CZ3005 Tutorial 4

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Question 4.1

Being able to prove the validity of logical sentences is the key to sound reasoning.

(a) Use *truth tables* to show that the following logical equivalences hold.

(i) $(P \Rightarrow Q) \Leftrightarrow (\neg P \vee Q)$

(ii) $(P \Leftrightarrow Q) \Leftrightarrow (P \Rightarrow Q) \wedge (Q \Rightarrow P)$

(b) Prove *without using a truth table* that the following equivalence holds.

(Hint: try applying and rewriting well-known logical equivalences.)

(iii) $(P \Leftrightarrow Q) \Leftrightarrow (P \wedge Q) \vee (\neg P \wedge \neg Q)$

Question 4.1

(a) (i) $P \Rightarrow Q \Leftrightarrow \neg P \vee Q$

P	Q	¬ P	¬ P ∨ Q	P ⇒ Q	P ⇒ Q ⇔ ¬ P ∨ Q
T	T	F	T	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

Question 4.1

(a) (ii) $P \Leftrightarrow Q \Leftrightarrow (P \Rightarrow Q) \wedge (Q \Rightarrow P)$

P	Q	$P \Leftrightarrow Q$	$P \Rightarrow Q$	$Q \Rightarrow P$	$(P \Rightarrow Q) \wedge (Q \Rightarrow P)$	$P \Leftrightarrow Q \Leftrightarrow (P \Rightarrow Q) \wedge (Q \Rightarrow P)$
T	**T**	**T**	**T**	**T**	**T**	**T**
T	F	F	F	**T**	F	**T**
F	**T**	F	**T**	F	F	**T**
F	F	**T**	**T**	**T**	**T**	**T**

Question 4.1

(b) (iii) $(P \Leftrightarrow Q) \Leftrightarrow (P \land Q) \lor (\neg P \land \neg Q)$

Proof using rewriting rules / equivalences

Using (ii):

$$(P \Leftrightarrow Q) \Leftrightarrow (P \Rightarrow Q) \land (Q \Rightarrow P)$$

Then, using (i):

$$(P \Leftrightarrow Q) \Leftrightarrow (\neg P \lor Q) \land (\neg Q \lor P)$$

Using distributivity of $\land$ over $\lor$:

$$(P \Leftrightarrow Q) \Leftrightarrow (\neg P \land \neg Q) \lor (\neg P \land P) \lor (Q \land \neg Q) \lor (Q \land P)$$

Simplifying:

$$(P \Leftrightarrow Q) \Leftrightarrow (\neg P \land \neg Q) \lor (Q \land P)$$

Finally, using commutativity of $\lor$:

$$(P \Leftrightarrow Q) \Leftrightarrow (P \land Q) \lor (\neg P \land \neg Q)$$

Question 4.2

Amy, Bob, Cal, Don, and Eve were invited to a party last night.
Whenever there is a party, Cal will always go if Amy and Bob go.
But Cal will not go if Don goes, and conversely.
We know that Amy went to the party with Eve.
And that Bob goes to every party that Eve goes to.

Use *Propositional Logic* and the *Modus Ponens* to infer logically whether Don went to the party or not.

Question 4.2

Constants: A ∴ "Amy went to the party", B, C, D, E

Knowledge base:

"Cal will always go if Amy and Bob go."

(1) $A \land B \Rightarrow C$ ($\neg A \lor \neg B \lor C$)

"Cal will not go if Don goes, and conversely."

(2) $D \Rightarrow \neg C$ ($\neg D \lor \neg C$)

(2b) $C \Rightarrow \neg D$

"Amy went to the party with Eve." $A \land E$

(3) A (4) E

"Bob goes to every party that Eve goes to."

(5) $E \Rightarrow B$ ($\neg E \lor B$)

Question 4.2

Proof:

(4)+(5) $E, E \Rightarrow B$ $\vdash B$ (6)

(3)+(6)+(1) $A, B, A \wedge B \Rightarrow C$ $\vdash C$ (7)

(7)+(2b) $C, C \Rightarrow \neg D$ $\vdash \neg D$

Don did not go to the party.

