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CZ2003 TUT 1
U1922243C

CZ2003 Tutorial 1 (2020/21, Semester 1)

Coordinate Systems and Vectors

1. A straight line is defined by equation $y = 4x + 2$ in Cartesian coordinate system XY.
   (i) Define this straight line in polar coordinates r, a as an explicit function $r = f(\alpha)$.
   (ii) Specify the domain for the polar coordinate α which is valid for this straight line.

2. (i) Define in polar coordinates $r = f(\alpha)$ the origin-centred circle with radius R. Specify the domain for the polar coordinate α.

   (ii) Define in polar coordinates $r = f(\alpha)$ a circle with radius R and the centre at the Cartesian coordinates (R, 0). Specify the domain for the polar coordinate α.

3. With reference to Figure Q34, write formulas deriving Cartesian coordinates x, y, z, from the cylindrical r, α, h and spherical coordinates r, α, β. Notice that the axes layout is different in the two cases.

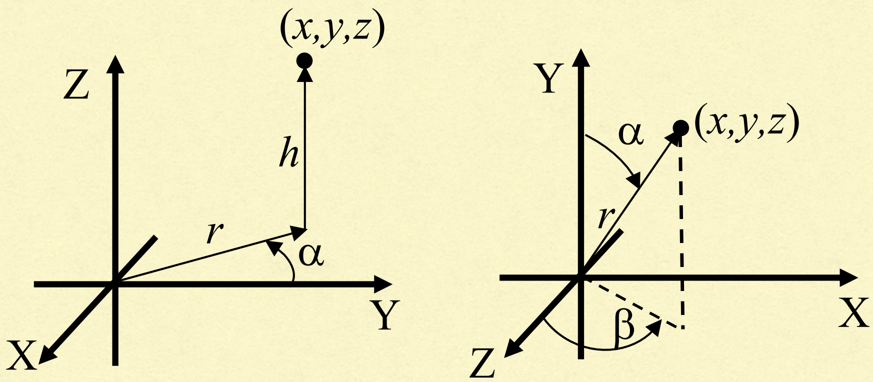


Figure Q3

4. (i) With reference to Figure Q4, calculate coordinates of the unit (magnitude is equal to 1) normal vector **N**.
   (ii) What are the coordinates of the unit normal vector to the opposite side of the triangle?

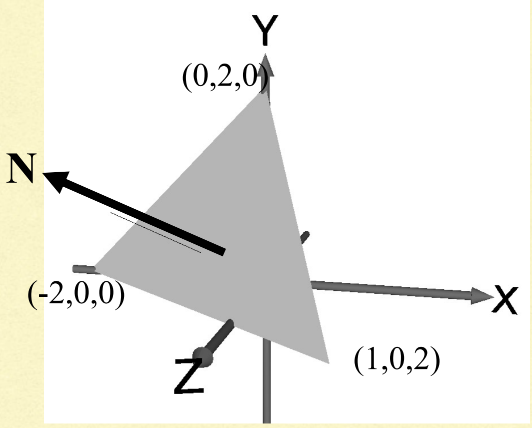


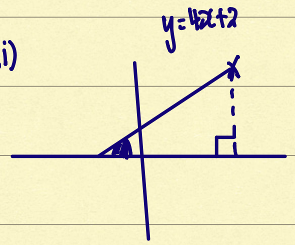
Figure Q4

1. A straight line is defined by equation $y = 4x + 2$ in Cartesian coordinate system XY.

(i) Define this straight line in polar coordinates r, a as an explicit function $r = f(\alpha)$.

(ii) Specify the domain for the polar coordinate α which is valid for this straight line.

1(i)



$y = 4x+2$

$x = r\cos(\alpha)$

$y = r\sin(\alpha)$

$r\sin(\alpha) = 4r\cos(\alpha)+2$

$r\sin(\alpha) - 4r\cos(\alpha) = 2$

$r(\sin(\alpha) - 4\cos(\alpha)) = 2$

$r = \dfrac{2}{\sin(\alpha)-4\cos(\alpha)}$ //

1(ii) $\tan^{-1}(4) = 1.32582$

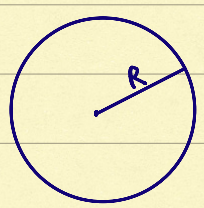
$\alpha = (1.32582, 1.32582+\pi)$

$D_\alpha = (1.32582, 4.46741)$ //

2. (i) Define in polar coordinates $r = f(\alpha)$ the origin-centred circle with radius R. Specify the domain for the polar coordinate α.

(ii) Define in polar coordinates $r = f(\alpha)$ a circle with radius R and the centre at the Cartesian coordinates $(R, 0)$. Specify the domain for the polar coordinate α.

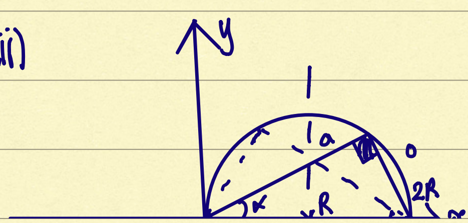
2(i)



$x^2+y^2=R^2$

$r=R$ with $\alpha \in [0,2\pi)$ //

2(ii)



since the diagonals are equal & all angles are 90.

from triangles inscribed into the circle.

