CZ2003 Tutorial 3 (2020/21, Semester 1)

Curves

1. Write **parametric** formulas *x(u), y(u)* for the ray cast from the point with coordinates (2, 3) through the point with coordinates (4, 5). Define the domain for the parameter *u*.

2. **Using an equation in intercepts**, obtain an **implicit** formula *f*(*x*,*y*)=0 for the straight line intersecting the coordinate axes X and Y at the points with coordinates (3, 0) and (0, -2), respectively.

3. With reference to Figure Q3, write parametric functions $x(u)$, $y(u)$, $u \in [0, 1]$ defining this spiral curve which has to be drawn clockwise from the point with coordinates (0, 0.2).

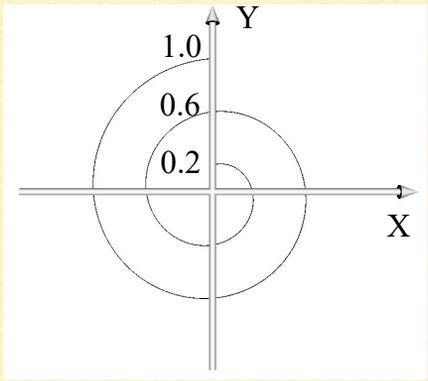


Figure Q3

4. Based on the way how polar coordinates are mapped to Cartesian, propose parametric functions $x(u)$, $y(u)$, $u \in [0, 1]$ which make the trigonometric sinusoidal curve (sine wave) follow a semicircle (half circle) with the radius of 1.5. The curve has to make 10 periodic oscillations (cycles) moving counterclockwise around the semicircle with the oscillations amplitude of ±0.5 as shown in Figure Q4.

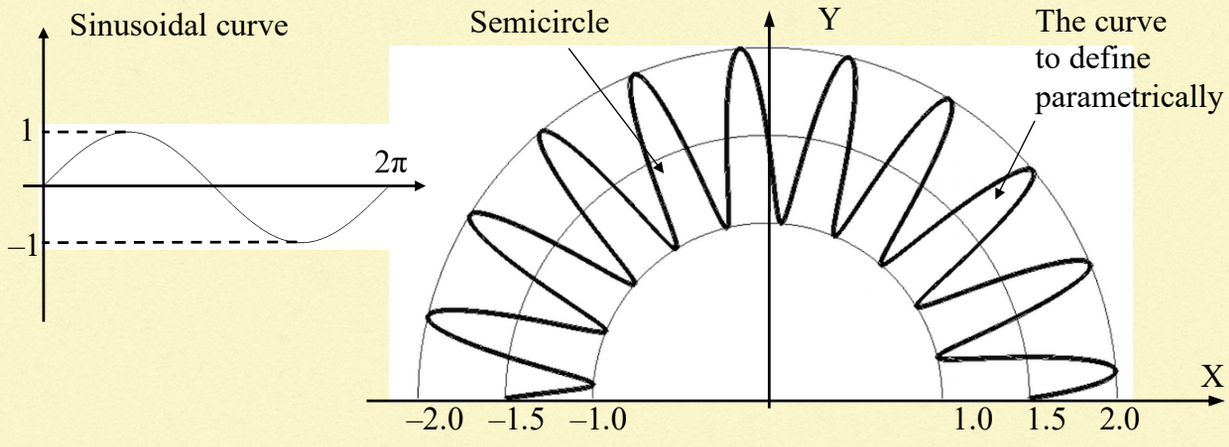


Figure Q4

1. Write **parametric** formulas *x(u), y(u)* for the ray cast from the point with coordinates (2, 3) through the point with coordinates (4, 5). Define the domain for the parameter *u*.

$x_1 = 2 \quad , x_2 = 4$

$x = x_1 + (x_2 - x_1)u$

$= 2 + (4-2)u$

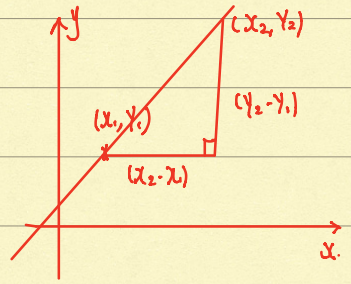
$= 2 + 2u$ #

$y_1 = 3 \quad , y_2 = 5$

$y = y_1 + u(y_2 - y_1)$

$= 3 + u(5-3)$

$= 3 + 2u$ #

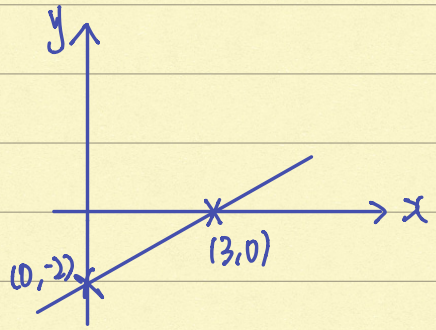


Domain for parameter u

$u = [0, \infty)$ ✓

$u = (-\infty, 1]$ ✗

2. **Using an equation in intercepts**, obtain an **implicit** formula $f(x,y)=0$ for the straight line intersecting the coordinate axes X and Y at the points with coordinates (3, 0) and (0, -2), respectively.

