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Computation of the GIT-fan using a massively parallel implementation

Masterarbeit

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Abstract

Fancy, english text.

Kurzzusammenfassung

Brillierender, deutscher Text.

Acknowledgements

Syn, Ack...

Contents

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Well, skip this and just read the rest. Less work for me.

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insbesondere beschreiben, wo die Unterschiede zwischen Single-Node und Cluster-enabled sind

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an Kommandozeilenparameter entlanghangeln

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Let $p = x_1x_3 + x_2x_4 + x_5 \in \mathbb{K}[x_1, x_2, x_3, x_4, x_5] =: \mathbb{K}[\mathbf{x}]$. Set

$$\mathfrak{a} := \langle x_3, x_4\rangle \cdot \langle p\rangle,$$

$$Q = (q_1, q_2, q_3, q_4, q_5) := \begin{pmatrix} 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 2 \end{pmatrix}.$$

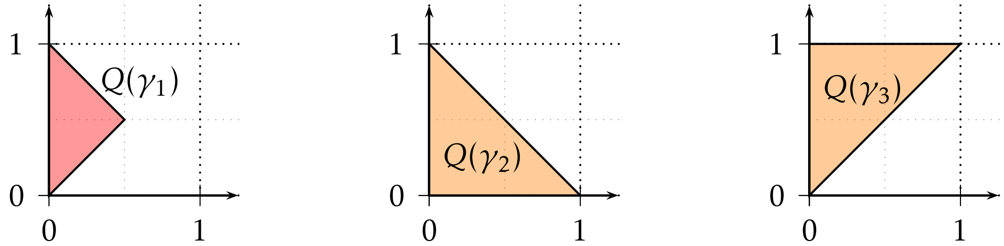
Note that Q defines a $\mathbb{Z}^3$ grading on ${}^{\mathbb{K}[x_1,x_2,x_3,x_4,x_5]}\!/_{\mathfrak{a}}$ since p and therefore $\mathfrak{a}$ is homogenous with respect to the grading $\deg(x_i) = q_i$. Thus, we obtain a torus action of $(\mathbb{K}^*)^3$ on $V(\mathfrak{a})$.

The $\mathfrak{a}$-faces with full dimensional image in $Q(\gamma)$, $\gamma = \mathbb{Q}^5_{\geq 0}$, are given by

$$\begin{aligned}
\gamma_1 &:= \langle e_1, e_2, e_5\rangle \\
\gamma_2 &:= \langle e_1, e_2, e_3, e_5\rangle \\
\gamma_3 &:= \langle e_1, e_2, e_4, e_5\rangle \\
\gamma_4 &:= \langle e_1, e_3, e_4, e_5\rangle \\
\gamma_5 &:= \langle e_2, e_3, e_4, e_5\rangle \\
\gamma_6 &:= \langle e_1, e_2, e_3, e_4\rangle \\
\gamma_7 &:= \langle e_1, e_2, e_3, e_4, e_5\rangle
\end{aligned}$$

This is easily seen by sending all variables x_i with $e_i \notin \gamma_j$ in $\mathfrak{a}$ to zero and check that the new ideal does not contain any monomials. (details see below)

The orbit cones $Q(\gamma_j) \in \mathbb{Q}^3_{z_1,z_2,z_3}$ have the following form when intersecting them with the $\{z_3 = 1\}$-plain (vice versa, the orbit cones are recovered by expanding the polytopes in z_3-direction to cones in $\mathbb{Q}^3$):



A Something appendixable

Theorem A.1 (Great theorem) *Problem in appendix.*

Proof: by intimidation. □

Bibliography

[DKV16] Arnaud Durand, Juha Kontinen, and Heribert Vollmer. 'Expressivity and complexity of dependence logic'. In: *Dependence Logic: Theory and Applications*. Birkhäuser, 2016, pp. 5–32. DOI: 10.1007/978-3-319-31803-5_2.

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Erklärung zu dieser Arbeit

Hiermit erkläre ich, die vorliegende Masterarbeit selbstständig erstellt zu haben. Weiterhin versichere ich, dass sämtliche von mir verwendeten Hilfsmittel und Quellen im Literaturverzeichnis aufgeführt sind.

Christian Reinbold, Hannover, den 11.01.2018