Tutorial 3 : Risk and Return I

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Key Points

- The average rate of return on a single risky asset from historical data is :

$$\text{Average Return}, \overline{\mathrm{R}} = \frac{\mathrm{R}_1 + \mathrm{R}_2 + \mathrm{R}_3 + \cdots . \mathrm{R}_n}{\mathrm{n}}$$

- The Geometric average return or geometric mean rate of return, (GM) on a single risky asset from historical data is :

$$\text{Geometric Mean}, \mathrm{GM} = \sqrt[n]{(1 + \mathrm{R}_1)(1 + \mathrm{R}_2)(1 + \mathrm{R}_3) \ldots . (1 + \mathrm{R}_n)} - 1$$

Which one to Use?

Consider the following investment :

Year	Beginning Value	Ending Value	Return (%)
1	\$50.0	\$100.0	100%
2	\$100.0	\$50.0	- 50%

We can calculate the arithmetic mean rate of return (AM) as follows :

$$AM = \frac{1.0 + (-\ 0.5)}{2} = 0.25 = 25\%$$

Note that we started with \$50 and ended with exactly the same amount at the end of 2 years. The investment brought no change in wealth and therefore no return. Yet, the AM computes a mean return of 25% !

Cont'd

From the previous data

Year	Beginning Value	Ending Value	Return (%)
1	$50.0	$100.0	100%
2	$100.0	$50.0	- 50%

We can calculate the geometric mean rate of return (GM) as follows :

$$GM = \{ [1 + r_1)(1 + r_2) \ldots\ldots\ldots]^{1/n} - 1 \} \times 100$$

$$GM = \{ [1 + 1)(1 - 0.5)]^{1/2} - 1 \} \times 100 = 0$$

Note that GM accurately measures that the investment has not yielded any return.

So which is correct, 0 percent or 25 percent?

Both are correct : They just answer different questions.

Cont'd

- The geometric average return answers the question "What was your average compound return per year over a particular period?"

- The arithmetic average return answers the question "What was your return in an average year over a particular period?"

- The variance and standard deviation of returns from historical data are calculated as :

$$\text{Variance}, \sigma^2 = \frac{\sum_{t=1}^{n}(R_t - \bar{R})^2}{n-1}$$

$$\text{Standard Deviation}, \sigma = \sqrt{\text{Variance}}$$

Key Points

- The expected rate of return from expectational data for a single risky asset is :

$$E(R) = \sum_{i-1}^{n} P_i R_i = P_1 R_1 + P_2 R_2 + \cdots P_n R_n$$

- The variance and standard deviation of rates of return from expectational data for an individual investment are calculated as :

$$\text{Variance}, \sigma^2 = \sum_{i=1}^{n} P_i \{(R_i - E(R)]^2$$

$$\text{Standard Deviation} = \sqrt{\text{Variance}}$$

Key Points

- Risk can be defined as the chance that some unfavorable event will occur.

- The stand-alone risk is the risk an investor would face if he or she held only one asset.

- The risk of an asset can be considered on a stand-alone basis (each asset by itself) or in a portfolio context, where the investment is combined with other assets and its risk is reduced through diversification.

- When we invest in Treasury bills, the investment's return can be estimated quite precisely as the investment is defined as being essentially risk free.

- The return of the risky assets would be much harder to predict. When evaluating the risky investment, we would estimate the return by first estimating the probability of certain events occurring. We then estimate the rate of return (outcome) for each of this event occurring.

Key Points

- We then multiply each possible outcome by its probability of occurrence and then sum these products to get the weighted average of outcomes. The weights are the probabilities, and the weighted average is the expected rate of return, , called "r-hat".

$$\text{Expected return} = \hat{R} = \sum_{i=1}^{n} R_i P_i = P_1 R_1 + P_2 R_2 + \cdots + P_n R_n$$

- If each investment opportunity can be looked at as a probability distribution of possible returns over a given investment horizon, we can use standard deviation of the different possible returns around the mean to measure risk.
- The expected return on an investment is the mean value of its probability distribution of returns.
- The greater the probability that the actual return will be far below the expected return, the greater the stand-alone risk associated with an asset.

Key Points

- The risk premium is the return over and above the risk-free rate.
- The historical risk premium is the excess return over the risk-free rate.
- The expected return on a portfolio of two assets is simply the weighted average of the returns on the individual assets, weighed by their portfolio weights:

$$(R_P) = \sum_{i-1}^{n} w_i E(R_i) = w_1 E(R_1) + w_2 E(R_2) + \cdots w_n E(R_n)$$

w_i is the percentage of our money invested in the asset i.

- The variance of a portfolio of two assets is a function of the correlation between the returns of the two assets. The portfolio standard deviation is the square root of the portfolio variance.

$$\sigma^2{}_p = w^2{}_1\sigma^2{}_1 + w^2{}_2\sigma^2{}_2 + 2\, w_1 w_2\, \rho_{1,2}\, \sigma_1\, \sigma_2$$

- The lower the correlation between the returns of the stocks in a portfolio, all else equal, the greater the diversification benefits.

