FIN 2704/2704X

Week 4 Slides

Return

Learning objectives

- Understanding investment return
- Understand the difference between nominal returns and real returns
- Be able to estimate expected return

A First Look at Risk and Return

We begin our look at risk and return by looking at historical return and risk (volatility) experienced by various investments.

Suppose your great-grandparents had invested $100 on your behalf in 1925 and instructed their broker to reinvest any dividends and/or interest earned in the account until the beginning of 2007. Consider how an investment would have grown if it were wholly invested in any one of the following investments:

A First Look at Risk and Return

1. **Standard & Poor's 500 (S&P 500):** A portfolio, constructed by Standard and Poor's, comprising 90 U.S. stocks up to 1957 and <u>500 U.S. stocks</u> after that. The firms represented are leaders in their respective industries and are among the largest firms, in terms of market capitalization (share price times the number of shares in the hands of the shareholders), traded on U.S. markets.
2. **Small Stocks:** A portfolio of stocks of U.S. firms whose <u>market capitalizations are in the bottom 10%</u> of all stocks traded on the NYSE. (As stocks' market values change, this portfolio is updated so it always consists of the smallest 10% of stocks.)

A First Look at Risk and Return

3. **World Portfolio:** A portfolio of international stocks from all of the world's major stock markets in North America, Europe and Asia.

4. **Corporate Bonds:** A portfolio of long-term, AAA-rated U.S. corporate bonds with maturities of approximately 20 years.

5. **Treasury Bills:** An investment in three-month U.S. Treasury Bills (reinvested as the bills mature).

A First Look at Risk and Return

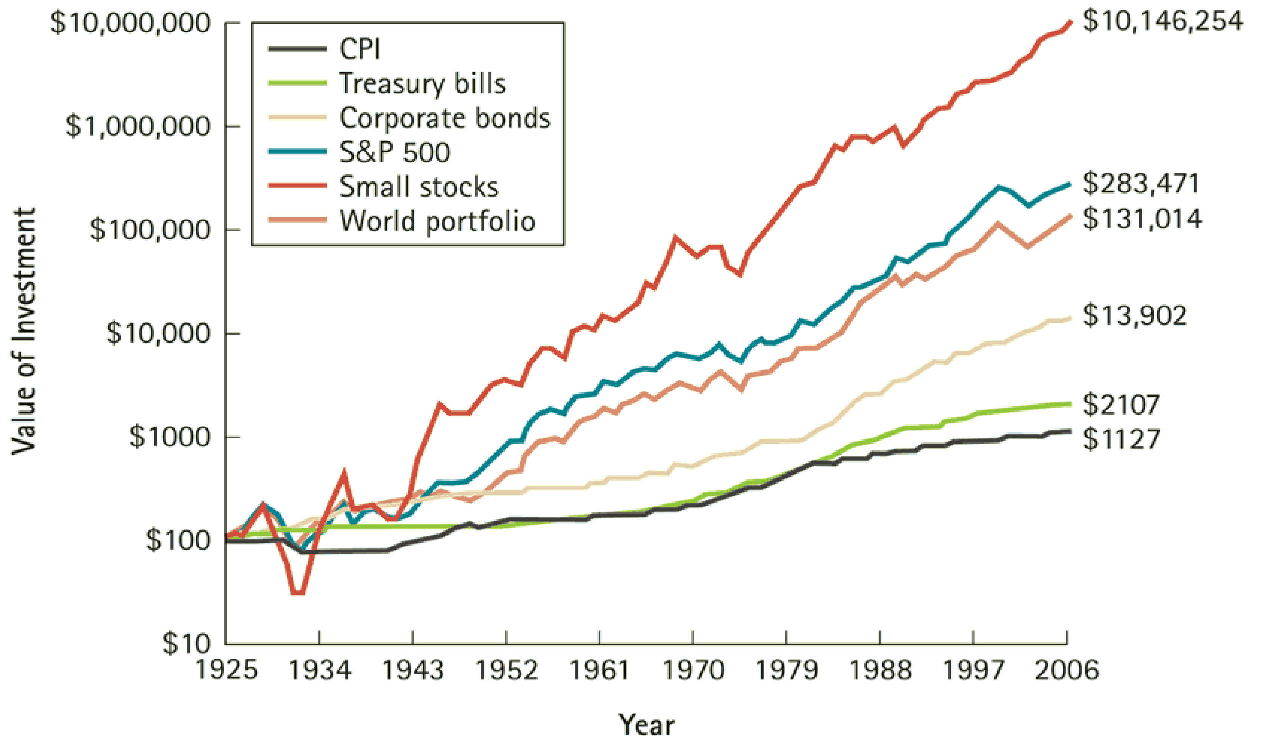
Value of $100 Invested at the End of 1925 in U.S. Large Stocks (S&P 500), Small Stocks, World Stocks, Corporate Bonds, and Treasury Bills

FIGURE 10.1

Value of $100 Invested at the End of 1925 in U.S. Large Stocks (S&P 500), Small Stocks, World Stocks, Corporate Bonds, and Treasury Bills

Note that the investments that performed the best in the long run also had the greatest fluctuations from year to year. The change in the consumer price index (CPI) is shown as a reference point.

Source: Global Financial Data.



What Are Investment Returns?

Investment returns measure the financial results of an investment

Returns may be **historical** or **prospective** (anticipated)

Any period's returns can be expressed in:

- Dollar terms:

  = Amt. received (End of Period) – Amt. invested (Beginning of Period)

- Percentage terms:

$$= \frac{\text{Amt. received (End of Period)} - \text{Amt. invested (Beginning of Period)}}{\text{Amount invested (Beginning of Period)}}$$

Example: PepsiCo Returns

When an investor buys a stock or a bond, their return comes in 2 forms:

1. Any **dividend or interest payment** (income) received, and
2. A **capital gain or a capital loss** (due to change in price)

Suppose you bought PepsiCo stocks at $43 a share. By the end of the year, the value of each share has appreciated to $49, giving a capital gain of $(49 - $43) = $6. In addition, PepsiCo paid a dividend of $0.56 a share.

Total dollar return = Dividend income + Capital gain/loss
= **$6.56**

Percentage Return for the Period

The percentage return is the sum of

$$1.\text{Dividend Yield} = \frac{\text{Dividend}}{\text{Initial Share Price}}$$

and

$$2.\text{Capital Gain Yield} = \frac{\text{Capital Gain}}{\text{Initial Share Price}}$$

Total percentage return = dividend yield + capital gains yield

Example: PepsiCo Percentage Return

$$\text{Dividend Yield} = \frac{0.56}{43}$$

$$= .013 \text{ or } 1.3\%$$

$$\text{Capital Gain Yield} = \frac{6}{43}$$

$$= .140 \text{ or } 14.0\%$$

$$\text{Total Percentage Return} = \frac{\text{Div} + \text{Cap Gain}}{\text{Initial Share Price}}$$

$$= \frac{0.56 + 6}{43} = .153 \text{ or } 15.3\%$$

What’s The Impact Of Inflation?

What we have calculated earlier is a **nominal return**. This refers to actual $ received versus actual $ given up.

But given inflation, are my ending dollars received able to buy the same or more goods when received as my initial $ investment could buy at the time I made the investment?

So, what was my return in terms of the increase in purchasing power relative to initial investment purchasing power?

The **real rate of return** tells you how much more you will be able to buy with your money at the end of the year.

What's The Impact Of Inflation?

To convert from a nominal to a real rate of return:

$$\boxed{1 + \text{real return} = \frac{1 + \text{nominal return}}{1 + \text{inflation rate}}}$$

A common approximation for the real rate of return is:

$$\boxed{\text{real return} \approx \text{nominal return} - \text{expected inflation}}$$

Example: PepsiCo

Let's go back to the PepsiCo. % return example. If the inflation for the year was 2.8 percent, what was the real rate of return on a share of PepsiCo. stock?

$$1 + \text{real return} = \frac{1 + \text{nominal return}}{1 + \text{inflation rate}}$$

$$1 + \text{real return} = \frac{1 + 0.153}{1 + 0.028} = 1.1216$$

real return = 0.1216 = 12.16%

Which Stocks To Invest In?

- First, you need to estimate stocks' expected returns.
- **Expected returns** are returns that take into account uncertainties that are present in different scenarios.
- With perfect quantifiable information, an investor must estimate the different return scenarios possible and the probability of each return scenario.

How Do You Calculate Expected Return?

IF we know the possible return scenarios and their probabilities

Expected Return:

$$\boxed{\hat{r} = \sum_{i=1}^{n} r_i P_i}$$

Summation

- r_i = Possible return
- P_i = Probability of possible return

Note: there are *n* possible returns

Example: Expected Return Using Probabilities

An asset has a

- 30% probability of a 10% return
- 10% probability of a -10% return
- 60% probability of a 25% return

What is the expected return?

Probability of Return	Possible Return
0.3	10%
0.1	-10%
0.6	25%

Total = 1.0

Expected Return = (0.30)(10%) + (0.10)(-10%) + (0.60)(25%)

= **17%**

Calculating *Arithmetic* Average Return

If using historical data on an asset (since we usually do not have information on future possible outcomes and their probabilities) to estimate average return, we can find the arithmetic average (the amount earned in a typical year) as follows:

Arithmetic average return or **Arithmetic mean**

$$\boxed{\bar{r} = \frac{\sum_{t=1}^{T} r_t}{T}}$$

Is This A Good Investment?

