FIN 2704/2704X

Week 3 Slides

Time Value of Money

Learning objectives

- Understand the concept of time value of money
- Know how to compute the value of cash flows that come in at very different times
  - Be able to compute the future value of an investment made today
  - Be able to compute the present value of cash to be received at some future date
  - Be able to draw and explain the use of a timeline

Time value analysis

- Cash is crucial in financial valuation. One of the main problems in finance is **determining what value we should place** on prospective cash profits that will accrue to a project/investment
- This has two components: **time** and **uncertainty**
  - For now, we will assume away uncertainty
  - Focus on how to devise techniques for evaluating the worth of certain cash flows that will come in at various dates in the future
- This is crucial, because it will allow us to compare the value of profits that may be coming in at very different times.

The main insight that we will start with is

A dollar paid today is worth more than a dollar paid tomorrow

What is the time value of money?

$1 received today is preferred to $1 received some time in the future

Why?

- Lost earnings: can invest the money to earn interest
- Loss of purchasing power: because of the presence of inflation
- Trade-off depends on the rate of return

Example: Earning interest on deposits

(it can become a complex decision)

A year of abundance starts with getting the most of your wealth and a rewarding relationship.

1.38 % p.a.#	9-month ANZ Instant Interest Time Deposit with a minimum placement of S$150,000
8.88 % p.a.+	1-month ANZ Instant Interest Time Deposit with minimum placement of S$50,000 and same amount in investments or insurance.

Also, receive these exclusive welcome privileges when you start your ANZ Signature Priority Banking relationship.

Example: Negative Interest Rates?
(how can that happen?)

Julius Baer's Singapore branch to impose negative interest rates on some deposits

The Singapore branch of Bank Julius Baer is set to impose negative interest rates on certain deposits.

The Swiss-based bank told clients last week that "in response to prevailing market conditions", it will introduce negative interest rates for cash balances in current accounts in certain currencies from June 1.

An interest rate of -0.4 per cent a year will be applied if a cash balance exceeds €100,000 (S$156,000), for instance, while a rate of -0.75 per cent will be levied for a cash balance of more than 500,000 Swiss francs (S$703,000). Other currencies affected include the Danish krone and Swedish krona.

Source: https://www.straitstimes.com/business/banking/julius-baers-singapore-branch-to-impose-negative-interest-rates-on-some-deposits posted on May 11, 2016

Example: Negative Interest Rates?

Danish mortgage costs in negative territory

Danish households are being paid to borrow, with interest on floating-rate loans sinking to the lowest on record, as Britain's decision to quit the European Union (EU) feeds a capital flight into markets considered safe. AAA-rated Denmark, which pegs its krone to the euro, has felt the weight of a sudden capital influx, sending its 10-year government yield close to zero and driving down rates on home loans.

Denmark's mortgage banks are resetting interest on loans tied to money market rates, and those have plunged. The result, said economist Lise Bergmann of Nordea Kredit, is thousands of home owners with ***negative*** rates. "This means investors will pay home owners to lend them money," she said.

"At first glance, it's good to be a home owner, but it really is a sign of sickness in the economy," Mr Beltoft, head of the Danish Mortgage Bankers Federation, said. "That's the flip side to the coin."

Source: https://www.straitstimes.com/business/danish-mortgage-costs-in-negative-territory posted on June 29, 2016

Timelines

Drawing timelines can be helpful in understanding time value of money problems

Year 0	**Year 1**	**Year 2**
	End of year 1	**End of year 2**
Today		
Beginning of year 1	**Beginning of year 2**	**Beginning of year 3**

0 1 2 3

Basic Definitions: PV and FV

Present Value (PV)

- The value of something today. On a timeline t = 0
- Also referred to as the **market value** of a cash flow to be received in the future.
- Translating a value that comes at some point in the future to its value in the present is referred to as **discounting**

Future Value (FV)

- The value of a cash flow sometime in the future. On a timeline t > 0
- Translating a value to the future is referred to as **compounding**

Meaning Of Future And Present Values

- Keep in mind that all that present values and future values do is to put cash flows which come in at different times on a **comparable basis**
- Once they are in the "same units", we will be able to compare and make decisions on which pattern of cash flows are preferable.

Image source: https://www.comparisonshopping.us/comparison.htm

Annuities & Perpetuities

Annuity

- A series of cash flows in which the **same cash flow** "CF" (or payment) takes place each period for a set number of periods
- An **ordinary annuity** is one in which the first cash flow occurs one period from now (end of the period 1)
- An **annuity due** is an annuity in which the first cash flow occurs immediately (at the beginning of period 1)

Perpetuity

- **A set of equal payments that are paid forever**, with the first cash flow occurring at the end of period 1
- A **growing perpetuity** is a set of payments which grow at a constant rate (g) each period and continue forever, with the first cash flow occurring at the end of period 1

More Definitions

- **Principal** in a loan context is the original amount borrowed

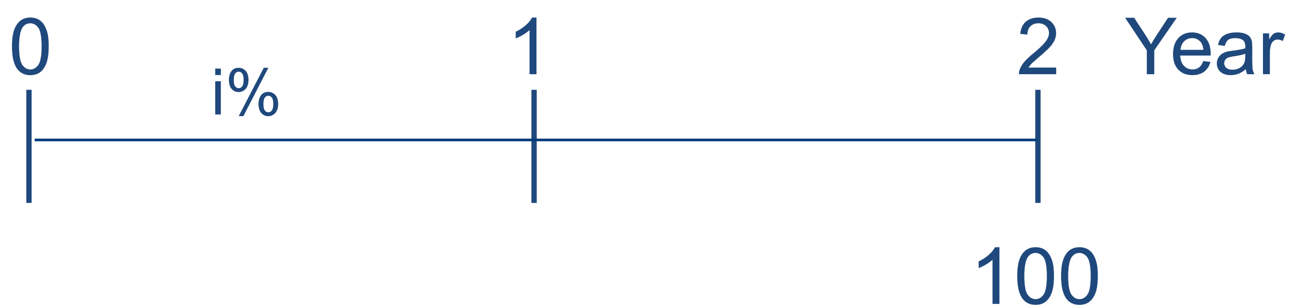
- **Interest** is the compensation for the opportunity cost of funds and the uncertainty of repayment of the amount borrowed. Sometimes, referred to as:
    - Discount rate
    - Cost of capital
    - Opportunity cost of capital
    - Required return

Timelines

General Timeline

Example:

Timeline for a $100 **lump sum** due at the end of Year 2



More timelines

Example 2:

Timeline for an **ordinary annuity** of $100 for 3 years

Example 3:

Timeline for **uneven CFs**: -$50 at t = 0 and $100, $75, and $50 at the end of Years 1 through 3.

A negative sign means it is a cash **outflow**

Back to future values

We can think of **future value** as an amount to which an investment will grow after earning interest.

Simple Interest:
Interest earned only on the original investment

Compound Interest:
Each period, in addition to interest earned on the original investment, interest is also earned on interest previously received (on the original investment)

Savings example: Simple interest

Today you deposit $100 into a fixed deposit account paying <u>5% simple interest</u>. How much should you have in 5 years?

Solution: $100 + 5 years * 100(5%) = $125

Principal

Interest earned at the end of each year: $5

Year 1: (5% of $100 = $5) + $100 = $105
Year 2: (5% of $100 = $5) + $105 = $110
Year 3: (5% of $100 = $5) + $110 = $115
Year 4: (5% of $100 = $5) + $115 = $120
Year 5: (5% of $100 = $5) + $120 = $125

Savings Example: Compound Interest

Suppose you deposit your $100 into a Savings Deposit offering 5% compound interest, i.e., interest is earned at 5% at the end of each year, based on the beginning of each year's balance:

Interest Earned Per Year = Prior Year Balance x 5%

Year 1: (5% of $100.00 = $5.00) + $100.00 = $105.00

Year 2: (5% of $105.00 = $5.25) + $105.00 = $110.25

Year 3: (5% of $110.25 = $5.51) + $110.25 = $115.76

Year 4: (5% of $115.76 = $5.79) + $115.76 = $121.55

Year 5: (5% of $121.55 = $6.08) + $121.55 = $127.63

Interest earned at the end of each year increases, as the beginning balance each period increases

Future Values: General Formula

$$\boxed{\mathbf{FV = PV(1 + r)^t = PV\,(1+i)^n}}$$

- FV = future value
- PV = present value
- r = i = period interest rate, expressed as a decimal (e.g. 5% =.05)
- t = n = number of periods

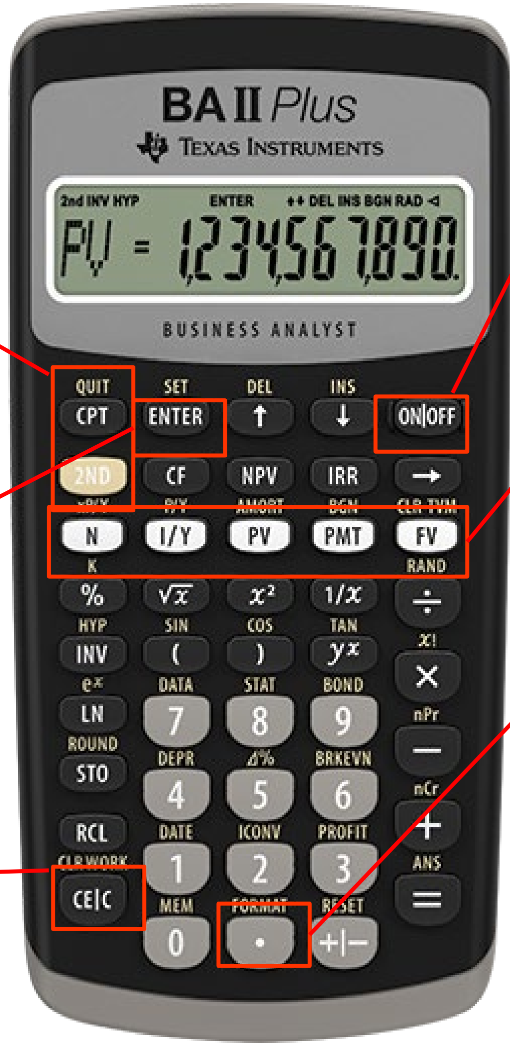
Future value interest factor = **FVIF = $(1 + r)^t$**

