

**NANYANG  
TECHNOLOGICAL  
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**SINGAPORE**

CZ2003: Computer Graphics and Visualization  
Lab Report 2:  
Parametric Surfaces

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1. In 4 separate files, define parametrically using functions  $x(u, v)$ ,  $y(u, v)$ ,  $z(u, v)$ ,  $u, v \in [0, 1]$  and display:
  - a. A plane passing through the points with coordinates  $(N, M, 0)$ ,  $(0, M, N)$ ,  $(N, 0, M)$ .

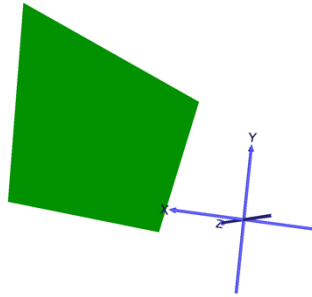


Fig 1.0 Snapshot of "Q1A1.wrl" which defines a plane with parameters [0 1 0 1] and resolution of [30 30]

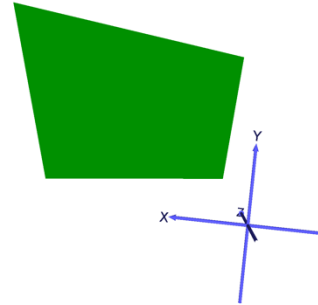


Fig 1.1 Snapshot of "Q1A2.wrl" which defines a plane with parameters [0 1 0 1] and resolution of [1 1]

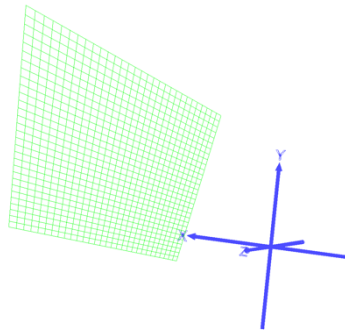


Fig 1.2 Snapshot of "Q1A1.wrl" when graphic mode is set to "wireframe".

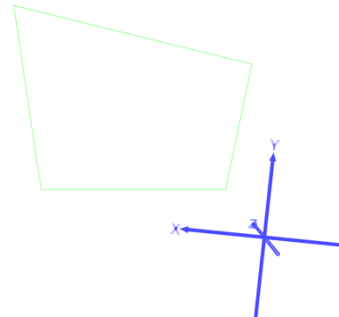


Fig 1.3 Snapshot of "Q1A2.wrl" when graphic mode is set to "wireframe".

Fig.1s are snapshots of a plane passing through the points with coordinates  $(4, 3, 0)$ ,  $(0, 3, 4)$ ,  $(4, 0, 3)$ .

Parametric Function Definition:

$$\begin{aligned} x &= 4 - 4 * u \\ y &= 3 - 3 * v \\ z &= 4 * u + 3 * v \\ u, v &\in [0, 1] \end{aligned}$$

Remarks

When changing the curve's resolution, there is no change in the shape's display because the plane does not contain any curved edges. The minimum resolution for this plane is [1, 1]. With a higher resolution of [30 30], it will sample 30 points on  $u$ ,  $v$  and form a wire mesh which contains  $30 \times 30$  polygons. The added polygons will not change the shape of the plane as observed above.

- b. A triangular polygon with the vertices at the points with coordinates  $(N, M, 0)$ ,  $(0, M, N)$ ,  $(N, 0, M)$ .

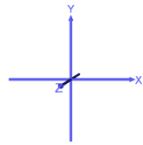


Fig 2.0 Snapshot of “Q1B1.wrl” which defines a triangular polygon with parameters  $[0 \ 1 \ 0 \ 1]$  and resolution of  $[75 \ 75]$

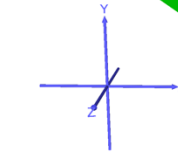


Fig 2.1 Snapshot of “Q1B2.wrl” which defines a triangular polygon with parameters of  $[0 \ 1 \ 0 \ 1]$  and resolution of  $[2 \ 2]$

