
- The graph below shows the expected relationship between risk.



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Movements along the SML

- Movement along the security market line indicates that the risk level of an individual security has changed.

- The change can be due to a change in one of its risk sources, such as business or financial risk.
- For example, if a company diversifies into a high-risk industry, an investment in its common stock (let us call this Asset A) will be perceived as being a more risky investment and therefore shareholders will require a higher rate of return. As an investment in a company’s common stock becomes more risky for shareholders, Asset A will change its position on the SML. Asset A will move up the SML, indicating a change in the risk-return relationship of Asset A. Any change in the environment that results in a change in the risk factors that affect a particular investment will induce the asset to move along the SML line. Investors will consequently require a higher rate of return to compensate them for the higher levels of risk.

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Parallel shifts of the SML

- The SML will undergo a parallel shift if capital market conditions change and/or if the rate of inflation changes.
- The SML will shift upward for increasing inflation and downward for decreasing inflation.
- The SML will also experience a parallel upward shift during periods of high economic growth (real R_{RF} increase) and a downward shift when capital markets tighten (real R_{RF} decreases)

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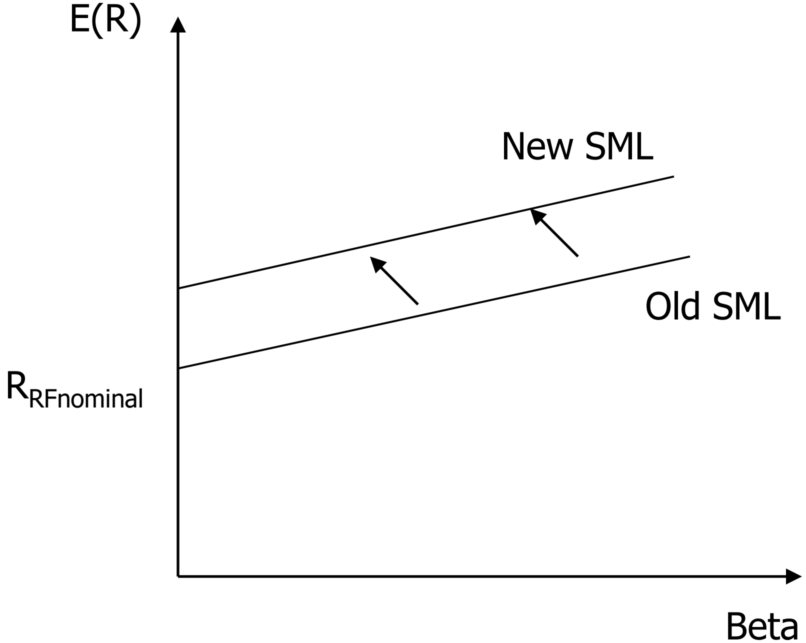
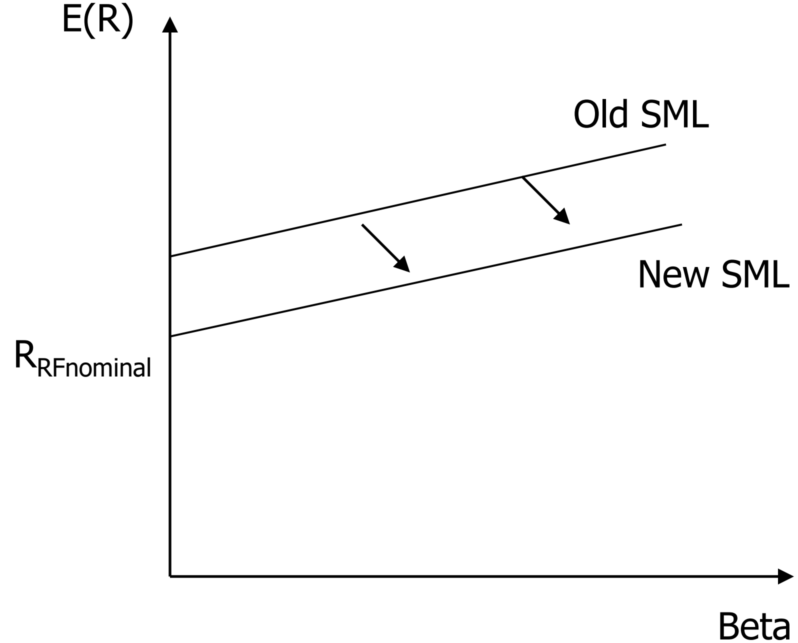
Increase in Expected Inflation



