

**NANYANG
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CZ3005 Tutorial 5

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N4-02c-109



Question 5.1

Find all extensions of the following default theories $T = \langle \Delta, \Phi \rangle$

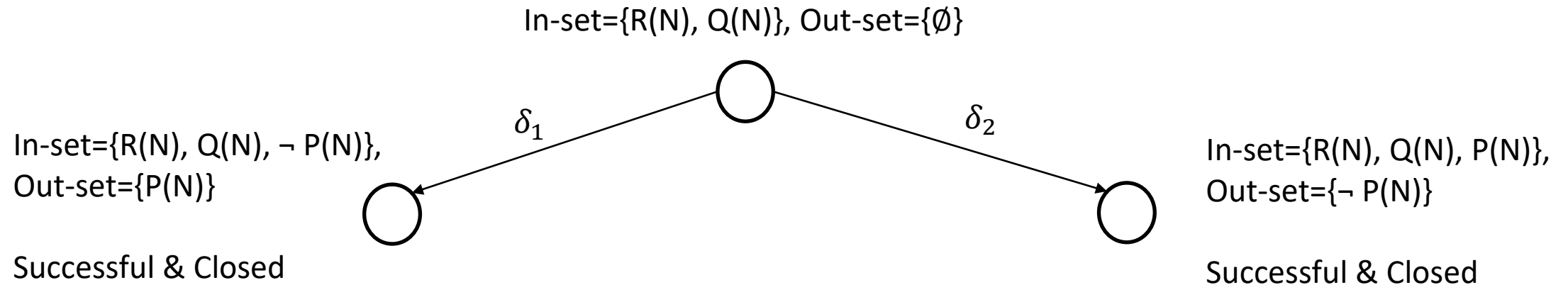
$$1. \ T = \langle \Delta = \left\{ \frac{R(x):\neg P(x)}{\neg P(x)}, \frac{Q(x):P(x)}{P(x)} \right\}, \Phi = \{R(N) \wedge Q(N)\} \rangle$$

$$2. \ T = \langle \Delta = \left\{ \frac{Summer:\neg Rain}{Sun_Shining} \right\}, \Phi = \{\neg Sun_Shining \wedge Summer\} \rangle$$



Question 5.1(1)

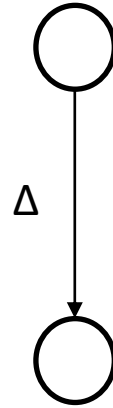
$$T = \langle \Delta = \left\{ \delta_1 = \frac{R(N): \neg P(N)}{\neg P(N)}, \delta_2 = \frac{Q(N): P(N)}{P(N)} \right\}, \Phi = \{R(N) \wedge Q(N)\} \rangle$$



Question 5.1(2)

$$T = \langle \Delta = \left\{ \frac{\text{Summer} : \neg \text{Rain}}{\text{Sun_Shining}} \right\}, \Phi = \{ \neg \text{Sun_Shining} \wedge \text{Summer} \} \rangle$$

In-set = $\{ \neg \text{Sun_Shining}, \text{Summer} \}$, Out-set = $\{ \emptyset \}$



In-set = $\{ \neg \text{Sun_Shining}, \text{Summer}, \text{Sun_Shining} \}$, Out-set = $\{ \text{Rain} \}$

Failed & Closed

Question 5.2

The fuzzy variable X is described by a set of fuzzy labels over the interval $[0, 21]$ as shown in the Figure Q5.2.

- Describe the fuzzy labels using the trapezoidal membership function (TPMF) denoted by: $\text{TPMF}[a, b, c, d]$ for each of the labels.
- State the type of fuzzy partitioning of the space provided by these four membership functions over the interval.