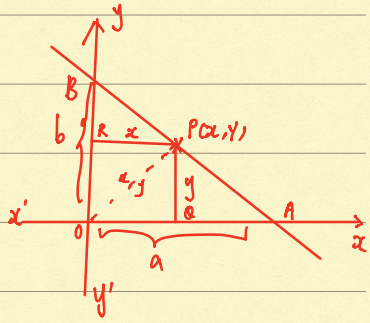


$\frac{x}{a} + \frac{y}{b} = 1$

$\frac{x}{3} + \frac{y}{-2} = 1$

$\frac{x}{3} - \frac{y}{2} - 1 = 0$

$2x - 3y - 6 = 0$ #



3. With reference to Figure Q3, write parametric functions $x(u)$, $y(u)$, $u \in [0, 1]$ defining this spiral curve which has to be drawn clockwise from the point with coordinates (0, 0.2).

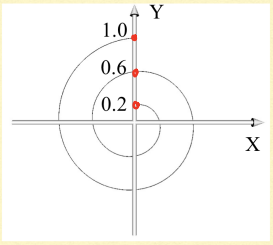


Figure Q3

$x = r\cos\theta$ (r: Radius, θ: Rotation Angle)

$y = r\sin\theta$

perform 2 full rotations : $2 \times 2\pi$

Add $\frac{\pi}{2}$ to offset Angle of rotation since we started from Y axis.

$x = ((1.0-0.2)u+0.2)\cos(-4\pi u + \frac{\pi}{2})$

$= (0.8u+0.2)\cos(-4\pi u + \frac{\pi}{2})$ #

$y = ((1-0.2)u+0.2)\sin(-4\pi u + \frac{\pi}{2})$

$= (0.8u+0.2)\sin(-4\pi u + \frac{\pi}{2})$ #

$u \in [0,1]$

Let's consider how the curve can be drawn by rotation clockwise from the point with coordinates (0, 0.2). It performs 2 full rotation $2\times2\pi$ while the dist from the origin linearly increments from 0.2 to 1.0. Also, we have to add $\frac{\pi}{2}$ offset for the angle of rotation since we started from x axis.

Another method. using y axis as reference or starting point at 1 → clockwise rotation has no effect

↳ $((1+0.2)-(1\times u))$

4. Based on the way how polar coordinates are mapped to Cartesian, propose parametric functions $x(u)$, $y(u)$, $u \in [0, 1]$ which make the trigonometric sinusoidal curve (sine wave) follow a semicircle (half circle) with the radius of 1.5. The curve has to make 10 periodic oscillations (cycles) moving counterclockwise around the semicircle with the oscillations amplitude of ±0.5 as shown in Figure Q4.

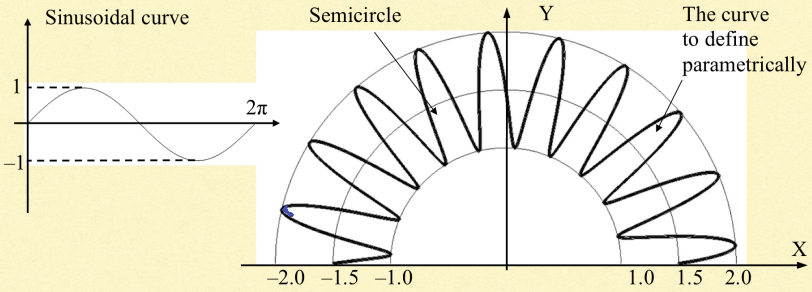


Figure Q4

Whenever circle should think of polar eqn

In form of Polar eqn.

Amplitude: 0.5

$r = 1.5 + 0.5\sin(20\theta)$

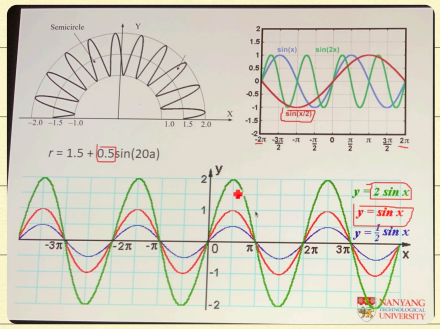
$\theta \in [0, \pi]$

To convert into u

$\frac{1}{2}$ circle to cycle

2π : 20 cycle.

frequency : 20



$r = 1.5 + 0.5 \sin(20\pi u)$

$u \in [0, 1]$

Conversion into parametric eqn.

using Pythathm $x = r\cos\theta$ $y = r\sin\theta$

$x = (1.5 + 0.5\sin(20\pi u))\cos(\pi u)$

$y = (1.5 + 0.5\sin(20\pi u))\sin(\pi u)$

$u \in [0, 1]$