Mediation & moderation

Readings for today

- Preacher, K. J., & Hayes, A. F. (2008). Asymptotic and resampling strategies for assessing and comparing indirect effects in multiple mediator models. Behavior research methods, 40(3), 879-891.

Topics

1. Graphs
2. Moderation
3. Mediation

Graphs

Graphs

Statistical model: $$f(X) = Y$$

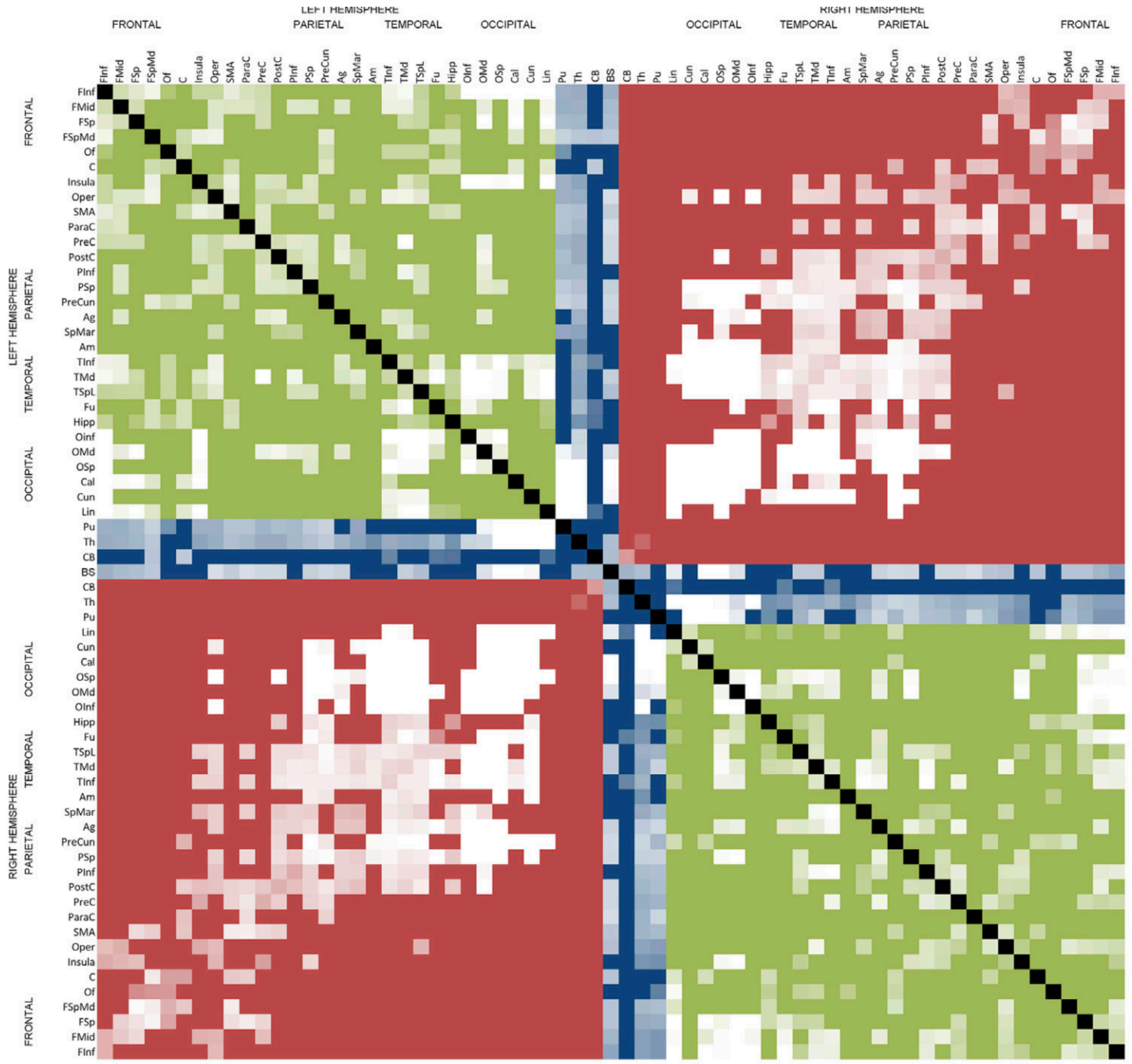
Graphical form: $$X \xrightarrow{f} Y$$

Nodes: The objects (variables)

