Module 6
Motions & Morphing

Learning Objectives

- Understand two aspects of animation
- Define functions to simulate the effects of acceleration, deceleration, and uniform speed
- Construct time-dependent functions for motions or animation
- Use linear interpolation to create morphing

Sources

- Textbook (Chapter 8: How Computer Animation Works)

Outline

1. Introduction
2. Speed specification
3. Motion-by-path
4. Morphing

1. Introduction

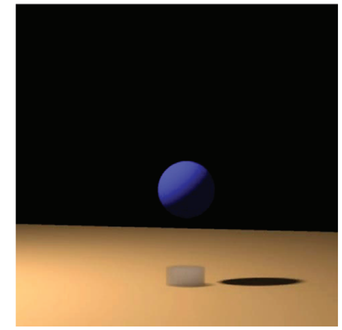
- Enjoy Pixar's movie "Geri's game"

This movie demonstrates three new techniques in computer graphics:

- *Modeling: arbitrary topology*
- *Animation: support for cloth dynamics*
- *Rendering: parametric texture mapping*

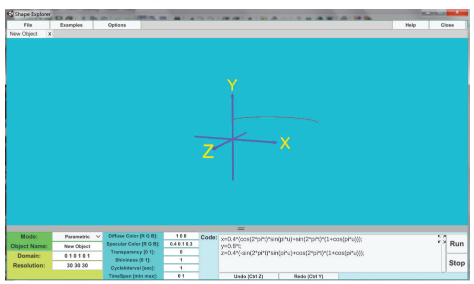
Animation

- Animation can be viewed as visual changes in a scene with time.
- An active research field in computer graphics
  - Motion caption
  - Physics-based simulation
  - Natural phenomena simulation

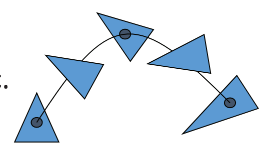


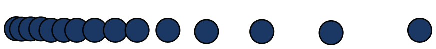
Basic concepts

- Basic idea is to introduce ***time*** into the definitions of shape, position, orientation, etc.



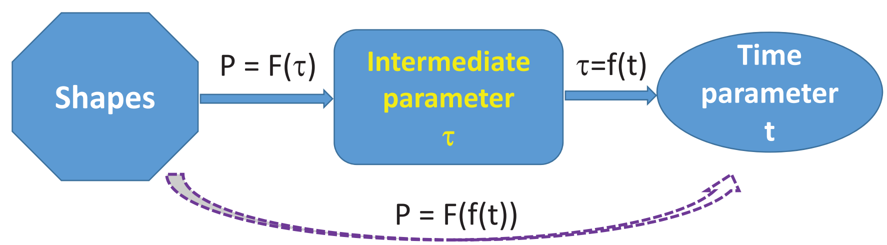
- Two fundamental aspects:
  - Change of shape, position, orientation, etc.
  - Speed control





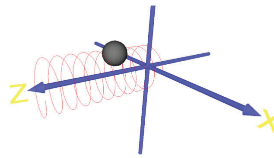
Our strategy

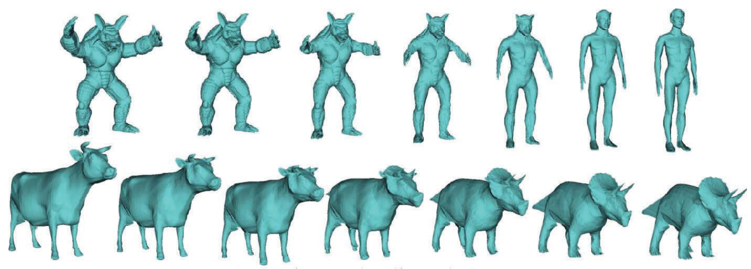
- Strategy for specifying **time-dependent** changes:
  - Define the shape, position, or orientation, etc., by functions of some parameter (say, τ)
  - Define function $\tau = f(t)$ to relate the change to time



Two techniques

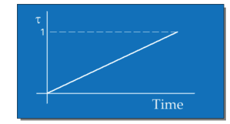
- In this module, we will learn two simple animation techniques
  - Motion-by-path: an object is moving along pre-defined path
  - Morphing: smooth transition between two shapes



