Geometric Shapes: Quadric Surfaces and Sweeping

Module 3

Lecture 5

We have learnt that

- Plane surfaces can be defined by explicit, implicit and parametric functions
- In implicit (linear) equation Ax+By+Cz+D=0, **N**=[A B C] while D defines displacement from the origin
- To get the plane equation, derive **N** first (e.g., by cross product of two vectors), then substitute any *x,y,z* on the plane to derive D.
- Implicit equation in intercepts can be easily written from x/a+y/b+z/c=1
- Parametric definition of plane is based on linear equations of two parameters
- Bilinear interpolation is an extension of linear interpolation for interpolating functions of two variables (e.g., *u, v*). It performs linear interpolation first in one direction, and then again in the other direction. Although each step is linear, the interpolation as a whole is not linear but rather quadratic in the sample location
- Bilinear surface defines a 4-sided polygon, including non-planar surfaces.
- Bilinear surface can be used for writing parametric functions of a triangle as well: two of the vertices are simply merged together which will also simplify the defining formulas.

Quadric Surfaces

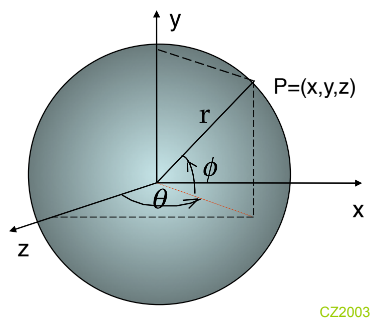
- $ax^2+by^2+cz^2+dxy+eyz+fxz+gx+hy+kz+m=0$
- Ellipsoids (including spheres), Elliptic Paraboloid, Hyperbolic Paraboloid, Hyperboloid of One Sheet, Hyperboloid of Two Sheets, Cone, Elliptic Cylinder (including circular cylinders), Hyperbolic Cylinder, Parabolic Cylinder
- Only highlighted surfaces are examinable

Sphere

- Implicit $r^2 - x^2 - y^2 - z^2 = 0$
- Explicit $z = \pm\sqrt{r^2 - x^2 - y^2}$
- Parametric

$$x = r\cos\varphi\sin\theta$$
$$y = r\sin\varphi$$
$$z = r\cos\varphi\cos\theta$$
$$-\frac{\pi}{2} \le \varphi \le \frac{\pi}{2}$$
$$-\pi \le \theta \le \pi$$

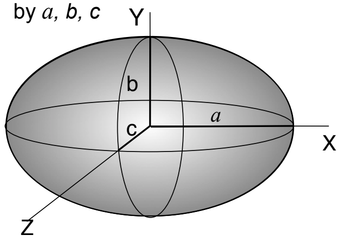
Two parameters !



Ellipsoid

- Implicit $1-(x/a)^2-(y/b)^2-(z/c)^2=0$
- Explicit $z=\pm c\sqrt{1-(x/a)^2-(y/b)^2}$
- Parametric

By scaling of a sphere with radius 1 by *a, b, c*

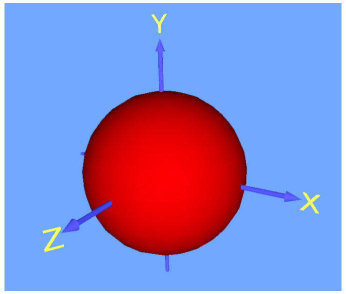


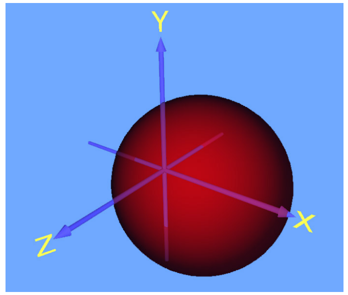
$$x=a*\cos\varphi\sin\theta$$
$$y=b*\sin\varphi$$
$$z=c*\cos\varphi\cos\theta$$
$$-\frac{\pi}{2}\le\varphi\le\frac{\pi}{2}$$
$$-\pi\le\theta\le\pi$$

Two parameters !

