BC2410 : Prescriptive Analytics

Assignment 3

(Due: 1 March 2022)

1. Consider the following optimization problem,

$$
\begin{aligned}
\max \quad & 2x_1 + 5x_2 + 2x_3 + 5x_4 & \\
\text{s.t} \quad & -x_1 - 3x_2 + 2x_4 \geq -5 & :A \\
& x_1 + x_2 + x_3 + x_4 \geq 5 & :B \\
& 4x_2 + 2x_3 + x_4 \leq 10 & :C \\
& x_2 \leq 100 & :D
\end{aligned}
$$

   Substituting the optimal solution, $x_1 = 25, x_2 = 0, x_3 = 0, x_4 = 10$ we have

$$
\begin{array}{ll}
\text{Objective} & 2x_1 + 5x_2 + 2x_3 + 5x_4 = 100 \\
\text{A} & -x_1 - 3x_2 + 2x_4 = -5 \\
\text{B} & x_1 + x_2 + x_3 + x_4 = 35 \\
\text{C} & 4x_2 + 2x_3 + x_4 = 10 \\
\text{D} & x_2 = 0
\end{array}
$$

   (a) (2 Marks) What is the objective value?

   (b) (2 Marks) Which constraints (A,B,C and D) are active or binding?

   (c) (2 Marks) What are the slacks or surpluses for the non-active constraints?

   (d) (2 Marks) Which constraints (A,B,C and D) MUST have zero shadow prices?

   (e) (2 Marks) Which constraints (A,B,C and D) MAY have positive shadow prices?

   (f) (2 Marks) Which constraints (A,B,C and D) MAY have negative shadow prices?

   (g) (8 Marks) Obtain the dual problem and verify your answers.

2. A company produces two kinds of products. A product of the first type requires 1/5 hours of assembly labor, 1/7 hours of testing, and \$3 worth of raw materials. A product of the second type requires 3/10 hours of assembly labor, 3/7 hours of testing and \$1 worth of raw materials. There can be at most 90 hours of assembly labor and 90 hours of testing, each day. Products of the first and second type have a market value of \$7 and \$9, respectively. The company must produce at least 200 products of the first type, and at least 100 products of the second type.

   (a) (10 Marks) Formulate a LOP that can be used to maximize the daily profit of the company.

(b) (10 Marks) Provide a LOP that can provide the shadow prices of assembly labor and testing.

3. (15 Marks) Sandy Arledge is the program scheduling manager for WCBN-TV. Sandy would like to plan the schedule of television shows for next Wednesday evening. Of the nine possible one-half hour television shows listed in the table below, Sandy must schedule exactly five of these shows for the period from 8:00pm to 10:30 pm next Wednesday evening. For each television show, its estimated advertising revenue in ($million) is shown in the second column of the table. Furthermore, each show has been categorized into one or more of the categories of "Public Interest", "Contains Violence", "Comedy", and "Drama".

Television Show	Advertising Revenue ($ million)	Public Interest	Contains Violence	Comedy	Drama
Cheers	6			Yes	Yes
Dynasty	10		Yes		Yes
L.A. Law	9	Yes	Yes		Yes
Jake	4		Yes		Yes
Bob Newhart	5			Yes	
News Special – The Middle East	2	Yes	Yes		
Focus on Science: The Fusion Issue	6	Yes			Yes
Beaches	7			Yes	
Urban Action for Education	8	Yes			

Sandy would like to determine a revenue-maximizing schedule of television shows for next Wednesday evening. However, she must be mindful of the following considerations:

(a) There must be at least as many shows scheduled that are categorized as public interest as there are shows scheduled that are categorized as containing violence.

(b) If Sandy schedules "Focus on Science – The Fusion Issue", then she must also schedule either Jake or L.A. Law (or both).

(c) Sandy cannot schedule both "Focus on Science" and "Urban Action for Education" as both of these shows are considered a bit on the dry side.

(d) If Sandy schedules two or more shows in the comedy category, then she must schedule at least one show in the drama category.

(e) If Sandy schedules more than three shows in the "contain violence" category, she will lose an estimated \$4 million in advertising revenues from family-oriented sponsors.

Construct a binary optimization model of Sandy's scheduling problem.

4. With reference to Figure 1, formulate LOPs for

   (a) (10 Marks) Maximizing the total flow from Node 1 to Node 9.

   (b) (8 Marks) Finding the shortest path from Node 1 to Node 9.

   (c) (7 Marks) Finding the minimum cost flows to satisfy the demands at the sink nodes 5, 7, 9, where the source node is 1 and demands at sink nodes 5, 7, 9 are respectively 10, 15, 5.

