

# **BC3402**

## **Financial Service Processes and Analytics**

Algorithm Trading (II)

### **Objectives**

- Evolution of trading algorithms
  - Impact driven (MI)
  - Cost driven
  - Opportunistic
- How these algorithms can be modified
- Implementing trading strategies: execution tactics



### **Types of Algorithms**

- Earliest form of algorithms (Impact driven)
  - TWAP
  - VWAP
  - Percent of Volume
- Modified VWAP and POV can be regarded as cost driven or even opportunistic
- Cost driven / Opportunistic
  - Adaptive shortfall
  - Price inline
  - Liquidity driven



### **TWAP**

- Allows trades to be “time-sliced” over a certain period of time.
- Same amount of trade regardless of market volume at any point in time → trade quantity spread equally throughout the time period.
- One of the easiest algorithm to implement
- Simple objective function



## TWAP (Illustration of 2 orders)

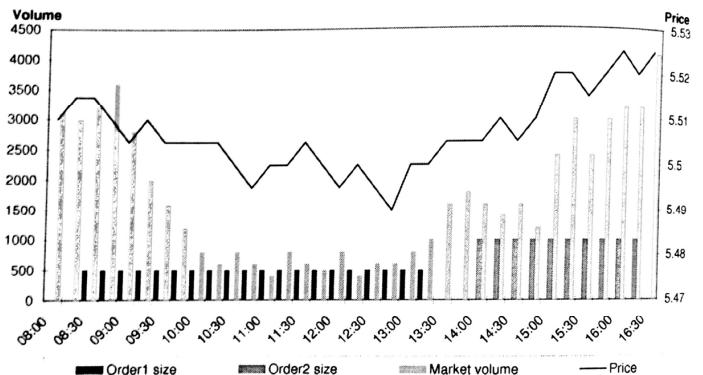


Figure 5-1 Two example TWAP orders

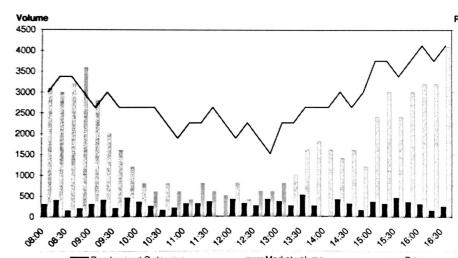
## Problems

- Poor execution quality due to rigid adherence to the time schedule esp. when prices become unfavorable
- Very predictable, high signaling risk. Only thing other market players don't know is the total size of the order
- So what are some modifications we can do?

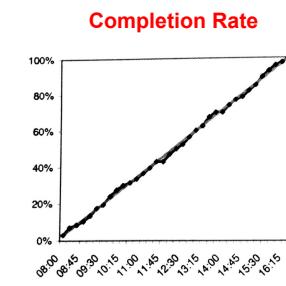
## Modification 1

- Add a random parameter to TWAP
  - TWAP +  $R$ , where  $R \sim N(0, \sigma^2)$
  - $R$  has to have a mean of 0, else completion time will be impacted
  - Main purpose is to reduce signaling risk
  - Number of time periods have to be relatively large. Smaller number of time periods results in greater uncertainty of completion

## Illustration of Randomized TWAP



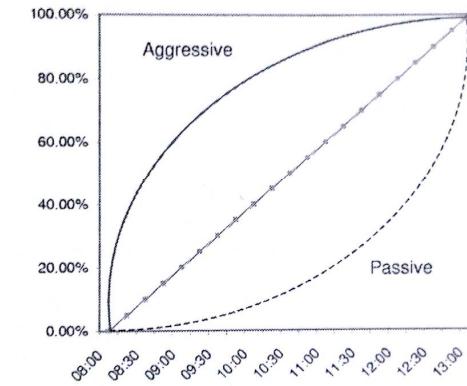
Order details



## Modification 2: Tilt

- Modify the quantum of trade for each time period
- Aggressive tilt:
  - +3.1%, +3.0%, +2.9%, ..., 0%, .. -2.9%, -3.0%, -3.1% (as an example)
  - Reduce timing risks, but potentially increase market impact
- Passive tilt:
  - -3.1%, -3.0%, -2.9%, ..., 0%, .. +2.9%, +3.0%, +3.1% (as an example)
  - Increase timing risks, but decrease market impact

## Modified TWAP (with Tilt) – Completion Rate



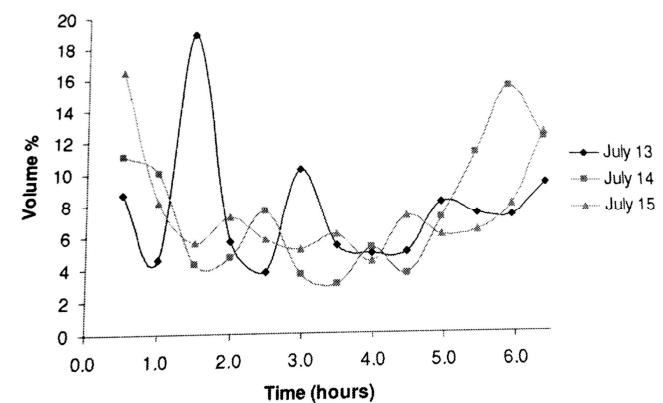
## VWAP

- Orders correspond to overall turnover for the day
- Where  $u_j$  represents the % of daily volume for time period  $j$
- $\bar{p}_j$  represents the average price for that period
- Use historical volume profile to estimate the volume for each time period
- Apply the estimates of volume to the equation:

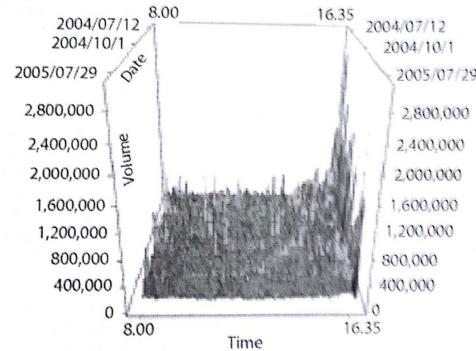
$$x_j = \hat{u}_j X$$

- Where  $\hat{u}_j$  is the predicted volume percentage for time period  $j$
- $X$  is the total intended volume,  $x_j$  is the intended volume for period  $j$ .

## How to estimate daily volume movement?

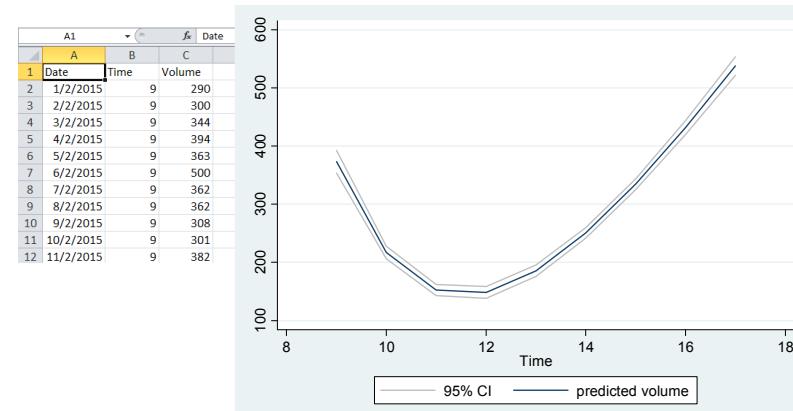


## How to estimate daily volume movement (cont'd)



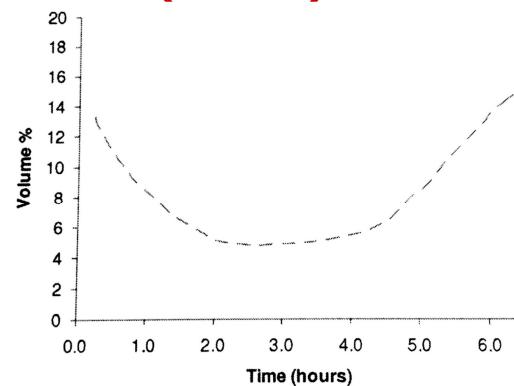
- Super impose all distribution over a fixed time period
- Aggregated data might be needed for fragmented markets
- For each time period compute the value which will minimize the sum of squares

## Estimating Trade Volume



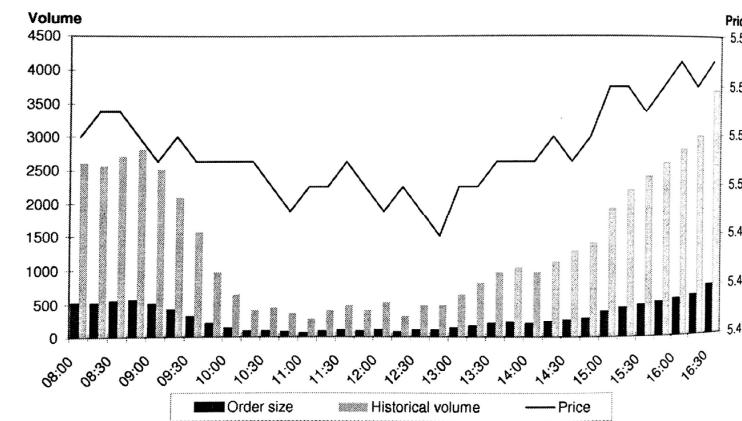
Darken line represents predicted volume. Lighter bands represent 95% CI