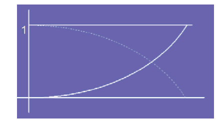


2. Speed specification: τ=f(t)

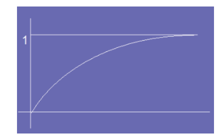
- How to define an appropriate function τ = f(t) to simulate:
  - Uniform $\tau = f(t) = \frac{t - t_1}{t_2 - t_1},\ t \in [t_1, t_2]$



  - Acceleration $\tau = f(t) = 1 - \cos\left(\frac{\pi}{2}\frac{t - t_1}{t_2 - t_1}\right),\ t \in [t_1, t_2]$

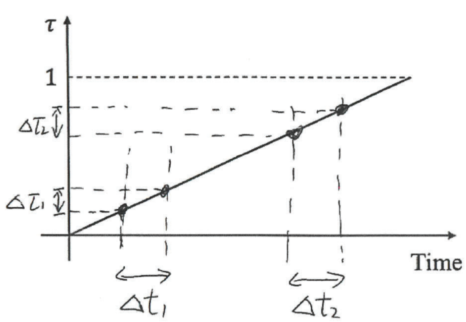


  - Deceleration $\tau = f(t) = \sin\left(\frac{\pi}{2}\frac{t - t_1}{t_2 - t_1}\right),\ t \in [t_1, t_2]$



Uniform speed

- Uniform $\tau = f(t) = \frac{t - t_1}{t_2 - t_1},\ t \in [t_1, t_2]$



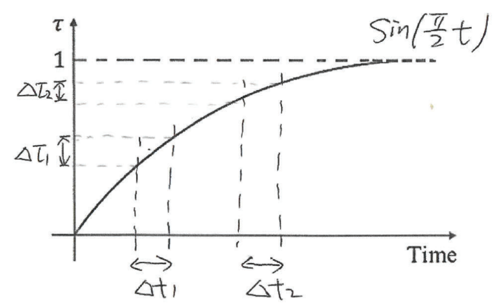
$\Delta t_1 = \Delta t_2$

$\Delta \tau_1 = \Delta \tau_2$

t Const speed

Deceleration

- Deceleration
$$\tau = f(t) = \sin\left(\frac{\pi}{2}\frac{t - t_1}{t_2 - t_1}\right),\ t \in [t_1, t_2]$$



$\Delta t_1 = \Delta t_2$

$\Delta \tau_1 > \Delta \tau_2$

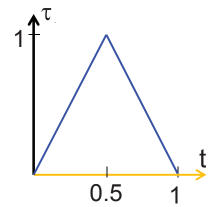
deceleration

- Similar analysis applies to acceleration.

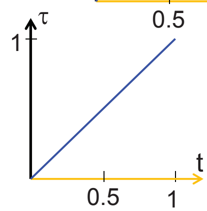
More complicated speed

- For simulating complicated speed, some general functions are needed. For example, cubic spline functions.
  - Professional animation software provides such a function.
- Example: how to simulate "back and forth"?
  - One solution

$$\tau = f(t) = 1 - |1 - 2t| = \begin{cases} 2t, & 0 \le t \le \frac{1}{2} \\ 2 - 2t, & \frac{1}{2} \le t \le 1 \end{cases}$$



  - As a comparison, $\tau = f(t) = t$



Two descriptions of animation models

- Use time parameter

$$\tau = f(t), \quad t \in [t_1, t_2]$$

(c) With reference to Figure Q4, propose parametric formulae which define a jumping solid cone. The cone has radius 1 and height 2. The cone jumps 3 times, moving in the path defined by $y=|\sin(x)|$, $z=0$. The motion takes 9 seconds.

(12 marks)

- Use frame index

$$\tau = f(k), \quad k = 1, 2, 3, \cdots$$

where *k* is the frame index, $1 \le k \le m$,
m is the total number of frames

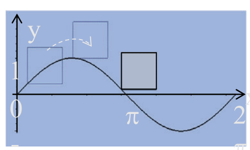
Example:

(c) A square polygon with vertices at coordinates (-1, 1), (1, 1), (1, -1), (-1, -1) transforms into an origin-centered disk with radius 2. Define this morphing by parametric functions. The animation takes 1024 frames and has to be done with acceleration. The frames are numbered 1, 2, 3, etc.

