Convection and Diffusion

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Introduction

The convection diffusion equation (1) is used to model systems where both diffusion and transport are occurring.

$$\nabla \cdot k\nabla T - \nabla \cdot \rho u C_P T - q = 0 \tag{1}$$

$$\int_V (\nabla \cdot k\nabla T - \nabla \cdot \rho u C_P T - q) dV = 0 \tag{2}$$

$$k\frac{dT}{dx} - \rho u C_P T \bigg|_{x_w}^{x_e} - qV = 0 \tag{3}$$

After applying the Finite Volume Method in equation (2), equation (3) is obtained.

$$k\frac{T_E - T_P}{x_E - x_P} - k\frac{T_P - T_W}{x_P - x_W} - \rho u C_P T_e + \rho u C_P T_w - qV = 0 \tag{4}$$

Finite Differences are applied giving equation (4). The interpolation scheme to determine T_e still remains to be selected.

Convection

A quantity called the Peclet number can be defined as the ratio of convective flux to diffusive flux (6). This is useful for concisely stating how much convection is occurring.

$$D = k\Delta x^{-1}; \;\; F = \rho u C_P \tag{5}$$

$$P = \frac{F}{D} = \frac{\rho u C_P}{k\Delta x^{-1}} \tag{6}$$

Central-Differences

The obvious interpolation scheme for the convective term is a central-difference (7). However, stability issues can result.

$$T_e = \frac{T_E + T_P}{2} \tag{7}$$

$$a_e = D_e - \frac{1}{2}F_e; \;\; a_w = D_w + \frac{1}{2}F_w \tag{8}$$

As made evident by the formulation of coefficients for a central-difference interpolation (8), a Peclet number above 2 will result in negative coefficients.

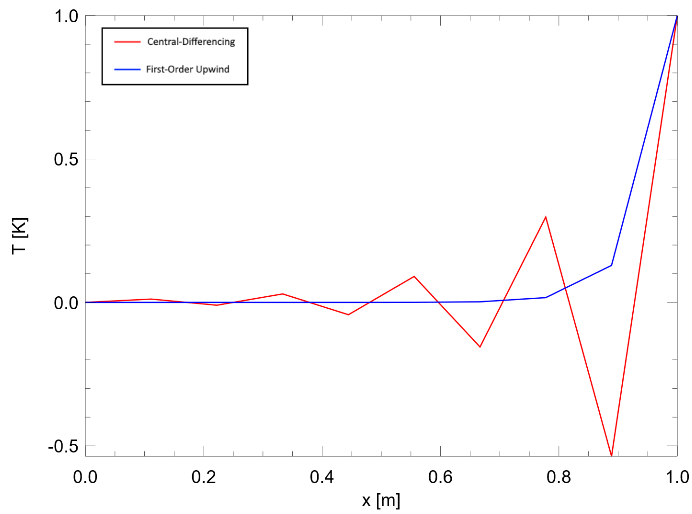

Figure 1: Solution with $P = 50$

Upwinding

Another option is to take the upwind value of T (9) as the value at the cell face.

$$T_e = \begin{cases} T_P & u > 0 \\ T_E & u \leq 0 \end{cases} \tag{9}$$

While this results in excess diffusion as demonstrated in figure 1, it is stable and will give positive coefficients.

Grid Refinement

In order for central-differencing to be a viable interpolation scheme, the Peclet number must be reduced. In the definition of Peclet number (6), it is evident that P is a function of grid spacing. Thus, a grid refinement will lower the Peclet number allowing the coefficients to become positive again: The grid must be refined to use central-differencing.

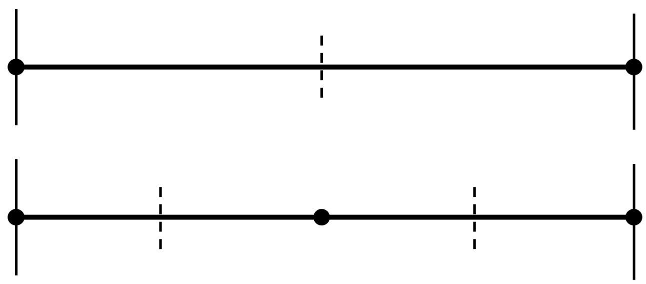

Figure 2: Cell Splitting

Selective Refinement

It is undesirable to refine the entire grid, because of memory and processing constraints. One approach involves inserting a node into the grid or splitting a cell in two.

Solution

The grid refinement provided an efficient usage of cells so that central-differencing could be used without instability. This method would also be useful to refine the grid such that cell size is a function of temperature gradient.