BC2410 Assignment 2
Sem 1, Group 3
Members: Ng Chi Hui, Lulu Sui, Lim Yan Di, Goh Ray Sam, Tan Jin Xian Cyrus

1.) $P_1 = \{(x, y, z) \in R^3 : |x| \le 1, |y| \le 1, |z| \le 1\}$

Polyhedron $= \{x \mid Ax \ge b\}$

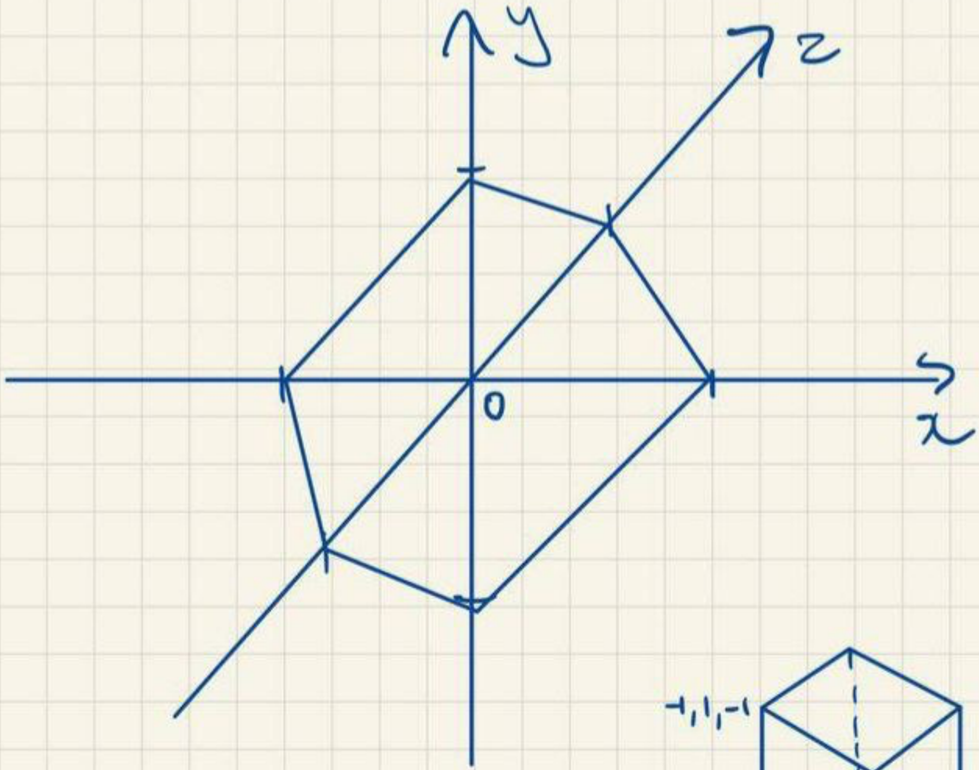
$P_1 = |x| \le 1, \quad |y| \le 1, \quad |z| \le 1$

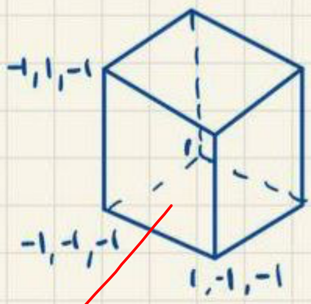
| | | |
|---|---|---|
| $|x| = x \le 1$ | $|y| = y \le 1$ | $|z| = z \le 1$ |
| $x \ge -1$ | $y \ge -1$ | $z \ge -1$ |
| $= -x \le 1$ | $= -y \le 1$ | $= -z \le 1$ |

$$A = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{pmatrix}, \quad m = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \quad B = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