Decrease in Expected Inflation



Cont'd - Summary

- The following is a summary of events causing changes in the required rate of return:
  - Movements along the SML reflect a change in the riskiness of the particular investment.
  - Changes in the slope of the SML reflect a change in investors' attitudes toward risk that affects the market risk premium.
  - Parallel shifts in the SML reflect changing market conditions, such as a change in expected real growth, changing inflation expectations or tightening of the money supply (a change in the real risk-free rate)

CML versus SML

- The capital market line and the security market line are similar, yet different concepts.

- The capital market line is the relationship between the required return on **efficient** portfolios (R_p) and their **total risk** (σ_p).

- The security market line is the relationship between the expected returns on individual assets (R_s) and the their risk as measured by their beta (β_s). This is because the relevant measure of risk for an individual security <u>held as a part of a well-diversified portfolio is not the security's standard deviation or variance</u>, but the contribution that it makes to the overall portfolio variance, measured by the asset's beta.

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- If the expected return-beta relationship is true for any individual security, it must also be true for any combination of securities.

- Therefore, the security market line is valid for both portfolios (**efficient or inefficient**) and individual securities. Thus, in equilibrium, **all securities and portfolios must lie on the security market line (SML)**.

- But, **only efficient portfolios lie on the capital market line (CML)** because standard deviation is a measure of risk for efficiently diversified portfolios that are candidates for an investor's overall portfolio.

Identifying Mispriced Securities With the SML

Example : Table below contains information based on analyst's forecasts for three stocks. Assume a risk-free rate of 7 percent and market return of 15 percent. Compare the expected and required return on each stock, determine whether each stock is undervalued, overvalued, or properly valued, and outline an appropriate trading strategy.

Stock	Price Today	E(Price) in 1 year	E(dividend) in 1 year	Beta
A	$25	$27	$1.00	1.0
B	40	45	2.00	0.8
C	15	17	0.50	1.2

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Expected and required returns computations are shown as follows :

Stock	Forecasted Return	Required Return (CAPM)
A	(\$27 - \$25 + \$1) / \$25 = 12.0%	0.07 + (1.0)(0.15 - 0.07) = 15.0%
B	(\$45 - \$40 + \$2) / \$40 = 17.5%	0.07 + (0.8)(0.15 - 0.07) = 13.4%
C	(\$17 - \$15 + \$0.50) / \$15 = 16.6%	0.07 + (1.2)(0.15 - 0.07) = 16.6%

Stock A is overvalued. It is expected to earn 12%, but based on its systematic risk it should earn 15%. Short sell stock A.

Stock B is undervalued. It is expected to earn 17.5%, but based on its systematic risk it should earn 13.4%. Buy stock B.

Stock C is properly valued. It is expected to earn 16.6%, but based on its systematic risk it should earn 16.6%. Buy, sell, or ignore stock C.

