CZ2003: Computer Graphics and Visualization

Lab Report 1:

Parametric Curves

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Note: For the whole lab experiment, N = 4 and M = 3 is used

1. Define parametrically in 4 separate files using functions $x(u)$, $y(u)$, $u \in [0,1]$ and display:
    a. Straight line segment spanning from the point with coordinates (**-N,-M**) to the point with coordinates (**M**,**N**).

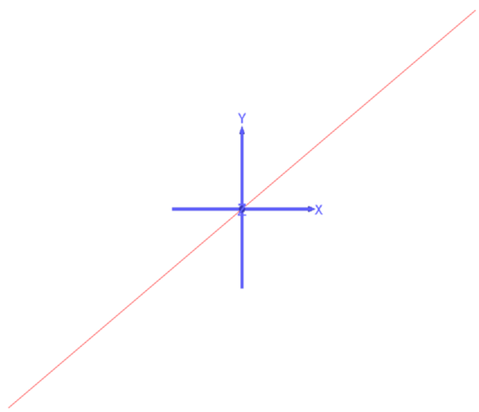


Fig 1.0 Straight line at resolution 1

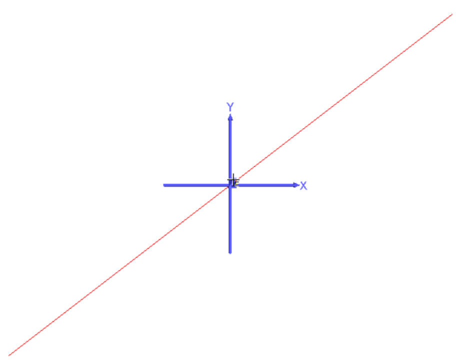


Fig 1.1 Straight line at resolution 100

Figure 1.0 is the screenshot of “Q1A.wrl” which define a straight line with parametric equations:
x = 4 – 8 * u
y = 3 – 6 * u
z = 0
at sampling resolution = 1

Parameter domain: u $\in$ [0, 1]

<u>The parametric is obtained by:</u>
$x = x_1 + u\,(x_2 - x_1) = 4 + u\,(-4-4)$
$= 4 - 8u$

$y = y_1 + u\,(y_2 - y_1) = 3 + u\,(-3-3)$
$= 3 - 6u$

Note: The minimum sampling resolution can be 1 because it is a straight line. Sampling at a higher resolution doesn’t change the image shown in Fig 1.0 vs Fig 1.1

b. A circular arc with radius **N**, centered at point with coordinates (**N**,**M**) with the angles $[\frac{\pi}{N}, 2\pi]$.

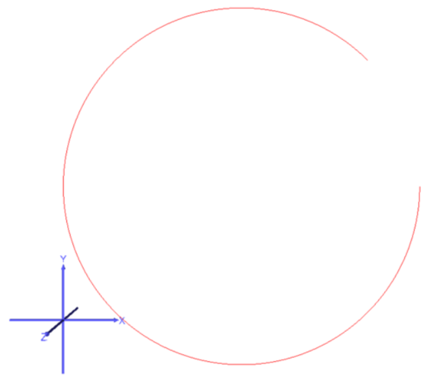


Fig 2.0 Arc with sampling resolution of 100

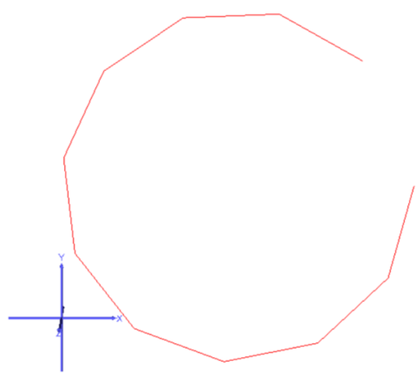


Fig 2.1 Arc with sampling resolution of 10

Fig 2.0 is a screenshot of "Q1B.wrl" which define an arc with parametric equations, taken at sampling resolution of 100:

x = 4 cos (u * pi * 7/4 + pi/4) + 4
y = 4 sin (u * pi * 7/4 + pi/4) + 3
z = 0
Parameter domain: u $\in$ [0, 1]