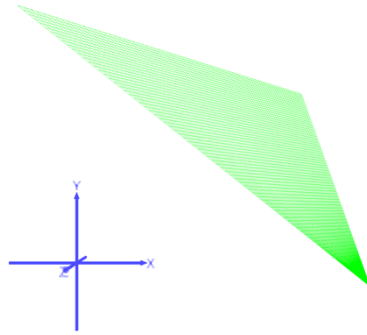


Fig 2.2 Snapshot of “Q1B1.wrl” when graphic mode is set to “wireframe”

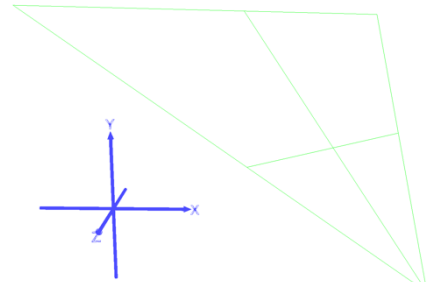


Fig 2.3 Snapshot of “Q1B2.wrl” when graphic mode is set to “wireframe”

Fig 2s are snapshots defining a triangular polygon with the vertices at the points with coordinates  $(4, 3, 0)$ ,  $(0, 3, 4)$ ,  $(4, 0, 3)$  at different resolutions and graphic viewing modes.

Parametric Function Definition:

$$\begin{aligned} x &= 4 - 4*u + 4*u*v \\ y &= 3 - 3*v \\ z &= 4*u + 3*v - 4*u*v \\ u, v &\in [0,1] \end{aligned}$$

Remarks

The minimum resolution for the polygon is  $[1 \ 1]$  because the triangular plane does not contain any curve edges like the concept mentioned in 1a. Thus, a higher resolution is not necessary to display it. When the curve resolution decreases to 2, the shape still looks like a triangular polygon. We also noted that a lower resolution affects the colour of the polygon as observed in Fig 2.1 where the area around the vertex of the triangle is visibly darker and contains shades of black.

- c. An origin-centered ellipsoid with the semi-axes  $N$ ,  $M$ ,  $(N+M)/2$

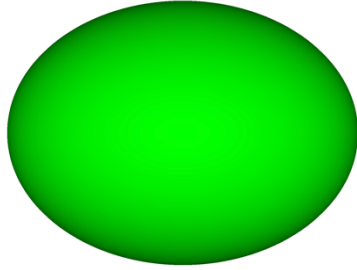


Fig 3.0 Snapshot of "1C1.wrl" which defines an origin centered ellipsoid with parameters  $[0\ 1\ 0\ 1]$  and resolution of  $[75\ 75]$

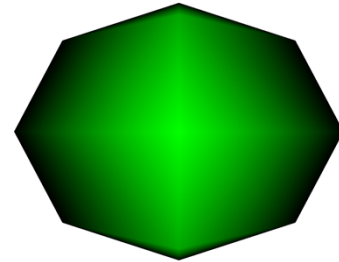


Fig 3.1 Snapshot of "1C2.wrl" which defines an origin centered ellipsoid with parameters  $[0\ 1\ 0\ 1]$  and resolution of  $[4\ 4]$

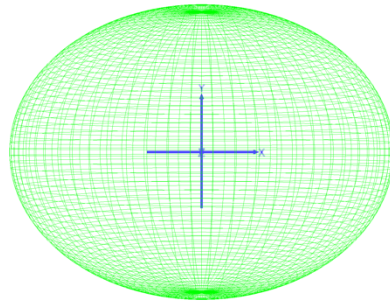


Fig 3.2 Snapshot of "1C1.wrl" which defines an origin centered ellipsoid with graphic mode set to "wireframe"

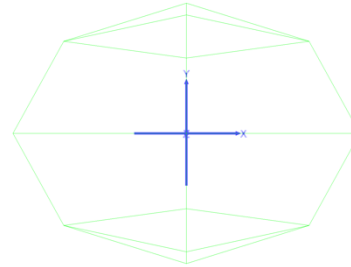


Fig 3.3 Snapshot of "1C2.wrl" which defines an origin centered ellipsoid with graphic mode set to "wireframe"

Fig 3 are snapshots of various origin-centered ellipsoid with the semi axes 4, 3, 3.5 at different resolutions and graphic modes.

