Lecture outline: Randomised and mixed strategies

Matching Pennies

		Player Odd	
		Heads	Tails
Player Even	Heads	1, -1	-1, 1
	Tails	-1, 1	1, -1

1. Solve the Matching Penny games: player Even plays Head with probability X and tails with $(1 - X)$, and Player Odd Y and $(1 - Y)$. What are X and Y?

   Expected utility of both players:

   $E_{Even}(H) = 1Y + -1(1 - Y) = 2Y - 1$

   $E_{Even}(T) = -1Y + 1(1 - Y) = 1 - 2Y$

   $E_{Odd}(H) = -1X + 1(1 - X) = 1 - 2X$

   $E_{Odd}(T) = 1X + -1(1 - X) = 2X - 1$

2. In equilibrium, a player should randomise their selection to make their opponent indifferent between their own stategy.

$$
\begin{aligned}
& E_{Even}(H) = E_{Odd}(T) \\
\Rightarrow\quad & 2Y - 1 = 1 - 2Y \\
\Rightarrow\quad & 4Y = 2 \\
\Rightarrow\quad & Y = \tfrac{1}{2} \\
\Rightarrow\quad & (1 - Y) = \tfrac{1}{2}
\end{aligned}
$$

   Similarly for X, therefore each player choose H and T with 50-50 probability.

Security games

		Adversary	
		Terminal 1	Terminal 2
Defender	Terminal 1	5, -3	-1, 1
	Terminal 2	-5, 5	2, -1

What are the strategies? They should not be just any random strategy – indifference rules!

1. For the Defender:

   $E(T1) = 5Y + -1(1 - Y) = 6Y - 1$

   $E(T2) = -5Y + 2(1 - Y) = 2 - 7Y$

   $$\begin{aligned} & E(T1) = E(T2) \\ \Rightarrow \quad & 6Y - 1 = 2 - YY \\ \Rightarrow \quad & 2 = 13Y \\ \Rightarrow \quad & Y = \tfrac{3}{13} \\ \Rightarrow \quad & (1 - Y) = \tfrac{10}{13} \end{aligned}$$

2. For the attacker:

   $E(T1) = -3X + 5(1 - X) = 5 - 8X$

   $E(T2) = 1X + -1(1 - X) = 2X - 1$

   $$\begin{aligned} & E(T1) = E(T2) \\ \Rightarrow \quad & 5 - 8X = 2X - 1 \\ \Rightarrow \quad & 6 = 10X \\ \Rightarrow \quad & X = \tfrac{6}{10} \\ \Rightarrow \quad & (1 - X) = \tfrac{4}{10} \end{aligned}$$

Mixed and pure equilibria example

		Player 2	
		Left	Right
Player 1	Up	3, 1	0, 0
	Down	0, 0	1, 3

1. There can be mixed strategy equilibria even if there are already pure strategy equilbria. See the example: 3,1 and 1,3 are pure strategy equilibria, BUT!:
   - Player 1: $E(U) = E(D) \to 3q = 1 - q \to 4q = 1 \to q = 1/4$
   - Player 2: $E(L) = E(R) \to p = 3(1 - p) \to 4p = 3 \to p = 3/4$
2. So, Player 1 with $(3/4U + 1/4D)$ and Player 2 with $(1/4L + 3/4D)$.