Experimenting with Quadrics

$2^2-x^2-y^2-z^2 = 0$ $\quad$ $2^2-(x-1)^2-y^2-z^2 = 0$

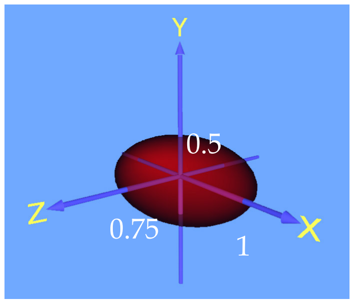




sphere

Experimenting with Quadrics

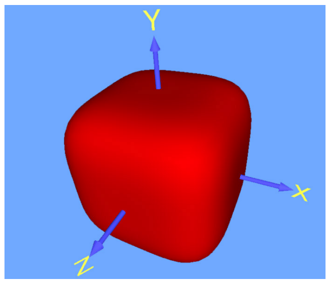
$1-(x/1)^2-(y/0.5)^2-(z/0.75)^2 = 0$

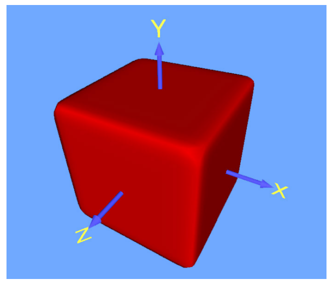


ellipsoid

Experimenting with Quadrics

$1^2-x^4-y^4-z^4 = 0$ $\quad$ $1^2-x^{16}-y^{16}-z^{16} = 0$



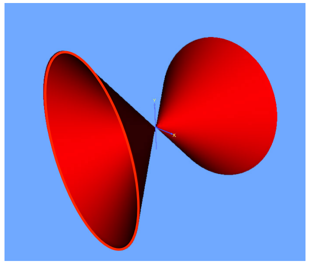


Super-ellipsoid

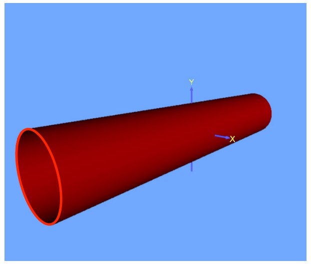
Experimenting with Quadrics

$(z/a)^2-(x/b)^2-(y/c)^2=0$

$1-(x/b)^2-(y/c)^2=0$



Cone

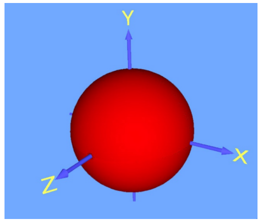


Elliptic Cylinder

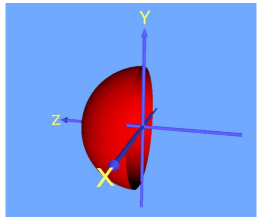
Experimenting with Parametric Sphere

$X=0.7*\cos(u)\cdot\cos(v)$
$y=0.7*\cos(u)\cdot\sin(v)$
$z=0.7*\sin(u)$

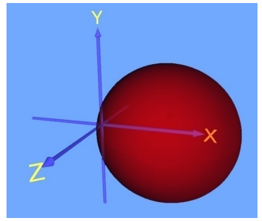
$x=0.7*\cos(u)\cdot\cos(v)+1$
$y=0.7*\cos(u)\cdot\sin(v)$
$z=0.7*\sin(u)$



$u=[0,2\pi]$ $v=[0,\pi]$



$u=[0,\pi]$ $v=[0,\pi]$



$u=[0,2\pi]$ $v=[0,\pi]$

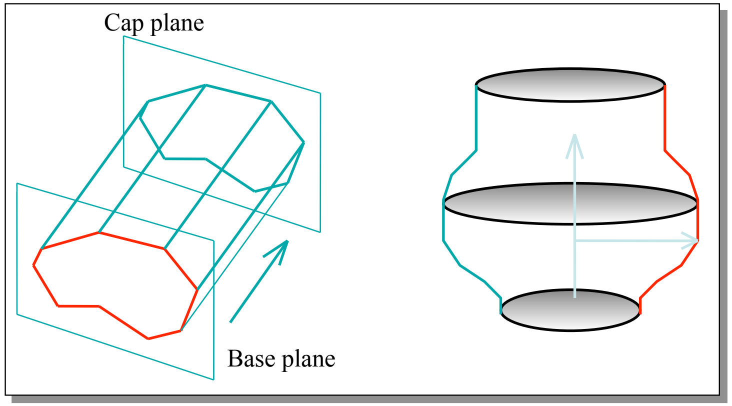
We have learnt that

- Plane and quadric surfaces can be defined by explicit, implicit and parametric functions
- Parametric definition of plane is based on linear equations of two parameters
- Bilinear interpolation is an extension of linear interpolation for interpolating functions of two variables (e.g., *u, v*). It performs linear interpolation first in one direction, and then again in the other direction. Although each step is linear, the interpolation as a whole is not linear but rather quadratic in the sample location
- Curves and surfaces were considered as objects sampled by a moving point:
  1 DOF for curves and 2 DOFs for surfaces
- Surfaces can also be though as objects sampled by moving curves:
  planes and bilinear surfaces – by moving straight lines
  spheres and ellipsoids – by rotating circles and ellipses, etc