(12 marks)

3. Motion-by-path

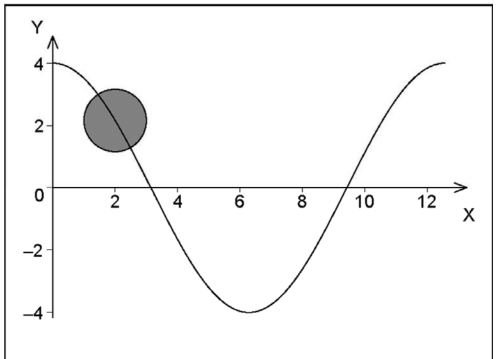
- "Motion by path" approach: define a motion or animation via explicitly specifying the motion path



- Method:
  - Step 1: Represent the path by parametric equations: $(x, y, z) = (x(\tau), y(\tau), z(\tau))$, $\tau \in [0, 1]$.
  - Step 2: By linking the path to the target animation, derive the representation of the motion object.
  - Step 3: Appropriately define $\tau = f(t)$, $t \in [t_1, t_2]$ or $\tau = f(k)$, $k = 1, \ldots, m$ to control the speed.

Example 1

Q: The following figure shows a unit disk moving on the XY plane. During the movement, the center of the disk moves along a trajectory defined by $y = 4\cos(x/2)$ from point (0, 4) to point (4π, 4). Propose a mathematical model in implicit representation for this motion. The motion consists of 100 frames and involves deceleration.



Example 1 (cont)

Hint:

Step 1. The equation of the trajectory is

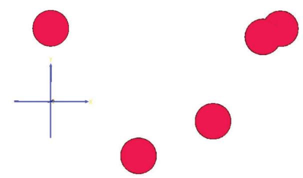
$$\begin{cases} x_o &= 4\pi u \\ y_o &= 4\cos(4\pi u/2) = 4\cos(2\pi u) \end{cases} \quad u \in [0, 1]$$

Step 2. The moving disk can thus be represented implicitly by

$$1-(x-4\pi u)^2-(y-4\cos(2\pi u))^2 \geq 0$$

Step 3. To specify the speed, let

$$u = \sin\left(\frac{\pi}{2}\frac{k-1}{100-1}\right), \quad k = 1,2,\cdots 100.$$



Recap

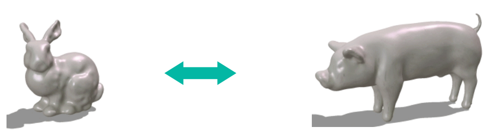
- Speed simulation by using an appropriate function in time
  - Uniform
  - Acceleration
  - Deceleration
- Motion-by-path
  - Derive parametric representation of the path
  - Derive the animation model
- Two ways to describe an animation
  - Time parameter
  - Frame index

4. Morphing by linear interpolation

- **Problem:** Given two items *A* and *B* of the ***same type***, we want to compute an intermediate item v(τ) which gradually changes from *A* to *B* at a constant rate.
- Linear interpolation model:

  $v(\tau) = (1 - \tau)A + \tau B, 0 \leq \tau \leq 1.$

  - When $\tau = 0$, $v(\tau) = A$
  - When $\tau = 1$, $v(\tau) = B$
  - For intermediate values of parameter τ, $v(\tau)$ is a linear combination of A and B



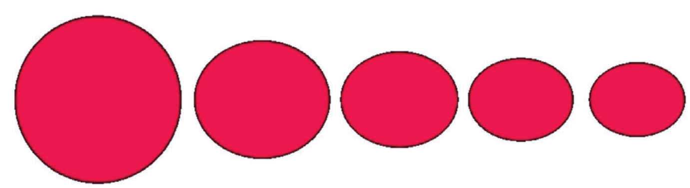
Example 1

Q: A mathematical model for a morphing between two shapes is given by $F(x, y, t) = 1+(1-2t)\, x^2+(3-4t)\, y^2 \geq 0$, where $t \in [1, 2]$ is the time parameter.

(i) Find the implicit representations for the two shapes corresponding to $t = 1$ and $t = 2$.

(ii) Modify the model to yield a morphing with deceleration.

(iii) Using the frame index as the parameter, express the morphing in (ii) as an animation that takes 100 frames.



