NANYANG TECHNOLOGICAL UNIVERSITY SINGAPORE

# BC2410

# Group Assignment 1

# Semester 2 AY 2021/2022

# Seminar Group No. 1

# Team No. 5

89

**Group Members:**

| Members | Matriculation Number |
|---|---|
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**Question 1**

1. $\underset{\sim}{a} = \begin{pmatrix} 1 \\ 8 \end{pmatrix}$, $\underset{\sim}{b} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

(a) $0.1\underset{\sim}{a} + 0.9\underset{\sim}{b}$

$= 0.1\begin{pmatrix} 1 \\ 8 \end{pmatrix} + 0.9\begin{pmatrix} 3 \\ 2 \end{pmatrix}$

$= \begin{pmatrix} 0.1 \\ 0.8 \end{pmatrix} + \begin{pmatrix} 2.7 \\ 1.8 \end{pmatrix}$

$= \begin{pmatrix} 2.8 \\ 2.6 \end{pmatrix}$

$\underset{\sim}{a} - \underset{\sim}{b}$

$= \begin{pmatrix} 1 \\ 8 \end{pmatrix} - \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

$= \begin{pmatrix} -2 \\ 6 \end{pmatrix}$

1

$\underset{\sim}{a}'\underset{\sim}{b}$

$= (1\ \ 8)\begin{pmatrix} 3 \\ 2 \end{pmatrix}$

$= (1\times3 + 8\times2)$

$= (19)$

-1 scalar, no need the brackets

(b) Let $\lambda \in [0,1]$

$\lambda\underset{\sim}{a} + (1-\lambda)\underset{\sim}{b}$

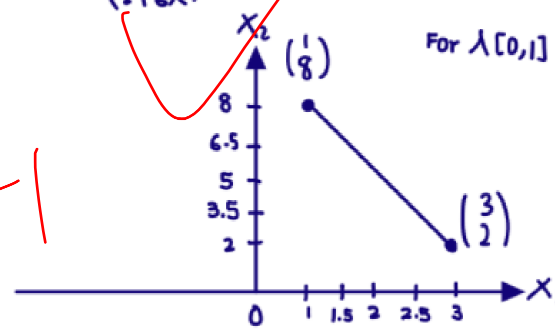
$= \lambda\begin{pmatrix} 1 \\ 8 \end{pmatrix} + (1-\lambda)\begin{pmatrix} 3 \\ 2 \end{pmatrix}$

$= \begin{pmatrix} \lambda \\ 8\lambda \end{pmatrix} + \begin{pmatrix} 3-3\lambda \\ 2-2\lambda \end{pmatrix}$

$= \begin{pmatrix} 3-2\lambda \\ 2+6\lambda \end{pmatrix}$

1.5

-1




All the plots lie in a straight line, it is a linear graph.

line segment

## Question 2

2. $\underline{A} = \begin{pmatrix} 1 & -2 & 3 \\ 4 & -5 & 7 \end{pmatrix}$, $\underline{B} = \begin{pmatrix} 1 & 2 \\ 1 & 1 \\ 2 & 0 \end{pmatrix}$

a) $\underline{AB} = \begin{pmatrix} 1\times1 + (-2)\times1 + 3\times2 & 1\times2 + (-2)\times1 + 3\times0 \\ 4\times1 + (-5)\times1 + 7\times2 & 4\times2 + (-5)\times1 + 7\times0 \end{pmatrix}$

$= \begin{pmatrix} 5 & 0 \\ 13 & 3 \end{pmatrix}$

$\underline{A} = \begin{pmatrix} 1 & -2 & 3 \\ 4 & -5 & 7 \end{pmatrix}$, $\underline{B} = \begin{pmatrix} 1 & 2 \\ 1 & 1 \\ 2 & 0 \end{pmatrix}$

$\underline{BA} = \begin{pmatrix} 1\times1+2\times4 & 1\times(-2)+2\times(-5) & 1\times3+2\times7 \\ 1\times1+1\times4 & 1\times(-2)+1\times(-5) & 1\times3+1\times7 \\ 2\times1+0\times4 & 2\times(-2)+0\times(-5) & 2\times3+0\times7 \end{pmatrix}$

