CZ2003 Tutorial 6 (2020/21, Semester 1)

Solid Objects

1. Define the two-dimensional polygon displayed in Figure Q1

   (i) by functions $x(u,v)$, $y(u,v)$, $u\in[0,1]$, $v\in[0,1]$

   (ii) by functions $f(x,y)\geq 0$

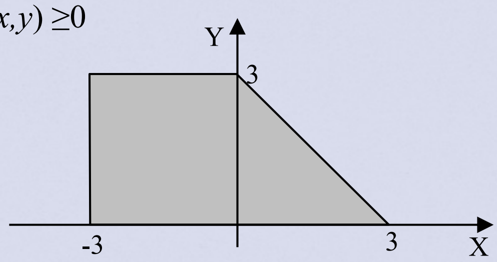


Figure Q1

2. A curve displayed in Figure Q2 (left) is defined in polar coordinates r and α by the function $r = 1.2\sin(2\alpha - 0.5\pi)$, $\alpha \in [0,2\pi]$. Propose parametric functions $x(u, v)$, $y(u, v)$, $u, v \in [0, 1]$ defining the 2D solid shape located in the XY Cartesian coordinates system as it is displayed in Figure Q2 (right).

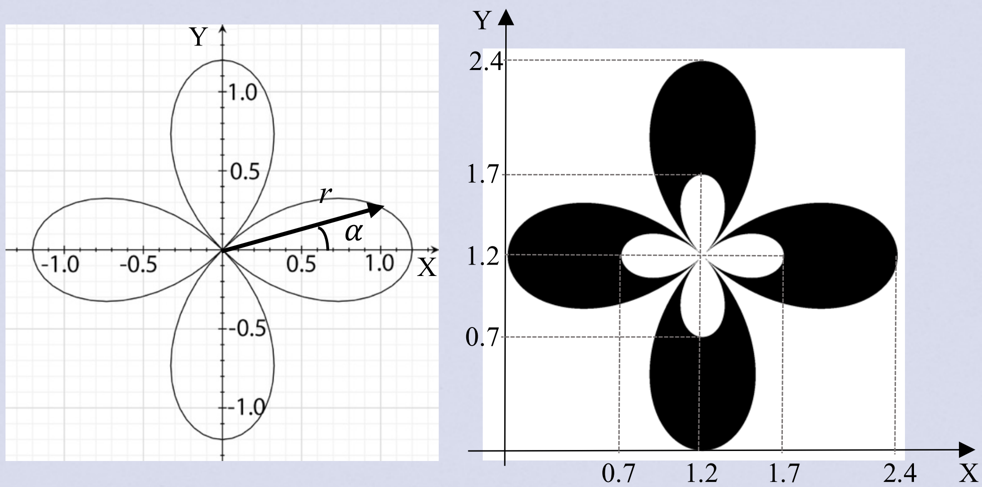


Figure Q2

3. Define parametrically with functions $x(u, v, w)$, $y(u, v, w)$, $z(u, v, w)$, $u, v, w \in [0, 1]$ the solid object displayed in Figure Q3. The object is created by rotational sweeping by $5\pi/4$ about axis Y of the sinusoidal curve followed by translational sweeping by 1.5 units parallel to axis Y.

