FIN 2004/2704/2004X/2704X

Week 5 Slides

Portfolio Return & Risk

Learning objectives

Understand the return and risk of a portfolio and how diversification affects these.

Portfolio returns example

Suppose you invest *$100,000.* You buy 200 shares of Apple at $200 per share *($40,000*) and 1000 shares of Coca-Cola at $60 per share *($60,000*).

At the end of the year, if Apple's stock goes up to $240 per share and Coca-Cola stock falls to $57 per share and neither paid dividends, what is the new value of the portfolio?

What ***return did the portfolio earn***? If you don't buy or sell any shares after the price change, what are the ***new portfolio weights***?

Portfolio returns example (cont.)

200 shares of Apple: \$200 → \$240 (\$40 capital gain/share)
1000 shares of Coca-Cola: \$60 → \$57 (\$3 capital loss/share)

New value:
Apple shares: 200 x \$240 = \$48,000
Coca-Cola shares: 1000 x \$57 = \$57,000

New value = \$105,000

Portfolio return = (105K – 100K)/100K = **5%**

New portfolio weight:
Apple: 48K/105K = **45.71%**
Coca-Cola: 57K/105K = **54.29%**

Note: old weightage = 40/60

Portfolio Returns Example (cont.)

Assume that Apple and Coca-Cola are the only assets in the economy

Originally, Apple original market cap of $400,000 (2000 shares at $200/share)

Coca-Cola had a market cap of $600,000 (10,000 shares at $60/share)

*The weightage of Apple vs. Coca-Cola in the economy: **40%** vs. **60%**.*

At the end of the year, Apple's market cap is $480,000 (2000 shares at $240/share).

Coca-Cola's market cap is $570,000 (10,000 shares at $57/share)

The new weightage of Apple vs. Coca-Cola in the economy: **45.71%** vs. **54.29%**

Our portfolio is <u>a mini portfolio </u>of the larger market. When the larger market valuations changed, so too did the mini market portfolio.

The Volatility of a Portfolio

- Generally, investors in companies like Apple care less about Apple's specific return and more about how Apple contributes to their portfolio's overall return. As well, they care about their **portfolio's overall risk**. Understanding how Apple's investors think about Apple's risk requires us to understand how to calculate the risk of a portfolio.
- When we combine stocks in a portfolio, some of the stock's risk is eliminated through diversification. The amount of risk that will remain in the portfolio depends upon the degree to which the stocks included in the portfolio share common risk (i.e., their correlation).

The **volatility of a portfolio** is the total risk of the portfolio, as measured by the portfolio standard deviation.

Returns for Three Stocks, & Portfolios of Pairs of Stocks

	Stock Returns			Portfolio Returns	
Year	North Air	West Air	Tex Oil	(1) Half N.A. and Half W.A.	(2) Half W.A. and Half T.O.
1998	21%	9%	−2%	15.0%	3.5%
1999	30%	21%	−5%	25.5%	8.0%
2000	7%	7%	9%	7.0%	8.0%
2001	−5%	−2%	21%	−3.5%	9.5%
2002	−2%	−5%	30%	−3.5%	12.5%
2003	9%	30%	7%	19.5%	18.5%
Avg. Return	10.0%	10.0%	10.0%	10.0%	10.0%
Volatility	13.4%	13.4%	13.4%	12.1%	5.1%

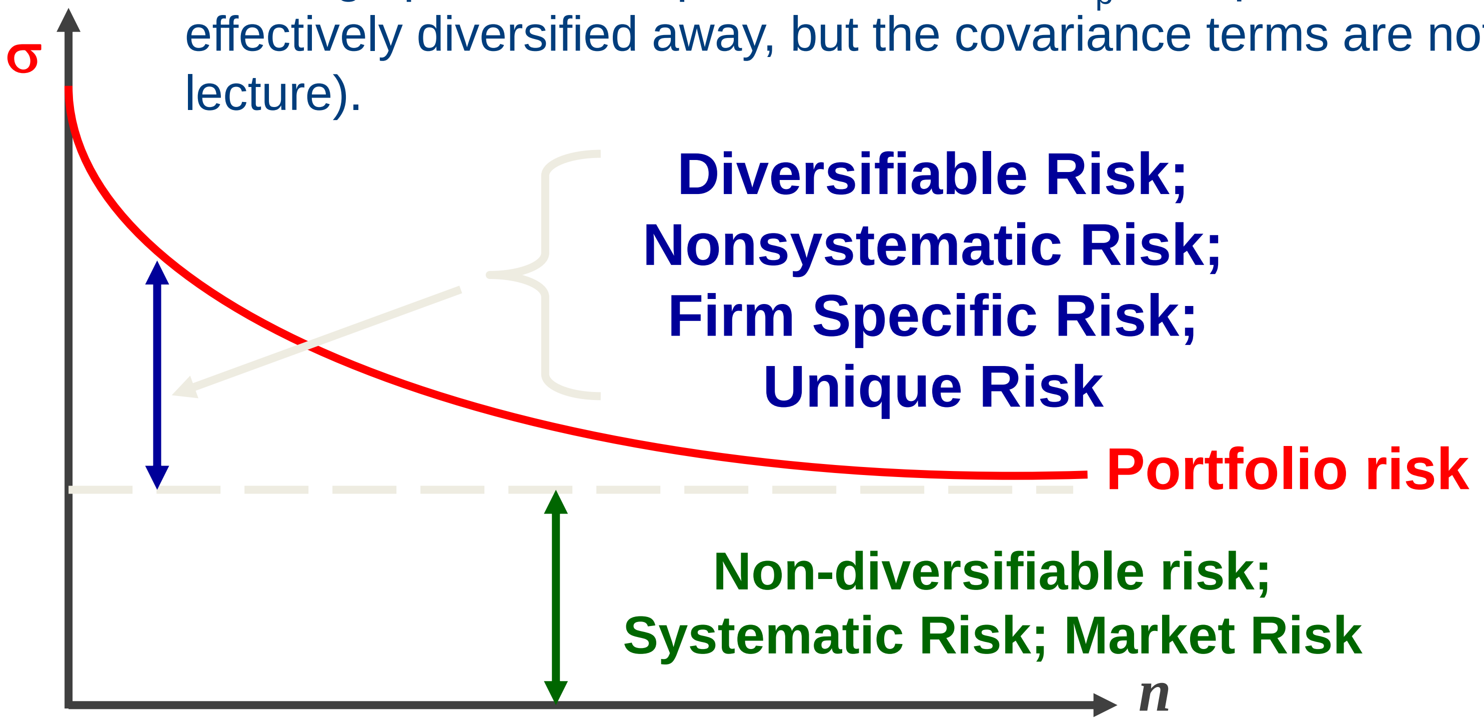
When will stock returns be highly correlated with each other?
Stock returns will tend to move together if they are affected similarly by ***economic events***. Thus, stocks in the same industry tend to have more highly correlated returns than stocks in different industries.

Diversification Summary

- Portfolio diversification involves investment in several different asset classes or sectors that are not perfectly correlated with each other.
- Diversification is not just holding a lot of assets.
  - For example, if you own 50 internet stocks, you are not diversified. However, if you own 50 stocks that span 20 different industries, then you are clearly much more diversified.
- **Diversification** can substantially <u>reduce the variability of returns without an equivalent reduction in expected returns</u>.
  - Because <u>worse-than-expected returns</u> from one asset are <u>offset by better-than-expected returns</u> from another
- However, there is a minimum level of risk that cannot be diversified away and that is the **systematic** portion.

Portfolio Risk as a Function of the Number of Stocks in the Portfolio

In a large portfolio, for portfolio variance σ_p, the 'pure' variance terms are effectively diversified away, but the covariance terms are not (Recall last lecture).



Thus diversification can <u>eliminate some *(i.e.,'pure variance' terms*), but not all</u> of the risk of individual securities (i.e., the individual securities' impact on the total portfolio variance via their covariance with other assets) because most assets are positively correlated with one another).

Total Risk

Total risk = Systematic risk + Unsystematic risk

- The standard deviation of returns is a measure of total risk.
- For well diversified portfolios, unsystematic risk is very small. Consequently:

- The total risk measure (σ) for a **well-diversified portfolio** is essentially *equivalent* to the systematic risk!

- This is ***not*** the case for an individual asset (i.e., the total risk measure (σ) for an individual asset is not equivalent to its systematic risk as most individual assets will have unsystematic risk as well).

Summary

- How diversification affects the volatility of a portfolio
- What are total risk, systematic risk, and unsystematic risk
  – Which of these risks can be diversified away?

Capital Asset Pricing Model (CAPM)

Learning objectives

- Learn how to calculate an asset's market risk (non-diversifiable, systematic risk), known as Beta
- Know how to calculate portfolio returns, betas, and required rates of return
- Understand the Capital Asset Pricing Model and the link between an asset's Beta and its required rate of return.
- Be able to use the Capital Asset Pricing Model to assess whether an asset is correctly priced, overpriced or underpriced.

