NANYANG TECHNOLOGICAL UNIVERSITY SINGAPORE

CZ3005 Artificial Intelligence

Week 11a – Default Logic

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Learning Goals

Understanding the:

- Default Rules
- Default Theory
- Default Logic Inference
  – Reiter Default Logic (RDL) inference
  – Limitations of RDL

Default Logic

- Standard logic can only express that something is true or that something is false.
  – **Explicitly given rules** (i.e., something is true if and only if it is explicitly stated as true in the knowledge base)
- Default logic is proposed to formalize reasoning with default assumptions.
  – **Common sense** (i.e., if there is nothing in the knowledge base that conflicts with our common sense, then we apply our common sense)
- It can express facts like
  – "by default, something is true".

Default Logic

- What do we see here?
  – A child is using his handphone
- What do we intuitively think is going on?
  – He is playing a mobile game
- What do we intuitively conclude?
  – Doing so might negatively affect his study

Default Logic

- Default Rules
- Knowledge Base

Default Theory

- Default Logic Inference
  - Reiter Default Logic Inference
  - Makinson Approach
  - Process Tree

Default Rule

As long as Just^n don't conflict ⇒ draw conclusion

$$\frac{\text{Prerequisite} : \text{Justification}_1, \ldots, \text{Justification}_n}{\text{Conclusion}}$$

According to this default:
- If we believe that Prerequisite is true;
- AND each of Justifications is consistent with our current beliefs;
- THEN, we are led to believe that the Conclusion is true.

Default Rule

If $\alpha(x)$ is T & $\beta_1(x) \ldots \beta_n(x)$ does not conflict with $\alpha(x)$
⇒ conclude $\gamma(x)$ is T

prerequisite

justification

$$\frac{\alpha(\boldsymbol{x}):\ \beta_1(\boldsymbol{x}), \ldots, \beta_n(\boldsymbol{x})}{\gamma(\boldsymbol{x})}$$

conclusion

where $\boldsymbol{x} = x_1, \ldots, x_m$, and $\alpha(\boldsymbol{x}), \beta_1(\boldsymbol{x}), \ldots, \beta_n(\boldsymbol{x}), \gamma(\boldsymbol{x})$ are formulae whose free variables are among $x_1, \ldots, x_m$.

e.g.,:

$$\frac{bird(\boldsymbol{x}):\ has\_wings(\boldsymbol{x})}{fly(\boldsymbol{x})}$$

⇒ Are there any thing in our current KB that says bird x doesn't have wings?

"By default, a bird can fly, unless we know that a particular bird has lost its wings"

Default Rule

The default is **applied** by substituting **c** (the ground instance) into α and β to infer γ:
- Trigger: $\alpha(\boldsymbol{c})$ belongs to our set of beliefs.
- Justification: the set of our beliefs is consistent with each $\beta(\boldsymbol{c})$.

Our KB does not entail $\neg\beta(\boldsymbol{c})$

e.g.,:

$$\frac{bird(\boldsymbol{x}):\ has\_wings(\boldsymbol{x})}{fly(\boldsymbol{x})} \quad \{bird(Tweety), \neg has\_wings(Tweety)\}$$

Tweety no wings

∴ cannot conclude that tweety can fly

① confirm c is a bird
⇓ if T
② is there any fact that c does not have wings

if no fact concludes Tweety has wings or not that we can conclude Tweety fly.

Types of Default Rules

- Normal Defaults: $\frac{\alpha(x)\ :\ \gamma(x)}{\gamma(x)}$

- Semi-Normal Defaults: $\frac{\alpha(x)\ :\ \beta(x)}{\gamma(x)}$, where $\beta(x) \vdash \gamma(x)$