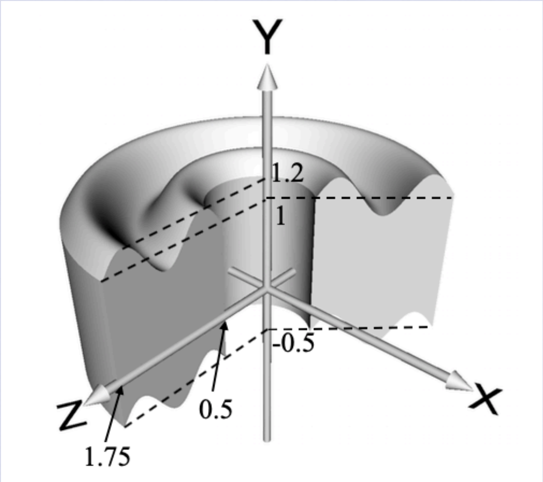
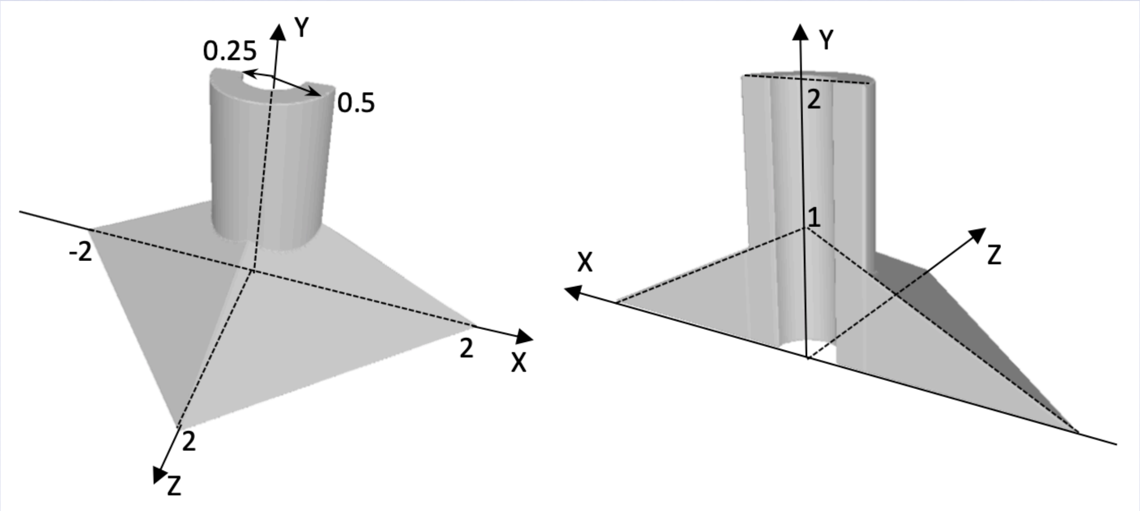


Figure Q3

4. The solid object displayed in Figure Q4 (front and back views) is constructed from a 3-sided pyramid with height 1 and a cylinder which has the height 2, the outer radius 0.5, and the inner radius 0.25.

   (i) Define the pyramid and the cylinder by functions $f(x, y, z) \geq 0$.

   (ii) Based on the definition obtained in part (i), define the final solid object.



Front view Back view

Figure Q4

1. Define the two-dimensional polygon displayed in Figure Q1

(i) by functions $x(u,v)$, $y(u,v)$, $u\in[0,1]$, $v\in[0,1]$

(ii) by functions $f(x,y) \geq 0$

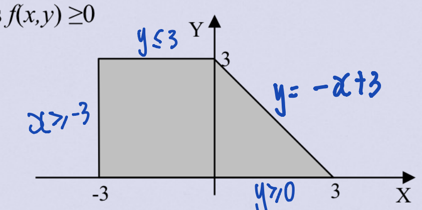


Figure Q1

(i) using the concept of a billinear surface.

$$\begin{pmatrix}-3\\0\\0\end{pmatrix} \begin{pmatrix}3\\0\\0\end{pmatrix} \begin{bmatrix}-3\\3\\0\end{bmatrix} \begin{pmatrix}0\\3\\0\end{pmatrix}$$

$$P = P_1 + u(P_2 - P_1) + v(P_3 - P_1 + u(P_4 - P_3 - (P_2 - P_1)))$$

$$x(u,v) = -3 + u(6) + v(u(0+3-(6))$$

$$= -3 + 6u + v(-3u)$$

$$= -3 + 6u - 3uv$$

$$y(u,v) = v(3-0 + u(0))$$

$$= 3v$$

$u \in [0,1]$

$v \in [0,1]$

one way is to make rectangular polygon where upper sides shrinke to the left. Rectangular polygon defined as $x = -3$ and $x = 3$

$x = -3 + u(3-(-3))$

$y = 3v$

$u = [0,1]$ $v = [0,1]$

Now the fixed coord will Δ to var coord $x(v)$ the changes from 3 to 0 following changes of v from 0 to 1 : $3 + v(0-3) = 3 - 3v$

Replacing $(3-(-3)))$ with $(3-3v)-(-3) = 6-3v$

$\therefore$ $x = -3 + u(6-3v)$

$y = 3v$

$u = [0,1]$ $v = [0,1]$

(ii) $f_1 = x \geq -3$ $f_1 = x + 3 \geq 0$

$f_2 = y \geq 0$

$f_3 = y \leq 3$ $f_3 = 3 - y \geq 0$

$f_4 = y = -x + 3$ $f_4 = -x - y + 3 \geq 0$

$\min(f_1, f_2, f_3, f_4)$

$= \min(x+3, y, 3-y, -x-y+3) \geq 0$

2. A curve displayed in Figure Q2 (left) is defined in polar coordinates r and α by the function $r = 1.2\sin(2\alpha - 0.5\pi)$, $\alpha \in [0, 2\pi]$. Propose parametric functions $x(u, v)$, $y(u, v)$, $u, v \in [0, 1]$ defining the 2D solid shape located in the XY Cartesian coordinates system as it is displayed in Figure Q2 (right).