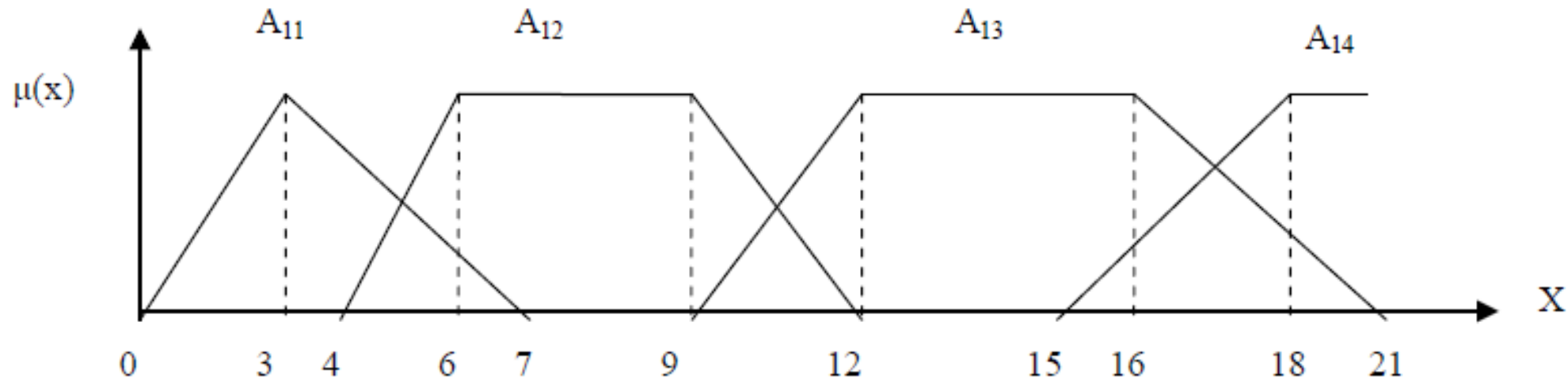
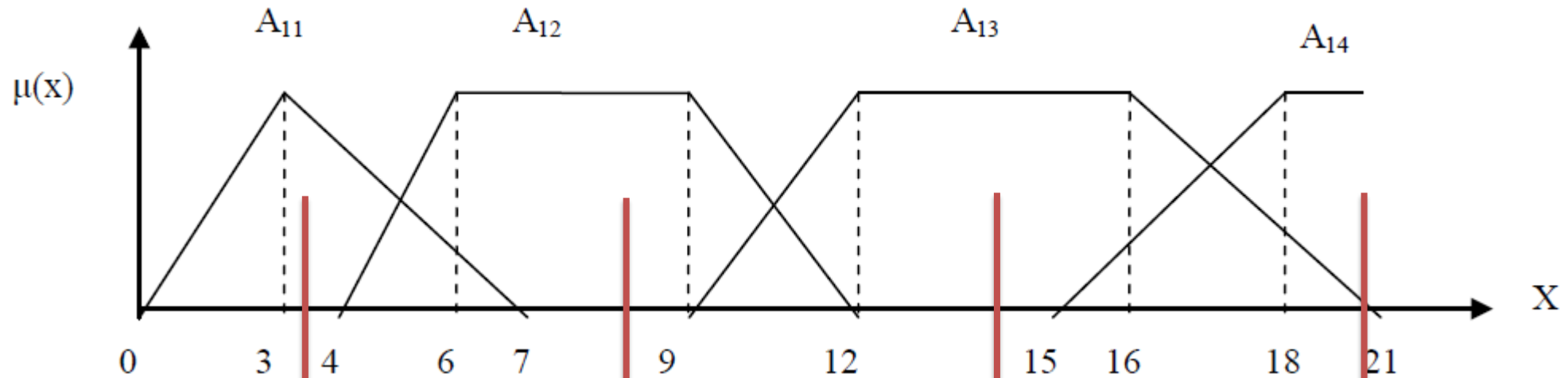


Figure Q5.2

Question 5.2



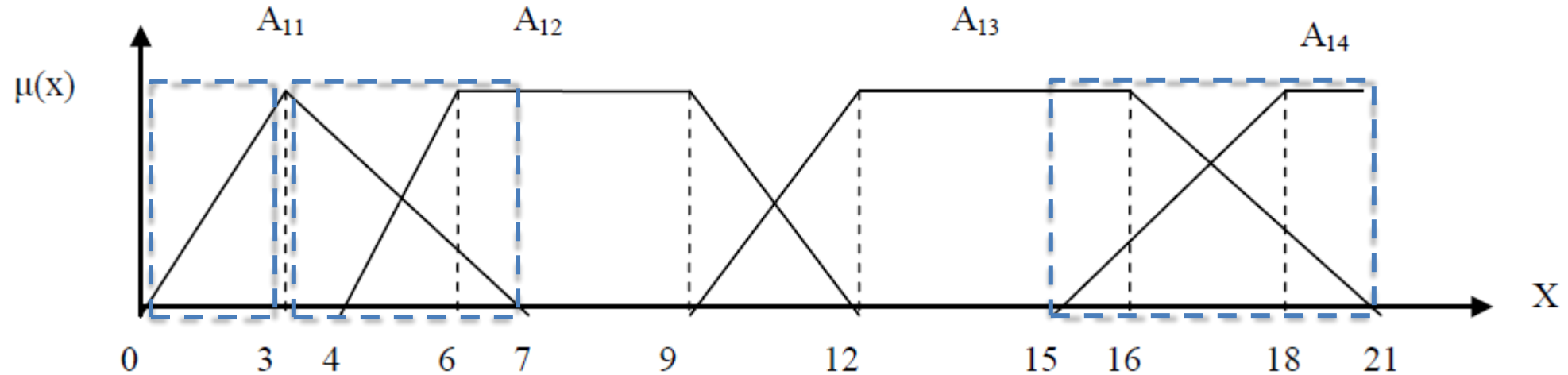
$A_{11} = \text{TPMF}[0, 3, 3, 7]$ – triangular MF

