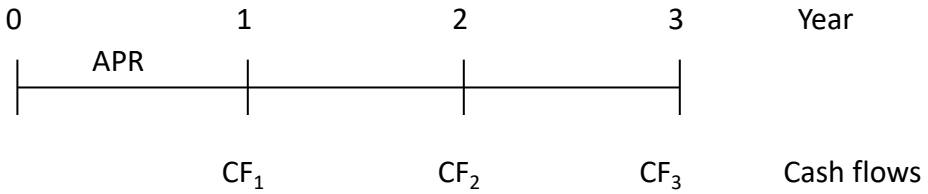
Tutorial 2 : Time Value of Money

Conducted by : Mr. Chong Lock Kuah, CFA

Key Takeaways

- The first step in time value analysis is to set up a time-line, which will help you to visualize what is happening in a particular problem



1. Time 0 is today; Time 1 is the end of period 1; or the beginning of period 2
2. Negative CF (-CF) are cash outflows
3. Annual periods $\Rightarrow$annual rate
4. Monthly periods $\Rightarrow$monthly rate

Key Takeaways

- FV is the amount to which a cash flow or series of cash flows will grow over a given period when compounded at a given interest rate

- PV is the value today of a future cash flow or series of cash flows

- Annual Percentage Rate (APR) "Nominal"
  - The annual rate quoted by law
  - APR= periodic rate x number of periods per year. APR must be compounded the same number of periods per year. For example, 6-month period rate = APR compounded semi-annually/2 since the number of periods per year is 2
  - Periodic rate = APR/periods per year

- Effective Annual Rate (EAR)
  - The interest rate expressed as if it were compounded once per year
  - Used to compare two alternative investments with different compounded periods

Computing APRs

What is the APR, if the monthly rate is 0.5%?

- 0.5%(12) = 6% (this APR is compounded monthly)

What is the APR, if the semiannual rate is 0.5%?

- 0.5%(2) = 1% (this APR is compounded semiannually)

What is the monthly rate, if the APR is 12% with monthly compounding?

- Monthly rate =12% / 12 = 1%
- Can you divide the above APR by 2 to get the semiannual rate?
  - NO. You need an APR based on semiannual compounding to find the semiannual rate.

Computing EAR and APR

- The effective annual rate is the real return paid on savings or the real cost of a loan as it considers the effects of compounding.
- The more frequent the compounding periods, the greater the return on your savings or the higher your real cost of loan.
- For example, consider these two offers: Investment A pays 10% interest, compounded monthly. Investment B pays 10.1% compounded semi-annually. Which is the better offer?

$$\mathrm{EAR} = (1 + \mathrm{APR}/\mathrm{m})^{\mathrm{m}} - 1$$

- Investment A: $\mathrm{EAR} = (1 + 0.10/12)^{12} - 1 = 10.47\%$
- Investment B: $\mathrm{EAR} = (1 + 0.101/2)^{2} - 1 = 10.36\%$

Computing APR from EAR

- Suppose if you are given APR of 12% based on monthly compounding and you require APR based semi-annual compounding.
- To convert APR based on monthly compounding to APR based on semiannual compounding, first step is to convert APR based on monthly compounding to $\mathrm{EAR} = (1 + \mathrm{APR}/12)^{12} - 1$.
- Next, convert EAR to APR based on semiannual compounding, using $\mathrm{EAR} = (1 + \mathrm{APR}/2)^{2} - 1$

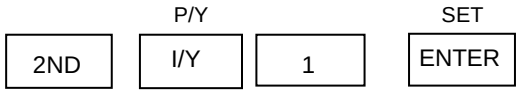
Semi-annual rate, $\mathrm{APR}/2 = (1 + \mathrm{EAR})^{\frac{1}{2}} - 1$

Texas Instruments BA-II Plus

Solving time value of money problems using financial calculator TI BII Plus is a preferred method

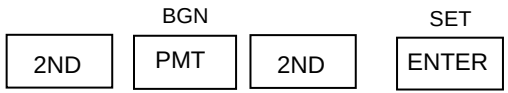
Before using a financial calculator, make sure that the calculator is set up as follows:

P/Y is set equal to 1, to set it

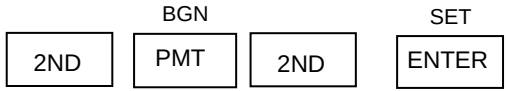


Calculator by default is in "End Mode"

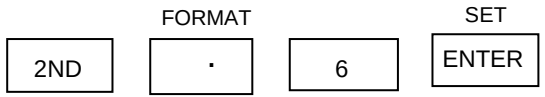
To change to "BGN Mode" hit



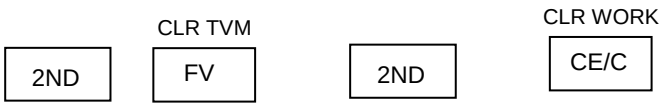
To change back to "End Mode" hit



Set the calculator to display 6 decimal places, hit

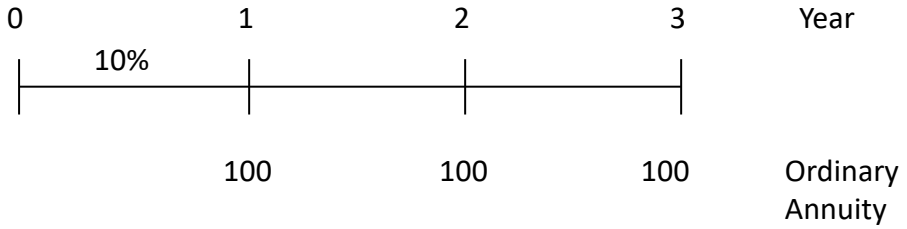


Clear all memory hit



Key Takeaways

- When calculating the PV of an ordinary annuity, .



Using a financial calculator input: N = 3, I/Y = 10, PMT =100, FV = 0, and then solve for PV = -$248.69

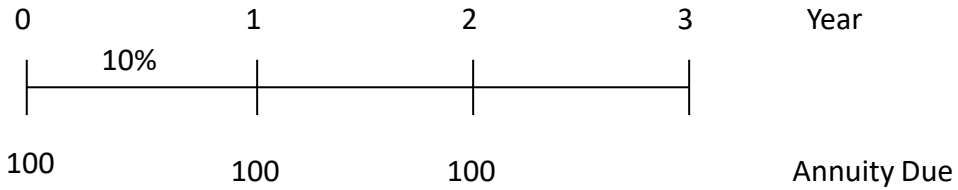
Alternatively, using present value of ordinary annuity,

$$\text{PVA} = \text{C}_1\left[\frac{1-\frac{1}{(1+\text{r})^{\text{n}}}}{\text{r}}\right] = 100\left[\frac{1-\frac{1}{(1+0.1)^3}}{0.1}\right] = \$248.69; \text{ or}$$

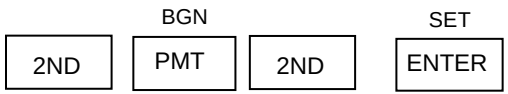
$$\text{PV} = \frac{100}{(1+0.1)} + \frac{100}{(1+0.1)^2} + \frac{100}{(1+0.1)^3} = \$248.69$$

Key Takeaways

- When calculating the PV of an annuity due,



Using a financial calculator

Hit  and then enter :

N = 3, I/Y = 10, PMT =100, FV = 0, and then solve for PV = -$273.55

Please remember to switch back to “END Mode”