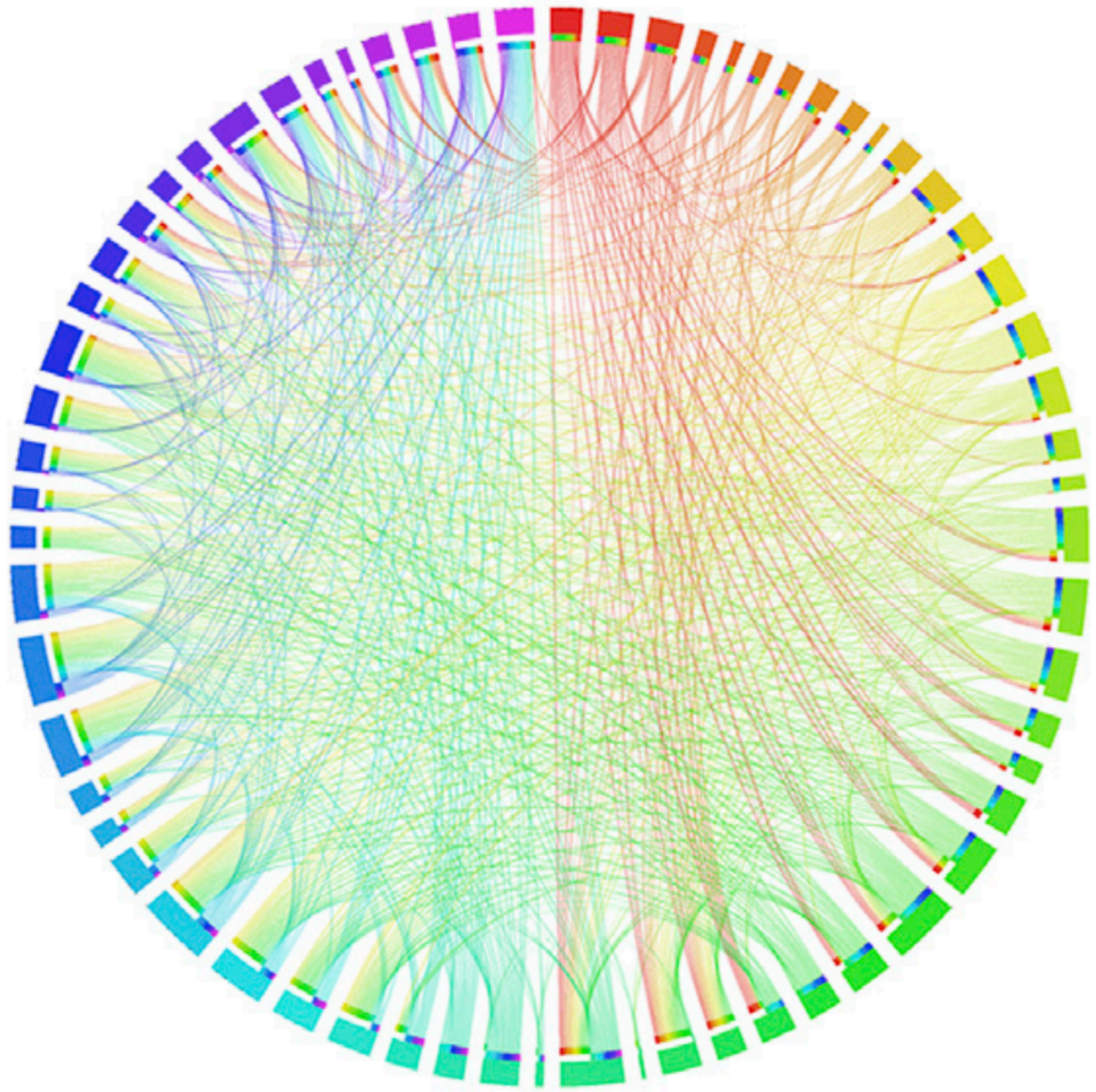
Edges: The connections (relations)