Risk When Investors Hold a Diversified Portfolio

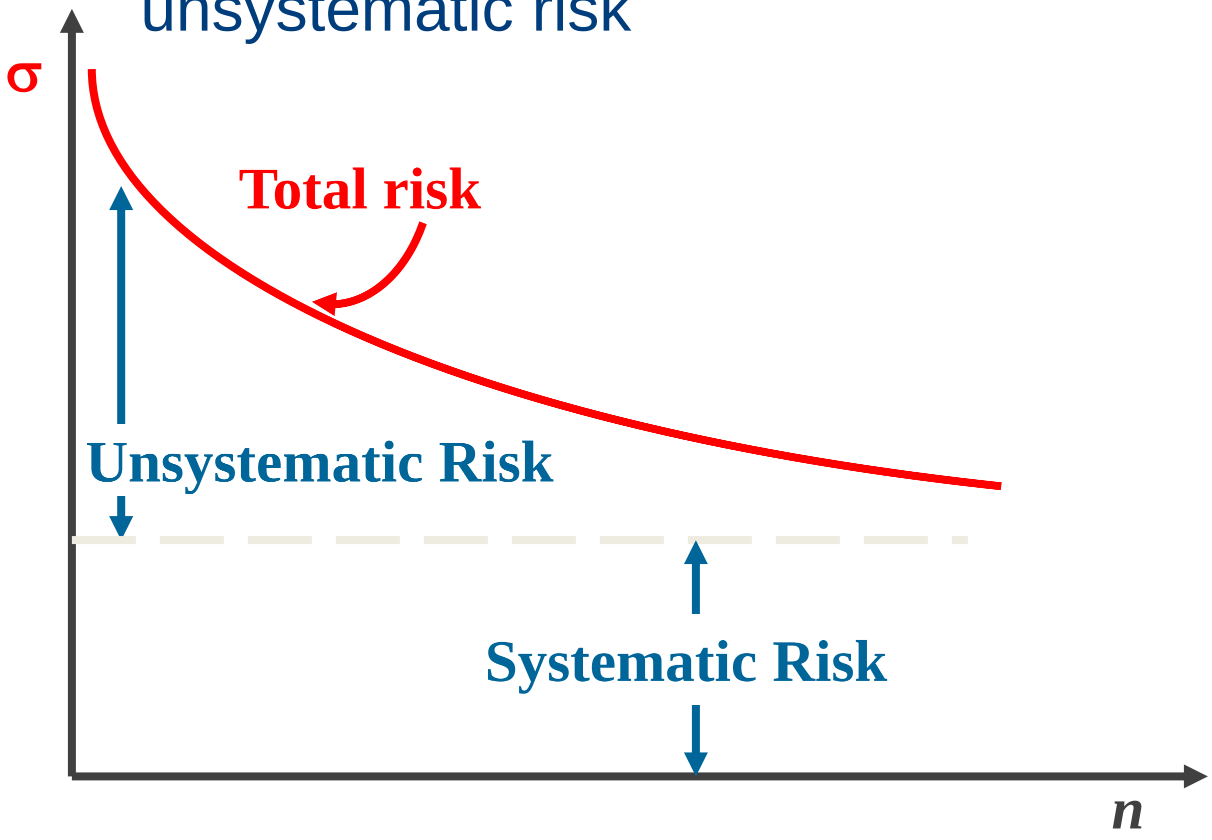
- Capital market history suggests: There **IS** a reward for bearing risk. But there is **NO** reward for bearing risk unnecessarily.
- Linking this to finance theory, the **required return** on a risky asset depends only on that asset's **systematic risk** (its market risk) since unsystematic (diversifiable) risk can be diversified away when the asset is placed in a portfolio.
- In finance theory, the best measure of the risk of a security when held in a large portfolio (i.e. its market risk) is the **beta (β_i)** of the security, defined as follows:

$$\boxed{\beta_i = \frac{Cov(r_i, r_M)}{\sigma_M^{\ 2}}}$$

- **Beta** measures the responsiveness of a security to movements in the market portfolio. Beta is the slope of the regression line of the asset's returns on the market portfolio's returns.

Risk: Systematic and Unsystematic

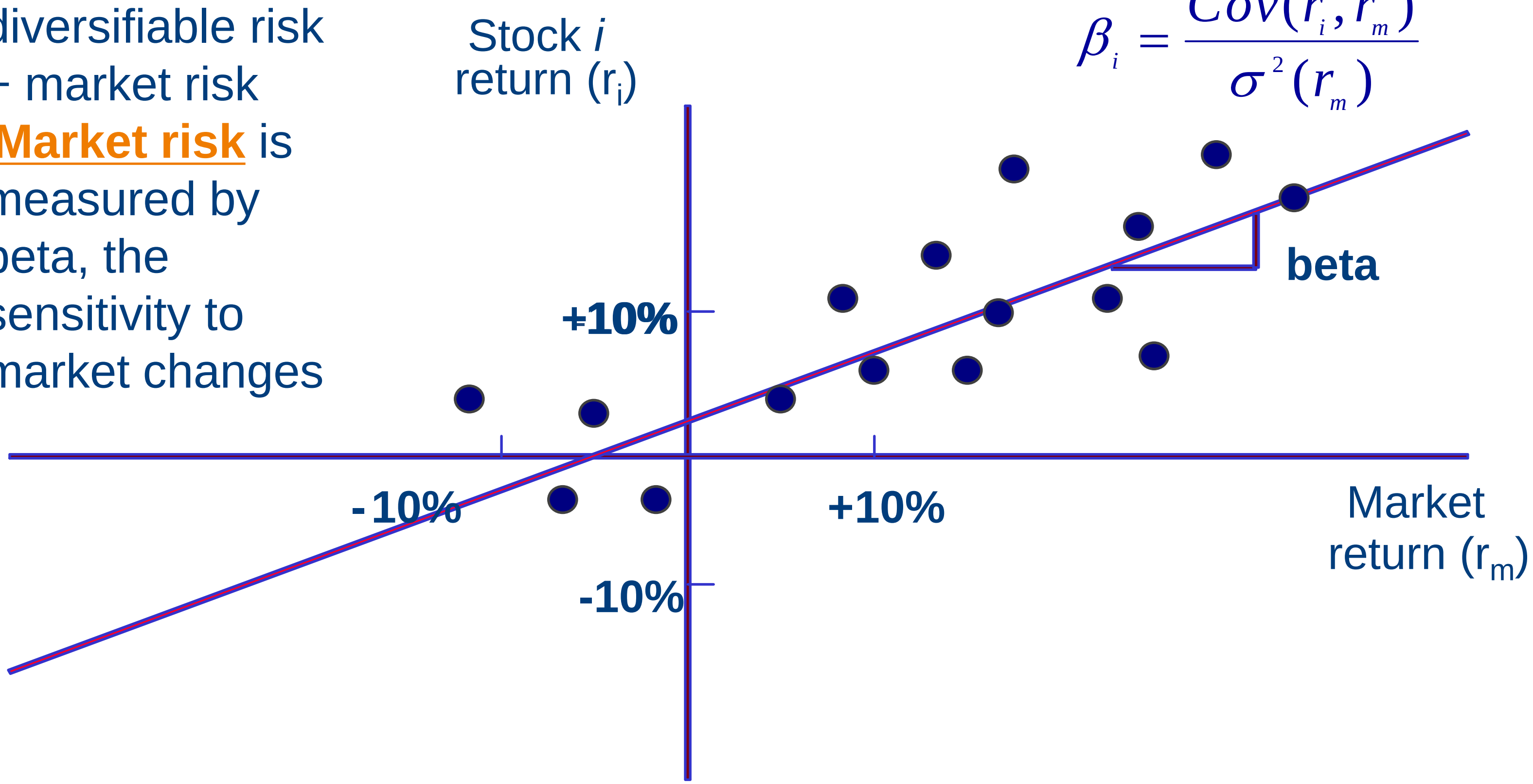
We can break down the total risk of holding stocks into the sum of two components: systematic risk and unsystematic risk



β Measurement

1. **Total risk:** diversifiable risk + market risk
2. **Market risk** is measured by beta, the sensitivity to market changes

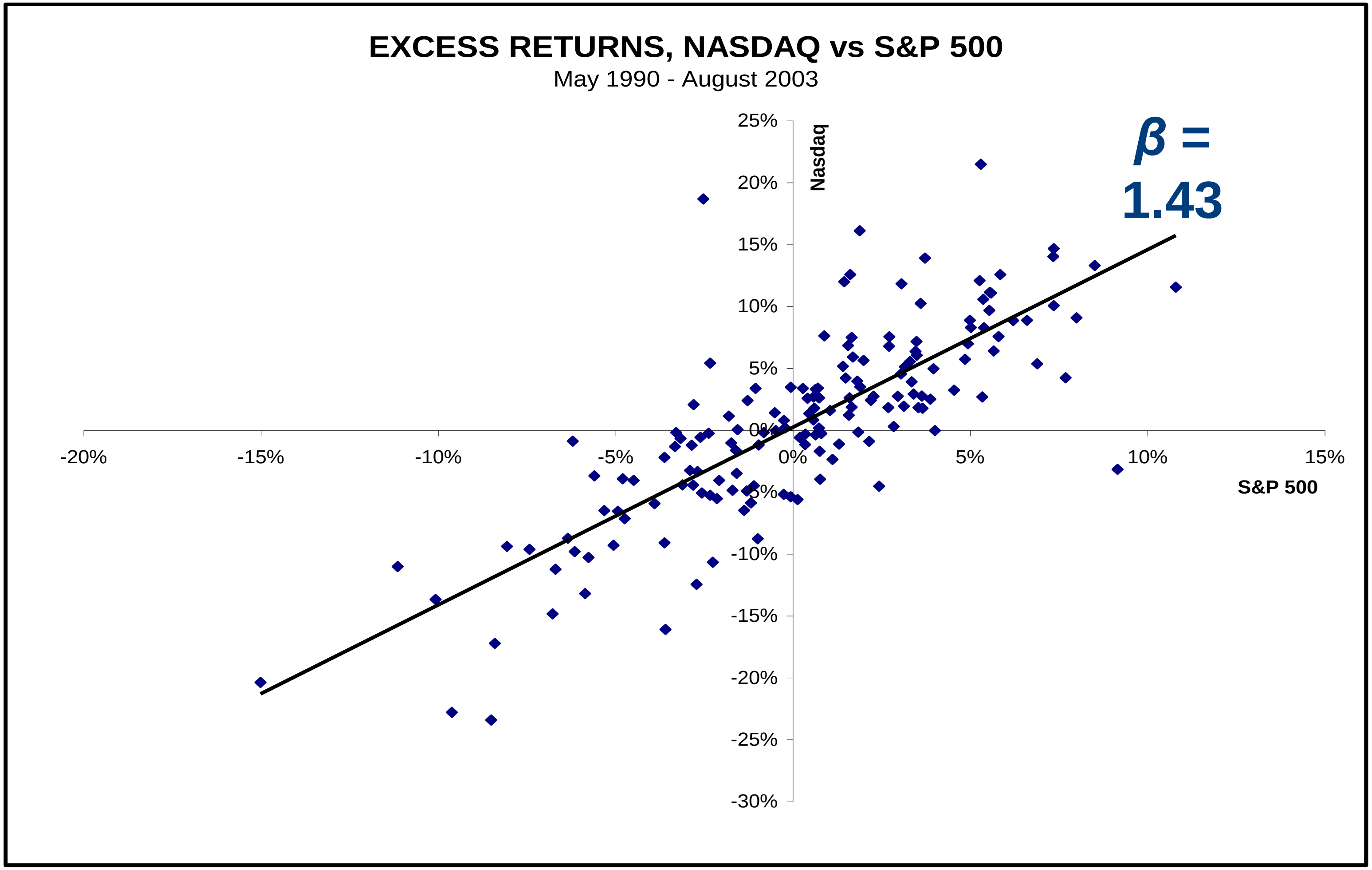
$$\beta_i = \frac{Cov(r_i, r_m)}{\sigma^2(r_m)}$$



β_i measures the sensitivity of stock's return to the return on the market portfolio.

