

Follow the notations of previous scans. ①
 Start with the isomorphism of Galois representations:

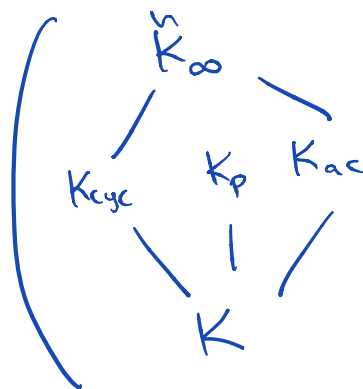
$$\text{Ind}_{G_K}^{G_F} \left[\text{Res}_{G_K}^{G_F} (V_g) \otimes_L L(\psi_p \varepsilon) \right] \cong V_g \otimes_L V_f$$

Branchi

Rankin-Selberg

This Galois representation (s) satisfies the Panchishkin condition of

Greenberg.



$$\tilde{\Gamma} = \text{Gal}(\tilde{K}_\infty / K)$$

$$\Gamma_{\text{Fyc}} = \text{Gal}(K_{\text{Fyc}} / K)$$

$$\Gamma_{\text{ac}} = \text{Gal}(K_{\text{ac}} / K)$$

Bianchi - perspective : For the Galois rep over G_K (2)

$\text{Res}_{G_K}^{\text{Gal}}(V_g) \otimes L(E)$ (without the φ_p twist)

$\hookrightarrow_{\text{Bianchi}}$ contains the homomorphisms:

$$\phi: \prod_{\mathbb{Z}}^S \longrightarrow \overline{\mathcal{G}}_p^{\times}$$

$\Gamma_{\text{cyc}} \times \Gamma_p$

(via restriction maps $\tilde{\Gamma} \rightarrow \Gamma_{\text{cyc}}, \tilde{\Gamma} \rightarrow \Gamma_p$)

given χ

$$\Sigma_{\text{cyc}}: \Gamma_{\text{cyc}} \rightarrow \overline{\mathcal{G}}_p^{\times} \quad \Sigma_p: \Gamma_p \rightarrow \overline{\mathcal{G}}_p^{\times}$$

finite characters

Denote this q -adic L -function by $\bigcirc_{\text{Bianchi}}$
in fraction field of $\mathbb{Q}_q[\tilde{\Gamma}]$.

(will discuss incorporating twists of Γ_p at the end).

Some generalities for Rankin Selberg 3

p -adic L -functions.

Suppose F, G are two Hida families

R_F, R_G are the normalizations of the
irreducible components of Hida's ordinary
Hecke algebra through F, G resp.

The p -adic L -functions

$L_p(\underline{F}, G, s)$ and $L_p(F, \underline{G}, s)$
are elements of $R_F \hat{\otimes} R_G[[\Gamma_{\text{yc}}]]$.

For the p -adic L -function

$$L_p(\underline{F}, G, s)$$

where F is the dominant Hida
family, the critical set

$\subset_{R.S.}^{(F)}$ is the set of homomorphisms

(4)

$$\phi : R_F \otimes R_a [U_{gc}] \rightarrow \overline{\mathcal{G}}_F$$

given by

classical specializations

$$\phi_{k,F} : R_F \rightarrow \overline{\mathcal{G}}_F, \quad \phi_{k,a} : R_a \rightarrow \overline{\mathcal{G}}_F, \quad \phi_{gc} : \Gamma_{gc} \rightarrow \overline{\mathcal{G}}_F^+$$

$$\text{with } \phi_{gc} = \left(\chi_{p,gc} \omega^{-1} \right)^{-s}$$

↑
p-adic
cyclotomic
character

↑
Teichmüller
character

with the condition

$$\text{wt}(\phi_{k,F}) > s+1 \geq \text{wt}(\phi_{k,a})$$

Similarly, define $\subset_{R.S.}^{(a)}$ for $L_F(F, \underline{a}, s)$.

And similarly if you specialize for eigenforms

⑤

f_k and/or g_k
to get two variable / one-variable
mod \equiv
L-functions.

Remark:

i) Suppose f is any eigenform
(ordinary or no), then

$L_p(\underline{f}, G, s)$ is defined!

ii) If f has weight two, for
 $L_p(\underline{f}, G, s)$ to be defined,

G needs to have infinitely many
weight one specializations.

This is only possible if G is a C.M.
Hida family.

⑥

Now consider the case when

\underline{E} : elliptic curve (so wt = 2).

G_K : C.M. Hida family corresponding to the Galois representation

$$\text{Ind}_{G_K}^{G_{\mathbb{Q}}}(\chi_p)$$

where $\chi_p: G_K \twoheadrightarrow \Gamma_p \hookrightarrow GL_1(\mathbb{Z}_p[\Gamma_p])$ is the p -adic character

And the critical set for the

$$L_p(\underline{E}, G_K, s) \rightarrow \text{Rankin - Selberg}$$

matches with the critical set for

the corresponding Bianchi p -adic

L -function.

$$\chi_p(\Theta_{\text{Bianchi}})$$

(I haven't worked out the exact twist here. not sure sorry.)

There must be a relation

⑦

(conjecturally, at least) between these

p -adic L -functions, since they

correspond to the same Galois

representation, (\Rightarrow the complex L -functions are the same)

they interpolate at the same critical set

(\Rightarrow At least conjecturally, the corresponding complex and p -adic periods must be related.)