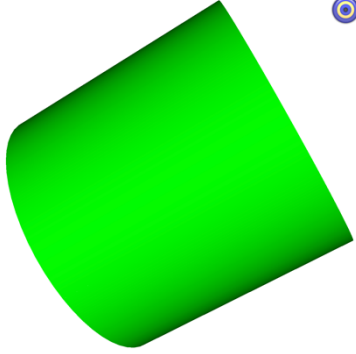
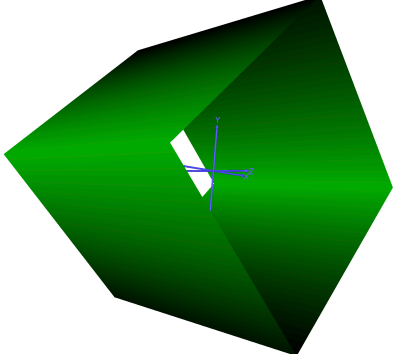
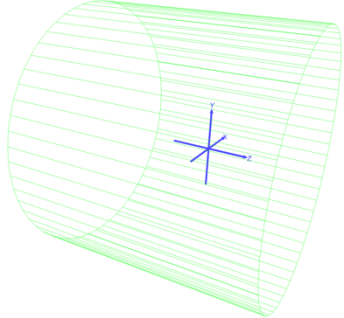
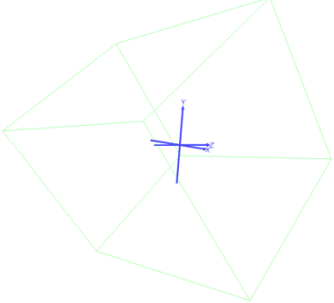
Parametric Function Definition:

$$\begin{aligned} x &= 4 * \cos((u-0.5)*\pi) * \sin((v-0.5)*2*\pi) \\ y &= 3 * \sin((u-0.5)*\pi) \\ z &= 3.5 * \cos((u-0.5)*\pi) * \cos((v-0.5)*2*\pi) \\ u, v &\in [0,1] \end{aligned}$$

Remarks

When the curve resolution decrease to smaller values, it affects the shape of the ellipsoid. There needs to be reasonably many points sampled in order to generate polygons with curves. If a smaller resolution of  $[4\ 4]$  is being used, the curve will look like a cuboid as it requires 4 points to generate a square or rectangle. This is observed in Fig 3.1.

- d. A cylindrical surface with radius  $N$  which is aligned with axis  $Z$ , and spans from  $z_1 = -N$  to  $z_1 = M$ .

 <p>Fig 4.0 Snapshot of "1D1.wrl" which defines a cylindrical surface with parameters [0 1 0 1] and resolution of [60 1]</p>	 <p>Fig 4.1 Snapshot of "1D2.wrl" which defines a cylindrical surface with parameters [0 1 0 1] and resolution of [4 1]</p>
 <p>Fig 4.2 Snapshot of "1D1.wrl" with graphic mode set to "wireframe"</p>	 <p>Fig 4.3 Snapshot of "1D2.wrl" with graphic mode set to "wireframe"</p>
<p>Fig 4s are snapshots of a cylindrical surface with the radius 4 aligned with <math>Z</math> axis spanning from <math>z_1 = -4</math> to <math>z_1 = 3</math>.</p> <p><u>Parametric Function Definition:</u></p> $x = 4 * \cos(2 * \pi * u)$ $y = 4 * \sin(2 * \pi * u)$ $z = -4 + 7 * v$ $u, v \in [0,1]$ <p>The minimum resolution for this surface is [60 1]. This means that it will sample 60 points for <math>u</math> and 1 point for <math>v</math> which creates a wireframe with 60x1 polygons. Number of sampling resolution for <math>u</math> is significantly larger because the parameter <math>u</math> is used for equation <math>x</math> and <math>y</math> which gives the cylinder its curve shape. Sufficient number of sampling points are needed to give a curve shape. Whereas only 1 sampling resolution is needed for parameter <math>v</math> because it is found in equation <math>z</math> which is used to form straight line.</p> <p>When a smaller resolution of [4 1] is used, it samples 4 points on <math>u</math> and 1 point on <math>v</math>. The curve will not be generated instead we will get a cuboid.</p>	

2. Define parametrically using functions  $x(u, v)$ ,  $y(u, v)$ ,  $z(u, v)$ ,  $u, v \in [0,1]$  a surface obtained by translational sweeping of the curve number **M** (Table 1) along axis Z so that it will span from  $z_1 = -N$  to  $z_1 = M$ .