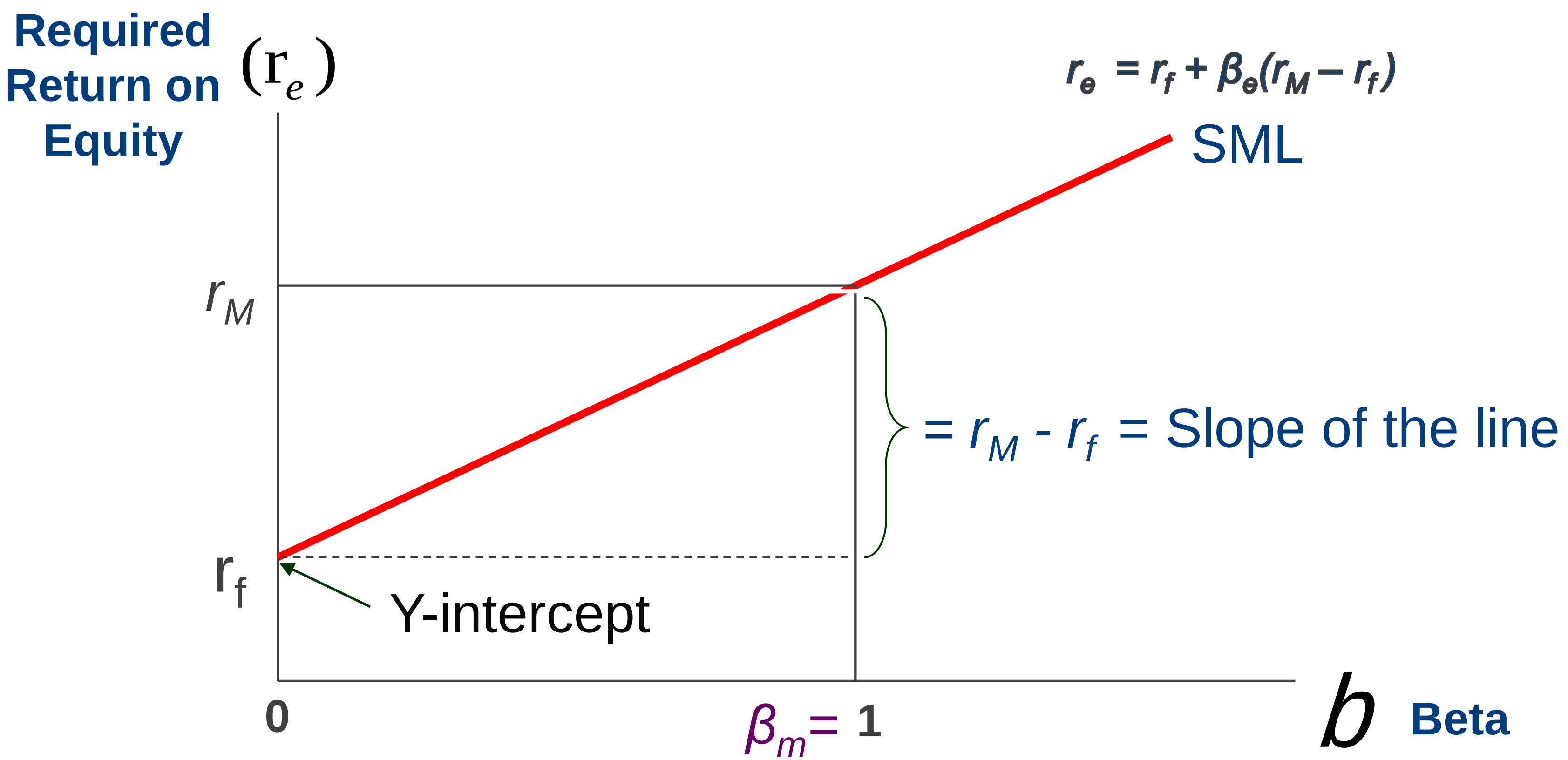
- r_m refers to the market portfolio return
- r_i refers to the stock i's return

β Measurement Example: $\boldsymbol{\beta}_{\text{NASDAQ}}$



The Security Market Line (SML)

Part of the *Capital Asset Pricing Model (CAPM)*



⇒ The SML describes the <u>risk return relationship</u> between the β of a security and its <u>required rate of return</u>. Thus the SML directly translates beta into an estimate of required rate of return. It is the most common method of estimating the required rate of return.

The Capital Asset Pricing Model (CAPM)

- CAPM defines the relationship between **market risk** and **required return**

$$(r_i) = r_f + \beta_i (r_M - r_f)$$

- If we know an asset's **systematic risk** measure (its beta), we can use the CAPM to determine its required return (which can then be used to price the asset).
- This is true whether we are talking about financial assets or physical assets.

Factors affecting required return

Capital Asset Pricing Model

$$(r_i) = r_f + \beta_i \underbrace{(r_M - r_f)}$$

1. Pure time value of money – measured by the risk-free rate

2. Reward (Risk Premium) for bearing systematic risk – determined by the

   - Systematic risk measure – captured by beta
   - Market risk premium

Application of CAPM's SML

Consider the betas for each of these assets. If the risk-free rate is 4.5% and the market risk premium is 8.5%, what is the required return for each?

Security	Beta	Required Return
DCLK	3.69	4.5 + 3.69(8.5) = 35.865%
KO	0.64	4.5 + 0.64(8.5) = 9.940%
INTC	1.64	4.5 + 1.64(8.5) = 18.440%
KEI	1.79	4.5 + 1.79(8.5) = 19.715%

Reward-to-Risk Ratio

- The **reward-to-risk ratio** is the slope of the line illustrated in the previous slides:

$$
\begin{aligned}
\text{Slope} &= (r_M - r_f)/(\beta_M - 0) \\
&= (r_M - r_f)/(1 - 0) \\
&= r_M - r_f
\end{aligned}
$$

- For every unit of beta (market risk taken), the required additional return over the risk-free rate is $r_M - r_f$
- Since the beta for the market is *ALWAYS* equal to one: Slope = $r_M - r_f$ = market risk premium.
- In equilibrium, all assets and portfolios must have the same reward-to-risk ratio. Thus under SML, all assets' excess return over the risk-free rate will be proportionate to their beta measure.

CAPM's Security Market Line

The Security Market Line: $r_i = r_f + \boldsymbol{\beta}_i\,(r_m - r_f)$

$\boldsymbol{\beta}_\mathbf{i}$ = the measured beta for asset i = $\beta_i = \dfrac{Cov(r_i, r_m)}{\sigma^2(r_m)}$

$\Rightarrow$ the risk premium on any asset *i* is proportional to its β_i

Recall that $\boldsymbol{\beta}_{\mathbf{market}} = \dfrac{Cov(r_m, r_m)}{\sigma^2(r_m)} = \dfrac{\sigma^2(r_m)}{\sigma^2(r_m)} = 1$ thus $\mathbf{r_i = r_m}$

$\boldsymbol{\beta}_{\textbf{risk free asset}} = \dfrac{Cov(r_f, r_m)}{\sigma^2(r_m)} = \dfrac{\rho_{fm}\sigma_f\sigma_m}{\sigma^2(r_m)} = \dfrac{0 \times 0 \times \sigma_m}{\sigma^2(r_m)} = 0$ thus $\mathbf{r_i = r_f}$

More on β (Systematic Risk Measure)

- How do we measure systematic risk?

  We use the beta coefficient to measure systematic risk

- What does beta tell us?
  - A beta = 1 implies the asset has the same systematic risk as the overall market
  - A beta < 1 implies the asset has less systematic risk than the overall market
  - A beta > 1 implies the asset has more systematic risk than the overall market

Example: Total Risk σ versus Systematic Risk β

- Consider the following information:

	Standard Deviation	Beta
Security C	20%	1.25
Security K	30%	0.95

- Which security has more total risk? **K**
- Which security has more systematic risk? **C**
- Which security should have the higher required return? **C**

Estimating Beta

$$(r_i) = r_f + \beta_i (r_M - r_f)$$

- Many analysts use the returns of the S&P 500 (or the MSCI) as a ‘proxy’ for the market portfolio returns (to be used in the CAPM)
  - The returns of the company of interest are then regressed on the S&P’s returns to find the company’s β
- Analysts typically use four or five years of monthly returns to establish the regression line.
- Some analysts use weekly returns instead of monthly returns and then use only two or three years of weekly data, rather than four or five years. Again, practices can vary, and none are set in stone.

Finding Betas

- Many companies provide company beta estimates (e.g., Moody's, S&P, Bloomberg), as do a number of internet sites
- Yahoo Finance provides company betas, as well as much additional information under its company profile link
- Try it out: Visit http://finance.yahoo.com/
  - Enter a ticker symbol and get a basic quote
  - Click on Key Statistics

(E.g., Compare the betas of Pepsi, Coca-Cola, Google & Apple)

Portfolio βs
Portfolio Systematic Risk Measure

Given a large number (m) of assets in a portfolio, we would multiply each asset's beta by its portfolio weight and then sum up the results to get the **portfolio's beta**:

$$\boxed{\beta_P = \sum_{j=1}^{m} w_j \beta_j}$$

Example: Portfolio Betas

Consider the following four securities in a portfolio:

Security	Weight	Beta	$w_i\beta_i$
DCLK	0.133	3.69	0.491
KO	0.2	0.64	0.128
INTC	0.267	1.64	0.438
KEI	0.4	1.79	0.716
			1.77

What is the portfolio beta (β_P)?

