CZ2003 Tutorial 10 (2020/2021, Semester 1)

Motions

1. Propose an animation model in implicit representation, which defines the movement of a unit sphere along a 3D helical curve in a uniform speed along the z-direction (see Figure Q1). The helical curve has radius 30, period 10, and 15 full rotations. The animation sequence should have 360 frames.

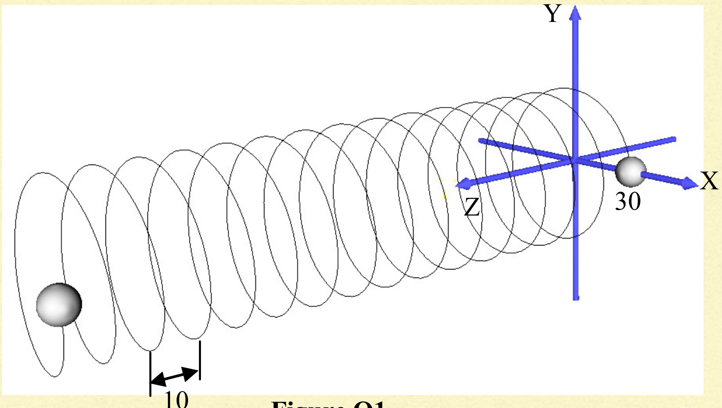


Figure Q1

2. A line segment bounded by two points (0,0,0) and (1,1,0) is rotated about the y-axis by 360 degrees. Using $x(u,t),\ y(u,t),\ z(u,t),\ u,t \in [0,1]$, define parametrically an animation of the rotating line such that it shows some deceleration.

3. Figure Q3 displays a sequence of images showing the shape changes from shape A to shape B, where A is an ellipse with semi-axes of 2 and 1, and B is an ellipse with semi-axes of 1 and 2. Propose a mathematical model using parametric functions to represent this animation.

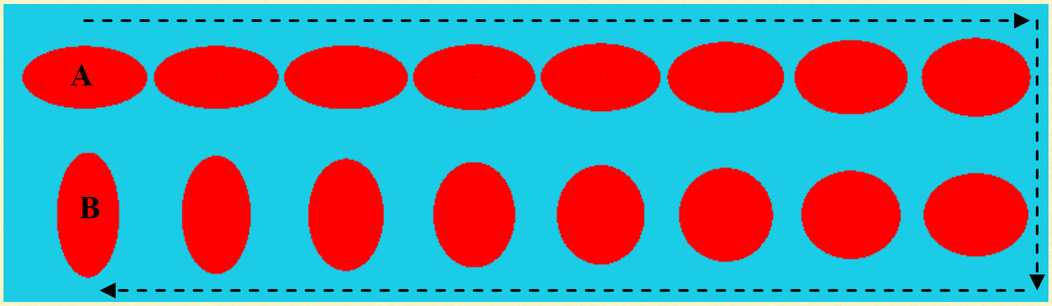


Figure Q3

4. Propose a mathematical model that implements morphing of transforming a solid unit sphere with center at the point (1, 1, 1) into a solid cylinder parallel to axes *y* with radius 3, center at the point (-2, -2, 2) and height of 4. The morphing sequence should have 160 frames and involve deceleration.

1. Propose an animation model in implicit representation, which defines the movement of a unit sphere along a 3D helical curve in a uniform speed along the z-direction (see Figure Q1). The helical curve has radius 30, period 10, and 15 full rotations. The animation sequence should have 360 frames.

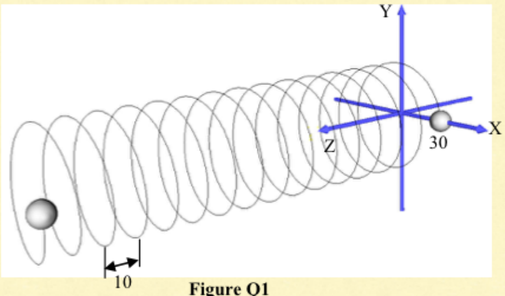


Figure Q1

1. Represent the path by parametric equations.

$x = 30\cos(15.2\pi u)$
$= 30\cos(30\pi u)$
$y = 30\sin(30\pi u)$

$z = z_1 + u(z_2 - z_1)$
$= 0 + u(10.(15) - 0)$
$= 150u$

2. linking path to target animation, derive representation of motion object. convert into implicit eqn.

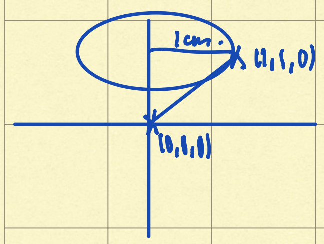
$1 - (x - 30\cos(30\pi u))^2 - (y - 30\sin(30\pi u))^2 - (z - 150u)^2 = 0$

3. define time using frame index. let parameter u represents time.

$u = f(k) = \frac{k-1}{m-1} = \frac{k-1}{360-1} = \frac{k-1}{359}$

where k is the frame index $1 \leq k \leq 360$
m is the total number of frames.

2. A line segment bounded by two points (0,0,0) and (1,1,0) is rotated about the y-axis by 360 degrees. Using $x(u,t)$, $y(u,t)$, $z(u,t)$, $u,t \in [0,1]$, define parametrically an animation of the rotating line such that it shows some deceleration.



