CE2001/CZ2001: Algorithms

Introduction to Algorithms

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Learning Objectives

At the end of this lecture, students should be able to:

- Explain what an algorithm is
- Explain the different designs of algorithms using examples
  - Summation function
  - Fibonacci Sequence
  - Sine function
- State the criteria to choose algorithms

What is an Algorithm?

- An algorithm is a sequence of unambiguous instructions for solving a problem, i.e., for obtaining a required output for any legitimate input in a finite amount of time.

  Introduction to The Design & Analysis of Algorithms

  -Anany Levitin

- An algorithm is any well-defined computational procedure that takes some value, or set of values, as input and produces some value, or set of values, as output.

  Introduction to Algorithms

  -T. H. Cormen et. al.

What is an Algorithm?

- A clearly specified set of simple instructions to be followed to solve a problem or compute a function.

How to make a cake?

Step 1:..............................
....
Step 2: ....
Step 3: ....
....
....
Step 4:...............
Step 5:.........
Step 6: ....
Step 7: ....
Step 8:................

Reference: Reference Stock Vectors, Clipart and Illustrations." *123RF Stock Photos.* N.p., n.d. Web. 16 May 2016.

What is an Algorithm?

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How to make a cake?

- ✓ **Step 1:**.........................
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  ....
  ....
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- ✓ **Step 5:**.........
- ✓ **Step 6:** ....
- ✓ **Step 7:** ....
- ✓ **Step 8:**...............

Reference: Reference Stock Vectors, Clipart and Illustrations." *123RF Stock Photos.* N.p., n.d. Web. 16 May 2016.

What is an Algorithm?

A **good algorithm** possesses the following properties:

1. **It must be correct.**
   - ✓ **Returns the correct results.**

How to make a cake?

- ✓ **Step 1:**.....................
- ✓ **Step 2:** ....
- ✓ **Step 3:** ....
  ....
- ✓ **Step 4:**...............
- ✓ **Step 5:**.........
- ✓ **Step 6:** ....
- ✓ **Step 7:** ....
- ✓ **Step 8:**...............

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What is an Algorithm?

A **good algorithm** possesses the following properties:

2. **It must be composed of a series of concrete steps.**
   - ✓ **There can be no ambiguity as to which step will be performed next.**

How to make a cake?

- ✓ **Step 1:**.........................
  ....
- ✓ **Step 2:** ....
- ✓ **Step 3:** ....
  ....
  ....
  Step 4:...............
- ✓ **Step 5:**.........
- ✓ **Step 6:** ....
- ✓ **Step 7:** ....
- ✓ **Step 8:**...............

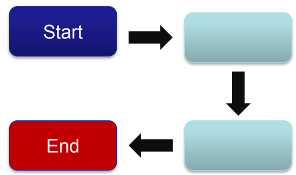
What is an Algorithm?

A **good algorithm** possesses the following properties:

3. **It must be composed of a finite number of steps.**
   - ✓ **It must terminate.**

How to make a cake?

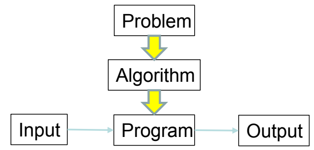
- ✓ **Step1:**.........................
  ....
- ✓ **Step 2:** ....
- ✓ **Step 3:** ....
  ....
  ....
  Step 4:...............
- ✓ **Step 5:**.........
- ✓ **Step 6:** ....
- ✓ **Step 7:** ....
- ✓ **Step 8:**...............



Computer Programs
Vs.
Algorithms

Computer Programs

- A **computer program** is an instance, or concrete representation of an algorithm in some programming language.
- **Programming constructs**
  - A composite collection of basic steps.
  - Commonly used ones:
    - ✓ for-loop, while-loop
    - ✓ if-else-statement, switch-case statement
    - ✓ functions (recursive and non-recursive)
- **Implementation** is the task of turning an algorithm into a computer program.



Design of Algorithms

There are often many ways (algorithms) to solve a problem.

Example 1: Summing up 1 to n

1+2+3+4+…….+n

Design of Algorithms

There are often many ways (algorithms) to solve a problem.

Example 1: Summing up 1 to n

1
```
Initialize sum to 0;
for (i = 1 to n)
add i to sum;
```

2
```
int sum;
sum = (1+n)*n/2;
```

3
```
int sum (int n);
{
      if (n is 1) return 1
      else return n + sum(n-1);
}
```

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Design of Algorithms

There are often many ways (algorithms) to solve a problem.

Example 1: Summing up 1 to n

Algorithm 1 Summing Arithmetic Sequence

function Method_One(n)
begin
$sum \leftarrow 0$
for $i = 1$ **to** n **do**
    $sum \leftarrow sum + i$
end

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Design of Algorithms

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```


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Design of Algorithms

There are often many ways (algorithms) to solve a problem.

Example 1: Summing up 1 to n

Algorithm 2 Summing Arithmetic Sequence

function Method_Two(n)
begin
$sum \leftarrow n * (1 + n)/2$
end

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Design of Algorithms

There are often many ways (algorithms) to solve a problem.

Example 1: Summing up 1 to n

1

```
Initialize sum to 0;
for (i = 1 to n)
add i to sum;
```

2

```
int sum;
sum = (1+n)*n/2;
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int sum (int n);
{
      if (n is 1) return 1
      else return n + sum(n-1);
}
```

Design of Algorithms

There are often many ways (algorithms) to solve a problem.