$A_{12} = \text{TPMF}[4, 6, 9, 12]$

$A_{13} = \text{TPMF}[9, 12, 16, 21]$

$A_{14} = \text{TPMF}[15, 18, 21, 21]$

Question 5.2



This is a non-pseudo fuzzy partition. The interval overlaps between A_{11} and A_{12} and between A_{13} and A_{14} (the bounded regions) are non pseudo fuzzy partition, i.e. summation of MF in the overlap region is $\neq 1$

Question 5.3

A set of fuzzy variables s-quality, f-quality and t-payment are defined by the respective set of membership functions:

s-quality:	fuzzy term/label "poor" μ_{sq1} :	$\text{tpmf}[0, 0, 4, 5]$
	fuzzy term/label "good" μ_{sq2} :	$\text{tpmf}[4, 5, 6, 7]$
	fuzzy term/label "excellent" μ_{sq3} :	$\text{tpmf}[6, 7, 10, 10]$
f-quality:	fuzzy term/label "lousy" μ_{fq1} :	$\text{tpmf}[0, 0, 2, 3]$
	fuzzy term/label "delicious" μ_{fq2} :	$\text{tpmf}[7, 8, 10, 10]$
t-quality:	fuzzy term/label "cheap" μ_{tq1} :	$\text{tpmf}[0, 2, 2, 3]$
	fuzzy term/label "average" μ_{tq2} :	$\text{tpmf}[3, 4, 4, 5]$
	fuzzy term/label "generous" μ_{tq3} :	$\text{tpmf}[4, 5, 5, 9]$

Question 5.3(i)

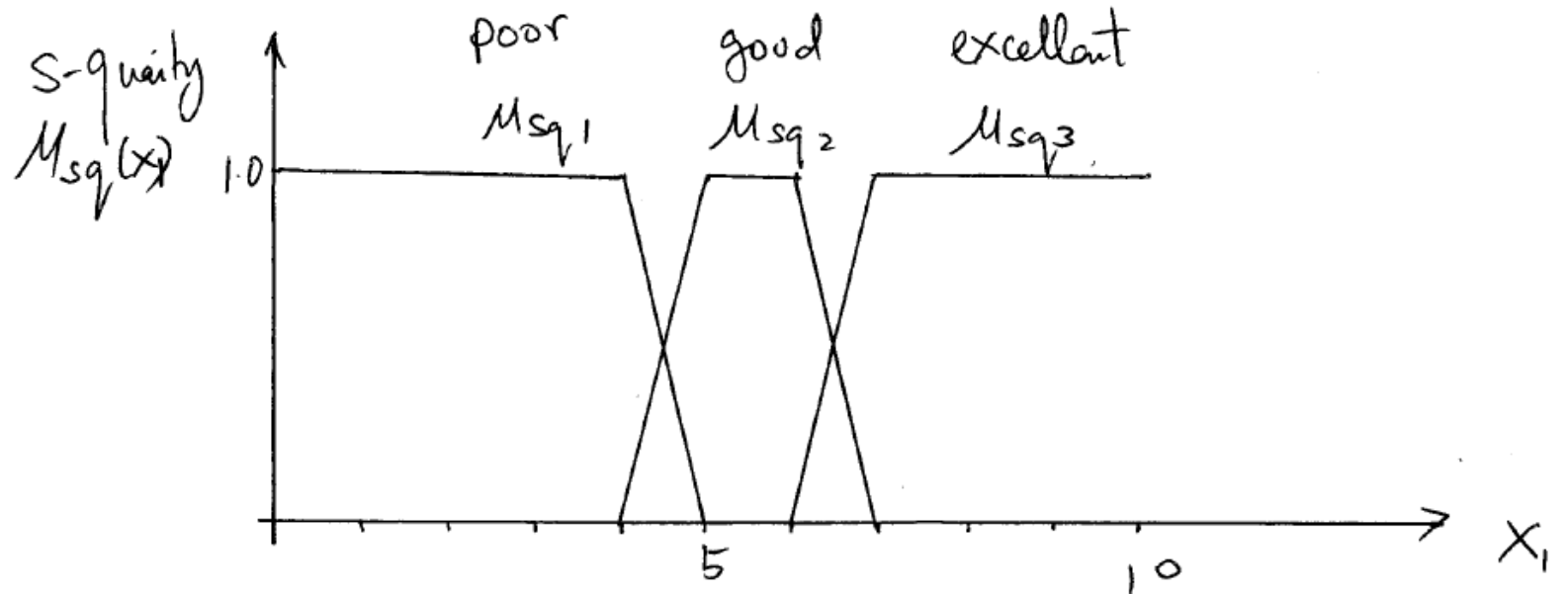
Draw the fuzzy partitions for each of the fuzzy variable over the domain $[0, 10]$. State the type of fuzzy partitioning for each of the dimensions.