<u>The parametric equation is obtain by:</u>

$x = r\cos(\theta) + x_0$
$\quad = 4\cos(\theta) + 4$
$y = r\sin(\theta) + y_0$
$\quad = 4\sin(\theta) + 3$

To convert to u such the u $\in$ **[0, 1]**

$x = 4\cos(\theta) + 4$
$\quad = 4\cos(u*\pi*{}^{7}/_{4} + {}^{\pi}/_{4}) + 4$

$y = 4\sin(\theta) + 3$
$\quad = 4\sin(u*\pi*{}^{7}/_{4} + {}^{\pi}/_{4}) + 4$

Note:
The higher the sampling resolution, the higher the accuracy and smoothness of the circle. This is because there will be more points sampled that can be joined together resulting in the appearance of a smooth curve

If a resolution of 10 is used, it will result in 10 jagged edges for the arc as observed in Fig 2.1 making it appear as though it is a 'decagon'.

A minimum resolution of 30 can be used for the arc to appear smooth but 100 was used in Fig 2.0 to make it look more accurate and smooth

c. Origin-centered 2D spiral curve which starts at the origin, makes **N+M** revolutions clockwise and reaches eventually the radius 2***M**.

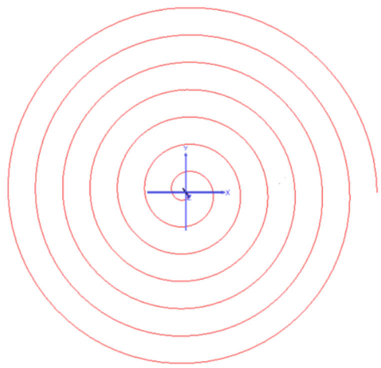


Fig 3.0 Spiral Curve at 500 sampling resolution

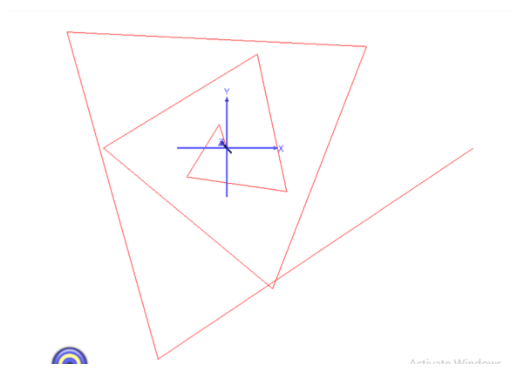


Fig 3.1 Spiral Curve at 10 sampling resolution

Fig 3.0 is the screenshot of “Q1C.wrl” which define a spiral curve with parametric equations, taken at a sampling resolution of 500:

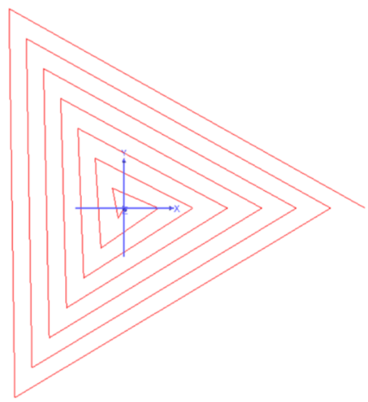
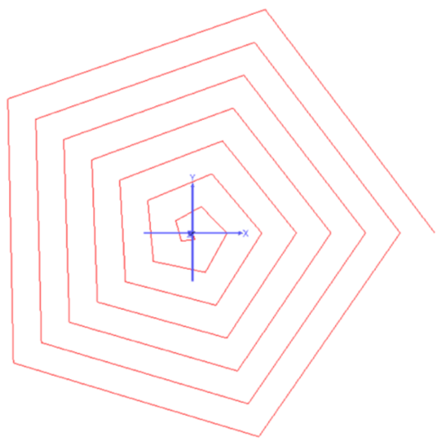
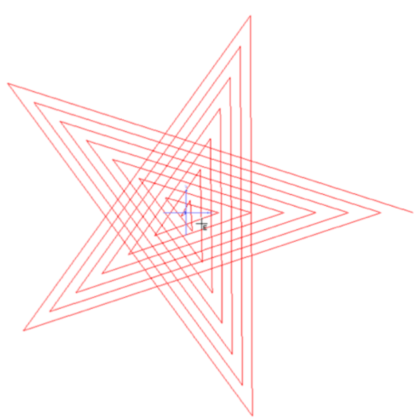
x = 6 * u * cos(-1 * u * 2 * 7 * pi)
y = 6 * u * sin(-1 * u * 2 * 7 * pi)
z = 0
Parameter Domain: u ∈ [0, 1]