So, let's say we've estimated an expected return:

- How does one determine whether the return is adequate?
  - By comparing to a <u>benchmark</u>
- What should be the benchmark?
  - It is the <u>required rate of return on the investment</u>
- What determines the required return?
  - Rate of return depends on the <u>risk</u> of the investment

Summary

- What is a return on investment?
  - Real return vs. nominal return
- How to do you calculate the expected return of an investment?
- Given an expected return for an investment, is it a good investment?

Risk

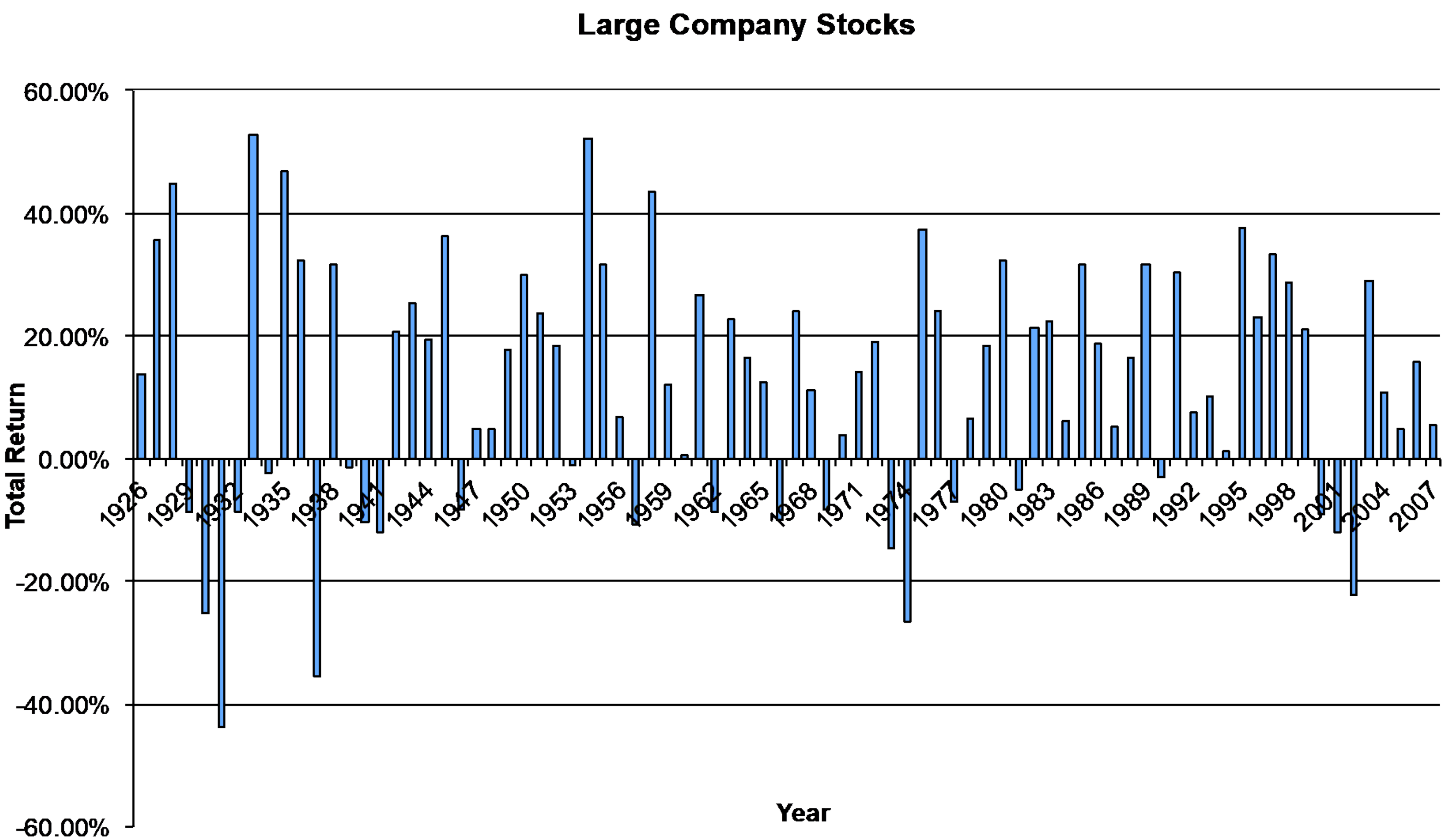
Learning objectives

- Understand what an investment risk is and what the risk and return trade-off is
- Understand how risk is measured and calculated
- Understand what risk aversion is and how it relates to risk premium

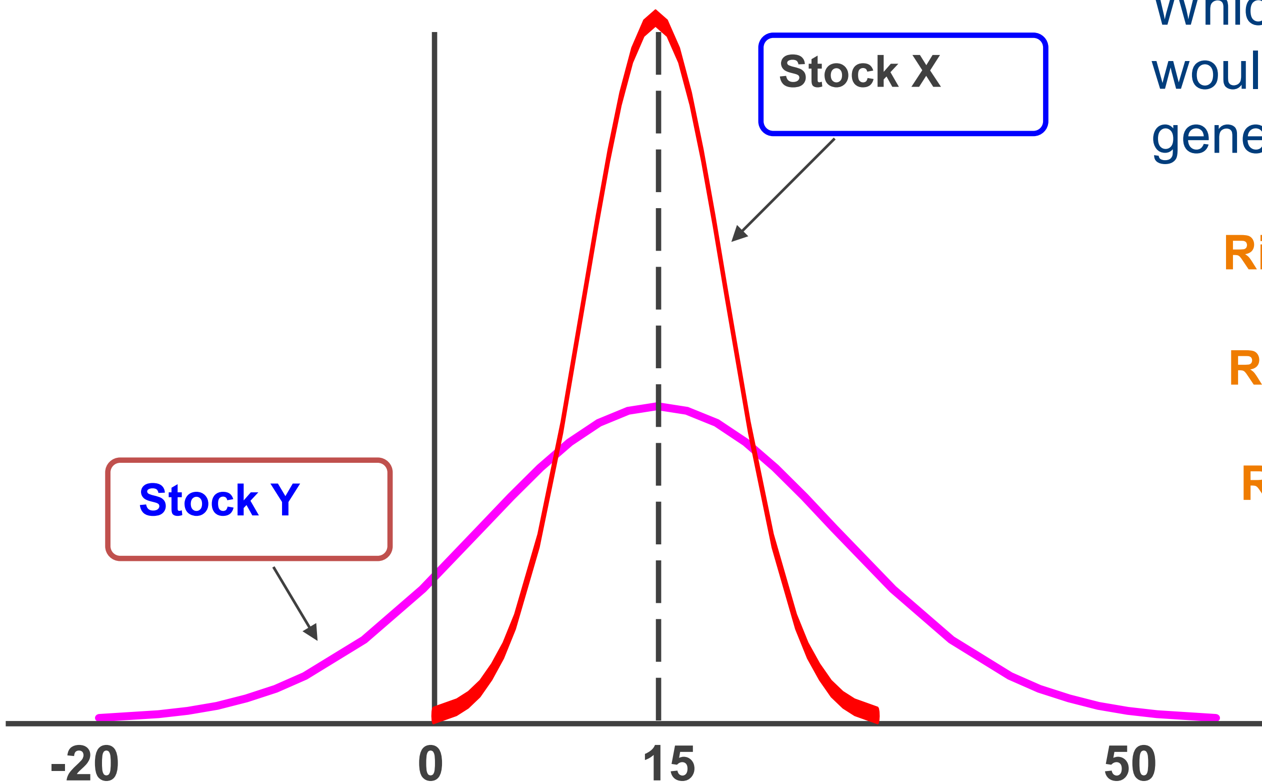
What is Risk?

- Risk is the **uncertainty** associated with future possible outcomes
- We are concerned with **investment risk**
- Investment risk refers to the potential for your **investment return to fluctuate** - go up or down - in value from year to year
- All investments carry some risk, but some carry more than others.
  - Generally, we will see that if you want higher returns, you need to be comfortable with accepting a higher level of risk

Rates of Return on Large Company Stocks



Probability Distribution



Which stock would investors generally prefer?

Risk Neutral
vs
Risk Averse
vs
Risk Lover

The greater the **chance of a return far below the expected return, the greater the risk**. In the slide, Stock Y is riskier than Stock X.

How Do We Measure Uncertainty?

How do we measure this variability or volatility in investment returns?

We calculate the **variance** or **standard deviation** of the possible returns.