Graphs

Matrix form



Graphical form

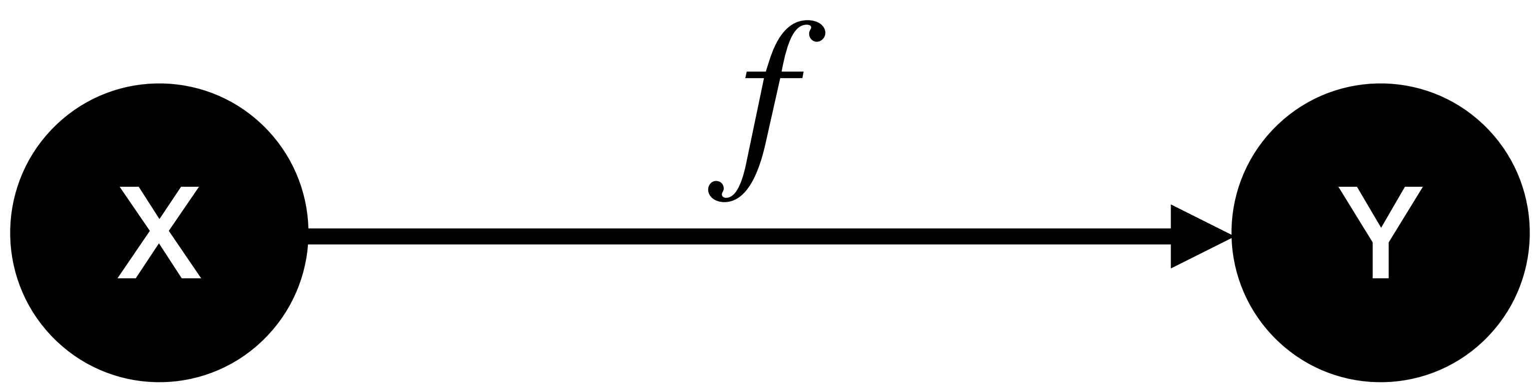


(Yeh et al. 2018)

Types of graphs

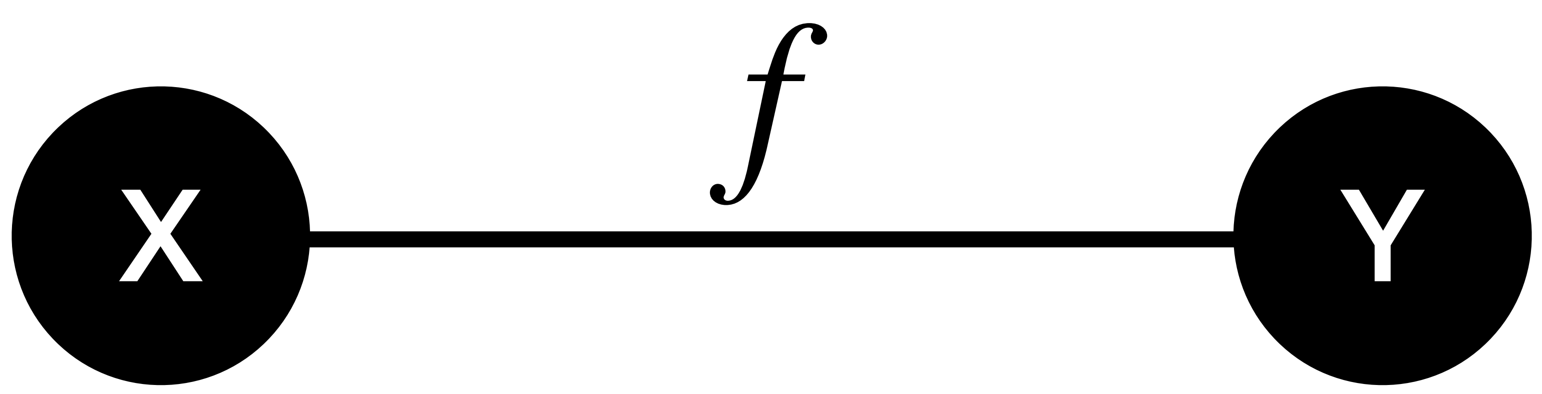
Directed graphs:

- “causal”
- regression



Undirected graphs:

- association
- correlation

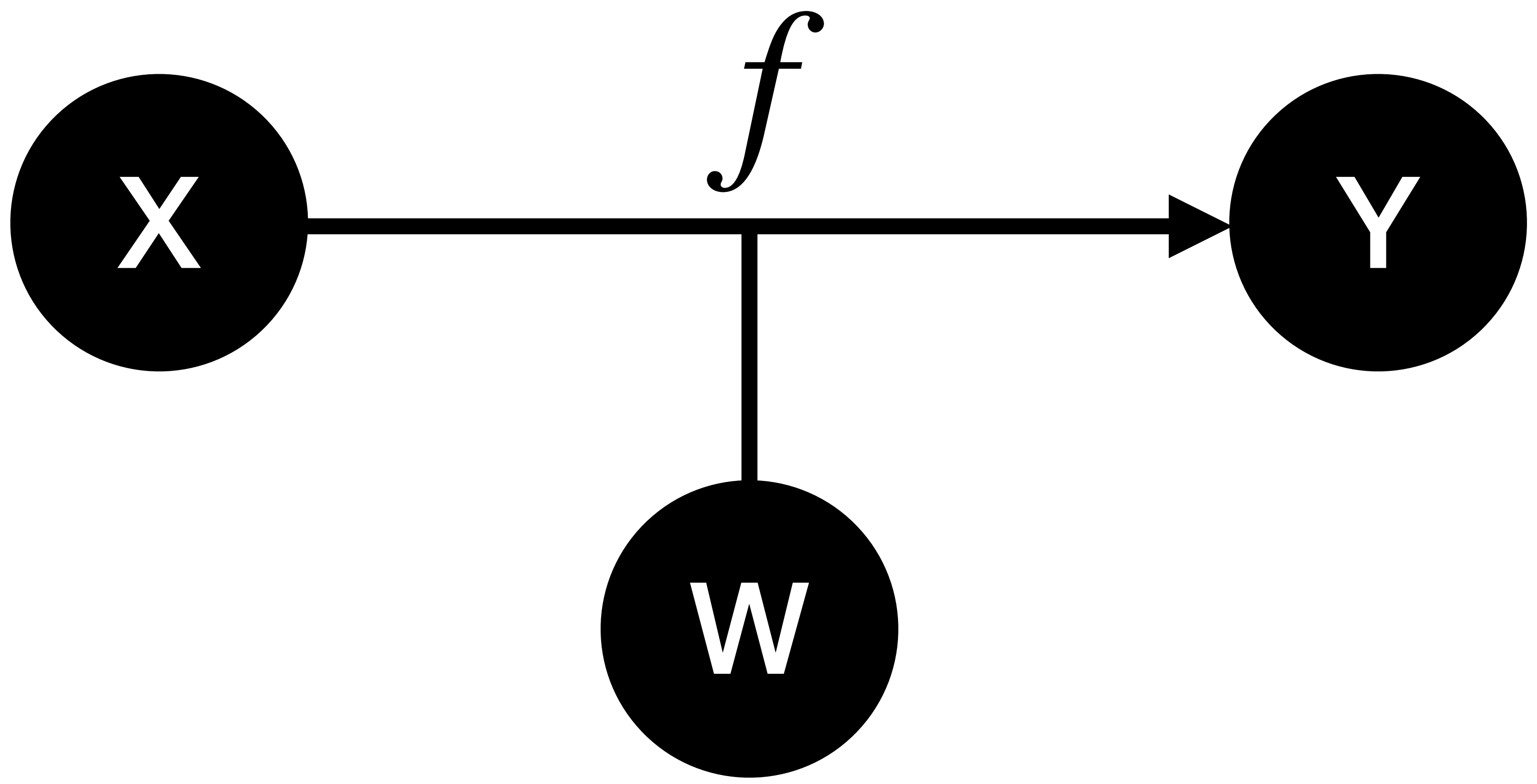


Utility

1. Visualization: Easily <u>see</u> the structure of relationships in the data.
2. Complexity: Captures the complex & hierarchical relationships in the data.

Moderation

Moderation models



X: Predictor (independent) variable(s).

Y: Response (dependent) variable(s).