$= \begin{pmatrix} 9 & -12 & 17 \\ 5 & -7 & 10 \\ 2 & -4 & 6 \end{pmatrix}$

$\therefore \underline{AB} \neq \underline{BA}$ meaning the commutative property does not hold true

b) $(\underline{AB})' = \begin{pmatrix} 5 & 13 \\ 0 & 3 \end{pmatrix}$ ; $A' = \begin{pmatrix} 1 & 4 \\ -2 & -5 \\ 3 & 7 \end{pmatrix}$ ; $B' = \begin{pmatrix} 1 & 1 & 2 \\ 2 & 1 & 0 \end{pmatrix}$

$\underline{B}'\underline{A}' = \begin{pmatrix} 1\times1 + 1\times(-2) + 2\times3 & 1\times4 + 1\times(-5) + 2\times7 \\ 2\times1 + 1\times(-2) + 0\times3 & 2\times4 + 1\times(-5) + 0\times7 \end{pmatrix}$

$= \begin{pmatrix} 5 & 13 \\ 0 & 3 \end{pmatrix}$

$\therefore \underline{B}'\underline{A}' = (\underline{AB})'$

## Question 3

3. $X = \{ \vec{x} \in \mathbb{R}^2 : \underline{A}\vec{x} \leq \vec{b} \}$

a) $\vec{x}$ has 2 elements so it is a $2\times1$ : $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$

Hence to satisfy $\underline{A}\vec{x}$,
$\underline{A}$ must be $m \times 2$ matrix
such that resulting $\vec{b}$
is a $m \times 1$
where $m$ is any integer $\geq 1$

$$\underline{A}\vec{x} \leq \vec{b}$$

$$\underset{m\times2}{\begin{bmatrix} a_{11} & a_{21} \\ \vdots & \vdots \\ a_{1m} & a_{2m} \end{bmatrix}} \underset{2\times1}{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}} \leq \underset{m\times1}{\begin{bmatrix} b_1 \\ \vdots \\ b_m \end{bmatrix}}$$

b) $\underline{A}\vec{x} \le \vec{b}$

$$\begin{pmatrix} 1 & 1 \\ -1 & 1 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \le \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} x_1 + x_2 \\ -x_1 + x_2 \\ -x_2 \end{pmatrix} \le \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix}$$

−5

$-x_2 \le 0$

$x_2 \le 0$

$-x_1 + x_2 \le 0$

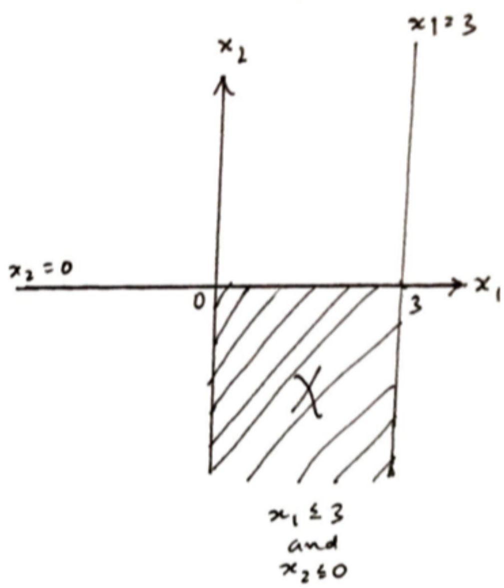
$x_2 \le x_1$

$x_1 + x_2 \le 3$

Since $x_2 \le x_1$ & $x_2 \le 0$ :

$x_1 \le 3$

∴ The set $X$ contains all possible vectors $\vec{x}$ with elements $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$, where $x_1 \le 3$ and $x_2 \le 0$.



**Question 4**

4. a) $\{Ax \leq b\} \subseteq \{Ax + s = b, s \geq 0\}$

$\{Ax = b, Ax < b\} \subseteq \{Ax + s = b, s \geq 0\}$

Starting with LHS;

if $Ax = b$; if $Ax < b$;

then $Ax + 0 = b$ then $Ax + s = b$

where $s = 0$ where $s > 0$

$\therefore \{Ax \leq b\} \subseteq \{Ax + s = b, s \geq 0\}$

b) $\{Ax + s = b, s \geq 0\} \subseteq \{Ax \leq b\}$

$\{Ax = b - s, s \geq 0\} \subseteq \{Ax \leq b\}$

Starting with LHS;

if $s = 0$; if $s > 0$;

then $b - 0 = Ax$ then $b - s = Ax$

$b = Ax$ $b > Ax$

$\therefore \{Ax + s = b, s \geq 0\} \subseteq \{Ax \leq b\}$

Correct but not rigorous

For any $x \in X_1$, we have $Ax \leq b$, so that $b - Ax \geq 0$

Let $s = b - Ax$, then $s \geq 0$. As such $x \in X_2$.

