Ng Chi Hui
Lulu Sui
Lim Yan Di
Goh Ray Sam
Tan Jin Xian Cyrus

68

Assignment 1

Q1) a. $a = \begin{pmatrix} 1 \\ 8 \end{pmatrix}$ $b = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

$0.1a + 0.9b$

$= 0.1\begin{pmatrix} 1 \\ 8 \end{pmatrix} + 0.9\begin{pmatrix} 3 \\ 2 \end{pmatrix}$

$= \begin{pmatrix} 0.1 \\ 0.8 \end{pmatrix} + \begin{pmatrix} 2.7 \\ 1.8 \end{pmatrix}$

$= \begin{pmatrix} 2.8 \\ 2.6 \end{pmatrix}$

a. $a - b$

$= \begin{pmatrix} 1 \\ 8 \end{pmatrix} - \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

$= \begin{pmatrix} -2 \\ 6 \end{pmatrix}$

$a'b$

$= (1 \quad 8)\begin{pmatrix} 2 \\ 6 \end{pmatrix}$

$= 3 + 16 = 19$ //

Q1) a. $\lambda \in [0,1]$

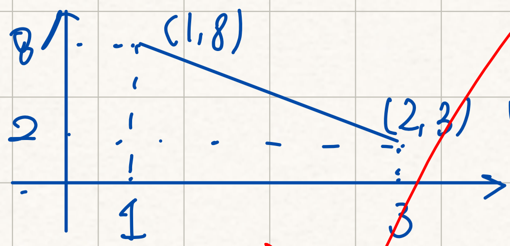
$y = \lambda a + (1-\lambda)b = (a-b)\lambda + b$

if $\lambda = 0$

$y = b$

if $\lambda = 1$

$y = a$



$y = (a-b)\lambda + b$

10

Q2) a. $A = \begin{pmatrix} 1 & -2 & 3 \\ 4 & -5 & 7 \end{pmatrix}$ $B = \begin{pmatrix} 1 & 2 \\ 1 & 1 \\ 2 & 0 \end{pmatrix}$

$AB = \underset{2\times3}{\begin{pmatrix} 1 & -2 & 3 \\ 4 & -5 & 7 \end{pmatrix}} \underset{3\times2}{\begin{pmatrix} 1 & 2 \\ 1 & 1 \\ 2 & 0 \end{pmatrix}}$

$= \begin{pmatrix} (1)(1)+(-2)(1)+(3)(2) & (1)(2)+(-2)(1) \\ (4)(1)+(-5)(1)+(7)(2) & (4)(2)+(-5)(1) \end{pmatrix}$

$= \begin{pmatrix} 5 & 0 \\ 13 & 3 \end{pmatrix}$ //

$BA = \underset{3\times2}{\begin{pmatrix} 1 & 2 \\ 1 & 1 \\ 2 & 0 \end{pmatrix}} \underset{2\times3}{\begin{pmatrix} 1 & -2 & 3 \\ 4 & -5 & 7 \end{pmatrix}}$

$= \begin{pmatrix} (1)(1)+(2)(4) & (1)(-2)+(2)(-5) & (1)(3)+(2)(7) \\ (1)(1)+(1)(4) & (1)(-2)+(1)(-5) & (1)(3)+(1)(7) \\ (2)(1) & (2)(-2) & (2)(3) \end{pmatrix}$

$= \begin{pmatrix} 9 & -12 & 17 \\ 5 & -7 & 10 \\ 2 & -4 & 6 \end{pmatrix}$ //

$AB \neq BA$. This is because matrix multiplication is not commutative in general

Q2) b. $(AB)'$

$= \begin{pmatrix} 5 & 0 \\ 13 & 3 \end{pmatrix}' = \begin{pmatrix} 5 & 13 \\ 0 & 3 \end{pmatrix}$ //

$B' = \begin{pmatrix} 1 & 1 & 2 \\ 2 & 1 & 0 \end{pmatrix}$ //

$A' = \begin{pmatrix} 1 & 4 \\ -2 & -5 \\ 3 & 7 \end{pmatrix}$ //

$B'A' = \underset{2\times3}{\begin{pmatrix} 1 & 1 & 2 \\ 2 & 1 & 0 \end{pmatrix}} \underset{3\times2}{\begin{pmatrix} 1 & 4 \\ -2 & -5 \\ 3 & 7 \end{pmatrix}}$

$= \begin{pmatrix} (1)(1)+(1)(-2)+(2)(3) & (1)(4)+(1)(-5)+(2)(7) \\ (2)(1)+(1)(-2) & (2)(4)+(1)(-5) \end{pmatrix}$

$= \begin{pmatrix} 5 & 13 \\ 0 & -3 \end{pmatrix}$ //

$B'A' = (AB)'$. The transpose of the product of two matrices is equivalent to the product of their transpose in reverse order.