  E.g., $\frac{bird(x)\ :\ has\_wings(x)}{flies(x)}$,

  where $has\_wings(x) \vdash flies(x)$

Types of Default Rules

- Open Defaults (Default Schemas) have unbounded variables, e.g., *x*

$$\frac{\alpha(x)\ :\ \beta_1(x), \dots, \beta_n(x)}{\gamma(x)}$$

- Closed (Grounded) Defaults use ground terms, e.g., *x=c*

$$\frac{\alpha(c)\ :\ \beta_1(c), \dots, \beta_n(c)}{\gamma(c)}$$

Default Theory

Delta: Set of Default Rules

$$\langle \Delta, \Phi \rangle$$

Phi: A given KB (a set of "Facts")

Example – the default rule that "birds typically fly":

- $\Delta = \left\{ \frac{bird(x)\ :\ flies(x)}{flies(x)} \right\}$

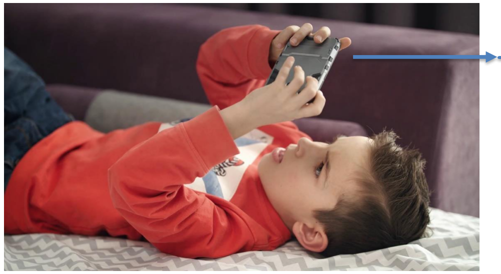
  - This rule means that, "*if x is a bird, and there is no fact in the knowledge base suggesting that x cannot fly, then we can conclude that x flies*".

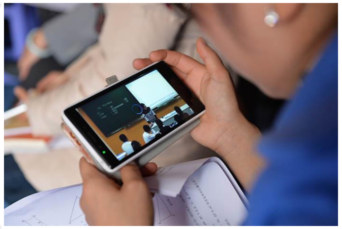
- $\Phi = \{bird(Tweety), cat(Sylvester)\}$

Example

- What do we see here?
  - A child is using his handphone (Pre-requisite)
- What do we intuitively think is going on?
  - He is playing a mobile game (Justification)
- What do we intuitively conclude?
  - Doing so might negatively affect his study (Conclusion)

Example





He is attending online class (not playing game)

- What do we see here?
  – A child is using his handphone (<u>Pre-requisite</u>)
- What do we intuitively think is going on?
  – He is playing a mobile game (<u>Justification</u>)
- What do we intuitively conclude?
  – Doing so might negatively affect his study (<u>Conclusion</u>)

Conflict with our default justification

Reiter Default Logic (RDL) Inference

- Guess the extension Ξ (pronounced as “Xi”)
- Initialise beliefs $\Xi^* = \Phi$ ⇒ initialise initial belief to fact part in KB.
- (loop over) c-ground instance of an (unused) default $\frac{\alpha(x)\ :\ \beta(x)}{\gamma(x)}$:
  - Check two conditions → is c a member of class α
    - Triggered?: $\Xi^* \vdash \alpha(c)$
    - Justified?: $\beta(c)$ is consistent with Ξ ⇒ look through KB to see if any fact contradict β(x)
  - If yes: update beliefs $\Xi^* \leftarrow \Xi^* \cup \{\gamma(c)\}$ → Add new sentence to KB that γ(c) is true
- (end loop)
- If $\Xi = \Xi^*$ then extension found/confirmed

The extension is added to our KB as new knowledge

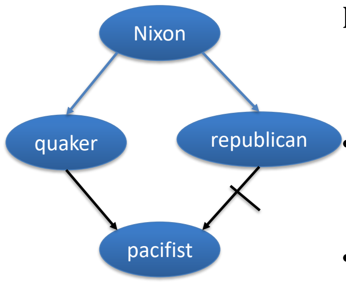
Example

Given Theory: $\mathrm{T} = \left\langle \Delta = \left\{\frac{bird(x)\ :\ flies(x)}{flies(x)}\right\}, \Phi = \{bird(Tweety), cat(Sylvester)\}\right\rangle$, (KB)

- Guess the extension $= \mathrm{Cn}(\{flies(Tweety)\} \cup \Phi)$
- Our initial knowledge is $F = \Phi$
- Sylvester-instance of default not applicable:
  - not hold $\Phi \vdash bird(Sylvester)$
- $\Phi \vdash bird(Tweety)$ and $flies(Tweety)$ is consistent with F
- $F = \Phi \cup \{flies(Tweety)\}$
- No more default rules to apply
- An extension is reached

