Power analysis via simulations

Readings for today

- Beaujean, A. A. (2014). Sample size determination for regression models using Monte Carlo methods in R. Practical Assessment, Research, and Evaluation, 19(1), 12.

Topics

1. Power

2. Monte Carlo methods

Power

Estimating data sufficiency

Q: Is my data sufficient to address my hypothesis?

Statistical Power: The ability to correctly reject a false H_0

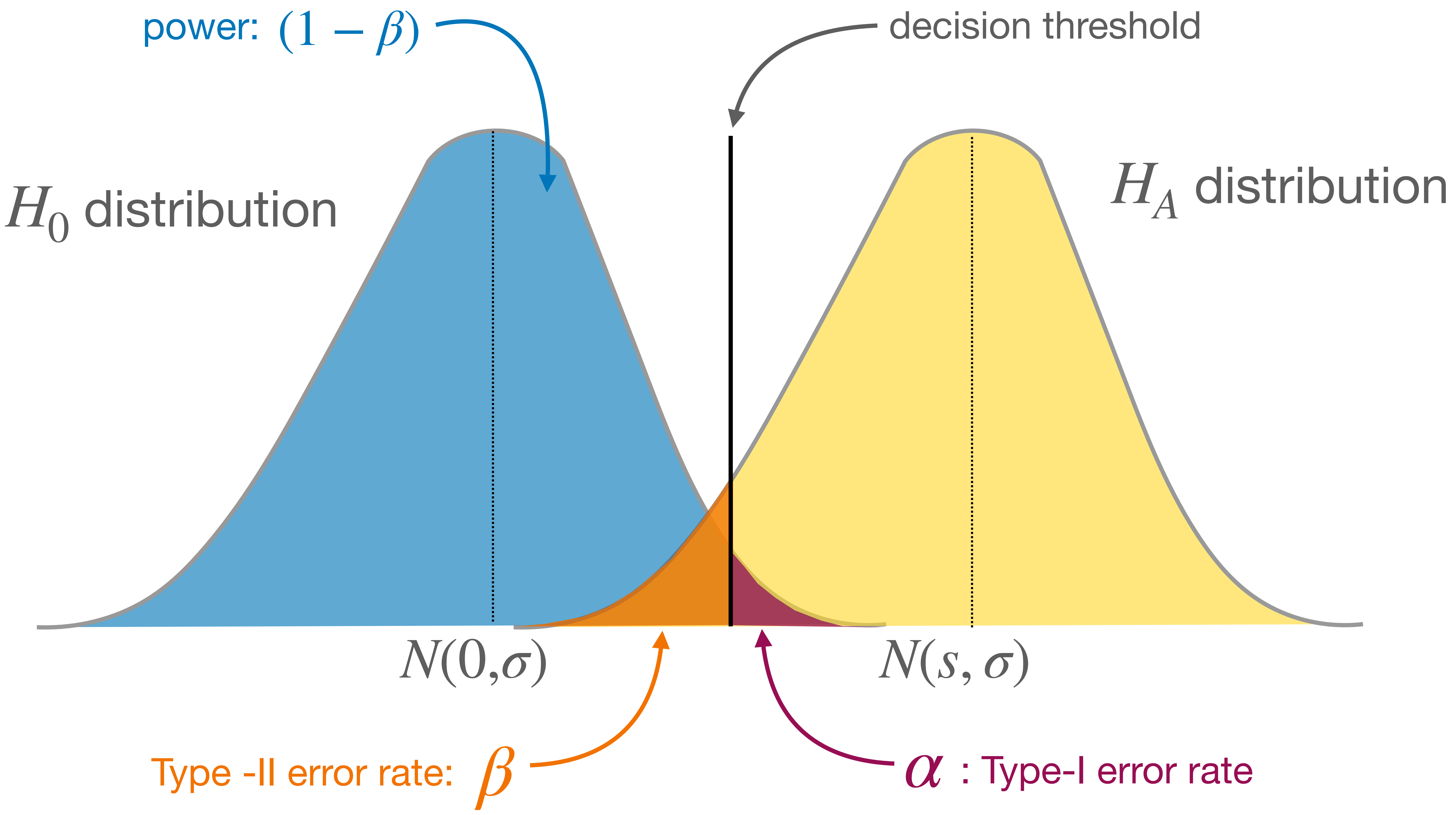
n : sample size

α : Type-I error (false positive) rate → p threshold

β : Type-II error (false negative) rate → Power = $1 - \beta$

s : effect size

Statistical power



(Beaujean 2014)