Example 1 (cont)

Hint:

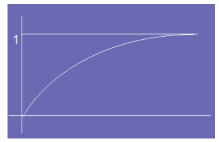
(i) t = 1: F1(x,y) = F(x,y,1) = 1+(1-2) x^2+(3-4) y^2 = 1- x^2–y^2 ≥ 0

t = 2: F2(x, y) = F(x,y,2) = 1+(1-4) x^2+(3-8) y^2 =1-3 x^2–5 y^2 ≥ 0

(ii) The modified model would be

F(x,y) = (1- x^2 -y^2) (1-s) + (1-3 x^2- 5 y^2) s ≥ 0

where $$s = \sin\left(\frac{t-1}{2-1}\frac{\pi}{2}\right), \quad t \in [1,2].$$



(iii) The model could be expressed as

F(x,y) = (1- x^2 -y^2) (1-s) + (1-3 x^2- 5 y^2) s ≥ 0

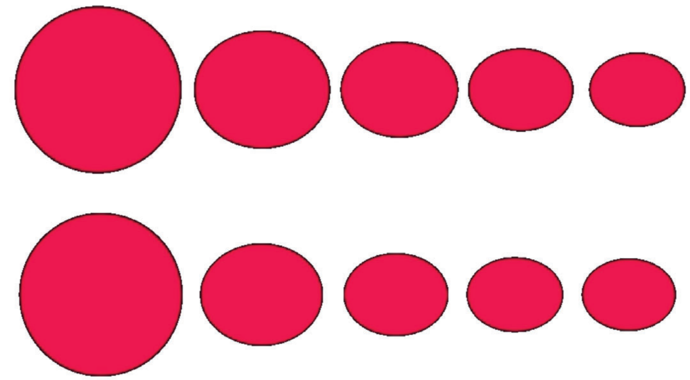
where $$s = \sin\left(\frac{k-1}{100-1}\frac{\pi}{2}\right), \quad k = 1,2,\cdots 100.$$

Example 1 (cont)

Visual comparison of the two morphing models

Original

New



Example 3: a hybrid situation

Q: There are 2 shapes A and B, where A is defined by

x=4cosθ, y=2sinθ, θ∈[0,2π], and B is defined by

.
$$\frac{(x-1)^2}{1^2} + \frac{(y-2)^2}{3^2} = 1$$

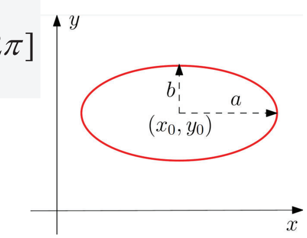
(1). Using implicit functions, propose an animation model which transforms shape A at time 10 to shape B at time 20 in a uniform speed.

(2). Using parametric functions, propose an animation model which transforms shape A at time 10 to shape B at time 20 in a uniform speed.

Example 3: a hybrid situation (cont)

Hints: Background knowledge – a coordinate axis-aligned ellipse can be defined implicitly by $1 - \frac{(x-x_0)^2}{a^2} - \frac{(y-y_0)^2}{b^2} = 0$

or parametrically by $$\begin{cases} x = x_0 + a\cos\theta \\ y = y_0 + b\sin\theta \end{cases}, \theta \in [0, 2\pi]$$



(1). Represent shape A in implicit function:

$$f(x,y,z) = 1 - \frac{x^2}{4^2} - \frac{y^2}{2^2} = 0$$

Define $g(x,y,z) = 1 - \frac{(x-1)^2}{1^2} - \frac{(y-2)^2}{3^2} = 0$ for shape B.

So the animation model could be

Example 3: a hybrid situation (cont)

h(x,y,z) = f(x,y,z) $(1-\tau)$+g(x,y,z) τ = 0, $\tau \in [0,1]$.

To specify the speed, let τ = (t-10)/(20-10), t $\in$[10,20].

(2). Represent shape B in parametric equations:

x=1+cosθ, y=2+3sinθ, $\theta \in [0,2\pi]$.

So now the animation model can be

$$\begin{cases} x = 4\cos\theta\,(1-\tau) + (1+\cos\theta)\,\tau \\ y = 2\sin\theta\,(1-\tau) + (2+3\sin\theta)\,\tau \end{cases}, \theta \in [0,2\pi], \tau \in [0,1]$$

To specify the speed, let τ = (t-10)/(20-10), t $\in$[10,20].

Example 4

Exam AY13/14 S2

(b) An animation transforms an origin-centered unit circle to a curve implicitly defined by 1 - |x| - |y| = 0 from time 0 to time 10 in acceleration and then further transforms from time 10 to 20 in uniform speed to an origin-centered ellipse with semi-major and semi-minor axes of 9 and 5, which coincide with axes X and Y. Propose a mathematical model for this animation.