s-quality:

$\text{tpmf}[0, 0, 4, 5]$

$\text{tpmf}[4, 5, 6, 7]$

$\text{tpmf}[6, 7, 10, 10]$



pseudo fuzzy partition

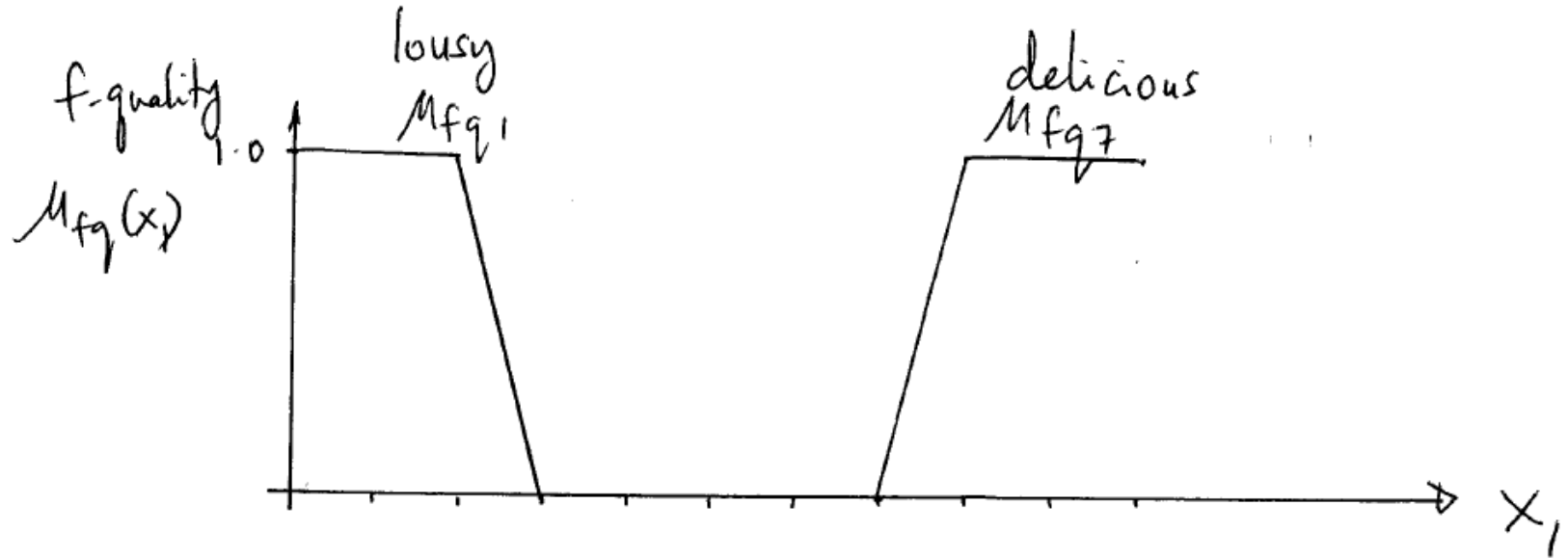
Question 5.3(i)

Draw the fuzzy partitions for each of the fuzzy variable over the domain $[0, 10]$. State the type of fuzzy partitioning for each of the dimensions.

f-quality:

$\text{tpmf}[0, 0, 2, 3]$

$\text{tpmf}[7, 8, 10, 10]$



non-pseudo fuzzy partition

Question 5.3(i)