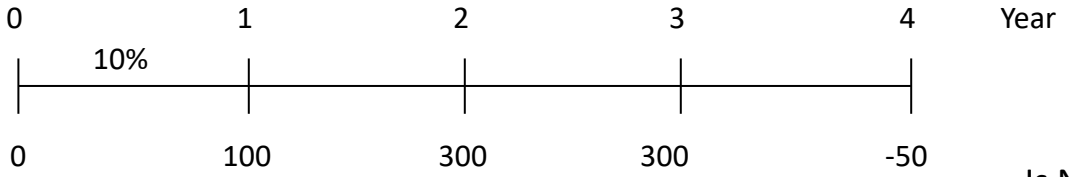
Alternatively,

$$\text{PVA}_{\text{Ord}} = C_1\left[\frac{1-\frac{1}{(1+r)^n}}{r}\right] = 100\left[\frac{1-\frac{1}{(1+0.1)^3}}{0.1}\right] = \$248.69;$$

$$\text{PVA}_{\text{Due}} = (248.69)(1.1) = \$273.55$$

Key Takeaways

- When calculating the PV of series of uneven cash flows,



Display	You Enter
	CF 2nd CE/C
C00	0 ENTER ↓
C01	100 ENTER ↓
F01	1 ENTER ↓
C02	300 ENTER ↓
F02	1 ENTER ↓
C03	300 ENTER ↓
F03	1 ENTER ↓
C04	-50 ENTER ↓
F04	1 ENTER ↓ NPV
I	10 ENTER ↓
NPV	CPT
530.09	

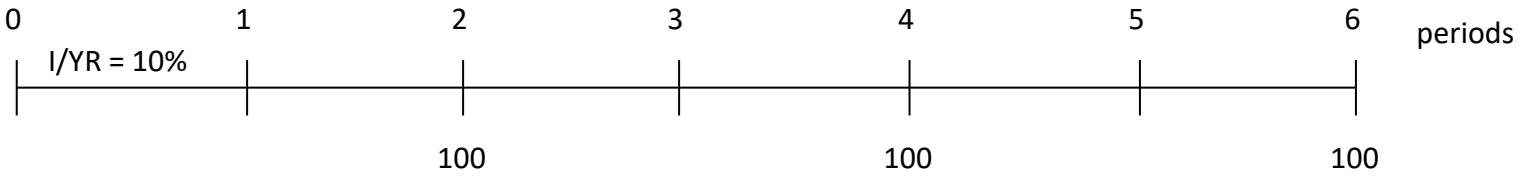
Is NPV equal to PV?

NPV = $-CF_0$ + PV(CFs)

Since $CF_0 = 0$, NPV=PV(CFs)

Key Takeaways

When calculating the FV of series of cash flows when payments occur annually, but compounding occurs each 6 months,



APR given in the question is APR compounded semiannually. Since cash flows occur annually, we require APR compounded annually. To convert APR compounded semiannually to EAR using $\mathrm{EAR} = (1 + \mathrm{APR}/\mathrm{m})^{\mathrm{m}} - 1$. EAR is the interest rate as if it is compounded annually, therefore, EAR is also equal to APR compounded annually.

APR=10% compounded semiannually , to compute EAR, using financial calculator,

hit

2ND	ICONV								
	2	calculator will display nom = 0.000000,							
	SET	INS	INS			SET	INS	INS	QUIT
10	ENTER	↓	↓	C/y	2	ENTER	↓	↓	CPT

EAR (or EFF) = 10.25

Finally, enter N=3, I/YR=10.25, P/Yr =1, PV=0, PMT=-100, to find FV=331.80

Amortized Loan with Fixed Payment Example

- Each payment covers the interest expense plus reduces principal
- Consider a 4-year loan with annual payments. The interest rate is 8% and the principal amount is $5000.
  - What is the annual payment?
    - $5{,}000 = PMT[1 - 1 / 1.08^4] / 0.08$, solving PMT = 1,509.60
    - Using financial calculator

    4 N; 8 I/Y; 5000 PV, 0 FV, CPT PMT = 1509.60

Amortized Loan with Fixed Payment - Example

Year	Beginning Balance	Total Payment Payment	Interest Paid	Principal Paid	Ending Balance
1	$ 5,000.00	$ 1,509.60	$ 400.00	$ 1,109.60	$ 3,890.40
2	$ 3,890.40	$ 1,509.60	$ 311.23	$ 1,198.37	$ 2,692.03
3	$ 2,692.03	$ 1,509.60	$ 215.36	$ 1,294.24	$ 1,397.79
4	$ 1,397.79	$ 1,509.60	$ 111.82	$ 1,397.79	$ -
Totals		$ 6,038.40	$ 1,038.42	$ 5,000.00	

Interest Paid = Beginning Balance * Rate (8%)

Principal Paid = Total Payment – Interest Paid

Ending Balance = Beginning Balance – Principal Paid

Loan with Interest calculated on “Flat” basis

- The interest payments are calculated based on the original loan amount.
- The monthly interest stays the same throughout, even though your outstanding loan reduces over time.
- A flat rate is commonly used for car loans and personal term loans.

