CZ2003 Tutorial 12 (2020/2021, Semester 1)

Surface mapping

1. Discuss how the texture mapping, bump mapping and displacement mapping use an image to change the appearance of a surface.

2. A bilinear surface is defined by $P(s, t) = P_1 + (P_2 - P_1)\, s + (P_3 - P_1)\, t + (P_1 - P_2 - P_3 + P_4)\, st$ with $P_1 = (10, 30, 0)$, $P_2 = (4, 20, 0)$, $P_3 = (16, 10, 2)$ and $P_4 = (12, 5, 2)$. With reference to Figure Q2(left), an image of pixels 101×161 is defined by Pixel(i,j) = (r(i,j),g(i,j),b(i,j)), i,=0,1,…,100, j=0,1,…,160, where

$$\begin{cases} r(i,j) = 0.9 \sin^2\left(\frac{\pi}{2}\left\lfloor \frac{i}{30} \right\rfloor\right) \\ g(i,j) = \frac{1}{8}\left\lfloor \frac{j}{20} \right\rfloor \\ b(i,j) = 0.5 \end{cases}$$

   i). The image is mapped to the bilinear surface as shown in Figure Q2(right). Which pixel on the image is mapped to the point with coordinates (9.8, 18, 0.8) on the surface? Compute the color values at that pixel.

   ii). If the image is used for displacement mapping, derive the parametric equations of the displaced surface.

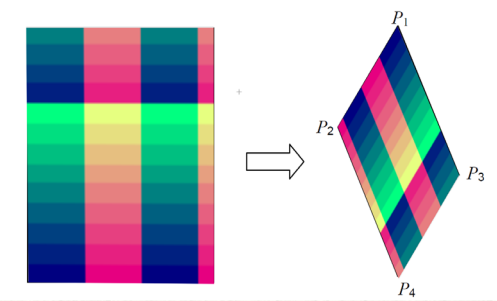

Figure Q2

3. With reference to Figure Q4, a displacement mapping is applied to a sphere, which creates a 3D solid object defined implicitly by:

$$f(x,y,z) = \left(4 - x^2 - y^2 - z^2\right) + 0.1 \cdot \left[\sin(20\pi x)\sin(20\pi y) + \sin(20\pi x)\sin(20\pi z) + \sin(20\pi y)\sin(20\pi z)\right] \geq 0$$

Find a tight sphere containing this object. What are the center and radius of the bounding sphere?

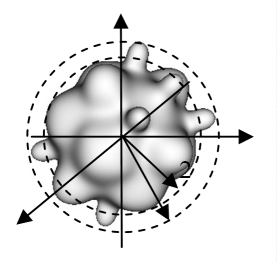

Figure Q4

1. Discuss how the texture mapping, bump mapping and displacement mapping use an image to change the appearance of a surface.

texture mapping: the addition of a seperately defined texture or pattern to a surface, which changes the colour patterns of the surface.

Bump mapping: roughening the surface of an object without actually changing the surface, which just makes the surface look as if it has bumps.

Displacement mapping: displacing the surface, which actually changes the geometry of the surface.

2. A bilinear surface is defined by $P(s, t) = P_1 + (P_2 - P_1)\, s + (P_3 - P_1)\, t + (P_1 - P_2 - P_3 + P_4)\, st$ with $P_1 = (10, 30, 0)$, $P_2 = (4, 20, 0)$, $P_3 = (16, 10, 2)$ and $P_4 = (12, 5, 2)$. With reference to Figure Q2(left), an image of pixels 101×161 is defined by Pixel(i,j) = (r(i,j),g(i,j),b(i,j)), i,=0,1,…,100, j=0,1,…,160, where

$$\begin{cases} r(i,j) &= 0.9 \sin^2\left(\frac{\pi}{2}\left\lfloor \frac{i}{30} \right\rfloor\right) \\ g(i,j) &= \frac{1}{8}\left\lfloor \frac{j}{20} \right\rfloor \\ b(i,j) &= 0.5 \end{cases}$$

i). The image is mapped to the bilinear surface as shown in Figure Q2(right). Which pixel on the image is mapped to the point with coordinates (9.8, 18, 0.8) on the surface? Compute the color values at that pixel.

ii). If the image is used for displacement mapping, derive the parametric equations of the displaced surface.