Example 1: Summing up 1 to n

Algorithm 3 Summing Arithmetic Sequence

```
function Method_Three(n)
begin
if n=1 then
    return 1
else
    return  n+Method_Three(n − 1)
end
```


Design of Algorithms

There are often many ways (algorithms) to solve a problem.

Example 2: Fibonacci Sequence

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, …

Algorithm 4 Fibonacci Sequence: A Simple Recursive Function

```
function Fibonacci_Recursive(n)
begin
if n<1 then
    return 0
if n==1 OR n==2 then
    return 1
return  Fibonacci_Recursive(n − 1)+Fibonacci_Recursive(n − 2)
end
```

Algorithm 5 Fibonacci Sequence: A Simple Iterative Function

```
function Fibonacci_Iterative(n)
begin
if n<1 then
    return 0
if n==1 OR n==2 then
    return 1
F_2 ← 1
F_1 ← 1
for i = 3 to n do
    begin
    F_i ← F_{i−2} + F_{i−1}
    F_{i−2} ← F_{i−1}
    F_{i−1} ← F_i
    end
return F_n
end
```


Design of Algorithms

Example 3: How to compute the Sine function?

1. By storing and referring to trigonometry table.

Trigonometric Functions

	sin	cos	tan	cot	sec	csc	
0°	0.0000	1.0000	0.0000	···	1.000	···	90°
1°	0.0175	0.9998	0.0175	57.29	1.000	57.30	89°
2°	0.0349	0.9994	0.0349	28.64	1.001	28.65	88°
3°	0.0523	0.9986	0.0524	19.08	1.001	19.11	87°
4°	0.0698	0.9976	0.0699	14.30	1.002	14.34	86°
5°	0.0872	0.9962	0.0875	11.43	1.004	11.47	85°
6°	0.1045	0.9945	0.1051	9.514	1.006	9.567	84°
7°	0.1219	0.9925	0.1228	8.144	1.008	8.206	83°
8°	0.1392	0.9903	0.1405	7.115	1.010	7.185	82°
9°	0.1564	0.9877	0.1584	6.314	1.012	6.392	81°
10°	0.1736	0.9848	0.1763	5.671	1.015	5.759	80°
11°	0.1908	0.9816	0.1944	5.145	1.019	5.241	79°
12°	0.2079	0.9781	0.2126	4.705	1.022	4.810	78°
13°	0.2250	0.9744	0.2309	4.331	1.026	4.445	77°
14°	0.2419	0.9703	0.2493	4.011	1.031	4.134	76°
15°	0.2588	0.9659	0.2679	3.732	1.035	3.864	75°
16°	0.2756	0.9613	0.2867	3.487	1.040	3.628	74°
17°	0.2924	0.9563	0.3057	3.271	1.046	3.420	73°
18°	0.3090	0.9511	0.3249	3.078	1.051	3.236	72°
19°	0.3256	0.9455	0.3443	2.904	1.058	3.072	71°
20°	0.3420	0.9397	0.3640	2.747	1.064	2.924	70°
21°	0.3584	0.9336	0.3839	2.605	1.071	2.790	69°
22°	0.3746	0.9272	0.4040	2.475	1.079	2.669	68°
23°	0.3907	0.9205	0.4245	2.356	1.086	2.559	67°
24°	0.4067	0.9135	0.4452	2.246	1.095	2.459	66°
25°	0.4226	0.9063	0.4663	2.145	1.103	2.366	65°

Reference: Abelsson. (2006). Illustration of CORDIC operation. Retrieved from https://commons.wikimedia.org/wiki/File:CORDIC-illustration.png.

Design of Algorithms

Example 3: How to compute the Sine function?

1. By storing and referring to trigonometry table.
2. By expanding Maclaurin series.

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots \textit{ for all } x$$

which is a special case of Taylor series with **a=0**.

$$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f^{(3)}(a)}{3!}(x-a)^3 \ldots$$

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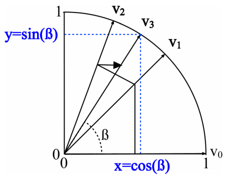
Design of Algorithms

Example 3: How to compute the Sine function?

3 For computers, easier to use **CORDIC**.

CORDIC (for **CO**ordinate **R**otation **DI**gital **C**omputer)

- To calculate hyperbolic and trigonometric functions.



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Design of Algorithms

Example 3: How to compute the Sine function?

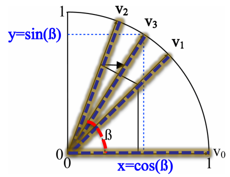
3 For computers, easier to use **CORDIC**.

$$v_i = Ki \begin{bmatrix} 1 & -\sigma_i 2^{-i} \\ \sigma_i 2^{-i} & 1 \end{bmatrix} \begin{bmatrix} x_{i-1} \\ y_{i-1} \end{bmatrix}$$

where

$$v_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad K_i = \frac{1}{\sqrt{1 + 2^{-2i}}}$$

and σ_i = -1 or 1 depending on direction of rotation.



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How do we choose among the different algorithms?

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Choosing the Right Algorithm

At the heart of computer program design, there are two (sometimes conflicting) goals:

1) To design an algorithm that is easy to understand, code and debug.

❖ **Software Engineering**

2) To design an algorithm that makes efficient use of the computer's resources.

❖ **Data structures and algorithm analysis**

Choosing the Right Algorithm

In this course, we favour the most efficient algorithms.

❖ **One that uses the least amount of resources**

Summary

- What an Algorithm is?
- Computer programs vs. Algorithms
- How do we choose among the different algorithms?