Assumptions of CAPM

The CAPM assumes that:

- All investors try to maximize economic utilities.
- All investors are rational and risk-averse.
- All investors are fully diversified across a range of investments.
- All investors are price takers, thus they cannot influence prices.
- All investors can lend <u>and</u> borrow unlimited amounts at the risk free rate of interest.
- All investors trade without transaction or taxation costs.
- All securities are highly divisible into small parcels.
- All information is available at the same time to all investors.
- The standard deviation of past returns is a perfect proxy for the future risk associated with a given security.

Recall the Comprehensive Example Last Class: Now Let's Look at *Expected* Returns & Beta

Investment	$\hat{r}$	beta
Alta	17.4%	1.29
Market	15.0	1.00
Am. F.	13.8	0.68
T-bonds	8.0	0.00
Repo Men	1.7	-0.86

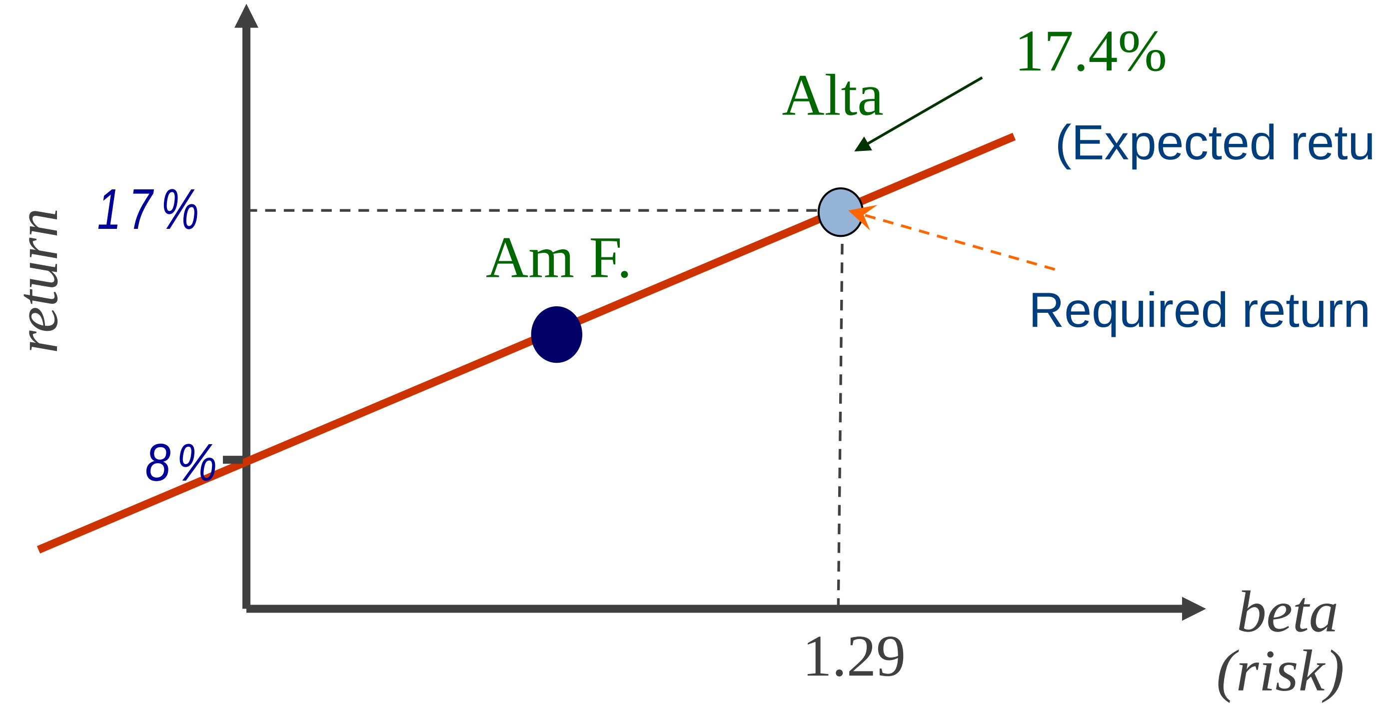
Given a risk-free rate of 8% and market return of 15%, we must first determine the required returns of the various investments, using the betas provided. Note that $\hat{r}$ here refers to expected return as extracted from current stock prices (not required return which is obtained from the CAPM's SML).

Comprehensive Example: *Expected* Returns & *Required* Returns

Investment	$\hat{r}$	*Required* return	Attractive?
Alta	17.4%	17.0%	Underpriced
Market	15.0	15.0	Fairly Valued
Am. F.	13.8	12.8	Underpriced
T-bonds	8.0	8.0	Fairly Valued
Repo Men	1.7	2.0	Overpriced

- ►For a fairly priced asset, the expected return is on the SML.
- ►For an underpriced asset, it would be above the SML.
- ►For an overpriced asset, it would be below the SML.

Relationship Between Risk & Required Return



$$\beta_i = 1.29 \qquad R_f = 8\% \qquad r_M = 15\%$$

Required return

$$(r_i) = 8\% + 1.29 \times (15\% - 8\%) = 17.0\%$$

Valuation of Project A

A project is expected to generate the following cash flows:

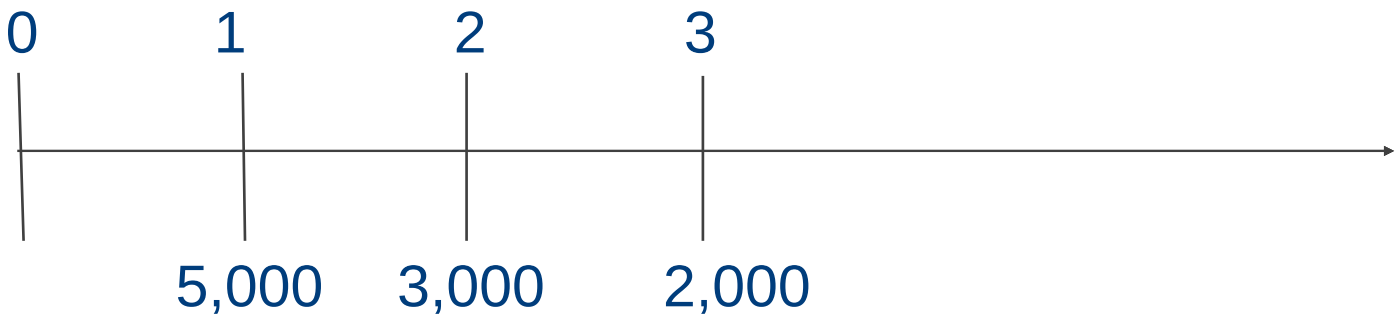
Year	Cash flows
1	$5,000
2	3,000
3	2,000

Given that the project beta is 1.5, the risk-free rate is 6% and the market risk premium is 8%, what is the maximum you should pay for this project?

Valuation of Project A

We use CAPM's SML to determine the *required* rate of return:

$$r_f + \beta_i\,(r_M - r_f) = 6\% + 1.5\,(8\%) = 18\%$$



4,237.29
2,154.55
1,217.26

7,609.10 = PV

Therefore, you shouldn't pay more than $7,609.10.

Example: Valuation of Project A

<u>Calculator Solution:</u>

The discount rate is the required rate of return:

$$r_f + \beta_i (r_M - r_f) = 6\% + 1.5\ (8\%) = 18\%$$

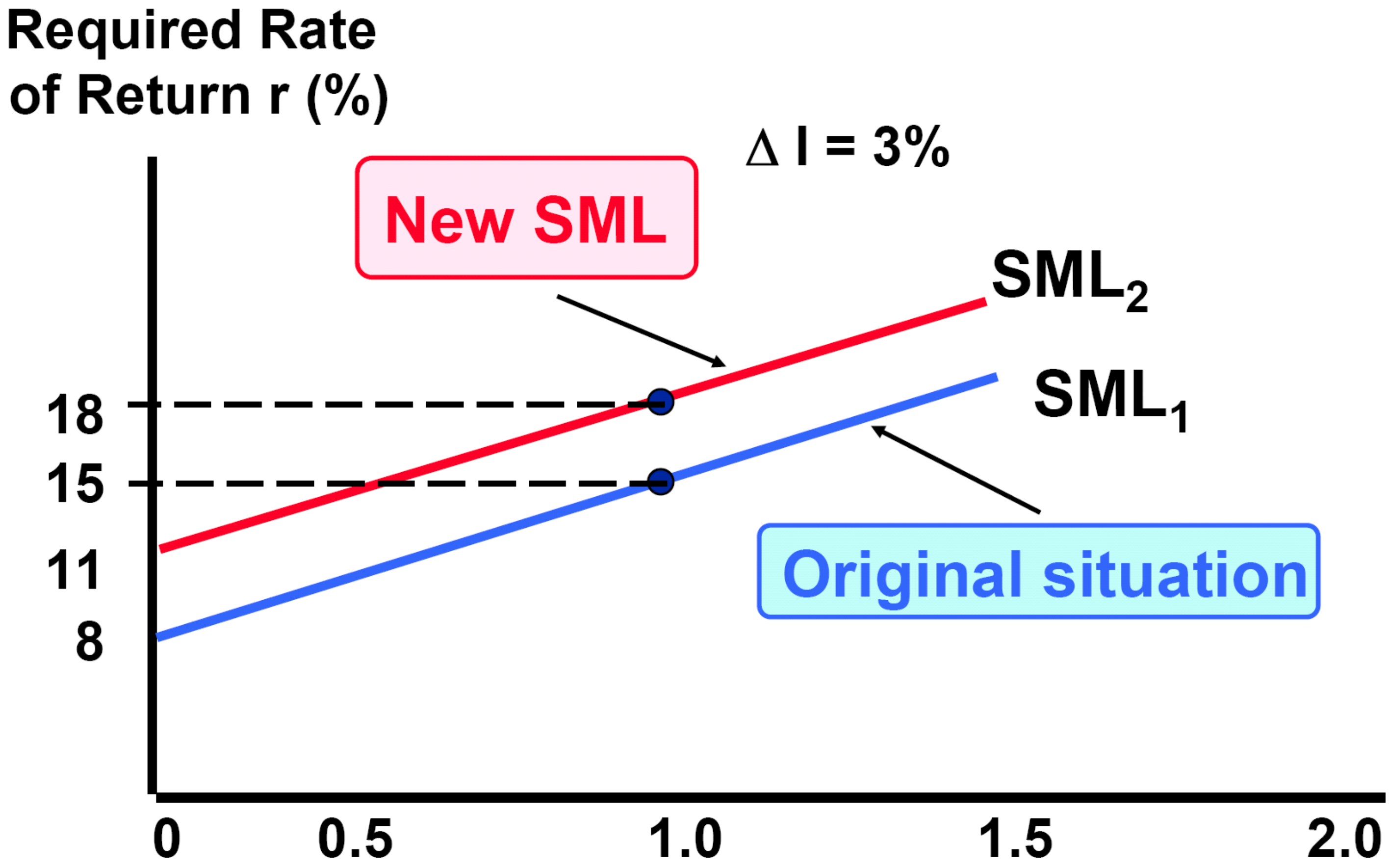
- Use the <CF> worksheet and <NPV> function to solve:
- <CF>: Brings up the worksheet
- <2ND><CLR WORK> : Clear values in worksheet
- Use <↓> and <↑> to enter the cash values according.
- Press <NPV> to display the current discount rate (I)
- **18 <ENTER>** → I = 18 : enters a discount rate of 18%

