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In[=]:= Id = {{1, 0}, {0, 1}};
sigX = {{0, 1}, {1, 0}};
sigY = {{0, -I}, {I, 0}};
sigZ = {{1, 0}, {0, -1}};
II = KroneckerProduct[Id, Id];
XX = KroneckerProduct[sigX, sigX];
IZ = KroneckerProduct[Id, sigZ];
IX = KroneckerProduct[Id, sigX];
XZ = KroneckerProduct[sigX, sigZ];
YY = KroneckerProduct[sigY, sigY];
ZZ = KroneckerProduct[sigZ, sigZ];
H = 1/Sqrt[2] {{1, 1}, {1, -1}};
Phi00 = 1/2 {{1, 0, 0, 1}, {0, 0, 0, 0}, {0, 0, 0, 0}, {1, 0, 0, 1}};
Phi01 = 1/2 {{0, 0, 0, 0}, {0, 1, 1, 0}, {0, 1, 1, 0}, {0, 0, 0, 0}};
Phi10 = 1/2 {{1, 0, 0, -1}, {0, 0, 0, 0}, {0, 0, 0, 0}, {-1, 0, 0, 1}};
Phi11 = 1/2 {{0, 0, 0, 0}, {0, 1, -1, 0}, {0, -1, 1, 0}, {0, 0, 0, 0}};

In[=]:= x * y (XX.YY + YY.XX)
Out[=]= {{-2 x y, 0, 0, 0}, {0, 2 x y, 0, 0}, {0, 0, 2 x y, 0}, {0, 0, 0, -2 x y}}

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In[=]:= v1 = 1/Sqrt[3] {{1}, {1}, {1}, {0}};
V1 = KroneckerProduct[v1, ConjugateTranspose[v1]];
v2 = 1/Sqrt[3] {{1}, {Exp[I*2 Pi/3]}, {Exp[I*4 Pi/3]}, {0}};
V2 = KroneckerProduct[v2, ConjugateTranspose[v2]];
v3 = 1/Sqrt[3] {{1}, {Exp[I*4 Pi/3]}, {Exp[I*8 Pi/3]}, {0}};
V3 = KroneckerProduct[v3, ConjugateTranspose[v3]];

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In[=]:= MatrixForm[V1[[1;;2, 1;;2]]]
Out[=]/MatrixForm=

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$$\begin{pmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} \end{pmatrix}$$


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In[=]:= MatrixForm[V2]
Out[=]/MatrixForm=

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$$\begin{pmatrix} \frac{1}{3} & \frac{1}{3} e^{-\frac{2i\pi}{3}} & \frac{1}{3} e^{\frac{2i\pi}{3}} & 0 \\ \frac{1}{3} e^{\frac{2i\pi}{3}} & \frac{1}{3} & \frac{1}{3} e^{-\frac{2i\pi}{3}} & 0 \\ \frac{1}{3} e^{-\frac{2i\pi}{3}} & \frac{1}{3} e^{\frac{2i\pi}{3}} & \frac{1}{3} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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In[]:= P1 = 1/4 (V1 + KroneckerProduct[Id, V1[[1;;2,1;;2]]] +
KroneckerProduct[V1[[1;;2,1;;2]], Id] + 1/3 KroneckerProduct[Id, Id]);
P2 = 1/4 (V2 + KroneckerProduct[Id, V2[[1;;2,1;;2]]] +
KroneckerProduct[V2[[1;;2,1;;2]], Id] + 1/3 KroneckerProduct[Id, Id]);
P3 = 1/4 (V3 + KroneckerProduct[Id, V3[[1;;2,1;;2]]] +
KroneckerProduct[V3[[1;;2,1;;2]], Id] + 1/3 KroneckerProduct[Id, Id]);