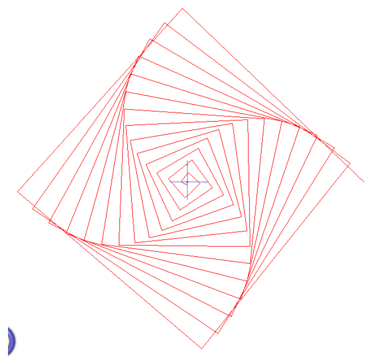
<u>The parametric equation is obtained by:</u>
Where a = 6, #of revolutions = 7
x = a cos((#of revolutions * 2) * -1 * π* u)
y = a sin ((#of revolutions * 2) * -1 * π* u)

Note: A minimum resolution of 500 has to be used to achieve a smooth curve. For a curve to be sufficiently approximated using straight lines, there needs to be enough sampling points. If a resolution of 10 is used it will look like the image in Fig 3.1

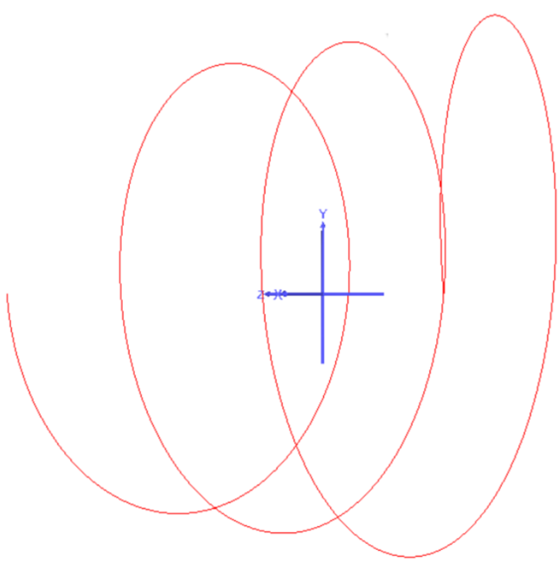
As per the question requirements, the curve should be spiralling outwards in a clockwise direction for 7 times ending at the coordinate of (6,0). To derive the final equation, we have to multiple by -1 else the direction will be anti-clockwise

Extra : By observations and from part 1B and 1C, we can create different types of interesting fix shapes by adjusting the resolutions

 *Fig 4.0 Spiral triangle at 21 sampling resolution*	Keeping the parametric the equation the same as the one used in 1C: x = 6 * u * cos(-1 * u * 2 * 7 * pi) y = 6 * u * sin(-1 * u * 2 * 7 * pi) z = 0 We can draw a triangle by reducing the resolution to 21. Fig 4.0 is a screenshot of "Triangle.wrl".
 *Fig 4.1 Spiral pentagon at 35 sampling resolution*	Keeping the parametric the equation the same as the one used in 1C: x = 6 * u * cos(-1 * u * 2 * 7 * pi) y = 6 * u * sin(-1 * u * 2 * 7 * pi) z = 0 We can draw a pentagon by reducing the resolution to 35. Fig 4.1 a snapshot of "Pentagon.wrl". There are 5 lines connecting per spiral (35 spirals total). After trying to change multiple resolutions by experimenting, we realise that a *n*-sided figure can be drawn, by taking number of spiral (7 in this case) multiplied by *n (5 sides in a pentagon)*. ∴ No of sampling Resolutions = 5 * 7 = 35 Note: As number of revolutions increase, the lines will start look more like a curve until eventually straight lines will now appear as curves similar to the notes mentioned above
 *Fig 4.2 Star in 2D*	By substituting u by 2 * u, rotation rate can increase resulting in appearance of a star. x = 6 * 2 * u * cos(-1 * 2 * u * 2 * 7 * pi) y = 6 * 2 * u * sin(-1 * 2 * u * 2 * 7 * pi) z = 0 Fig 4.2 is a screenshot of the "Star 2D.wrl". Sampling resolution was set to 35.