- Finding FVs (moving to the right on a timeline) is called **compounding**

Using the TI BA II Plus

How to setup your calculator: https://www.youtube.com/watch?v=maD6HuaA-2k

1. The On/Off button!
2. Return to Home screen
   - "2nd" "Cpt"
3. TMV factor inputs
4. Default P/Y to 1
   - "2nd" "I/Y"
   - "1" "Enter"
5. Default the Display
   - "2nd" "."
   - "# of decimals" "Enter"
6. Clearing the Memory
   - "2nd" "CE/C"
   - "2nd" "FV"



Financial Calculator Solution

Financial calculators solve this equation:

$$FV = PV(1+i)^n$$

4 Variables

If 3 are known, the calculator will solve for the 4th

- **Excel**: For an Excel Solution Example, please see here

Compounding example with calculator

Suppose one of your ancestors deposited $10 at 5.5% interest 200 years ago. How much would the investment be worth today?

$FV = 10(1.055)^{200} = \mathbf{447{,}189.84}$

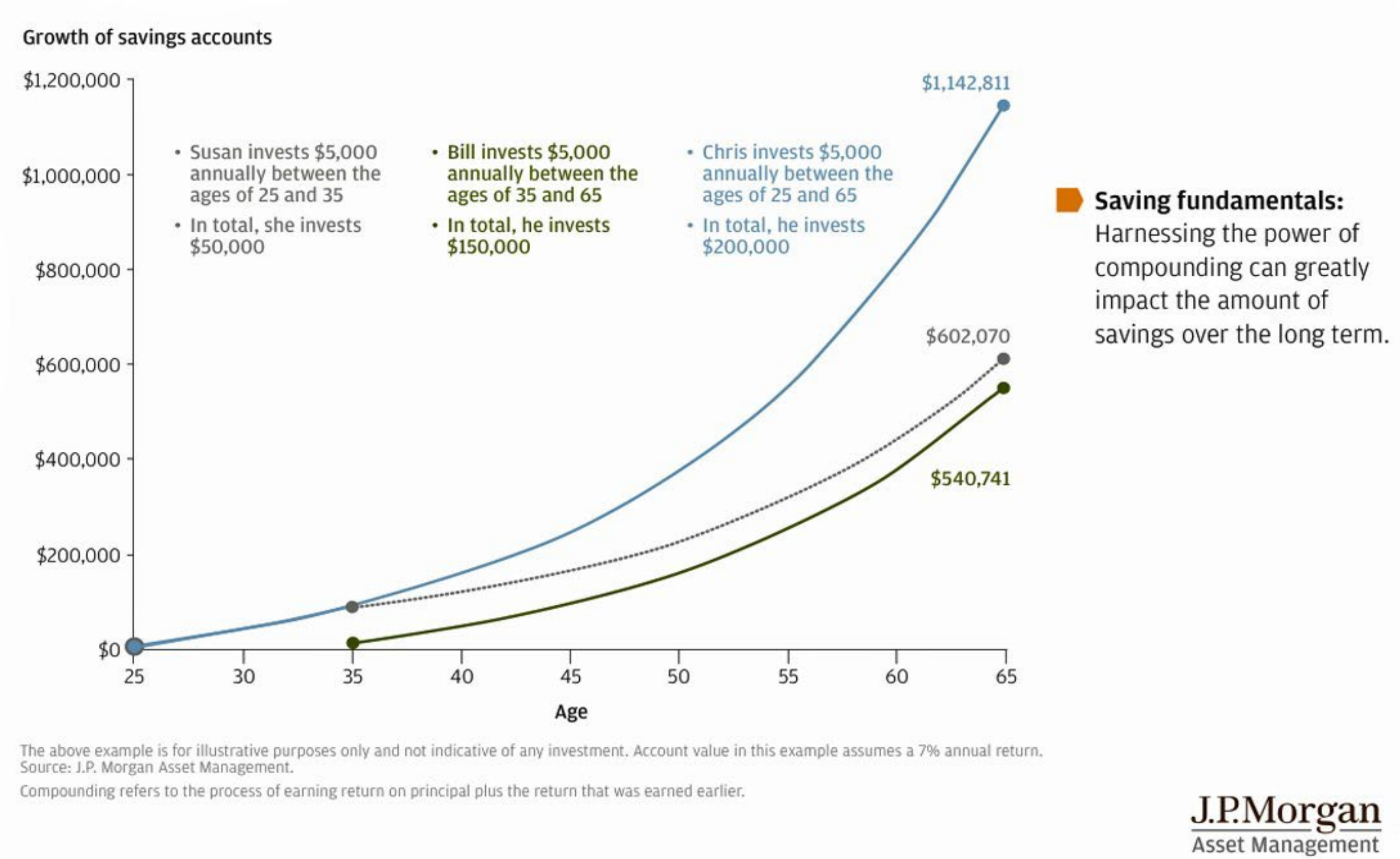
INPUTS	200	5.5	-10		
	N	I/YR	PV	PMT	FV
OUTPUT					447,189.84

Tutorial: https://www.youtube.com/watch?v=qSc2zM0LZNQ

What is the effect of compounding?

- Simple interest = 10 + 200(10)(.055) = **120.00**
- Compounding added $**447,069.84** to the value of the investment!

Example: Effects of Compounding



- Intuitively, it makes sense that Chris would end up with the most money. But the amount he has saved is astronomically larger than the amounts saved by Susan or Bill.
- Interestingly, Susan, who saved for just 10 years, has more wealth than Bill, who saved for 30 years.

Example: Compound Growth in General

Suppose your company expects to increase unit sales by 15% per year for the next 5 years. If you currently sell 3 million cars in one year, how many cars do you expect to sell in 5 years?

- FV = $3{,}000{,}000(1.15)^5$ = **6,034,072**

	N	I/YR	PV	PMT	FV
INPUTS	5	15	3,000,000		
OUTPUT					-6,034,072

	A	B	C	D	E
1	**FUTURE VALUE OF A SINGLE PAYMENT AT DIFFERENT INTEREST RATES** **How $100 at time 0 grows at 0%, 6%, 12%**				
2	Initial deposit	100			
3	Interest rate	0%	6%	12%	
4					
5	**Year**	**FV at 0%**	**FV at 6%**	**FV at 12%**	
6	0	100.00	100.00	100.00	<-- =$B$2*(1+D$3)^$A6
7	1	100.00	106.00	112.00	<-- =$B$2*(1+D$3)^$A7
8	2	100.00	112.36	125.44	
9	3	100.00	119.10	140.49	
10	4	100.00	126.25	157.35	
11	5	100.00	133.82	176.23	
12	6	100.00	141.85	197.38	
13	7	100.00	150.36	221.07	
14	8	100.00	159.38	247.60	
15	9	100.00	168.95	277.31	
16	10	100.00	179.08	310.58	
17	11	100.00	189.83	347.85	
18	12	100.00	201.22	389.60	
19	13	100.00	213.29	436.35	
20	14	100.00	226.09	488.71	
21	15	100.00	239.66	547.36	
22	16	100.00	254.04	613.04	
23	17	100.00	269.28	686.60	
24	18	100.00	285.43	769.00	
25	19	100.00	302.56	861.28	
26	20	100.00	320.71	964.63	
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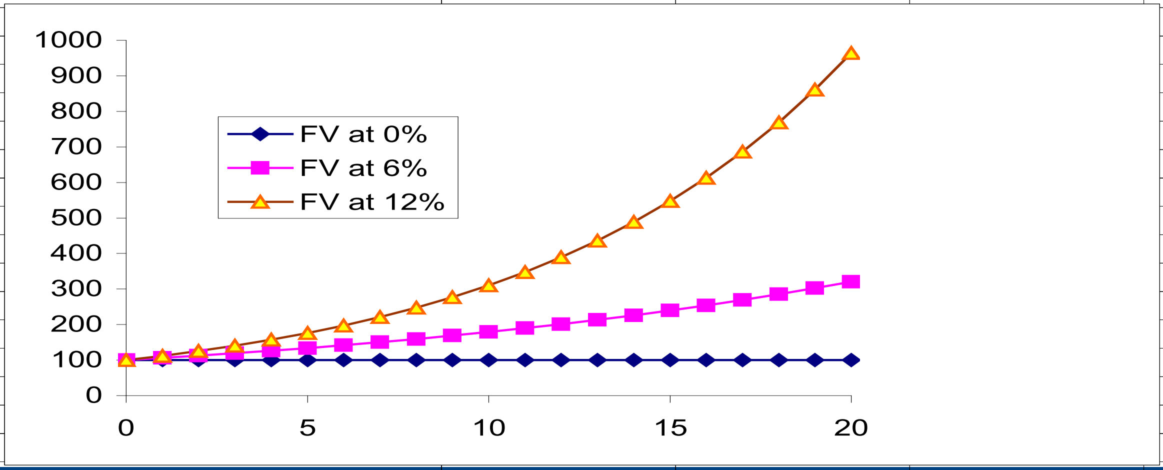


Summary – Part 1

What is the difference between simple and compound interest?

