SC3000/CZ3005
Artificial Intelligence

Game Theory

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What is Game Theory?

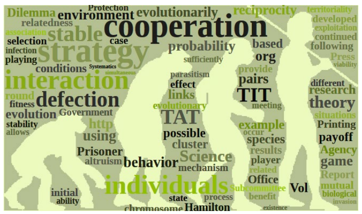
- Game theory studies settings where multiple parties (agents) each have
  - different preferences (utility functions),
  - different actions that they can take
- Each agent's utility (potentially) depends on all agents' actions
  - What is optimal for one agent depends on what other agents do
  - Very circular!
- Game theory studies how agents can rationally form beliefs over what other agents will do, and (hence) how agents should act
  - Useful for acting as well as predicting behavior of others
- John von Neumann

What is Game Theory?

Economics

Politics


Games

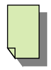
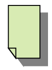


Biology

Normal Form Games – An Example

- List of players, strategies, payoffs
- Simultaneous
- Zero-sum here but not necessary win-lose - super competitive.
- Analysis:
  - What should they do?
  - Advice to player A

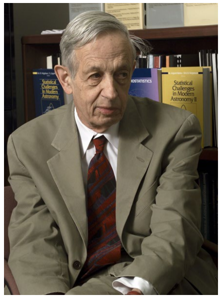
Player B

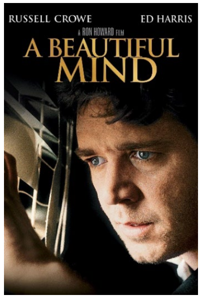
			Rock	
		0, 0	1, -1	-1, 1
Player A	Rock	-1, 1	0, 0	1, -1
		1, -1	-1, 1	0, 0

no optimal strategy.

Nash Equilibrium

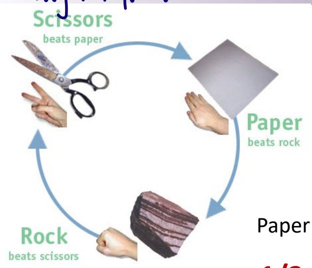
- Each agent is selfish
- Each agent makes decision based on what he thinks others would do
- No one can do better by changing strategy solely





Nash Equilibrium

No pure strategy N.E
only mix for



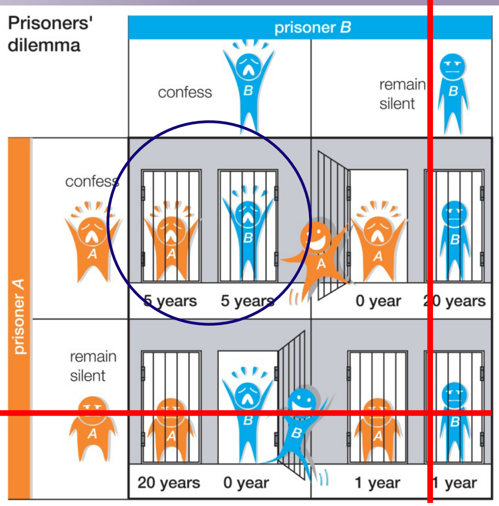
		Player B		
		Paper **1/3**	Rock **1/3**	Scissors **1/3**
Player A	Paper **1/3**	0, 0	1, -1	-1, 1
	Rock **1/3**	-1, 1	0, 0	1, -1
	Scissors **1/3**	1, -1	-1, 1	0, 0

Nash Equilibrium

Nash equilibrium

confess ***dominates*** silent

Dominant strategy.



Cooperation would be better for both!
But, rational for both to defect!

Both can do better, but difficult to happen.

Nash Equilibrium

- In general, we will say that two strategies s_1 (for i) *and* s_2 (for j) are in Nash equilibrium if:
  1. under the assumption that agent i plays s_1, agent j can do no better than play s_2; and
  2. under the assumption that agent j plays s_2, agent i can do no better than play s_1.
- *Neither agent has any incentive to deviate from a Nash equilibrium*
- Unfortunately:
  1. *Not every interaction scenario has a pure strategy Nash equilibrium*
  2. *Some interaction scenarios have more than one pure strategy Nash equilibrium*

		i	
		defect	coop
j	defect	1 1	4 1
	coop	1 4	4 4

Matching Pennies

- Players *i* and *j* simultaneously choose the face of a coin, either “heads” or “tails”.
- If they show the same face, then *i* wins, while if they show different faces, then *j* wins.
- The Payoff Matrix:

	i heads	i tails
j heads	-1, 1	1, -1
j tails	1, -1	-1, 1

No pure strategy N.E

→ Both players have incentive to deviate.