Question 4.3

Translate each of the following statements into a *First Order Logic sentence*, using a consistent vocabulary (which you must define).

i. Not all students take both History and Biology.

ii. Politicians can fool all of the people some of the time, they can even fool some of the people all of the time, but they can't fool all of the people all of the time.

Question 4.3

i. Not all students take both History and Biology.

<u>Constants</u>:

History, Biology

<u>Predicates</u>:

Student(x) “x is a student”

Takes(x, c) “x takes course c”

<u>Sentence</u>:

$\neg (\forall x\ (Student(x) \Rightarrow Takes(x, History) \land Takes(x, Biology)))$

<u>Equivalent to</u>:

$\exists x\ Student(x) \land (\neg Takes(x, H) \lor \neg Takes(x, B))$

Question 4.3

ii. Politicians can fool all of the people some of the time, they can even fool some of the people all of the time, but they can't fool all of the people all of the time.

Predicates:

- Politician(x) "x is a politician"
- Person(x) "x is a person"
- Time(t) "t is a time"
- Fools(x, y, t) "x fools y at t"

Sentence:

$\forall x$ Politician(x) $\Rightarrow$ (

$\forall y$ Person(y) $\Rightarrow \exists t$ Time(t) $\wedge$ Fools(x, y, t))

$\wedge$ ($\exists y$ Person(y) $\wedge$ ($\forall t$ Time(t) $\Rightarrow$ Fools(x, y, t)))

$\wedge \neg$ ($\forall y \forall t$ Person(y) $\wedge$ Time(t) $\Rightarrow$ Fools(x, y, t))

)

Or 3 sentences:

$\forall x$ Politician(x) $\Rightarrow$ ($\forall y$ Person(y) $\Rightarrow \exists t$ Time(t) $\wedge$ Fools(x, y, t))

$\forall x$ Politician(x) $\Rightarrow$ ($\exists y \forall t$ Person(y) $\wedge$ Time(t) $\Rightarrow$ Fools(x, y, t))

$\forall x$ Politician(x) $\Rightarrow \neg$ ($\forall y,t$ Person(y) $\wedge$ Time(t) $\Rightarrow$ Fools(x, y, t))

Question 4.4

Consider the following two First Order Logic sentences:

(i) $\forall x \; (\; Boy(x) \Rightarrow \exists y \; (Girl(y) \wedge Likes(x, y)) \;)$

(ii) $\exists y \; (\; Girl(y) \wedge \forall x \; (Boy(x) \Rightarrow Likes(x, y)) \;)$

(a) Give a concise *interpretation* of each sentence in plain English.

(b) Explain informally whether the two sentences are *equivalent* or, if not, whether (i) logically *entails* (ii), or whether (ii) logically *entails* (i), or else if they are unrelated.

(c) Describe how to prove whether (i) logically *entails* (ii) using *logical reasoning* (A detailed proof is not required).

Question 4.4(a)

(i) $\forall x \, (\, Boy(\, x \,) \Rightarrow \exists y \, (\, Girl(\, y \,) \wedge Likes(\, x, y \,) \,) \,)$

"Every boy likes a girl"

(ii) $\exists y \, (\, Girl(\, y \,) \wedge \forall x \, (\, Boy(\, x \,) \Rightarrow Likes(\, x, y \,) \,) \,)$

"There is a girl that all boys like"

Question 4.4(b)

The above two sentences are not equivalent.

(i) does not entail (ii):

Every boy likes a girl, but not necessarily the same girl.

(ii) entails (i):

Every boy likes a particular girl, so every boy likes a girl.

Question 4.4(c)

Proof by counter example:

P is true, but Q is not. Then, P does not entail Q.

Counter example:

2 boys, Al and Ben

2 girls, Chloe and Dora

Likes (Al, Chloe)

Likes(Ben, Dora)

In this case, (i) is true, but (ii) is not.

Thank you!