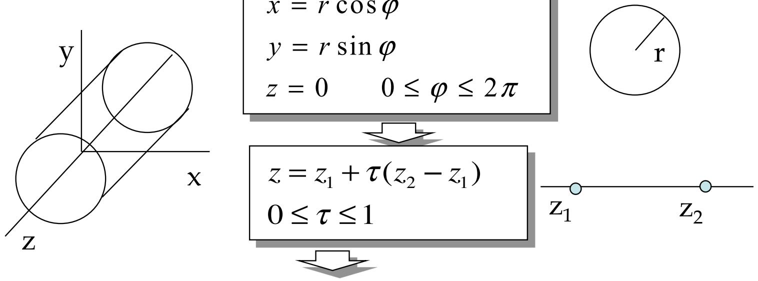
Sweeping

- Shapes are created by curve moving along some path
- Two particular cases of sweeping--*translational* and *rotational sweeping*--can be easily defined parametrically

Translational and Rotational Sweeping



Parametric Representation of Translational Sweeping



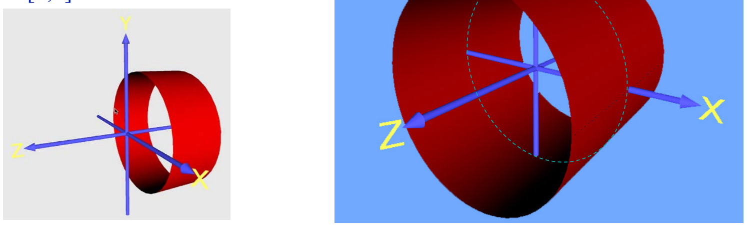
$$x = r\cos\varphi$$
$$y = r\sin\varphi$$
$$z = 0 \qquad 0 \le \varphi \le 2\pi$$

$$z = z_1 + \tau(z_2 - z_1)$$
$$0 \le \tau \le 1$$

$$x = r\cos\varphi$$
$$y = r\sin\varphi \qquad 0 \le \varphi \le 2\pi$$
$$z = z_1 + \tau(z_2 - z_1) \qquad 0 \le \tau \le 1$$

Cylinder by Translational Sweeping a Circle

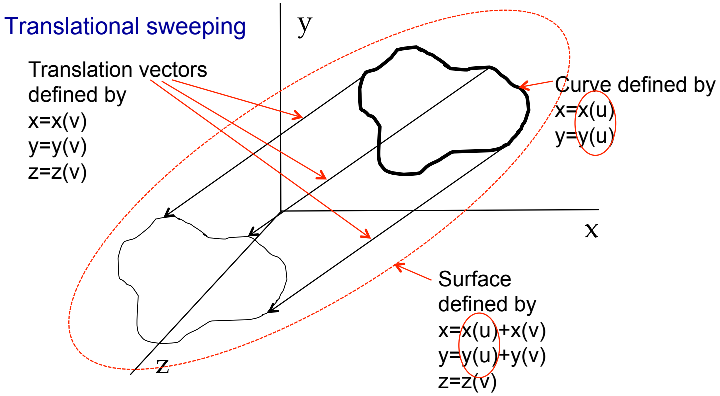
*x=0.5*cos(u*2π)*
*y=0.5*sin(u*2π)*
*z=-1+(1-(-1))*v*
u=[0,1]
v=[0,1]