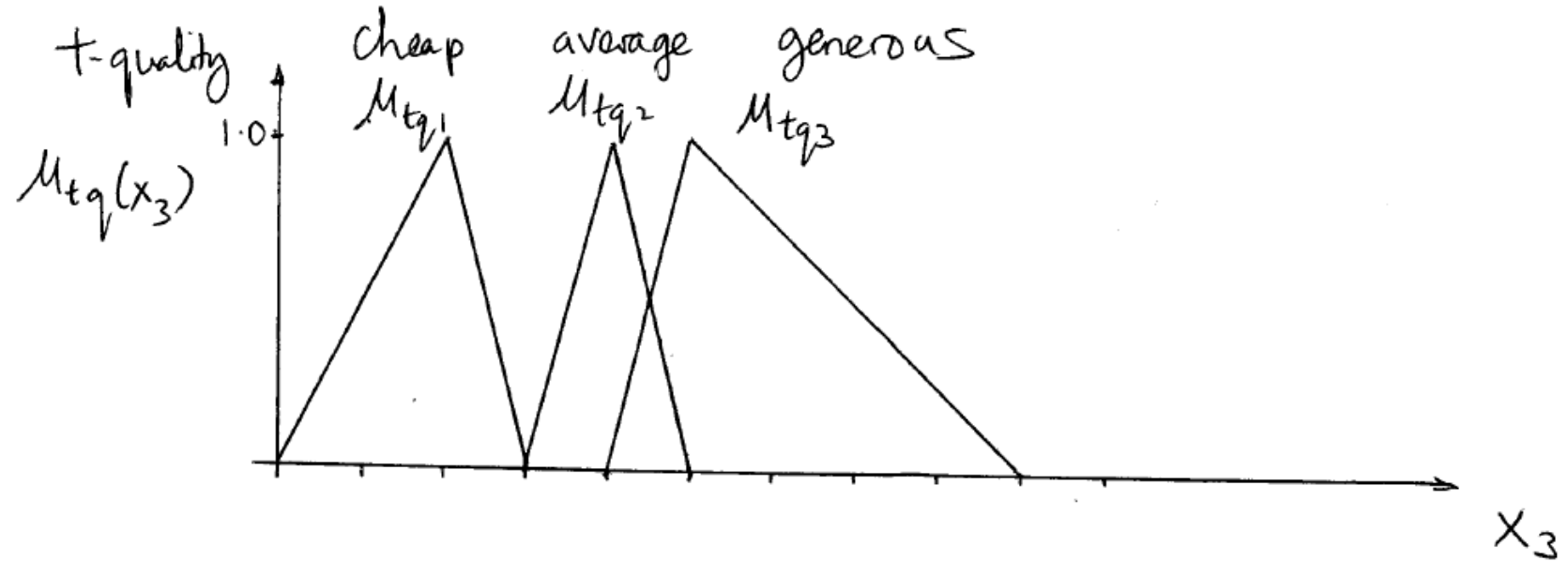
Draw the fuzzy partitions for each of the fuzzy variable over the domain $[0, 10]$. State the type of fuzzy partitioning for each of the dimensions.

t-quality:

$\text{tpmf}[0, 2, 2, 3]$

$\text{tpmf}[3, 4, 4, 5]$

$\text{tpmf}[4, 5, 5, 9]$



non-pseudo fuzzy partition

Question 5.3(ii)

The above fuzzy labels are used in the formulation of a fuzzy expert rule system for tipping. The amount of tips (t-quality) derived from the fuzzy rules are based on the service quality (s-quality) and the food quality (f-quality).