$\cos(\alpha) = \dfrac{r}{2R}$

$r = 2R\cos(\alpha)$

$\therefore \alpha \in [-0.5\pi, 0.5\pi]$

3. With reference to Figure Q34, write formulas deriving Cartesian coordinates *x*, *y*, *z*, from the cylindrical *r*, α, *h* and spherical coordinates *r*, α, β. Notice that the axes layout is different in the two cases.

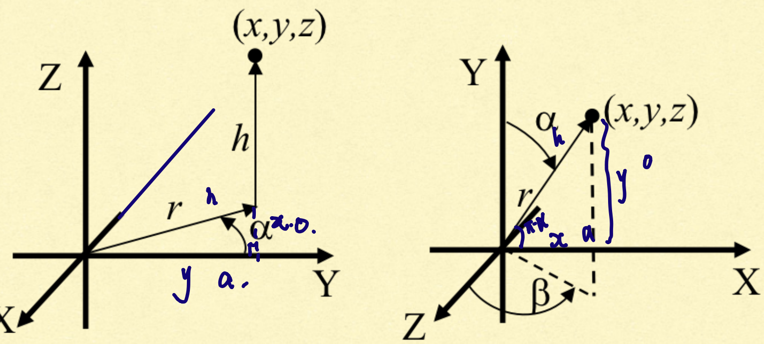


Figure Q3

<u>Cylindrical</u>

$\cos(\alpha) = \frac{y}{r}$

$y = r\cos(\alpha)$

$\sin(\alpha) = \frac{x}{r}$

$x = r\sin(\alpha)$

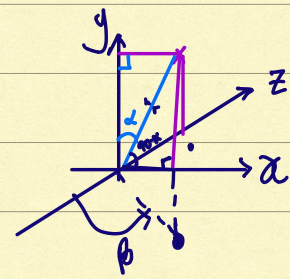
$z = h$

<u>Spherical</u>

$y = r\cos(\alpha)$

$x = r\sin(\alpha)\sin(\beta)$

$z = r\sin(\alpha)\cos(\beta)$



4. (i) With reference to Figure Q4, calculate coordinates of the unit (magnitude is equal to 1) normal vector **N**.

(ii) What are the coordinates of the unit normal vector to the opposite side of the triangle?

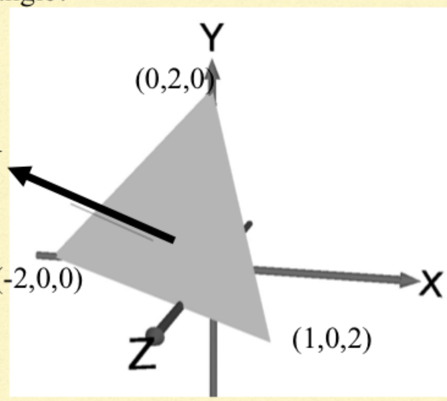
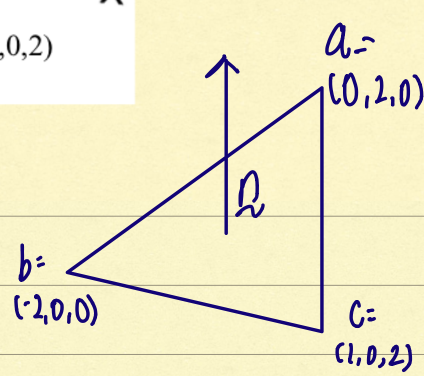


Figure Q4



(i) $\vec{ab} = (-2,-2,0)$

$\vec{ac} = (1,-2,2)$

$\vec{N} = \vec{ab} \times \vec{ac}$

$= \det \begin{bmatrix} i & j & k \\ -2 & -2 & 0 \\ 1 & -2 & 2 \end{bmatrix}$

$= i \det \begin{bmatrix} -2 & 0 \\ -2 & 2 \end{bmatrix} - j \det \begin{bmatrix} -2 & 0 \\ 1 & 2 \end{bmatrix} + k \det \begin{bmatrix} -2 & -2 \\ 1 & -2 \end{bmatrix}$

$= i(-4) - j(-4) + k(4+2)$

$= \underline{-4i + 4j + 6k}$ //

$|\vec{N}| = \sqrt{(-4)^2 + (4)^2 + (6)^2}$

$= \sqrt{68}$

$\vec{N} = \left(\frac{-4}{\sqrt{68}}, \frac{4}{\sqrt{68}}, \frac{6}{\sqrt{68}} \right)$

$= \left(\frac{-2}{\sqrt{17}}, \frac{2}{\sqrt{17}}, \frac{3}{\sqrt{17}} \right)$

$= \underline{(-0.49, 0.49, 0.73)}$ // (2d.p)

(ii) Coordinates of normal vector to the opp of Δ

$= \left(\frac{2}{\sqrt{17}}, \frac{-2}{\sqrt{17}}, \frac{-3}{\sqrt{17}} \right)$

$= \underline{(0.49, -0.49, -0.73)}$ //