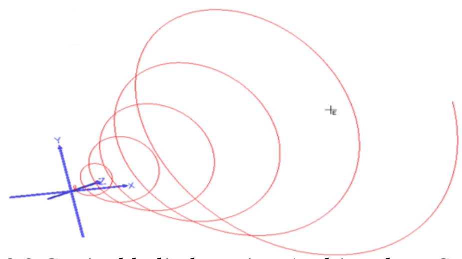
 *Fig 4.3 Rotating Square*	Using the same equation to form the star and by changing the resolution to 55. x = 6 * 2 * u * cos(-1 * 2 * u * 2 * 7 * pi) y = 6 * 2 * u * sin(-1 * 2 * u * 2 * 7 * pi) z = 0 An image of a rotating square can be achieved. Fig 4.3 is a screenshot "Rotating Square.wrl".

d. 3D cylindrical helix with radius **N** which is aligned with axis Z, makes **M** counterclockwise revolutions about axis Z while spanning from $z_1 = -\boldsymbol{N}$ to $z_1 = \boldsymbol{M}$.

 *Fig 5.0 3D Cylindrical Helix*	Fig 5.0 is a screenshot of "Q1D.wrl" defines which defines a counter-clockwise cylindrical helix with parametric equations, taken at a sampling resolution of 500: x = 4 * cos(u * 2 * 3 * pi) y = 4 * sin(u * 2 * 3 * pi) z = (7 * u) - 4 Parameter Domain: u ∈ [0, 1]

Extra: By adapting from the Archimedean Spiral and Equiangular spiral, we are able to make a conical helix where the base closest to the axes is the thinnest and it increase in diameter as it expands out making it look like a "cone".

By using Archimedean Spiral: x = u * cos (6*u*pi) y = u * sin (6*u*pi) z = u * pi Parameter Domain: u ∈ [0, 2] Fig 6.0 is a screenshot of "Conical Helix from Archimedean Spiral.wrl"	By using Equiangular Spiral: x = 0.5*exp (0.15*u*pi) * cos (2*u*pi) y = 0.5*exp (0.15*u*pi) * sin (2*u*pi) z = 0.5 exp (0.15*u*pi) Parameter Domain: u ∈ [0, 8] Fig 6.1 is a screenshot of "Conical Helix from Equiangular Spiral.wrl"

<table>
<tr>
<td>
Fig 6.0 Conical helix by using Archimedean Spiral</td>
<td>
Fig 6.1 Conical helix by using equiangular spiral</td>
</tr>
</table>

2. With reference to Table 1, convert the explicitly defined curve number **M** to parametric representations $x(u)$, $y(u)$, $u \in [0,1]$ and display it. Note that sketches of the curves in Table 1 are done not to the actual scale since the values of **N** and **M** are different in each variant.

<table>
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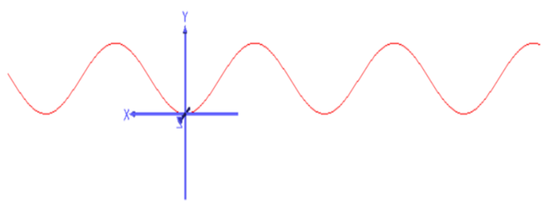
<td>
Fig 7.0 sin curve for Q2</td>
<td>Fig 7.0 is a snapshot of “Q2.wrl” which define the parametric graph
x = -8 + 12 * u
y = (sin(u*4*pi))^2
z = 0
Parameter Domain: u ∈ [0, 1]</td>
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</table>

3. With reference to Figure 5, a curve is defined in polar coordinates by:

$$r = \boldsymbol{N} - (\boldsymbol{M} + 5)\cos\alpha \qquad \alpha \in [0, 2\pi]$$

Define the curve parametrically as $x(u)$, $y(u)$, $u \in [0,1]$ and display it.