RDL Limitation - Nixon Diamond

The default rules may be applied in <u>different orders</u>, and this may lead to <u>different extensions</u>. E.g.:



Let Theory $\mathrm{T} = \langle \Delta, \Phi \rangle$

- $\Delta = \left\{ \frac{quaker(x)\ :\ pacifist\ (x)}{pacifist(x)}, \frac{republican(x)\ :\neg pacifist\ (x)}{\neg pacifist(x)} \right\}$
- $\Phi = \{quaker(Nixon), republican(Nixon)\}$

RDL Limitation - Nixon Diamond

Given $\langle \Delta, \Phi \rangle$

- $\Delta = \left\{ \frac{quaker(x)\ :\ pacifist(x)}{pacifist(x)}, \frac{republican(x)\ :\neg pacifist(x)}{\neg pacifist(x)} \right\}$
- $\Phi = \{quaker(Nixon), republican(Nixon)\}$

There are **two** extensions:

1. One that contains: **pacifist(Nixon)**
2. .. and the one that contains: **¬ pacifist(Nixon)**

Conflicting conclusions

if analyse Republican first
↳ the conclusion will be diff ∴ conflicts

Addressing the RDL Limitation

- A default theory can have 0, 1 or more extensions.
- Entailment of a formula from a default theory can be defined in one of two ways:

decision maker's properties

  - Skeptical:
    - a formula is entailed by a default theory if it is entailed by all its extensions.

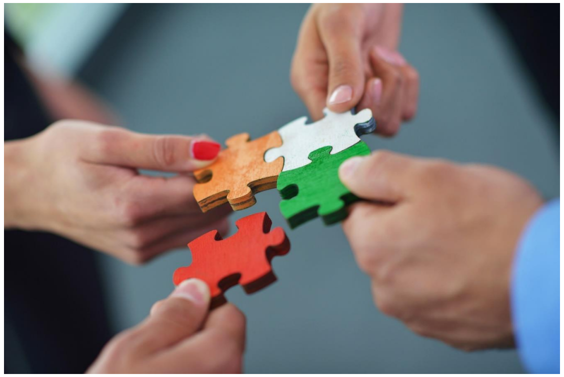
  OR

  - Credulous: (Relaxed)
    - a formula is entailed by a default theory if it is entailed by at least one of its extensions.

Example

- The Nixon diamond example theory has two extensions:
  - one in which Nixon is a pacifist; and
  - one in which Nixon is not a pacifist.
- Thus, we have:
  - Neither Pacifist(Nixon) nor ¬Pacifist(Nixon) are skeptically entailed.
  - Both Pacifist(Nixon) and ¬Pacifist(Nixon) are credulously entailed.
- The credulous extensions of a default theory can be inconsistent with each other.

Thank you!



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Understanding the:

- Default Logic Inference
  - Makinson Approach
  - Process Tree

Recap

- A default rule can be applied to a theory
  - if its precondition is entailed by the theory; and
  - its justifications are all consistent with the theory.
- The application of a default rule leads to the addition of its consequence to the theory.
- Other default rules may then be applied to the resulting theory.
- When the theory is such that no other default can be applied, the theory is called an <u>extension</u> of the default theory.
- The default rules may be applied in <u>different orders</u>, and this may lead to <u>different extensions</u>.

Makinson Approach

- Order ***ground*** instances of defaults in Δ: $d_1, d_2, \ldots$
- Initialize beliefs $\Xi_0 = \Phi$ and used defaults set $\Delta_0 = \emptyset$
- Define Ξ_{n+1} from Ξ_n,
  - Find d $= \frac{\alpha(c)\,:\,\beta_1(c),\ldots,\beta_n(c)}{\gamma(c)} \notin \Delta_n$ such that
    - Triggered?: $\Xi_n \vdash \alpha(c)$
    - Justified?: Ξ_n is consistent with $\beta_1(c), \ldots, \beta_m(c)$
  - If $\Xi_n \cup \{\gamma(c)\}$ is consistent with each $\beta'(c')$ in $\Delta_n \cup \{d\}$
    - $\Xi_{n+1} = \Xi_n \cup \{\gamma(c)\}, and\ \Delta_{n+1} = \Delta_n \cup \{d\}$
  - else abort -- no extension **for this order of defaults**
- The extension is $\Xi = \cup_{i \geq 0} \Xi_i$