That is, $X_1 \subseteq X_2$

**Question 5**

5.

(a) Let the amount of grains(kg) consume be $X_1$ for G1 and $X_2$ for G2

minimise $0.6X_1 + 0.35X_2$

s.t. $5X_1 + 7X_2 \geq 8$

$4X_1 + 2X_2 \geq 15$

$2X_1 + X_2 \geq 3$

$X_1, X_2 \geq 0$

Using rsome

$X_1 = 3.75$

$X_2 = 0$

(b) $c = \begin{pmatrix} c_1 \\ \vdots \\ c_n \end{pmatrix}$ $c_j$: cost of food $j$

$A = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \vdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix}$ $a_{ij}$ : nutritional content of nutrient $i$ of food $j$

2

$b = \begin{pmatrix} b_1 \\ \vdots \\ b_n \end{pmatrix}$ careful $b_i$ :required amount of nutrient $i$

$x = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$

min $c'x$

s.t. $Ax \geq b$

$x \geq 0$

**Question 6**

Approach 1 (Assuming franchise is for a new store with existing manufacturing productions):

---

Assume in the long run (2nd year and beyond)

let $X_i$ be the number of workers hired in each quarter

let $E_i$ be the excess pairs of shoes in each quarter

| 1 | 2 | 3 | 4 |
|---|---|---|---|
| $X_1$ | $X_1$ | $X_1$ | |
| | $X_2$ | $X_2$ | $X_2$ |
| $X_3$ | | $X_3$ | $X_3$ |
| $X_4$ | $X_4$ | | $X_4$ |

$$\min \quad 1500\,(X_1+X_2+X_3+X_4)+50\,(E_1+E_2+E_3)$$

$$50\,(X_1+X_3+X_4)-E_1 = 600$$

$$50\,(X_1+X_2+X_4)-E_2+E1 = 300$$

$$50\,(X_1+X_2+X_3)-E_3+E_2 = 800$$

$$50\,(X_2+X_3+X_4)+E3 = 100$$

$-2$

$X_i \geq 0$, $i=1,2,3,4$

$E_i \geq 0$ $\quad i=1,2,3$

```
In [3]: model = ro.Model('assignment 1 qn 6')        # create a Model object
        x1 = model.dvar()
        x2 = model.dvar()
        x3 = model.dvar()
        x4 = model.dvar()
        e1 = model.dvar()
        e2 = model.dvar()
        e3 = model.dvar()

        model.min(1500*x1 + 1500*x2 + 1500*x3 + 1500*x4 + 50*e1 + 50*e2 + 50*e3) # min the objective function
        model.st(50*(x1 + x3 + x4) - e1 == 600)
        model.st(50*(x1 + x2 + x4) - e2 + e1 == 300)            # specify the 3rd constraints
        model.st(50*(x2 + x3 + x4) + e3 == 100)
        model.st(x1 >= 0)
        model.st(x2 >= 0)
        model.st(x3 >= 0)
        model.st(x4 >= 0)
        model.st(e1 >= 0)
        model.st(e2 >= 0)
        model.st(e3 >= 0)

        model.solve()
```

```
Being solved by the default LP solver...
Solution status: 0
Running time: 3.9748s
```

```
In [4]: print('x1:', x1.get())
        print('x2:', x2.get())
        print('x3:', x3.get())
        print('x4:', x4.get())
        print('e1:', e1.get())
        print('e2:', e2.get())
        print('e3:', e3.get())
        print('Objective:', round(model.get(), 2))
```

```
x1: [10.]
x2: [1.66495748e-12]
x3: [2.]
x4: [1.6528152e-11]
e1: [8.73640023e-11]
e2: [200.]
e3: [3.01863555e-10]
Objective: 28000.0
```

$X_1 = 10$ min cost = \$28,000

$X_2 = 0$ //

$X_3 = 2$

$X_4 = 0$

Approach 2 (Assuming franchise is for a new manufacturing plant, we will optimizing cost for the 1st year of operations of the newly opened factory):

let $x_i$ -> no. of workers hired starting that quarter

$E_i$ -> excess pairs of shoes per quarter

start

| | Q1 | Q2 | Q3 | Q4 |
|---|---|---|---|---|
| $Q_1$ | 1 | 0 | 0 | 0 |
| $Q_2$ | 1 | 1 | 0 | 0 |
| $Q_3$ | 1 | 1 | 1 | 0 |
| $Q_4$ | 0 | 1 | 1 | 1 |

