Geometric Shapes: Solid Objects

Module 3
Lecture 6

Geometric Shapes

- Geometry has no color and texture
- Points
- Curves
- Surfaces
- Solid objects

Learning Objectives

- To understand how solids can be used in solving data visualization problems
- To understand solids as objects with *3 degree of freedom*
- To understand what mathematical representations are the most efficient for defining and displaying solids
- To understand how different coordinate systems can be used together for deriving mathematical representations of solids
- To understand solids as objects created *by moving surfaces*
- To understand how complex solids can be created from *combinations of other solids*

Geometric Shapes

- Geometry has no color and texture
- Points – 0 degree of freedom shape
- Curves – 1 degree of freedom shape
- Surfaces – 2 degree of freedom shape
- Solid objects – 3 degree of freedom shape

- 2 and 3 dimensional spaces
- Time is yet another dimension

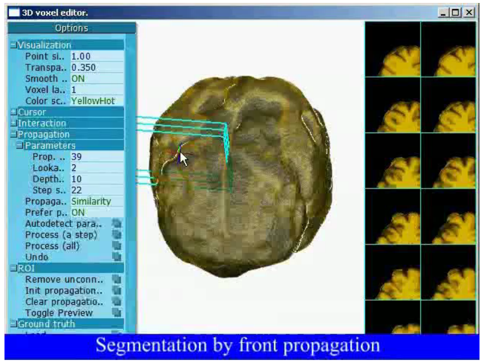
- Displayed as pixels, voxels, polylines, and shaded polygons

Solid Objects

- Voxels (volume elements)
- Parametric representation
- Explicit (variant of implicit) representation

Voxels

- Voxel (volumetric pixel or Volumetric Picture Element) is a volume element, representing a value on a regular grid in three dimensional space.



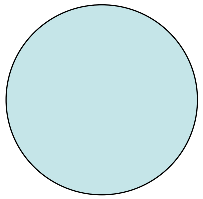
Solid Objects

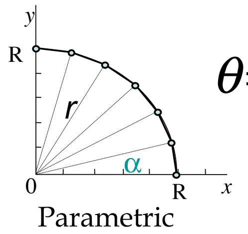
- Voxels (volume elements)
- Parametric representation
- Explicit (variant of implicit) representation

Using Parametric Functions for Defining Solid Objects

- Let’s add one more degree of freedom which is an additional parameter

Circular Disk. Parametric Representation





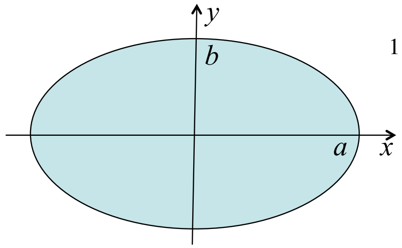
$\theta = \alpha$

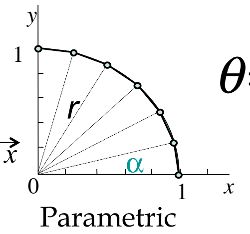
$$x = r\cos(\theta) \qquad y = r\sin(\theta)$$

r =[0, R], θ =[0, 2π]

Two parameters to define a 2D disk!

Elliptical Disk. Parametric Representation





$\theta \neq \alpha$

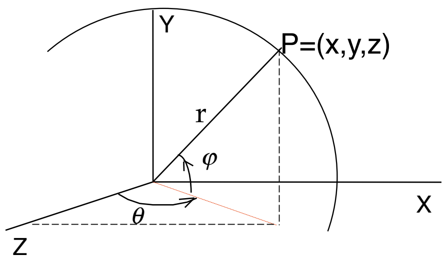
$$x = a*k*\cos(\theta) \qquad y = b*k*\sin(\theta)$$

k =[0, 1], θ =[0, 2π]

Two parameters to define an elliptical disk!

Solid Sphere

- Parametric

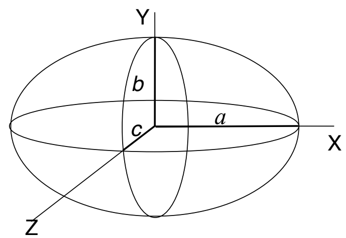


$$x = r\cos\varphi\sin\theta$$
$$y = r\sin\varphi$$
$$z = r\cos\varphi\cos\theta$$
$$-\frac{\pi}{2} \le \varphi \le \frac{\pi}{2}$$
$$-\pi \le \theta \le \pi$$
$$0 \le r \le R$$

Three parameters !