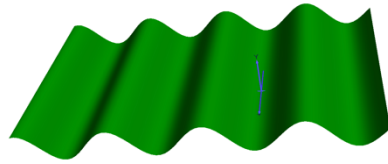


Fig 5.0 Snapshot of "2A.wrl" which defines a sin curved surfaced with parameters [0 1 0 1] and resolution of [60 1]

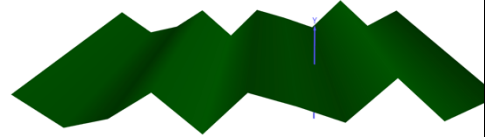


Fig 5.1 Snapshot of "2B.wrl" which defines a sine curved surface with parameters [0 1 0 1] and resolution of [10 1]

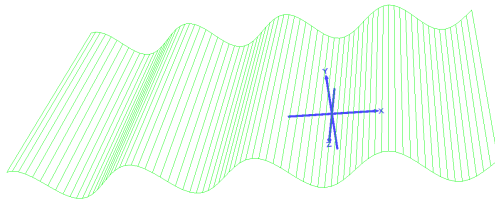


Fig 5.2 Snapshot of "2A.wrl" which defines a sine curved surface with graphic mode set to "wireframe"

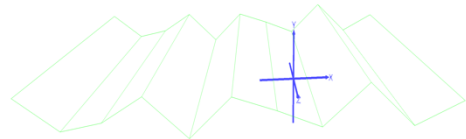


Fig 5.3 Snapshot of "2B.wrl" which defines a sine curved surface with graphic mode set to "wireframe"

Fig 5 are snapshots for Q2 which is a surface obtained by translational sweeping of curve number 3 along z axis, it spans from  $z_1 = -4$  to  $z_1 = 3$ .

Parametric Function Definition:

$$x = -8 + (12 * u)$$

$$y = (\sin(u*4*\pi))^2$$

$$z = -4 + 7*v$$

$$u, v \in [0,1]$$

3. Define parametrically using functions  $(u, v)$ ,  $y(u, v)$ ,  $z(u, v)$ ,  $u, v \in [0,1]$  a surface created by rotational sweeping of the curve defined in Fig. 5. The curve has to be first translated by  $(-N, 0, 0)$  and then subjected to rotational sweeping about axis Y clockwise by angle  $\frac{\pi}{N}$  starting the rotation at the angle  $+\frac{3\pi}{2M}$  away from the coordinate plane YZ.