• **NPV:** <↓> → NPV = 0

• **NPV: <CPT>** → NPV = **7,609.10**

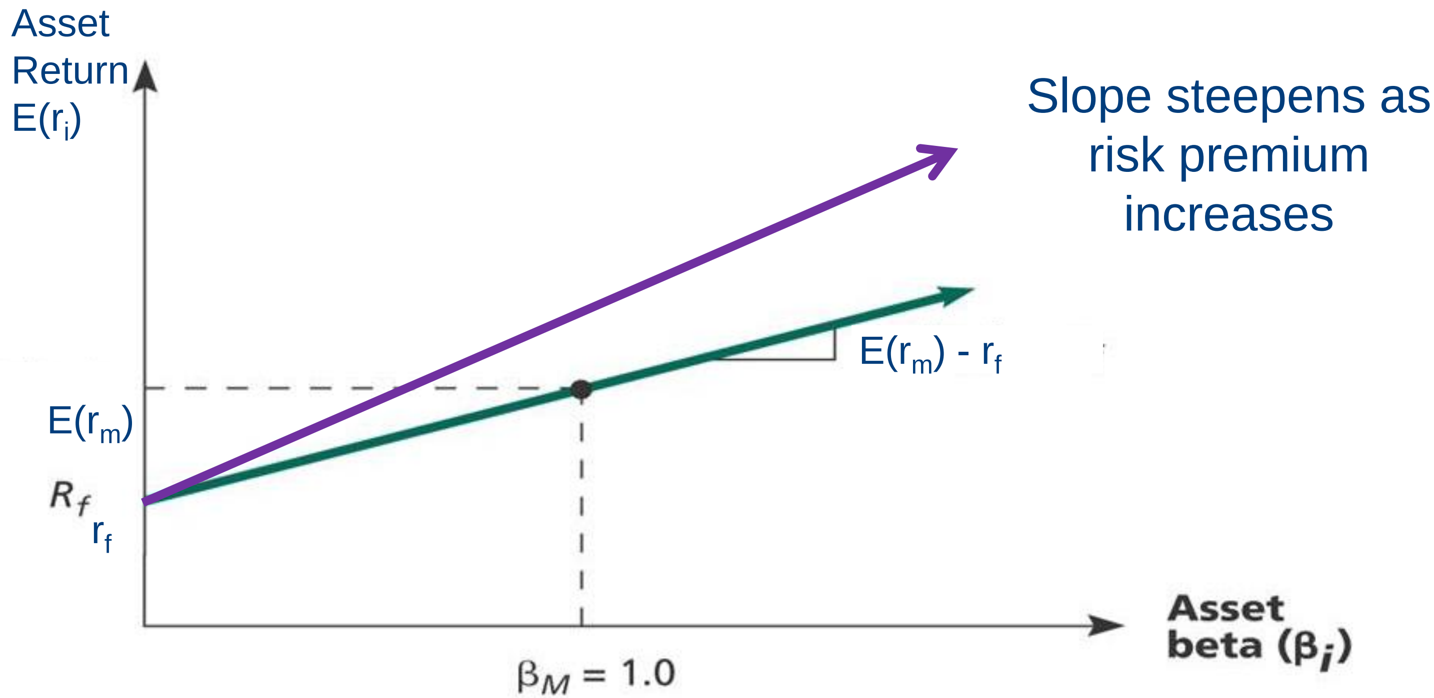
Again, you shouldn't pay more than $7,609.10.

Impact of Inflation Change on SML



Impact of a Risk Aversion Change

What happens when investors become more risk averse?



The slope of the security market line is equal to the market risk premium, i.e. the reward for bearing an average amount of systematic risk. The equation describing the SML can be written:

$$E(r_i) = r_f + \beta_i [E(r_M) - r_f]$$

which is the capital asset pricing model (CAPM).

Summary

- How do you compute the beta of an individual asset? For a portfolio?
- What is the effect of inflation to the SML?
- What is the effect of a change in the risk aversion to the SML?

Markowitz Portfolio Theory

Learning objectives

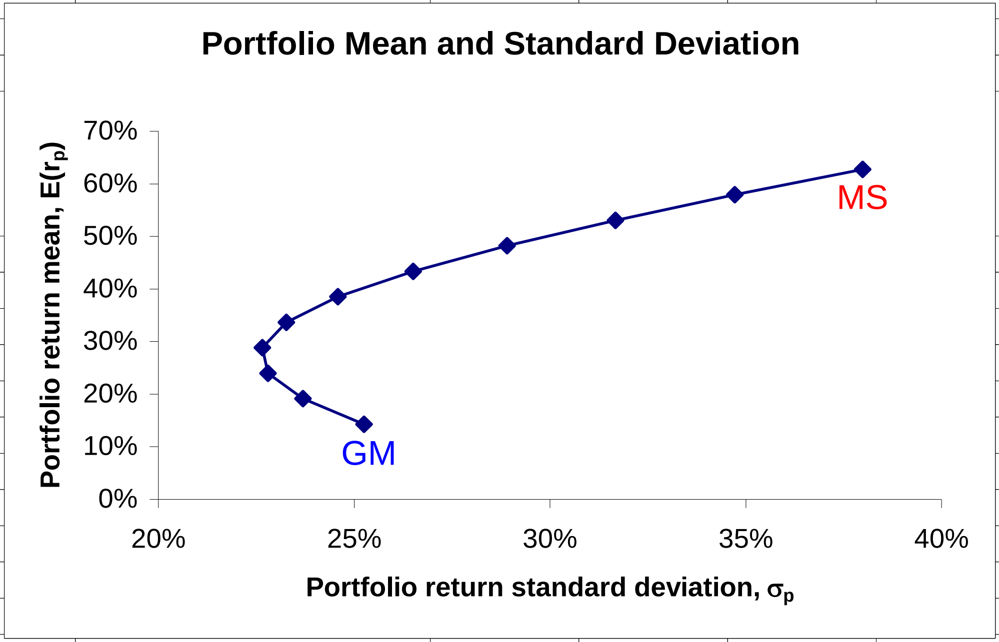
- Understand what an efficient portfolio is
- Understand what an efficient frontier is and how they are constructed
- Understand where the market portfolio is located on the efficient frontier

Markowitz Portfolio Theory

- As we’ve seen, combining stocks into portfolios can reduce standard deviation below the level obtained from a simple weighted average calculation.
- This is because correlation coefficients make this possible.
- A portfolio that provides the greatest expected portfolio return for a given level of portfolio standard deviation (risk), or equivalently, the lowest portfolio risk for a given expected portfolio return is called an **efficient portfolio.**
- The line representing all efficient portfolios is called the **efficient frontier.**

Portfolios – Two Asset Example

	A	B	C	D
1	**CALCULATING PORTFOLIO RETURNS AND THEIR STATISTICS FROM THE FORMULAS**			
2		**General Motors GM**	**Microsoft MSFT**	
3	Mean	14.25%	62.72%	
4	Variance	6.38%	14.43%	
5	St. dev.	25.25%	37.99%	
6	Covariance		-5.52%	
7				
8	**Proportion of GM in portfolio**	**Portfolio Variance**	**Portfolio standard deviation**	**Portfolio mean**
9	0%	14.43%	37.99%	62.72%
10	10%	12.06%	34.72%	57.87%
11	20%	10.03%	31.67%	53.03%
12	30%	8.36%	28.91%	48.18%
13	40%	7.03%	26.51%	43.33%
14	50%	6.05%	24.59%	38.49%
15	60%	5.42%	23.28%	33.64%
16	70%	5.14%	22.66%	28.79%
17	80%	5.20%	22.81%	23.95%
18	90%	5.62%	23.70%	19.10%
19	100%	6.38%	25.25%	14.25%