## How to estimate daily volume movement (cont'd)

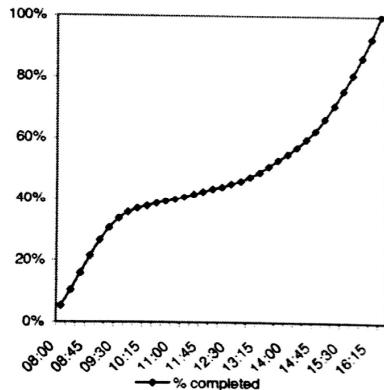


Use the estimates as  $\hat{u}_j$  for the relevant time period

## Illustration of VWAP



## Percentage of Completion (VWAP)



## Modification of VWAP - Tracking

- Tracking whether one is getting ahead or falling behind the schedule
- If that happens what are the remedial actions
- At end of each time period, the tracking option will do the following:
  - Add up the cumulated trades executed
  - If cumulated trades executed > cumulated ideal quantum (ahead of schedule)
  - If cumulated trades executed < cumulated ideal quantum (behind schedule)

## Remedial Actions

- If ahead of schedule:
  - Surplus quantity (difference between ideal and executed at that point in time) have to be proportion to the remaining time periods (weighted by historical volume) in the form of a reduction in trade quantity.
  - If the original trade is buy using VWAP, this is equivalent to a sell using VWAP for the excess quantities and we net of the “buy” quantity with the “sell” quantity
- If behind schedule:
  - The deficit (difference between ideal and executed at that point in time) have to proportion to the remaining time periods (weighted by historical value) in the form of increase in trade quantities
  - This is equivalent to putting in another VWAP order of the same direction for the remaining time period

## VWAP – Estimating Prices

- How do we estimate the potential VWAP price with the knowledge of a price trend?
- How about market impact?

$$E(VWAP) = p_0 + \Delta p \frac{(n+1)}{2}$$

$$\sigma^2(VWAP) = \frac{(n+1)}{2} \sigma_p^2$$

Kissell & Glantz

$p_0$  = arrival price  
 $\Delta p$  = change in price per trade period  
 $n$  = total number of trade periods  
 $\sigma_p^2$  = variance of price per trade period

## Example

- Assume a broker intend to perform a trade using VWAP algorithm at the start of the trading day when the opening price is \$25.00. The number of trading periods in the VWAP algorithm is 1000 for the trade horizon and the price is assume to increase by \$1.00 for the entire duration of the trade. The variance of the price is **\$0.009** per trading period. Compute the probability of which the VWAP price is less than \$27.

Answer: Probability of VWAP less than \$27 is 0.76

## Solution

$$E(VWAP) = p_0 + \Delta p \frac{(n+1)}{2} = \$25 + \$0.001 \frac{(1000+1)}{2} = \$25.5005$$

$$\sigma^2(VWAP) = \frac{(n+1)}{2} \sigma_p^2 = \frac{(1000+1)}{2} \cdot \$0.009 = \$4.5045$$

$$\sigma(VWAP) = \sqrt{\$4.5045} = \$2.1224$$

$$z = \frac{X - \bar{x}}{\sigma_x} = \frac{27 - 25.5005}{2.1224} = \frac{1.4995}{2.1224} = 0.7065$$

$$P(VWAP < 27) = P(z_i < 0.7065) = F(0.7065) = 76\%$$

## VWAP Modification - Tilt

- Some brokers believe they can accurately predict price movement over the day.
- If the price trend is significant, this might provide some motivation to modify the VWAP using a tilt
- Two types of tilt:
  - Morning tilt: to trade more in earlier time periods and less in later time periods
    - Buying in +ve price trend
    - Selling in -ve price trend
  - Afternoon tilt: to trade more in later time periods and less in earlier time periods
    - Selling in +ve price trend
    - Buying in -ve price trend

## Tilt Computation

$$w_j = \frac{u_j a_j}{\sum u_j a_j}$$

$u_j$  = % of daily volume for time period  $j$ , (i.e. original VWAP %)

$a_j$  = adjustment factor

$w_j$  = percentage of trades to be executed at time period  $j$

### Morning Tilt

$$a_j = \frac{(p_0 + nk) - k \cdot j}{n^2 + n(2p_0 - 1)}$$

### Afternoon Tilt

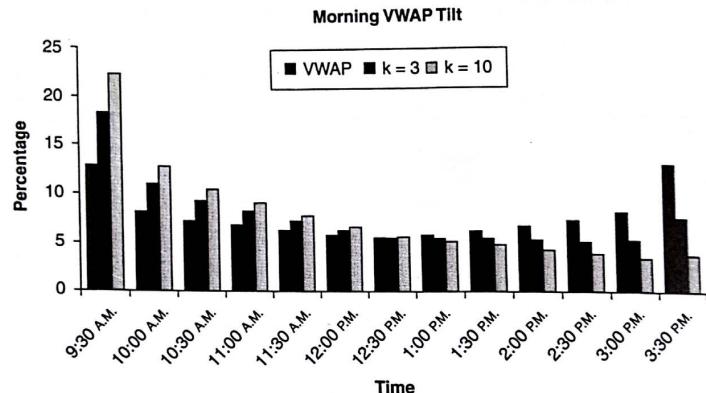
$$a_j = \frac{p_0 + k \cdot (j - 1)}{n^2 + n(2p_0 - 1)}$$