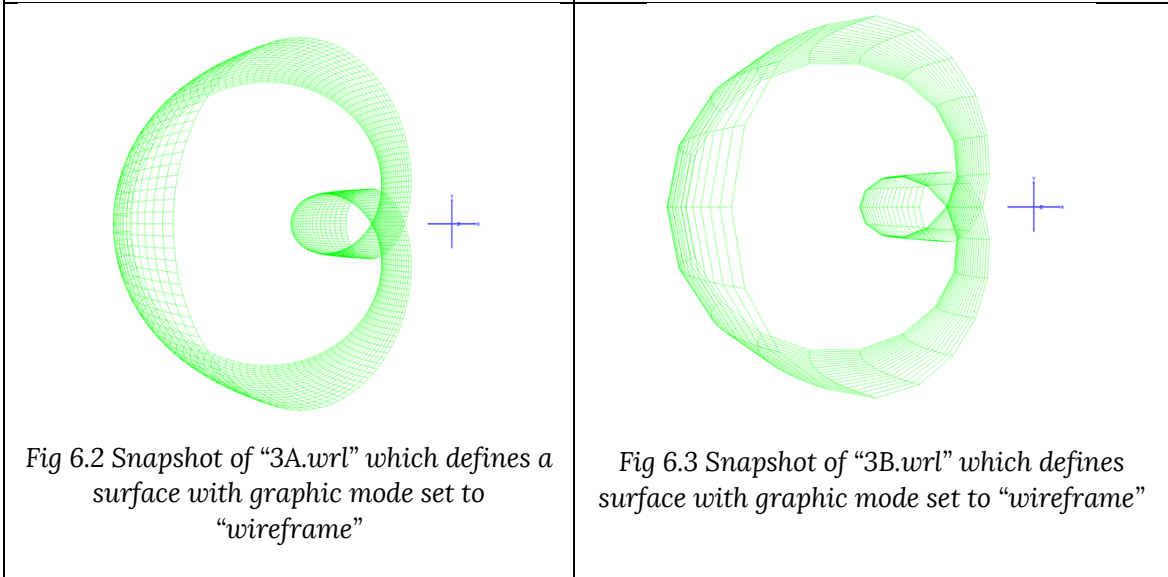
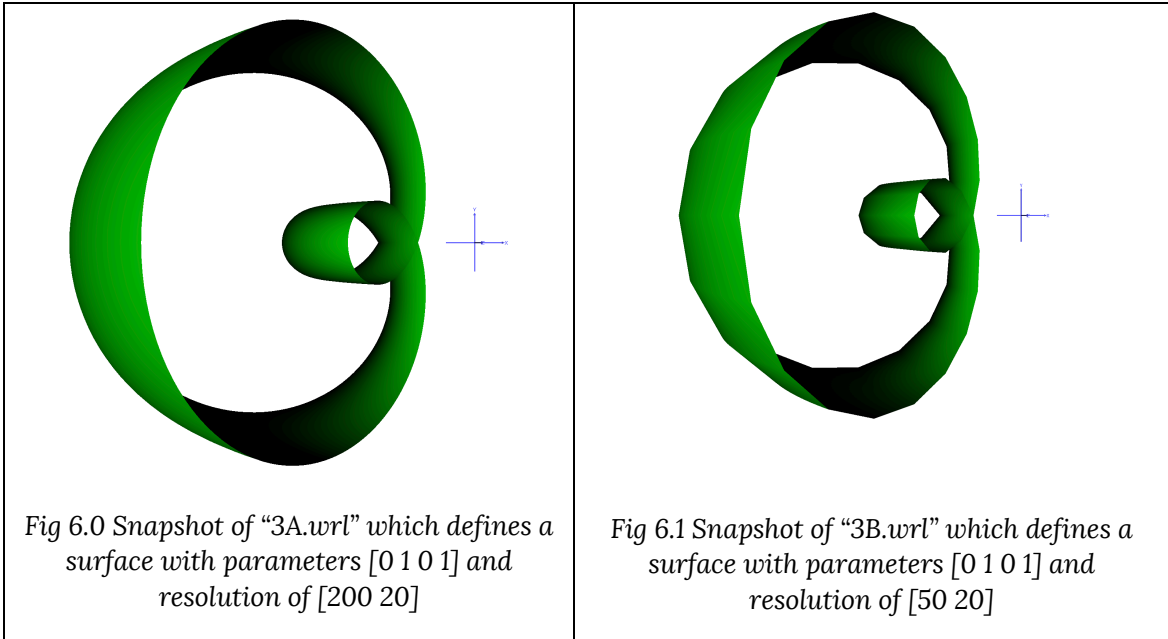


Fig 6s are snapshots of a limaçon curve translated by  $(-4, 0, 0)$  and then subjected to rotational sweeping about axis Y clockwise by angle  $\frac{\pi}{4}$  starting the rotation at the angle  $+\frac{\pi}{2}$  away from the coordinate plane YZ.