IF we know the future possible returns and the probability of each possibility:

$$\sigma = \sqrt{Variance} = \sqrt{\sigma^2}$$

$$= \sqrt{\sum_{i=1}^{n} \left(r_i - \hat{r}\right)^2 P_i}$$

Expected return

Variance and Standard Deviation

IF we only have historical past data, then variance and standard deviation may be **estimated using the realized rate of return at any time t**

– t ranges from 1 to n

– n is the number of historical observations

$$\text{Estimated } \sigma = \sqrt{\frac{\sum_{t=1}^{n}\left(r_t - \bar{r}_{Avg}\right)^2}{n-1}}$$

The average annual return (arithmetic mean) over the last n years

Example – Estimating Standard Deviation from Historical Data

Year	Actual Return	Average Return	Deviation from the Mean	Squared Deviation
1	.15	.105	.045	.002025
2	.09	.105	-.015	.000225
3	.06	.105	-.045	.002025
4	.12	.105	.015	.000225
Total	.42			.0045

Variance = .0045 / (4-1) = .0015 Standard Deviation = .03873

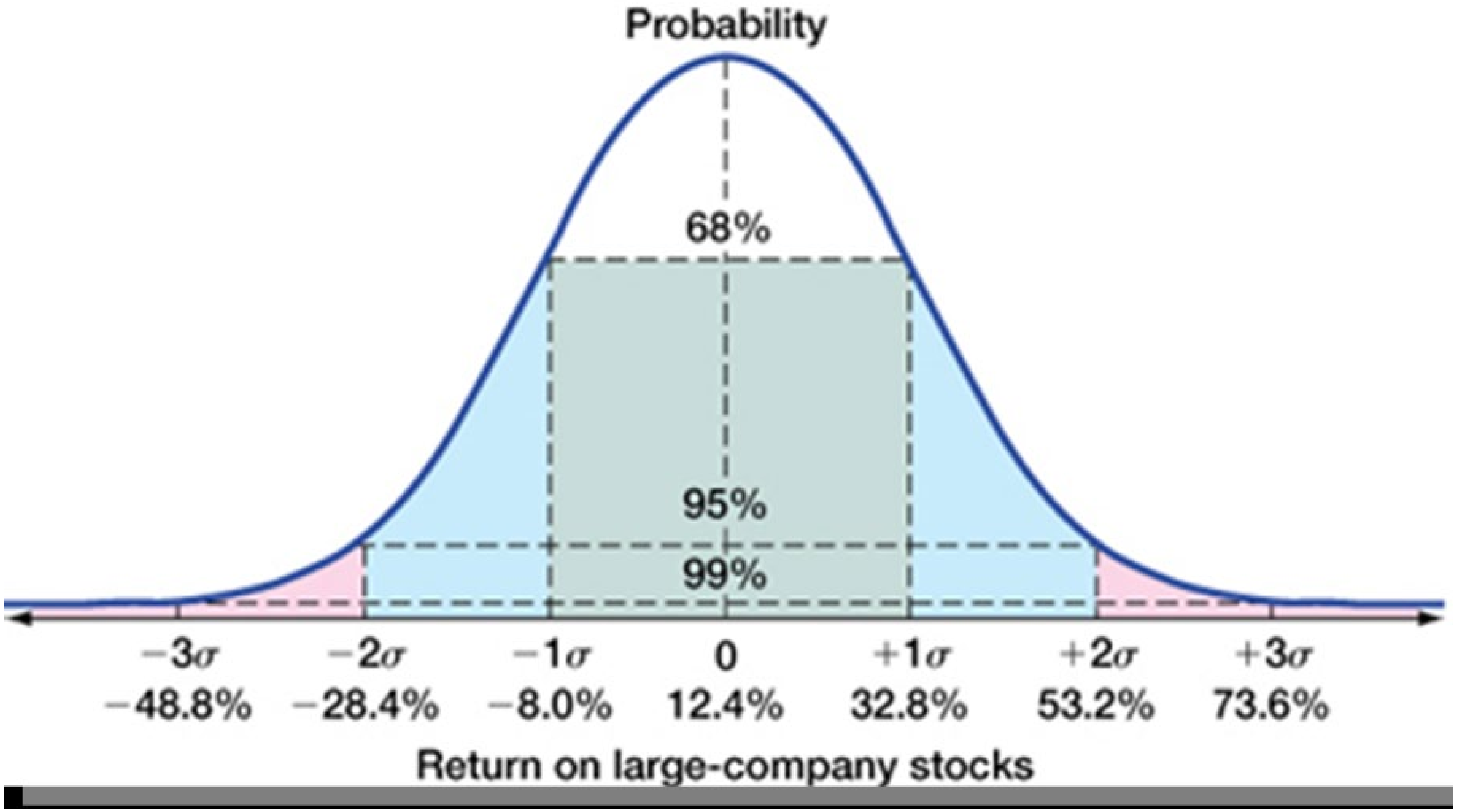
Stand-Alone Risk

Standard deviation measures **total risk**, also called the **stand-alone risk** of an investment.

- **Stand-alone risk** is the risk an investor would face if he/she held only this one asset
- The larger the standard deviation, the higher the probability that actual returns will be far away from the expected return
  - Thus standard deviation measures dispersion around an expected value

The Normal Distribution

The graph below is based on historical return and standard deviation for a *portfolio* of large-firm common stocks



Risk & Return: Comprehensive Example

Economy	Prob.	T-Bill	Alta	Repo	Am F.	MP
Recession	0.10	8.0%	-22.0%	28.0%	10.0%	-13.0%
Below avg.	0.20	8.0	-2.0	14.7	-10.0	1.0
Average	0.40	8.0	20.0	0.0	7.0	15.0
Above avg.	0.20	8.0	35.0	-10.0	45.0	29.0
Boom	0.10	8.0	50.0	-20.0	30.0	43.0
	1.00					

Calculate the expected returns and standard deviations of the above investment alternatives.

Comprehensive example: Calculation for Alta

Expected Return

$$\hat{r} = \sum_{i=1}^{n} r_i P_i$$

$$\hat{r}_{Alta} = 0.1(-22\%) + 0.2(-2\%) + 0.4(20\%) + 0.2(35\%) + 0.1(50\%)$$

$$= 17.4\%$$

Economy	Prob.	Alta
Recession	0.10	-22.0%
Below avg.	0.20	-2.0
Average	0.40	20.0
Above avg.	0.20	35.0
Boom	0.10	50.0
	1.00	

Comprehensive example: Calculation for Alta

Standard Deviation

$$\sigma_{Alta} = \sqrt{\sum_{i=1}^{n} \left(r_i - \hat{r}\right)^2 P_i}$$

$$\sigma^2_{Alta} = 0.1\,(-22\% - 17.4\%)^2 + 0.2\,(-2\% - 17.4\,\%)^2 + 0.4\,(20\% - 17.4\%)^2 + 0.2\,(35\% - 17.4\%)^2 + 0.1\,(50\% - 17.4\%)^2$$

= 401.44

$$\sigma_{\text{Alta}} = 20\%$$

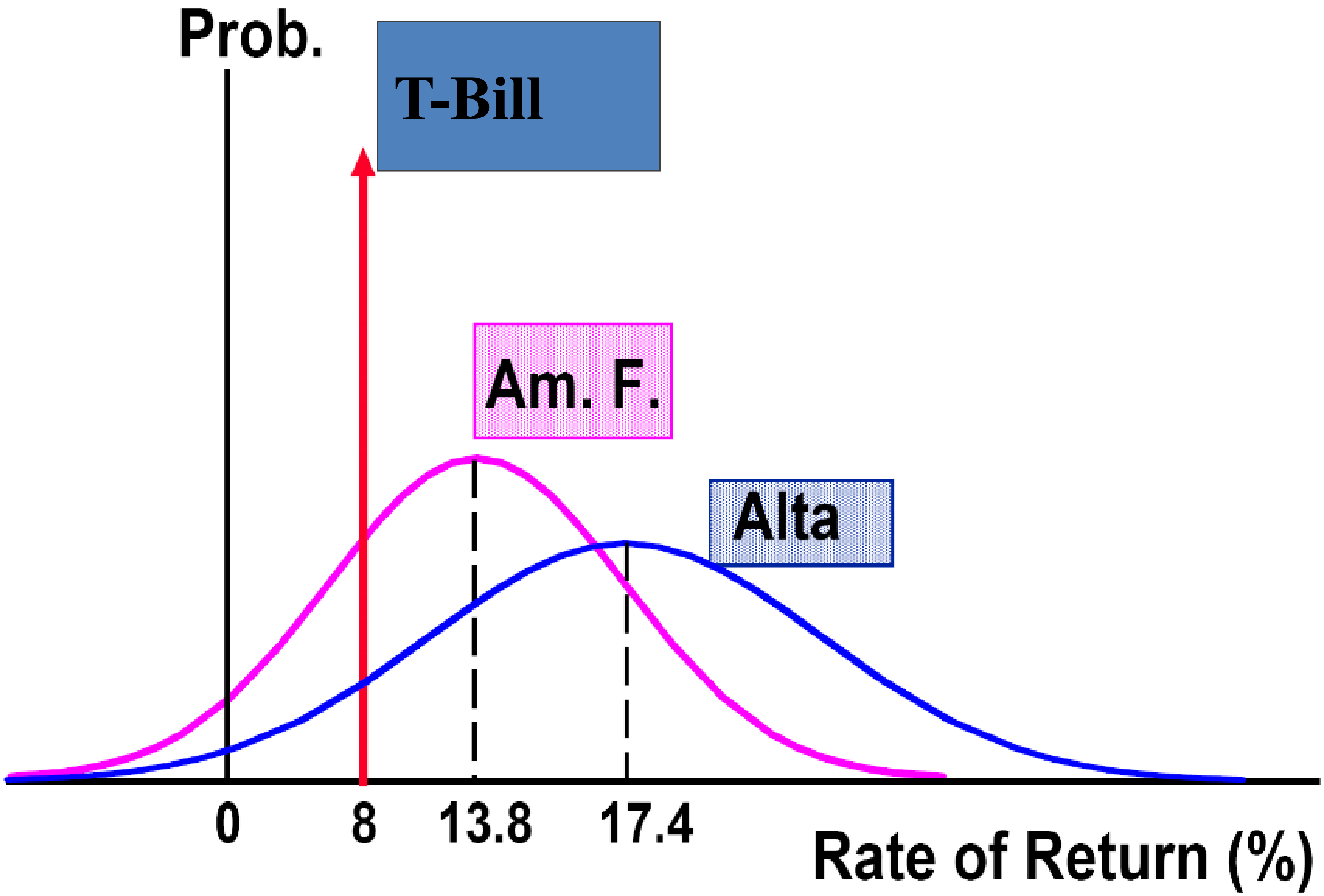
Economy	Prob.	Alta
Recession	0.10	-22.0%
Below avg.	0.20	-2.0
Average	0.40	20.0
Above avg.	0.20	35.0
Boom	0.10	50.0
	1.00	

