CZ2003
Computer Graphics and Visualization

Revision of
Modules 1, 2, 3

AY 2020/2021, Sem. 1

Module 1: Introduction and Foundation Mathematics

- Definition of computer graphics
- Visualization steps (objects, materials, viewpoints, light sources)
- Various coordinate systems → coordinate mapping between systems (Cartesian 2D and 3D (right/left handed), 2D polar, 3D spherical, 3D cylindrical). Right/Left-handed as well as order of coordinates in other systems is important.
- Definition and types of mathematical functions: explicit, implicit parametric functions. How to convert between them.
- Pythagoras' theorem and consequences: angles and distances
- Matrices, vectors and operations on them
- Definition of coordinates by vector operations (sum of scaled vectors)
- Trigonometric functions: any harmonic oscillations (geometry, colors, motions)
- Dot product: angle between vectors
- Cross product: normal to the plane surface

Module 1: Introduction and Foundation Mathematics

- Conversion from polar, cylindrical and spherical coordinates to parametric functions in Cartesian coordinates is based on the right triangle rules:

  $x = r\cos\alpha \qquad y = r\sin\alpha$
- Inverse conversion from parametric functions in Cartesian coordinates to polar coordinates, etc. is still based on the Pythagoras triangle that

  $r^2 = x^2 + y^2 \quad \tan\alpha = y/x$
- Note that polar coordinate α is computed about the origin of the coordinate system.

Module 2: Programming Computer Graphics and Visualization

- Classification of the software tools into imperative and declarative, pixel-based and polygon-based.

Module 3: Geometric Shapes 1/8

- Should be able
  - to define curves parametrically
  - to define surfaces and solids parametrically and implicitly
- Define straight line, ray, and segment by:
  - coordinates of any 2 points on it
  - coordinates of intercept points on the coordinate axes
  - one point and a vector along the line
- Define plane by:
  - coordinates of 3 points on it
  - coordinates of intercept points on the coordinate axes
  - one point and a normal vector to the plane
  - two vectors/lines (intersecting or parallel)
- Remember that coefficients A,B,C of the plane equation A*x*+B*y*+C*z*+D=0 define coordinates of the normal vector to the plane

Module 3: Geometric Shapes 2/8

- In parametric representation:
  - curves – 1 parameter,
  - surfaces – 2 parameters,
  - 3D solids – 3 parameters
- Plane is defined parametrically based on sum of two vectors
- Bilinear surfaces are used for defining 4-sided patches of surfaces (not the whole plane).
- Triangle can be defined as a bilinear patch by putting together two of the vertices in the 4-sided bilinear patch.
- The concept of sweeping is used for defining curves as moving points, surfaces as moving curves and solids as moving surfaces.
- When defining rotational sweeping of plane objects:
  - Do not change the definition of the coordinate corresponding to the axis of rotation,
  - The other coordinate definition is used as a radius of a circle defined in the right-hand sense: 1st axis: *r**cos(), 2nd axis: *r**sin()

Module 3: Geometric Shapes 3/8

- Fast forward:

General rotational and translational sweeping can be defined using matrix transformations for rotation and translation (module 5):

[x,y,z formulas]=[Transformation matrix] [x,y,z formulas].

$$x' = ax + by + cz + l$$
$$y' = dx + ey + fz + m$$
$$z' = gx + hy + kz + n$$

$$\mathbf{P}' = \begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} ax+by+cz+l \\ dx+ey+fz+m \\ gx+hy+kz+n \\ 1 \end{bmatrix} = \begin{bmatrix} a & b & c & l \\ d & e & f & m \\ g & h & k & n \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Module 3: Geometric Shapes 4/8

- Example: Scaling by S_x, S_y and S_z with reference to the point (l, m, n)

$$\begin{bmatrix} x'(u) \\ y'(u) \\ z'(u) \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 & l(1-s_x) \\ 0 & s_y & 0 & m(1-s_y) \\ 0 & 0 & s_z & n(1-s_z) \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x(u) \\ y(u) \\ z(u) \\ 1 \end{bmatrix}$$

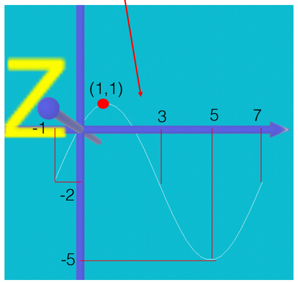
Scaling by 2, 3 and 1 with reference to the point (1, 1, 0)

