```
In[256]:=  v1 = 1 / Sqrt[3] {{0}, {1}, {1}, {1}};
           V1 = KroneckerProduct[v1, ConjugateTranspose[v1]];
           v2 = 1 / Sqrt[3] {{0}, {1}, {Exp[I * 2 Pi / 3]}, {Exp[I * 4 Pi / 3]}};
           V2 = KroneckerProduct[v2, ConjugateTranspose[v2]];
           v3 = 1 / Sqrt[3] {{0}, {1}, {Exp[I * 4 Pi / 3]}, {Exp[I * 8 Pi / 3]}};
           V3 = KroneckerProduct[v3, ConjugateTranspose[v3]];
           V4 = {{1, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}};

In[187]:=  Phi00 = 1 / 2 {{1, 0, 0, 1}, {0, 0, 0, 0}, {0, 0, 0, 0}, {1, 0, 0, 1}};
           Phi01 = 1 / 2 {{0, 0, 0, 0}, {0, 1, 1, 0}, {0, 1, 1, 0}, {0, 0, 0, 0}};
           Phi10 = 1 / 2 {{1, 0, 0, -1}, {0, 0, 0, 0}, {0, 0, 0, 0}, {-1, 0, 0, 1}};
           Phi11 = 1 / 2 {{0, 0, 0, 0}, {0, 1, -1, 0}, {0, -1, 1, 0}, {0, 0, 0, 0}};

In[308]:=  E0 = Sqrt[p] {{1, 0}, {0, Sqrt[1 - t]}};
           E1 = Sqrt[p] {{0, 0}, {Sqrt[t], 0}};
           E2 = Sqrt[1 - p] {{Sqrt[1 - t], 0}, {0, 1}};
           E3 = Sqrt[1 - p] {{0, Sqrt[t]}, {0, 0}};

In[312]:=  M0 = KroneckerProduct[E0, E0];
           M1 = KroneckerProduct[E0, E1];
           M2 = KroneckerProduct[E0, E2];
           M3 = KroneckerProduct[E0, E3];
           M4 = KroneckerProduct[E1, E0];
           M5 = KroneckerProduct[E1, E1];
           M6 = KroneckerProduct[E1, E2];
           M7 = KroneckerProduct[E1, E3];
           M8 = KroneckerProduct[E2, E0];
           M9 = KroneckerProduct[E2, E1];
           M10 = KroneckerProduct[E2, E2];
           M11 = KroneckerProduct[E2, E3];
           M12 = KroneckerProduct[E3, E0];
           M13 = KroneckerProduct[E3, E1];
           M14 = KroneckerProduct[E3, E2];
           M15 = KroneckerProduct[E3, E3];


In[328]:=  Ch[X_] := M0.X.Transpose[M0] + M1.X.Transpose[M1] + M2.X.Transpose[M2] +
              M3.X.Transpose[M3] + M4.X.Transpose[M4] + M5.X.Transpose[M5] + M6.X.Transpose[M6] +
              M7.X.Transpose[M7] + M8.X.Transpose[M8] + M9.X.Transpose[M9] +
              M10.X.Transpose[M10] + M11.X.Transpose[M11] + M12.X.Transpose[M12] +
              M13.X.Transpose[M13] + M14.X.Transpose[M14] + M15.X.Transpose[M15]

In[300]:=  Clear[p, t]

In[329]:=  Simplify[Ch[V1] + Ch[V2] + Ch[V3] + Ch[V4]]

Out[·]=   {{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}}
```
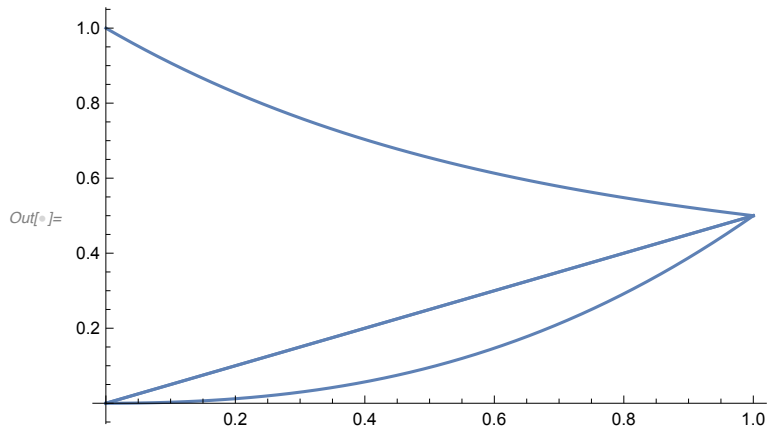
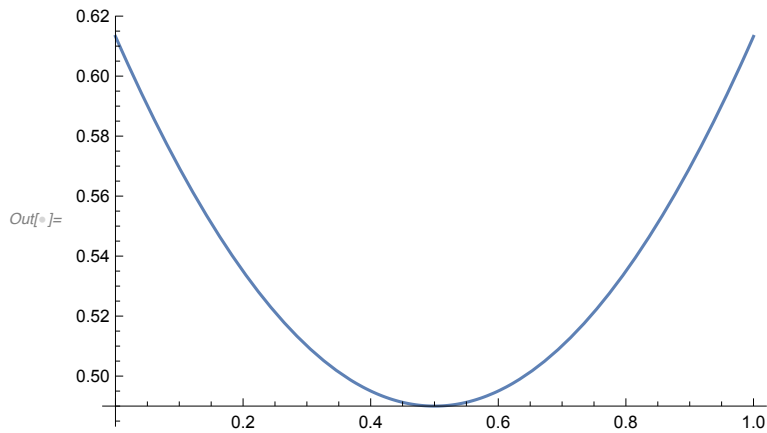In[153]:= `Plot[Simplify[Eigenvalues[Ch[Phi00]]], {t, 0, 1}]`

