**CZ2003 Tutorial 5 (2020/21, Semester 1)**

**Surfaces obtained by sweeping**

1. Using rotational sweeping counterclockwise, define by parametric functions $x(u,v)$, $y(u,v)$, $z(u,v)$, $u,v \in [0,1]$ the surface displayed in Figure Q1.

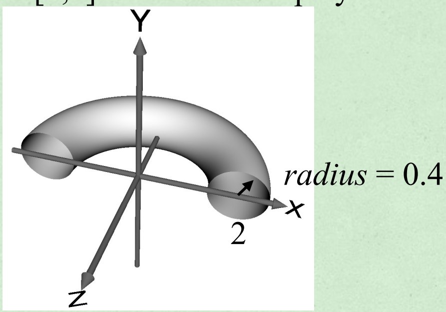



**Figure Q1**

2. Write parametric equations $x(u,v)$, $y(u,v)$, $z(u,v)$, $u, v, \in [0, 1]$ defining the surface created by sweeping (counterclockwise rotation by $3\pi/2$ and vertical displacement by 2) of the curve which is defined in polar coordinates by $r = 0.5\sin(4\alpha)$, $\alpha \in [0, 2\pi]$ (Figure Q2).

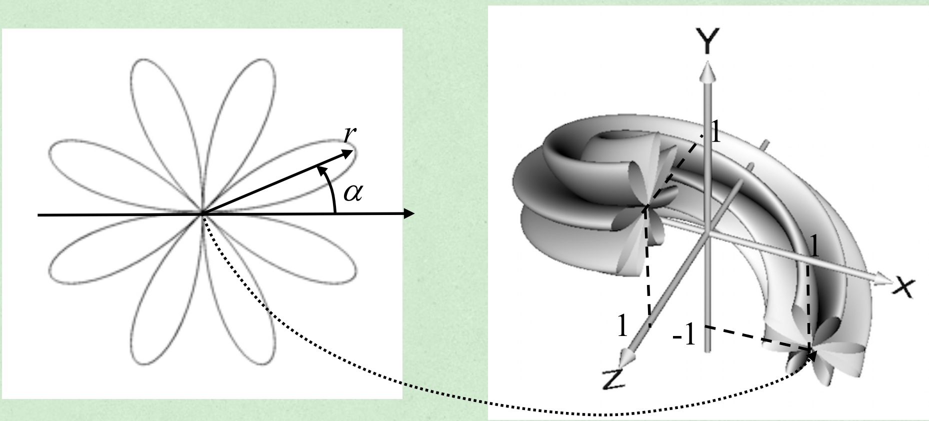



**Figure Q2**

1. Using rotational sweeping counterclockwise, define by parametric functions $x(u,v)$, $y(u,v)$, $z(u,v)$, $u,v \in [0,1]$ the surface displayed in Figure Q1.

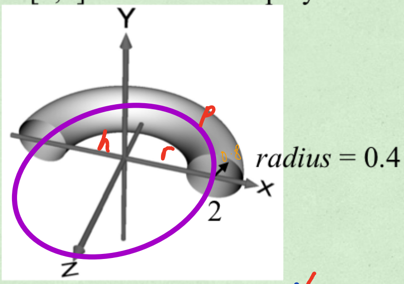



**Figure Q1**

## To form the 2D Circle

$x' = r\cos\varphi$

$= 0.4\cos\varphi + 2 \qquad 0 \le \varphi \le 2\pi$

$y' = r\sin\varphi$

$= 0.4\sin\varphi$

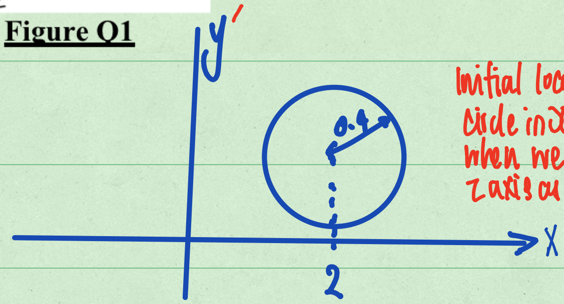
$z' = 0$

To convert to $u \in [0,1]$

$x(u) = 0.4\cos(2\pi u) + 2$ ✓

$y(u) = 0.4\sin(2\pi u)$ ✓

$z' = 0$




Initial location of circle in $x, y$ plane, when we start to sweep using $z$ axis as ref, we already have offset of $90°$

