Geometric Shapes

Module 3
Lecture 1

Geometric Shapes

- Geometry has no color and texture
- Points – 0 degree of freedom shape
- Curves – 1 degree of freedom shape
- Surfaces – 2 degree of freedom shape
- Solid objects – 3 degree of freedom shape

- 2 and 3 dimensional spaces
- Time is yet another dimension however different

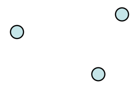
- At the display level, drawn as pixels (picture elements), connected segments (polylines), and shaded polygons (polygon meshes)

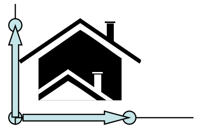
Lecture 1: Learning Objectives

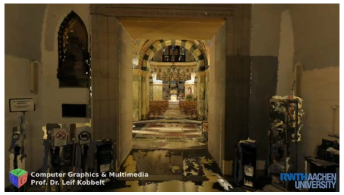
- To understand how points and curves can be used in solving data visualization problems
- To understand curves as objects with 1 degree of freedom
- To understand what mathematical representation is the most efficient for defining and displaying curves
- To understand how different coordinate systems can be used together for deriving mathematical representations of curves

Points

- Individual points
- Reference points
- Point rendering
- Splats (e.g. little oriented disks) rendering
  example (6 MP, 120M) rays https://www.youtube.com/watch?v=X_wyoroo4co
- 2D pixels (picture elements) and 3D voxels (volume elements)
- Defined by Cartesian coordinates (x, y, z), polar (r, α), spherical (r, α, β) or cylindrical (h, r, α) coordinates







Curves

- 2D and 3D
- Polylines – interpolation by connected straight line segments
- Implicit (only 2D)

  $f(x,y)=0$
- Explicit (only 2D)

  $y=f(x)$ *or* $x=f(y)$
- Parametric (2D and 3D)

  $x=x(t)$
  $y=y(t)$
  $t=[t_1, t_2]$

  $x=x(t)$
  $y=y(t)$
  $z=z(t)$
  $t=[t_1, t_2]$

2D Curves: Study by Example

- Straight Line (Segment, Ray)
- Circle (Arc)
- Ellipse (Arc)

Straight Line.
Implicit Representation

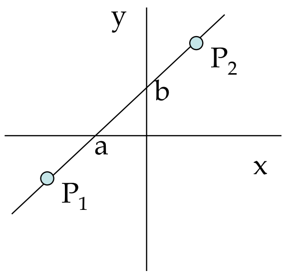
- Straight line

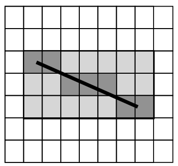
$$Ax+By+C=0$$

$$\frac{y-y_1}{x-x_1}-\frac{y-y_2}{x-x_2}=0$$

- Segment: $x\in[x_1,x_2], y\in[y_1,y_2]$
- Straight line: $x,y\in(-\infty,\infty)$
- Ray: $x\in[x_1,\infty),\quad y\in[y_1,\infty)$

- Drawing is done by sampling points (pixels) within the x and y domains. It is slow since most of the points within the domain do not belong to the segment.





Straight Line.
Implicit Representation

- Straight line

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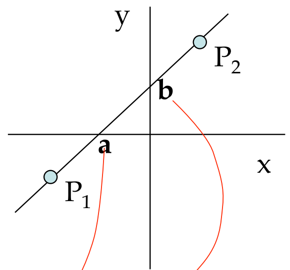


- Equation in intercepts

$$\frac{x}{a}+\frac{y}{b}=1\Rightarrow\frac{x}{a}+\frac{y}{b}-1=0$$

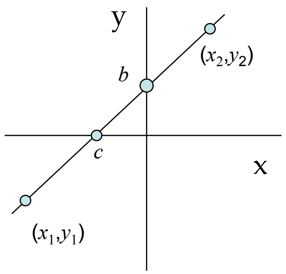
Signed coordinates of points **a** and **b**

Straight Line.
Explicit Representation

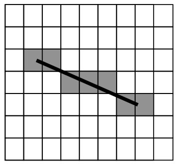
- Straight line

  $y = ax + b$ or $x = dy + c$

  $$a = \frac{y - y_1}{x - x_1} = \frac{y - y_2}{x - x_2} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_1 - y_2}{x_1 - x_2}$$
- Segment: $x \in [x_1, x_2], y \in [y_1, y_2]$
- Straight line: $x, y \in (-\infty, \infty)$
- Ray: $x \in [x_1, \infty)$, $y \in [y_1, \infty)$



- Drawing is done by incrementing x or y and obtaining y and x, respectively. Fast. Integer version used for drawing segments in all computers.
- Axes dependency: special cases for drawing vertical and horizontal lines $x=c$, $y=b$