Key Points

- Nominal rates are interest rates or rate of return that have not been adjusted for inflation.
- Real rates are interest rates or rates of return that have been adjusted for inflation.
- The relationship between nominal returns, real returns, and inflation is called the Fisher effect. If R is the nominal rate, h is the inflation rate and r is the real rate, the Fisher effect can be written as :

$$1 + R = (1 + r) (1 + h)$$

- For example, if the nominal rate was 15.5% and the inflation rate was 5%. What was the real rate?

$$r = \frac{(1+R)}{(1+h)} - 1 = \frac{1.155}{1.05} - 1 = 0.10$$

Key Points

- We take another look at the Fisher effect; we can rearrange things a little as follows :

  $1 + R = (1 + r) (1 + h)$

  $R = r + h + rh$

  rh is usually very small, so it is often dropped, the nominal rate is then approximately equal to the real rate plus the inflation rate.

  $R \approx r + h, \quad \text{or } r \approx R - h$

- For example, if investors require a 10% real rate of return, and the inflation rate is 8%, what must be the approximate nominal rate? The exact nominal rate?

  $R_{approx} \approx r + h = 10\% + 8\% = 18\%$

  Using Fisher equation,

  $1 + R = (1 + r) (1 + h) = 1.10 \times 1.08 = 1.188$

  $R_{exact} = 18.8\%$

#1:

You've observed the following returns on Crash-n-Burn Computer's stock over the past five years: 7 percent, –12 percent, 11 percent, 38 percent, and 14 percent.

a. What was the arithmetic average return on Crash-n-Burn’s stock over this five-year period?
b. What was the variance of Crash-n-Burn’s return over his period? The standard deviation?

Explanation:

a. To find the average return, we sum all the returns and divide by the number of returns, so:

Average return = (0.07 –0.12 +0.11 +0.38 +0.14)/5 =0.1160 or 11.60%

b. Using the equation to calculate variance, we find:

$$\text{Variance}, \sigma^2 = \frac{\sum_{t=1}^{n} (R_t - \bar{R}\,)^2}{n-1}$$

Variance = 1/4[(0.07 – 0.116)^2 + (–0.12 – 0.116)^2 + (0.11 – 0.116)^2 + (0.38 – 0.116)^2 + (0.14 – 0.116)^2] = 0.032030

So, the standard deviation is:

Standard deviation = (0.03230)$^{1/2}$ = 0.1790 or 17.90%

#2:

For problem #1, suppose the average inflation rate over this period was 3.5 percent and the average rate T-bill rate over the period was 4.2 percent.

a. What was the average real return on Crash-n-Burn’s stock?
b. What was the average nominal risk premium on Crash-n-Burn’s stock?

a. To calculate the average real return, we can use the average return of the asset, and the average inflation in the Fisher equation. Doing so, we find:

$$\text{Average nominal return} = (1 + \text{average real return})(1 + \text{average inflation})$$

$$\left(1+\overline{\mathrm{R}}\right) = (1+\overline{\mathrm{r}})(1+\overline{\mathrm{h}})$$

$$\overline{\mathrm{r}} = \frac{1+\overline{\mathrm{R}}}{1+\overline{\mathrm{h}}} - 1 = \frac{1.1160}{1.035} = 0.0783 = 7.83\%$$

b. The average risk premium is simply the average return of the asset, minus the average risk-free rate, so, the average risk premium for this asset would be:

$$\text{Average risk premium} = \text{average nominal return} - \text{average risk free rate}$$

$$\overline{\mathrm{RP}} = \overline{\mathrm{R}} - \overline{\mathrm{R}}_{\mathrm{F}} = 0.1160 - 0.042 = 0.0740 = 7.4\%$$

#3:

Given the information in Problem #2, what was the average real risk-free rate over this time period? What was the average real risk premium?

We can find the average real risk-free rate using the Fisher equation. The average real risk-free rate was:

$$(1+\bar{R}_f) = (1+\bar{r}_f)(1+\bar{h})$$
$$\bar{r}_f = (1.042/1.035) - 1 = 0.0683 \text{ or } 0.68\%$$

And to calculate the average real risk premium, we can subtract the average risk-free rate from the average real return. So, the average real risk premium was:

$$\overline{rp} = \bar{r} - \bar{r}_F = 7.83\% - 0.68\% = 7.15\%$$

#4:

A stock has had the following year-end prices and dividends:

Year	Price	Dividend
1	$60.18	-
2	73.66	$.60
3	94.18	0.64
4	89.35	0.72
5	78.49	0.80
6	95.05	1.20

What are the arithmetic and geometric returns for the stock?