Solid Ellipsoid

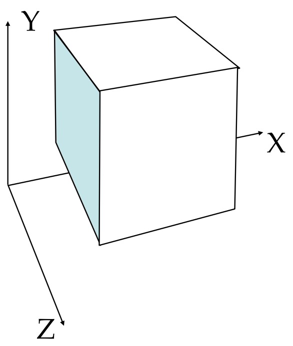
- Parametric



$$x = k*a*\cos\varphi\sin\theta$$
$$y = k*b*\sin\varphi$$
$$z = k*c*\cos\varphi\cos\theta$$
$$-\frac{\pi}{2} \le \varphi \le \frac{\pi}{2}$$
$$-\pi \le \theta \le \pi$$
$$0 \le k \le 1$$

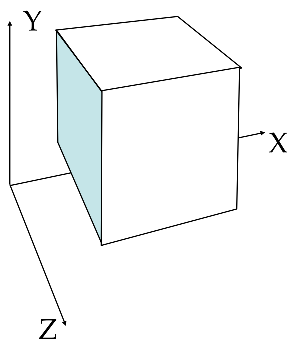
Three parameters !

Parametrically-defined Solid Box



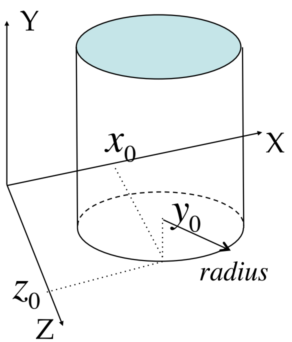
$$x = u$$
$$y = v$$
$$z = w$$
$$u_1 \le u \le u_2$$
$$v_1 \le v \le v_2$$
$$w_1 \le w \le w_2$$

Parametrically-defined Solid Box



$$x = u_1 + u\left(u_2 - u_1\right)$$
$$y = v_1 + v\left(v_2 - v_1\right)$$
$$z = w_1 + w\left(w_2 - w_1\right)$$
$$0 \le u \le 1$$
$$0 \le v \le 1$$
$$0 \le w \le 1$$

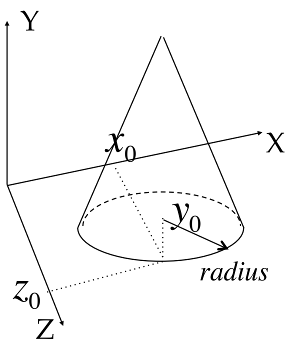
Parametrically-defined Solid Cylinder



$$x = u \cdot radius \cdot \sin(2\pi v) + x_0$$
$$y = w \cdot height + y_0$$
$$z = u \cdot radius \cdot \cos(2\pi v) + z_0$$
$$0 \le u \le 1$$
$$0 \le v \le 1$$
$$0 \le w \le 1$$

Parametrically-defined Solid Cone

$$cone\_radius(\tau) = radius + \tau(0 - radius) = radius(1-\tau), \tau = [0,1]$$

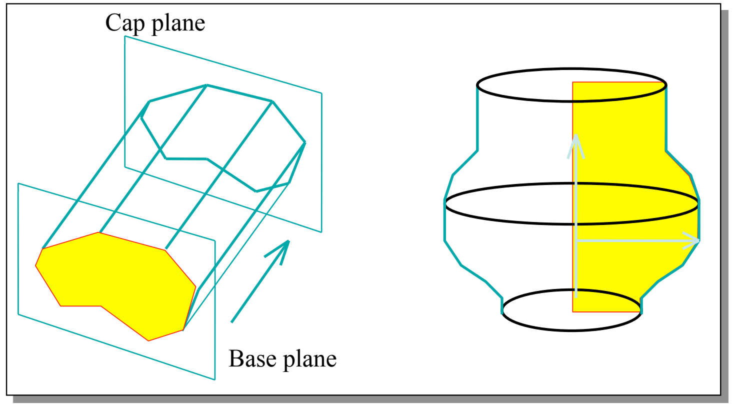


$$x = u \cdot radius \cdot (1-w) \cdot \sin(2\pi v) + x_0$$
$$y = w \cdot height + y_0$$
$$z = u \cdot radius \cdot (1-w) \cdot \cos(2\pi v) + z_0$$
$$0 \le u \le 1$$
$$0 \le v \le 1$$
$$0 \le w \le 1$$

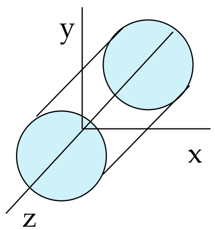
Solid Sweeping

- Shapes are created by moving curve, surface or solid along some path
- Sweeping of curves generates surfaces, sweeping of surfaces produces solids, sweeping of solids creates solids
- Two particular cases of sweeping--*translational* and *rotational sweeping*--can be easily defined parametrically

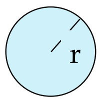
Translational and Rotational Sweeping



Parametric Representation of Translational Sweeping. Solid Cylinder.