$Am \le b$ and $-Am \ge -b$,
$P_1$ is a polyhedra

Each row of matrix A corresponds to a vector pointing outwards




Extreme point

$= (-1,-1,-1), (1,-1,-1), (-1,1,-1), (1,1,-1)$
$(-1,-1,1), (1,-1,1), (-1,1,1), (1,1,1)$

$$P_2 = \{(x,y,z) \in \mathbb{R}^3 : |x| + |y| + |z| \leq 1\}$$

$$|x| + |y| + |z| \leq 1$$

$$= \quad x + y + z \leq 1 \qquad -x - y + z \leq 1$$

$$-x + y + z \leq 1 \qquad x - y - z \leq 1$$

$$x + -y + z \leq 1$$

$$x + y - z \leq 1$$

$$-x + y - z \leq 1$$

$$-x - y - z \leq 1$$

$$A = \begin{pmatrix} 1 & 1 & 1 \\ -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \\ -1 & -1 & 1 \\ 1 & -1 & -1 \\ -1 & 1 & -1 \\ -1 & -1 & -1 \end{pmatrix} \quad m = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \quad b = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

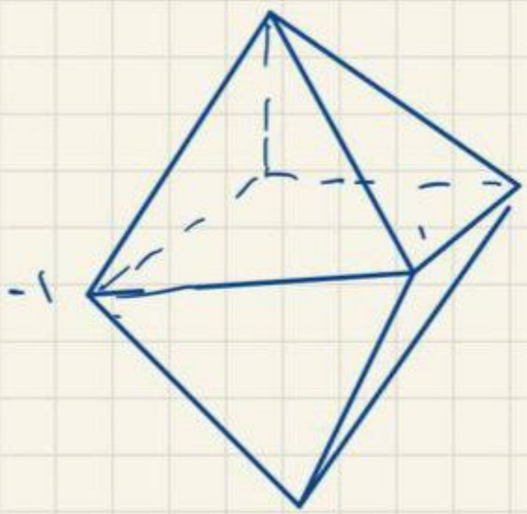
Since $Am \leq b$ and $-Am \geq -b$, $P_2$ is a polyhedron.

# Extreme points

$(1,0,0)$, $(-1,0,0)$,

$(0,1,0)$, $(0,-1,0)$,

$(0,0,1)$, $(0,0,-1)$



Q2

let $x$ be the amount of the 6-month loan

let $y_i$ be the amount of the monthly loan for month $i$, $i=1,\dots,6$

let $s_i$ be the surplus left at the end of the month $i$, $i=1,\dots,6$

Obj: $\min \ 0.12x + 0.04\sum_{i=1}^{6} y_i$

s.t.

$$x + y_1 + 20{,}000 - s_1 = 50{,}000$$
$$s_1 + y_2 + 30{,}000 - s_2 = 60{,}000 + y_1$$
$$s_2 + y_3 + 40{,}000 - s_3 = 50{,}000 + y_2$$
$$s_3 + y_4 + 50{,}000 - s_4 = 60{,}000 + y_3$$
$$s_4 + y_5 + 80{,}000 - s_5 = 50{,}000 + y_4$$
$$s_5 + y_6 + 100{,}000 - s_6 = 30{,}000 + y_5$$
$$1.12x + \overset{1.04}{y_6} \le s_6$$
$$x,\ y_i,\ c_i \ge 0,\quad i=1,\dots,6$$

pay back principal of previous month

-2

| Month | Revenues | Liabilities |
|---|---|---|
| July | $20,000 | $ 50,000 |
| August | 30,000 | 60,000 |
| September | 40,000 | 50,000 |
| October | 50,000 | 60,000 |
| November | 80,000 | 50,000 |
| December | 100,000 | 30,000 |

Answer: (from python)

$x = 60{,}000$

$y_1 = 0$

$y_2 = 0$

$y_3 = 10{,}000$

$y_4 = 20{,}000$

$y_5 = 0$

$y_6 = 0$

min interest = 8400

Code -2

3) $C_1$ : unit storage cost

$C_2$ : unit cost for switching

$x_i$ : No. of units produced

$S_i$ : No. of units stored.

$$\text{Min}: \sum_{i=1}^{11} C_1 S_i + \sum_{i=0}^{11} C_2 |x_{i+1} - x_i|$$

s.t. $x_i + S_{i-1} = d_i + S_i$ for $i = 1, \dots, 12$.