Suppose you have \$500 to invest and you believe that you can earn 8% per year over the next 15 years. How much would you have at the end of 15 years:

- Using simple interest
  $\$500 + 15(\$500)(.08) = \$1,100.00$
- Using compound interest
  $\$500(1.08)^{15} = \$500(3.172169) = \$1,586.08$

	N	I/YR	PV	PMT	FV
INPUTS	15	8	500		
OUTPUT					-1,586.08

Present Values

How much do I have to invest today to have some amount in the future?

We re-arrange $FV = PV(1 + r)^n$

$\Rightarrow$ to solve for **$PV = FV / (1 + r)^n$**

- When we talk about **discounting**, we mean finding the present value of some future amount
  - Present value (where we are discounting) is inversely related to future value (where we are compounding)
- When we simply talk about the "value" of something, we are talking about the present value. If we want future value, we specifically indicate that we want the future value.

Present Value

$$PV = \frac{FV}{(1+i)^n} = FV\left(\frac{1}{1+i}\right)^n$$

PV Factor

Suppose you have $100 today. How much did you save three years ago if interest rates were 10%?

$$PV = \$100\left(\frac{1}{1.10}\right)^3$$

$$= \$100\ (0.7513) = \$75.13$$

PV Factor for n = 3 and i = 10%

Present Value – Example 1

You want to begin saving for your daughter's university education and you estimate that she will need $150,000 in 17 years. If you feel confident that you can earn 8% per year, how much do you need to invest today?

FV = $150,000, r = 8%, n = 17

PV = 150,000 / $(1.08)^{17}$ = **40,540.34**

INPUTS	17	8			150,000
	N	I/YR	PV	PMT	FV
OUTPUT			-40,540.34		

Present Values – Example 2

Your parents set up a trust fund for you 10 years ago that is now worth $19,671.51. If the fund earned 7% per year, how much did your parents invest?

We know the value today is $19,671.51 and we want to know what it was worth 10 years ago if the annual compound interest it has been receiving is 7%, so

PV = 19,671.51 / $(1.07)^{10}$ = **10,000**

INPUTS	10	7			19,671.51
	N	I/YR	PV	PMT	FV
OUTPUT			-10,000		

Summary – Part 2

What is the relationship between present value and future value?

Suppose you need $15,000 in 3 years. If you can earn 6% annually, how much do you need to invest today?

$$PV = \$15{,}000 / (1.06)^3 = \$15{,}000(.8396) = \$12{,}594.29$$

INPUTS	3	6			15000
	N	I/YR	PV	PMT	FV
OUTPUT			-12,594.29		

Summary – Part 2 (cont.)

If you could invest the money at 8%, would you have to invest more or less than if you invested it at 6%? By how much?

$PV = \$15,000 / (1.08)^3 = \$15,000(0.7938) = \$11,907.48$

INPUTS	3	8			15000
	N	I/YR	PV	PMT	FV
OUTPUT			-11,907.48		

Difference = \$12,594.29 - \$11,907.48 = \$686.81

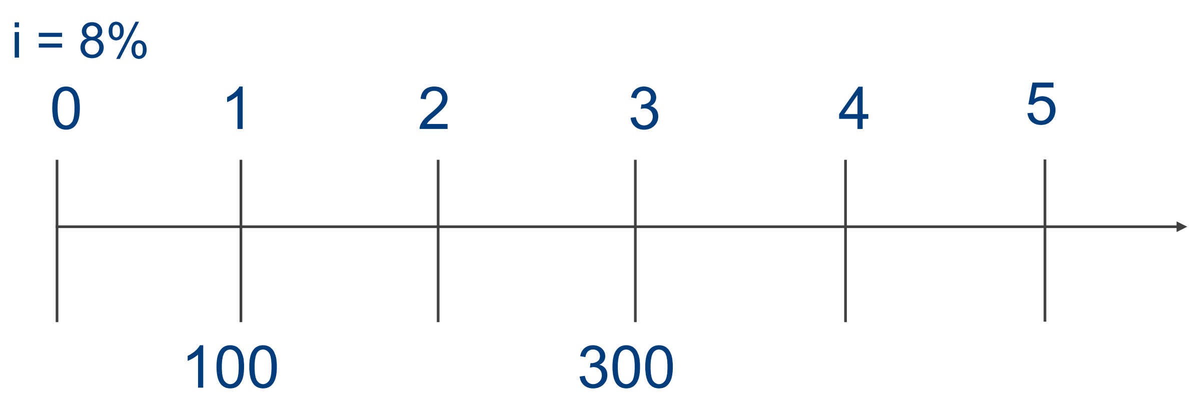
Multiple Cash Flows

Learning objectives

- Be able to draw a timeline for cash flows with different amount occurring at different times
- Be able to calculate the present value and future value of multiple cash flows

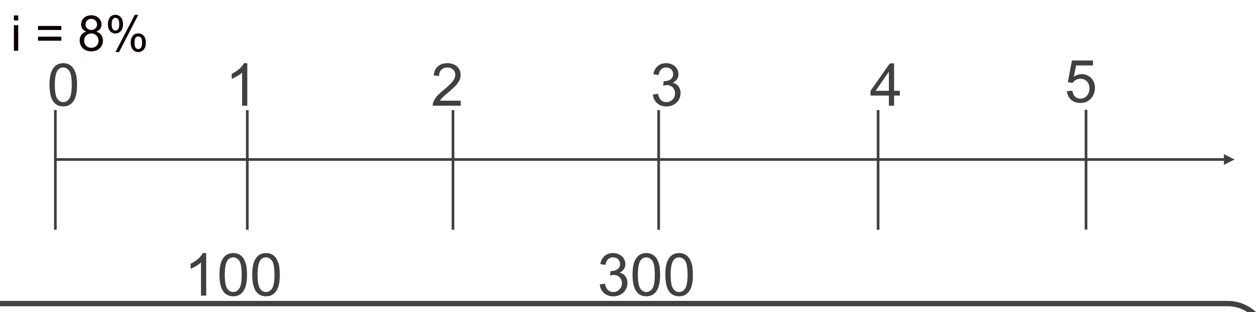
Multiple cash flows example 1

Suppose you plan to deposit $100 into an account in one year's time and $300 into the account in three years from now. How much will be in the account <u>in five years</u> if the interest rate is 8%?



Multiple cash flows example 2 (cont.)

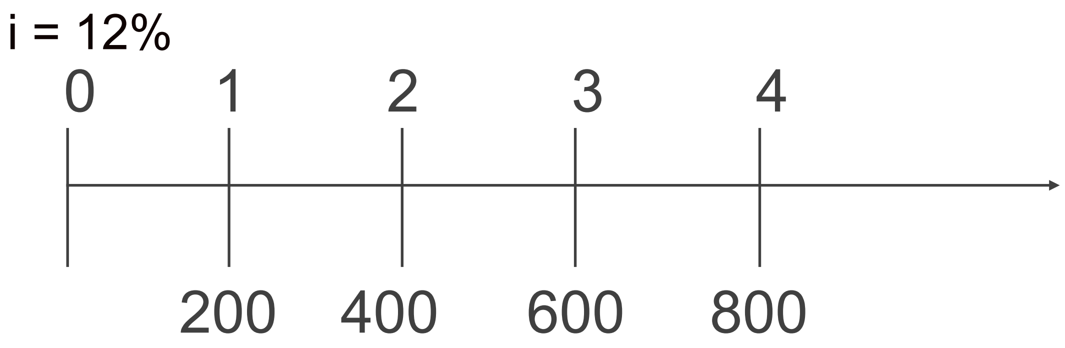
FV = $\$100(1.08)^4 + \$300(1.08)^2 = \$136.05 + \$349.92 =$ **$\$485.97$**



INPUTS	4	8	-100		
	N	I/YR	PV	PMT	FV
OUTPUT					136.05

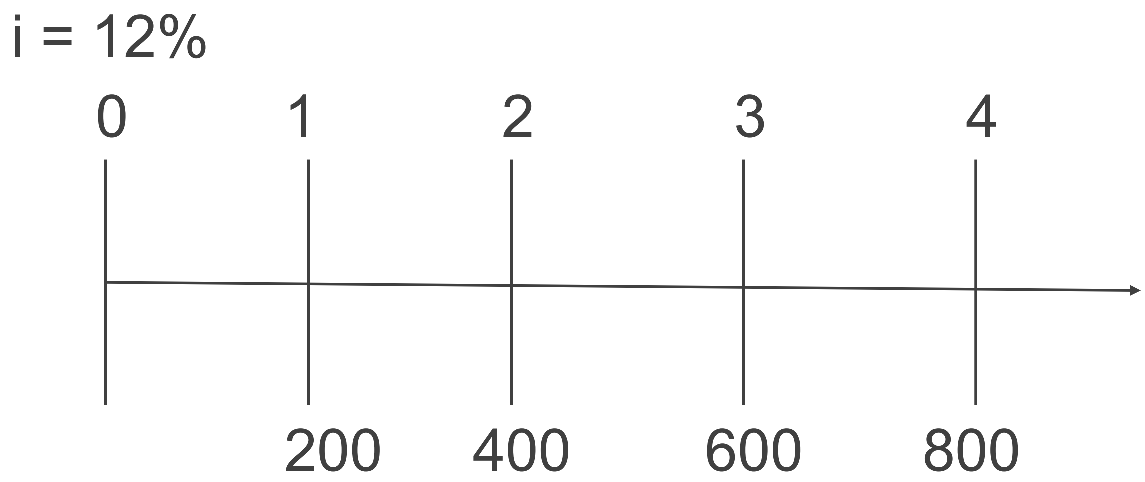
INPUTS	2	8	-300		
	N	I/YR	PV	PMT	FV
OUTPUT					349.92

Multiple cash flows example 2



What is the PV of the entire cash flow?