## Rotating about Y axis

$z = r\cos(\theta)$

$x = r\sin(\theta)$

$\theta \in [0, \pi]$

Each point P of the circle rotates about axis Y by the circle with radius r equal to its $x'$ coord & height h equals to its $y'$ coord

$z = r\cos(0.5\pi + v\pi)$ ✓ $\qquad v \in [0,1]$

$x = r\sin(0.5\pi + v\pi)$ ✓

$y = h$

**surface defined by.** sub $r = 0.4\cos(2\pi u) + 2$, $h = 0.4\sin(2\pi u)$

$x = x(u)\sin v = (0.4\cos(2\pi u) + 2)(\sin(0.5\pi + v\pi))$ ✓ //

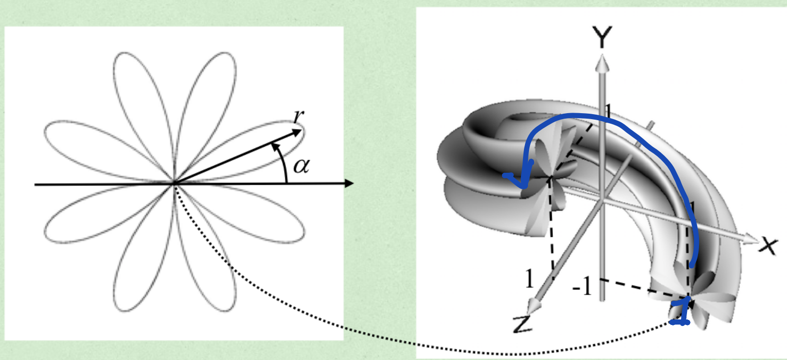
$y = y(u) = 0.4\sin(2\pi u)$ ✓ //

$z = x(u)\cos(v) = (0.4\cos(2\pi u) + 2)(\cos(0.5\pi + v\pi))$ ✓ //

$u = [0,1]$, $v = [0,1]$

2. Write parametric equations $x(u,v)$, $y(u,v)$, $z(u,v)$, $u, v, \in [0, 1]$ defining the surface created by sweeping (counterclockwise rotation by $3\pi/2$ and vertical displacement by 2) of the curve which is defined in polar coordinates by $r = 0.5\sin(4\alpha)$, $\alpha \in [0, 2\pi]$ (Figure Q2).

4cycle




$r = 0.5\sin(4\alpha)$ , $\alpha \in [0, 2\pi]$

sweeping

1. counterclockwise rotation by $\frac{3\pi}{2}$

2. vertical displacement 2.

1. convert polar function to parametric

$x = r\cos\alpha = (0.5\sin(4\alpha))(\cos\alpha)$

$x(u) = (0.5\sin(8\pi u))(\cos(2\pi u))$

$y = r\sin\alpha = (0.5\sin(4\alpha))(\sin\alpha)$

$y(u) = (0.5\sin(8\pi u))(\sin(2\pi u))$

2. Translation of $x$ by 1

$x = (0.5\sin(8\pi v))(\cos(2\pi u)) + 1$

$y = (0.5\sin(8\pi v))(\sin(2\pi u))$

3. Rotate about axis Y

$z = r\cos(v)$    $x = r\sin(v)$

$x = r\sin(1.5\pi v + \frac{\pi}{2})$

$z = r\cos(1.5\pi v + \frac{\pi}{2})$

$x = (0.5\sin(8\pi v)\ \cos(2\pi u) + 1)(\sin(1.5\pi v + \frac{\pi}{2}))$

$y = (0.5\sin(8\pi v)\ \sin(2\pi u))$

$z = (0.5\sin(8\pi v)\ \cos(2\pi u) + 1)(\cos(1.5\pi v + \frac{\pi}{2}))$

4. Translational sweeping // to Y axis.

Start: 1    $y = -1 + (1 - (-1))(v)$

End: -1    $= -1 + 2v$

How come cannot be $\sin(\pi v - \frac{3\pi}{2})$

$x = (0.5\sin(8\pi v)\ \cos(2\pi u) + 1)(\sin(1.5\pi v + \frac{\pi}{2}))$

$y = (0.5\sin(8\pi v)\ \sin(2\pi u)) + (-1 + 2v)$

$z = (0.5\sin(8\pi v)\ \cos(2\pi u) + 1)(\cos(1.5\pi v + \frac{\pi}{2}))$

$V, U \in [0,1]$