In[]:= N[Eigenvalues[P1]]
Out[]= {0.636119, 0.269288, 0.25, 0.0945929}

In[]:= N[Eigenvalues[P2]]
Out[=] {0.563644, 0.34377, 0.25, 0.0925859}

In[]:= N[Eigenvalues[P3]]
Out[=] {0.563644 - 1.66533 × 10-16 I, 0.34377 - 5.55112 × 10-17 I,
0.25 - 5.55112 × 10-17 I, 0.0925859 - 2.08167 × 10-17 I}

In[]:= Simplify[P1 + P2 + P3]
Out[=] {{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 3/4} }

In[]:= Simplify[P1 + P2 + P3];
MatrixForm[%]

Out[=]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$


In[]:= M00 = 1/4 (Phi00 + 1/2 (KroneckerProduct[Id, {{1, 0}, {0, 0}}] +
KroneckerProduct[{{1, 0}, {0, 0}}, Id]) + 1/2 * II);
In[]:= M01 = 1/4 (Phi01 + 1/2 (KroneckerProduct[Id, {{0, 0}, {0, 1}}] +
KroneckerProduct[{{0, 0}, {0, 1}}, Id]));
In[]:= M10 = 1/4 (Phi10 + 1/2 (KroneckerProduct[Id, {{1, 0}, {0, 0}}] +
KroneckerProduct[{{1, 0}, {0, 0}}, Id]) + 1/2 * II);
M11 = 1/4 (Phi11 + 1/2 (KroneckerProduct[Id, {{0, 0}, {0, 1}}] +
KroneckerProduct[{{0, 0}, {0, 1}}, Id]));

In[]:= Simplify[M00 + M01 + M10 + M11]
Out[=] {{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1} }

In[]:= Eigenvalues[M00]
Out[=] {1/8 (3 + √2), 1/4, 1/4, 1/8 (3 - √2) }

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In[1]:= **Eigenvalues[M01]**

$$\text{Outf}= \left\{ \frac{3}{8}, \frac{1}{4}, \frac{1}{8}, 0 \right\}$$

In[2]:= **Eigenvalues[M10]**

$$\text{Outf}= \left\{ \frac{1}{8} (3 + \sqrt{2}), \frac{1}{4}, \frac{1}{4}, \frac{1}{8} (3 - \sqrt{2}) \right\}$$

In[3]:= **Eigenvalues[M11]**

$$\text{Outf}= \left\{ \frac{3}{8}, \frac{1}{4}, \frac{1}{8}, 0 \right\}$$

In[4]:= **N[1/4 (3 + Sqrt[2]) + 3/4]**

$$\text{Outf}= 1.85355$$

In[5]:= **N[9/4]**

$$\text{Outf}= 2.25$$

In[6]:= **N[1/10 (15 - Sqrt[29])]**

$$\text{Outf}= 0.961484$$

In[7]:= **Eigenvalues[II + 1/2 (IZ + ZZ + XX - YY)]**

$$\text{Outf}= \left\{ \frac{1}{2} (3 + \sqrt{5}), 1, \frac{1}{2} (3 - \sqrt{5}), 0 \right\}$$

In[8]:= **Eigenvalues[II + 9/10 * IZ + 2/10 * (ZZ + XX - YY)]**

$$\text{Outf}= \left\{ \frac{1}{10} (12 + \sqrt{97}), \frac{17}{10}, \frac{1}{10} (12 - \sqrt{97}), -\frac{1}{10} \right\}$$

In[9]:=

$$1 + 1/2 + 1/16$$

$$\text{Outf}= \frac{25}{16}$$

In[10]:= **2 + 1 / 4**

$$\text{Outf}= \frac{9}{4}$$

In[11]:= **1 / 2 + 1 / 2 (1 / 4 + 1 / 16 + 1 / 16) + 1 / 8**

$$\text{Outf}= \frac{13}{16}$$

In[5]:= $1 / 2 + 1 / 4 + 1 / 4 (3 / 4)^2$

Out[5]= $\frac{57}{64}$