$x_0 = 0$.

$S_0 = 0$.

$x_i, S_i \ge 0$ for $i = 0, \dots, 12$.

$\rightarrow$

$$\text{Min} \sum_{i=1}^{11} C_1 S_i + \sum_{i=0}^{11} C_2 z_i$$

s.t. $z_i \ge x_{i+1} - x_i$ for $i = 0, \dots, 11$

$z_i \ge x_i - x_{i+1}$ for $i = 0, \dots, 11$

$x_i + S_{i-1} = d_i + S_i$ for $i = 1, \dots, 12$.

$x_0 = 0$.

$S_0 = 0$.

$x_i, S_i \ge 0$ for $i = 0, \dots, 12$.

Q4

Q4 primal

$$\max \sum_{i=1}^{500} c_i x_i$$

$$\text{st} \quad \sum_{i=1}^{500} x_i = 30 \ : \ p_1$$

$$0 \le x_i \le 1, \ i = 1, \dots, 500,$$

$$\begin{cases} x_i \ge 0 \\ x_i \le 1, \ i = 1, \dots, 500 \end{cases}$$

$$\max \quad c_1x_1 + c_2x_2 + c_3x_3 + c_4x_4 \cdots + c_{500}x_{500}$$

st

$$x_1 + x_2 + x_3 + x_4 \cdots + x_{500} = 30 \ : \ p_1$$

$$x_1 \le 1 \ : \ p_2$$

$$x_2 \le 1 \ : \ p_3$$

$$x_3 \le 1 \ : \ p_4$$

$$\vdots$$

$$x_{500} \le 1 \ : \ p_{501}$$

$$x_i \ge 0, \ i = 1, \dots, 500$$

dual

$$\min \quad 30p_1 + p_2 + p_3 + \cdots + p_{501}$$

st.

$$p_1 + p_2 \ge c_1$$

$$p_1 + p_3 \ge c_2$$

$$p_1 + p_4 \ge c_3$$

$$\vdots$$

$$p_1 + p_{501} \ge c_{500}$$

$p_1$ free

$$p_2, \dots, p_{501} \ge 0$$

better not to use this table. You need to understand the logic

Primal

$$\min \sum_{i=1}^{500} c_i x_i$$

s.t. $\sum_{i=1}^{500} x_i = 30$

$0 \le x_i \le 1, i=1,\cdots,500$

$\Downarrow$

min $\quad c_1x_1 + c_2x_2 + \cdots + c_{500}x_{500}$

st $\quad x_1 + x_2 + x_3 + \cdots + x_{500} = 30 \; : \; p_1$

$x_1 \le 1 \; : \; p_2$

$x_2 \le 1 \; : \; p_3$

$x_3 \le 1 \; : \; p_4$

$\vdots$

$x_{500} \le 1 \; : \; p_{501}$

$x_i \ge 0, i = 1, \cdots, 500$

| PRIMAL | minimize | maximize | DUAL |
|---|---|---|---|
| **constraints** | $\geq b_i$ | $\geq 0$ | **variables** |
| | $\leq b_i$ | $\leq 0$ | |
| | $= b_i$ | free | |
| **variables** | $\geq 0$ | $\leq c_j$ | **constraints** |
| | $\leq 0$ | $\geq c_j$ | |
| | free | $= c_j$ | |

dual

max $\quad 30p_1 + p_2 + p_3 + \cdots + p_{501}$

st $\quad p_1 + p_2 \le c_1$

$p_1 + p_3 \le c_2$

$p_1 + p_4 \le c_3$

$\vdots$

$p_1 + p_{501} \le c_{500}$

$p_1$ free

$p_2, \cdots, p_{501} \le 0$