$x(u,t) = u\sin\left(\sin\left(\frac{\pi}{2}t\right) \cdot 2\pi\right)$

$y(u,t) = u$

$z(u,t) = u\cos\left(\sin\left(\frac{\pi}{2}t\right) \cdot 2\pi\right)$

To specify speed: $t \in [0,1]$

$$t = \sin\left(\frac{\pi}{2} \cdot \frac{t-0}{1-0}\right) = \sin\left(\frac{\pi}{2} \cdot t\right)$$

First the equations the the line are:

$$[x, y, z] = [u, u, 0], \quad u \in [0,1]$$

Second, the rotation matrix is:

$$\begin{bmatrix} \cos(2\pi v) & 0 & \sin(2\pi v) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(2\pi v) & 0 & \cos(2\pi v) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad v \in [0,1]$$

Third, applying the rotation matrix to the line gives:

$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} \cos(2\pi v) & 0 & \sin(2\pi v) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(2\pi v) & 0 & \cos(2\pi v) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y \\ y \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} u\cos(2\pi v) \\ u \\ -u\sin(2\pi v) \\ 1 \end{bmatrix}$$

To make the animation show some deceleration

let $v = \sin\left(\frac{\pi}{2}t\right)$ $t \in [0,1]$

Thus, we finally have the equations:

$$\begin{cases} x(u,t) = u\cos\left(2\pi\sin\left(\frac{\pi}{2}t\right)\right) \\ y(u,t) = u \\ z(u,t) = -u\sin\left(2\pi\sin\left(\frac{\pi}{2}t\right)\right) \end{cases}$$

3. Figure Q3 displays a sequence of images showing the shape changes from shape A to shape B, where A is an ellipse with semi-axes of 2 and 1, and B is an ellipse with semi-axes of 1 and 2. Propose a mathematical model using parametric functions to represent this animation.

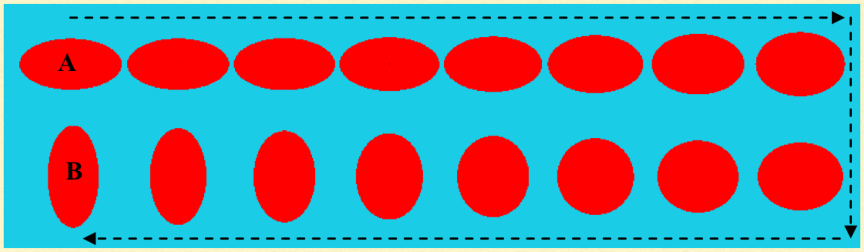


Figure Q3

Shape A to B:

A: ellipse with semi axis of 2 & 1

B: ellipse with semi axis of 1 & 2.

if w/o the u parameter ⇒ disk shape will not be filled ◯

Shape A $\qquad u, v \in [0,1]$ $\qquad$ Shape B

$x' = 2u\cos\theta = 2u\cos(2\pi v)$ $\qquad$ $x'' = 1u\cos\theta = u\cos(2\pi v)$

$y' = 1u\sin\theta = u\cos(2\pi v)$ $\qquad$ $y'' = 2u\sin\theta = 2u\sin(2\pi v)$

let no. of frames be 16

$$t = \frac{k-1}{16-1}, \quad t \in [1, 16]$$

$$x = 2\cos\theta(1-t) + (\cos\theta)t$$

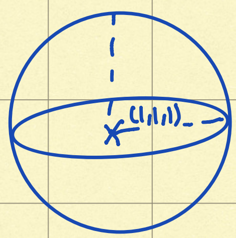
$$y = \sin\theta(1-t) + (2\cos\theta)t$$

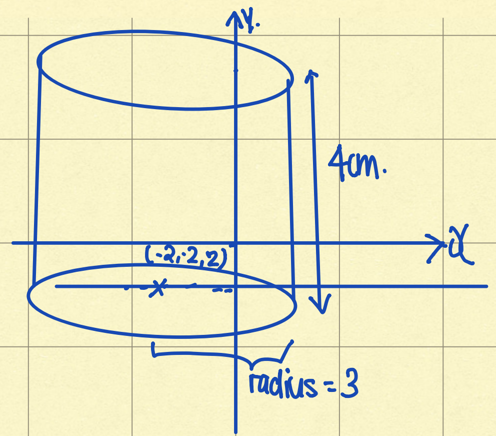
morphing model:

$x = 2u\cos(2\pi v)(1-t) + u\cos(2\pi v)t = u\cos(2\pi v)(2-t)$

$y = u\sin(2\pi v)(1-t) + 2u\sin(2\pi v)t = u\sin(2\pi v)(1+t)$

4. Propose a mathematical model that implements morphing of transforming a solid unit sphere with center at the point (1, 1, 1) into a solid cylinder parallel to axes *y* with radius 3, center at the point (-2, -2, 2) and height of 4. The morphing sequence should have 160 frames and involve deceleration.





1. define the solid sphere

$$f_s = 1-(x-1)^2-(y-1)^2-(z-1)^2 \geq 0$$

2. define the solid cylinder.

$$f_c = \min(\overset{-y}{\cancel{X}},\ \overset{y+4}{\cancel{4-z}},\ 9-(x+2)^2-(y+2)^2) \geq 0$$

3. Finally, define the morphings.

$$f(x,y,z) = f_s\,(1-t) + f_c\,(t)$$

where $t = \sin\left(\frac{\pi}{2}\cdot\frac{K-1}{159}\right)$ $K = 1, 2, \ldots, 160.$