To calculate the arithmetic and geometric average returns, we must first calculate the return for each year. The return for each year is:

R_2 = (\$73.66 – 60.18 + 0.60) / \$60.18 = 0.2340 or 23.40%
R_3 = (\$94.18 – 73.66 + 0.64) / \$73.66 = 0.2873 or 28.73%
R_4 = (\$89.35 – 94.18 + 0.72) / \$94.18 = –0.0436 or –4.36%
R_5 = (\$78.49 – 89.35 + 0.80)/ \$89.35 = –0.1126 or – 11.26%
R_6 = (\$95.05 – 78.49 + 1.20) / \$78.49 = 0.2263 or 22.63%

The arithmetic average return was:
R_A = (0.2340 + 0.2873 – 0.0436 – 0.1126 + 0.2263)/5 = 0.1183 or 11.83%

And the geometric average return was:
$R_G = [(1 + 0.2340)(1 + 0.2873)(1 – 0.0436)(1 – 0.1126)(1 + 0.2263)]^{1/5} – 1$
$= 0.1058$ or 10.58%

#5:

Consider the following information on three stocks:

		Rate of Return if State Occurs		
State of Economy	Probability of State of Economy	Stock A	Stock B	Stock C
Boom	0.35	0.24	0.36	0.55
Normal	0.50	0.17	0.13	0.09
Bust	0.15	0.00	-0.28	-0.45

a. If your portfolio is invested 40 percent each in A and B and 20 percent in C, what is the portfolio expected return? The variance? The standard deviation?

b. If the expected T-bill rate is 3.80 percent, what is the expected risk premium on the portfolio?

Explanation:

(a) If your portfolio is invested 40% each in A and B and 20% in C

We consider a single portfolio instead of a portfolio of three stocks.

Boom: $E(R_p) = 0.4(0.24) + 0.4(0.36) + 0.2(0.55) = 0.3500$ or 35.00%
Normal: $E(R_p) = 0.4(0.17) + 0.4(0.13) + 0.2(0.09) = 0.1380$ or 13.80%
Bust: $E(R_p) = 0.4(0.00) + 0.4(-0.28) + 0.2(-0.45) = -0.2020$ or -20.20%

And the expected return of the portfolio is:
$E(R_p) = 0.35(0.35) + 0.50(0.138) + 0.15(-0.202) = 0.1612$ or 16.12%

Since we consider single portfolio, we can use variance of single asset to calculate the single portfolio variance as follows:
So, the variance and standard deviation of the portfolio is:
$\sigma^2_p = 0.35(0.35 – 0.1612)^2 + 0.50(0.138 – 0.1612)^2 + 0.15(-0.202 – 0.1612)^2$
$\sigma^2_p = 0.03253$
$\sigma_p = (.03253)^{1/2} = 0.1804$ or 18.04%

(b) The **expected** risk premium of the portfolio is the **expected** return of a portfolio minus the **expected** risk-free rate. T-bills are often used as the risk-free rate, so:

Expected risk premium = **Expected** return – **Expected** T-bill rate (risk-free rate)

$E(RP_P) = E(R_p) - R_f = 0.1612 - 0.0380 = 0.1232$ or 12.32%

#6:

Consider the following information on 2 stocks:

Year	Stock A return	Stock B return
1	0.12	0.07
2	0.09	0.11
3	0.15	-0.02
4	0.05	0.04

a. What is the arithmetic average return for Stocks A and B?
b. What is the estimated standard deviation of returns for Stocks A and B?
c. What is the covariance of returns for Stock A and Stock B?
d. What is the correlation coefficient of returns for Stock A and Stock B?

$$\text{Variance}, \sigma^2 = \frac{\sum_{t=1}^{n}(R_t - \overline{R})^2}{n-1}$$

#6:

Year	Stock A			Stock B		
	Return	Deviation from Avg	Sq Deviation from Avg	Return	Deviation from Avg	Sq Deviation from Avg
1	0.12	0.0175	0.00030625	0.07	0.02	0.0004
2	0.09	-0.0125	0.00015625	0.11	0.06	0.0036
3	0.15	0.0475	0.00225625	-0.02	-0.04	0.0049
4	0.05	-0.0525	0.00275625	0.04	-0.01	0.0001
	Avg = 0.1025		Sum = 0.005475	Avg = 0.05		Sum = 0.009

a. Average Return of Stock A = 0.1025 or 10.25%
   Average Return of Stock B = 0.05 or 5.00%

b. Standard Deviation of return for Stock A = SQRT(0.005475/3) = 0.04272
   Standard Deviation of return for Stock B = SQRT(0.009/3) = 0.05477

c. Covariance of returns

$$\text{Covariance} = \frac{(r_{X1} - \bar{r}_X)(r_{Y1} - \bar{r}_Y) + (r_{X2} - \bar{r}_X)(r_{Y2} - \bar{r}_Y) + \cdots (r_{Xn} - \bar{r}_X)(r_{Yn} - \bar{r}_Y)}{n-1}$$

= [(0.0175)*(0.02) + (-0.0125)*(0.06) + (0.0475)*(-0.04) + (-0.0525)*(-0.01)]/3 = -0.001067

d. Correlation Coefficient
   = -0.001067/(0.04272*0.05477)
   = -0.4559

$$\rho_{A,B} = \frac{\mathrm{Cov}(r_A, r_B)}{\sigma_A \sigma_B}$$