Parametric Function Definition:

$$\begin{aligned}
 x &= ((\cos(2\pi u))^*(4-8\cos(2\pi u))-4)*\sin(0.25-v\pi+\pi/2) \\
 y &= (\sin(2\pi u))^*(4-8\cos(2\pi u)) \\
 z &= ((\cos(2\pi u))^*(4-8\cos(2\pi u))-4)*\cos(0.25-v\pi+\pi/2)
 \end{aligned}$$

How to derive the final parametric function:

**First, let's define a solid curve on curve  $z = 0$**

Using equations derived in Q3 of Lab 1:

$$r = 4 - (8\cos(\alpha))$$

$$x = (4 - 8\cos(2\pi u)) * (\cos(2\pi u))$$

$$y = (4 - 8\cos(2\pi u)) * (\sin(2\pi u))$$

$$z = 0$$

$$u \in [0,1]$$

**Second, translation by  $(-4,0,0)$**

$$x = ((4 - 8\cos(2\pi u)) * (\cos(2\pi u)) - 4)$$

$$y = (4 - 8\cos(2\pi u)) * (\sin(2\pi u))$$

$$z = 0$$

**Third, the limaçon curve will be rotated about the Y axis. The rotation will be defined as**

$$z = r\cos(w)$$

$$x = r\sin(w)$$

$$w \in [0.5\pi, 0.75\pi]$$

To convert in terms of  $v$

$$z = r\cos(\pi/4 - v + \pi/2)$$

$$x = r\sin(\pi/4 - v + \pi/2)$$

$$v \in [0,1]$$

Note: The rotation should be in clockwise direction so we will need to multiply -1 to  $v$

**Finally, substituting the various equations**

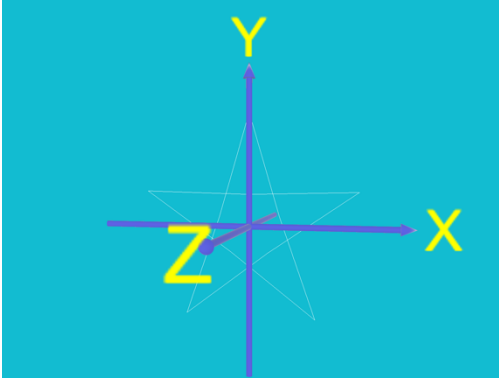
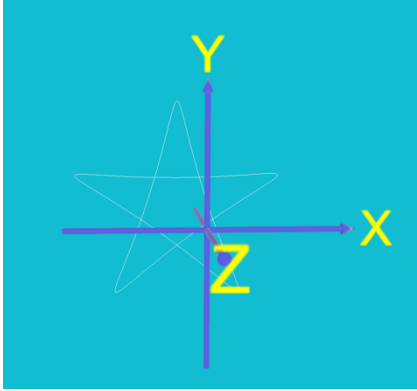
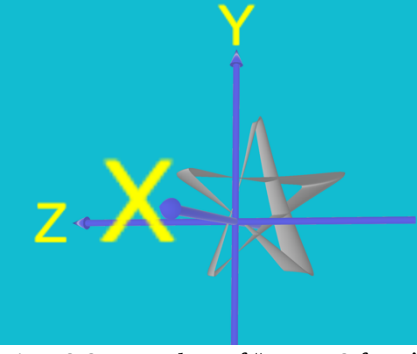
$$x = x(u)\sin(v) = ((4 - 8\cos(2\pi u)) * (\cos(2\pi u)) - 4) * \sin(\pi/4 - v + \pi/2)$$

$$y = y(u) = (4 - 8\cos(2\pi u)) * (\sin(2\pi u))$$

$$z = x(u)\cos(v) = ((4 - 8\cos(2\pi u)) * (\cos(2\pi u)) - 4) * \cos(\pi/4 - v + \pi/2)$$



4. Besides the above compulsory part, you are welcome to add any other shapes of parametric surfaces into folder Lab2/Extras. These extra exercises may increase your total mark.