-> as NBS Pte Ltd is opening a new franchise (production facility), we assume no workers carry over from the year before (i.e. Quarter 1 workers won't include Y-1 Q3 & Q4)

Obj fn minimize $1500x_1 + 1500x_2 + 1000x_3 + 500x_4 + 50(E_1 + E_2 + E_3)$

Constraints:

$$50x_1 - E_1 = 600$$

$$50(x_1 + x_2) + E_1 - E_2 = 300$$

$$50(x_1 + x_2 + x_3) + E_2 - E_3 = 800$$

$$50(x_2 + x_3 + x_4) + E_3 = 100$$

$$x_i \geq 0 \qquad E_i \geq 0$$

-2

In [3]:
```
model.min( 1500*x1 + 1500*x2 + 1000*x3 + 500*x4 + 50*e1 + 50*e2 + 50*e3)  #objective fx
model.st( 50*x1 - e1 == 600)
model.st( 50*x1 + 50*x2 + e1 - e2 == 300)
model.st( 50*x1 + 50*x2 + 50*x3 + e2 - e3 == 800)
model.st( 50*x2 + 50*x3 + 50*x4 + e3 == 100)

model.st( x1 >= 0)
model.st( x2 >= 0)
model.st( x3 >= 0)
model.st( x4 >= 0)
model.st( e1 >= 0)
model.st( e2 >= 0)
model.st( e3 >= 0)

model.solve()
```

```
Being solved by the default LP solver...
Solution status: 0
Running time: 0.0316s
```

In [4]:
```
print('x1:', x1.get())
print('x2:', x2.get())
print('x3:', x3.get())
print('x4:', x4.get())
print('e1:', e1.get())
print('e2:', e2.get())
print('e3:', e3.get())
print('Objective:', round(model.get(), 2))
```

```
x1: [12.]
x2: [1.80198623e-12]
x3: [4.89202402e-12]
x4: [1.70439514e-11]
e1: [2.56944433e-10]
e2: [300.]
e3: [100.]
Objective: 38000.0
```

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SINGAPORE

# BC2410
# Group Assignment 2
# Semester 2 AY 2021/2022
# Seminar Group No. 1
# Team No. 5

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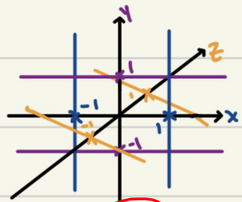
| Han Xiao | U1920108G |
| Lim He Ying | U2011832J |

**Question 1**

1\. a) $P_1 = \{(x,y,z) \in \mathbb{R}^3 : |x| \le 1,\ |y| \le 1,\ |z| \le 1\}$

graphically:



$-1 \le x \le 1$

2 half spaces

/10

$\therefore$ there are 3 halfspaces : $\{x \mid |x| \le 1\}$, $\{y \mid |y| \le 1\}$, $\{z \mid |z| \le 1\}$

Since $P_1$ is the union of these 3 halfspaces, $P_1$ is a polyhedra

more specifically, it is a cube with sides of $2 \times 2$ units$^2$ each

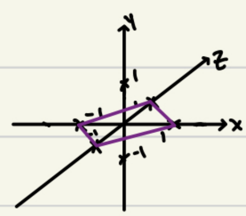
$P_2 = \{(x,y,z) \in \mathbb{R}^3 : |x| + |y| + |z| \le 1\}$

is this a linear constraint?

graphically

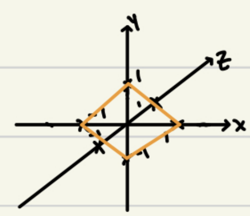
if $x = 0$:

hyperplane $|y| + |z| \le 1$ is created



if $y = 0$:

hyperplane $|x| + |z| \le 1$ is created

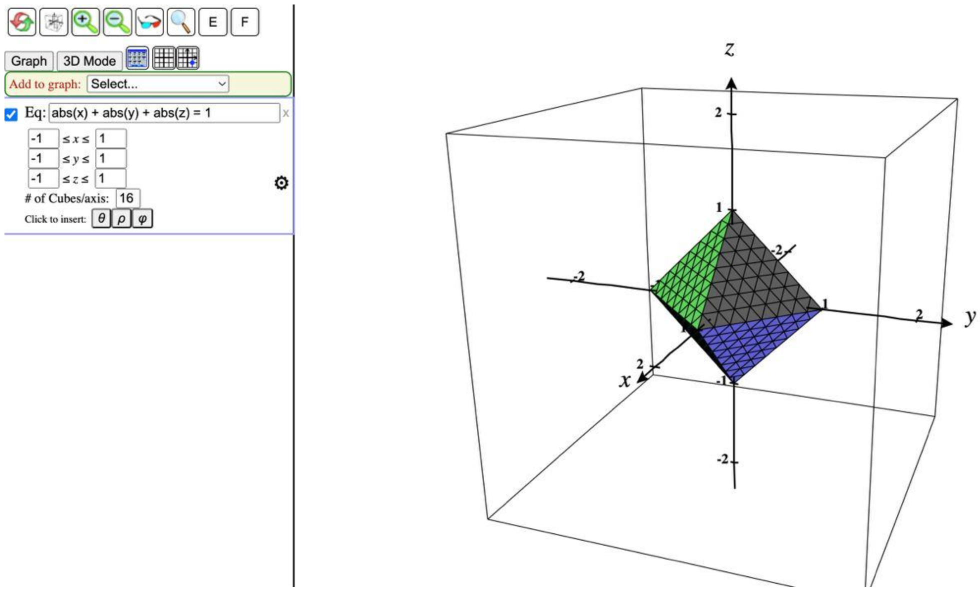


if $z = 0$:

hyperplane $|x| + |y| \le 1$ is created

Since $P_2$ is the intersection of these 3 hyperplanes, $P_2$ is a polyhedra

Graphical representation of $P_2$:




b) Extreme Points of $P_1$: $(1,1,1)$, $(-1,1,1)$, $(1,-1,1)$, $(1,1,-1)$,

$(-1,-1,1)$, $(1,-1,-1)$, $(-1,1,-1)$, $(-1,-1,-1)$

Extreme Points of $P_2$: $(1,0,0)$, $(0,1,0)$, $(0,0,1)$,

$(-1,0,0)$, $(0,-1,0)$, $(0,0,-1)$

**Question 2**

let $x_i$ be amount of 6 months loan and $y_i$ be amount of 1 month loan at start of month i where $i = 1, \dots, 6$. Keep $C_i$ at the end of the month where $i = 1, \dots, 6$

$$\min \; 0.12x_1 + 0.04\sum_{i=1}^{6} y_i$$

s.t.

$$x_1 + y_1 + 20000 - C_1 = 50000$$
$$C_1 + y_2 + 30000 - C_2 = 60000 + 1.04y_1$$
$$C_2 + y_3 + 40000 - C_3 = 50000 + 1.04y_2$$
$$C_3 + y_4 + 50000 - C_4 = 60000 + 1.04y_3$$
$$C_4 + y_5 + 80000 - C_5 = 50000 + 1.04y_4$$
$$C_5 + y_6 + 100000 - C_6 = 30000 + 1.04y_5$$
$$1.12x_1 + 1.04y_6 \leq C_6$$
$$x_1, y_i, C_i \geq 0$$

no need to spend all the surplus.

-2

```
In [70]: R= [20000,30000,40000,50000,80000,100000]
         L= [50000,60000,50000,60000,50000,30000]
         model = ro.Model('assignment 2 qn 2')
         x1 = model.dvar()
         y = model.dvar(6)
         c = model.dvar(6)

         model.min(0.12*x1 + 0.04*(y[0]+y[1]+y[2]+y[3]+y[4]+y[5]))
         model.st(x1 + y[0] + 20000 - c[0] == 50000)
         for i in range(5):
             model.st(c[i] + y[i+1] + R[i+1] - c[i+1] == L[i+1] + 1.04*y[i])
         model.st(1.12*x1 + 1.04*y[5] == c[5])
         model.st(x1 >= 0)
         model.st(y >= 0)
         model.st(c >= 0)

         model.solve()

         Being solved by the default LP solver...
         Solution status: 0
         Running time: 0.0300s
```

y.sum() or sum(y)

-2

```
In [71]: print('x1:', x1.get())
         print('y:', y.get())
         print('c:', c.get())

         print('Objective:', round(model.get(), 2))

         x1: [50148.15767626]
         y: [46560.95284107 60077.21774639 67939.81251007 76655.92129856
          59411.11634699 38910.5062278 ]
         c: [66709.11051735 48362.93730904 43822.44336288 39820.95965097
          49509.91784744 96632.86307431]
         Objective: 20000.0
```

**Question 3**

Q3

obj : minimize tot pdtn cost + inv cost for 12 mnths

$d_i$ = demand (month $i$)