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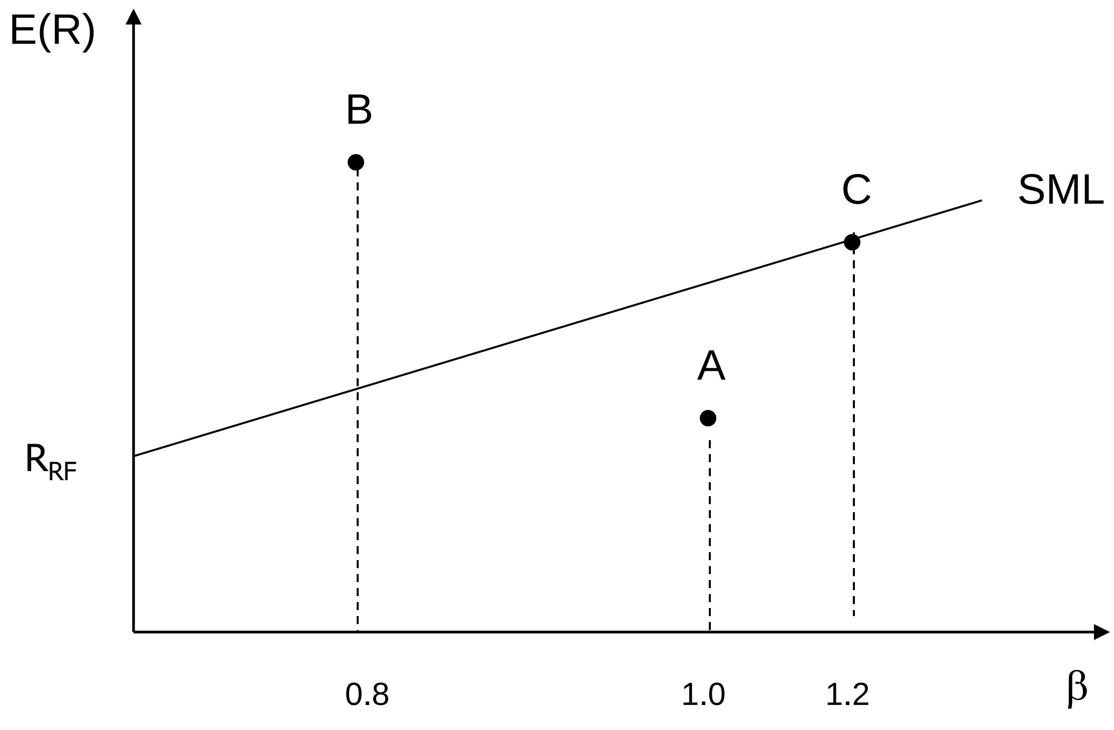
We can do this same analysis graphically. The expected return/beta combinations of all three stocks are graphed as follows relatively to SML.



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- Remember, all stocks should plot on the SML; any stock not plotting on the SML is mispriced.
- Stock A falls below the SML, Stock B lies above the SML, and Stock C is on the SML.
- If you plot a stock's expected return and it falls below the SML, and the stock is overpriced. That is, the stock's expected return is too low given its systematic risk.
- If the stock plots above the SML, it is underpriced and is offering an expected return greater than required for its systematic risk.
- If it plots on the SML, the stock is properly priced.

Summary

Total Risk, σ = Systematic Risk + Unsystematic Risk

In general, if $\sigma_1 > \sigma_2$, then asset 1 is riskier than asset 2.

Now, if asset 2 has more systematic risk than asset 1, then asset 2 is riskier than asset 1, since unsystematic risk can be diversified away.

For two-asset portfolio:

$$E(R_p) = w_1 E(R_1) + w_2 E(R_2)$$

$$\sigma_P = \sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1 w_2 COV_{1,2}}$$

$$\sigma_p = \sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1 w_2 \rho_{1,2}\sigma_1\sigma_2}$$ where $COV_{1,2} = \rho_{1,2}\sigma_1\sigma_2$

$$\sigma_p \neq w_1\sigma_1 + w_2\sigma_2 \text{ since } \rho_{1,2} \neq 1$$

$$\beta_p = w_1\beta_1 + w_2\beta_2 \text{ where } w_1 + w_2 = 1$$

Summary

Capital Asset Pricing Model (CAPM):

For a single risky asset: (assume that risky asset is part of well-diversified portfolio.

$$E(R_i) = R_f + \beta_i[E(R_m) - R_f)]$$

This is also true for stock portfolio. Hence,

$$E(R_p) = R_f + \beta_p[E(R_m) - R_f)]$$

CAPM is an equilibrium model. When capital market is at equilibrium, the security or portfolio are correctly priced, and they lie on a Security Market Line (SML). The required rate of return is the same as the expected rate of return.

Security or portfolio lies above (under) the SML is underpriced (overpriced).

CAPM is a single factor model as it assumes that the return on stock or portfolio is a function of a single factor, i.e., the market risk.

#1:

In broad terms, why is some risk diversifiable? Why are some risks nondiversifiable? Does it follow that an investor can control the level of unsystematic risk in a portfolio, but not the level of systematic risk?

Some risk is diversifiable as this risk is often associated with or is unique to the type of securities or the firms which issued them. Some examples of this risk include business risk, financial risk, management risk and liquidity risk. These risks can be minimised by investing in a broad spectrum of securities involving a number of different issuers.

On the other hand, there are some risks that affect all investments. They are caused by general factors such as changes in inflation rates, interest rates, and market sentiments. This portion of the total risk of an asset cannot be costlessly eliminated. In other words, systematic risk can be controlled, but only by a costly reduction in expected returns. To control the systematic risk in the portfolio, replace high beta stocks with a low beta stocks.