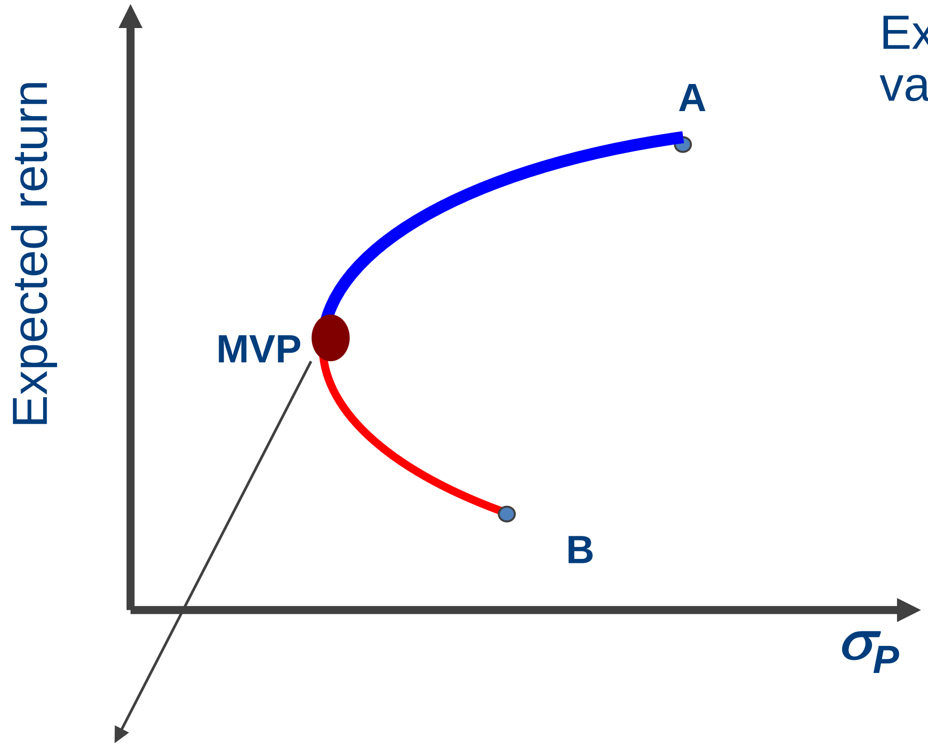
=SQRT(B19)

=A19*B3+(1-A19)*C3

=A19^2*B4+(1-A19)*C4+2*A19*(1-A19)*C6

^2

Example of Efficient Frontier of a 2-asset Portfolio of Assets A and B



MVP = minimum variance portfolio

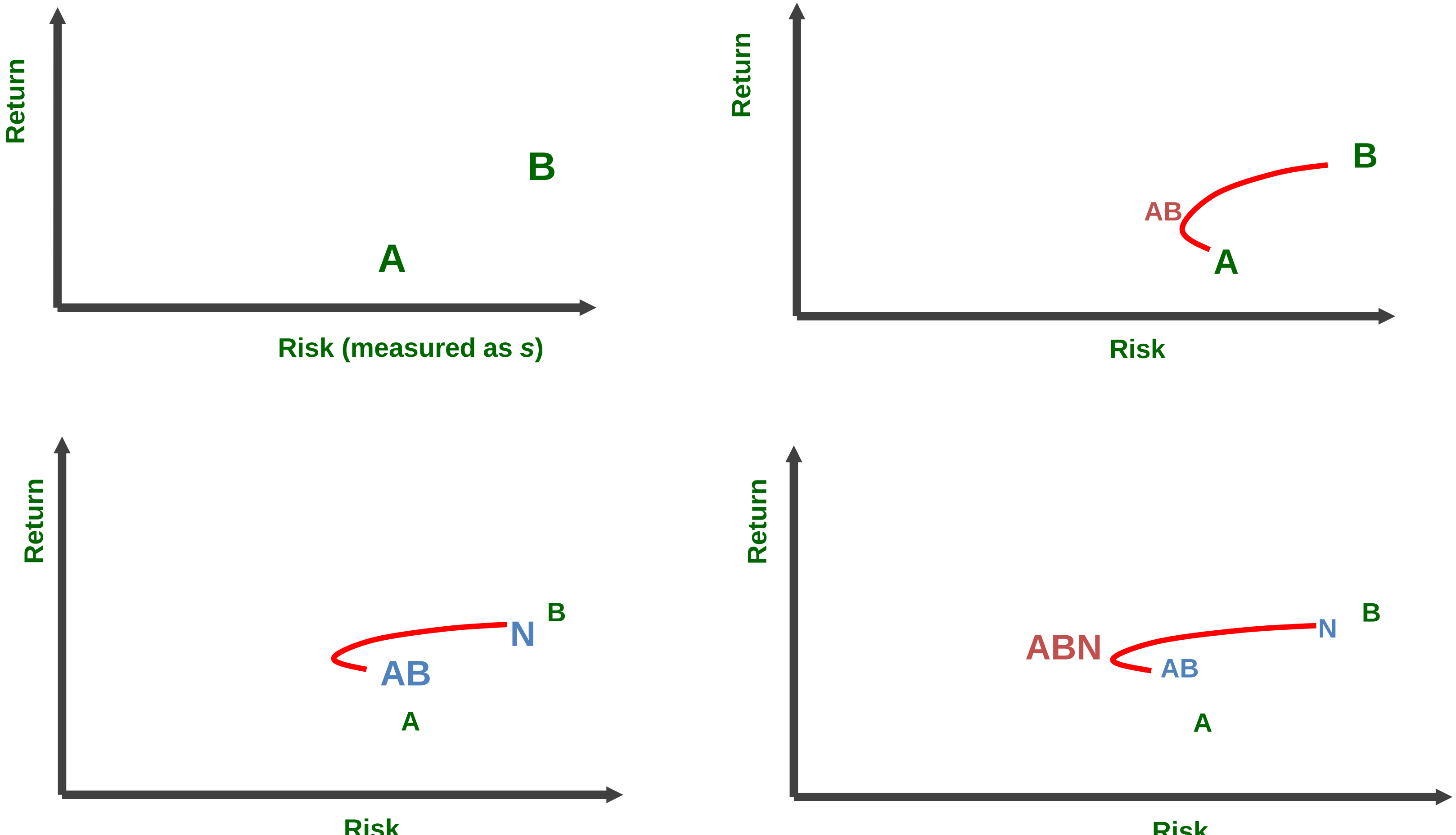
Efficient Frontier:

Extends from the MVP (minimum variance portfolio to A).

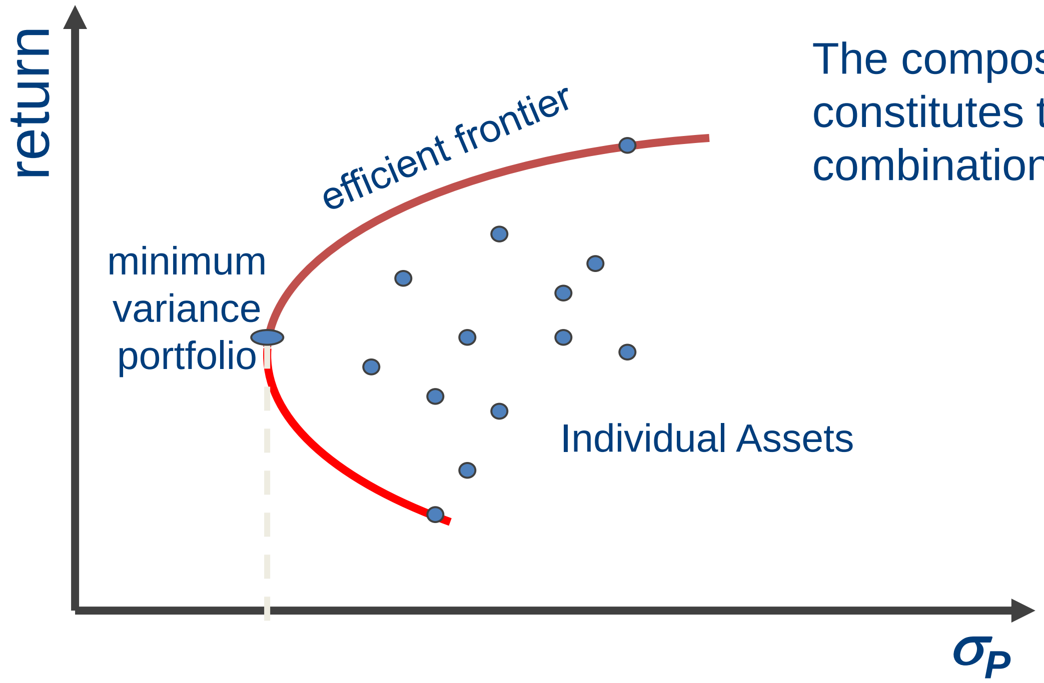
- For the same return, risk is minimized.
- For the same risk, return is maximized.
- **Efficient portfolios** are the ones we get maximum expected return for a given level of risk.
- Rational investor should not choose any other portfolios.

Developing the Efficient Frontier of All Possible Assets

Start with 2 assets A and B, and keep adding assets to new MVP formed



Efficient Set for Many Securities

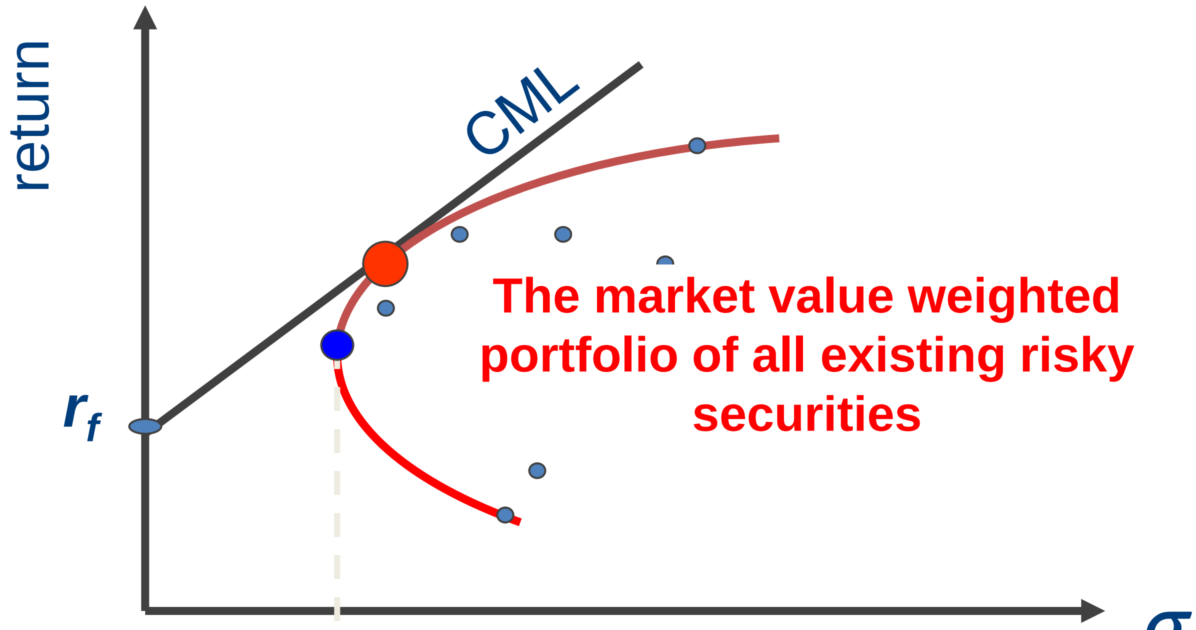


The composite of all risky asset sets constitutes the frontier of all combinations of all risky assets.