$p_0$  = Arrival price

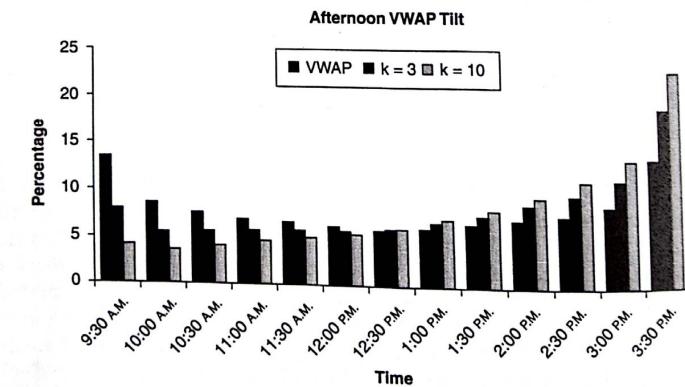
$n$  = number of time periods in VWAP strategy

$k$  = intensity parameter – how confident you are in the price trend & how strong you think the trend will be

## Morning vs. Afternoon Tilt (with different intensity parameters)



## Morning vs. Afternoon Tilt (with different intensity parameters)



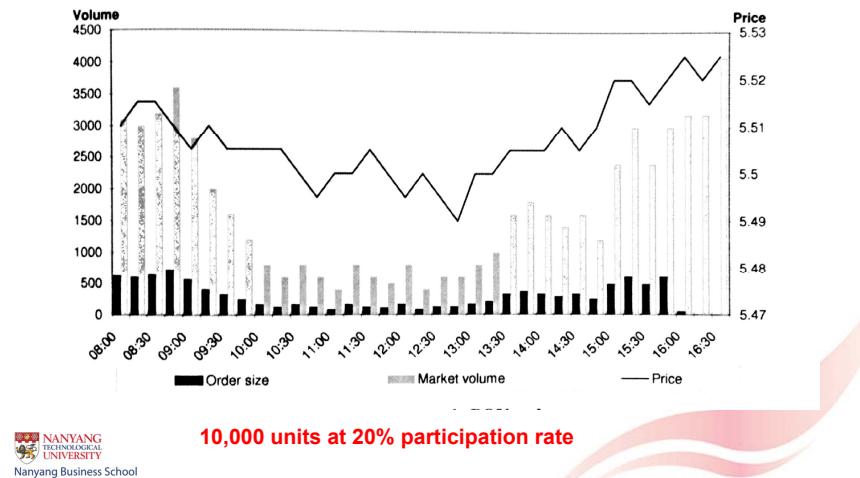
## Percent of Volume (POV)

- POV algorithms are similar to VWAP as trade intensity is positively correlated to the volume of trade in the market during the time period (more market volume, more trade)
- Main difference is that it “go along” with the volume dynamically and not have a pre-determined trading schedule (unlike the VWAP)
- Sometimes called volume inline, participation, target volume or follow algorithm
- The end time of the algorithm need not be fixed. Can complete as soon as the market volume allows or with a fixed end time whichever arrives first

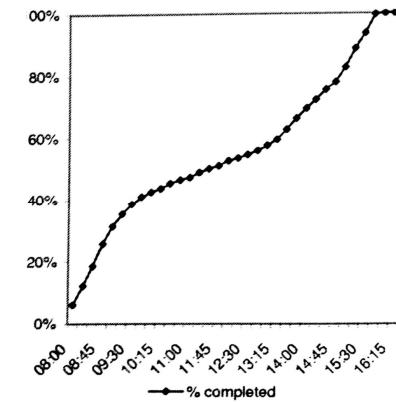
## Mechanics of Percent of Volume

- Determine ex-ante the percentage of volume (participation rate) one intends to target,  $P$  e.g.  $P = 20\%$  of total market volume
- Time slice the trade horizon to smaller time periods e.g. 15 minutes
- After each time period has elapse, obtain the total trade volume for that time period
- Compute the amount to trade for the next time period by using the formula:
 
$$- \text{Quantum to trade } t + 1 = \frac{1}{1-P} * \text{Volume in time } t$$
- The adjustment factor ( $1/1 - P$ ) is significant if  $P$  is large given that one's trade will also change the market volume

## Percent of Volume (illustration)



## Percentage of Completion (POV)



## Drawbacks of Percent of Volume

- It is backward looking. Can be mitigated if time period is made smaller
- End time may be unpredictable
- If several POV algo are in the market competing for an illiquid asset, might drive the price up. Hence, common to place price limits on the trade
- Poor performance if market experience large block trades (volume spikes), as it might be too late to execution after the trades are booked

## Modifications of POV – Price Adaption

- The POV is mainly volume driven and does not have intrinsic price dependency. One way to introduce price dependency is to adjust the participation rate based on how current market price compares to a benchmark price.
- When market prices vary from a particular benchmark by a particular level, the volume of participation varies.
- This is analogous to combining a POV with adaptive shortfall (AS) algorithm. Details of AS later.