You need a car loan of $100,000 from a bank in Singapore. The bank charges an annual interest rate of 6% (stated) for a 5-year loan with monthly payments.
How much is your monthly car loan payment?
How much is EAR of this loan?

Stated rate, 6% is a flat rate and is not APR compounded monthly

Loan principal = 100,000
Total interest = 5 x 0.06 x 100,000 = 30,000
Monthly car loan payment = (100,000 + 30,000)/ 60 = $2,166.67

$$\mathrm{PVA} = \mathrm{C}_1\left[\frac{1-\frac{1}{(1+\mathrm{r})^{\mathrm{n}}}}{\mathrm{r}}\right] = 2{,}166.67\left[\frac{1-\frac{1}{(1+\mathrm{r}/12)^{60}}}{\mathrm{r}/12}\right]$$

$$\mathrm{r}/12 = \text{effective monthly rate} = 0.90\%$$

$$\mathrm{EAR} = (1+\mathrm{APR}/\mathrm{m})^{\mathrm{m}} - 1 = (1+0.009)^{12} - 1 = 11.4\%$$

#1:

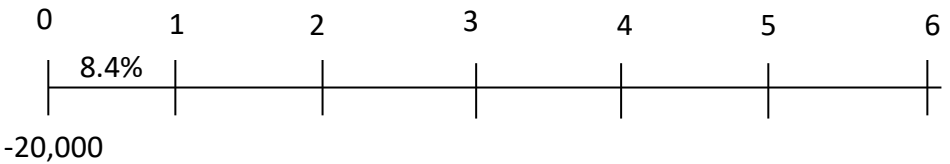
First City Bank pays 8 percent simple interest on its savings account balances, whereas Second City Bank pays 8 percent interest compounded annually. If you made a $5,000 deposit in each bank, how much more money would you earn from your Second City Bank account at the end of 10 years?

The simple interest per year is:
$5,000 × 0.08 = $400
So after 10 years you will have: $400 × 10 = $4,000 in interest.
The total balance will be $5,000 + $4,000 = $9,000

With compound interest we use the future value formula:
$FV = PV(1 + r)^t$
$FV = \$5,000(1.08)^{10} = \$10,794.62$
The difference is:$10,794.62 − $9,000 = $1,794.62

#2: (Using Financial Calculator)

You are scheduled to receive $20,000 in two years. When you receive it, you will invest it for six more years at 8.4 percent per year. How much will you have in eight years?



6	xP/Y N	20,000	RESET +/-	AMORT PV	0	BGN PMT	8.4	P/Y I/Y

QUIT CPT	CLR TVM FV

FV=$32,449.33

#2: (Formula)

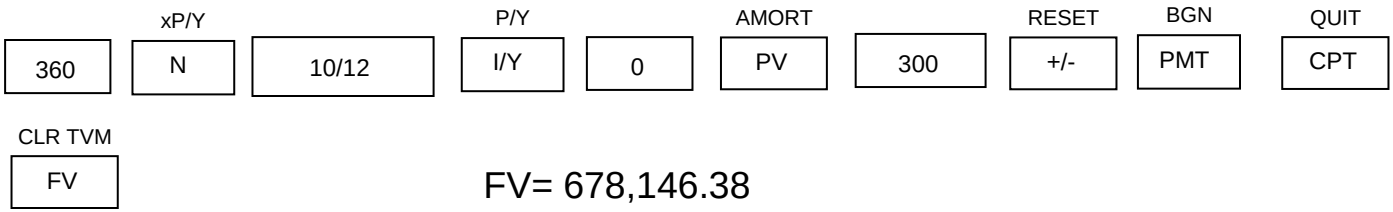
You are scheduled to receive $20,000 in two years. When you receive it, you will invest it for six more years at 8.4 percent per year. How much will you have in eight years?

