Assignment 1

(i)   Set up the programming environment that you are going to use in the course (no need to report anything on this).

1. Implement a GGL random number generator and run it to test that it works as it should. Then implement a RAN3 generator so that the initial set of elements (seeds) are generated by the GGL algorithm you wrote. Finally, simulate $X \sim U(0,1)$ and $Y \sim U(0,1)$ using your GGL, your RAN3 and the built-in random number generator in Python (Mersenne-Twister). For each RNG, place the consecutive random numbers as points in (x, y)-coordinates. (More precisely, take consecutively generated random numbers in the order $x, y, x, y, x, y, \dots$) Inspect a sufficiently small interval (slice) $x \in [x_A, x_B]$ $(x_B - x_A < 10^{-3})$ and $y \in (0, 1]$ to see if there are any differences. (**Note**: you must generate *a lot* of points, well over 20000 - even up to 10^6 - random number pairs for the visual tests in order to see differences.) Name what you find.
(Things to be shown for peergrading:
Notebook showing clearly the implementations of the required algorithms and plots and explaining the findings -at least for ggl - using correct terminology .)

2. In a (ridiculously) simplistic model a person's optimism/pessimism is characterised by a number that changes through interactions with others. A large number means strong optimism and a small number strong pessimism. You want to find out the probability density resulting from a large number of interactions. You take 500 000 agents (persons) in the model and set the number describing their initial state at 50. Each of these agents has 100 interactions in a) and b). In c) due to the nature of the resulting distribution take only 10 interactions for each agent. The interactions change an agent's state according to the following models:
   a) Independently of previous interactions, each new interaction shifts the agent's state by equal probability towards pessimism by -1, or towards optimism by +1.
   b) Independently of previous transitions, each new interaction shifts the agent's state by equal probability towards pessimism by -0.5, or towards optimism by +10.
   c) The agent's response is dependent on previous interactions such that the new state is determined by multiplying the present state by 0.7 if the interaction strengthens pessimism and by 1/0.7 if the interaction strengthens optimism.
   In a) and b) name the probability mass function (PMF) (= the discrete probability density function) and the probability density function (pdf) it approaches in the limit of infinite number of agents (and also in the limit of continuous shifts of states). In c), based on your simulation and knowledge of distributions, argue what the probability density function in the limit of infinite number of agents is; in other words, name the stochastic process. Getting a representative pdf in c) would require much larger statistics than in a) and b). Why is that?
   You can use ready random generators of python. You get PMF's by plotting histograms (hist-command in matplotlib, with density='true'). In some case(s) it may help to change axis/axes to logarithmic in order to determine the pdf.