1. Let
$$U = \{(x, y) : x^2 + y^2 = 1\}, F = \{(x, y) : 2x + y = 1\}$$

and
$$G = \{(x, y) : |x| + |y| = 1\}, H = \{(x, y) : |x| + |y| \leq 1\}.$$

   (a) Determine $U \cap F$, $U \cap G$, $F \cap G$, $F \cap H$, and $G \cup H$.

   (b) True or False? $(0, 1) \subseteq U$, $H \supseteq G$, $U \cap F \subseteq H$.

   Solution.   1. Set U contains all of the points on the unit circle; Set F contains all of the points on a line; Set G contains all of the points on a rhombus; Set G contains all of the points on a rhombus and its interior points.

   (a) $U \cap F = \{(0, 1), (\frac{4}{5}, -\frac{3}{5})\}$, $U \cap G = \{(0, 1), (0, -1), (1, 0), (-1, 0)\}$, $F \cap G = \{(0, 1), (\frac{2}{3}, -\frac{1}{3})\}$, $F \cap H = \{(x, y) : x = \frac{2(1-\lambda)}{3}, y = \frac{4\lambda-1}{3}, \lambda \in [0, 1]\}$, and $G \cup H = H$.

   (b) False, True, False.

2. Let
$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}, B = \begin{pmatrix} -1 & 1 & 0 \\ 1 & -1 & 1 \\ 0 & 1 & -1 \end{pmatrix}, c = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix},$$

   determine

   a) A', $A + 3B'$, $A - 3B'$;

   b) AB, $B'A'$;

   c) Bc, $B^\top c$, $c^\top A$.

   Solution.

   a)
$$A' = \begin{pmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{pmatrix}, A + 3B' = \begin{pmatrix} -2 & 5 & 3 \\ 7 & 2 & 9 \\ 7 & 11 & 6 \end{pmatrix}, A - 3B' = \begin{pmatrix} 4 & -1 & 3 \\ 1 & 8 & 3 \\ 7 & 5 & 12 \end{pmatrix}$$

b)

$$AB = \begin{pmatrix} 1 & 2 & -1 \\ 1 & 5 & -1 \\ 1 & 8 & -1 \end{pmatrix}, B'A' = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 5 & 8 \\ -1 & -1 & -1 \end{pmatrix}$$

c)

$$Bc = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}, A^\top c = \begin{pmatrix} 30 \\ 36 \\ 42 \end{pmatrix}, c^\top A = (30, 36, 42)$$

3. Let $U = \{1, 2, 3\}$ and $T = \{4, 5\}$, determine the RHS:

a) $\displaystyle\sum_{i \in S} \sum_{j \in T} a_{ij} x_{ij} =$

b) $V = \{(i, j) : i \in S, j \in T\}, \displaystyle\sum_{(i,j) \in V} x_{i,j} =$

Solution.

a) $\displaystyle\sum_{i \in S} \sum_{j \in T} a_{ij} x_{ij} = a_{14} x_{14} + a_{24} x_{24} + a_{34} x_{34} + a_{15} x_{15} + a_{25} x_{25} + a_{35} x_{35}$

b) $V = \{(i, j) : i \in S, j \in T\}, \displaystyle\sum_{(i,j) \in V} x_{i,j} = x_{1,4} + x_{2,4} + x_{3,4} + x_{1,5} + x_{2,5} + x_{3,5}$