$FV = PV(1 + r)^t$

$FV = \$20,000(1.084)^6 = \$32,449.33$

#3: Using Financial Calculator

You are to make monthly deposits of $300 into a retirement account that pays 10 percent interest compounded monthly. If your first deposit will be made one month from now, how large will your retirement account be in 30 years?



FV= 678,146.38

#3: (Formula)

You are to make monthly deposits of $300 into a retirement account that pays 10 percent interest compounded monthly. If your first deposit will be made one month from now, how large will your retirement account be in 30 years?

This problem requires us to find the FVA. The equation to find the FVA is:

$$\mathrm{FVA} = \mathrm{C}_1 \left[\frac{(1+\mathrm{r})^{\mathrm{n}}-1}{\mathrm{r}}\right]$$

$$\mathrm{FVA} = \$300 \left[\frac{(1+0.10/12)^{360}-1}{0.10/12}\right] = \$678{,}146.38$$

#4: Using Financial Calculator

You are planning to save for retirement over the next 30 years. To do this, you will invest $700 a month in a stock account and $300 a month in a bond account. The return of the stock account is expected to be 11 percent, and the bond account will pay 6 percent. When you retire, you will combine your money into an account with a 9 percent return. How much can you withdraw each month from your account assuming a 25-year withdrawal period?

We need to find the annuity payment in retirement. Our retirement savings ends, and the retirement withdrawals begin, so the PV of the retirement withdrawals will be the FV of the retirement savings. So, we find the FV of the stock account and the FV of the bond account and add the two FVs.

Stock a/c

360	xP/Y N	11/12	P/Y I/Y	700	RESET +/-	BGN PMT	QUIT CPT	CLR TVM FV

FV= $1,963,163.82

Bond a/c

360	xP/Y N	6/12	P/Y I/Y	300	RESET +/-	BGN PMT	QUIT CPT	CLR TVM FV

FV= $301,354.51

So, the total amount saved at retirement is:
$1,963,163.82 + 301,354.51 = $2,264,518.33

300	xP/Y N	9/12	P/Y I/Y	2,264,518.33	RESET +/-	AMORT PV	QUIT CPT	BGN PMT

PMT= $19,003.763 withdrawal per month

#4: (Formula)

You are planning to save for retirement over the next 30 years. To do this, you will invest $700 a month in a stock account and $300 a month in a bond account. The return of the stock account is expected to be 11 percent, and the bond account will pay 6 percent. When you retire, you will combine your money into an account with a 9 percent return. How much can you withdraw each month from your account assuming a 25-year withdrawal period?

We need to find the annuity payment in retirement. Our retirement savings ends, and the retirement withdrawals begin, so the PV of the retirement withdrawals will be the FV of the retirement savings. So, we find the FV of the stock account and the FV of the bond account and add the two FVs.

Stock account: $FVA = \$700[\{[1 + (0.11/12)]^{360} - 1\} / (0.11/12)] = \$1,963,163.82$
Bond account: $FVA = \$300[\{[1 + (0.06/12)]^{360} - 1\} / (0.06/12)] = \$301,354.51$
So, the total amount saved at retirement is:

$\$1,963,163.82 + 301,354.51 = \$2,264,518.33$

Solving for the withdrawal amount in retirement using the PVA equation gives us:
$PVA = \$2,264,518.33 = \$C_1[1 - \{1 / [1 + (0.09/12)]^{300}\} / (0.09/12)]$
$C1 = \$2,264,518.33 / 119.1616 = \$19,003.763$ withdrawal per month

#5: (formula)

Your job pays you only once a year for all the work you did over the previous 12 months. Today, December 31, you just received your salary of $50,000 and you plan to spend all of it. However, you want to start saving for retirement beginning next year. You have decided that one year from today you will begin depositing 5 percent of your annual salary in an account that will earn 11 percent per year. Your salary will increase at 4 percent per year throughout your career. How much money will you have on the date of your retirement 40 years from today?