Example: Multiple Cash Flows Timeline



Year 1 CF: $200 / (1.12)^1 =$ 178.57

Year 2 CF: $400 / (1.12)^2 =$ 318.88

Year 3 CF: $600 / (1.12)^3 =$ 427.07

Year 4 CF: $800 / (1.12)^4 =$ 508.41

1,432.93 = PV

Summary

Be careful with **“when”** the cash flows happen

- Highly recommend drawing a timeline
- Calculate the present or future value of the cash flow for each period
- Then sum up the present or future value from each period to get the total

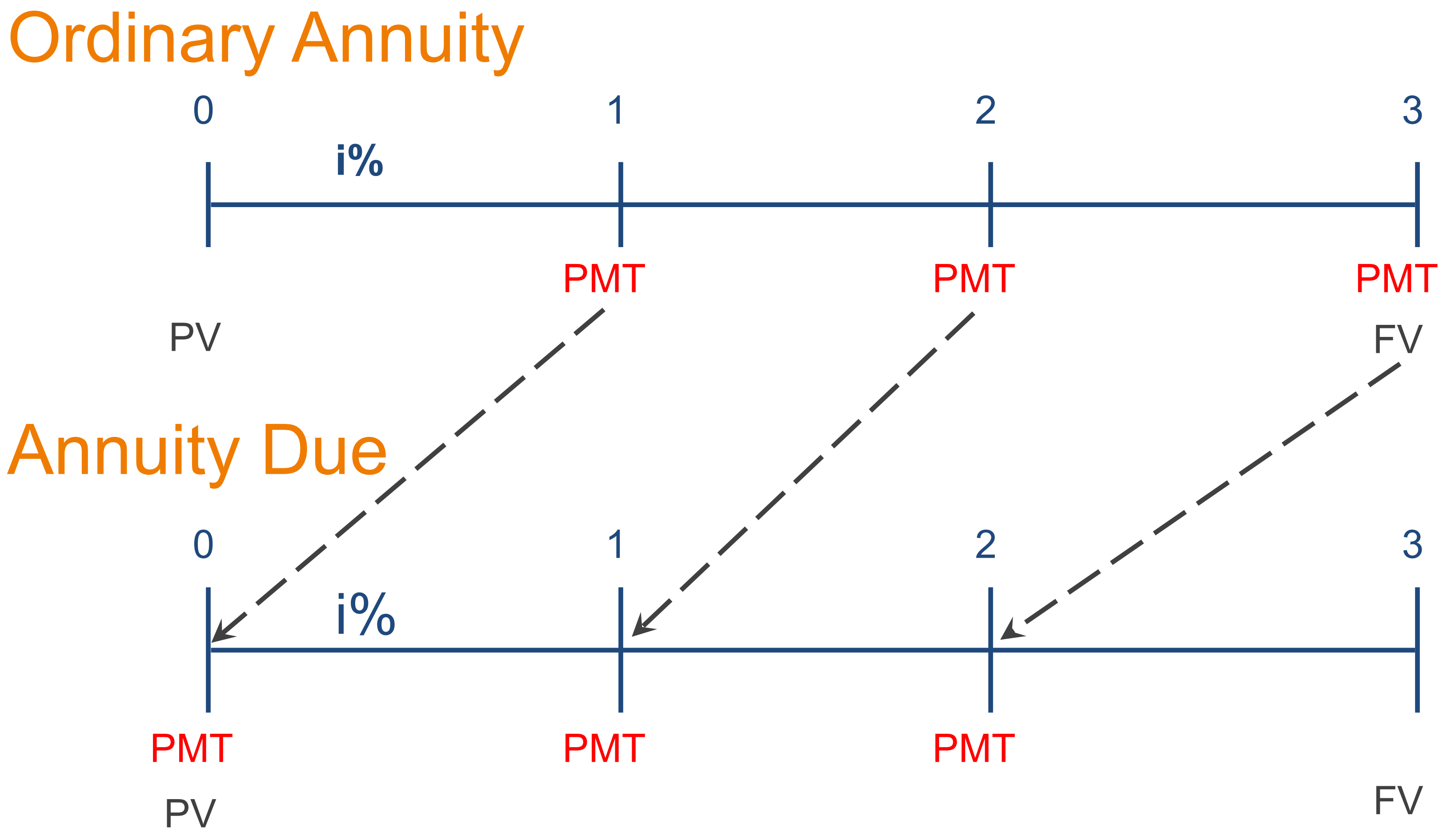
Annuity & Perpetuity

Learning objectives

Know how to calculate present value and future value of:

- Annuity & Annuity due
- Perpetuity
- Growing annuity

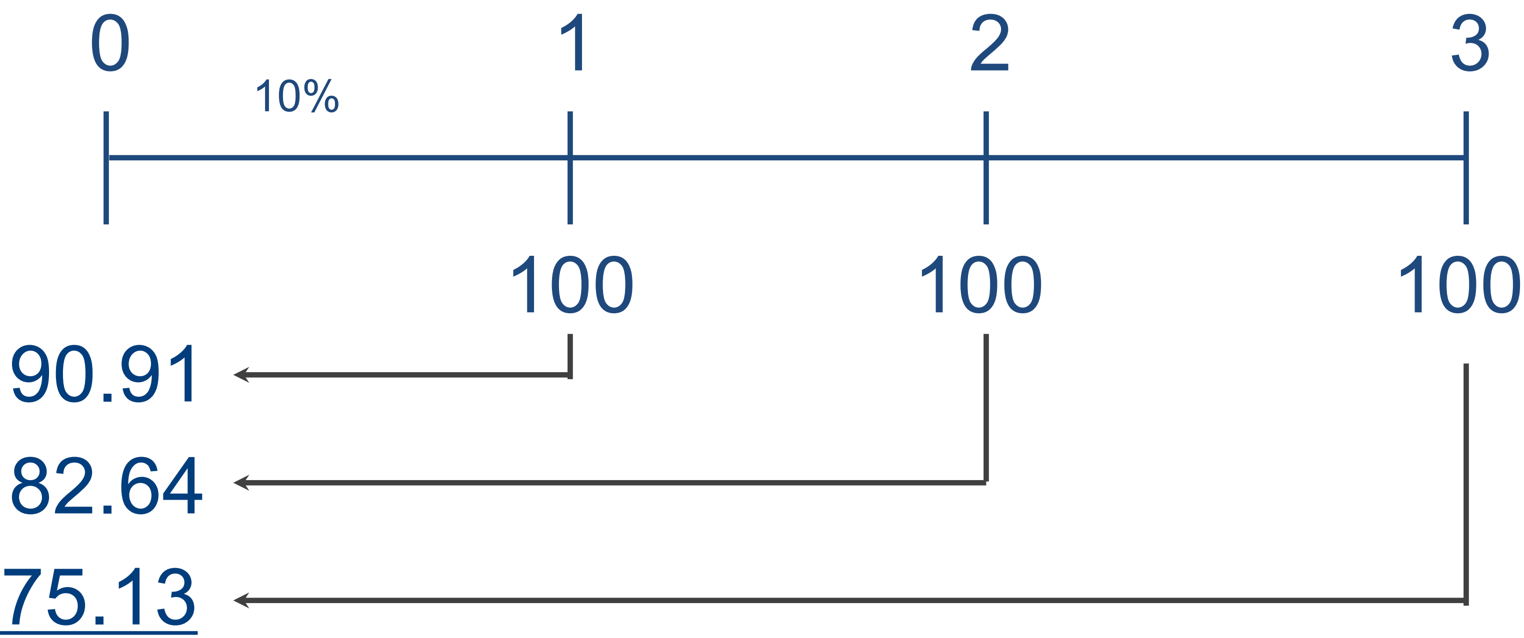
Annuity & Annuity Due



The difference between the two is (1+i)

Example: PV of an Ordinary Annuity

What’s the PV of a 3-year ordinary annuity of $100 at 10%?