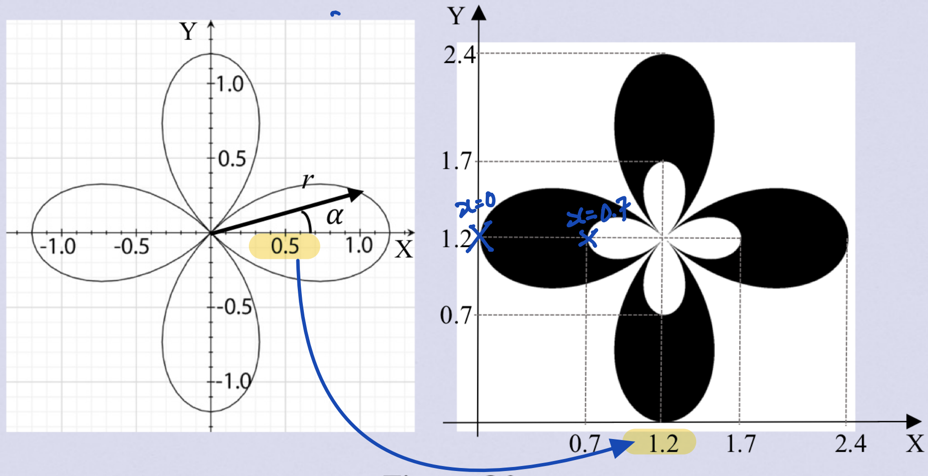


Figure Q2

curve from 0 to 1.2.

$r = 1.2\sin(2\alpha - 0.5\pi)$

To convert into parametric eqn where $u \in [0,1]$

$x = r\cos\alpha$

$= 1.2\sin(2\alpha - 0.5\pi)\cos(\alpha)$

$= 1.2\sin(4\pi v - 0.5\pi)\cos(2\pi v)$

$y = r\sin\alpha$

$= 1.2\sin(4\pi u - 0.5\pi)\sin(2\pi u)$

Now you want to make curve populate it i.e become black from 0 to 0.7 the after you want white from 0.7 to 1.2, diff 0.5

$x = (0.5 + 0.7v)\sin(4\pi u - 0.5\pi)\cos(2\pi u)$

$y = (0.5 + 0.7v)\sin(4\pi u - 0.5\pi)\sin(2\pi u)$

Translate along y axis by 1.2 and along x axis by 1.2

$x = (0.5 + 0.7v)\sin(4\pi u - 0.5\pi)\cos(2\pi u) + 1.2$

$y = (0.5 + 0.7v)\sin(4\pi u - 0.5\pi)\sin(2\pi u) + 1.2$ //

3. Define parametrically with functions $x(u, v, w)$, $y(u, v, w)$, $z(u, v, w)$, $u, v, w \in [0, 1]$ the solid object displayed in Figure Q3. The object is created by rotational sweeping by $5\pi/4$ about axis Y of the sinusoidal curve followed by translational sweeping by 1.5 units parallel to axis Y.

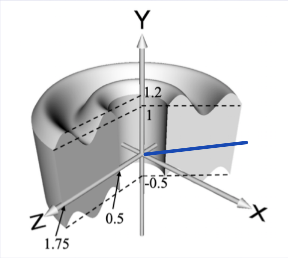


Figure Q3

1. To make the 2D surface of the sine wave

$x = 0.5 + u(1.75 - 0.5)$
$= 0.5 + u(1.25)$
$= 0.5 + 1.25u$ //

$y = 0.2\sin(3\pi u)$ $u \in [0,1]$

$z = 0$

//

2. Rotational sweeping by $\frac{5\pi}{4}$ about axis Y

$x' = \sin\left(\frac{3\pi}{4} + \frac{5\pi}{4}v\right)$

$z' = \cos\left(\frac{3\pi}{4} + \frac{5\pi}{4}v\right)$

Start $\left[\frac{3\pi}{4}, 2\pi\right]$

Surface defined by:

$x = (0.5 + 1.25u)\left(\sin\left(\frac{3\pi}{4} + \frac{5\pi}{4}v\right)\right)$