<table>
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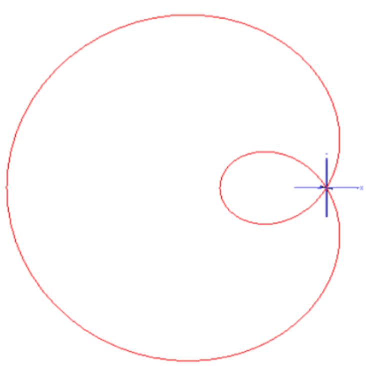
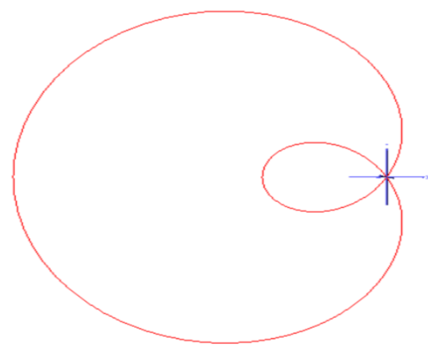
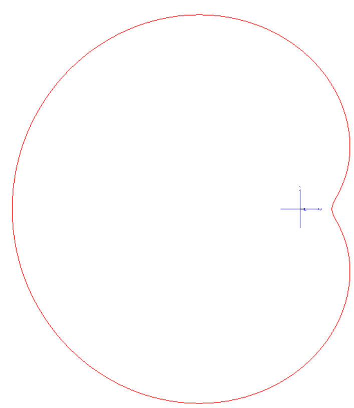
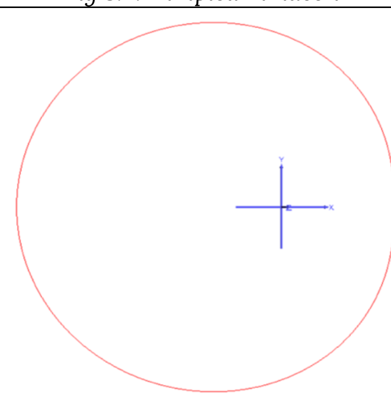
<td>
Fig 8.0 Looped Limacon at resolution 100</td>
<td>Figure 8.0 is a screenshot of “Q3.wrl” which define a Limacon by parametric equation
x=R*cos(2*pi*u);
y=R*sin(2*pi*u);

x = (4 - 8*cos(2*pi*u)) * (cos(2*pi*u))
y = (4 - 8*cos(2*pi*u)) * (sin(2*pi*u))
z = 0
$u \in [0,1]$

<u>The parametric equation is obtained by:</u>
Given the polar equation:
r = N - ((M+5)*cos(α))
= 4 – ((3+5)* cos(α))
= 4 – (8cos(α))
Using Pythagoras theorem and substituting R
x = rcos(α)
= (4 – (8cos(α))) * cos(α)
y = rsin(α)</td>
</tr>
</table>

	$= (4 - (8\cos(\alpha))) * \sin(\alpha)$ $\alpha \in [0, 2\pi]$ ***To convert it to be in terms of u*** Replace α = 2*pi*u Note: This type of Limacons is also known as looped Limacons

Extra

Limacons can also be represented in the polar form where the equation of the other forms such as r = a $\pm$ b sin θ and a $\pm$ b cos θ. Using this knowledge, we can examine what happens for various values of a and b by modifying the parametric equation we already found in Q3	
Fig 8.1 is the same as Figure 8.0 is screenshot of a "Q3.wrl" which is a Looped Limacon. This happens when the value of a is less than b ○ Using a = 4 and b = 8 ○ x = (cos(2*pi*u))*(4 - 8*cos(2*pi*u)) ○ y= (sin(2*pi*u))*(4 - 8*cos(2*pi*u)) ○ z=0 ○ $u \in [0,1]$	 *Fig 8.1 Looped Limacon*
Fig 8.2 is a screenshot of "Dimpled Limacon.wrl" This happens which the value of a greater than b ○ Set a = 10 and b = 8 ○ x = (cos(2*pi*u))*(10 - 8*cos(2*pi*u)) ○ y= (sin(2*pi*u))*(10 - 8*cos(2*pi*u)) ○ z=0 ○ $u \in [0,1]$	 *Fig 8.2: Dimpled Limacon*
Fig 8.3 is a screenshot of "Convex Limacon.wrl" This happens which the value of a greater than or equals to 2B ○ Set a = 5 and b = 2 ○ x = (cos(2*pi*u))*(5 - 2*cos(2*pi*u)) ○ y= (sin(2*pi*u))*(5 - 2*cos(2*pi*u)) ○ z=0 ○ $u \in [0,1]$	 *Fig 8.3: Convex Limacon*