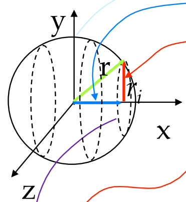
$$x = r\cos\varphi \qquad y = r\sin\varphi \quad 0 \le r \le R \qquad z = 0 \quad 0 \le \varphi \le 2\pi$$



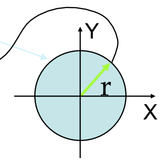
$$z = z_1 + \tau(z_2 - z_1) \qquad 0 \le \tau \le 1$$

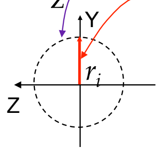
$$x = r\cos\phi \quad 0 \le r \le R$$
$$y = r\sin\phi \quad 0 \le \phi \le 2\pi$$
$$z = z_1 + \tau(z_2 - z_1) \quad 0 \le \tau \le 1$$

Parametric Representation of Rotational Sweeping. Solid Sphere.

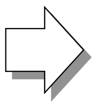


$$x = r\cos\varphi \qquad y = r\sin\varphi \qquad 0 \le \varphi \le 2\pi \qquad 0 \le r \le R$$



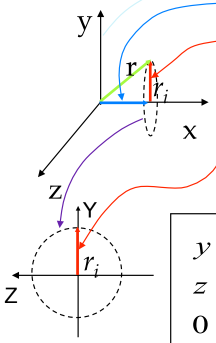


$$y = r_i\cos\theta \qquad z = r_i\sin\theta \qquad 0 \le \theta \le \pi$$

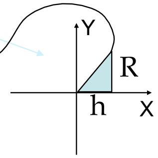


$$x = r\cos\varphi$$
$$y = r\sin\varphi\cos\theta$$
$$z = r\sin\varphi\sin\theta$$
$$0 \le \varphi \le 2\pi$$
$$0 \le r \le R$$
$$0 \le \theta \le \pi$$

Parametric Representation of Rotational Sweeping. Solid Cone



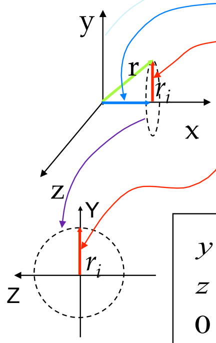
$$x = h * u$$
$$y = R * v * u$$
$$0 \le u \le 1$$
$$0 \le v \le 1$$



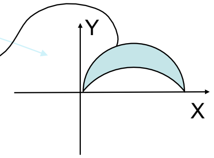
$$y = r_i \cos\theta$$
$$z = r_i \sin\theta$$
$$0 \le \theta \le 2\pi$$

$$x = h * u$$
$$y = R * v * u \cos\theta$$
$$z = R * v * u \sin\theta$$
$$0 \le u \le 1$$
$$0 \le v \le 1$$
$$0 \le \theta \le 2\pi$$

Parametric Representation of Rotational Sweeping. Solid Anything



$$x = f_x(u,v)$$
$$y = f_y(u,v)$$
$$0 \le u \le 1$$
$$0 \le v \le 1$$

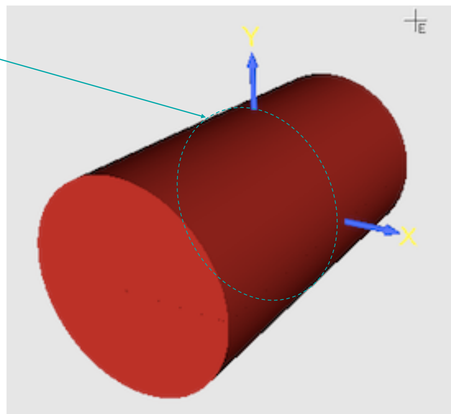


$$y = r_i \cos\theta$$
$$z = r_i \sin\theta$$
$$0 \le \theta \le 2\pi$$

$$x = f_x(u,v)$$
$$y = f_y(u,v)\cos\theta$$
$$z = f_y(u,v)\sin\theta$$
$$0 \le u \le 1$$
$$0 \le v \le 1$$
$$0 \le \theta \le 2\pi$$

Experimenting with Sweeping

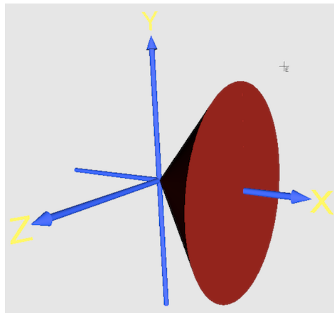
*x=0.6*w*cos(u*2*π)*
*y=0.6*w*sin(u*2*π)*
z=-0.3+v(1-(-0.3))
u=[0, 1]
v=[0, 1]
w=[0,1]



