BC2410: Prescriptive Analytics 19 Feb. 2022

Linear Optimization Problems

1. Consider a portfolio selection problem with the expected annual return of investments as shown in Table 1:

Table 1: Expected annual return rate of investments

Investment	Expected annual return rate (%)
Share A – manufacturing sector	15.4
Share B - manufacturing sector	19.2
Share C - food and beverage sector	18.7
Share D – food and beverage sector	13.5
Mutual fund E	17.8
Mutual fund Z	16.3

Please help build a portfolio satisfying the following requirements:

   (a) Total amount =$ 90000

   (b) Amount in shares of a sector no larger than 50% of total available

   (c) Amount in shares with the larger return of a sector less or equal to 80% of sector's total amount

   (d) Amount in manufacturing company B less or equal to 10% of the whole share amount

   (e) Amount in mutual funds less or equal to 25% of the amount in manufacturing shares

2. Please solve the LOP of problem 1 in RSOME.

3. Please derive the dual problem of your LOP in problem 1.

4. Now suppose there is a fixed cost of f_i for investing product i, $i \in \{A, B, C, D, E, Z\}$ and in addition, the portfolio should meet the following requirements:

   (a) Cannot choose both share C and D;

(b) If Mutual fund E is chosen, then mutual fund Z must be selected;

(c) Share A should be selected if both share B and share C are selected;

(d) Share B should be selected if either share C or mutual fund E is selected.

Solutions:

1. Let x_i be the invested amount and r_i the expected annual return rate for product $i, i \in \mathcal{P} \triangleq \{A, B, C, D, E, Z\}$. We have

$$\begin{aligned}
\max \ & \sum_{i \in \mathcal{P}} r_i x_i \\
\text{s.t.} \ & \sum_{i \in \mathcal{P}} x_i \leq 90,000 \\
& x_A + x_B \leq 45,000 \\
& x_C + x_D \leq 45,000 \\
& 0.2x_B \leq 0.8x_A \\
& 0.2x_C \leq 0.8x_D \\
& -0.1x_A + 0.9x_B - 0.1x_C - 0.1x_D \leq 0 \\
& -0.25x_A - 0.25x_B + x_E + x_Z \leq 0 \\
& x_i \geq 0, i \in \mathbb{P}
\end{aligned}$$

2. Code is omitted. Solution: $\boldsymbol{x} = (27900, 8100, 36000, 9000, 9000, 0)$ with a total expected return of \$15400.8.

3. The dual problem is as follows:

$$\begin{aligned}
\min \ & 90,000p_1 + 45,000p_2 + 45,000p_3 \\
\text{s.t.} \ & p_1 + p_2 - 0.8p_4 - 0.1p_6 - 0.25p_7 \geq r_1 \\
& p_1 + p_2 + 0.2p_4 + 0.9p_6 - 0.25p_7 \geq r_2 \\
& p_1 + p_3 + 0.2p_5 - 0.1p_6 \geq r_3 \\
& p_1 + p_3 - 0.8p_5 - 0.1p_6 \geq r_4 \\
& p_1 + p_7 \geq r_5 \\
& p_1 + p_7 \geq r_6 \\
& p_i \geq 0, i = 1, ..., 7
\end{aligned}$$

4. Let $y_i \in \{0,1\}$ denote the decision of selection product $i, i \in \mathbb{P}$. Now the LOP becomes:

$$
\begin{aligned}
\max \ & \sum_{i \in \mathcal{P}} (r_i x_i - f_i y_i) \\
\text{s.t.} \ & \sum_{i \in \mathcal{P}} x_i \leq 90,000 \\
& x_A + x_B \leq 45,000 \\
& x_C + x_D \leq 45,000 \\
& 0.2x_B \leq 0.8x_A \\
& 0.2x_C \leq 0.8x_D \\
& -0.1x_A + 0.9x_B - 0.1x_C - 0.1x_D \leq 0 \\
& -0.25x_A - 0.25x_B + x_E + x_Z \leq 0 \\
& x_i \leq 90,000 y_i \quad \forall i \in \mathbb{P} \\
& y_C + y_D \leq 1 \\
& y_E \leq y_Z \\
& y_A \geq y_B + y_C - 1 \\
& 2y_B \geq y_C + y_E \\
& x_i \geq 0 \quad \forall i \in \mathbb{P}
\end{aligned}
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