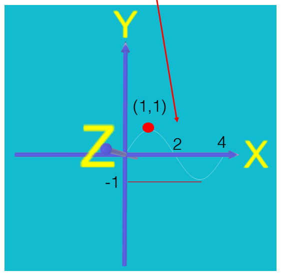
$$\begin{bmatrix} x'(u) \\ y'(u) \\ z'(u) \\ 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 & 1(1-2) \\ 0 & 3 & 0 & 1(1-3) \\ 0 & 0 & 1 & 0(1-1) \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x(u) \\ y(u) \\ z(u) \\ 1 \end{bmatrix}$$

Module 3: Geometric Shapes 5/8

- Scaling by 2, 3 and 1 with reference to point (1, 1, 0)

$$\begin{bmatrix} 8u-1 \\ 3\sin(2\pi u)-2 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 & -1 \\ 0 & 3 & 0 & -2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4u \\ \sin(2\pi u) \\ 0 \\ 1 \end{bmatrix}$$





Module 3: Geometric Shapes 4/8

- Example: Scaling by S_x, S_y and S_z with reference to the point (l, m, n)

$$\begin{bmatrix} x'(u) \\ y'(u) \\ z'(u) \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 & l(1-s_x) \\ 0 & s_y & 0 & m(1-s_y) \\ 0 & 0 & s_z & n(1-s_z) \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x(u) \\ y(u) \\ z(u) \\ 1 \end{bmatrix}$$

Animating the scaling by 2, 3 and 1 with reference to the point (1,1,0)

$$\begin{bmatrix} x'(u) \\ y'(u) \\ z'(u) \\ 1 \end{bmatrix} = \begin{bmatrix} 1+1t & 0 & 0 & 1(1-(1+1t)) \\ 0 & 1+2t & 0 & 1(1-(1+2t)) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x(u) \\ y(u) \\ z(u) \\ 1 \end{bmatrix}$$

Module 3: Geometric Shapes 5/8

- Example: Scaling by S_x, S_y and S_z with reference to the point (l, m, n)

$$\begin{bmatrix} 4u(1+t)-t \\ (1+2t)\sin(2\pi u)-2t \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1+t & 0 & 0 & -t \\ 0 & 1+2t & 0 & -2t \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4u \\ \sin(2\pi u) \\ 0 \\ 1 \end{bmatrix}$$

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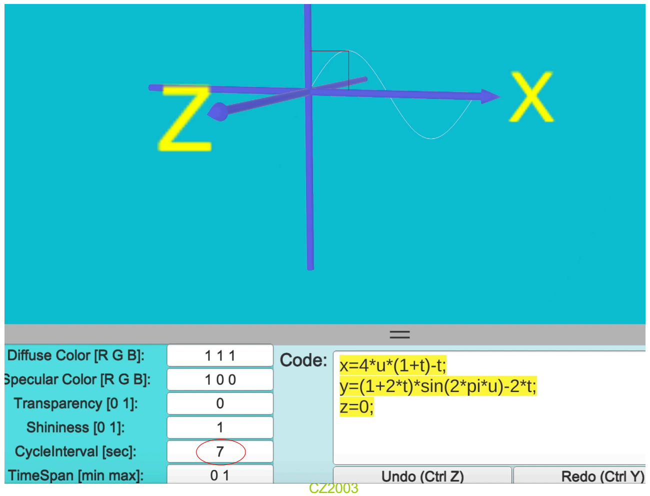
Module 3: Geometric Shapes 6/8



Module 3: Geometric Shapes 7/8

- Boolean (Set-Theoretic) operations for solid objects can be defined by inequalities $f(x, y, z) \geq 0$.
  Only in this case,
  $\min(f_1, f_2) \geq 0$ defines intersection,
  $\max(f_1, f_2) \geq 0$ - union,
  $-f \geq 0$ - complement
  $\min(f_1, -f_2) \geq 0$ - difference operations.
- Be able to find an intersection point of a straight line (ray) with a surface (Modules 7-8):
  - Define the straight line parametrically, while the surface – implicitly
  - Substitute parametric definitions $x(u)\ y(u)\ z(u)$ into implicit formula for the surface $f(x, y, z) = 0$ and solve it in terms of parameter u.
  - Among several u that can be produced, select the lowest value.
  - Obtain x, y, z by substituting the derived u.
- Blobby shapes add new opportunities and illustrate how implicit definitions cab be further enriched

End of Part 1

Module 3: Geometric Shapes 8/8

- How to draw curves and surfaces: formulas have to be sampled to calculate coordinates of the polyline and polygon vertices on curves and surfaces. Sampling is controlled by bounding boxes (domains of the coordinates) and resolutions for implicit functions and by parametric domains and resolutions for parametric functions.
  ★ Formula,
  ★ Domain (bounding box or parameters range),
  ★ Sampling (resolution vales)