Translational sweeping

Experimenting with Sweeping

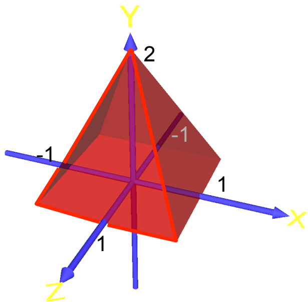
$$x = u$$
$$y = w * u \cos v$$
$$z = w * u \sin v$$
$$0 \le u \le 1$$
$$0 \le v \le 2\pi$$
$$0 \le w \le 1$$



Rotational sweeping

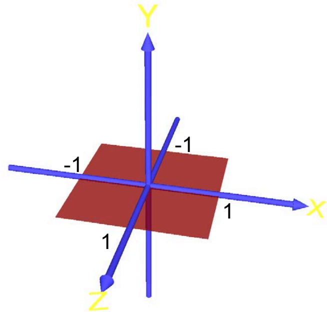
Experimenting with Sweeping

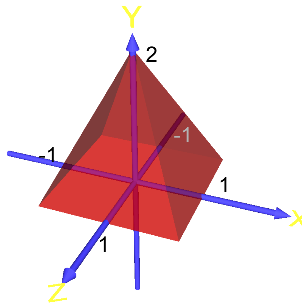
x= (-1 + 2*u)*(1-w)
y= 2*w
z= (-1 + 2*v)*(1-w)
u=[0,1], v=[0,1], w=[0,1]



Experimenting with Sweeping

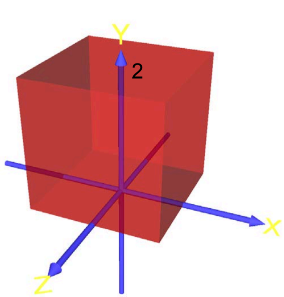
x= -1 + 2*u
y= 0
z= -1 + 2*v
u=[0,1], v=[0,1]

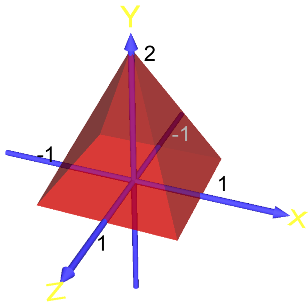




Experimenting with Sweeping

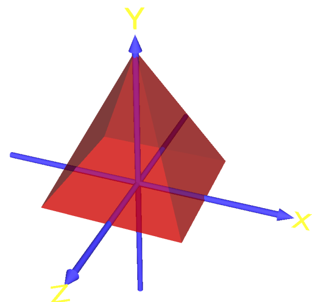
x= -1 + 2*u
y= 2*w
z= -1 + 2*v
u=[0,1], v=[0,1], w=[0,1]

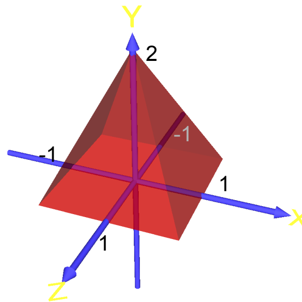




Experimenting with Sweeping

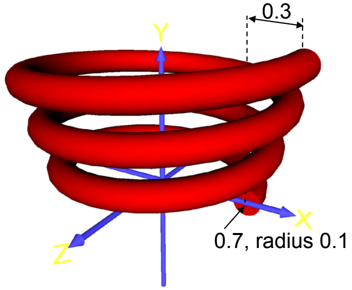
x= (-1 + 2*u)*(1-w)
y= 2*w
z= (-1 + 2*v)*(1-w)
u=[0,1], v=[0,1], w=[0,1]





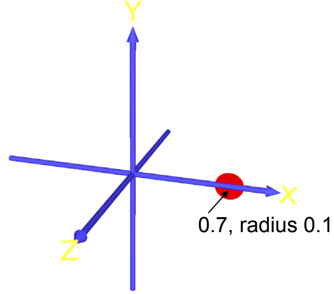
Experimenting with Sweeping

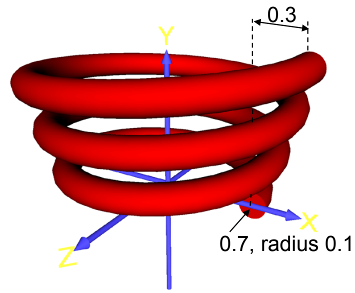
x= (0.1*v*cos(u*2*π) + 0.7 + 0.3*w) * sin(w*6*π + π/2)
y= 0.1*v*sin(u*2*π) + 1*w
z= (0.1*v*cos(u*2*π) + 0.7 + 0.3*w) * cos(w*6*π + π/2)
u=[0,1], v=[0,1], w=[0,1]