## Modifications of POV – Excluding Trades

- Volume filters:
  - Excluding block trades in order books
  - Impose maximum volume limits
- Price filters:
  - Ignore trades that are out of a particular range of price
  - Used especially when price limit is set for the POV algorithm. To prevent errors in volume signals.

## Difference between VWAP and POV

- VWAP is schedule driven (forward looking), POV is dynamically driven (backward looking)
- VWAP normally has a set end time, POV can end earlier or have a set end time
- VWAP (that last entire trading day) generally end of high trade volume, POV generally end with low trade volume

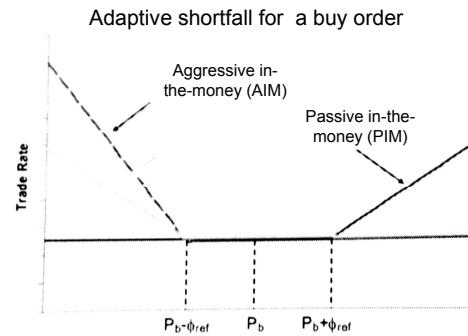
## Adaptive Shortfall

- “Newer” form of execution market impact algorithms is more dynamic
- Commonly used as an addition / modification on a VWAP or POV strategy
- Introduces the price component into most market impact algorithms which are volume driven.
- Adapts the trading schedule based on changes to the market price
  - Two strategies: AIM & PIM

## Adaptive Shortfall

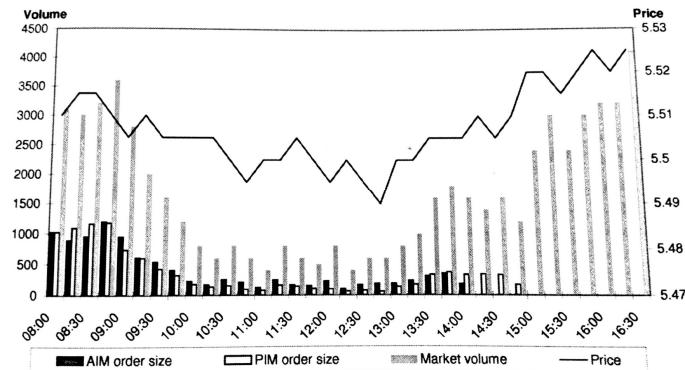
- AIM (short-term view)
  - Trade more aggressively (more trade) with favorable prices
  - Favorable => price below a benchmark for buys; price above a benchmark for sells
  - Unfavorable => price above a benchmark for buys; price below a benchmark for sells
  - AIM strategy achieve better prices than PIM, but at a higher risk
- PIM (long-term view)
  - Trade more passively (less trade) with favorable prices
  - Unfavorable price => more trade
  - Why?
  - Relies on trend persisting
  - PIM strategy is quicker to cut losses short so has lower risks, but may achieve poorer prices

## Adaptive Shortfall



- For sell order, the AIM becomes the PIM and vice versa
- The slope represents the tilt factor (or percentage of increased trades for the condition)
- The slope of the strategy can vary, steeper => more intense tilt

## Illustration of Execution



What do you think are the underlying algorithms for this execution?

## Price Inline

- Adapts its trading to the market price in a similar way the POV algorithm works on volume
- Dynamic trading pattern, but backward looking (similar to POV)
- A benchmark price is defined and trading quantum is adjusted every trading period  $j$ , based on the difference between the mid-price at the start of the trading period and the benchmark

## Price Inline (cont'd)

- Favorable price for buy => mid-price lower than benchmark, unfavorable otherwise
- Favorable price for sell => mid-price higher than benchmark, unfavorable otherwise
- Generally used the “AIM” method similar to adaptive shortfall – transact **more** when the prices are favorable.
- Straightforward adjustment to the participation rate based on changes to market price
- Similar to POV, time horizon for completion is unknown (based on price movement)

## Price Inline

$$\text{Participation rate} = u_j + k \cdot \text{sign}(X) \cdot \left( \frac{p_0 - p_j}{p_0} \right)$$

$u_j$  = baseline proportion of trade for time period  $j$  (e.g. 0.3 of  $X$ )

$k$  = adjustment factor, higher values more sensitive towards price changes

$X$  = total intended volume of trade (+ for buys, - for sells)

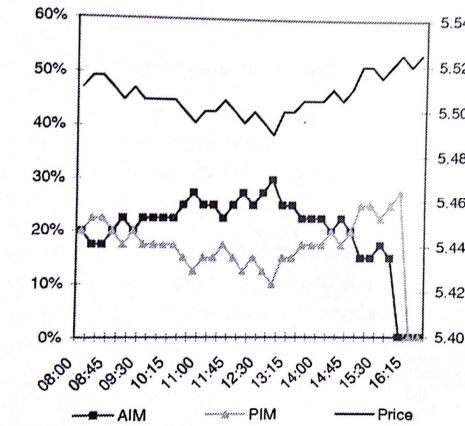
$p_0$  = benchmark price (e.g. arrival price)

$p_j$  = price at start of trading period  $j$ .