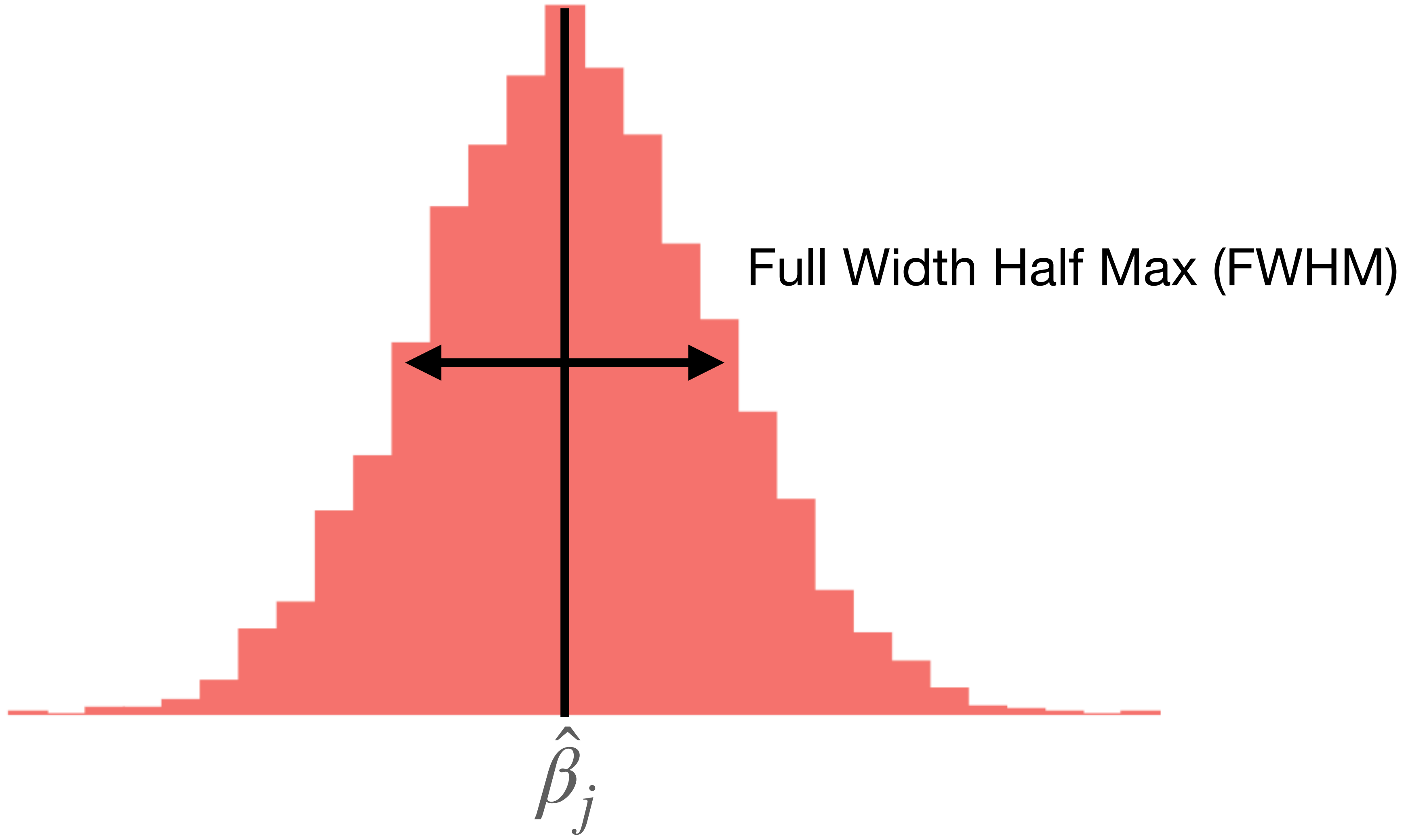
Goals of power analysis

Null Hypothesis Test Statistics (NHTS): Do you have sufficient power to detect whether a test statistic is "statistically significant"?

$$P(\hat{H}_0 \text{ rejected} \mid H_0 = \text{false})$$

Accuracy in Parameter Estimation (AIPE): Do you have sufficient power to accurately recover the value of a statistical parameter (regardless of "statistical significance")?

$$MSE(\hat{\beta}_j - \beta_j)$$



(Beaujean 2014)

Traditional approaches to estimating power

<u>Goal:</u> Sample size (n) determination.

<u>Step 1</u>: Search the literature for estimates of effect size (s).

<u>Step 2</u>: Determine best expectation of effective $E[s]$.

<u>Step 3</u>: Determine your ideal Type-I (α) and Type-II (β) error rates.

<u>Step 4</u>: Use *parametric* methods to calculate the sample size needed to achieve your α and β, assuming specific distributions of your data.

(Beaujean 2014)

Traditional approaches to estimating accuracy

<u>Goal:</u> Parameter estimation accuracy ($\hat{s}$) determination.

<u>Step 1</u>: Determine the distributions of your X and Y.

<u>Step 2</u>: Review the literature to find the best estimates of $\hat{s}$ (i.e., $\hat{f}(X)$).