(8 marks)

Solution:

Shape 1: f(x,y) = 1-x^2-y^2 = 0 (1 mark)

Shape 2: g(x,y) = 1-|x|-|y| =0 (1 mark)

Shape 3: h(x,y) = 1-x^2/81-y^2/25 = 0 (1 mark)

Example 4 (cont)

For time t from 0 to 10, the animation model could be

F(x,y, t) = f(x,y)(1-s) + g(x,y)s = 0 (1 mark)

where

$$s = 1 - \cos\left(\frac{\pi}{2}\frac{t-0}{10-0}\right) = 1 - \cos\left(\frac{\pi}{20}t\right)$$ (2 marks)

For time t from 10 to 20, the animation model could be

F(x,y, t) = g(x,y)(1-s) + h(x,y)s = 0 (1 mark)

where

$$s = \frac{t-10}{10}$$ (1 mark)

Hands-on experiments

Q1: Derive a matrix representation for a rotation by 1.5π about an axis defined from the origin to the point (0,1,1).

Ans: Applying the method of "aligning vector to z-axis" gives

post-process $R_x(-\alpha)$ rotation $R_z(1.5\pi)$ pre-process $R_x(\alpha)$

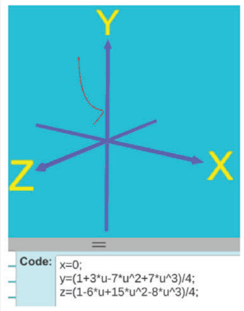
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(1.5\pi) & -\sin(1.5\pi) & 0 & 0 \\ \sin(1.5\pi) & \cos(1.5\pi) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}} & \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Hands-on experiments

Q2: A parametric curve is defined by

$$\begin{cases} x = 0 \\ y = \dfrac{1+3u-7u^2+7u^3}{4} \\ z = \dfrac{1-6u+15u^2-8u^3}{4} \end{cases} \quad u \in [0,1]$$



Apply the rotation in Q1 to this curve. Derive the rotated curve and display it.

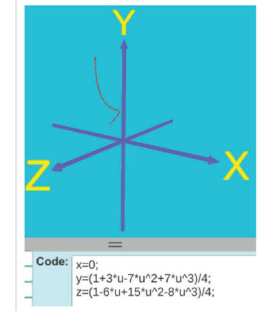
Ans:

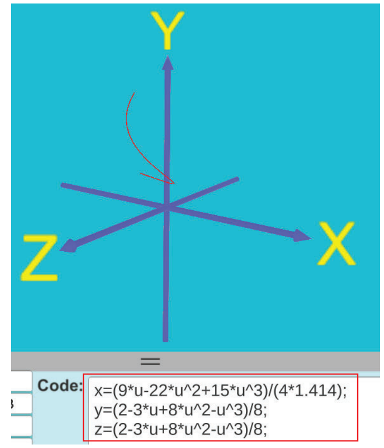
$$\begin{bmatrix} 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}} & \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ \frac{1+3u-7u^2+7u^3}{4} \\ \frac{1-6u+15u^2-8u^3}{4} \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{9u-22u^2+15u^3}{4\sqrt{2}} \\ \frac{2-3u+8u^2-u^3}{8} \\ \frac{2-3u+8u^2-u^3}{8} \\ 1 \end{bmatrix}$$

Hands-on experiments

Ans:

use “Shape Explorer”





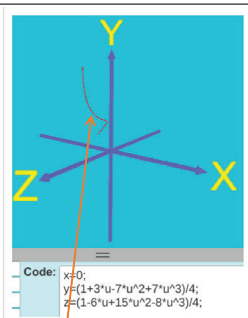
Hands-on experiments

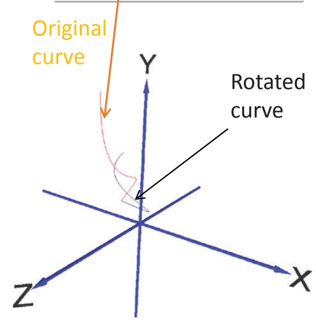
Ans: just use FShape from FVRML

```
FShape {
    polygonizer "analytical_curve"
    geometry FGeometry {
        definition "x=(9*u-22*u^2+15*u^3)/(4*1.414);
            y=(2-3*u+8*u^2-u^3)/8;
            z=(2-3*u+8*u^2-u^3)/8;"
        parameters [0 1]
        resolution [100]
    }
```

or use Transform from FVRML

```
Transform {
    rotation 0 1 1 4.712
    children[
        FShape {
            polygonizer "analytical_curve"
            geometry FGeometry {
                definition "x=0;
                    y=(1+3*u-7*u^2+7*u^3)/4;
                    z=(1-6*u+15*u^2-8*u^3)/4;"
                parameters [0 1]
                resolution [100]
        }
```





