CZ2003 Tutorial 4 (2020/21, Semester 1)

Planes, polygons and bilinear surfaces

1. **Using an equation in intercept**s, write an implicit equation of the plane which intersects the Cartesian coordinate axes X, Y and Z at the three points with coordinates $P_1=(1, 0, 0)$, $P_2=(0, 2, 0)$ and $P_3=(0, 0, 3)$, respectively.

2. Write an implicit equation of a plane which passes through the point with Cartesian coordinates (1, 2, 3) while being orthogonal to the straight line defined by $x = u + 2, \quad y = u - 1, \quad z = 3u + 1, \quad u \in (-\infty, \infty)$.

3. Propose how to define parametrically with functions $x(u,v), y(u,v), z(u,v)$ a plane passing through points with coordinates (-2,0,0), (0,3,0), (0,0,4).

4. (a) A bilinear surface is defined by four points $P_1=(-1, 1, -1)$, $P_2=(1, 0, -1)$, $P_3=(-1, 0, 1)$ and $P_4=(1, 0.5, 1)$ and two parametric coordinates $u \in [0,1]$ and $v \in [0,1]$, as illustrated in Figure Q3. Write parametric equations defining the bilinear surface.

   (b) What are the coordinates of the point with the parametric coordinates 0.4, 0.8?

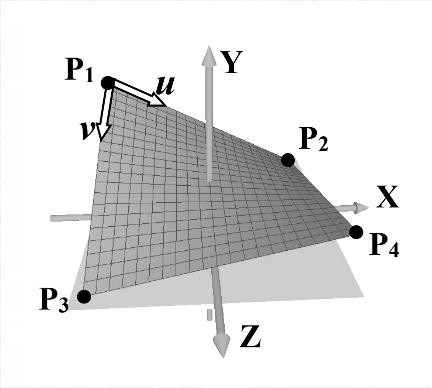


Figure Q3

5. Write parametric equations $x(u, v)$, $y(u, v)$, $z(u, v)$, $u, v \in [0, 1]$ defining a triangular polygon which is bounded by the three segments defined by:

$$x = 1 + 2u \qquad y = 1 + u \qquad z = 1 - u \qquad u \in [0, 1]$$
$$x = 3 - u \qquad y = 2 + u \qquad z = 4u \qquad u \in [0, 1]$$
$$x = 2 - u \qquad y = 3 - 2u \qquad z = 4 - 3u \qquad u \in [0, 1].$$

1. **Using an equation in intercept**s, write an implicit equation of the plane which intersects the Cartesian coordinate axes X, Y and Z at the three points with coordinates $P_1=(1, 0, 0)$, $P_2=(0, 2, 0)$ and $P_3=(0, 0, 3)$, respectively.

implicit equation in intercepts.

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}-1=0$$

$P_1=\begin{pmatrix}1\\0\\0\end{pmatrix}$

$P_2=\begin{pmatrix}0\\2\\0\end{pmatrix}$

$P_3=\begin{pmatrix}0\\0\\3\end{pmatrix}$

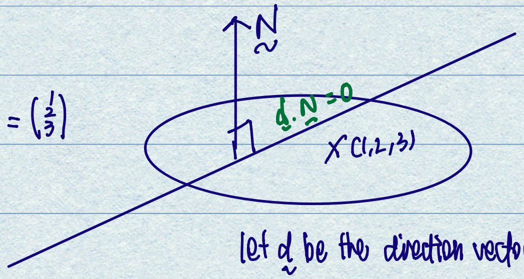
$$\frac{x}{1}+\frac{y}{2}+\frac{z}{3}-1=0 \quad //$$

$$\Rightarrow 6x+3y+2z-6=0$$

2. Write an implicit equation of a plane which passes through the point with Cartesian coordinates (1, 2, 3) while being orthogonal to the straight line defined by $x = u + 2$, $y = u - 1$, $z = 3u + 1$, $u \in (-\infty, \infty)$.

at right angles. Perpendicular

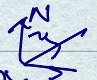
implicit equation of a plane:



$x=u+2$

$y=u-1$

$z=3u+1$



For any point $\underset{\sim}{r_0}=(x_0,y_0,z_0)$ $\therefore \underset{\sim}{r_0}=\begin{pmatrix}1\\2\\3\end{pmatrix}$

$\underset{\sim}{N}\cdot(r-r_0)=0$

$\underset{\sim}{N}=\begin{pmatrix}A\\B\\C\end{pmatrix}$ A,B,C corresponds to normal vector of plane

$Ax+By+Cz+D=0$ ← EQN of plane

↳ specific D gives us a specific planar surface

point $r_0=\begin{pmatrix}1\\2\\3\end{pmatrix}$ lies on the plane

let $\underset{\sim}{d}$ be the direction vector of the line

$\underset{\sim}{d}=\begin{pmatrix}1\\1\\3\end{pmatrix}$ ← to get this, you can also sub any no. of u into the line, find 2 pts then subtract

$(1)(x)+(1)(y)+3(z)+D=0$

$x+y+3z+D=0$

sub $r_0=\begin{pmatrix}1\\2\\3\end{pmatrix}$

$1+2+3(3)+D=0$

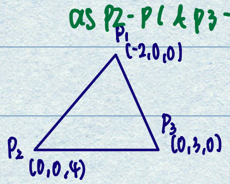
$D=-12$

$\therefore$ the equation is: $x+y+3z-12=0$ //

3. Propose how to define parametrically with functions $x(u,v)$, $y(u,v)$, $z(u,v)$ a plane passing through points with coordinates (-2,0,0), (0,3,0), (0,0,4).

Fixed formula given by lec. i.e each point on the plane can be computed as a sum of 2 vectors defined by 3 pts as P2-P1 & P3-P1 & scaled by the parameters u,v.

$P = P_1 + U * (P_2 - P_1) + V * (P_3 - P_1)$



$x(u,v) = -2 + u(0+2) + v(0+2)$

$= -2 + 2u + 2v$ #

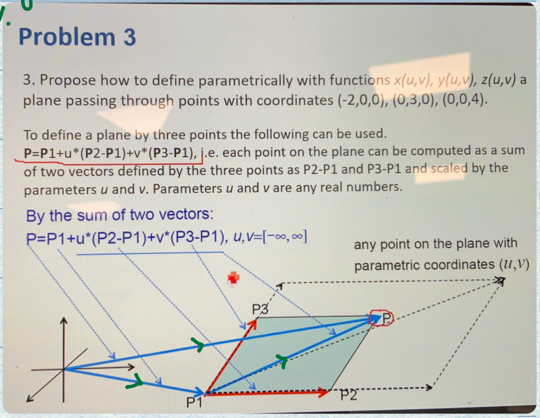
$y(u,v) = 0 + u(0-0) + v(3-0)$

$= 3v$ #

* $u, v \in \mathbb{R}$

$z(u,v) = 0 + u(4-0) + v(0)$

$= 4u$ #



4. (a) A bilinear surface is defined by four points $P_1=(-1, 1, -1)$, $P_2=(1, 0, -1)$, $P_3=(-1, 0, 1)$ and $P_4=(1, 0.5, 1)$ and two parametric coordinates $u \in [0,1]$ and $v \in [0,1]$, as illustrated in Figure Q3. Write parametric equations defining the bilinear surface.