<u>Step 3</u>: Determine your best expected model $E[\hat{f}(X)]$.

<u>Step 4</u>: Set desired confidence interval range for a given Type-I error rate (α)

<u>Step 5</u>: Use *parametric* methods to determine the sample size needed to achieve your target $(1 - \alpha)\,\%$ confidence intervals.

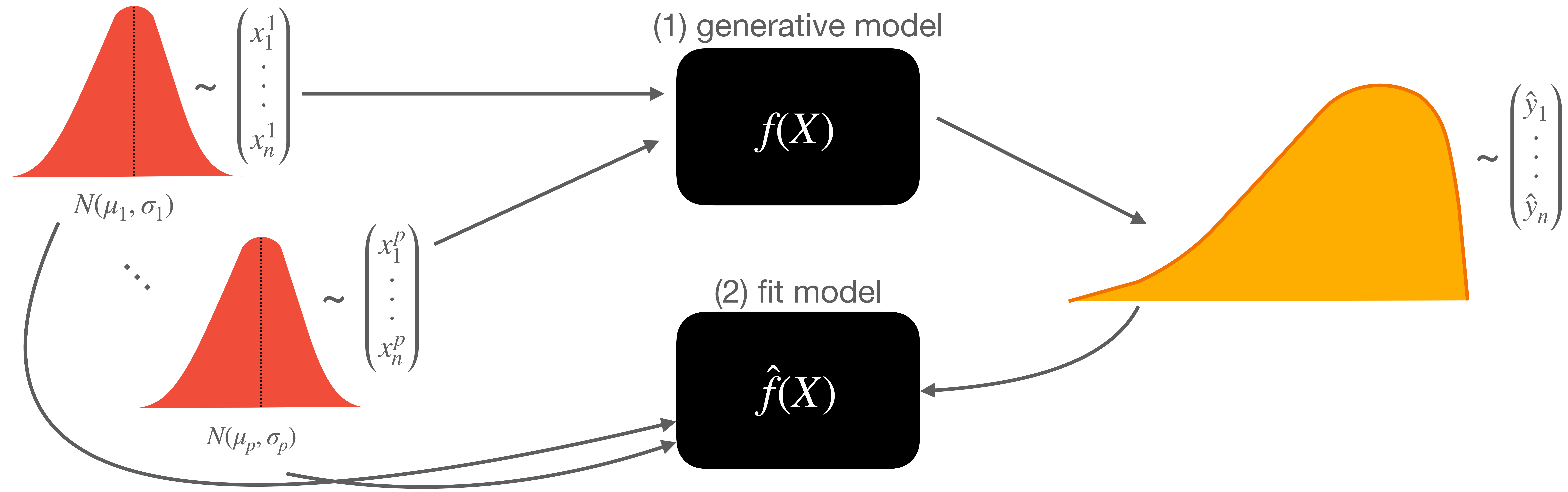
(Beaujean 2014)

Monte Carlo methods

Monte Carlo (MC) method

Goal: Generate *empirical* distributions of your expected effects, instead of estimated distributions, to determine sufficiency of your data set.

Monte Carlo method: Use random sampling to simple data for numerical analyses.



(Beaujean 2014)

MC power analysis

Step 1: Determine model of interest (e.g., regression, t-test)

Step 2: Decide on values for all parameters.

- Sample distributions: $X_p \sim N(\mu_p, \sigma_p)$, $Y \sim N(\mu_y, \sigma_y)$
- Effect sizes: e.g., β_j

Step 3: Identify data "quirks" (e.g., missing value rate)

Step 4: Decide on aspects of MC simulations.

- $\alpha \rightarrow$ Type-I error rate
- $(1 - \beta) \rightarrow$ power
- $m \rightarrow$ number of iterations/simulations
- $n \rightarrow$ sample size
- $c \rightarrow$ number of different random seeds (at least 2)

(Beaujean 2014)

MC power analysis

Step 5: Run m simulations using the parameters in Step 2.

Step 6: Evaluate performance of each simulation using quality metrics.

- If poor, $\uparrow n$ and repeat Step 5.

Step 7: Repeat Steps 5-6 using different random seeds. Check to see if results converge across seeds.

(Beaujean 2014)

MC power analysis algorithm

For $i = 1$ to m

i) Sample: e.g., $\underbrace{X_1 \sim N(\mu_1, \sigma_1)}_{normal}, \ldots, \underbrace{X_j \sim B(n, p)}_{binomial}$

ii) Sim: $Y = f(x, \hat{\theta}) \rightarrow \beta_0 + \sum_{j=1}^{p} \beta_j X_j + \epsilon$

iii) Evaluate: $T(y, x) \rightarrow \dfrac{\hat{\beta}_j}{SE(\hat{\beta}_j)}$

iv) Track: $T = [T_1(Y_1, X_1), \ldots, T_i(Y_i, X_i)]$

(Beaujean 2014)

MC simulation quality metrics

<u>Relative parameter estimate bias:</u>

$$\theta_{bias} = \frac{\hat{\theta} - \theta_H}{\theta_H}$$

- θ_H : pre-set (real) effect size
- $\hat{\theta}$: average fit parameter across simulations.