Given the *opportunity set* we can identify the **minimum variance portfolio**. The section of the opportunity set above the minimum variance portfolio is the efficient frontier.

Now Introduce Riskless Borrowing and Lending to Efficient Frontier of All Possible Risky Assets

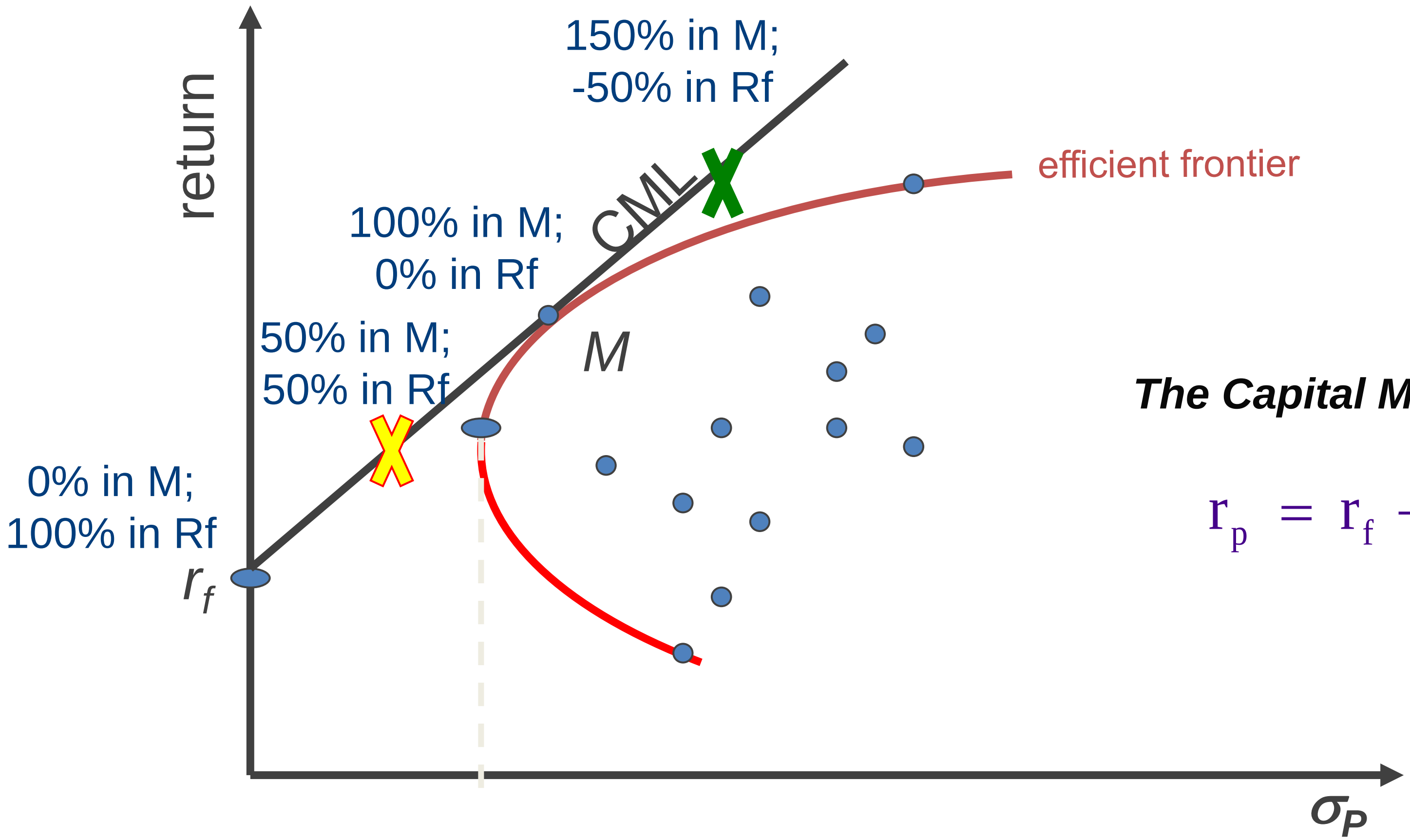
If there is *riskless borrowing and lending* then we draw the steepest line from the risk-free rate with the efficient frontier. Investors are able to allocate their money across the *risk-free asset* and the *market portfolio* with optimal result.



The Market Portfolio on the Efficient Frontier

- The **market portfolio** is the portfolio at the tangent line of the risk-free asset with the efficient frontier of all risky assets available.
- In theory, all risky assets are included in the true market portfolio in proportion to their market value (in practice, we use proxies for this portfolio, e.g., the S&P 500, MSCI, etc.)
- The **market portfolio**, because it contains all risky assets, is a completely diversified portfolio, which means that all the unique risk of individual assets (unsystematic risk) is diversified away, and thus only the systematic risk of all the assets remains in the portfolio (and gives a beta of 1).

Efficient Frontier for All Possible Risky Assets with Borrowing/Lending at the Risk-Free Asset

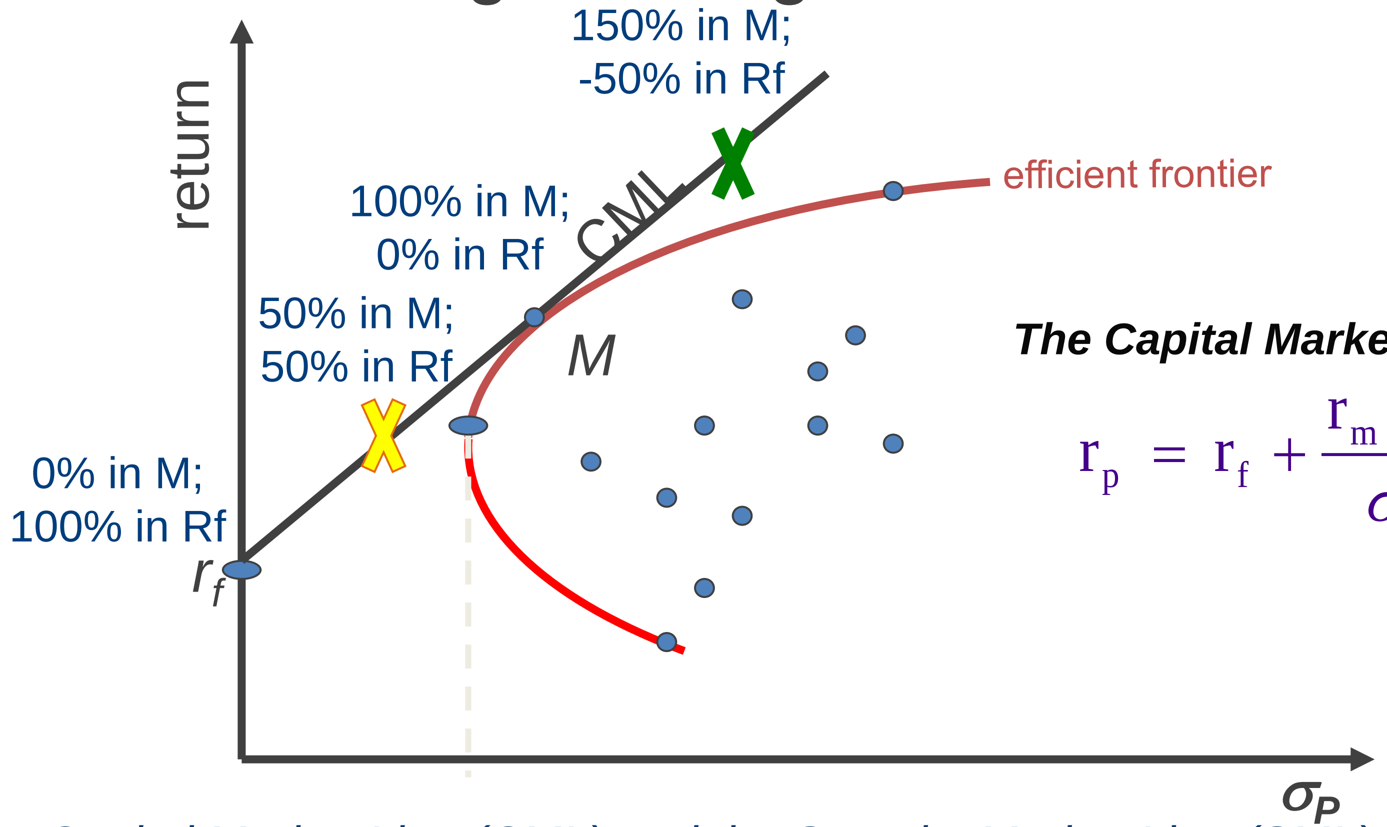


The Capital Market Line (CML):

$$r_p = r_f + \frac{r_m - r_f}{\sigma_m}\sigma_p$$

With the capital allocation line identified, all investors choose a point along the line – some combination of the risk-free asset and the market portfolio *M*. In a world with homogeneous expectations, *M* is the same for all investors.