$C_1$ = Storage cost per unit / month

$C_2$ = unit cost to switch pdtn levels

$x_i$ = units produced

$V_i$ = inventory surplus per month

each month

$$V_{i-1} + x_i = d_i + V_i$$

5

$$\text{obj} : \min \sum_{i=1}^{11} C_1 V_i + \sum_{i=\cancel{1}}^{11} C_2 \cancel{|x_{i+1} - x_i|}\ y_i$$

$y_i \geq x_{i+1} - x_i$ for $i = \cancel{1}$ to 11

$y_i \geq -x_{i+1} + x_i$ for $i = \cancel{1}$ to 11

5

$V_1 = x_1 - d_1$

$V_{i-1} + x_i = d_i + V_i$ for $i =$ 2 to 12

$V_{i-1} \leq d_i$ why you need this

2

$x_i, V_i \geq 0$ for $i = \cancel{1}$ to 11

2

$C_1, C_2 \geq 0$

$d_i \geq 0$

data

**Question 4**

4a)

Primal

$$\max \quad c_1 x_1 + c_2 x_2 + \dots c_{500} x_{500}$$

st
$1 x_1 + x_2 + \dots x_{500} = 30 \quad : P_1 \quad \text{free}$

$1 x_i \le 1 \quad \text{for } i = 1, \dots, 500 \quad : P_{i+1} \ge 0$

$x_i \ge 0 \quad \text{for } i = 1, \dots, 500$

Dual

$$\min \quad 30 P_1 + \sum_{i=1}^{500} P_{i+1}$$

st
$P_1 + P_{i+1} \ge c_i \quad \text{for } i = 1, \dots, 500 \quad : x_i$

$P_1 = \text{free}, \quad P_{i+1} \ge 0 \quad \text{for } i = 1, \dots, 500$

4b) Primal

$$\min \quad c_1x_1 + c_2x_2 + \dots + c_{500}x_{500}$$

st $\quad x_1 + x_2 + \dots + x_{500} = 30 \quad : p_1 \text{ free}$

$\quad x_i \le 1 \quad \text{for } i = 1, \dots, 500 \quad : p_{i+1} \le 0$

$\quad x_i \ge 0$

Dual

$$\max \quad 30p_1 + \sum_{i=1}^{500} p_{i+1}$$

st $\quad p_1 + p_{i+1} \le c_i \quad \text{for } i = 1, \dots, 500 : x_i$

$\quad p_1 = \text{free}, \quad p_{i+1} \le 0 \quad \text{for } i = 1, \dots, 500$

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# BC2410
# Group Assignment 3
# Semester 2 AY 2021/2022
# Seminar Group No. 1
# Team No. 5

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| Han Xiao | U1920108G |
| Lim He Ying | U2011832J |

Question 1:

1(a) Objective value = 100

(b) Binding constraints : A, C

(c) For B,

$-35-(-5)=-30$

There is a surplus of 30

For D,

$100-0=100$

There is a slack of 100

(d) Constraints B and D

(e) Constraints C

(f) Constraints A

(g) Primal

$$\max\ 2x_1+5x_2+2x_3+5x_4$$

$$\begin{aligned} \text{s.t. } -x_1-3x_2+2x_4 &\geq -5 \quad : A \quad P_A \leq 0 \\ x_1+x_2+x_3+x_4 &\geq 5 \quad : B \quad P_B \leq 0 \\ 4x_2+2x_3+x_4 &\leq 10 \quad : C \quad P_C \geq 0 \\ x_2 &\leq 100 \quad : D \quad P_D \geq 0 \end{aligned}$$

Dual $\min\ -5P_A+5P_B+10P_C+100P_D$

s.t.

$$\begin{aligned} -P_A+P_B &= 2 \\ -3P_A+P_B+4P_C+P_D &= 5 \\ P_B+2P_C &= 2 \\ 2P_A+P_B+P_C &= 5 \end{aligned}$$

$$P_A \leq 0, P_B \leq 0, P_C \geq 0, P_D \geq 0$$

From the dual results, it is verified that constraint C has positive shadow price constraint A has negative shadow price, and constraint B and D shadow price must be 0.