Red: original curve
Blue: rotated curve

Hands-on experiments

Q3: Construct a parametric surface by a rotational sweeping using the rotation in Q1 and the profile in Q2 and display it.

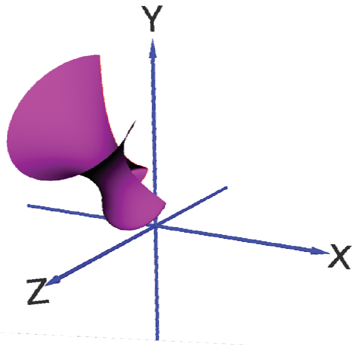
Ans: just replace 1.5π by 1.5π v, v∈ [0,1], in the previous rotation matrix.

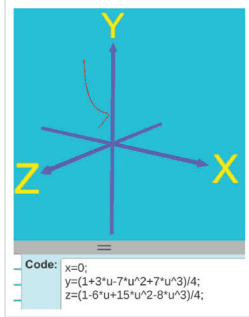
$$\begin{bmatrix} x(u,v) \\ y(u,v) \\ z(u,v) \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(1.5\pi v) & -\sin(1.5\pi v) & 0 & 0 \\ \sin(1.5\pi v) & \cos(1.5\pi v) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ \frac{1+3u-7u^2+7u^3}{4} \\ \frac{1-6u+15u^2-8u^3}{4} \\ 1 \end{bmatrix}$$

$$= \frac{1}{8} \begin{bmatrix} -\sqrt{2}(9u-22u^2+15u^3)\sin(1.5\pi v) \\ 2-3u+8u^2-u^3+(9u-22u^2+15u^3)\cos(1.5\pi v) \\ 2-3u+8u^2-u^3-(9u-22u^2+15u^3)\cos(1.5\pi v) \\ 1 \end{bmatrix}$$

Hands-on experiments

Ans: FVRML.






```
FShape {
    geometry FGeometry {
        definition "x=-sqrt(2)*sin(1.5*pi*v)*(9*u-22*u^2+15*u^3)/8;
         y=(2-3*u+8*u^2-u^3+(9*u-22*u^2+15*u^3)*cos(1.5*pi*v))/8;
         z=(2-3*u+8*u^2-u^3-(9*u-22*u^2+15*u^3)*cos(1.5*pi*v))/8;"
        parameters [0 1 0 1]
        resolution [100 50]
    }
```


Hands-on experiments

Q4: Propose an animation model using functions x(u,t), y(u,t) and z(u,t) to show the rotation in Q1 applied to the profile in Q2.

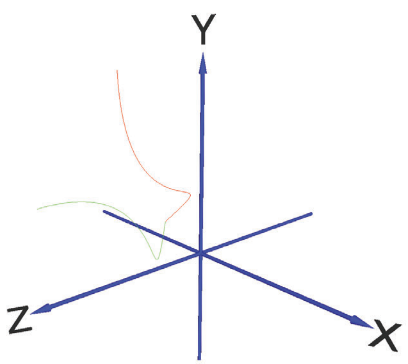
Ans: just replace v in Q3 by t.