Straight Line.
Parametric Representation

- Parametric definition of a straight line segment

  $P = P_1 + \tau(P_2 - P_1)$

  $x = x_1 + \tau(x_2 - x_1) = x_1(1-\tau) + \tau x_2$

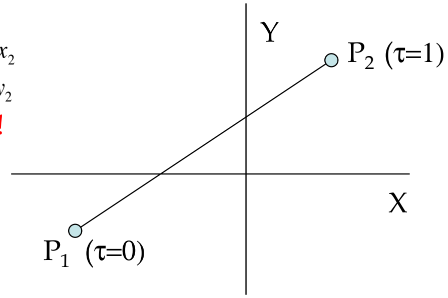
  $y = y_1 + \tau(y_2 - y_1) = y_1(1-\tau) + \tau y_2$

  $\tau = [0,1]$ One parameter !
- Straight line

  $\tau = (-\infty, \infty)$
- Straight line ray, e.g.,

  $\tau = [0, \infty)$

  $\tau = (-\infty, 1]$



- Drawing is done by incrementing parameter τ and obtaining x and y - axes independent. Fast.

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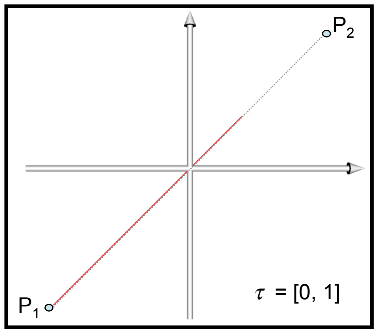
  $\tau = [0,1]$

The animation illustrates drawing of the segment by computing coordinates of its points for every value of the parameter being incremented



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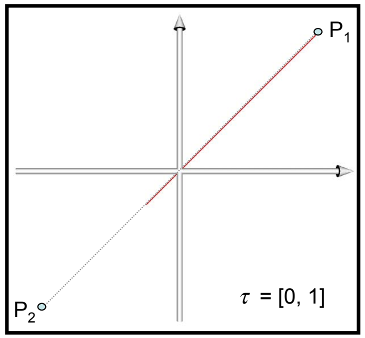

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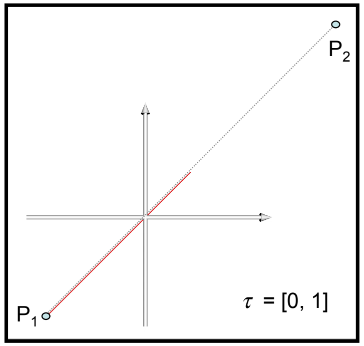

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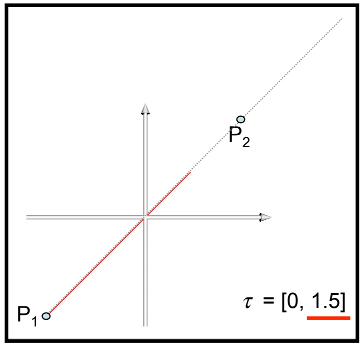

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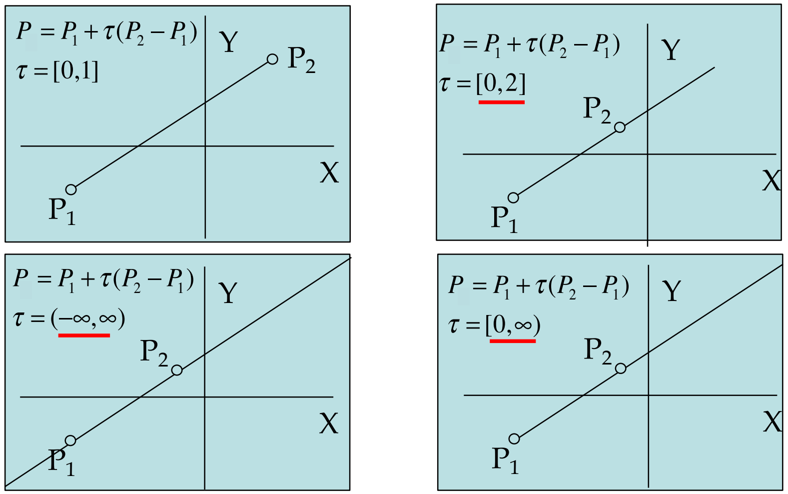

Summary

2D straight lines, segments and rays can be defined analytically by

- **Implicit functions**
  $f(x,y)=0$ – Slow for rendering
- **Explicit functions**
  $y=f(x)$ *or* $x=f(y)$ – Fast but axes dependent
- **Parametric functions**
  One parameter only
  $x=x(t),\quad y=y(t)\quad t=[t_1, t_2]$ – Fast and axes independent