248.69 = PV

Example: PV of an Ordinary Annuity

Using a Financial Calculator:

	N	I/YR	PV	PMT	FV
INPUTS	3	10		100	0
OUTPUT			-248.69		

Note: Have payments but no lump sum FV, so enter nothing or 0 for future value

PV of an Ordinary Annuity

$$\text{PV of } \underline{\text{Annuity}} = \text{PMT} * \left[\frac{1 - \text{PV Factor}}{r}\right] \qquad \frac{1}{(1+r)^n}$$

Regular **PV Factor**

$$PV\ of\ Annuity = PMT * \frac{1}{r} * \left(1 - \frac{1}{(1+r)^n}\right)$$

PV <u>Annuity Due</u> = PV Annuity * (1+r)

PV of Ordinary Annuity

$$PV = \boxed{PMT} * (\boxed{\frac{1}{(1+r)^1} + \frac{1}{(1+r)^2} + \ldots} + \frac{1}{(1+r)^n})$$

$$(1+r) * PV = (1+r) * PMT * (\frac{1}{(1+r)^1} + \frac{1}{(1+r)^2} + \ldots + \frac{1}{(1+r)^n})$$

$$(1+r) * PV = \boxed{PMT} * (1 + \boxed{\frac{1}{(1+r)^1} + \ldots + \frac{1}{(1+r)^{n-1}}})$$

$$(1+r) * PV = PMT + \boxed{\boxed{PV} - \frac{PMT}{(1+r)^n}}$$

$$PV * r = PMT - \frac{PMT}{(1+r)^n}$$

$$PV = PMT * \frac{1}{r} * (1 - \frac{1}{(1+r)^n})$$

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FV of an Ordinary Annuity

$$\boxed{\text{FV of } \underline{\text{Annuity}} = \text{PV of Annuity} * (1+r)^n}$$

$$\mathrm{FV} = \mathrm{PMT} * \left[\frac{1}{\mathrm{r}} * \left(1 - \frac{1}{(1+\mathrm{r})^{\mathrm{n}}}\right)\right] * (1+\mathrm{r})^{\mathrm{n}}$$

$$\mathrm{FV} = \boxed{PMT \times \left[\frac{1}{r} * \left[\left((1+r)^n - 1\right)\right]\right]}$$

$$\boxed{\text{FV } \underline{\text{Annuity Due}} = \text{FV Annuity} * (1+r)}$$

Example: FV of an Ordinary Annuity

DEPOSITS AT END OF YEAR

		End of year 1	End of year 2	End of year 3	End of year 4	End of year 5	End of year 6	End of year 7	End of year 8	End of year 9	**End of year 10**
0	6%	1	2	3	4	5	6	7	8	9	10
		$100	$100	$100	$100	$100	$100	$100	$100	$100	$100

	168.95	<-- =100*(1.06)^9
	159.38	<-- =100*(1.06)^8
	150.36	<-- =100*(1.06)^7
	141.85	<-- =100*(1.06)^6
	133.82	<-- =100*(1.06)^5
	126.25	<-- =100*(1.06)^4
	119.10	<-- =100*(1.06)^3
	112.36	<-- =100*(1.06)^2
	106.00	<-- =100*(1.06)^1
	100.00	<-- =100*(1.06)^0
Total	1318.08	<-- Sum of the above

$$FV = PMT \times \left[\frac{1}{r} * \left[\left((1+r)^n - 1 \right) \right] \right]$$

$$FV = 100 \times \frac{1}{.06} \left[(1.06)^{10} - 1 \right]$$

$$FV = 100 \times 13.1808 = 1{,}318.08$$

Example: FV of an Annuity Due

DEPOSITS AT BEGINNING OF YEAR

Beginning of year 1	Beginning of year 2	Beginning of year 3	Beginning of year 4	Beginning of year 5	Beginning of year 6	Beginning of year 7	Beginning of year 8	Beginning of year 9	Beginning of year 10	End of year 10
0	1	2	3	4	5	6	7	8	9	10
$100	$100	$100	$100	$100	$100	$100	$100	$100	$100	

6%

	179.08	<-- =100*(1.06)^10
	168.95	<-- =100*(1.06)^9
	159.38	<-- =100*(1.06)^8
	150.36	<-- =100*(1.06)^7
	141.85	<-- =100*(1.06)^6
	133.82	<-- =100*(1.06)^5
	126.25	<-- =100*(1.06)^4
	119.10	<-- =100*(1.06)^3
	112.36	<-- =100*(1.06)^2
	106.00	<-- =100*(1.06)^1
Total	1397.16	<-- Sum of the above

$$\text{FV of Annuity Due} = \text{FV of Annuity} * (1+r)$$

$$FV = 100 \times \frac{1}{.06}\left[(1.06)^{10} - 1\right] \times (1.06)$$

$$FV = 100 \times 13.1808 \times (1.06) = 100 \times 13.9716 = 1{,}397.16$$

Annuity – Lottery Example

Suppose you win a $10 million lottery prize. The money is paid in equal annual end-of-year installments of $333,333.33 over 30 years. If the appropriate discount rate is 5%, how much is the sweepstakes actually worth today?

$$PV = \$333{,}333.33[1 - 1/1.05^{30}] / .05 = \$5{,}124{,}150.29$$

	N	I/YR	PV	PMT	FV
INPUTS	30	5		333,333.33	
OUTPUT			-5,124,150.29		

Perpetuities

A **perpetuity** is a set of equal payments that are paid each period forever, with the first payment at the end of the first period. If the periodic payment is $C, then the present value of the perpetuity is:

$$\boxed{PV = \frac{C}{r}}$$

A **growing perpetuity** is a set of payments which grow at a **constant rate** (g) forever, with the 1st payment at the end of the 1st period. For example:

$$C_2 = C_1 \times (1+g) \quad \text{and} \quad C_3 = C_2 \times (1+g) = C_1 \times (1+g)^2$$

If the first payment is $\$C_1$, then the present value of the perpetuity is:

$$\boxed{PV = \frac{C_1}{r-g}}$$

PV of Growing Annuity

A **growing annuity** is a set of payments which grow at a constant rate, g, up to a certain maturity date.

If the first payment is \$C1, then the present value of the growing annuity is:

$$\boxed{PV = C_1 \times \left[\frac{1 - \left(\frac{1+g}{1+r} \right)^t}{r-g} \right]}$$

FV of Growing Annuity

If the first payment is \$C1, then the future value of the growing annuity is:

$$FV = C_1 \times \left[\frac{(1+r)^t - (1+g)^t}{r-g} \right]$$

Summary

- What are annuity, annuity due, perpetuity, and growing perpetuity?
  - The difference between annuity and annuity due
  - The difference between annuity and perpetuity
- Computing the present and future values of annuity, annuity due, perpetuity, growing perpetuity, and growing annuity

APR and EAR

Learning objectives

- Understand the difference between EAR and APR and be able to convert between the two
- Be able to compute period payments, PV, FV, EAR, and APR

For Citi Credit Card or Citibank Ready Credit customer (with minimum Citi Quick Cash amount of S$20,000 and above):					
Tenure (months)	12	24	36	48	60
Nominal Interest Rates (per annum)	4.11%	4.00%	3.99%	4.01%	4.05%
Effective Interest Rates (per annum)	7.50%	7.50%	7.50%	7.50%	7.50%

Effective Annual Rate (EAR)

The **effective annual rate** of interest refers to the **actual rate paid** (or received) after taking into consideration any **compounding** that may occur during the year

- If interest is compounded (or applied) exactly once a year, then the effective annual rate will be equal to the stated rate
- If interest is compounded more than once a year, then the stated rate will be different from the effective rate

If you want to compare two alternative investments with different compounding periods you need to compute the EAR and use that for comparison

Annual Percentage Rate (APR)

This is the annual rate that is quoted by law.

Also called **Nominal Annual Rate, Quoted Rate, Stated Rate**.

APR = Period rate * the number of periods per year

Consequently, to get the period rate, we rearrange the APR equation:

Period rate = APR / number of periods per year

Note that you should **NEVER** divide the **effective annual rate** by the number of periods per year – it will **NOT** give you the period rate

- How to use your calculator do the conversion: https://www.youtube.com/watch?v=OA3j3kpfMUA

Computing APRs

- What is the APR if the monthly rate is 0.5%?

  $\Rightarrow$ 0.5% * (12) = 6%

- What is the APR if the semiannual rate is 4%?

  $\Rightarrow$ 4% * (2) = 8%

- What is the monthly rate if the APR (based on the monthly rate) is 12%?

  $\Rightarrow$ 12% / 12 = 1%

Things to Remember

- You ALWAYS need to make sure that the **interest rate** and the **time period match**.
  - If you are looking at annual periods, you need an annual rate.
  - If you are looking at monthly periods, you need a monthly rate.

- E.g., If you have an APR based on monthly compounding, you have to use monthly periods for lump sums, or adjust the interest rate appropriately if you have payments other than monthly.

EAR General Formula

$$\boxed{EAR = \left[1 + \frac{APR}{m}\right]^{m} - 1}$$

- m = compounding frequency per year
- APR = the quoted or stated rate

Example: How do we find EAR for a nominal rate of 10%, compounded semiannually?

$$EAR = (1 + (0.1 / 2)^{2})-1$$
$$= (1.05)^{2}-1 = 0.1025 = 10.25\%$$

Computing EAR example

Below is the amount earned on a dollar invested at APR = 8% at different compounding intervals:

Compounding Intervals	Final Sum	EAR
1	**$1.0800**	**8.00%**
2	**$1.0816**	**8.16%**
12	**$1.0830**	**8.30%**
365	**$1.0833**	**8.33%**
∞	**$1.0833**	**8.33%**

$e^r = e^{0.08}$

Comparing Savings Accounts

You are looking at two savings accounts:

- Account 1: pays 5.25%, with daily compounding
- Account 2: pays 5.3% with semiannual compounding

Which account should you select?

- Account 1:

  $$EAR = (1 + 0.0525 / 365)^{365} – 1 = 5.39\%$$

- Account 2:

  $$EAR = (1 + 0.053 / 2)^{2} – 1 = 5.37\%$$

Given the above information, which account should you choose and why?