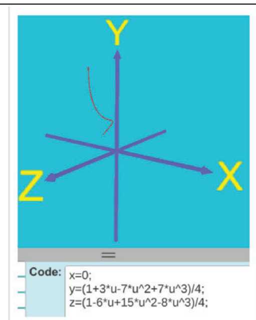
$$\begin{bmatrix} x(u,t) \\ y(u,t) \\ z(u,t) \\ 1 \end{bmatrix} = \frac{1}{8} \begin{bmatrix} -\sqrt{2}(9u - 22u^2 + 15u^3)\sin(1.5\pi t) \\ 2 - 3u + 8u^2 - u^3 + (9u - 22u^2 + 15u^3)\cos(1.5\pi t) \\ 2 - 3u + 8u^2 - u^3 - (9u - 22u^2 + 15u^3)\cos(1.5\pi t) \\ 1 \end{bmatrix}$$

where $t \in [0, 1]$

Hands-on experiments

Ans: FVRM






```
FShape {
    polygonizer "analytical_curve"
    loop TRUE
    cycleInterval 10
    geometry FGeometry {
        definition "x=-sqrt(2)*sin(1.5*pi*t)*(9*u-22*u^2+15*u^3)/8;
         y=(2-3*u+8*u^2-u^3+(9*u-22*u^2+15*u^3)*cos(1.5*pi*t))/8;
         z=(2-3*u+8*u^2-u^3-(9*u-22*u^2+15*u^3)*cos(1.5*pi*t))/8;"
        parameters [0 1]
        resolution [100]
    }
```


Hands-on experiments

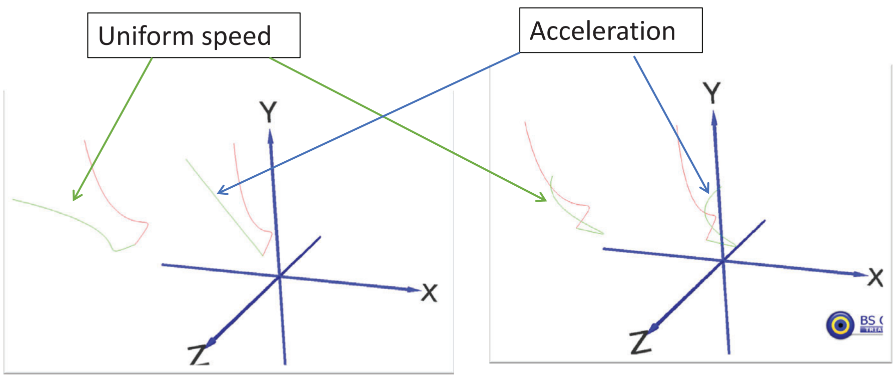
- **Q5:** Modify the proposed model in Q4 to involve some acceleration.

Ans: Just replace t in Q4 by $t \leftarrow 1 - \cos\left(\frac{\pi}{2}\, t\right)$

```
FShape {
    polygonizer "analytical_curve"
    loop TRUE
    cycleInterval 5
    geometry FGeometry {
        definition "
                x=-sqrt(2)*sin(1.5*pi* (1-cos(t*pi/2)) )*(9*u-22*u^2+15*u^3)/8;
                y=(2-3*u+8*u^2-u^3+(9*u-22*u^2+15*u^3)*cos(1.5*pi* (1-cos(t*pi/2))))/8;
                z=(2-3*u+8*u^2-u^3-(9*u-22*u^2+15*u^3)*cos(1.5*pi* (1-cos(t*pi/2))))/8;"
        parameters [0 1]
        resolution [100]
    }
    appearance FAppearance {
        material FMaterial {
            diffuseColor "r=0; g=1; b=0;"
        }
    }
}
```

Hands-on experiments

Ans: The figures below show the rotating curves at two time points. Comparing to the curves with uniform speed, we can see the effect of acceleration.

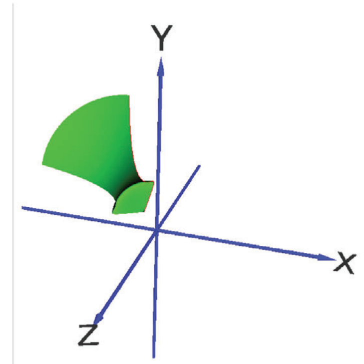


t is close to 0 (starting time). t is close to 1 (ending time).

Hands-on experiments

Q6: Propose an animation model to show the process of the rotation sweeping in Q3.