### Modifications and Calibration of Price Inline:

- The calibration of this algorithm will require the testing of different  $k$  values.
- Equation is true for AIM strategy (the dominant one for price inline)
- For PIM strategy (rare), just replace the  $+ k$  with  $- k$ .
- Can have an asymmetric model, larger  $k$  values for favorable and smaller  $k$  for unfavorable

## Completion Rates of Price Inline

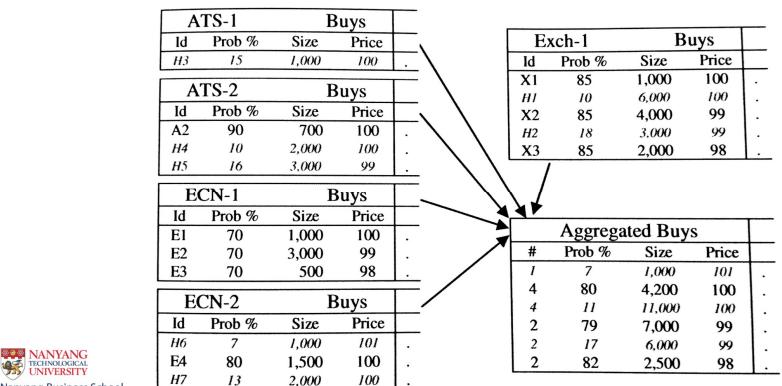


## Liquidity Driven (Liquidity Seeking)

- Trading decisions based on available order book depth rather than the best bid and ask
- Originally created to use more illiquid assets in fragmented markets
- More market depth => less spread costs for illiquid assets
- Challenges faced by this algorithm
  - Fragmented markets (multiple venues)
  - Hidden or iceberg orders

## Aggregation of Order Books

- Obtain order book details from different execution venues
- Rank them by price and then probability
- Probability obtained from historical execution success rates, latency, past history of cancellations
- This is used to establish the choice of execution venue or venues



## How to Predict Hidden Liquidity

- Using seeking or sniping micro orders to ping the market (details later)
- Using historical data to build an empirical estimation model
- Or a combination of both
  - Historical data to plan out the investment schedule
  - Seeking/ sniping micro orders to ping the market

## Presence of Hidden Liquidity

Buys				Sells	
Id	Time	Size	Price	Price	Volume
B3	8:25:00	1,000	100	101	800
B4	8:25:25	500	100	102	1,500
B1	8:20:25	2,000	99		
B2	8:24:09	1,500	98		

(a) before

Buys			
Id	Time	Size	Price
B3	8:25:00	1,000	100
B4	8:25:25	500	100
<b>B5</b>	<b>8:26:05</b>	<b>1,000</b>	<b>100</b>
B1	8:20:25	2,000	99

(b) after

- For example, “moments” after we observe (a), a trade of **2000** is executed
- The surplus of 500 of the executed trade and the excess of 1000 left in the order book has to come from either a new order, hidden order or iceberg order
- The smaller the time frame (“moment”), the increased in likelihood of this the order is from a hidden or iceberg source

## Estimating Hidden Liquidity

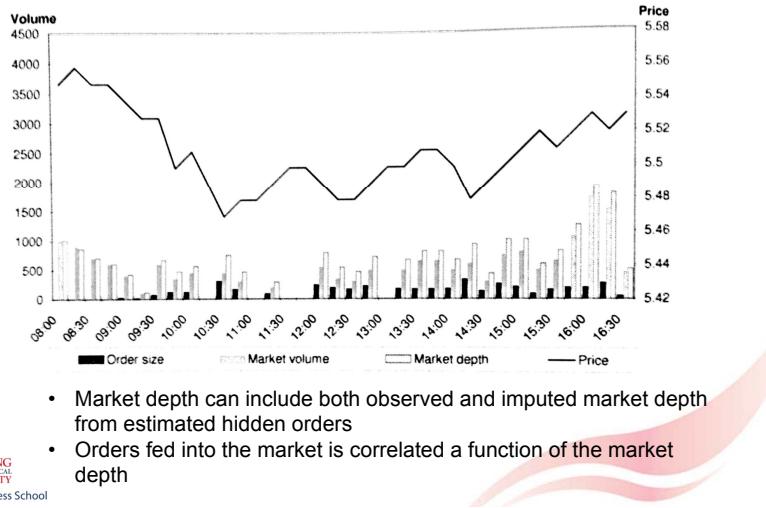
- Obtain best quotes and quantity for the execution venue for a particular ticker for over a period of time (e.g. 10 days). The best quotes values are obtained at regular small intervals e.g. one snapshot every 250 milliseconds.
- Capture the immediate time period after the quote values are captured. Observed if there is a trade with trade quantum greater than the bid or ask quantities. Flag those instances as 1 i.e. presence of hidden liquidity.