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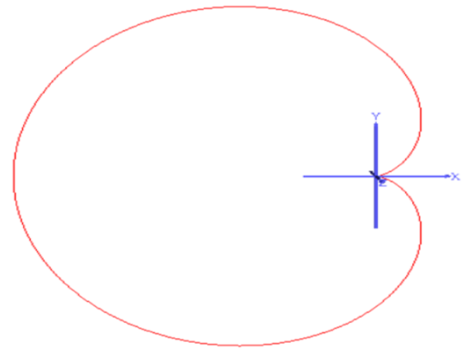
<td>Fig 8.4 is a screenshot of “Cardioid Limacon.wrl”

This happens when the value of equals to b. This is a special case of the Limacon.
Set a = 3 and b = 3
x = (cos(2*pi*u))*(3 - 3*cos(2*pi*u))
y = (sin(2*pi*u))*(3 - 3*cos(2*pi*u))
z = 0
$u \in [0,1]$</td>
<td>
Fig 8.4 Cardioid Limacon</td>
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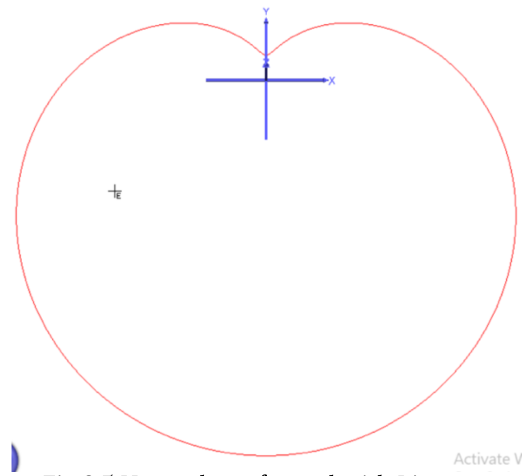
<td>The sign will determine if the limacon faces up or down left or right
The addition sign
With the sine function, limacon faces down. With the cosine function, limacon faces left.

The subtraction sign
With the sine function, limacon faces up. With the cosine function, limacon faces right.

With this knowledge, we could change the sign to generate a heart shape with limacon.

Using the Polar equation:
R = 4 - 3.5 sin(pi*u)
Fig 8.5 is a screenshot of a heart shaped made by a limacon using the parametric equation:
X = (4-3.5*sin(pi*u)) * cos(u*pi)
Y = (4-3.5*sin(pi*u)) * sin(u*pi)
Z = 0
$u \in [0,2]$