Makinson Approach

- No extension guessing
  - **Choose** the order of defaults in Δ: $d_1, d_2, \ldots$
- There still may be more than one possible extension
  - **Different orders** of defaults can lead to different Ξ
- We get the same extensions as in Reiter's approach
  - If they exist at all

Remember …

Delta: Set of Defaults

$\langle \Delta, \Phi \rangle$

Phi: Set of "Facts"

$\mathrm{Cn}()$

Cn means applying any known inference rules to expand the KB

Operational Semantics

Given a default theory $\mathrm{T} = \langle \Delta, \Phi \rangle$, let $\Pi = (\delta_0, \delta_1, \ldots)$ be (a finite or infinite) sequence of (closed) defaults from Δ without multiple occurrences.

$\Pi[\mathrm{k}]$ denotes the initial segment of sequence Π with length k.

Model of the world

initial set of facts

union w conclusions for

Each sequence Π is associated with two sets:

- $\mathrm{In}(\Pi) = Cn(\Phi \cup \{consequence(\delta) \mid \delta \text{ occurs in } \Pi\})$
- $\mathrm{Out}(\Pi) = \{\neg\phi \mid \phi \in justifications(\delta) \text{ for some } \delta \text{ in } \Pi\}$

Everything I have denied the existence of as a result of my assumptions (i.e., the negation of all the justifications)

For any default rule that we have … negation of justification

Example

if α T

$\neg\beta$ is T

β : T

Consider $\mathrm{T} = \langle \Delta, \Phi \rangle$ with $\Phi = \{\alpha\}$ and defaults from Δ:

nothing conflicts them

γ : T

$$\delta_1 = \frac{\alpha \ : \neg\beta}{\neg\beta}, \quad \delta_2 = \frac{\beta \ : \ \gamma}{\gamma}$$

$\neg\beta$: T

Added to in set

Adds negation of justification.

For $\Pi_\mathrm{a} = (\delta_1)$ we have

$$\mathrm{In}(\Pi_a) = Cn(\{\alpha, \neg\beta\}), \mathrm{Out}(\Pi_a) = \{\beta\}$$

For $\Pi_\mathrm{b} = (\delta_2, \delta_1)$ we have

$$\mathrm{In}(\Pi_b) = Cn(\{\alpha, \neg\beta\}), \mathrm{Out}(\Pi_b) = \{\beta\}$$

R_2 triggered first but is not supported by KB

∴ Apply R_1 ∴ same concl

concluded ⇒ Add negation of just...
portion of those rules

Process, Successful, Closed

Π is a process of $T = \langle \Delta, \Phi \rangle$ iff default δ_k is applicable to $In(\Pi[k])$ for every k such that δ_k occurs in Π.

Let Π be a process. We define:

- Π is **successful** iff $In(\Pi) \cap Out(\Pi) = \emptyset$ (*Nothing in the out set can be inferred from the in set*); Otherwise, it **fails**.
- Π is **closed** iff every $\delta \in \Delta$ that is applicable to $In(\Pi)$ already occurs in Π.

If every default rule in the current default theory that's applicable has already occured and been extended

Example

Consider $T = \langle \Delta, \Phi \rangle$ with $\Phi = \{\alpha\}$ and defaults from Δ:

$$\delta_1 = \frac{\alpha \ : \neg\beta}{\eta}, \quad \delta_2 = \frac{true \ : \ \gamma}{\beta}$$

$\Pi_1 = (\delta_1)$ is successful, $In(\Pi_1) = Cn(\alpha, \eta)$ and $Out(\Pi_1) = \{\beta\}$ but not closed, since δ_2 is applicable, too.