Comprehensive Example: Expected Returns & Standard Deviations

Investment	$\hat{r}$	σ
Alta	17.4%	20%
Market	15.0	15.3
Am. F.	13.8	18.8
T-Bill	8.0	0
Repo Men	1.7	13.4

Looking at these stocks, we do not see a consistent trade off between expected return and the total risk measure σ of the individual stocks.

Comprehensive Example Graphically



Coefficient of Variation

A widely used measure of relative variability is the **Coefficient of Variation (CV)**.

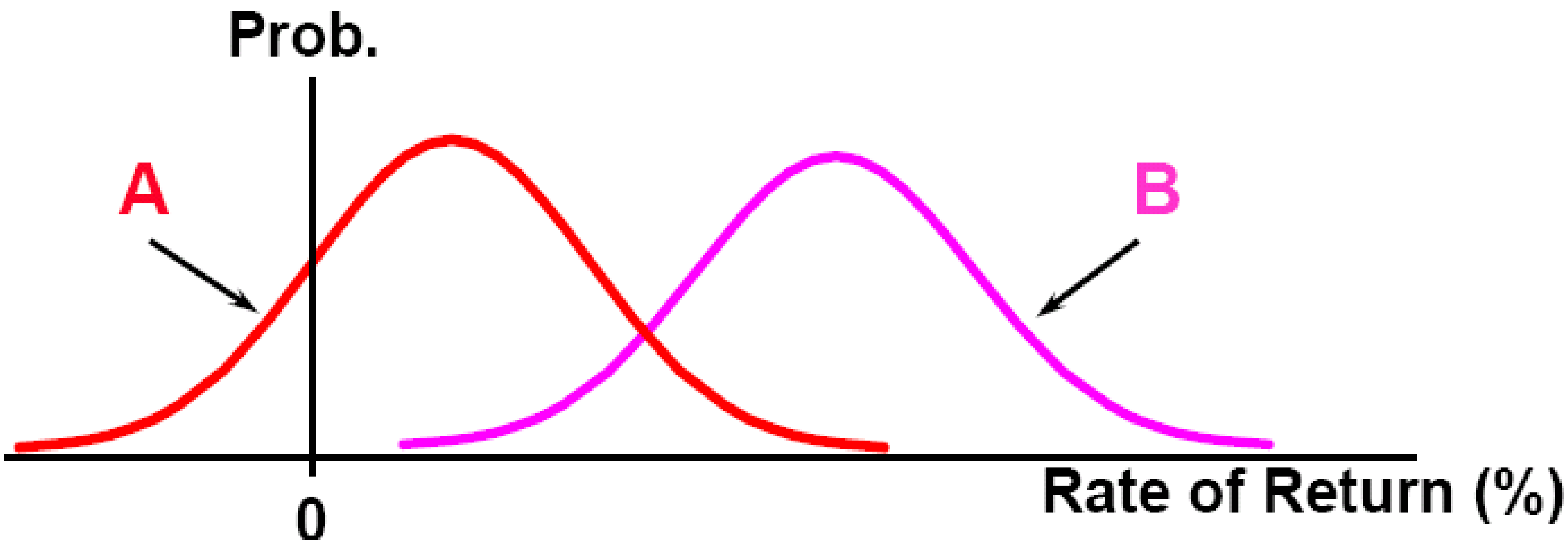
CV is a standardized measure of dispersion about the expected value, that shows the risk per unit of return.

$$CV = \frac{\textit{Standard Deviation}}{\textit{Expected Rate of Return}}$$

or

$$\boxed{CV = \frac{\textit{Standard Deviation}}{\textit{Mean}} = \frac{\sigma}{\hat{r}}}$$

Illustrating CV As A Measure Of Relative Risk



$$\sigma_A = \sigma_B$$

But A is riskier because of a larger probability of losses. In other words, the same amount of risk (as measured by σ) for less returns.

An Example of CV

Stock X and Stock Y have widely differing rates of return and standard deviations of return.

If you compare standard deviations, Stock X seems to be less risky than Stock Y.

	Stock X	Stock Y
Expected Return	7%	12%
Standard Deviation	5%	7%

Comparing CVs, the results are different:

- CV of Stock X = 5% / 7% = 0.714
- CV of Stock Y = 7% / 12% = 0.583

The CV figure shows that Stock Y has less relative variability or lower risk per unit of expected return.

Comprehensive Example: Coefficient of Variation

Security	Expected Return	σ	CV
Alta Inds	17.4%	20.0%	1.1
Market	15.0	15.3	1.0
Am F	13.8	18.8	1.4
T-Bill	8.0	0	0
Repo Men	1.7	13.4	7.9

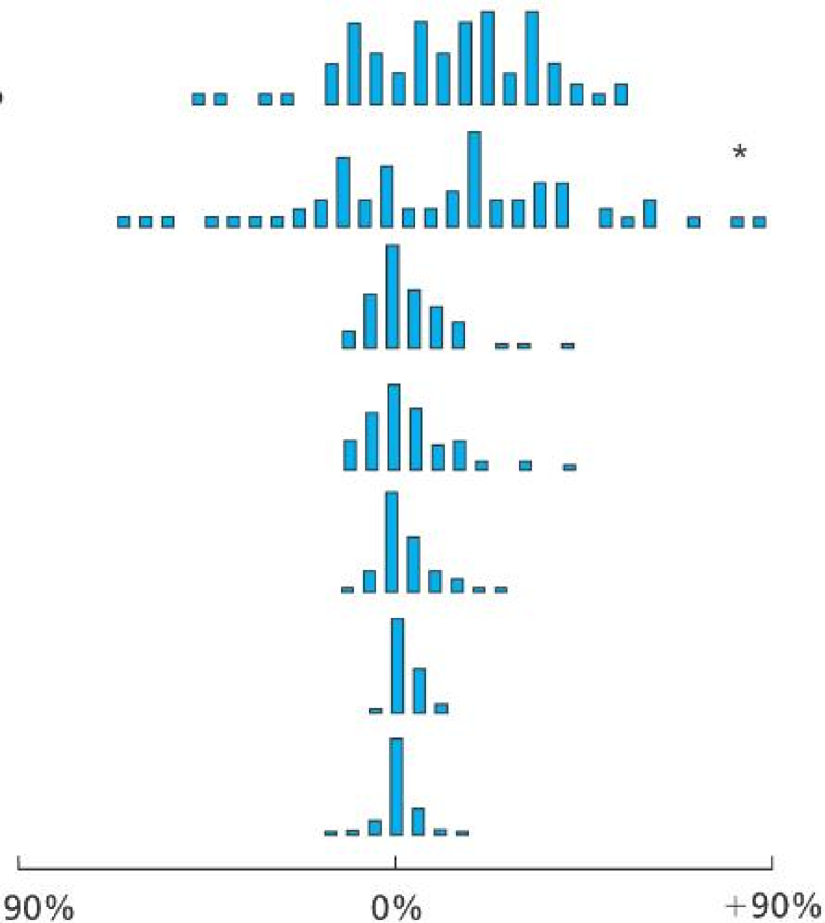
Risk, Return and Financial Markets

Lesson from capital market history:

- When examining *portfolios* of assets as per our example of 5 possible historical portfolios, we find:
  - There is a reward for bearing risk
  - The greater the potential reward, the greater the risk
  - This is called the **risk-return trade-off**
- Investors face a trade-off between risk and expected return
- Historical data confirm our intuition that assets with lower degrees of risk provide lower returns on average than do those of higher risk.
- Thus, low levels of uncertainty (low risk) are associated with low potential returns, whereas high levels of uncertainty (high risk) are associated with high potential returns.