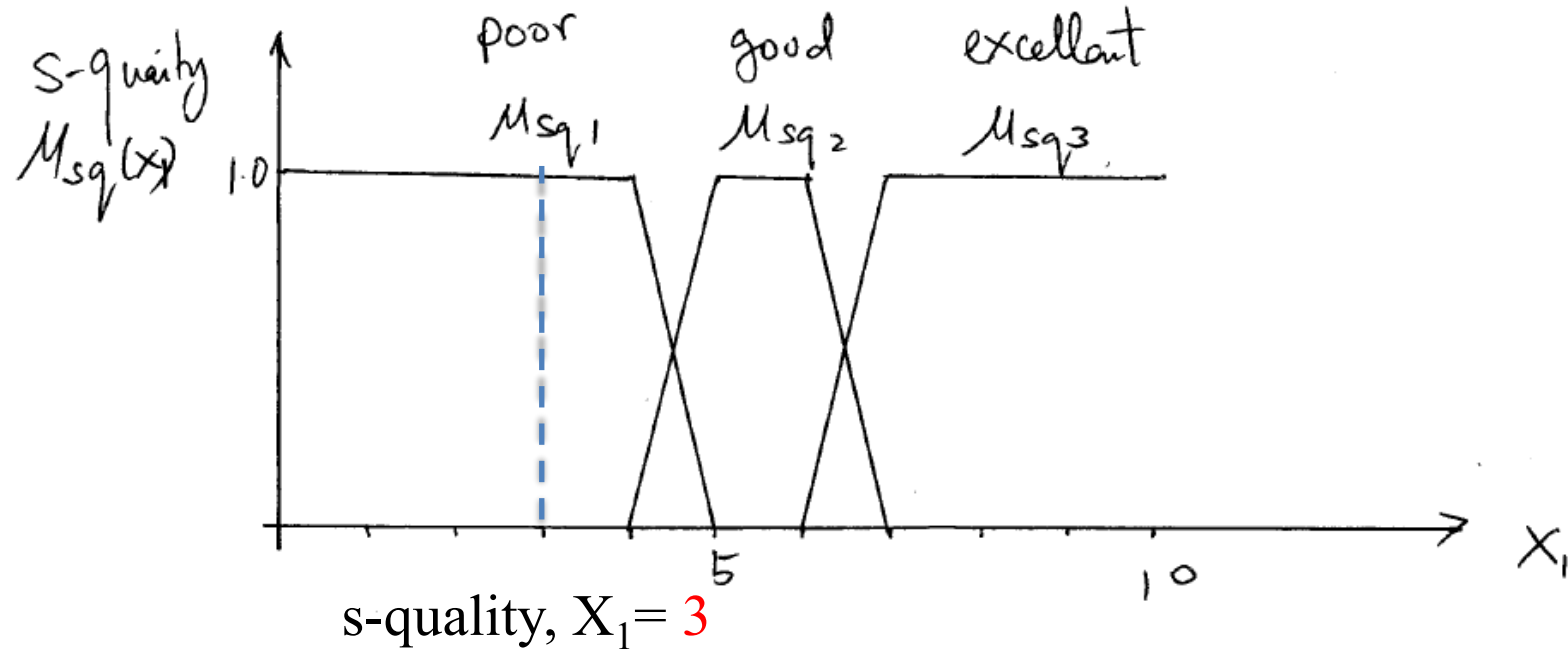
Here are 4 fuzzy rules:

- R1. If service is poor then tip is cheap.
- R2. If service is excellent and food is delicious then tip is generous.
- R3. If food is lousy then tip cheap.
- R4. If service is good and food is delicious then tip is average.

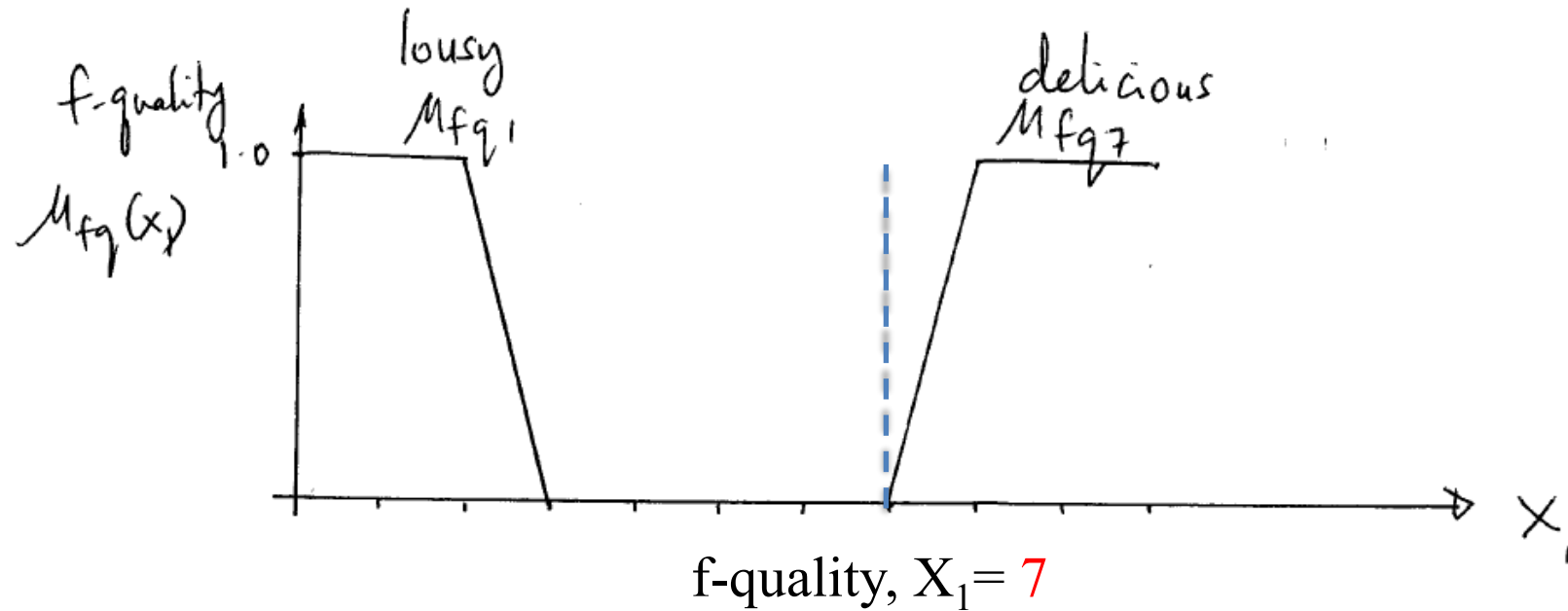
Determine the membership for the resultant tip if the scores for s-quality is 3 and f-quality is 7.



