i) $\mathrm{Rank}_{\mathbb{Z}} E(K) = 2$

K: imaginary quad field where p splits.

In this case, the fine Selmer group is NOT finite.

(there's a file with egs where $E/\mathbb{Q}$ elliptic curve
$\mathrm{Rank}_{\mathbb{Z}} E(\mathbb{Q}) = 1$, $\mathrm{Rank}_{\mathbb{Z}} E_{twist}(\mathbb{Q}) = 1$.

---

ii) $\mathrm{Rank}_{\mathbb{Z}} E(K) = 1$.

K: imag. quad field where p splits.

p is a prime of supersingular reduction.

(there's a file with $E(K)$ has rank 1.

- e.g. of $E/\mathbb{Q}$ with $\mathrm{rk}\, E(K) = 1$, but E is not defined over $\mathbb{Q}$.
- egs where

This case is interesting since

(·) $\Pi_{ac}(\Theta^{++}) = 0$, $\Pi_{ac}(\Theta^{--}) = 0$

So this could be a nice sanity check.

(·) So, is $\Pi_{ac}(\Theta^{+-}) \neq 0$?

$\Pi_{ac}(\Theta^{-+})$

This is not known.

(·) Suppose $E/\mathbb{Q}$

(base change)

Sanity check

$$\Pi_{cyc}(\Theta^{++}) = \Theta^{+}_{E} \cdot \Theta^{+}_{E_{twist}}$$

$$\Pi_{cyc}(\Theta^{--}) = \Theta^{-}_{E} \cdot \Theta^{-}_{E_{twist}}.$$

Question :

Is $\pi_{cyc}(\Theta^{+-}) \stackrel{?}{=} \Theta^{+}_{E} \cdot \Theta^{-}_{E_{twist}}$

$$\pi_{cyc}(\Theta^{-+}) = \Theta^{-}_{E} \cdot \Theta^{+}_{E_{twist}}.$$

This is not known!

	$p = 3$	$p = 5$	$p = 7$	$p = 11$	$p = 13$
$\mathbb{Q}(\sqrt{-1})$		✓			✓
$\mathbb{Q}(\sqrt{-2})$	✓			✓	
$\mathbb{Q}(-3)$			✓		✓
$\mathbb{Q}(\sqrt{-7})$				✓	
$\mathbb{Q}(\sqrt{-11})$	✓	✓			