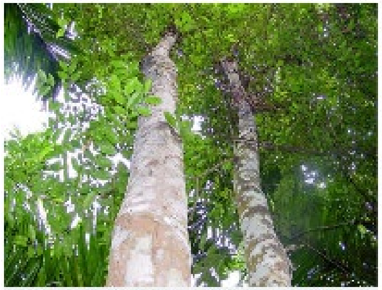
Historical Returns, Standard Deviations, and Frequency Distributions: 1926 - 2000

Series	Average Annual Return	Standard Deviation	Distribution
Large-company stocks	13.0%	20.2%	
Small-company stocks	17.3	33.4	
Long-term corporate bonds	6.0	8.7	
Long-term government	5.7	9.4	
Intermediate-term government	5.5	5.8	
U.S. Treasury bills	3.9	3.2	
Inflation	3.2	4.4	



Example: High Return Low Risk a Red Flag

Investors cry foul over tree investments gone wrong



ST, Feb 2016

- 70 investors in Singapore were crying foul over tree investments gone wrong. Some of them had invested as much as $60,000.
- Investors would put money into buying saplings or semi-matured agarwood trees with promises of up to 700% after 6-7 years.

Investors caught in '$60m ponzi scam'

ST, May 2015

- $60 million ponzi scam had been running for 15 years.
- This scam involved investing money with a “renowned” agent to buy distressed properties in prime districts and selling them to a pool of buyers in China.
- Duped more than 100 investors with promises of 10% to 48% over a period of four to eight months

More than 100 investors make police reports on gold buyback scheme at investment firm

ST, Feb 2015

- 250 people with approximately $35 million in investments were conned by Suisse International’s gold buyback scheme.
- Under this scheme, investors would invest in physical gold with the promise of a return of about 2% every month. Investors thought they could earn over 20% per annum without taking much risk.

The Importance of Financial Markets

Financial markets allow companies, governments and individuals to increase their utility by matching borrowers with savers

- Savers have the ability to invest in financial assets so that they can defer consumption and earn a return to compensate them for doing so
- Borrowers have better access to the capital that is available so that they can invest in productive assets

Financial markets also provide us with information about the **returns that are required for various levels of risk**

Risk Aversion

Risk aversion: assumes investors dislike risk and require higher rates of return to encourage them to hold riskier securities

- The "extra" return earned for taking on risk is referred to as **risk premium**
- Asset's risk premium is equal to its required return less the rate of return available on treasury securities (i.e., the risk-free asset)

Risk Premiums

The risk premium is the return over and above the risk-free rate

But what is the risk-free rate benchmark to use?

1. **Treasury Bills:** In most developed markets, where the government can be viewed as default free, Treasury Bills (maturity of 1 year or less) are considered to be risk-free.

2. **Treasury Bonds:** However, for analysis of longer-term projects or company valuation, where the long-term risk premium is investor’s main concern, many argue that the risk-free rate should be the long-term government bond rate.

Historical Risk Premiums

- The historic risk premium is defined as the excess return over the risk-free rate*.
- For the sample 1925 – 2007, the historical risk premiums are:
  - Large stocks: 13.0 – 3.9 = 9.1%
  - Small stocks: 17.3 – 3.9 = 13.4%
  - Long-term corporate bonds: 6.0 – 3.9 = 2.1%

* For this class, we will define US Treasury Bills as a riskless asset and hence we will use its returns (3.9%) as the risk-free rate.

Summary

- Investment risk measurement: variance & standard deviation
- Use coefficient of variance to compare the risk and return of different investment options
- Low risk are associated with low potential returns, whereas high risk are associated with high potential returns
- All investors dislike risk (to a varying degree), so, they require “extra” return for risky assets compared to risk-free assets.

Portfolio

Learning objectives

Understand the effects of diversification on portfolio risk and return

Portfolio

A set of assets hold by an investor

Portfolio: Alta & Repo

Assume you invest an equal amount of $50,000 each in Alta Inds. and Repo Men.

Calculate $\hat{r}_p$ and σ_p for the portfolio.

Portfolio risk

Expected return on a portfolio

Expected portfolio return

- It is a weighted average of the expected returns on the individual stocks $\hat{r}_p$

$$\boxed{\hat{r}_p = \sum_{i=1}^{n} w_i \hat{r}_i}$$

w_i = the fraction of the portfolio's dollar value invested in Stock *I*

Note, the w_i's must add up to 1

- Example:

$$\hat{r}_p = 0.5(17.4\%) + 0.5(1.7\%) = 9.6\%$$

Note that $\hat{r}_p$ is between $\hat{r}_{Alta}$ and $\hat{r}_{Repo}$.

Expected Portfolio Return: Alternative Method

		Estimated Return		
Economy	**Prob.**	**Alta**	**Repo**	**Port.**
Recession	0.10	-22.0%	28.0%	3.0%
Below avg.	0.20	-2.0	14.7	6.4
Average	0.40	20.0	0.0	10.0
Above avg.	0.20	35.0	-10.0	12.5
Boom	0.10	50.0	-20.0	15.0

$$\hat{r}_p = (3.0\%)0.10 + (6.4\%)0.20 + (10.0\%)0.40 + (12.5\%)0.20 + (15.0\%)0.10 = 9.6\%.$$

Here we look at the porfolio's return in each scenario and the probability of that scenario occurring.

What about the portfolio's risk?

Applying the **standard deviation** & **CV** formulas:

$$\sigma_p = \begin{bmatrix} 0.10\,(3.0 - 9.6)^2 \\ +\,0.20\,(6.4 - 9.6)^2 \\ +\,0.40\,(10.0 - 9.6)^2 \\ +\,0.20\,(12.5 - 9.6)^2 \\ +\,0.10\,(15.0 - 9.6)^2 \end{bmatrix}^{1/2} = 3.3\%$$

$$CV_p = \frac{3.3\%}{9.6\%} = 0.34$$

The standard deviation of the portfolio can be derived the same as for an individual asset. We look at the portfolio's scenario returns and the probability of those scenarios taking place. The next slide shows yet another way of calculating the portfolio standard deviation.

Portfolio Standard Deviation: Alternative Way

By definition, the standard deviation of a 2-stock portfolio is:

$$\sigma_p = \sqrt{w_1^2\sigma_1^2 + (1-w_1)^2\sigma_2^2 + 2w_1(1-w_1)Corr(R_1,R_2)\sigma_1\sigma_2}$$
$$= \sqrt{w_1^2\sigma_1^2 + (1-w_1)^2\sigma_2^2 + 2w_1(1-w_1)\rho_{12}\sigma_1\sigma_2}$$

$$\sigma_p = \sqrt{\left(1/2\right)^2 20^2 + \left(1/2\right)^2 13.4^2 + 2\left(1/2\right)\left(1/2\right)\rho(20)(13.4)}$$

We need to know the **covariance** and/or the **correlation coefficient** between the two assets to calculate standard deviation in this way. So how do we get the correlation coefficient?

Closer look at portfolio risk

Before we fully define covariance and correlation coefficient, let's look at the numbers.

Note that the standard deviation of the portfolio, σ_p, at 3.3%, is much lower than

- The standard deviation of either stock Alta, 20%, or Repo, 13.4%.
- The average of Alta and Repo's standard deviations (16.7%)

The portfolio provides a return that is the average of the returns of both stocks, at 9.6%, but the portfolio has a much lower risk as measured by the standard deviation of the portfolio, at 3.3%.

How does this happen?

The two stocks have a negative covariance which arises from the negative correlation between the stocks.

Portfolios – Historical Risk => Covariance

When looking at the risk of a portfolio of assets, it is important to recognize and consider the interaction between the individual stocks with one another. This leads us to the concept of **covariance**; that is, how the performance of two assets "move" or "do not move" together.

The covariance of the annual rates of return of 2 different investments is used to measure **how the two assets' rates of return vary together over the same mean time period**. For example, for n periods of measured returns of stock X and stock Y, the *covariance of X and Y* is found as follows:

$$\text{Covariance} = \frac{(r_{X1} - \bar{r}_X)(r_{Y1} - \bar{r}_Y) + (r_{X2} - \bar{r}_X)(r_{Y2} - \bar{r}_Y) + \cdots (r_{Xn} - \bar{r}_X)(r_{Yn} - \bar{r}_Y)}{n - 1}$$

and

$$Cov_{i,j} = \sum_{s=1}^{S} \Pr(s) * \left(R_{i,s} - \mathrm{E}(R_i)\right) * \left(R_{j,s} - \mathrm{E}(R_j)\right)$$