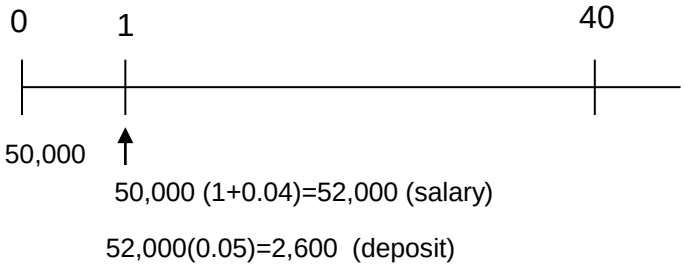
Since your salary grows at 4 percent per year, your salary next year will be:

Next year's salary = $50,000(1 + 0.04)

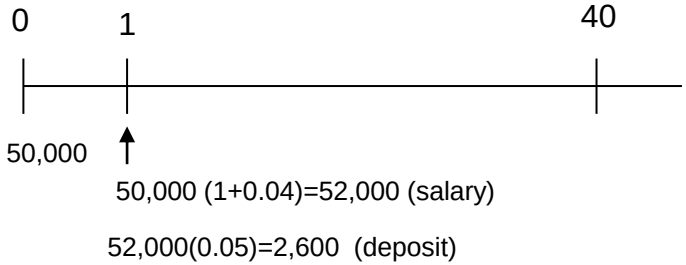
Next year's salary = $52,000

This means your deposit next year will be:

Next year's deposit = $52,000(0.05) = $2,600



Since your salary grows at 4 percent, your deposit will also grow at 4 percent. We can use the future value of a growing annuity equation to find the value of your deposits in year 40



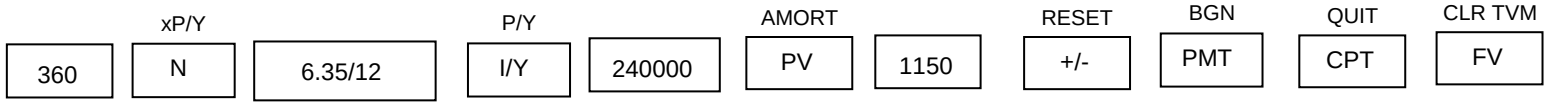
Future value of growing annuity formula

$$FV = C_1\left[\frac{(1+r)^n-(1+g)^n}{r-g}\right]$$

$$FV = 2600\left[\frac{(1+0.11)^{40}-(1+0.04)^{40}}{0.11-0.04}\right] = \$2{,}235{,}994$$

#6: Using Financial Calculator

You need a 30-year, fixed-rate mortgage to buy a new home for $240,000. Your mortgage bank will lend you the money at a 6.35 percent APR for this 360-month loan. However, you can afford monthly payments of only $1,150, so you offer to pay off any remaining loan balance at the end of the loan in the form of a single balloon payment. How large will this balloon payment have to be for you to keep your monthly payments at $1,150?

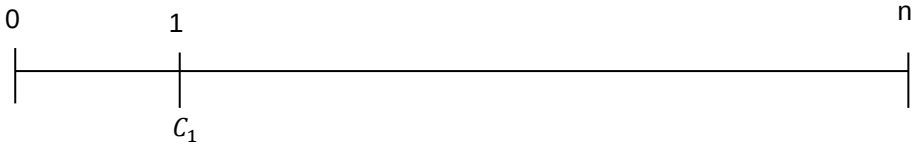


FV= -368,936.54

#6: (Formula)

You need a 30-year, fixed-rate mortgage to buy a new home for $240,000. Your mortgage bank will lend you the money at a 6.35 percent APR for this 360-month loan. However, you can afford monthly payments of only $1,150, so you offer to pay off any remaining loan balance at the end of the loan in the form of a single balloon payment. How large will this balloon payment have to be for you to keep your monthly payments at $1,150?