Question 5.3(ii)



Question 5.3(ii)



Question 5.3(ii)

R1. If service is poor then tip is cheap. // Fuzzy implication is the AND operator (i.e. $\min()$).

s-quality $X_1=3$: $\mu_{sq1}(X_1=3) = 1$

$$\min(1, \mu_{tq1}) = \mu_{tq1}$$

R2. If service is excellent and food is delicious then tip is generous.

$$\min(\mu_{sq3}(X_1=3) = 0, \mu_{fq2}(X_2=7) = 0) = 0$$

$$\min(0, \mu_{tq3}) = 0$$

R3. If food is lousy then tip cheap.

$$\text{f-quality } X_2=7: \mu_{fq1}(X_2=7) = 0$$

$$\min(0, \mu_{tq1}) = 0$$

R4. If service is good and food is delicious then tip is average.

$$\min(\mu_{sq2}(X_1=3) = 0, \mu_{fq2}(X_2=7) = 0) = 0$$

$$\min(0, \mu_{tq2}) = 0$$



Question 5.3(ii)

Fuzzy output is the aggregate of the outputs of each fuzzy rule.

Union operation - each of the rules is an alternative match.

$$U(\mu_{\text{tq1}}, 0, 0, 0) = \text{Max}(\mu_{\text{tq1}}, 0, 0, 0) = \mu_{\text{tq1}}$$

Either take maximum membership or Centroid defuzzification for the output μ_{tq1} (i.e. tip is cheap).



Question 5.3(iii)

Linguistic modifiers or hedges are used to change the semantics of the linguistic labels. What will the fuzzy memberships for s-quality and t-quality be like if a rule is given as:

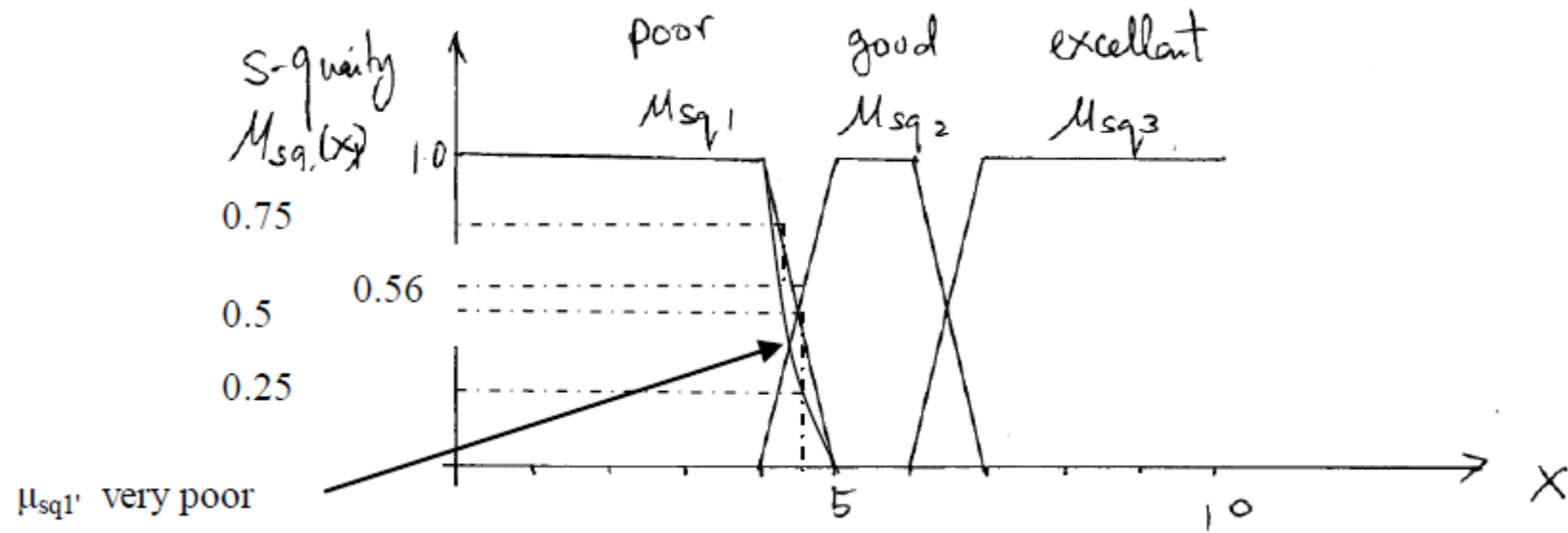
R1'. If service is very poor then tip is very cheap.