and

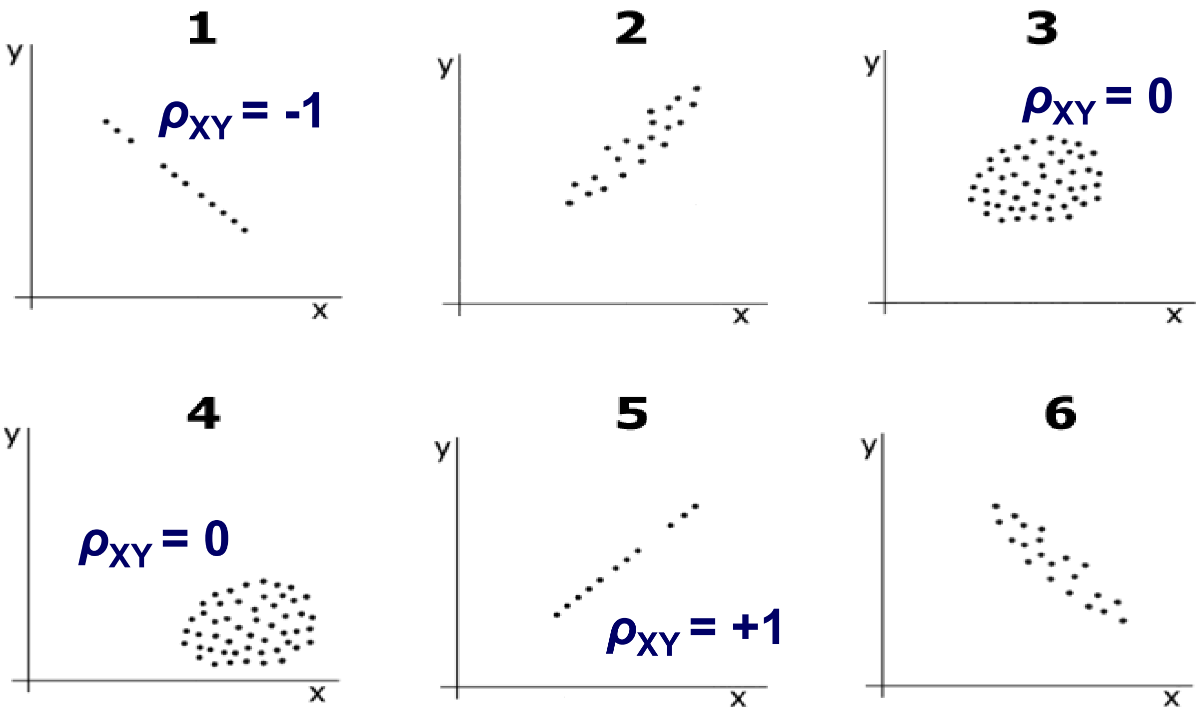
$$\text{Covariance} = \sigma_{XY} = \rho_{XY}\sigma_X\sigma_Y$$

ρ_{XY} = the correlation coefficient between stock X and stock Y

Portfolios – Correlation Coefficient

The **correlation coefficient** between two stocks (X and Y), denoted by ρ_{XY} measures the <u>extent to which two securities X and Y move together</u>

Rho – pronounced "roe"



In general

$1 \geq \rho_{XY} \geq -1$

Perfectly positively correlated

Perfectly negatively correlated

The variance of a portfolio depends on the correlation coefficients between the assets included in the portfolio.

Note: the correlation coefficient standardizes the units of covariance measure.

Example: Covariance & Correlation Coefficient

A security analyst has prepared the following probability distribution of the possible returns on 2 stocks: Compu-Graphics Inc (CGI) and Data Switch Corp (DSC).

Probability	Return on CGI	Return on DSC
0.30	10%	40%
0.50	14%	16%
0.20	20%	20%

Example: Covariance & Correlation Coefficient

For CGI, expected return is:

$$E(r_{CGI}) = \sum_{s=1}^{3} p_s r_{CGI,s}$$

$$= 0.30(10\%) + 0.50(14\%) + 0.20(20\%) = 14\%$$

The expected return for DSC is 24.00%

Example: Covariance & Correlation Coefficient

Recall that probabilities of potential returns are known, so we use the corresponding formula

Probability	Return on CGI	Return on DSC
0.30	10%	40%
0.50	14%	16%
0.20	20%	20%

$$Cov(CGI, DSC) = \sigma_{CGI,DSC} = \sum_{s=1}^{3} p_s (r_{CGI,s} - \bar{r}_{CGI})(r_{DSC,s} - \bar{r}_{DSC})$$

$$= 0.30 \times (10 - 14)(40 - 24) + 0.50 \times (14 - 14)(16 - 24)$$
$$+ 0.20 \times (20 - 14)(20 - 24) = -24.00$$

Example: Covariance & Correlation Coefficient

The correlation coefficient between CGI and DSC is :

$$\rho_{CGI,DSC} = \frac{Cov(r_{CGI}, r_{DSC})}{\sigma_{CGI}\sigma_{DSC}}$$

$$= \frac{-24.00}{3.46 \times 10.58}$$

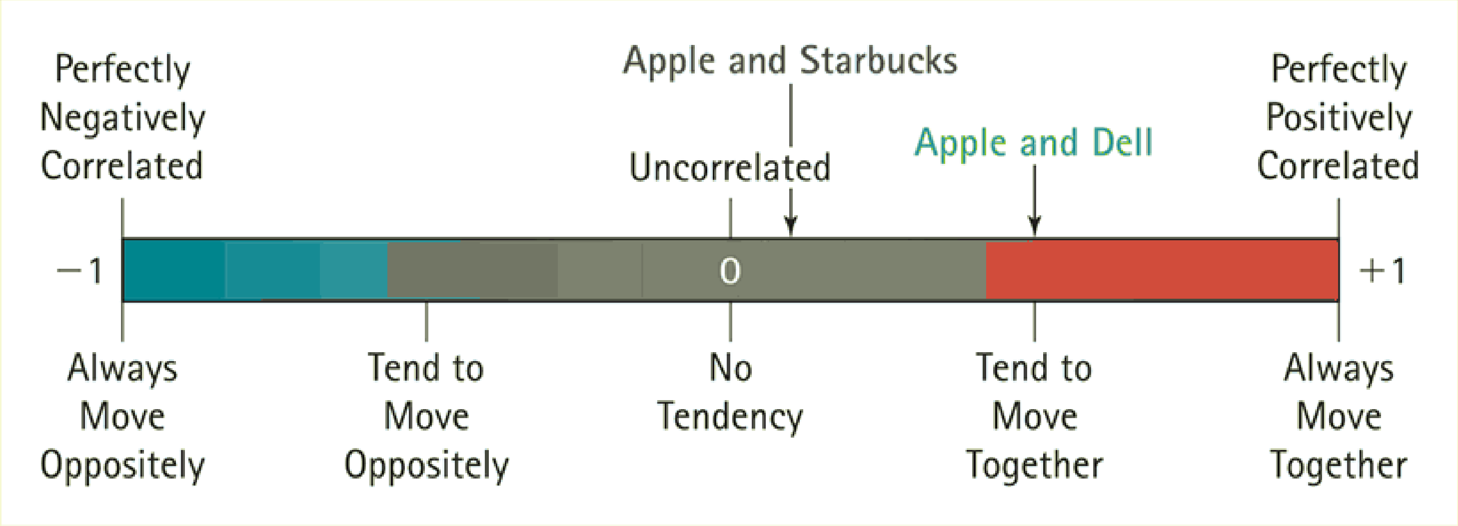
$$= -0.655$$

Correlation Coefficient – More Information

Correlation

The correlation measures how returns move in relation to each other. The correlation is between $+1$ (returns always move together) to -1 (returns always move oppositely). Independent risks have no tendency to move together and have zero correlation. The correlations of Apple with Starbucks and Dell are indicated on the continuum. Note that Apple is more correlated with another computer seller and less correlated with a coffee company. See Table 11.3 for more examples of correlations.

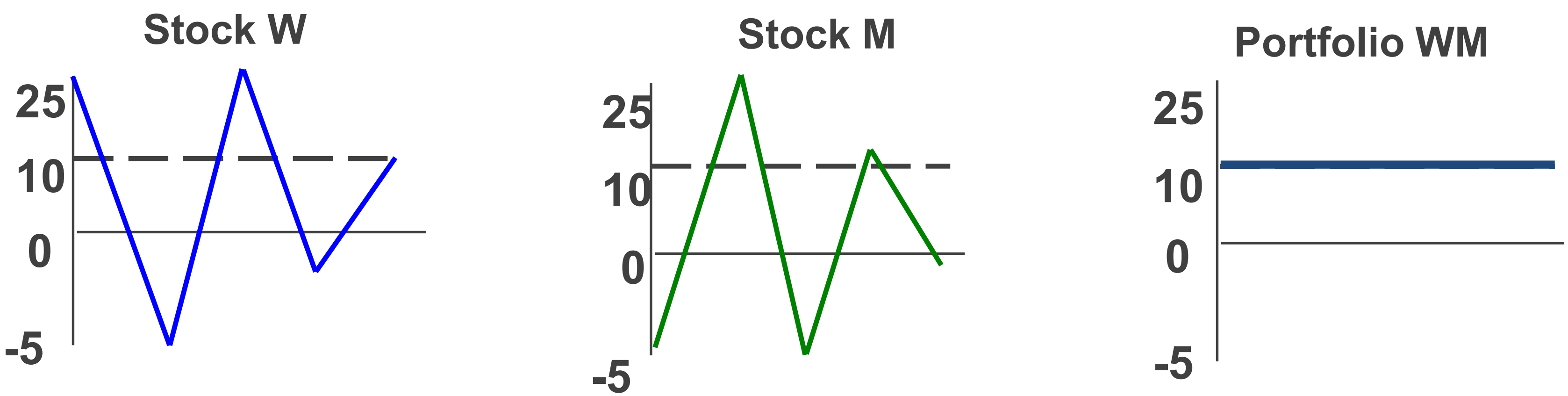
Source: Authors' calculations based on data from moneycentral.msn.com.