Question 2:

(a) let $X_1$ and $X_2$ be the number of products of first & second type of product produced respectively

Profit of Product 1 = 7 − 3 = \$4

Profit of product 2 = 9 − 1 = \$8

$$\max \quad 4X_1 + 8X_2$$

s.t.

$$\frac{1}{5}X_1 + \frac{3}{10}X_2 \le 90 \quad : P_1 \ge 0$$

$$\frac{1}{7}X_1 + \frac{3}{7}X_2 \le 90 \quad : P_2 \ge 0$$

$$X_1 \ge 200 \quad : P_3 \le 0$$

$$X_2 \ge 100 \quad : P_4 \le 0$$

(b) $$\min \quad 90P_1 + 90P_2 + 200P_3 + 100P_4$$

s.t.

$$\frac{1}{5}P_1 + \frac{1}{7}P_2 + P_3 \ge 4 \quad : X_1 \ge 0$$

$$\frac{3}{10}P_1 + \frac{3}{7}P_2 + P_4 \ge 8 \quad : X_2 \ge 0$$

$$P_1, P_2 \ge 0 \qquad P_3, P_4 \le 0$$

Question 3:

let $Y_i$ be the revenue in million earned from television shows, where $i=1,\dots,9$

let $X_i$ be whether the show $i$ is selected, where $i=1,\dots,9$

let $E$ be whether \$4 million in advertising revenues from family oriented sponsors is lost.

max $$\sum_{i=1}^{9} Y_i X_i - 4E$$

s.t. $$\sum_{i=1}^{9} X_i \cancel{\le} = 5$$ → max 5 shows exactly five -1

a) $X_2 + X_3 + X_4 + X_6 \le X_3 + X_6 + X_7 + X_9$

b) $X_4 \le X_3 + X_4$

c) $X_7 + X_9 \le 1$

d) $\frac{X_1 + X_5 + X_8 - 1}{2} \le X_1 + X_2 + X_3 + X_4 + X_7$

e) max $\sum_{i=1}^{9} Y_i X_i - 4E$

E binary ; $E=1$ if $>3$ shows contain violence

$E=0$ if $\le 3$ shows contain violence

$E \ge X_2 + X_3 + X_4 + X_6 - 3$

-2

$X_i$ binary

$X_i \ge 0$ $Y_i \ge 0$ $E \ge 0$

$i=1,\dots,9$

-1

Question 4:

4. a) Let $X_{ij}$ be the number of units flowing from node i to node j

$C_{ij}$ be the capacity of path flowing from node i to node j

Objective: $\max X_{69} + X_{79} + X_{49}$

s.t. $X_{12} = X_{26} + X_{27} + X_{23}$

$X_{13} + X_{23} = X_{34}$

$X_{15} = X_{56} + X_{57}$

$X_{26} + X_{56} = X_{69}$

$X_{27} + X_{57} + X_{47} = X_{79}$

$X_{34} = X_{47} + X_{49}$

$X_{ij} \le C_{ij}$ $(i,j) \in A$, $A = \{ \quad \ldots \quad \}$

$X_{ij} \ge 0$

b) Let $Y_{ij}$ be the path starting at node i & ending at node j

assuming equal distance between each node (since no info in question) distance is given

$\min \sum_{i=1}^{9} \sum_{j=1}^{9} Y_{ij}$ $-2$

s.t. $Y_{12} + Y_{15} + Y_{13} = 1$

$Y_{69} + Y_{79} + Y_{49} = 1$

$Y_{12} = Y_{23} + Y_{27} + Y_{26}$

$Y_{15} = Y_{56} + Y_{57}$

$Y_{13} + Y_{23} = Y_{34}$

$Y_{26} + Y_{56} = Y_{69}$

$Y_{27} + Y_{57} + Y_{47} = Y_{79}$

$Y_{34} = Y_{47} + Y_{49}$

$Y_{ij}$ is binary

network opt does not need the binary requirement.

$-1$

$y_{ij} \in \{0, 1\}$ for $(i,j) \in A$, where $A = \{ \quad \ldots \quad \}$.

c) let $p_{ij}$ be the cost needed per unit to flow from node i to node j.