4(a) using the general formula: $P = P' + (P'' - P')V$

$P = P_1 + u(P_2 - P_1) + V(P_3 - P_1 + u(P_4 - P_3 - P_2 + P_1))$

$P_1 = (-1, 1, -1)$ $P_2 = (1, 0, -1)$ $P_3 = (-1, 0, 1)$ $P_4 = (1, 0.5, 1)$

$x(u,v) = -1 + u(2) + v(\cancel{-1+1} + u(\cancel{1+1-1-1}))$

$= -1 + 2u$ #

$y(u,v) = 1 + u(0-1) + v(0 - 1 + u(0.5 - 0 - (0-1)))$

$= 1 - u + v(-1 + u(1.5))$

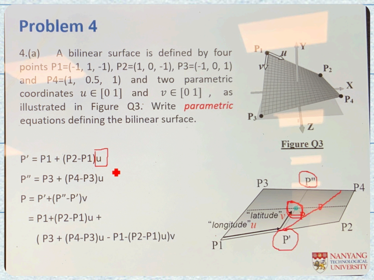
$= 1 - u - v + 1.5uv$ #

$z(u,v) = -1 + \cancel{u(-1-(-1))} + v(1-(-1) + u(\cancel{1-1} - (\cancel{-1}-(-1))))$

$= -1 + v(2)$

$= -1 + 2v$ #

$u = [0,1]$, $v = [0,1]$



(b) What are the coordinates of the point with the parametric coordinates 0.4, 0.8?

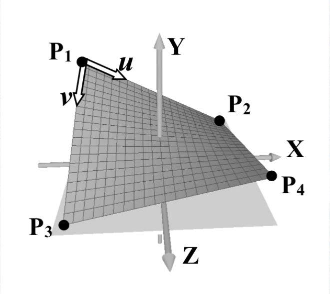


Figure Q3

$u=0.4$ $\quad x = -1+2u$ $\quad y = 1-u-v+1.5uv$ $\quad z = -1+2u$

$v=0.8$ $\quad = -1+2(0.4)$ $\quad = 1-0.8-0.4+1.5(0.4)(0.8)$ $\quad = -1+2v$

$= -0.2$ $\quad = 0.28$ $\quad = -1+2(0.8)$

$= 0.6$

5. Write parametric equations $x(u, v)$, $y(u, v)$, $z(u, v)$, $u, v \in [0, 1]$ defining a triangular polygon which is bounded by the three segments defined by:

$x = 1 + 2u$	$y = 1 + u$	$z = 1 - u$	$u \in [0, 1]$
$x = 3 - u$	$y = 2 + u$	$z = 4u$	$u \in [0, 1]$
$x = 2 - u$	$y = 3 - 2u$	$z = 4 - 3u$	$u \in [0, 1]$.

first define coordinates of the 3 vertices.

when $u=0$

$x=1+2u = 1+2(0) = 1$

$y = 1+u = 1+0 = 1$

$z = 1-u = 1$

$P_1 = (1,1,1)$

when $u=1$

$x = 1+2(1) = 3$

$y = 1+1 = 2$

$z = 1-1 = 0$

$P_2 = (3,2,0)$

when $u=1$

$x = 3-u = 3-1 = 2$

$y = 2+u = 2+1 = 3$

$z = 4u = 4(1) = 4$

$P_3 = (2,3,4)$

using the general eqn:

$P = P_1 + u(P_2 - P_1) + v(P_3 - P_1 + u(P_4 - P_3 - (P_2 - P_1)))$

to form a triangle polygon, make $P_3 = P_4$

$x(u,v) = 1 + u(3-1) + v(2-1 + u(2-2 - (3-1)))$

$= 1+2u + v(1+u(-2))$

$z(u,v) = 1+u(-1) + v(4-1 + u(4-4 - (0-1)))$

$= 1-u+v(3+u)$

$$= 1 + 2u + v - 2uv \quad \#$$

$$= 1 - u + 3v + vu \quad \#$$

$$y(u,v) = 1 + u(2-1) + v(3 - 1 + u(\cancel{3-}3 - (2-1)))$$

$$u, v \in [0, 1]$$

$$= 1 + u + v(2-u)$$

$$= 1 + u + 2v - uv$$

$P4 = P3.$

$P' = P1 + u(P2 - P1)$, $P'' = P3.$

$$P = P' + v(P'' - P')$$

$$= P1 + u(P2 - P1) + v(P3 - P1 - u(P2 - P1))$$

$$= P1 + u(P2 - P1) + v(P3 - P1) - uv(P2 - P1)$$