symbol	date	time	bid	ask	bsize	asize	exchange
QQQQ	2010-01-04	09:30:23	46.32	46.33	258	242	T
QQQQ	2010-01-04	09:30:23	46.32	46.33	260	242	T
QQQQ	2010-01-04	09:30:23	46.32	46.33	264	242	T
QQQQ	2010-01-04	09:30:24	46.32	46.33	210	271	P
QQQQ	2010-01-04	09:30:24	46.32	46.33	210	271	P
QQQQ	2010-01-04	09:30:24	46.32	46.33	161	271	P

## Estimating Hidden Liquidity

- Do the same for different tickers and aggregate the data
- Develop an empirical model with the following logic:
- $H = f(\text{qty. of bid \& ask, ticker, exchange, ....})$  where you can include other factors which might influence the probability of having “hidden order” e.g. market depth, time of day etc.
- A discrete choice model may be used here
- Parameter estimates of this model are to be retained and the model is used as a predictive tool for estimating the probability of hidden liquidity in future scenarios
- Parameters found to be strong predictors
  - Market depth
  - Ticker
  - Execution venue
  - Pass success rate

## Liquidity Driven Algorithm



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## Modification of Algorithms

- Modifications
  - Changes to input parameters derived from other estimation models
  - E.g. VWAP tilt, whereby the tilt parameter  $k$  is dependent on price trend forecast from market depth (seen last week)
- Combinations of algorithms
  - Most common between price-dependent and volume-dependent algorithms
  - E.g. VWAP with price inline



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The mini-algorithm

## EXECUTION TACTICS



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## What are Execution Tactics

- Order placement and management
- After the overall algorithm for execution (e.g. VWAP) is determined, orders are sliced and ready to be placed in the market at small time intervals (e.g. 2000 units per 15-minute time period)
- How do we put these orders in for the 15 min time period?
- All at once? Further strategies available?
- Sometimes called micro-trader, order placing algorithms, execution tactics
- Mainly to reduce spread costs, reduce signaling and be opportunistic (seek hidden liquidity)



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## Types of Execution Tactics

- Slicing
  - Hiding
  - Layering
  - Seeking/ Sniping
- 
- These are execution tactics that can be done by a human broker
  - Some of these tactics have similar traits as the algorithms we looked at earlier e.g. slicing similar to TWAP

## Slicing

- E.g. you use a VWAP strategy to trade 100,000 units of ABC over numerous 10-minute period. For a particular 10-minute period, you need to execute 3000 units.
- Using slicing, you slice 3000 into smaller parts
- Introduce a randomize component into the slicing to reduce signaling risks (as in TWAP with randomization)
- Eventual outcome = a synthetic iceberg order
- But some time lag compare to a native iceberg order as iceberg orders are still in order book (but without time priority), so can still cross
- It is not a synthetic hidden order as time priority is not granted

## "Micro" Slicing = Iceberg Orders



The screenshot shows a news article from SGX dated January 3, 2012. The headline reads "SGX proposes removal of iceberg order functionality to improve market transparency". The text explains that SGX is consulting the public on its proposal to remove engine-level iceberg order functionality. It notes that the functionality facilitates the execution of large orders in a manner that minimizes adverse market impact by releasing the order gradually into the market. The article states that the functionality is little used by the marketplace as similar execution functionalities are offered in order management systems of brokers and automated execution desks to execute block trades. It concludes that the removal of this functionality will enhance transparency in the opening and closing routines, as well as, in the calculation of the equilibrium price.

## Hiding

- Reducing market signaling risks
- Choice between discretionary, iceberg and fully hidden. Vary degrees of hiding. In ascending order.
- Discretionary
  - Only for small orders as order size is fully revealed
  - Limit price shown slightly less favorable to market e.g. actual price limit for "sell" is 101, but noted as 102 in the order book. If market order of 5000 units will 8000 will result in 4000 sold at 102. If limit order of 101 of 8000 units will result in 4000 sold at 101.

Sells		
Price	Size	Id
101	1,000	S1
101	3,000	S2
<b>102</b>	<b>4,000</b>	<b>S3</b>
102	2,000	S4

(a) Discretionary

Sells		
Price	Size	Id
101	2,000	S1
101	3,000	S2
<b>101</b>	<b>1,000</b>	<b>S3</b>
101	7,000	H1
102	4,000	S4

(b) Iceberg

Sells		
Price	Size	Id
101	1,000	S1
101	2,000	S2
<b>101</b>	<b>4,000</b>	<b>H1</b>
102	1,000	S4

(c) Fully hidden

## Hiding

- Iceberg
  - Show only a percentage of the trades to participants
  - Low percentage: longer to complete as only visible component has price **and** time priority
  - High percentage: shorter time to complete, but risk signaling
- Hidden
  - Lowest signaling risks
  - Order in the books but hidden from others, subj. to execution venue permission and requirements

Sells		
Price	Size	Id
101	1,000	S1
101	3,000	S2
<b>102</b>	<b>4,000</b>	<b>S3</b>
102	2,000	S4

(a) Discretionary

Sells		
Price	Size	Id
101	2,000	S1
101	3,000	S2
<b>101</b>	<b>1,000</b>	<b>S3</b>
101	7,000	<i>H1</i>
102	4,000	S4

(b) Iceberg

Sells		
Price	Size	Id
101	1,000	S1
101	2,000	S2
<b>101</b>	<b>1,000</b>	<b>S3</b>
101	7,000	<i>H1</i>
102	1,000	S4

(c) Fully hidden

## Layering

- Place orders at different tick levels
  - To maintain time priority
- Important for liquid stocks with densely populated order books
- Time priority essential to get orders done, else have to incur more aggressive prices to gain priority
- How to manage the layers? If prices changes?