$$\min \sum_{i=1}^{9}\sum_{j=1}^{9} p_{ij} x_{ij}$$

$x_{12}+x_{13}+x_{15}=30$

$-2$

s.t. $x_{12} = x_{26}+x_{27}+x_{23}$

$x_{13}+x_{23} = x_{34}$

$x_{15} = x_{56}+x_{57}+10$

$x_{26}+x_{56} = x_{69}$

$x_{27}+x_{57}+x_{47} = x_{79}+15$

$x_{34} = x_{47}+x_{49}$

$x_{49}+x_{69}+x_{79} = 5$

$x_{ij} \le C_{ij}$ $(i,j)\in A$, where $A = \{ \ldots \}$

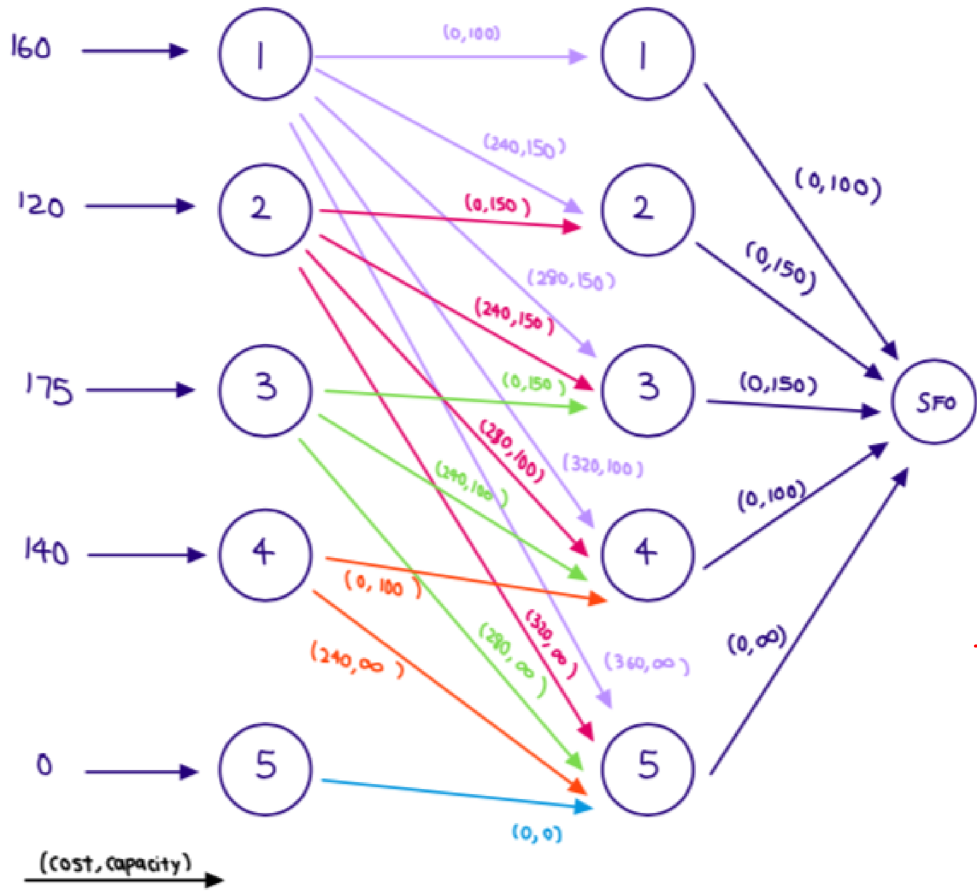
$x_{ij} \ge 0$

$-1$

Question 5:

5.

Let $x_{ij}$ represents the number of delayed passengers that are suppose to take flight $i$ but end up taking flight $j$. Where $i=1,2,3,4,5$ and $j=2,3,4,5$




595

-2

Let $x_{ij}$ represents the number of delayed passengers that are suppose to take flight i but end up taking flight j. Where i=1,2,3,4,5 and j=2,3,4,5

$$\min\ 240x_{12}+280x_{13}+320x_{14}+360x_{15}+240x_{23}+280x_{24}+320x_{25}+240x_{34}+280x_{35}+240x_{45}$$

s.t.

$$x_{11}+x_{12}+x_{13}+x_{14}+x_{15}=160$$
$$x_{22}+x_{23}+x_{24}+x_{25}=120$$
$$x_{33}+x_{34}+x_{35}=175$$
$$x_{44}+x_{45}=140$$
$$x_{11}\le 100 \qquad x_{11}=X_{1,SFO}$$
$$x_{12}+x_{22}\le 150 \qquad x_{12}+x_{22}=X_{2,SFO}$$
$$x_{13}+x_{23}+x_{33}\le 150\ X_{3,SFO}$$
$$x_{14}+x_{24}+x_{34}+x_{44}\le 100\ X_{4,SFO}$$
$$x_{ij}\ge 0,\ \text{where } i=1,2,\dots,5 \text{ and } j=1,2,\dots,5$$

$-5$

$$X_{1,SFO}\le 100$$
$$X_{2,SFO}\le 150$$
$$X_{3,SFO}\le 150$$
$$X_{4,SFO}\le 100$$

$$\sum_{i=1}^{5} X_{i,SFO}=595$$