 <p>Fig 7.0 Screenshot “Bonus1.func” of an origin-centered 2D star shape that is drawn with a self-intersecting closed curved with a resolution of 100 and parameter of <math>u</math> <math>[0, 1]</math></p>	<p>Making use of then concepts learnt from lab 1. I have managed to come up with a parametric equation of a 2D image of a star.</p> <p><u>Parametric Equation:</u></p> $\begin{aligned}x &= -9 \cdot \sin(2 \cdot u \cdot 2\pi) - 5 \cdot \sin(3 \cdot u \cdot 2\pi) \\y &= 9 \cdot \cos(2 \cdot u \cdot 2\pi) - 5 \cdot \cos(3 \cdot u \cdot 2\pi) \\z &= 0 \\u &\in [0, 1]\end{aligned}$
 <p>Fig 7.2 Screenshot of “Bonus2.func” start with the center <math>(-4, 3, 0)</math> with resolution 100</p>	<p>Using the concepts, we have learnt about translation, we could translate along x by <math>(-4, 0, 0)</math> and along y axis by <math>(0, 3, 0)</math>. The final position for the center of the star will be <math>(-4, 3, 0)</math>.</p> <p><u>Parametric Equation:</u></p> $\begin{aligned}x &= (-9 \cdot \sin(2 \cdot u \cdot 2\pi) - 5 \cdot \sin(3 \cdot u \cdot 2\pi) - 4) \\y &= (9 \cdot \cos(2 \cdot u \cdot 2\pi) - 5 \cdot \cos(3 \cdot u \cdot 2\pi) + 3) \\z &= 0 \\u &\in [0, 1]\end{aligned}$
 <p>Fig 7.3 Screenshot of “Bonus3.func”</p>	<p>Using the concept of rotation, we will rotate the star about the y axis which increases the depth so that it will look 3D.</p> <p><u>Parametric Equation:</u></p> $\begin{aligned}x &= (-9 \cdot \sin(2 \cdot u \cdot 2\pi) - 5 \cdot \sin(3 \cdot u \cdot 2\pi) - 4) \cdot \sin(v) \\y &= (9 \cdot \cos(2 \cdot u \cdot 2\pi) - 5 \cdot \cos(3 \cdot u \cdot 2\pi) + 3) \\z &= (-9 \cdot \sin(2 \cdot u \cdot 2\pi) - 5 \cdot \sin(3 \cdot u \cdot 2\pi) - 4) \cdot \cos(v) \\u, v &\in [0, 1]\end{aligned}$

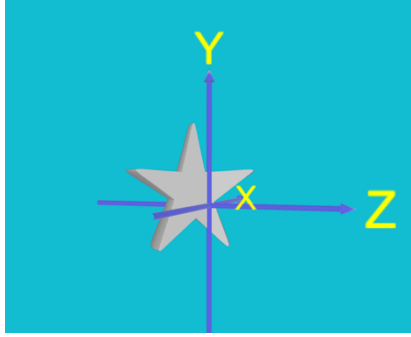


Fig 7.4 Screenshot of “Bonus4.func”  
which is an image of solid star

After experimenting around with shape explorer, I realize that by multiplying another parameter,  $w$  to all the  $x$ ,  $y$  and  $z$  equation we can achieve a solid star.

Parametric Equation:

$$\begin{aligned}x &= w * (-9 * \sin(2 * u * 2 * \pi) - 5 * \sin(3 * u * 2 * \pi) - 4) * \sin(v) \\y &= w * (9 * \cos(2 * u * 2 * \pi) - 5 * \cos(3 * u * 2 * \pi) + 3) * \cos(v) \\z &= w * (-9 * \sin(2 * u * 2 * \pi) - 5 * \sin(3 * u * 2 * \pi) - 4) * \cos(v) \\u, v &\in [0, 1]\end{aligned}$$

Please open “Bonus5.func” to see the rotation.

Using the time function, we can see the motion of the solid star that we have made.

Parametric Equation:

$$\begin{aligned}x &= w * (-9 * \sin(2 * u * 2 * \pi) - 5 * \sin(3 * u * 2 * \pi) - 4) * (\sin(v) * t) \\y &= w * (9 * \cos(2 * u * 2 * \pi) - 5 * \cos(3 * u * 2 * \pi) + 3) * (\cos(v) * t) \\z &= w * (-9 * \sin(2 * u * 2 * \pi) - 5 * \sin(3 * u * 2 * \pi) - 4) * (\cos(v) * t)\end{aligned}$$