The very fuzzy linguistic hedge is the **square** operator.

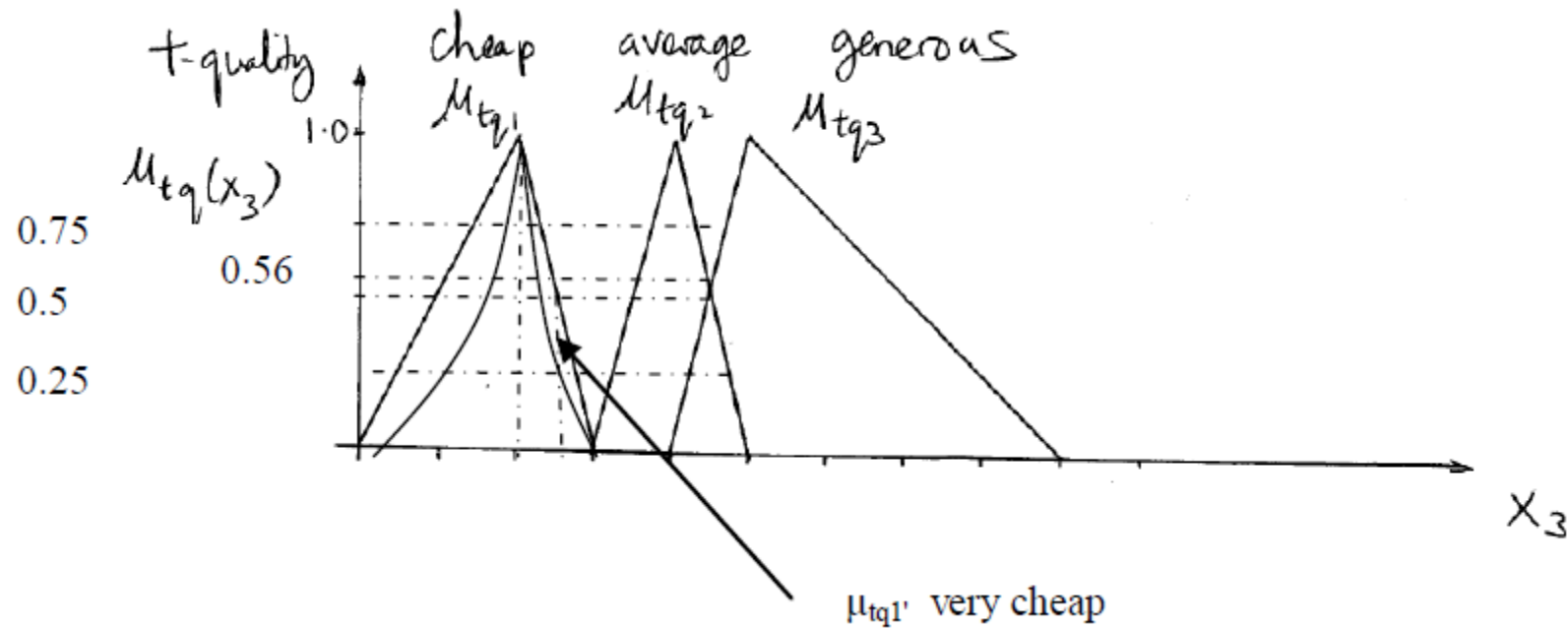
Therefore the membership functions for very poor and very cheap will be modified using the square operator on μ_{tq1} and μ_{tq3} .



Question 5.3(iii)



Question 5.3(iii)



Thank you!