<u>Relative standard error bias:</u>

$$\sigma_{bias} = \frac{\hat{\sigma}_\theta - \sigma_{\hat{\theta}}}{\sigma_{\hat{\theta}}}$$

- $\hat{\sigma}_\theta$: average of fit parameter SE across simulations.
- $\sigma_{\hat{\theta}}$: standard deviation of $\hat{\theta}$ across simulations.

<u>Coverage:</u>

$$c = \frac{1}{m} I(\theta \neq 0)$$

$$I(\theta \neq 0) = \begin{cases} 1, \text{ if } H_0 = \text{rejected} \\ 0, \text{ otherwise} \end{cases}$$

(Beaujean 2014)

Example: Regression

Model:

$$Y = \beta_0 + \beta_1 X + \epsilon$$

$$Y^* \sim N(\beta_0, \sigma_y)$$

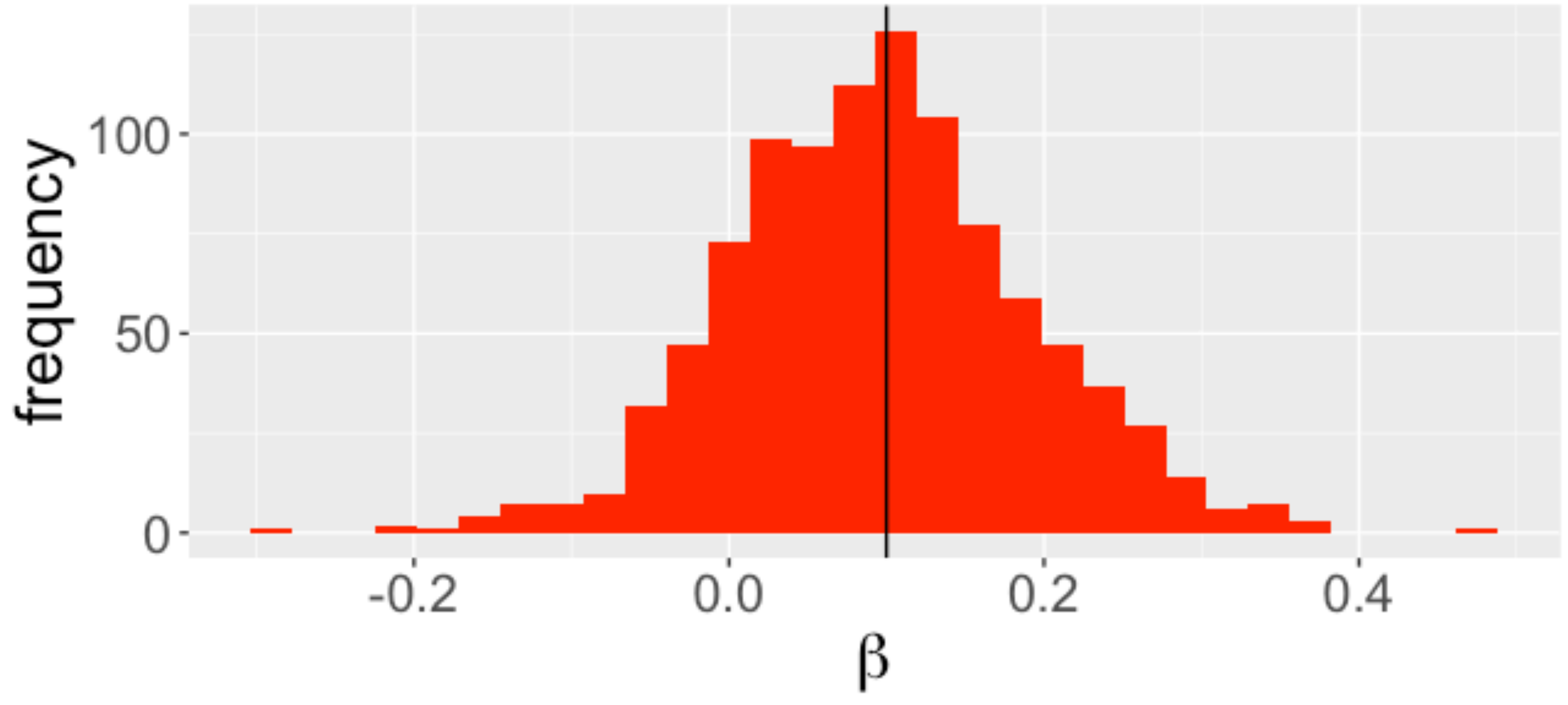
Parameters:

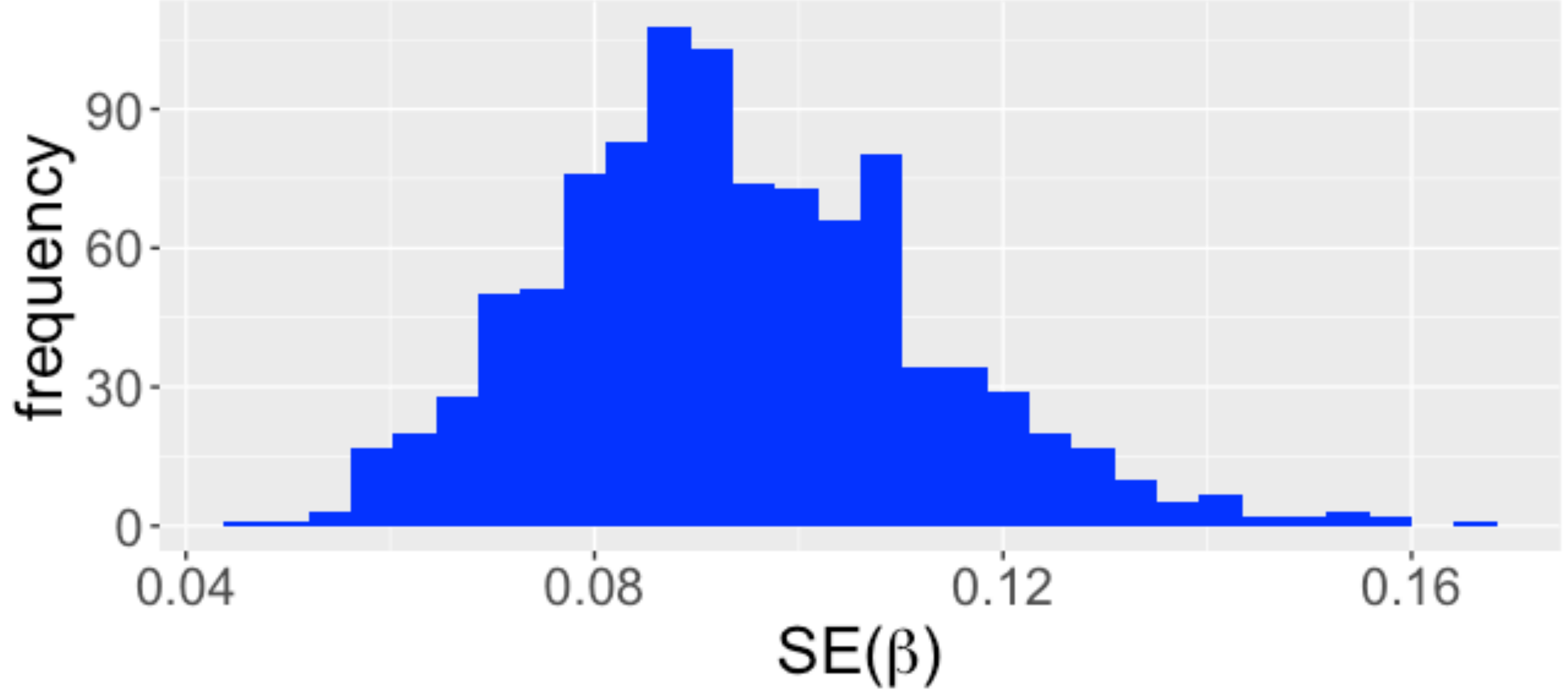
$\alpha = 0.05$ $\quad m = 1000$

$X \sim N(3,2)$ $\quad n = 30$

$\beta_0 = 9$ $\quad \beta_1 = 0.1$

$\epsilon \sim N(0,1) = N(0,\sigma_y)$





Example: Regression

Relative parameter estimate bias:

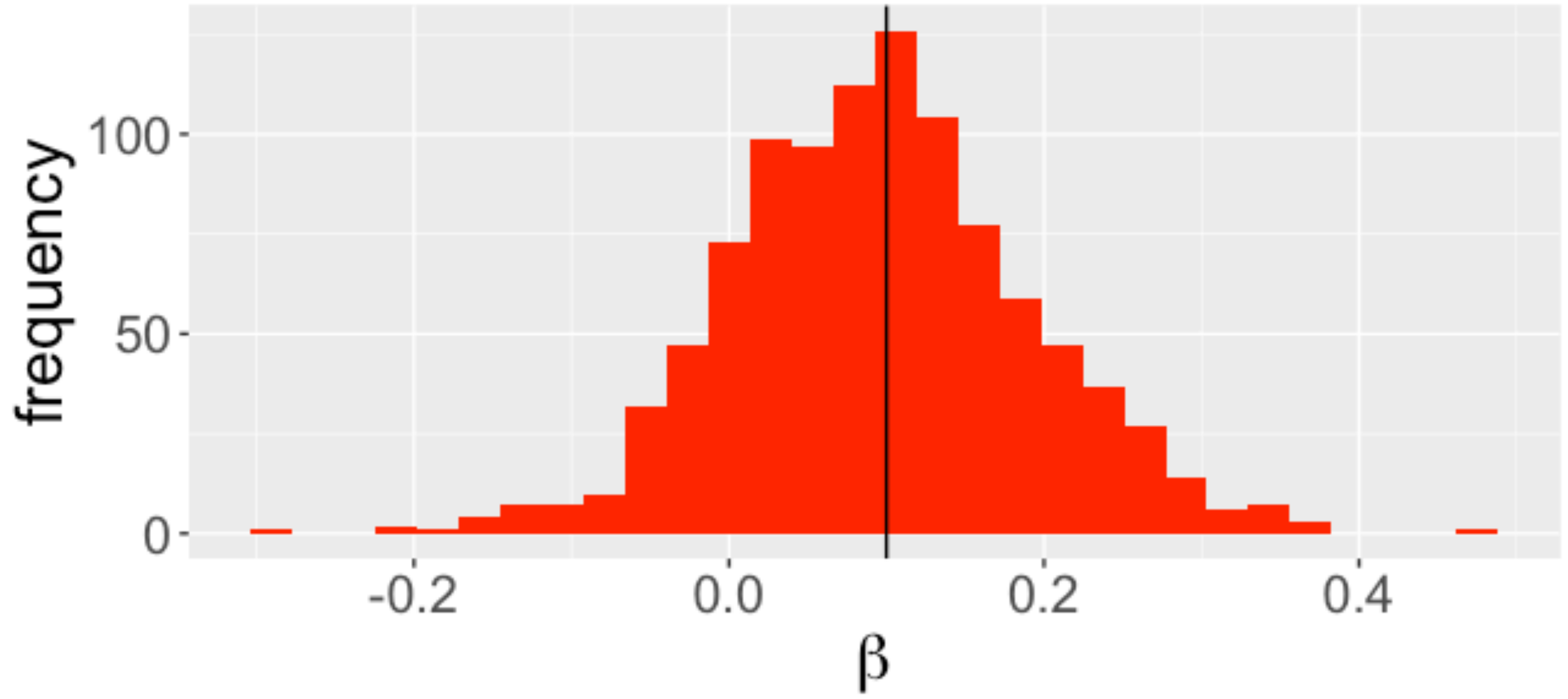
$$\theta_{bias} = \frac{\hat{\theta} - \theta_H}{\theta_H} = -0.045$$

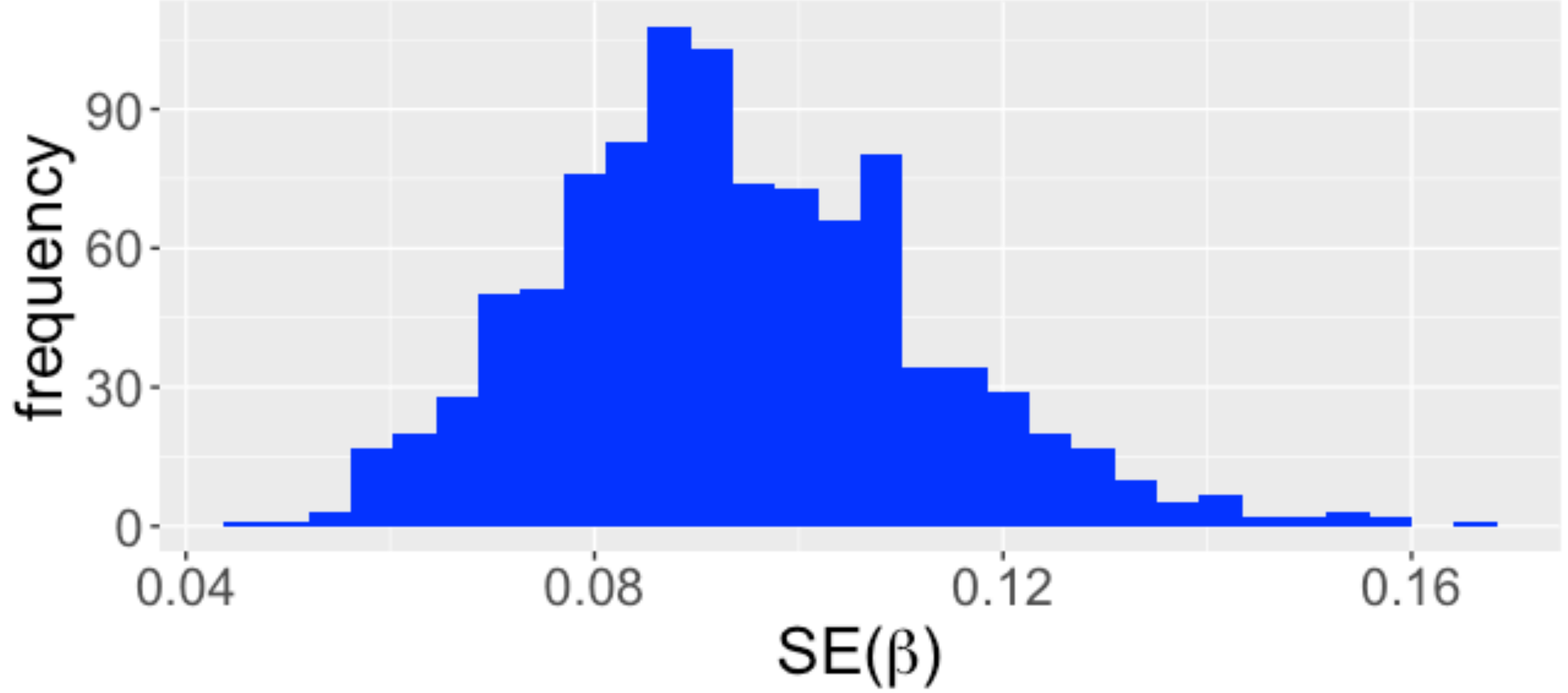
Relative standard error bias:

$$\sigma_{bias} = \frac{\hat{\sigma}_\theta - \sigma_{\hat{\theta}}}{\sigma_{\hat{\theta}}} = -0.010$$

Coverage:

$$c = \frac{1}{m} I(\theta \neq 0) = 0.173$$





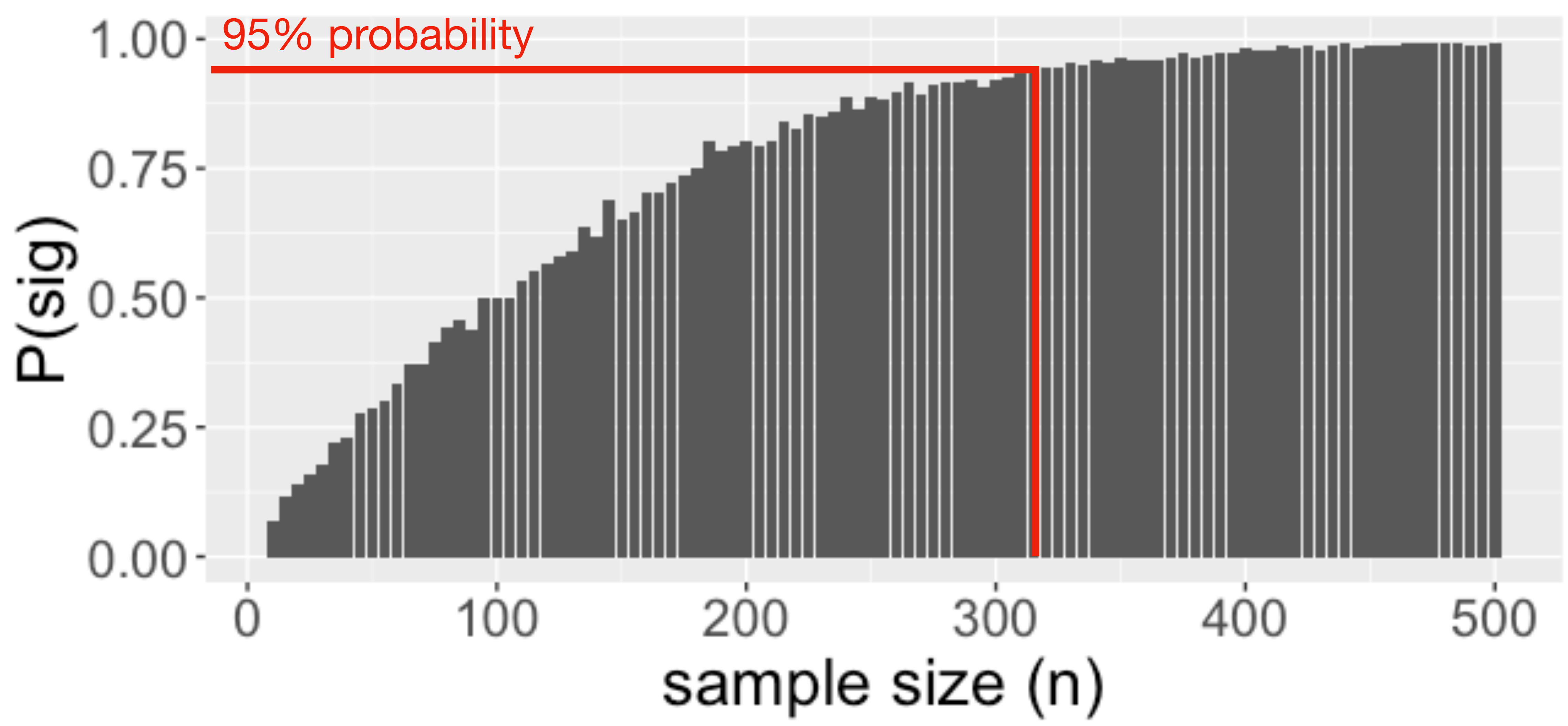
Example: power & accuracy

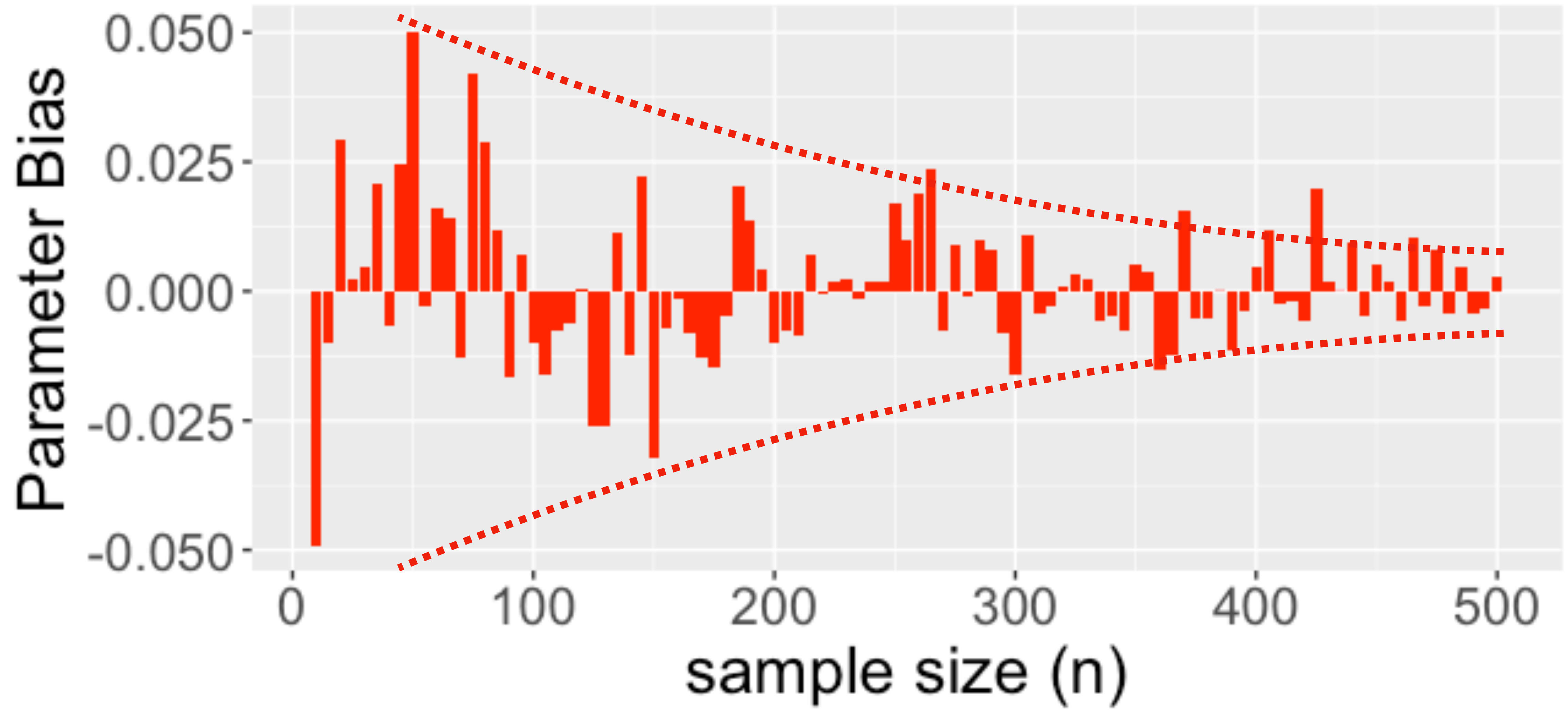
Goal: Determine the sample size (n) needed in order to:

1. Detect a significant effect of β_1 95% of the time.
2. Have a parameter bias $< |0.01|$

Approach: Repeat model sims from before, across a range of sample sizes (n).

$$n = \underbrace{10 - 500}_{range}$$





Take home message

- Because they rely on relatively few assumptions, Monte Carlo methods provide a robust and flexible way of estimating the statistical power of your model.