Mixed Strategies for Matching Pennies

- NO pair of strategies forms a pure strategy NE: whatever pair of strategies is chosen, somebody will wish they had done something else.
- The solution is to allow mixed strategies:
  - play “heads” with probability 0.5
  - play “tails” with probability 0.5.
- This is a NE strategy.

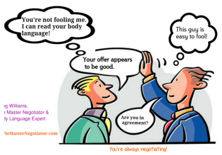
Games: Complete Information v.s. Incomplete Information

Games with complete information

Chess

Go

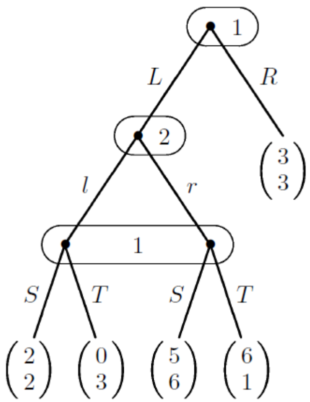
Games with incomplete information



Negotiation

Poker

Games: Normal Form v.s. Sequence Form



$$A = \begin{array}{c} l \quad r \\ \begin{bmatrix} 2 & 5 \\ 0 & 6 \\ 3 & 3 \\ 3 & 3 \end{bmatrix} \end{array} \begin{array}{l} \\ \langle L,S\rangle \\ \langle L,T\rangle \\ \langle R,S\rangle \\ \langle R,T\rangle \end{array} \qquad B = \begin{array}{c} l \quad r \\ \begin{bmatrix} 2 & 6 \\ 3 & 1 \\ 3 & 3 \\ 3 & 3 \end{bmatrix} \end{array} \begin{array}{l} \\ \langle L,S\rangle \\ \langle L,T\rangle \\ \langle R,S\rangle \\ \langle R,T\rangle \end{array}$$

$$A = \begin{array}{c} \emptyset \quad l \quad r \\ \begin{bmatrix} & & \\ & & \\ 3 & & \\ & 2 & 5 \\ & 0 & 6 \end{bmatrix} \end{array} \begin{array}{l} \\ \emptyset \\ L \\ R \\ LS \\ LT \end{array} \qquad B = \begin{array}{c} \emptyset \quad l \quad r \\ \begin{bmatrix} & & \\ & & \\ 3 & & \\ & 2 & 6 \\ & 3 & 1 \end{bmatrix} \end{array} \begin{array}{l} \\ \emptyset \\ L \\ R \\ LS \\ LT \end{array}$$

Texas Hold'em Poker



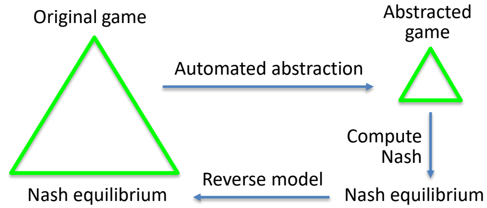

- Preflop: two private cards are dealt to each player, followed by a betting round; players can either check, bet or fold
- Flop: three public cards are dealt, followed by a betting round
- Turn: a fourth public card is dealt, followed by a betting round
- River: a last public card is dealt, followed by a betting round
- The game ends when
  - Only one player is left, all the other players fold
  - A showdown; a hand with the best 5 cards using both the two private cards and the five public cards wins

Architecture of Libratus

Abstraction (offline)
- action abstraction
- card abstraction
- took the game size from 10^{161} to 10^{12}

Equilibrium Finding (offline)
- CFR
- CFR+
- Monte Carlo CFR

Decomposition and Subgame Refinement (online)
- endgame solving
- subgame re-solving
- max-margin subgame refinement



