

CZ2003 Tutorial 2 (2020/21, Semester 1)

Mathematical functions in computer graphics

1. Give a definition of mathematical function.
2. What ways of analytic definition of mathematical functions do you know?
3. Given an explicit function $y = 2\sin(x) + \cos(3x)$, propose how to convert it to the respective parametric functions $x = f_1(t)$ $y = f_2(t)$?
4. (i) Given parametric functions $x = \sin(t)$ and $y = \cos(t)$, obtain the respective implicit function $f(x,y) = 0$.
(ii) Given parametric functions $x = 3 + 2t$ and $y = 3 + t$, obtain the respective implicit function $f(x,y) = 0$.

1. Give a definition of mathematical function.

A function associates one quantity, the argument of the function, also known as the input, with another quantity, the value of the function, also known as the output. A function assigns exactly one output to each input. Values from the input domain map to the values in the function range.

2. What ways of analytic definition of mathematical functions do you know?

1. Explicit functions.

e.g. $y=f(x)$. The ordered pair (x, y) is obtained easily. It's a function in which the dependent variable is expressed in terms of some independent variable.

- E.g. $x=f(y)$, $y=f(x)$, $z=f(x,y)$, $g=f(x,y,z)$
- E.g. $y=5x^3-3$

2. Implicit functions

is a function in which the dependent variable has not been given "explicitly" in terms of the independent variable. Or it is a function in which the dependent variable is not expressed in terms of some independent variable.

E.g. $f(x)=0$, $f(x,y)=0$, $f(x,y,z)=0$

E.g. $x^3+y^3-1=0$

Although it is theoretically possible to find all the functions represented, it may also be impossible to find them

3. Parametric functions.

For example, the ordered pair (x, y) is defined in terms of a parameter t , such as

$$x=t+1, y=t^2+t$$

For different values of the parameter t , we get different ordered pairs (x, y) .

It defines a group of quantities as functions of one or more independent variables called parameters.

E.g. $x=f_1(t)$ $y=f_2(t)$ $z=f_3(t)$ $T=[T_1, T_2]$

$$x=f_x(u, v) \quad y=f_y(u, v) \quad z=f_z(u, v) \quad u=[u_1, u_2] \quad v=[v_1, v_2]$$

3. Given an explicit function $y = 2\sin(x) + \cos(3x)$, propose how to convert it to the respective parametric functions $x = f_1(t)$ $y = f_2(t)$?

$$y = 2\sin(x) + \cos(3x)$$

Assign $x = t$

$$y = 2\sin(t) + \cos(3t) //$$

4. (i) Given parametric functions $x = \sin(t)$ and $y = \cos(t)$, obtain the respective implicit function $f(x,y) = 0$.

- (ii) Given parametric functions $x = 3 + 2t$ and $y = 3 + t$, obtain the respective implicit function $f(x,y) = 0$.

4(i)

$$x = \sin(t)$$

$$y = \cos(t)$$

$$4(ii) \quad x = 3 + 2t \Rightarrow t = \frac{x-3}{2}$$

$$x^2 = \sin^2(t)$$

$$y^2 = \cos^2(t)$$

$$y = 3 + t$$

$$\text{Since } \sin^2(t) + \cos^2(t) = 1$$

$$y = 3 + \frac{x-3}{2}$$

$$\therefore x^2 + y^2 = 1$$

$$= \frac{6+x-3}{2} = \frac{3+x}{2}$$

$$\text{implicit function: } x^2 + y^2 - 1 = 0 //$$

$$\text{implicit function: } y - \frac{3+x}{2} = 0 //$$

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