$y = 0.2\sin(3\pi u)$

$z = (0.5 + 1.25u)\left(\cos\left(\frac{3\pi}{4} + \frac{5\pi}{4}v\right)\right)$

3. translational sweeping by 1.5 units // to Y axis

$x = (0.5 + 1.25u)\left(\sin\left(\frac{3\pi}{4} + \frac{5\pi}{4}v\right)\right)$

$y = 0.2\sin(3\pi u) + 1 - 1.5w$

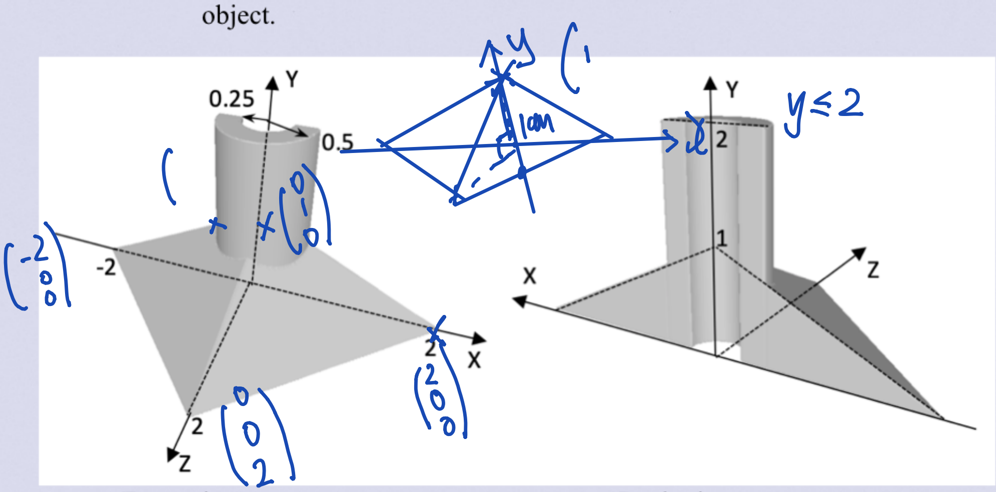
$z = (0.5 + 1.25u)\left(\cos\left(\frac{3\pi}{4} + \frac{5\pi}{4}v\right)\right)$

$u, v, w \in [0, 1]$

4. The solid object displayed in Figure Q4 (front and back views) is constructed from a 3-sided pyramid with height 1 and a cylinder which has the height 2, the outer radius 0.5, and the inner radius 0.25.

(i) Define the pyramid and the cylinder by functions $f(x, y, z) \geq 0$.

(ii) Based on the definition obtained in part (i), define the final solid object.



Front view Back view

Figure Q4

1)

4(i) To get the pyramid.

using the equation of plane bounded by half space: $-\frac{x}{a}-\frac{y}{b}-\frac{z}{c}+1=0$

$f_1 = -\frac{x}{2}-\frac{y}{1}-\frac{z}{2}+1=0 \quad \therefore f_1 = -0.5x-y-0.5z+1 \geq 0$

$f_2 = -\frac{x}{-2}-\frac{y}{1}-\frac{z}{2}+1=0 \quad \therefore f_2 = 0.5x-y-0.5z+1 \geq 0$

pyramid $= \min(-0.5x-y-0.5z+1,\ 0.5x-y-0.5z+1,\ y,\ z) \geq 0$

To obtain the cylinder

$r^2-x^2-y^2-z^2 \geq 0$

$f_1 = 0.5^2 - x^2 - z^2 \geq 0$ cylinder $= \min(0.5^2 - x^2 - z^2, 2-y, -(0.25^2 - x^2 - z^2), y, z) \geq 0$

$f_2 = 0.25^2 - x^2 - z^2 \geq 0$

$f_3 = 2 - y \geq 0$

$f_3 = z \geq 0$

To get a hollow bottom, deduct the larger cylinder with radius 0.5

int pyramid $= \min(\text{pyramid}, -(0.5^2 - x^2 - z^2)) \geq 0$

then union with the double cylinder on top

final pyramid $= \max(\text{int pyramid}, \text{cylinder}) \geq 0$

$$= \max\Big(\min\big(\min(-0.5x - y - 0.5z + 1, 0.5x - y - 0.5z + 1, y, z), -(0.5^2 - x^2 - z^2)\big),$$
$$\min(0.5^2 - x^2 - z^2, 2 - y, -(0.25^2 - x^2 - z^2), y, z)\Big)$$