Out[ ]=

In[267]:= `t = .6;`

In[333]:= `FullSimplify[Eigenvalues[Ch[Phi00]]]`

Out[ ]= $\left\{ \dfrac{t}{2} + (-1 + p) \, p \, t^2, \dfrac{t}{2} + (-1 + p) \, p \, t^2, \right.$

$\dfrac{1}{2} \left(1 + t \, (-1 + t + 2 \, (-1 + p) \, p \, t) - \sqrt{(-1 + t)^2 \left(1 + (1 - 2 \, p)^2 \, t^2 \right)} \right),$

$\left. \dfrac{1}{2} \left(1 + t \, (-1 + t + 2 \, (-1 + p) \, p \, t) + \sqrt{(-1 + t)^2 \left(1 + (1 - 2 \, p)^2 \, t^2 \right)} \right) \right\}$

In[ ]:= `t = .6;`

`Plot[` $\dfrac{1}{2} \left(1 + t \, (-1 + t + 2 \, (-1 + p) \, p \, t) + \sqrt{(-1 + t)^2 \left(1 + (1 - 2 \, p)^2 \, t^2 \right)} \right)$ `, {p, 0, 1}]`

Out[ ]=

In[ ]:= `FullSimplify[N[Eigenvalues[Ch[V3]]]]`

Out[ ]= $\left\{ 0.995565, 0.00772978, 0.00333333, 4.81281 \times 10^{-6} \right\}$

In[ ]:= `FullSimplify[N[Eigenvalues[Ch[V4]]]]`

Out[ ]= $\{ 0.9801, 0., 0., 0. \}$

In[ ]:= `(1 + Sqrt[1 - t]) ^2`

Out[ ]= 2.66491

`FullSimplify[N[Eigenvalues[Ch[V2]]]]`

In[ ]:= `FullSimplify[N[Eigenvalues[Ch[V1]]]]`

In[ ]:= `Simplify[Eigenvalues[Ch[Phi10]]]`

Out[ ]= $\left\{ \frac{t}{2}, \frac{t}{2}, \frac{1}{2}\left(1 - t + t^2 - \sqrt{(-1+t)^2(1+t^2)}\right), \frac{1}{2}\left(1 - t + t^2 + \sqrt{(-1+t)^2(1+t^2)}\right) \right\}$

In[ ]:= `Simplify[Eigenvalues[Ch[Phi11]]]`

Out[ ]= $\{1 - t, 0, 0, -(-1+t)\,t\}$

In[ ]:= $\text{Plot}\left[\left\{2(1-t) + 1 - t + t^2 + \sqrt{(-1+t)^2(1+t^2)}, (1 + \text{Sqrt}[1-t])\,{}^\wedge 2\right\}, \{t, 0, 1\}\right]$

Out[ ]=