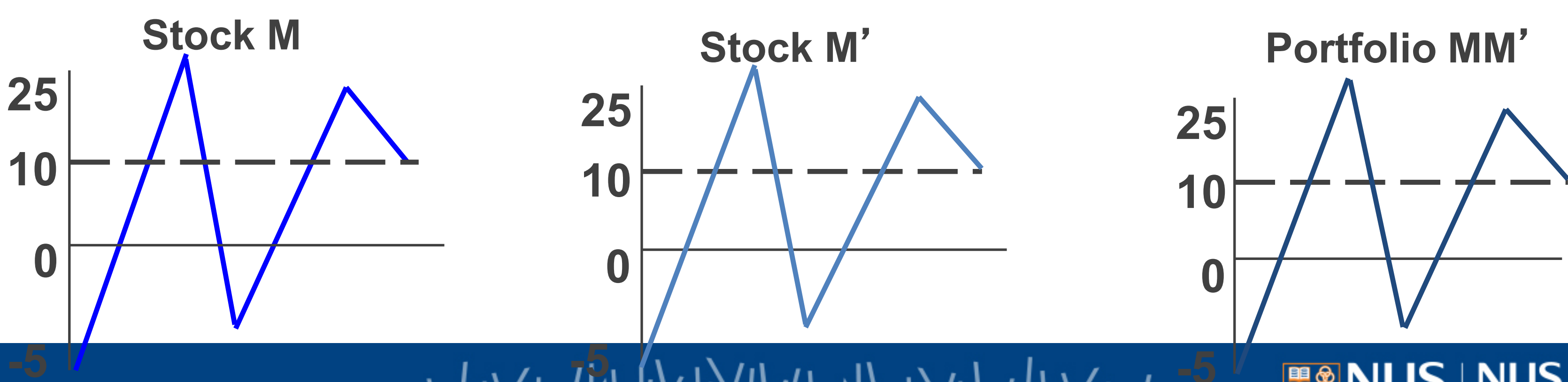
Returns Distribution

Invest an equal amount in both Stock W and Stock M

(i) For two perfectly negatively correlated stocks (ρ = -1.0)



(ii) For two perfectly positively correlated stocks (ρ = 1.0)



Estimated annual volatilities and correlations for selected stocks. (Based on monthly returns, 1996–2006)

	Apple	Microsoft	Best Buy	Target	Starbucks	Dell	HP
STANDARD DEVIATION	54%	38%	63%	30%	41%	50%	41%
Apple	1.00	0.32	0.31	0.17	0.14	0.48	0.40
Microsoft	0.32	1.00	0.36	0.36	0.25	0.63	0.39
Best Buy	0.31	0.36	1.00	0.41	0.12	0.40	0.27
Target	0.17	0.36	0.41	1.00	0.33	0.37	0.22
Starbucks	0.14	0.25	0.12	0.33	1.00	0.19	0.21
Dell	0.48	0.63	0.40	0.37	0.19	1.00	0.52
HP	0.40	0.39	0.27	0.22	0.21	0.52	1.00

Source: Authors' calculations based on data from moneycentral.msn.com.

Two-stock portfolio: ρ & risk reduction

Two stocks can be combined to form a riskless portfolio if $\rho = -1$

- Risk is not reduced at all if the two stocks have $\rho = +1$
- In general, stocks have $\rho \approx 0.65$, so risk is lowered but not eliminated.
- $\sigma \approx 35\%$ for an average stock.

Ability to get rid of risk increases as $\rho \rightarrow -1$

NONE $\rho = +1$

ALL $\rho = -1$

Risk of portfolio gets smaller as $\rho \rightarrow -1$

We looked at 2-security portfolios, results are essentially the same for N-security portfolios!

NOT EXAMINABLE

Portfolio Risk: 3-asset portfolio formula

Variance terms

$$Var(r_p) = \underbrace{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + w_3^2\sigma_3^2}_{\text{Variance terms}}$$

$$+ \underbrace{2w_1w_2\sigma_{12} + 2w_2w_3\sigma_{23} + 2w_1w_3\sigma_{13}}_{\text{Co-variance terms}}$$

Co-variance terms

	w_1	w_2	w_3
w_1	$w_1^2\sigma_1^2$	$w_1 w_2\sigma_{12}$	$w_1 w_3\sigma_{13}$
w_2	$w_1 w_2\sigma_{12}$	$w_2^2\sigma_2^2$	$w_2 w_3\sigma_{23}$
w_3	$w_1 w_3\sigma_{13}$	$w_2 w_3\sigma_{23}$	$w_3^2\sigma_3^2$

Note that there are 3 pure variance terms and 3^2-3 = 6 co-variance terms

NOT EXAMINABLE

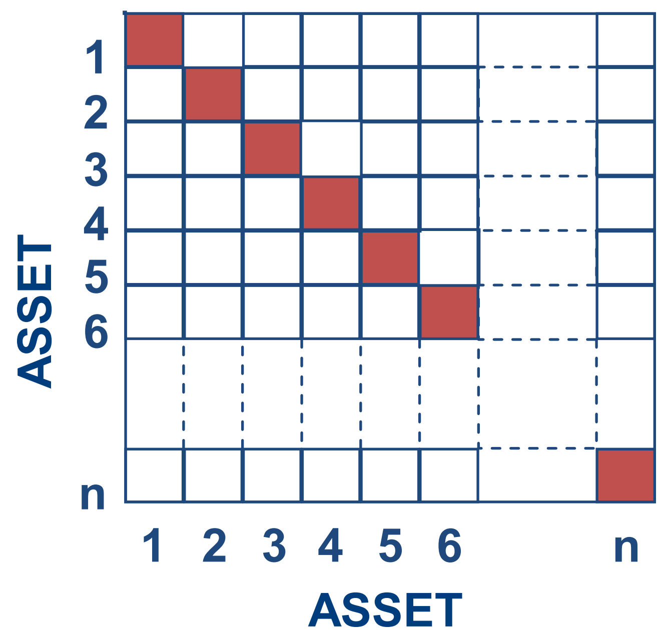
Portfolio risk: Variance of n-asset portfolio

Extending the 2- and 3-asset Variance definition to n-assets. The shaded boxes contain variance terms; the remainder contain covariance terms.

2 Asset Portfolio Variance

	Asset 1	Asset 2
Asset 1	$w_1^2\sigma_1^2$	$w_1 w_2 \sigma_{12} = w_1 w_2 \rho_{12}\sigma_1\sigma_2$
Asset 2	$w_1 w_2 \sigma_{12} = w_1 w_2 \rho_{12}\sigma_1\sigma_2$	$w_2^2\sigma_2^2$

n Asset Portfolio Variance



To calculate portfolio variance add up the boxes

As the number of securities increases, ***the variance of the portfolio approaches the average covariance***

Diversifiable & Non-diversifiable Risk

The 2 security portfolio example results suggest that there are two types of risk associated with each security:

Total Risk = Company-Specific Risk + Market Risk

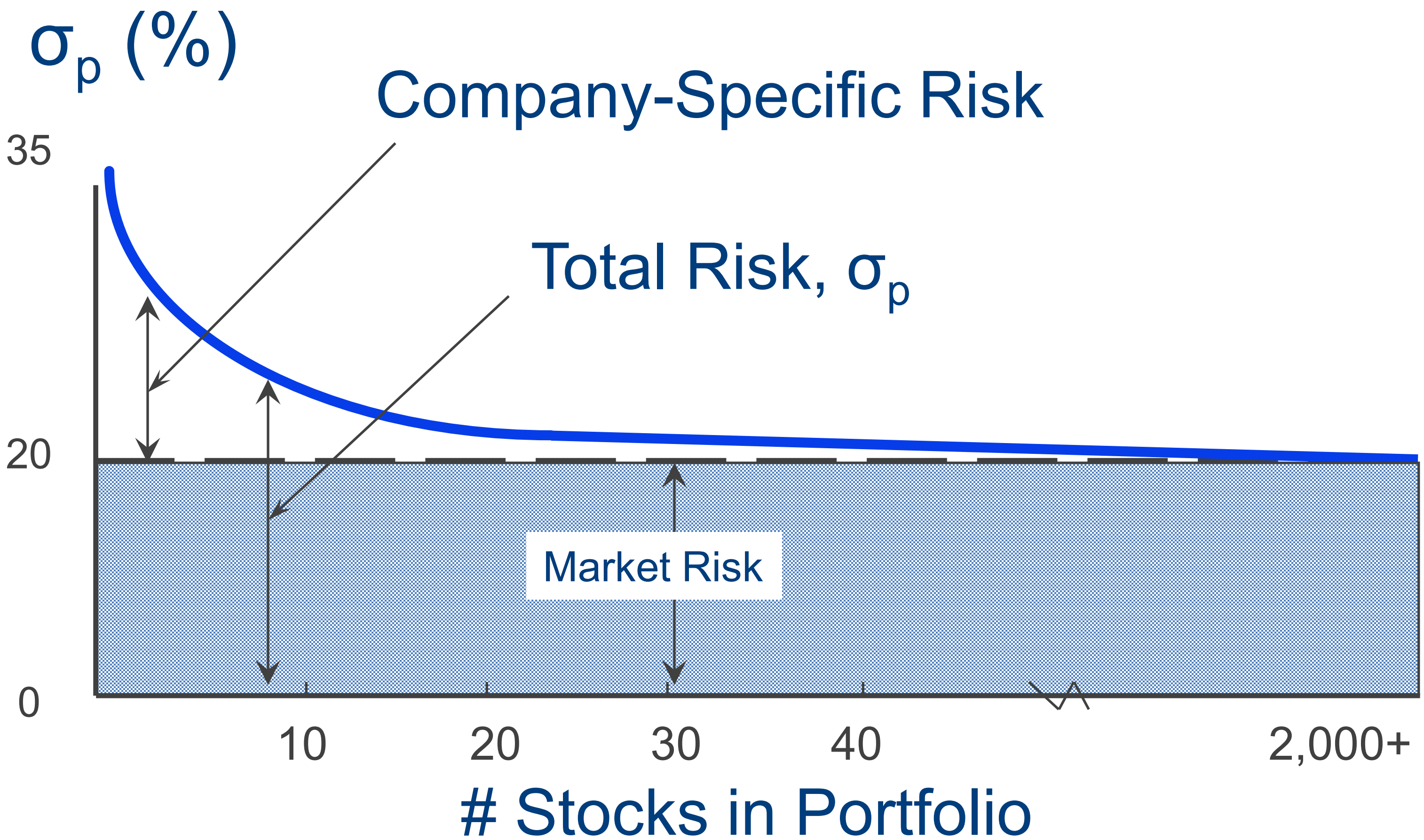
Also known as

"Unsystematic Risk" "Systematic Risk"