Fig 8.5 is a screenshot of “Heart with Limacon”</td>
<td>
Fig 8.5 Heart shape formed with Limacon</td>
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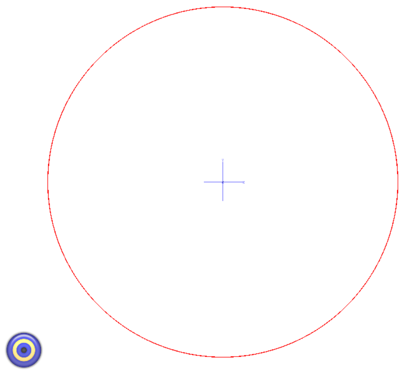
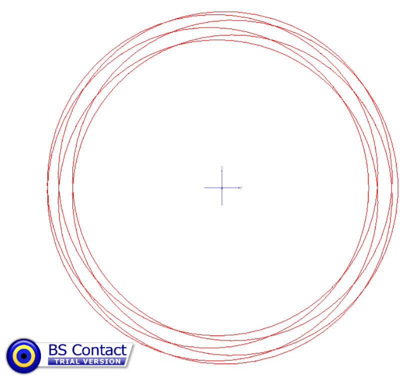
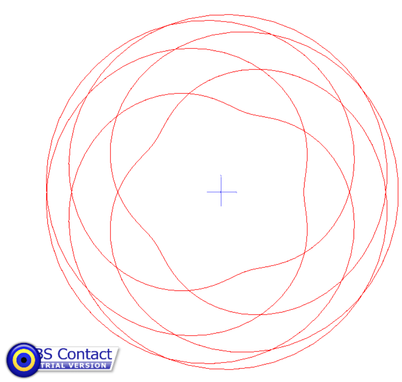
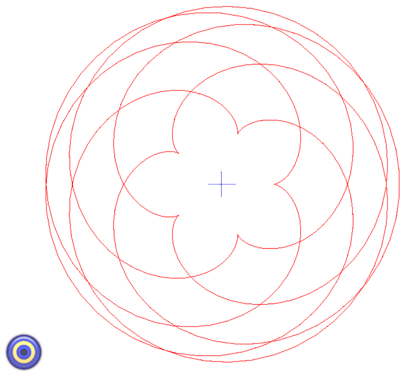
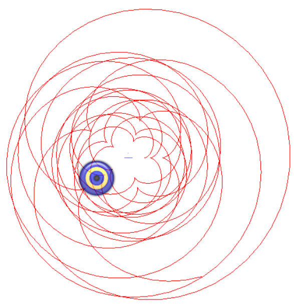
Extra: Using all the things that we have learnt above, we could try to draw a nice flower-liked shape by using the equation of a circle.

 *Fig 9.0 Screenshot of a circle*	We can start off by creating a circle with radius of 11. The parametric equation used is: $x = 11 * \cos(u)$ $y = 11 * \sin(u)$ $Z = 0$ $u \in [0, 100]$ Fig 9.0 is a screenshot of "Extra (Circle).wrl".
 *Fig 9.1 Screenshot of overlapping circles*	Following that, we can try to make overlapping circles. To make overlapping circle we can use the equation below. $x = 11 * \cos(u) - \cos(11/6 * u)$ $y = 11 * \sin(u) - \sin(11/6 * u)$ $Z = 0$ $u \in [0, 100]$ Here we are trying to take the big circle with the radius 11 subtracted by a smaller circle with radius 1 to create a path of motion. We multiply by 11/6 because we want the smaller circle to be "moving at a faster rate" where each increment of the parameter u is a bigger step up. Fig 9.1 is a screenshot of "Extra(Overlapping Circle).wrl".
 *Fig 9.2 Screenshot of a image that looks like star*	By changing the radius of the smaller circle from 1 to 4. The bigger circle is now subtracting a smaller circle that is bigger than previously. This creates a star liked shaped that looks a little like petals. $x = 11 * \cos(u) - 4 * \cos(11/6\ u)$ $y = 11 * \sin(u) - 4 * \sin(11/6\ u)$ $z = 0$ $u \in [0, 100]$ Fig 9.2 is a screenshot of "Extra(Test Flower).wrl".

 *Fig 9.3 Screenshot of the flower image*	By increasing the radius of the smaller circle to 6 instead, the flower petals will become more pronounced. X = 11 * cos(u) – 6 * cos(11/6 * u) Y = 11 * sin(u) - 6 * sin(11/6 * u) Z = 0 $u \in [0, 100]$ Fig 9.3 is a screenshot of “Extra(Final Flower).wrl”.
 *Fig 9.4 Screenshot of a 3D liked flower*	X = 11 cos(u) - 6 cos(11/6 * u) Y = 11 sin(u) - 6 * sin(11/6 * u) Z = u $u \in [0, 100]$ We change z to be equals u to make the image appear more 3D liked where the image is extended along the z axis Fig 9.4 is a screenshot of “Extra(Final Flower 3D).wrl”.