-No over lap b/w In set & out set
-No rules that says if α or μ T ⇒ then β ⇒ T

$\Pi_2 = (\delta_1, \delta_2)$ is closed, but not successful
$In(\Pi_2) = Cn(\alpha, \eta, \boxed{\beta})$ and $Out(\Pi_2) = \{\boxed{\beta}, \neg\gamma\}$,
$In(\Pi_2) \cap Out(\Pi_2) = \beta$

All rules explored

conflict

Nothing concludes γ ⇒ ∴ β ⇒ T ⇒ Add to In & negation of $\neg\beta$ out

$\Pi_3 = (\delta_2)$ is a closed and successful process T
$In(\Pi_3) = Cn(\alpha, \beta)$ and $Out(\Pi_3) = \{\neg\gamma\}$,
$In(\Pi_3) \cap Out(\Pi_3) = \emptyset$

δ_1 : α applies but
- Justification β : T conflicts w δ_2
$\neg\beta$ ∴ μ non conclusive
- δ_1 triggered but not successfully extended : N.A

Extension

– Let $T = \langle \Delta, \Phi \rangle$ be a default theory. A set of formulae Ξ is an **extension** of T iff there is some ***closed and successful*** Π such that $\Xi = In(\Pi)$.

– To **find a successful** process: generate a process Π, test whether $in(\Pi) \cap Out(\Pi) = \emptyset$. If not, then backtrack (try another process).

Process Tree

$T = \langle \Delta, \Phi \rangle$ be a default theory. A **process tree** is a tree $G = (V, E)$ such that all nodes $v \in V$ are labelled with two sets of formulae:

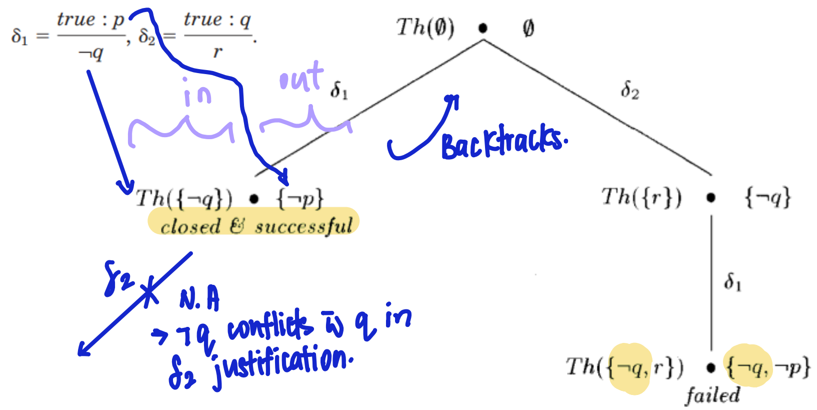
- an In-set $In(v)$ and
- an Out-set $Out(v)$.

The root of G is labelled with $Cn(\Phi)$ as the In-set and $\emptyset$ as the Out-set. Every $e \in E$ denotes a default application and is labelled by it.

A process is thus a path in G starting from the root.
A node $v \in V$ is **expanded** if $In(v) \cap Out(v) = \emptyset$. Otherwise, it is a "failed" leaf of the tree.

Process Tree Example

Let $T = (W, D)$ be the default theory with $W = \emptyset$ and $D = \{\delta_1, \delta_2\}$ with



Process Tree: Properties

- A process is thus a path in G starting from root.
- A node $v \in V$ is **expanded** if $In(v) \cap Out(v) = \emptyset$.
- Otherwise, it is a “failed” leaf of the tree.
- Expanded $v \in V$ has a child node, w_δ, for every $\delta = \frac{\alpha : \beta_1, \ldots, \beta_n}{\gamma}$
  - w_δ does not appear on the path from the root to v
  - δ is applicable to $In(v)$
  - w_δ connected to v by an edge labelled with δ
  - w_δ is labelled with $In(w_\delta) = Cn(In(v) \cup \{\gamma\})$ and $Out(w_\delta) = Out(v) \cup \{\neg\beta_1, \ldots, \neg\beta_n\}$

Thank you!