The Capital Market Line (CML): From the Efficient Frontier for All Possible Risky Assets with Borrowing/Lending at the Risk-Free Asset



The Capital Market Line (CML):

$$r_p = r_f + \frac{r_m - r_f}{\sigma_m}\sigma_p$$

The Capital Market Line (CML) and the Security Market Line (SML) are both part of the Capital Asset Pricing Model (CAPM). In fact, the CML is used to derive the SML. The SML is what we actually *use* to determine required return for a given level of market risk.

Summary

- An efficient frontier is constructed from the efficient set.
- To be included in the efficient set, a portfolio must have
    1. The highest expected return at a given standard deviation relative to all other portfolios in the investment opportunity set

       OR

    2. The lowest standard deviation at a given expected return relative to all other portfolios in the investment opportunity set

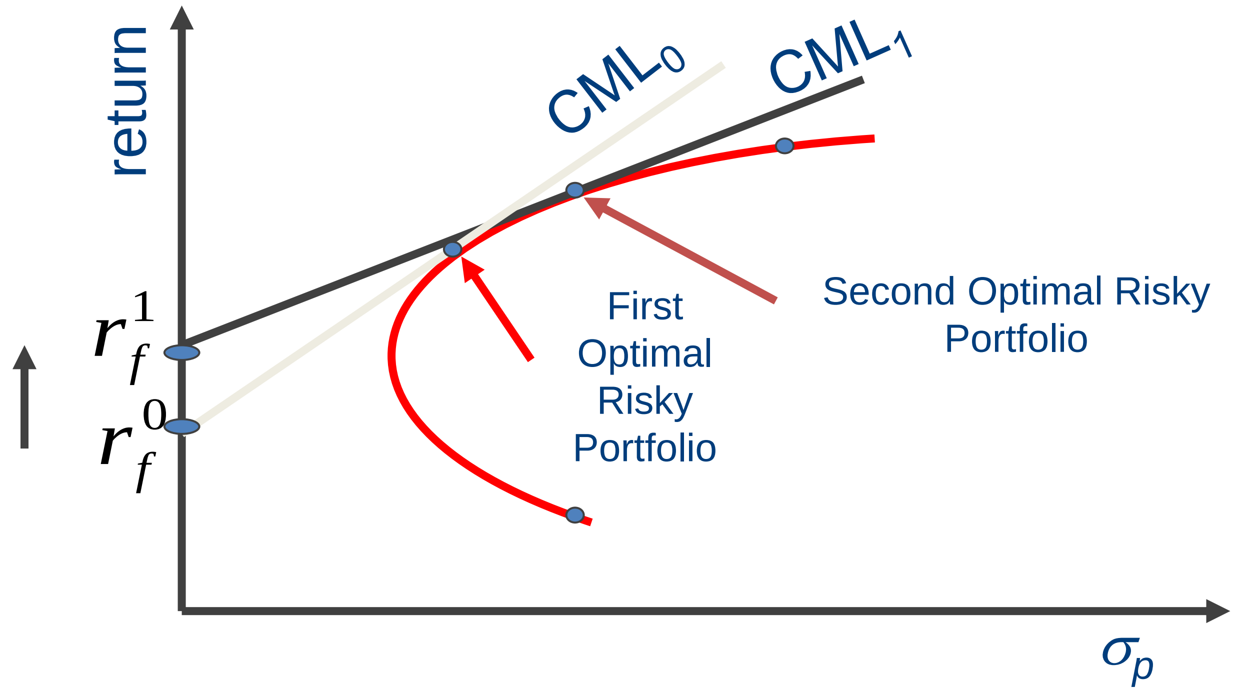
- Market portfolio includes all risky assets in the market (in theory)
    – Completely diversified portfolio

Additional Materials

Week 5 Additional Materials

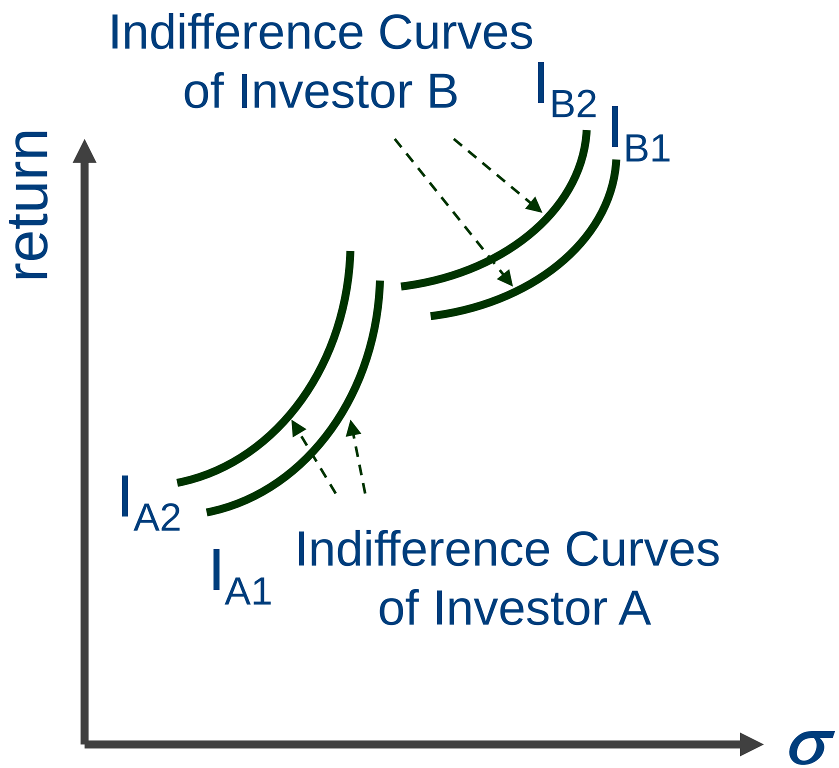
NOT EXAMINABLE

CAPM - Optimal Risky Portfolio with a Risk-Free Asset



Note also that the optimal risky portfolio depends on the risk-free rate as well as the efficient set of risky assets.

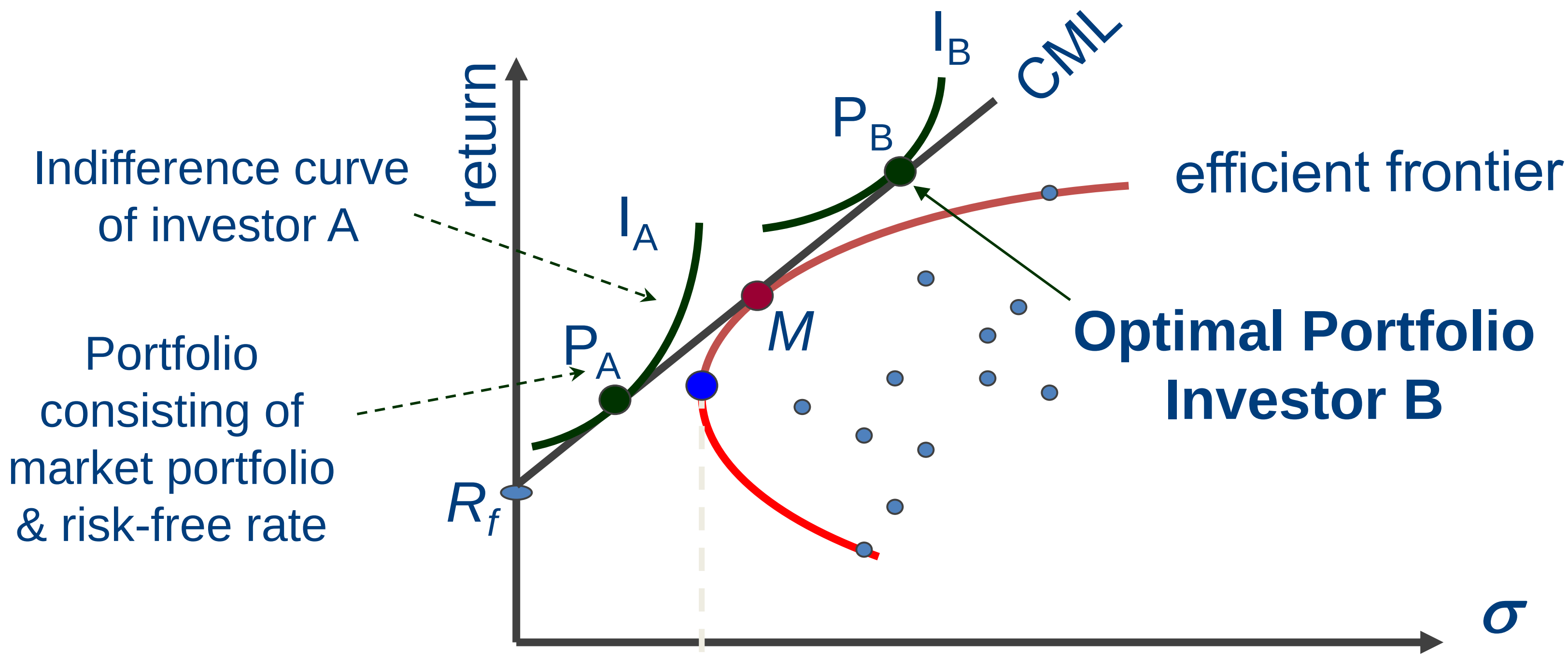
Indifference Curves



- Indifference curves reflect an investor's attitude toward risk as reflected in his or her risk/return tradeoff function.
- They differ among investors because of differences in the degree of risk aversion.

Note: Investor A is more risk-averse than B, as reflected by the former's steeper indifference curves.

The Separation Property



The **Separation Property** states that the market portfolio, *M*, is the same for all investors - they can separate their risk aversion from their choice of the market portfolio. According to portfolio theory, investor risk preferences determine where to stay along the capital market line – but does not affect their choice of the line.