Implied Discount Rate

Sometimes we will want to know what the implied interest rate is for an investment

$$\boxed{FV = PV(1 + i)^n \quad \Rightarrow \quad i = (FV / PV)^{1/n} - 1}$$

Example:

You are looking at an investment that will pay $1200 in 5 years if you invest $1000 today. What is the implied rate of interest?

Using a Calculator – (note that the sign convention matters!)

- N = 5
- PV = -1000 (you pay 1000 today)
- FV = 1200 (you receive 1200 in 5 years)
- Compute I/Y = 3.714%

Number of periods example

An investment costs $15,000 and offers a 7.5% annual return. At the end of the life of the investment, it will provide $21,750. How long must you hold the investment?

Using the Calculator - compute the number of periods

- PV = -15,000
- FV = 21,750
- I/Y = 7.5
- Compute N = 5.14 years

Amortized loan calculation example

Supposed you need a car loan of $100,000 from a bank. The bank charges an APR of 6% for a 5-year loan with monthly payments.

- How much is your monthly car loan payment?
- How much is EAR of this loan?

Answers:

Loan principal = 100,000; $r = 0.06 / 12 = 0.005$

$PV = PMT/r * (1 – (1 / (1+r)^N)$

$100{,}000 = PMT / 0.005 * (1 – (1/(1+0.005)^{60}))$

PMT = **$1,933.28**

EAR = **6.2%**

Summary

- APR vs. EAR
- Saving examples
  - Evaluate different saving accounts
- Loan examples
  - Evaluate loan accounts

Understanding APR vs. EAR is very useful for your personal finance!

Week 3 Additional Materials

Please review these slides on your own.
You are responsible for all the material

Different Types of Loans

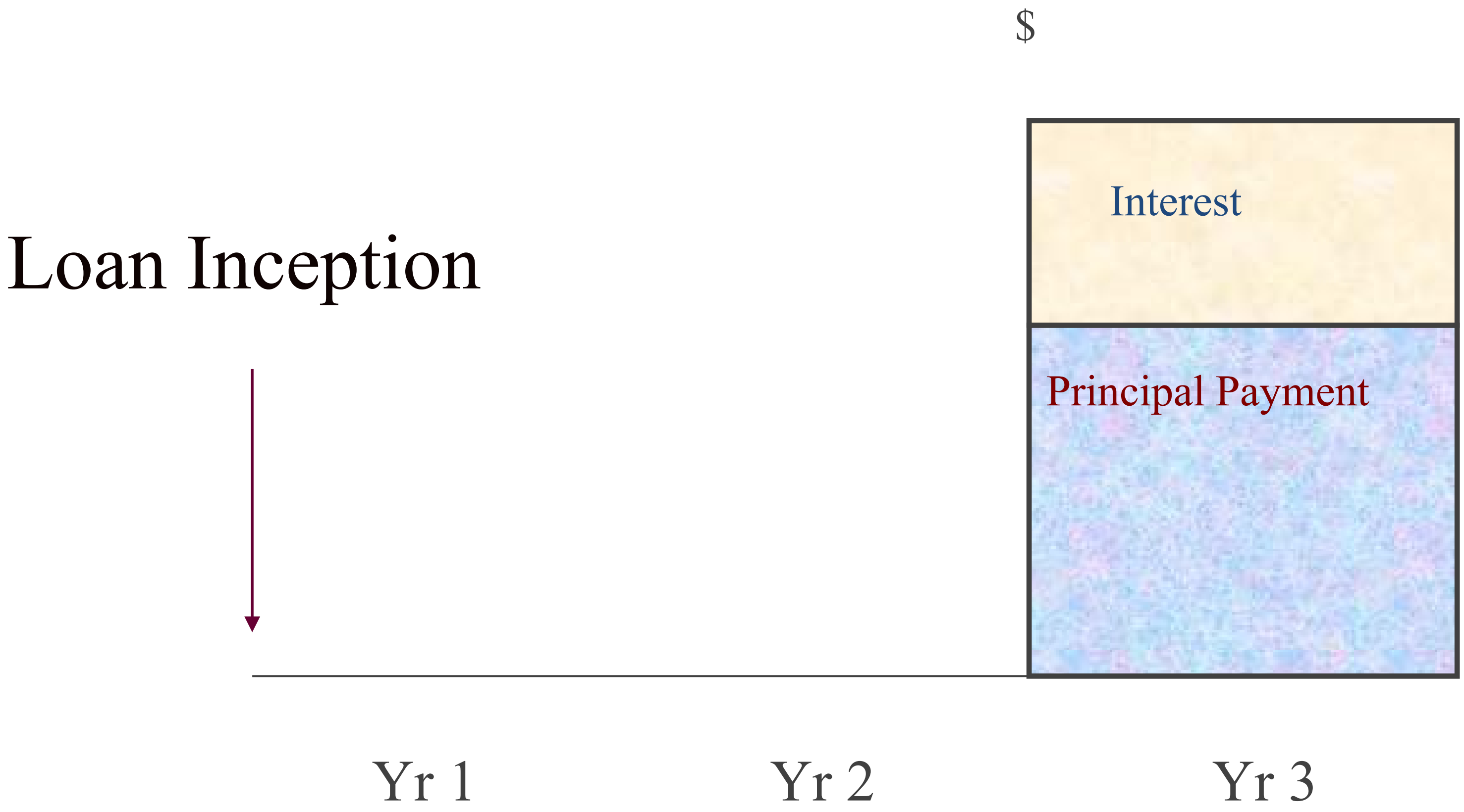
a. **Pure Discount Loans**
   – No interim interest; entire original 'principal' and accumulated interest are paid at maturity; The loan is issued at discount which means initial funds received are less than the total amount paid at maturity

b. **Interest Only Loans**
   – Interest paid throughout the loan period; principal entirely paid at maturity

c. **Loans with Fixed Principal Payments**
   – Interest and fixed amount of principal paid throughout the loan period.

d. **Amortized Loans**
   – Interest and a portion of the principal paid throughout the loan period

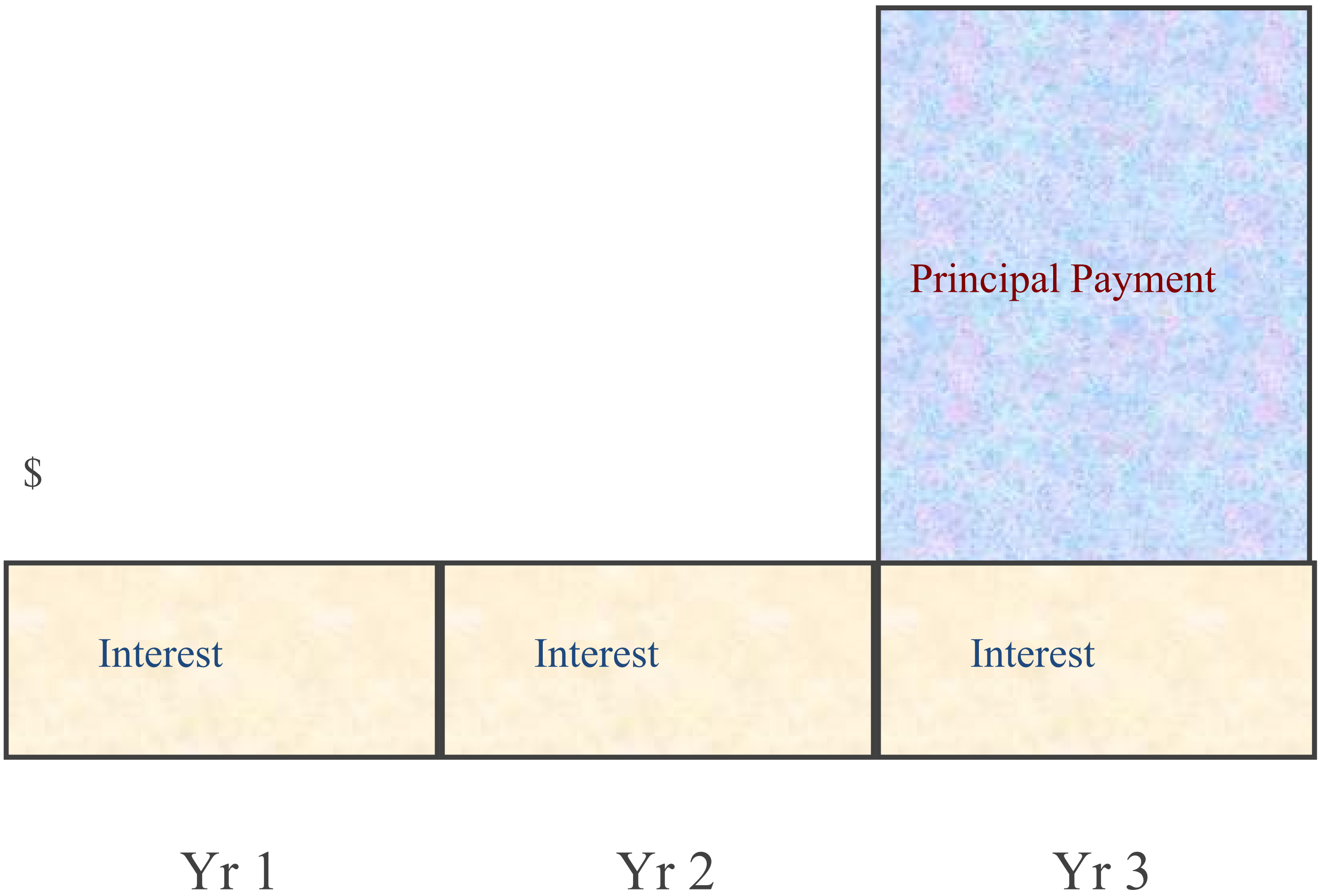
a. Pure Discount Loans



Pure Discount Loans - Example

- Treasury bills are excellent examples of pure discount loans. The total final amount is repaid at some future date, without any periodic interest payments. If a T-bill promises to repay $10,000 in 12 months and the market interest rate is 7 percent, how much will the bill sell for in the market?
    - PV = 10,000 / 1.07 = 9345.79
- Another pure discount loan example: Loan amount to be paid in year 4 is $6,802.44. Applicable annual interest is 8%. No payments are made until the end of year 4. What is the original principal amount of the load to be received today? What is the total interest to be paid in year four?
    - PV = $6,802.44 / $(1.08)^4$ = $5,000
    - Interest Paid is = $6,802.44 - $5,000 = $1,802.44