#2:

Indicate which of the following events might cause stocks in general to change price, and whether they might cause Big Widget Corp's stock to change price.

a. The government announces that inflation unexpectedly jumped by 2 percent last month.

A change in systematic risk has occurred; market prices in general will most likely decline.

b. Big Widget's quarterly earnings report, just issued, generally fell in line with analysts' expectations.

No change in unsystematic risk; company price will most likely stay constant.

c. The government reports that economic growth last year was at 3 percent, which was generally agreed with most economists' forecasts.

No change in systematic risk; market prices in general will most likely stay constant.

d. The directors of Big Widget die in a plane crash.

A change in unsystematic risk has occurred; company price will most likely decline.

e. Congress approves changes to the tax code that will increase the top marginal corporate tax rate. The legislation had been debated for the previous six months.

No change in systematic risk; market prices in general will most likely stay constant.

#3:

Stock Y has a beta of 1.3 and an expected return of 18.5 percent. Stock Z has a beta of 0.70 and an expected return of 12.1 percent. If the risk-free rate is 8 percent and the market risk premium is 7.5 percent, are these stock correctly priced?

Solution:

There are two ways to correctly answer this question. We will work through both. First, we can use the CAPM. Substituting in the value we are given for each stock, we find:

$$E(R_Y) = 0.08 + 0.075(1.30) = 0.1775 \text{ or } 17.75\%$$

It is given in the problem that the expected return of Stock Y is 18.5 percent, but according to the CAPM, the return of the stock based on its level of risk, the expected return should be 17.75 percent. This means the stock return is too high, given its level of risk. Stock Y plots above the SML and is undervalued. In other words, its price must increase to reduce the expected return to 17.75 percent

For Stock Z, we find:

$$E(R_Z) = 0.08 + 0.075(0.70) = 0.1325 \text{ or } 13.25\%$$

The return given for Stock Z is 12.1 percent, but according to the CAPM the expected return of the stock should be 13.25 percent based on its level of risk. Stock Z plots below the SML and is overvalued. In other words, its price must decrease to increase the expected return to 13.25 percent.

We can also answer this question using the reward-to-risk ratio. All assets must have the same reward-to-risk ratio. The reward-to-risk ratio is the risk premium of the asset divided by its β. We are given the market risk premium, and we know the β of the market is one, so the reward-to-risk ratio for the market is 0.075, or 7.5 percent. Calculating the reward-to-risk ratio for Stock Y, we find:

Reward-to-risk ratio Y = (0.185 – 0.08) / 1.30 = 0.0808

The reward-to-risk ratio for Stock Y is too high, which means the stock plots above the SML, and the stock is undervalued. Its price must increase until its reward-to-risk ratio is equal to the market reward-to-risk ratio.

For Stock Z, we find:

Reward-to-risk ratio Z = (0.121 – 0.08) / 0.70 = 0.0586

The reward-to-risk ratio for Stock Z is too low, which means the stock plots below the SML, and the stock is overvalued. Its price must decrease until its reward-to-risk ratio is equal to the market reward-to-risk ratio.

#4:

A stock has a beta of 1.35 and an expected return of 16%. A risk-free asset currently earns 4.8%.

a. What is the expected return on a portfolio that is equally invested in the two assets?

$$E(r_p) = w_s E(r_s) + w_{rf} E(r_{rf})$$

$$E(r_p) = 0.5\,(16\%) + 0.5\,(4.8\%)$$
$$= 10.4\%$$

Stock and risk-free asset

b. If a portfolio of the two assets has a beta of 0.95, what are the portfolio weights? ? Given: $\beta_{\text{stock}} = 1.35$

$$\beta_p = w_s\beta_s + w_{rf}\beta_{rf}$$

$\beta_p = 0.95 = w_S(1.35) + (1 – w_S)(0)$

$0.95 = 1.35w_S + 0$

$w_S = 0.95/1.35$

$w_S = 0.7037$

And, the weight of the risk-free asset is:

$w_{Rf} = 1 – 0.7037 = 0.2963$

Stock and risk-free asset

c. If a portfolio of the two assets has an expected return of 8%, what is its beta? Given: $\beta_{\text{stock}} = 1.35$; $E(R_{\text{stock}})$ =16%.; $r_{rf} = 4.8\%$

$$E(r_P) = w_s E(r_s) + (1 - w_s) r_{rf}$$

$$E(r_s) = 16\% \;\; ; \;\; r_{rf} = 4.8\%$$

$E(r_P) = 0.08 = 0.16w_S + 0.048(1 – w_S)$

$0.08 = 0.16w_S + 0.048 – 0.048w_S$

solving, $w_S = 0.2857$

So, the β of the portfolio will be:

$$\beta_P = w_s \beta_s + w_{rf} \beta_{rf}$$

$\beta_P = 0.2857(1.35) + (1 – 0.2857)(0) = 0.386$

Stock and risk-free asset

d. If a portfolio of the two assets has a beta of 2.7, what are the portfolio weights? How do you interpret the weights for the two assets in this case? Explain. ? Given: $\beta_{\text{stock}} = 1.35$

Assume we invest w_S in stock and $(1 – w_S)$ in risk-free asset

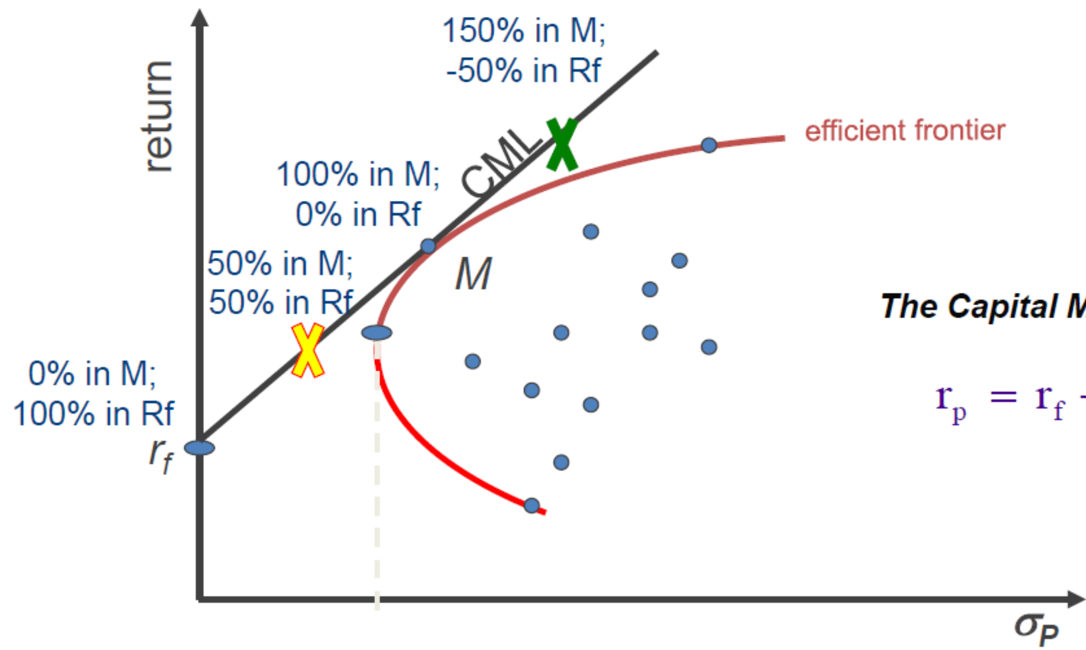
$\beta_p = 2.70 = w_S(1.35) + (1 – w_S)(0)$

$w_S = 2.70/1.35 = 2$

$w_{Rf} = 1 – 2 = –1$

The portfolio is invested 200% in the stock and –100% in the risk-free asset. This represents borrowing at the risk-free rate (or selling risk free asset) to buy more of the stock.

Efficient Frontier for All Possible Risky Assets with Borrowing/Lending at the Risk-Free Asset



The Capital Market Line (CML):

$$r_p = r_f + \frac{r_m - r_f}{\sigma_m}\sigma_p$$

With the capital allocation line identified, all investors choose a point along the line – some combination of the risk-free asset and the market portfolio *M*. In a world with homogeneous expectations, *M* is the same for all investors.

If you invest between the point r_f and M, you allocate a portion of your money to buy risk-free asset (lending) and the remaining to buy market portfolio. Between r_f and M Is lending. Suppose you have $100, If you invest beyond the point M, you invest more than $100 in market portfolio, say $150. You have $100 and you need to borrowing another $50 at risk-free rate. To borrow, you sell $50 worth of risk-free asset. A point beyond M is called borrowing. In this case, your portfolio is a leverage portfolio since you use leverage (borrowing).