Global Challenges for Security




Key challenges: Limited resources, surveillance

Stackelberg Games

Randomization: Increase Cost and Uncertainty to Attackers

- Security allocation
  - Target weights
  - Opponent reaction
- Stackelberg: Security forces commit first
- Optimal allocation: Weighted random
  - Strong Stackelberg Equilibrium

Attacker

Defender

	Target #1	Target #2
Target #1	4, -3	-1, 1
Target #2	-5, 5	2, -1

Game Theory for Security: Applications

Game Theory + Optimization + Uncertainty + Learning + …

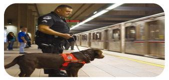
Infrastructure Security Games


Coast Guard


Coast Guard: Ferry


LAX

TSA


LA Sheriff


USC

Argentina Airport


Chile Border

Green Security Games

Coast Guard

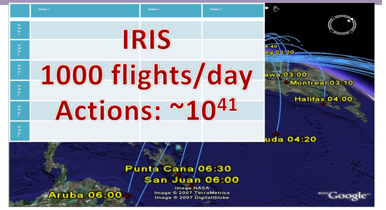

Panthera/WWF

India

Opportunistic Crime Games

Cyber Security Games

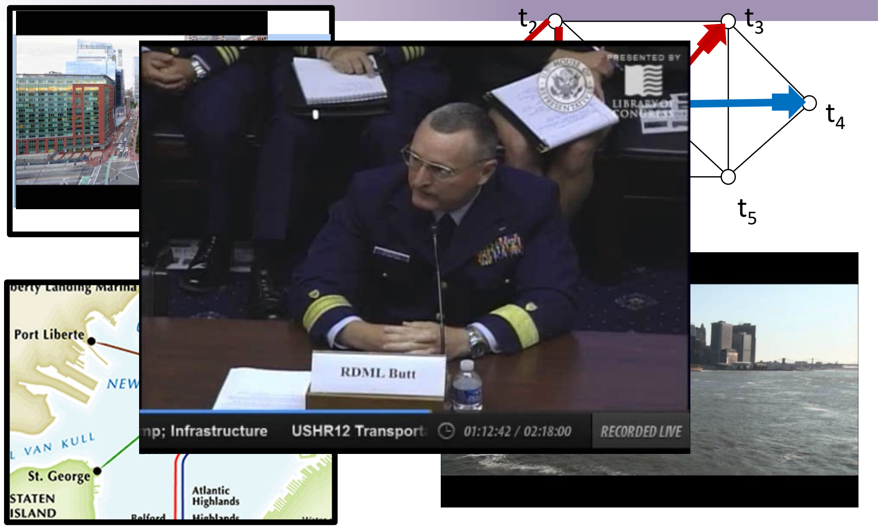
IRIS: Federal Air Marshals Service [2009]
Scale Up Number of Defender Strategies



- 1000 Flights, 20 air marshals: 10^{41} combinations
  - ARMOR out of memory
- Not enumerate all combinations combinations
  - Branch and price
    - Branch & bound + column generation

PROTECT: Randomized Patrol Scheduling [2011]

Coordination (Scale-up) and Ferries (Continuous Space/time)

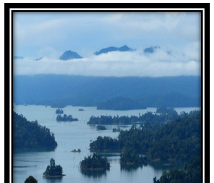


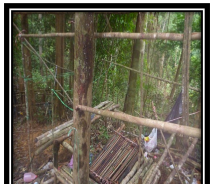
PAWS: Protection Assistant for Wildlife Security Trials in Uganda and Malaysia [2014]

- **Important lesson: Geography!**


Uganda


Andrew Lemieux


Malaysia

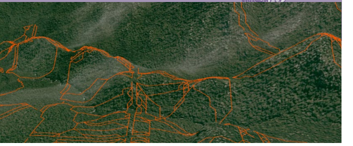

Panthera


Malaysia

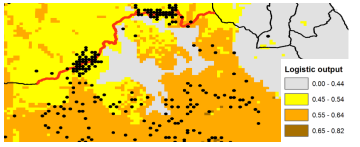
PAWS Deployed in 2015 in Southeast Asia (with Panthera and WWF)

PAWS Version 2: Features

- Street map
  - Ridgelines, rivers/streams



- Species Distribution Models (SDMs)
  - From data points to distribution map



Indonesia

Malaysia