b. Interest Only Loan



Interest Only Loan - Example

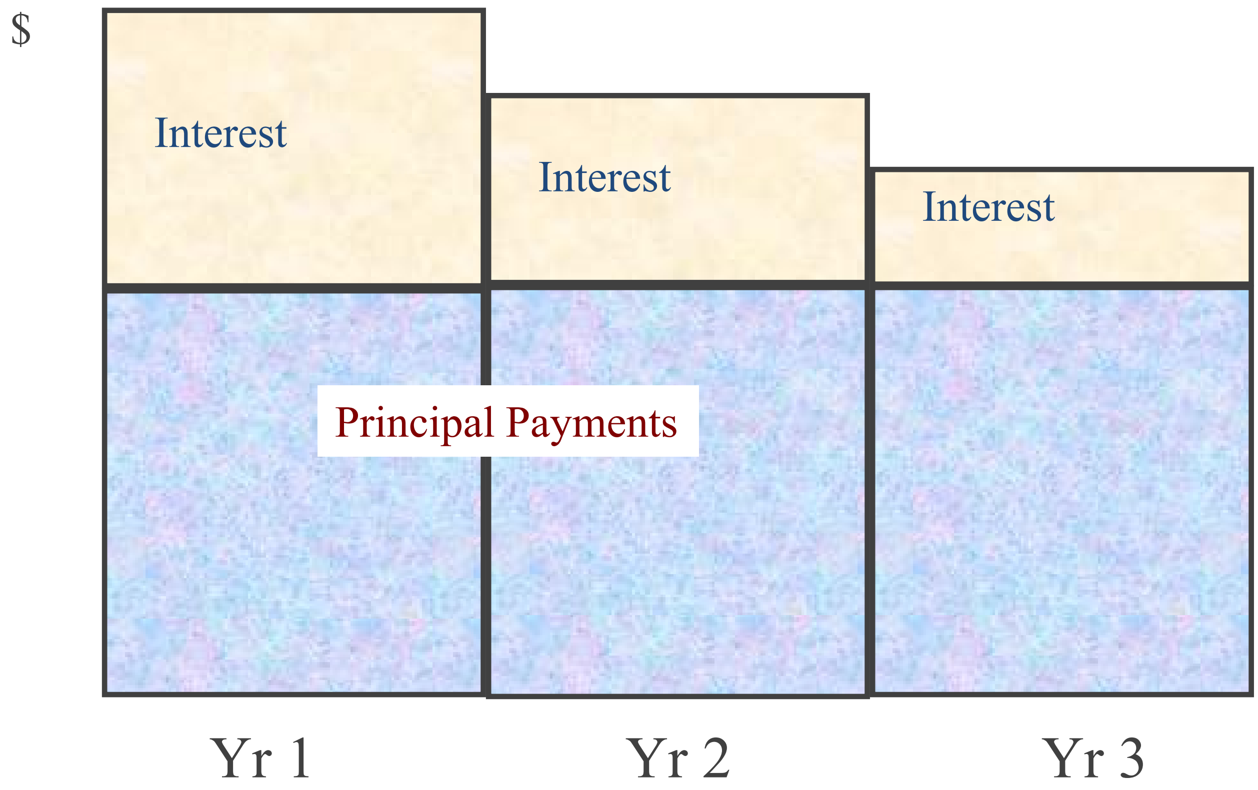
Consider a 4-year, interest only loan with an 8% interest rate. The principal amount is $5,000. Interest is paid annually.

- What would the stream of cash flows be?
  - Years 1 – 3: Interest payments of .08($5,000) = $400 each year
  - Year 4: Interest + principal = $5,400

- This cash flow stream is similar to the cash flows on corporate bonds.
- Total Interest Paid = $400 * 4 years = $1,600
- Total Principal Paid = $5,000

c. Loan with Fixed Principal Payment

Each payment covers the period's interest expense plus a ***fixed*** principal portion



Loan with Fixed Principal Payment - Example

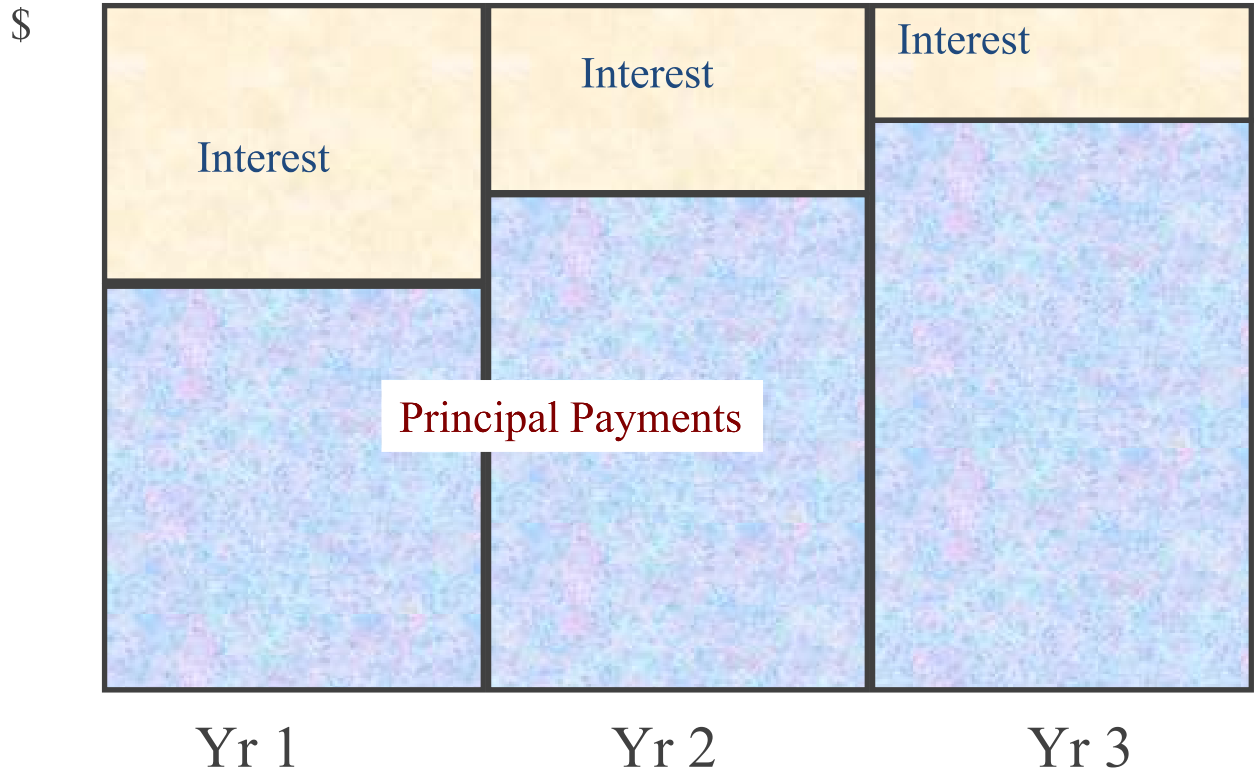
Consider a $5,000, 4-year loan at 8% interest. The loan agreement requires the firm to pay *$1,250 in principal* each year plus interest for that year.

Fixed Principal Payment Amortization Schedule

Year	Beginning Balance	Interest Payment	Principal Payment	Total Payment	Ending Balance
1	5,000	400	1,250	1,650	3,750
2	3,750	300	1,250	1,550	2,500
3	2,500	200	1,250	1,450	1,250
4	1,250	100	1,250	1,350	-
Totals		**1,000**	**5,000**	**6,000**	

d. Amortized Loan

Each ***equal*** payment covers both the period's interest expense and reduces principal



Amortized Loan - Example

Each payment covers the period's interest expense and reduces principal

Example: Consider a 4-year loan with annual payments. The interest rate is 8% and the principal amount is $5000.

- What is the annual payment (via financial calculator)?