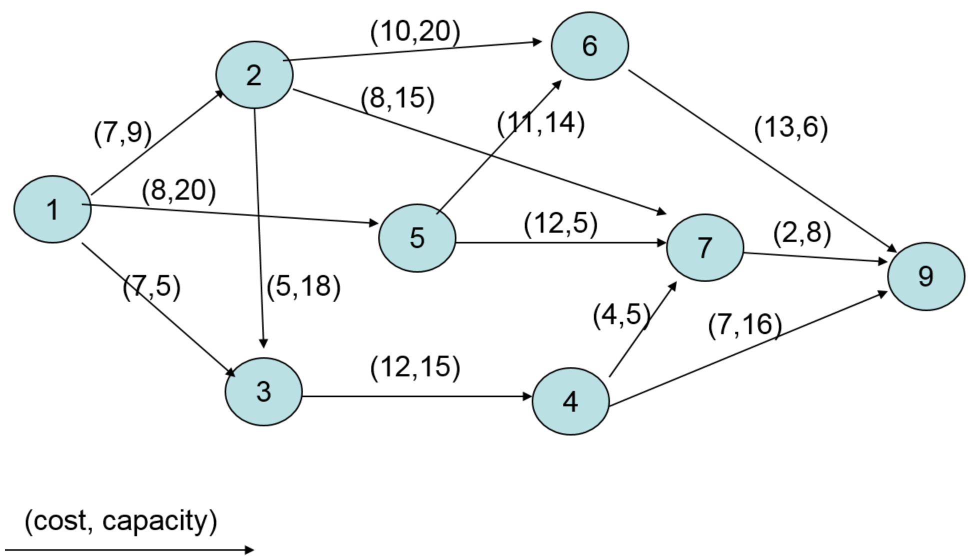

Figure 1: Network for Question 4

5. United Airlines has five daily flights from Boston (BOS) to San Fransisco (SFO), every two hours starting at 7am. The capacities of these flights are respectively 100, 150, 150, 100, and ∞. Passengers suffering from over booking are diverted to later flights. Delayed passengers get \$200 plus \$20 for every hour of delay. Suppose that today the first four flight have 160, 120, 175 and 140 confirmed reservation, the airline need to determine the most economical passenger routing strategy.

(a) (10 Marks) Draw the network flows model for the problem.

(b) (10 Marks) Formulate the mathematical optimization problem to determine the most economical passenger routing strategy.

6. (Optional) Let us look at a manufacturing problem with production vector $\boldsymbol{x} = (x_1, x_2, \ldots, x_n)$ and resource availability vector $\boldsymbol{b} = (b_1, b_2, \ldots, b_m)$. Each product x_j requires a_{ij} units of resource b_i and has a price c_j in the market. The manufacturer hopes to maximize his profit subjecting to the resource constraints. The optimization problem can be formulated as follows:

$$\begin{aligned} \max \quad & \sum_{j=1}^{n} c_j x_j \\ \text{s.t.} \quad & a_{11}x_1 + \cdots + a_{1n} \leq b_1 \\ & \vdots \\ & a_{m1}x_1 + \cdots + a_{mn} \leq b_m \\ & x_1, \ldots, x_n \geq 0. \end{aligned}$$

Prove that for any basic feasible solution, there are at most m positive productions.

7. (Optional) Find all basic feasible solutions to the following polyhedra.

$$P_1 = \{\boldsymbol{x} \in \mathbb{R}^n : 0 \leq x_i \leq 1, i = 1, \ldots, n\}$$

$$P_2 = \left\{\boldsymbol{x} \in \mathbb{R}^n : \sum_{i=1}^{n} x_i = \Gamma, 0 \leq x_i \leq 1, i = 1, \ldots, n\right\}$$

where Γ is a positive integer not larger than n.

8. (Optional) The office manager of a large law firm needs to replace the aging and out-of-style office furniture in their Singapore offices. The firm has decided to purchase 2,000 new desk/chair/credenza furniture sets for all offices in Singapore. The company has received bids from four different furniture companies who are willing to supply the furniture sets, as follows:

- Tim Woodworks has bid to deliver up to 1,000 furniture sets at a cost of \$2,500 per set and with a one-time delivery charge of \$10,000.
- Millworks has bid to supply up to 1,200 furniture sets at a cost of \$2,450 per set and with a one-time delivery charge of \$20,000.
- ABC Designs has bid to deliver up to 800 furniture sets at a cost of \$2,510 per set but with no delivery charge.

- Lim Artisan has bid to deliver up to 1,100 furniture sets at a cost of \$2,470 per set and with a one-time delivery charge of \$13,000.

Construct a discrete optimization model to determine how many furniture sets to purchase from each of these four potential suppliers.