Explanation:
The amount of principal paid on the loan is the PV of the monthly payments you make. So, the present value of the $1,150 monthly payments is:



$$\text{PVA} = \text{C}_1\left[\frac{1-\frac{1}{(1+\text{r})^{\text{n}}}}{\text{r}}\right] = \$1{,}150\left[\frac{1-\frac{1}{(1+0.0635/12)^{360}}}{0.0635/12}\right] = \$184{,}817.42$$

Cont'd

The monthly payments of $1,150 will amount to a principal payment of $184,817.42. The amount of principal you will still owe is:
$240,000 − 184,817.42 = $55,182.58

This remaining principal amount will increase at the interest rate on the loan until the end of the loan period. So the balloon payment in 30 years, which is the FV of the remaining principal will be:

Balloon payment = $55,182.58 $[1 + (0.0635/12)]^{360}$ = $368,936.54

#7: Using Financial Calculator

The present value of the following cash flow stream is $6,550 when discounted at 10 percent annually. What is the value of the missing cash flow?

Year	Cash Flow
1	$1,700
2	?
3	$2,100
4	$2,800

We have to enter individual cash flows using [CF] key to find NPV

[CF] [0] [SET ENTER] [INS ↓] [1700] [SET ENTER] [INS ↓] [SET ENTER] [INS ↓] [0] [SET ENTER] [INS ↓]

[INS ↓] [2100] [SET ENTER] [INS ↓] [SET ENTER] [INS ↓] [2800] [SET ENTER] [INS ↓] [SET ENTER] And then hit

[NPV] calculator will display I=0.000000, hit [10] [SET ENTER] [INS ↓] [QUIT CPT]

NPV = 5,035.65

Given that the present value of cash stream is $6,550, the present value of year 2 cash flow = $6,550 – 5,035.65=$1,514.35

The value of the missing CF is: $\$1,514.35(1.10)^2 = \$1,832.36$

#7: (Formula)

The present value of the following cash flow stream is $6,550 when discounted at 10 percent annually. What is the value of the missing cash flow?

Year	Cash Flow
1	$1,700
2	?
3	$2,100
4	$2,800

Explanation:
We are given the total PV of all four cash flows. If we find the PV of the three cash flows we know, and subtract them from the total PV, the amount left over must be the PV of the missing cash flow. So, the PV of the cash flows we know are:

Cont'd

PV of Year 3 CF: $\$2,100 / 1.10^3 = \$1,577.76$

PV of Year 4 CF: $\$2,800 / 1.10^4 = \$1,912.44$

So, the PV of the missing CF is:
$\$6,550 - 1,545.45 - 1,577.76 - 1,912.44 = \$1,514.35$

The question asks for the value of the cash flow in Year 2, so we must find the future value of this amount.

The value of the missing CF is:$\$1,514.35(1.10)^2 = \$1,832.36$

#8: Using Financial Calculator

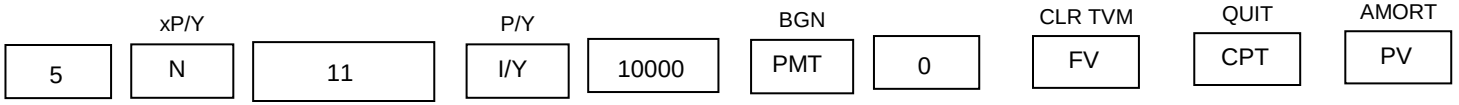
Suppose you are going to receive $10,000 per year for five years. The appropriate interest rate is 11 percent.

a. What is the present value of the payments if they are in the form of an ordinary annuity? What is the present value if the payments are an annuity due?

b. Suppose you plan to invest the payments for five years. What is the future value if the payments are an ordinary annuity? What if the payments are an annuity due?

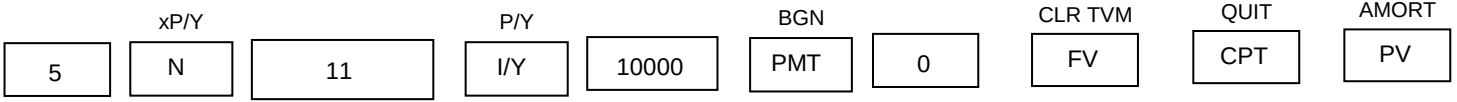
c. Which has the highest present value, the ordinary annuity or annuity due? Which has the highest future value? Will this always be true?

a(i).