10

Q3) $\infty \times 2$ $\quad 2\times 1$

$A\begin{pmatrix} R_1 \\ R_2 \end{pmatrix} \le b$ $\quad 2\times 2 \quad 2\times 1$

13

(a) matrix A

# of rows = $\{x \in \mathbb{Z}^+ \mid x \ge 1\}$

# of rows can be any postive integer that's more than or equal to one

# of columns = 2

Vector b

# of rows = $\{x \in \mathbb{Z}^+ \mid x \ge 1\}$

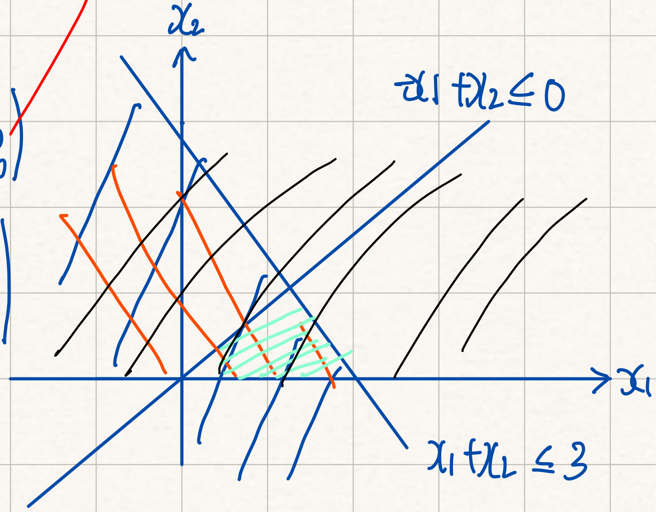
# of rows can be any postive integer that's more than or equal to one

# of cols = 2

b is a vector

(b) $\begin{pmatrix} 1 & 1 \\ -1 & 1 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \le \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix}$

$\begin{pmatrix} x_1 + x_2 \\ -x_1 + x_2 \\ -x_2 \end{pmatrix} \le \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix}$



Q4) a. $X_1 = \{x \in \mathbb{R}^4 : Ax - b \le 0\}$

10

$X_2 = \{x \in \mathbb{R}^n : Ax - b = -s,\ s \ge 0 \text{ for some } s\}$

satisfying $Ax \le b$

$\therefore x \in X_1$

since $s \ge 0$ where did you get $s \ge 0$?

there exists s such that

$Ax + s = b$

$\therefore x \in X_2$

$X_1 \subseteq X_2$

$x \in X_1$, we should have

$Ax \le b$, that is $Ax - b \le 0$

Let $s = b - Ax$, then $s \ge 0$ given

that $Ax \le b$.

In other words, for any $x \in X_1$,

there exists $s = b - Ax \ge 0$ such

that $Ax + s = b$, $s \ge 0$ for some s.

That is, x is also in set X_2.

4b. satisfying $Ax + s = b$

$\therefore x \in X_2$

$Ax + s = b$

$Ax = b - s$

$\therefore Ax \le b$, if $s \ge 0$

$\therefore x \in X_1$

$\therefore X_2 \subseteq X_1$

Q5) a. 20

let x_1: amount of grains G1 to be consumed per day. (in kg)

x_2: amount of grains G2 to be consumed per day. (in kg)

objective function: min $0.6x_1 + 0.35x_2$

constraints:

s.t
$$5x_1 + 7x_2 \geq 8$$
$$4x_1 + 2x_2 \geq 15$$
$$2x_1 + x_2 \geq 3$$
$$x_1 \geq 0$$
$$x_2 \geq 0$$

$x_1 = 3.75$ $g_2 = 0$

Ans: G_1 consume per day: 3.75 kg

G_2 consumed per day: 0 kg.

b. $x = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$

where x_j represents the numer of Food i, $j = 0, \ldots, n$ to be consumed per day

$A = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \\ a_{m1} & \cdots & a_{mn} \end{pmatrix}$

where a_{ij} # of nutrient i in Food j

$R = \begin{pmatrix} r_1 \\ \vdots \\ r_m \end{pmatrix}$

where r_j refers to requirements of each nutrient

$C = \begin{pmatrix} c_1 \\ \vdots \\ c_n \end{pmatrix}$ where c_j refers to cost of Food j

obj min $c_1x_1 + c_2x_2 + \ldots + c_nx_n$

$\therefore$ obj min C^Tx

s.t $x_1a_{11} + x_2a_{12} + \ldots x_na_{1n} \geq r_1$

$\vdots$

$x_1a_{m1} + x_2a_{m2} + \ldots x_{mn} \geq r_m$

$$Ax \geq R$$
$$x \geq 0$$

Q6)

Let W_j be the # of workers needed to be hired in quater j ($j = 1, \ldots, 4$)

Let S_j be the # of units in inventory brought over after quater j ($j = 1, \ldots, 3$)

5

min $1500 \sum_{j=1}^{4} W_j + 50 \sum_{j=1}^{3} S_j$

$$50W_1 + 50W_2 + 50W_3 = 600 + S_1 \qquad W_1 \geq 0$$
$$50W_2 + 50W_3 + 50W_4 = 300 + S_2 \qquad W_2 \geq 0$$
$$50W_3 + 50W_4 + 50W_1 = 800 + S_3 \qquad W_3 \geq 0$$
$$50W_4 + 50W_1 + 50W_2 = 100 \qquad W_4 \geq 0$$

$S_4 = 0$

1	2	3	4
w_1	w_1	w_1	
	w_2	w_2	w_2
w_3		w_3	w_3
w_4	w_4		w_4
600	300	800	100

$$50(w_1 + w_3 + w_4) \geq S_1 + 600$$
$$50(w_1 + w_2 + w_4) + S_1 \geq S_2 + 300$$
$$50(w_1 + w_2 + w_3) + S_2 \geq S_3 + 800$$
$$50(w_2 + w_3 + w_4) + S_3 \geq S_4 + 100$$
$$S_4 = 0$$