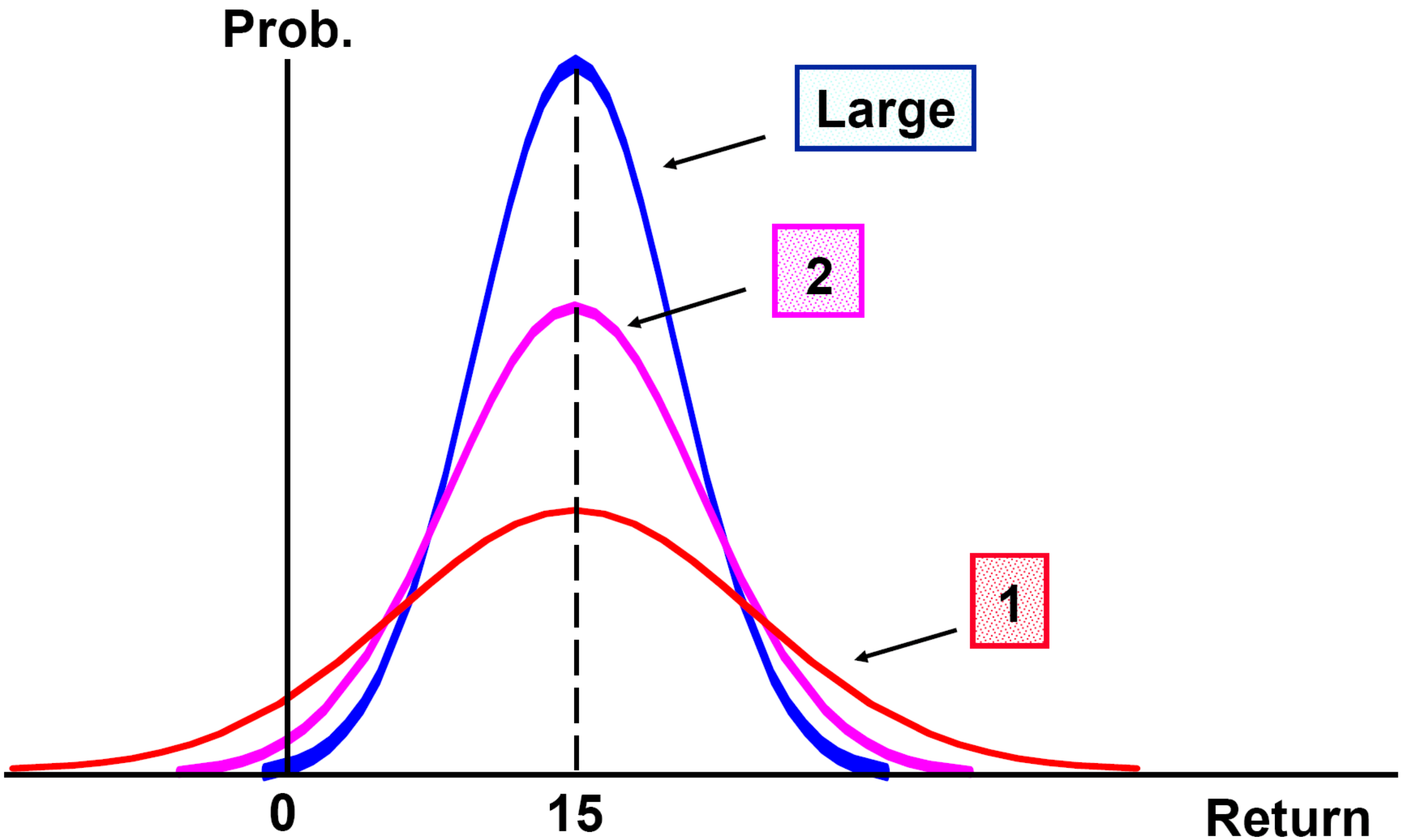
- Which risk can be diversified away?

Illustrating diversification effects of a stock portfolio

This is a general example



Returns distribution for 1-asset vs. large portfolio



$\sigma_1 \approx 35\%$; $\sigma_{Large} \approx 20\%$.

Diversifiable Risk

- These are caused by random events. E.g., lawsuits, unsuccessful marketing program, losing a major contract, and other events unique to a specific firm.

- Since the bad events in one firm can be offset by good events in another, their effects are eliminated in a portfolio.

Market risk (non-diversifiable)

- Market risk stems from factors that systematically affect most firms. E.g.,:
  - War
  - Inflation
  - Recessions
  - High interest rates

- Most, if not all stocks, are affected by these factors. Thus market risk cannot be diversified away by combining stocks into a portfolio. Stocks will generally all move in the same direction (all benefit or all suffer, but in varying degrees).

Important Lesson Of The Day: Diversify Your Investments

If you choose to hold a one-stock portfolio (exposed to more risk than a diversified investor), would you be compensated for the additional risk?

- NO!
- Stand-alone risk is not important to a well-diversified investor.
- Rational, risk-averse investors are concerned with σ_p, which is based upon market risk.
- No compensation should be earned for holding unnecessary, diversifiable risk.

Measuring market risk

- If the portfolio is well-diversified, the firm-specific risk is close to zero and the variance will come basically from the market risk (also called systematic risk and non-diversifiable risk).
- Therefore market risk remains in all portfolios. Some investments (portfolios or securities) will be more sensitive to market factors than others and will therefore have higher market risk.
- How do we measure this market risk?

Summary

Portfolio

- How to calculate portfolio return & risk
- Total risk = unsystematic risk + systematic risk
- Diversifiable vs. non-diversifiable risks

Arithmetic vs. Geometric Mean

Week 4 Additional Materials

Please review these slides on your own.
You are responsible for all the material

Calculating average return

Given a string of returns, average return can be calculated in 2 ways

1. **Geometric average** return also called Geometric mean

$$\bar{r} = \sqrt[T]{(1+r_1)(1+r_2)(1+r_3)(1+r_4)......(1+r_T)} - 1$$

$$\textit{where } r_t \textit{ are the actual nominal returns in year 1, year 2,} \ldots$$

2. **Arithmetic average** return also called Arithmetic mean

$$\bar{r} = \frac{\sum_{t=1}^{T} r_t}{T}$$

Geometric vs. Arithmetic

If an investor buys an asset at time 0 and holds it till time T:

1. The ***geometric mean*** is what s/he actually earned per year on average compounded annually
   – Also known as the mean holding period return or average compound return earned per year over a multi-year period.
2. The ***arithmetic mean*** is what s/he earned in a typical year.

Example: Geometric mean

Suppose you bought an investment and held it for 4 years. It provided the following returns over the 4-year period:

Year	Return
1	10%
2	-5%
3	20%
4	15%

Holding period = 4 years

$$\text{Holding period return} =$$
$$= (1+r_1)(1+r_2)(1+r_3)(1+r_4) - 1$$
$$= (1.10)(.95)(1.20)(1.15) - 1$$
$$= .4421 = 44.21\%$$

Example: Geometric mean

You would have actually realized an annual return of 9.58%:

Year	*Return*
1	10%
2	-5%
3	20%
4	15%

$$\text{Geometric average return} =$$

$$r_g = \sqrt[T]{(1+r_1)(1+r_2)(1+r_3)\ldots(1+r_T)} - 1$$

$$r_g = \sqrt[4]{(1.10)(.95)(1.20)(1.15)} - 1$$

$$= .095844 = 9.58\%$$

So, you realized 9.58% annual return on your money for 4 years or equivalently a holding period return of 44.21%

Example: Geometric vs. Arithmetic Mean

Note the difference between geometric average and arithmetic average.

Year	Return
1	10%
2	-5%
3	20%
4	15%

$$\text{Arithmetic average return} = \frac{r_1 + r_2 + r_3 + r_4}{4}$$

$$= \frac{10\% - 5\% + 20\% + 15\%}{4} = 10\%$$

10% is the average return in a typical year. Also known as ex-post average return, observed average return and historical average return.