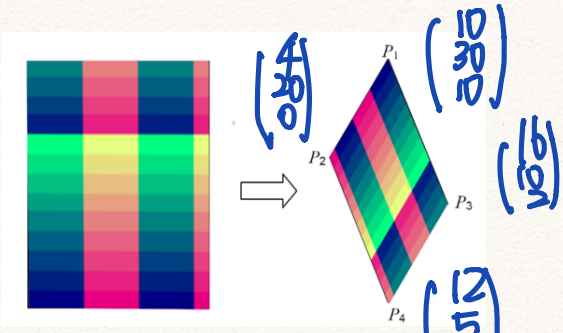

Figure Q2

Step 1: parameterize the texture

parameterize image I(i,j) by parameters(i,j) where $i \in [0,100]$, $j \in [0,160]$

step 2: Parameterize the eqn of bilinear surface.

$P(s,t) = P_1 + (P_2 - P_1)s + (P_3 - P_1)t + (P_1 - P_2 - P_3 + P_4)st$

$= \begin{pmatrix}10\\30\\10\end{pmatrix} + \left(\begin{pmatrix}4\\20\\0\end{pmatrix} - \begin{pmatrix}10\\30\\0\end{pmatrix}\right)s + \left(\begin{pmatrix}16\\10\\2\end{pmatrix} - \begin{pmatrix}10\\30\\10\end{pmatrix}\right)t + \left(\begin{pmatrix}10\\30\\10\end{pmatrix} - \begin{pmatrix}4\\20\\0\end{pmatrix} - \begin{pmatrix}16\\10\\2\end{pmatrix} + \begin{pmatrix}12\\5\\2\end{pmatrix}\right)st$

$= \begin{pmatrix}10 - 6s + 6t + 2st \\ 30 - 10s - 20t + 5st \\ 2t\end{pmatrix}$ $s, t \in [0,1]$.

For point (9.8, 18, 0.8) on the surface

$$10 - 6s + 6t + 2st = 9.8$$

$$30 - 10s - 20t + 5st = 18$$

$$2t = 0.8$$

$t = 0.4$

$$10 - 6s + 6(0.4) + 2(0.4)s = 9.8$$

$s = 0.5$

Step 3: define the mapping functions.

$s = \frac{i-0}{100-0}$ $\quad t = \frac{j-0}{160-0}$

$\therefore i = 100s$ $\quad j = 160t$.

Sub $s = 0.5$ and $t = 0.4$, $i = 50$, $j = 64$.

$\therefore$ Pixel I(50,64) with $i = 50$ and $j = 64$ is mapped to the given point.

using the given formula, we can compute the colour values for pixel.

Pixel(i,j) = (r(i,j),g(i,j),b(i,j)), i,=0,1,…,100, j=0,1,…,160, where

$$\begin{cases} r(i,j) = 0.9\sin^2\left(\frac{\pi}{2}\left\lfloor\frac{i}{30}\right\rfloor\right) \\ g(i,j) = \frac{1}{8}\left\lfloor\frac{j}{20}\right\rfloor \\ b(i,j) = 0.5 \end{cases}$$

Pixel(50,64), $i = 50$ $j = 64$

$r(50,64) = 0.9\sin^2\left(\frac{\pi}{2}\left\lfloor\frac{50}{30}\right\rfloor\right) = 0.9$

$g(50,64) = \frac{1}{8}\left[\frac{64}{360}\right] \approx 0.375$

$B(50,64) = 0.5$

$\therefore Pixel(50,64) \approx (0.9, 0.375, 0.5)$

ii). If the image is used for displacement mapping, derive the parametric equations of the displaced surface.

displacement mapping:

$x = 10 - 6s + 6t + 2st + 0.9\sin^2\left(\frac{\pi}{2}\left[\frac{100s}{360}\right]\right)$

$y = 30 - 10s - 20t + 5st + \frac{1}{8}[80]$

$z = 2t + 0.5$

3. With reference to Figure Q4, a displacement mapping is applied to a sphere, which creates a 3D solid object defined implicitly by:

$$f(x,y,z) = \left(4 - x^2 - y^2 - z^2\right) + 0.1 \cdot \left[\sin(20\pi x)\sin(20\pi y) + \sin(20\pi x)\sin(20\pi z) + \sin(20\pi y)\sin(20\pi z)\right] \geq 0$$

Find a tight sphere containing this object. What are the center and radius of the bounding sphere?

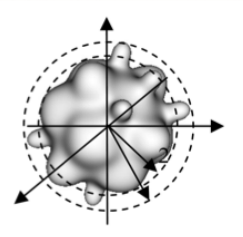

Figure Q4

initial sphere

radius: 2

centre: (0,0,0)

∴ coords: [-2, +2]

texture function takes values from 0.1 (-3) to 0.1 (3) ∴ [-0.3, +0.3]

max value of 0.3 contributes to the radius and the radius of the enlarged sphere will be $\sqrt{2^2 + 0.3} = \sqrt{4.3}$

∴ tight bounding sphere is origin centered with radius $\sqrt{4.3}$.