Buys		Sells			
Size	Price	Price	Size	Time	Id
1,000	99	101	1,000	8:25:20	S5
500	99	101	800	8:25:00	S4
1,800	98	102	2,500	8:24:25	S3
2,400	98	102	500	8:24:09	S2
900	97	103	1,000	8:23:00	S1
2,500	97				

(a) before

Sells			
Price	Size	Time	Id
100	500	8:26:20	S7
101	1,000	8:25:20	S5
101	800	8:25:00	S4
102	2,500	8:24:25	S3
102	500	8:24:09	S2
102	3,000	8:26:05	S6
103	1,000	8:23:00	S1

(b) a new price level is set

## Layering (Update of prices)

- Update price of S5 (lower it)
  - Reduce qty of S5 and split some as S8 at a better (\$100) price
  - Reduce qty of S1 and S2 and split some for S8 at a better price (\$100)
  - Reduce qty of S1 and split some with S8 at a better price (\$100)
- Option 4 is the “best” as it reduces order with the least probability of execution.

Buys		Sells			
Size	Price	Price	Size	Time	Id
1,000	99	101	1,000	8:25:20	S5
500	99	101	800	8:25:00	S4
1,800	98	102	2,500	8:24:25	S3
2,400	98	102	500	8:24:09	S2
900	97	103	1,000	8:23:00	S1
2,500	97				

(a) before

Sells			
Price	Size	Time	Id
100	500	8:26:20	S7
101	1,000	8:25:20	S5
101	800	8:25:00	S4
102	2,500	8:24:25	S3
102	500	8:24:09	S2
102	3,000	8:26:05	S6
103	1,000	8:23:00	S1

(b) a new price level is set

## Seeking & Sniping

- Micro orders to detect hidden liquidity
- Can be used as orders to ping for liquidity after which will be used to impute liquidity for the “higher order” liquidity seeking algorithms as seen earlier (liquidity driven algorithm)
- Can also be used to seek hidden liquidity and fill orders
- Dual objective:
  - To see liquidity (**Seeking & Sniping**), but also
  - To hide intentions: How to seek without others knowing about it? (**Sniping**)

## Seeking

An example of searching for iceberg/ hidden orders, H1 is hidden, how can we know if it exists?

Buys		Sells			
Size	Price	Price	Size	Time	Id
800	100	101	1,000	8:25:00	S1
1,000	99	101	6,000	8:20:25	H1
2,500	98	102	2,000	8:21:25	S2

(a) before

Sells			
Price	Size	Time	Id
101	4,000	8:25:00	S4
101	2,000	8:20:25	H4
101	1,000	8:28:00	S4
101	3,000	8:20:25	H4
102	2,000	8:21:25	S2

(b) after

Send into the market with a limit buy order of 3000 units at \$101. If the trade executes and if an iceberg order exists, the resultant order book will look like (b)

But what is the problem of this seeking strategy?

## Sniping Order

- Alternative to send an Immediate-or-cancel (IOC) order OR a Fill-or-Kill (FOK)
- Both will not add (unexecuted parts) to the book. Both expires shortly after submission.
- Difference between IOC and FOK – partial fulfilment not permitted for FOK.
- FOK reduces signaling risks but increases risks of non-completion (compared to IOC) & vice versa

Buys		Sells			
Size	Price	Price	Size	Time	Id
1,000	100	101	1,000	8:25:00	S1
1,700	99	102	800	8:20:25	S2
2,200	98	104	1,100	8:19:09	S3

(a) before

Sells			
Price	Size	Time	Id
101	900	8:25:00	S4
101	100	8:25:00	S1
102	800	8:20:25	S2
104	1,100	8:19:09	S3

(b) after

## Sniping

- Same situation here. H1 is hidden (iceberg).
- Sending a FOK buy order of 1001 allows you to ping & at the same time execute. If H1 exist, the bid and ask levels remains intact. If H1 is absent, nothing happens to the order book (bid and ask levels remains intact).
- Sending a IOC buy order of 1001 allows you to ping too. If H1 exists, the outcome is similar to FOK. if H1 is absent, 1000 units will be executed, but the best ask is now at 102. The spread increases. This cause some signaling due to changes in the spread through an increase in the best ask. Trade-off completion rate with signaling risk.

Buys		Sells			
Size	Price	Price	Size	Time	Id
800	100	101	1,000	8:25:00	S1
1,000	99	101	6,000	8:20:25	H1
2,500	98	102	2,000	8:21:25	S2
		102	1,500	8:24:09	S3

(a) before