#5:

Consider the following information on stock I and II:

State of Economy	Probability of State of Economy	Rate of Return if State Occurs	
		Stock I	Stock II
Recession	0.25	0.11	-0.40
Normal	0.50	0.29	0.10
Irrational exuberance	0.25	0.13	0.56

The market risk premium is 8%, and the risk-free rate is 4%. What stock has the most systematic risk? Which one has the most unsystematic risk? Which stock is “riskier”? Explain.

Stock I

$E(R_I) = 0.25(0.11) + 0.50(0.29) + 0.25(0.13) = 0.2050$ or 20.50%

Using the CAPM to find the β of Stock I, we find:

$0.2050 = 0.04 + 0.08\ \beta_I$

$\beta_I = 2.06$

$$E(R_i) = R_f + \beta_i \underbrace{(R_m - R_f)}_{\text{Market risk premium}}$$

$\sigma_I^{\ 2} = 0.25(0.11 - 0.2050)^2 + 0.50(0.29 - 0.2050)^2 + 0.25(0.13 - 0.2050)^2$

$\sigma_I^{\ 2} = 0.00728$; $\sigma_I = (0.00728)^{1/2} = 0.0853$ or 8.53%

Stock II

$E(R_{II}) = 0.25(-0.40) + 0.50(0.10) + 0.25(0.56) = 0.0900$ or 9.00%

$0.0900 = 0.04 + 0.08\ \beta_{II}$

$\beta_{II} = 0.63$

$\sigma_{II}^{\ 2} = = 0.25(-0.40 - 0.09)^2 + 0.50(0.10 - 0.09)^2 + 0.25(0.56 - 0.09)^2$

$\sigma_{II}^{\ 2} = 0.11530$; $\sigma_{II} = (0.11530)^{1/2} = 0.3396$ or 33.96%

Although Stock II has more total risk than stock I, it has much less systematic risk, since its beta is much smaller than stock I’s.

Thus, stock I has more systematic risk, and stock II has more unsystematic and more total risk.

Since unsystematic risk can be diversified away, stock I is actually the “riskier” stock despite the lack of volatility in its returns. Stock I will have a higher risk premium and a greater expected return.

#6:

Suppose you observe the following situation :

Security	Beta	Expected Return
Pete Corp	1.35	0.132
Repete Co.	0.80	0.101

Assume these securities are correctly priced. Based on the CAPM, what is the expected return on the market? What is the risk-free rate?

$E(R_{Pete}) = r_f + \beta_{Pete}\,(r_m - r_f)$; $0.132 = r_f + 1.35\,(r_m - r_f)$; …… (1)

$E(R_{Repete}) = r_f + \beta_{Repete}\,(r_m - r_f)$; $0.101 = r_f + 0.8\,(r_m - r_f)$; ……(2)

Solving (1) and (2),

$r_f = 0.0559 = 5.59\%$ and $r_m = 0.1123 = 11.23\%$

#7:

You are managing a portfolio of 6 stocks, which are held in equal dollar amounts. The current beta of the portfolio is 1.4, and the betas of Stock A is 1.8 and of Stock B is 2.0, respectively. If Stock A and Stock B are sold and the proceeds used to purchase 2 replacement stocks, what does the average beta of these 2 replacement stocks have to be to reduce the portfolio beta to 1.25?

Solution:

$$\beta_P = \sum_{i=1}^{n} w_i B_i$$

Since 6 stocks in the portfolio are equally weighted, $w_i = \frac{1}{n}$

$$\beta_P = \frac{1}{6}\sum_{i=1}^{6} B_i = 1.4 \quad \Rightarrow \sum_{i=1}^{6} B_i = 8.4$$

Total Sum of Beta of original 6 stocks = 6*1.4 = 8.4

Sum of remaining 4 stocks after sale of Stocks A and B = 8.4 – 1.8 – 2.0 = 4.6

Given that the target portfolio beta $\beta_P = \frac{1}{6}\sum_{i=1}^{6} B_i = 1.25$

So, target Sum of Beta of new portfolio = 6*1.25 = 7.5

Sum of Beta’s of 2 replacement stocks = 7.5 – 4.6 = 2.9

Average Beta of replacement stocks = 2.9/2 = **1.45**