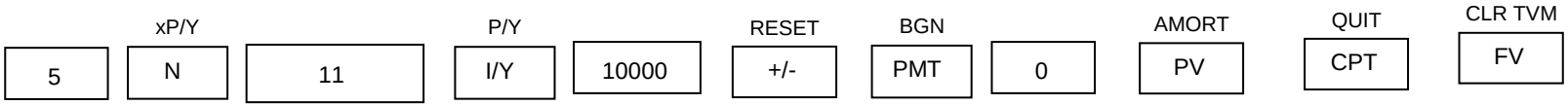
PV= -36,958.97

b(ii).



PV= -41,024.46

b(i).



FV= 62,278.01

b(ii).

FV= 69,128.60

c.

Annuity due will have the highest PV and FV.

#8: (Formula)

Suppose you are going to receive $10,000 per year for five years. The appropriate interest rate is 11 percent.

a. What is the present value of the payments if they are in the form of an ordinary annuity? What is the present value if the payments are an annuity due?

b. Suppose you plan to invest the payments for five years. What is the future value if the payments are an ordinary annuity? What if the payments are an annuity due?

c. Which has the highest present value, the ordinary annuity or annuity due? Which has the highest future value? Will this always be true?

a(i) If the payments are in the form of an ordinary annuity, the present value will be:

$PVA = C_1(\{ 1 - [1/(1 + r)^t] \} / r))$

$PVA = \$10,000[\{ 1 - [1 / (1 + 0.11)^5] \}/ 0.11]$

$PVA = \$36,958.97$

a(ii) If the payments are an annuity due, the present value will be:

$PVA_{due} = (1 + r)\ PVA$

$PVA_{due} = (1 + 0.11)\$36,958.97$

$PVA_{due} = \$41,024.46$

b(i) We can find the future value of the ordinary annuity as:

$FVA = C_1\{[(1 + r)^t - 1] / r\}$

$FVA = \$10,000\{[(1 + 0.11)^5 - 1] / 0.11\}$

$FVA = \$62,278.01$

Cont'd

b(ii) If the payments are an annuity due, the future value will be:

$FVA_{due} = (1 + r)\ FVA$

$FVA_{due} = (1 + 0.11)\$62,278.01$

$FVA_{due} = \$69,128.60$

c.

Annuity due will have the highest PV and FV provided r is positive.

#9:

Suppose you have a $1.5 million loan with semi-annual instalments over 10 years. How much do you pay towards the principal in the second instalment if the interest rate of this loan is 1.8% compounded quarterly?

We pay semi-annual instalments. APR used should be compounded semi-annually. But we are given APR compounded quarterly. We need calculate APR compounded semi-annually.

Step 1)
Find semi-annual period rate:
$EAR = (1+0.018/4)^4 - 1 = 1.8122\%$
$(1 + APR/2)^2 - 1 = 1.8122\%$
$APR/2 = (1.018122)^{1/2} - 1 = 0.9020\%$

Step 2)
Find the semi-annual installment:
N = 10*2 = 20
I/Y = 0.9020%
PV = $1,500,000
PMT = ?
$\rightarrow$ PMT = $82,305.12

Step 3)
Construct amortization table:

	Beg Principal	PMT	Interest	Principal Paid	End Principal
1	1,500,000.00	82,305.12	13,530.00	68,775.12	1,431,224.88
2	1,431,224.88	82,305.12	12,909.65	69,395.47	

→ Principal paid down in 2nd installment is $69,395.47.

Alternatively,
PV=$1,500,000; I/Y=0.9020%; N=1; PMT =$82,305.12; CPT FV_1 = 1,431,224.88

PV =$1,500,000; I/Y=0.9020; N=2; PMT=$82,305.12; CPT= FV_2=1,361,829.41

Principal paid in second instalment =1,361,829.41 – 1,431,224.88 = 69,395.47