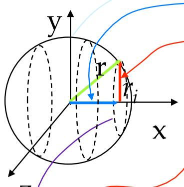
Translational sweeping

Translational Sweeping. Summary

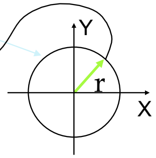
- Translational sweeping

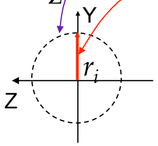


Parametric Representation of Rotational Sweeping. Making a Sphere



$$x = r\cos\varphi$$
$$y = r\sin\varphi$$
$$0 \le \varphi \le 2\pi$$

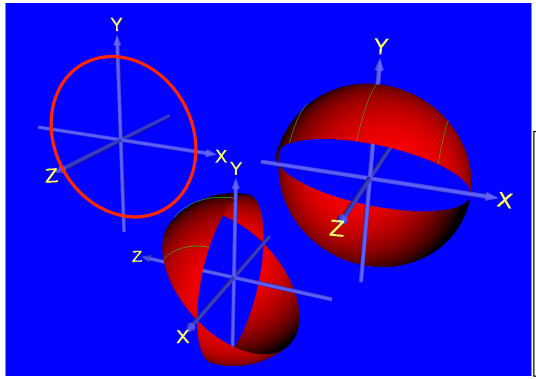




$$y = r_i \cos\theta$$
$$z = r_i \sin\theta$$
$$0 \le \theta \le \pi$$

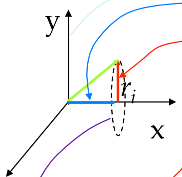
$$x = r\cos\varphi$$
$$y = r\sin\varphi\cos\theta$$
$$z = r\sin\varphi\sin\theta$$
$$0 \le \varphi \le 2\pi$$
$$0 \le \theta \le \pi$$

Parametric Representation of Rotational Sweeping. Making a Sphere

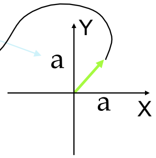


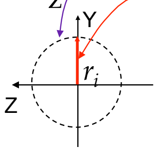
$$x = r\cos\varphi$$
$$y = r\sin\varphi\cos\theta$$
$$z = r\sin\varphi\sin\theta$$
$$0 \le \varphi \le 2\pi$$
$$0 \le \theta \le \pi$$

Parametric Representation of Rotational Sweeping. Making a Cone



$$x = u$$
$$y = u$$
$$0 \le u \le a$$

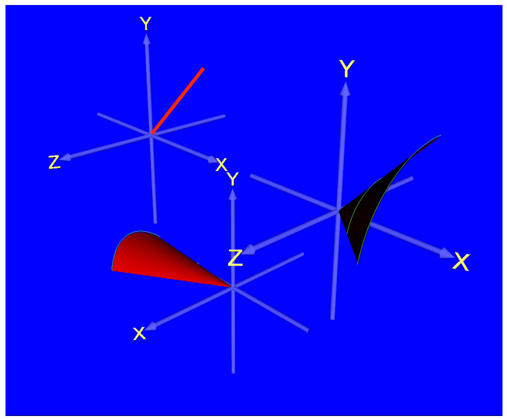




$$y = r_i \cos\theta$$
$$z = r_i \sin\theta$$
$$0 \le \theta \le 2\pi$$

$$x = u$$
$$y = u\cos\theta$$
$$z = u\sin\theta$$
$$0 \le \theta \le 2\pi$$
$$0 \le u \le a$$

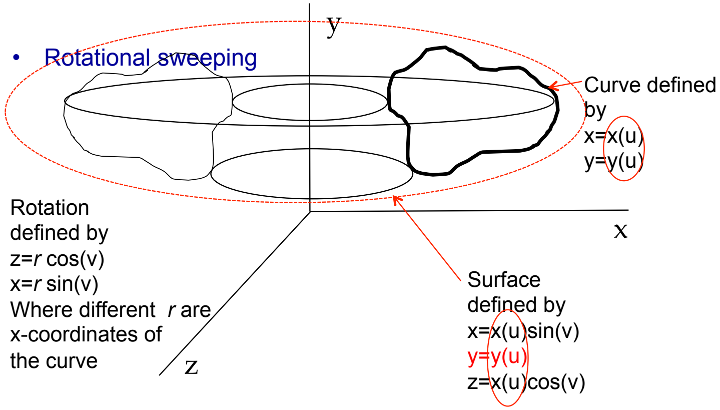
Parametric Representation of Rotational Sweeping. Making a Cone



$$x = u$$
$$y = u\cos\theta$$
$$z = u\sin\theta$$
$$0 \le u \le r/\sqrt{2}$$
$$0 \le \theta \le 2\pi$$

Rotational Sweeping. Summary

- Rotational sweeping



Surfaces. Summary

- Polygonal representation - polygon meshes
- Analytic representations
  Define how a point can move along the surface with 2 degrees of freedom (forward, backward, left, right).
  - Explicit representation.
    Seldom used because of axes dependency. Can be derived from implicit functions if needed.
  - Implicit representation
  - Parametric representation.
    Two parameters must be used to define a surface. Often, the parameters can be considered as a kind of latitude and longitude coordinates for locating a point on the surface.