  4 <N>

  8 <I/YR>

  5000 <PV>

  cpt <PMT> -1509.60

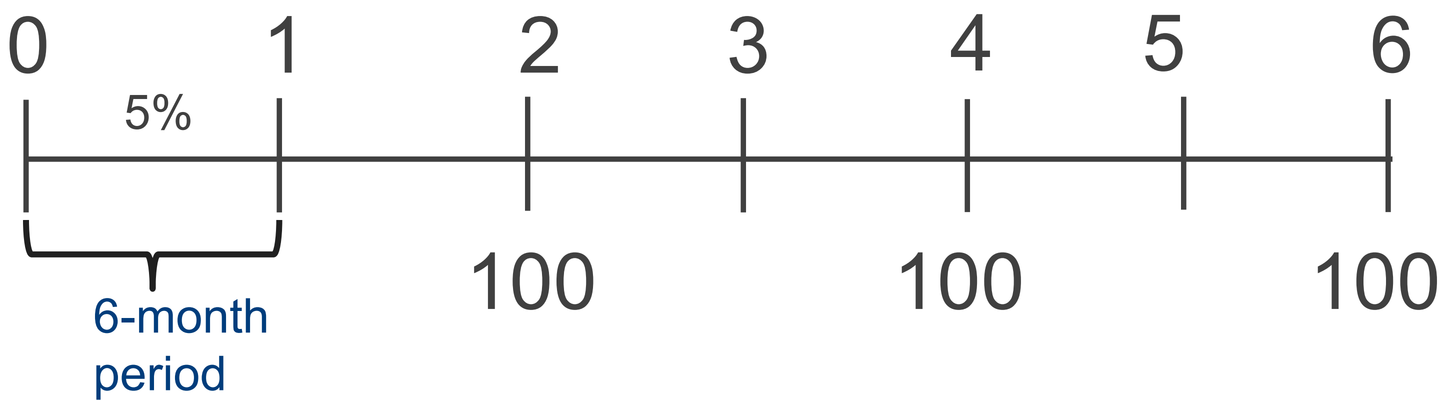
Fixed Payment Amortization Schedule

Year	Beginning Balance	Total Payment	Interest Paid	Principal Paid	Ending Balance
1	5,000.00	1,509.60	400.00	1,109.60	3,890.40
2	3,890.40	1,509.60	311.23	1,198.37	2,692.03
3	2,692.03	1,509.60	215.36	1,294.24	1,397.79
4	1,397.79	1,509.60	111.82	1,397.79	0.00
Totals		6,038.40	1,038.41	5,000.00	

Example – As long as you are **consistent (i.e., match period rate with the same period duration),** many paths lead to correct solution

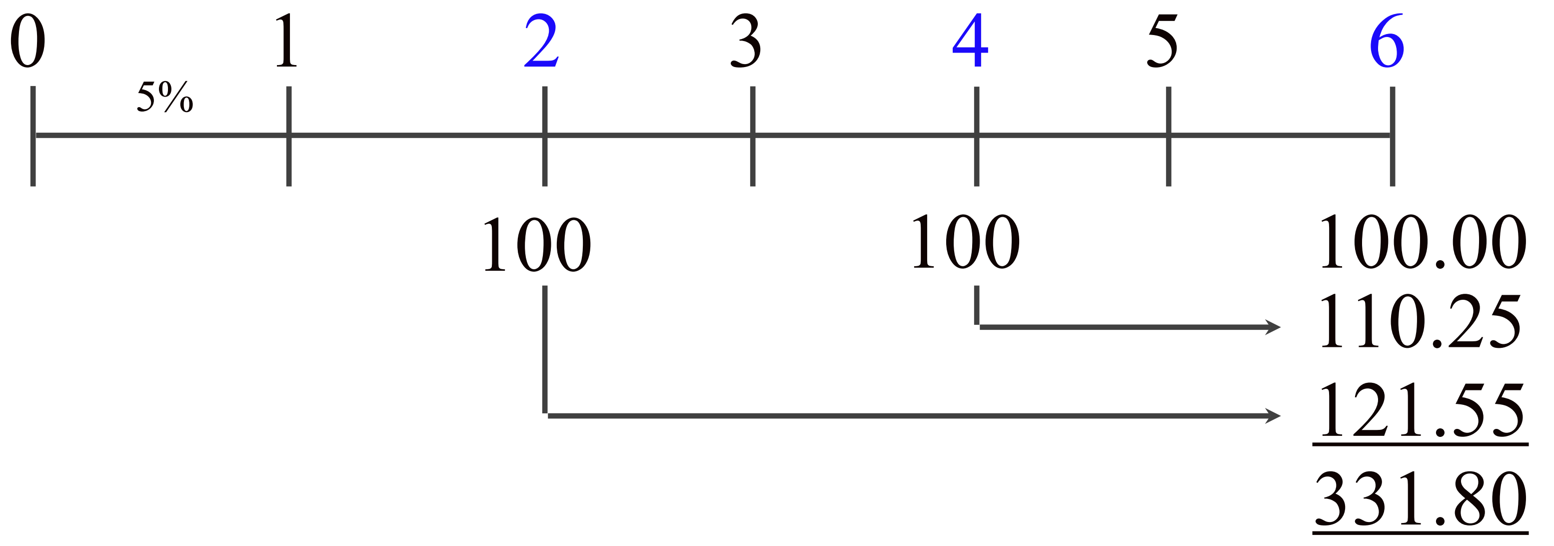
What's the value at the end of Year 3 of the following CF stream if the quoted interest rate is *10%, compounded semiannually?*

APR



- Payments occur annually, <u>**but**</u> ***compounding*** occurs each 6 months.
- Can we use normal annuity valuation techniques?

1st Method: Time is in **units of 6-month periods** - Can Compound Each Cash Flow



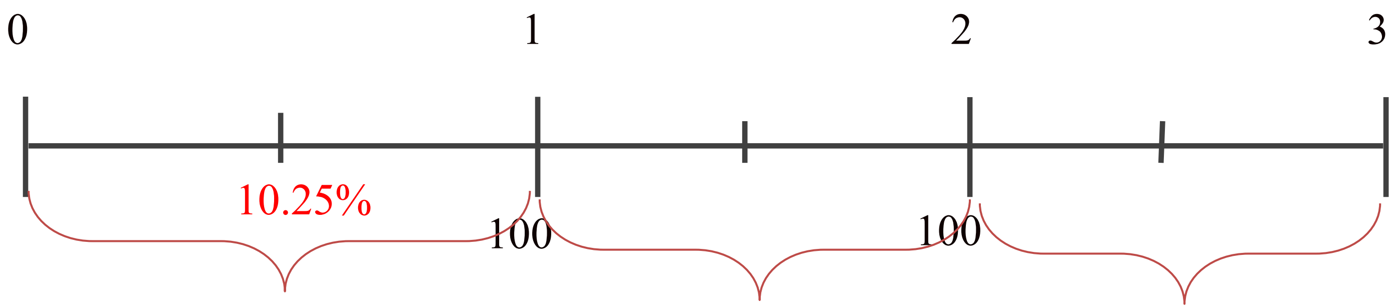
$$FV_6 = \$100(1.05)^4 + \$100(1.05)^2 + \$100$$

$$= \mathbf{\$331.80}$$

2nd Method: Time is in units of 1-year periods – Treat as an Annuity

You must first find the EAR for the quoted rate:

$$\mathbf{EAR} = \left(1 + \frac{0.10}{2}\right)^2 - 1 = 10.25\%.$$



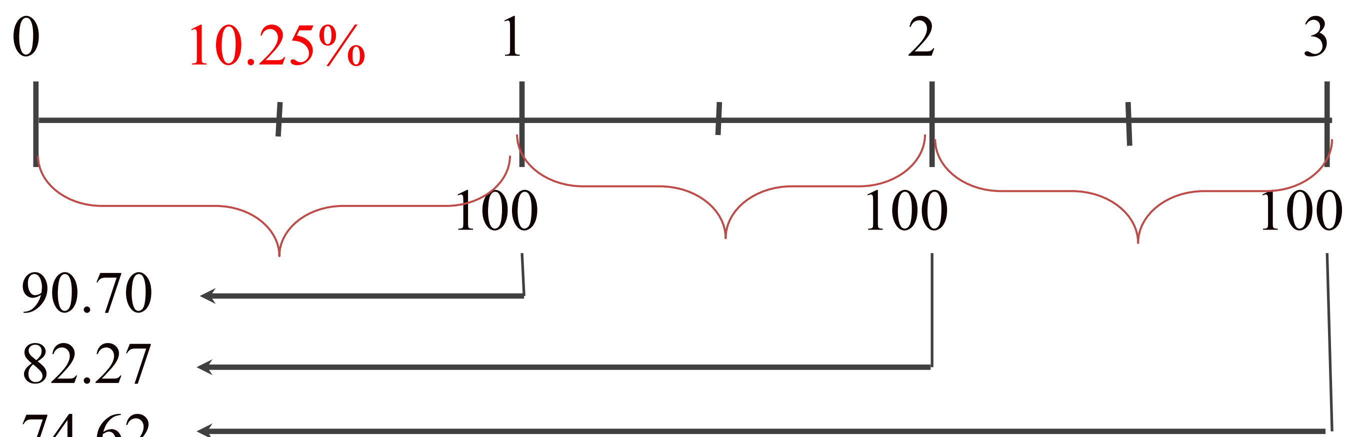
$$\text{FV} = PMT \times \left[\frac{1}{r} * \left[\left(1+r\right)^n - 1\right)\right]\right]$$

FV3 = 100 * [(1 / 0.1025) * ((1.1025)^3-1)] = \$331.80

3rd Method: Time is in units of 1-year periods - Treat as an Annuity

Find the EAR for the quoted rate: $\text{EAR} = \left(1 + \frac{0.10}{2}\right)^2 - 1 = 10.25\%.$

What's the PV of this stream of cash flows?



90.70
82.27
74.62

PV = 247.59

Then $FV_3 = \$247.59 * (1.1025)^3 = \$331.80$