Experimenting with Sweeping

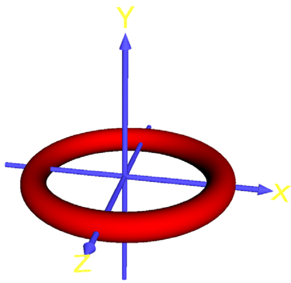
x= 0.1*v*cos(u*2*π) + 0.7
y= 0.1*v*sin(u*2*π)
z= 0
u=[0,1], v=[0,1]

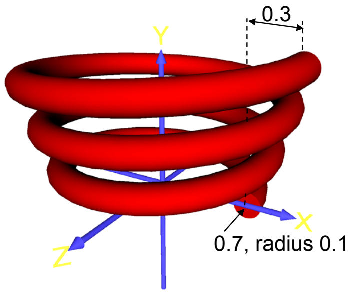




Experimenting with Sweeping

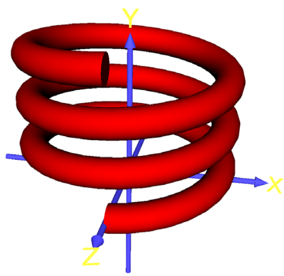
x= (0.1*v*cos(u*2*π) + 0.7) * sin(w*2*π)
y= 0.1*v*sin(u*2*π)
z= (0.1*v*cos(u*2*π) + 0.7) * cos(w*2*π)
u=[0,1], v=[0,1], w=[0,1]

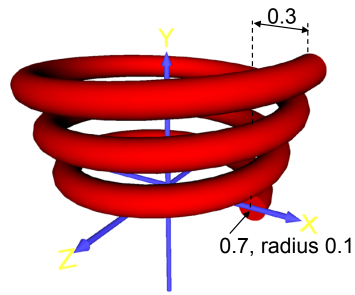




Experimenting with Sweeping

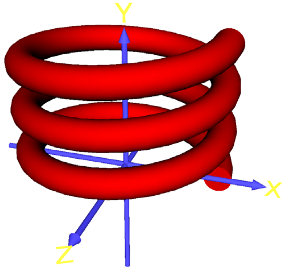
x= (0.1*v*cos(u*2*π) + 0.7) * sin(w*6*π)
y= 0.1*v*sin(u*2*π) + 1*w
z= (0.1*v*cos(u*2*π) + 0.7) * cos(w*6*π)
u=[0,1], v=[0,1] , w=[0,1]

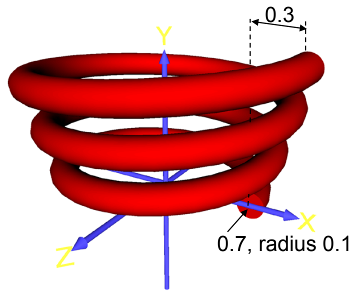




Experimenting with Sweeping

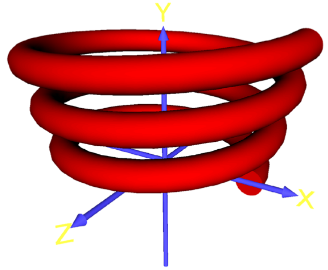
x= (0.1*v*cos(u*2*π) + 0.7) * sin(w*6*π + π/2)
y= 0.1*v*sin(u*2*π) + 1*w
z= (0.1*v*cos(u*2*π) + 0.7) * cos(w*6*π + π/2)
u=[0,1], v=[0,1] , w=[0,1]

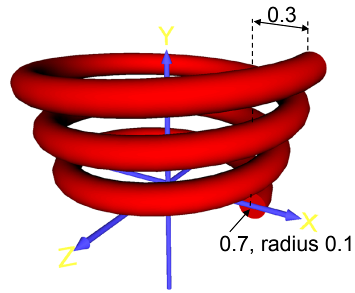




Experimenting with Sweeping

x= (0.1*v*cos(u*2*π) + 0.7 + 0.3*w) * sin(w*6*π + π/2)
y= 0.1*v*sin(u*2*π) + 1*w
z= (0.1*v*cos(u*2*π) + 0.7 + 0.3*w) * cos(w*6*π + π/2)
u=[0,1], v=[0,1], w=[0,1]





Summary

Solid objects can be defined by

- Sets of voxels
- Parametric Representation with 3 parameters