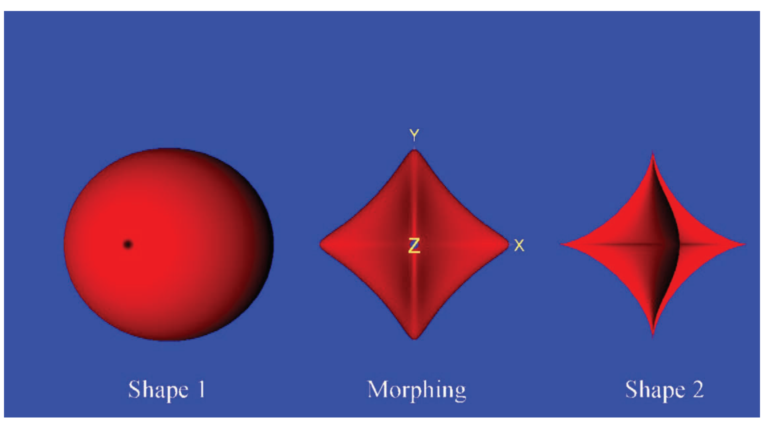
Ans: Replace "v" in Q3 by "v*t". Then the rotating curve becomes a sweeping surface.




```
FShape {
   loop TRUE
   cycleInterval 5
   geometry FGeometry {
       definition "
                  x=-sqrt(2)*sin(1.5*pi*v* t )*(9*u-22*u^2+15*u^3)/8;
                  y=(2-3*u+8*u^2-u^3+(9*u-22*u^2+15*u^3)*cos(1.5*pi*v* t))/8;
                  z=(2-3*u+8*u^2-u^3-(9*u-22*u^2+15*u^3)*cos(1.5*pi*v* t))/8;"
       parameters [0 1 0 1]
       resolution [100 100]
   }
```


Hands-on experiments

- **Problem:** Given two parametric surfaces, define an animated transformation (morphing) between them.



Morphing of parametric surfaces

Input:

surface A: $x = h_1(a,b)$, $y = h_2(a,b)$, $z = h_3(a,b)$

$a \in [a_0, a_1]$, $b \in [b_0, b_1]$

surface B: $x = g_1(r,s)$, $y = g_2(r,s)$, $z = g_3(r,s)$

$r \in [r_0, r_1]$, $s \in [s_0, s_1]$

Method:

Step 1. Reparameterize both surfaces to normalize their parameter domains. Let

$(a-a_0)/(a_1-a_0) = u \in [0,1] \rightarrow a = a(u)$

$(b-b_0)/(b_1-b_0) = v \in [0,1] \rightarrow b = b(v)$

$\rightarrow$ A: $x = h_1(a(u),b(v))$, $y = h_2(a(u),b(v))$, $z = h_3(a(u),b(v))$

Similarly,

B: $x = g_1(r(u),s(v))$, $y = g_2(r(u),s(v))$, $z = g_3(r(u),s(v))$

Morphing of parametric surfaces (cont)

Step 2. Linear interpolation for C = A→B:

$$x = h_1(a(u),b(v))\,(1-\tau) + g_1(r(u),s(v))\,\tau$$
$$y = h_2(a(u),b(v))\,(1-\tau) + g_2(r(u),s(v))\,\tau$$
$$z = h_3(a(u),b(v))\,(1-\tau) + g_3(r(u),s(v))\,\tau \qquad \tau \in [0,1]$$

Step 3. Specify the speed by defining $\tau = f(t)$:

for uniform speed, $\tau = f(t) = t$, $t \in [0,1]$

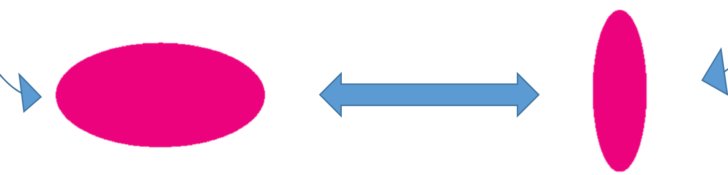
or τ = f(t) = 1-fabs(1-2*t) for the "back and forth" effect

Question for you

Q: There are 2 shapes A and B where

- A is defined by $x = 4\,a\cos(2\pi b)$, $y = 2\,a\sin(2\pi b)$, $a,b \in [0,1]$;
- B is defined by $x = s\cos(2\pi t)$, $y = 3\,s\sin(2\pi t)$, ,$s,t \in [0,1]$.

1) Pairing a⇔s and b⇔t, propose an animation model which transforms shape A to shape B in a uniform speed.
2) How about switching the pairing such that a⇔t and b⇔s?
3) Can you get any idea from this example for your lab 5 Q3?



Recap

- Morphing using linear interpolation
  - Implicit function
  - Parametric functions
- Motion by time-dependent transformations

END