W: Moderator variable(s).

$$Y = \hat{\beta}_0 + \hat{\beta}_1 X + \hat{\beta}_2 \underbrace{W}_{\text{moderating variable}} + \underbrace{\hat{\beta}_3}_{\text{moderating effect}} XW$$

Interpretation

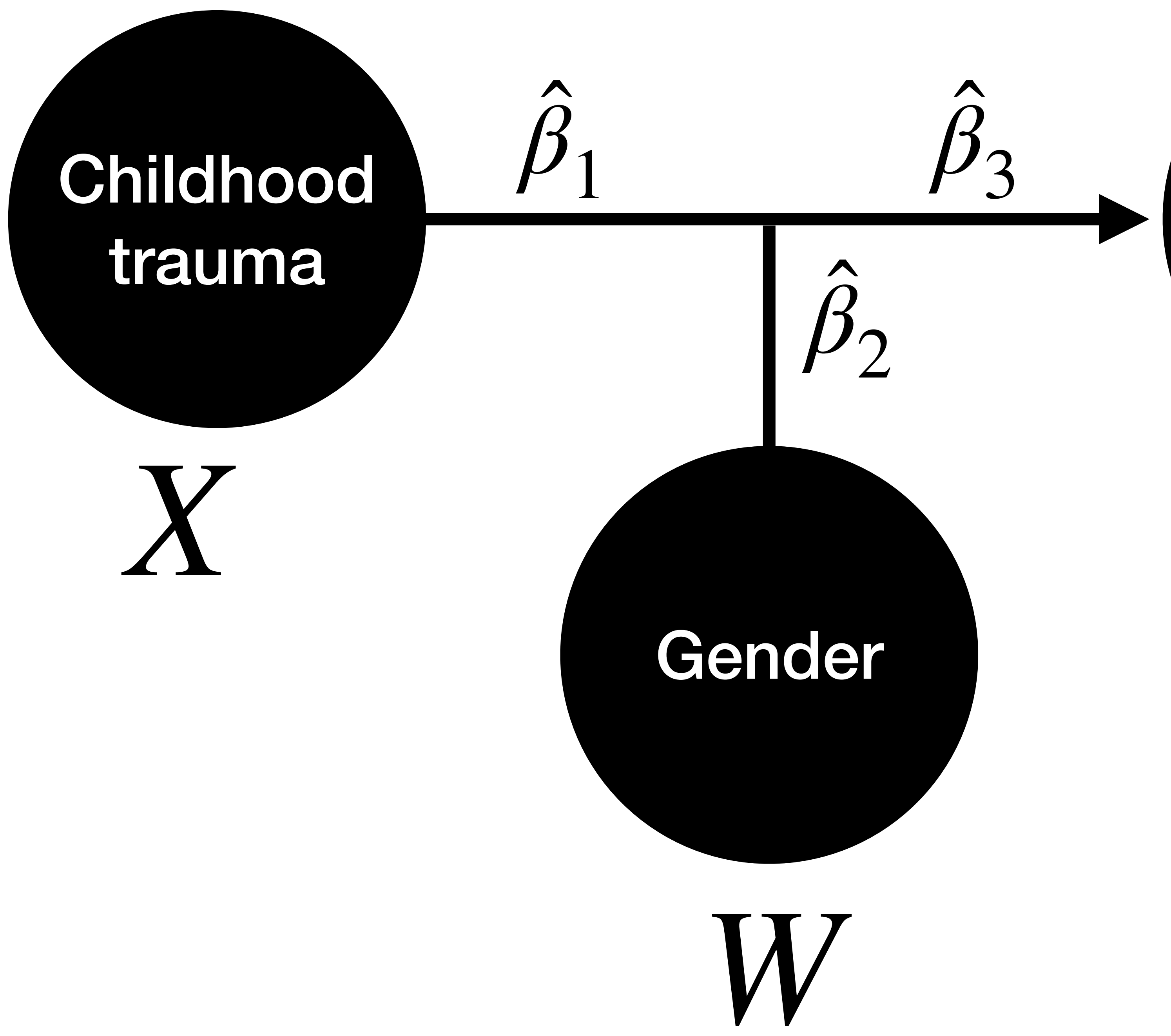
$\hat{\beta}_1$: Units that Y changes with X.

$\hat{\beta}_2$: Units that Y changes with W.

$\hat{\beta}_3$: Units that Y changes with X contingent on changes in W.

Example: moderation

Q: Is the effect of childhood trauma on risk aversion moderated by gender?

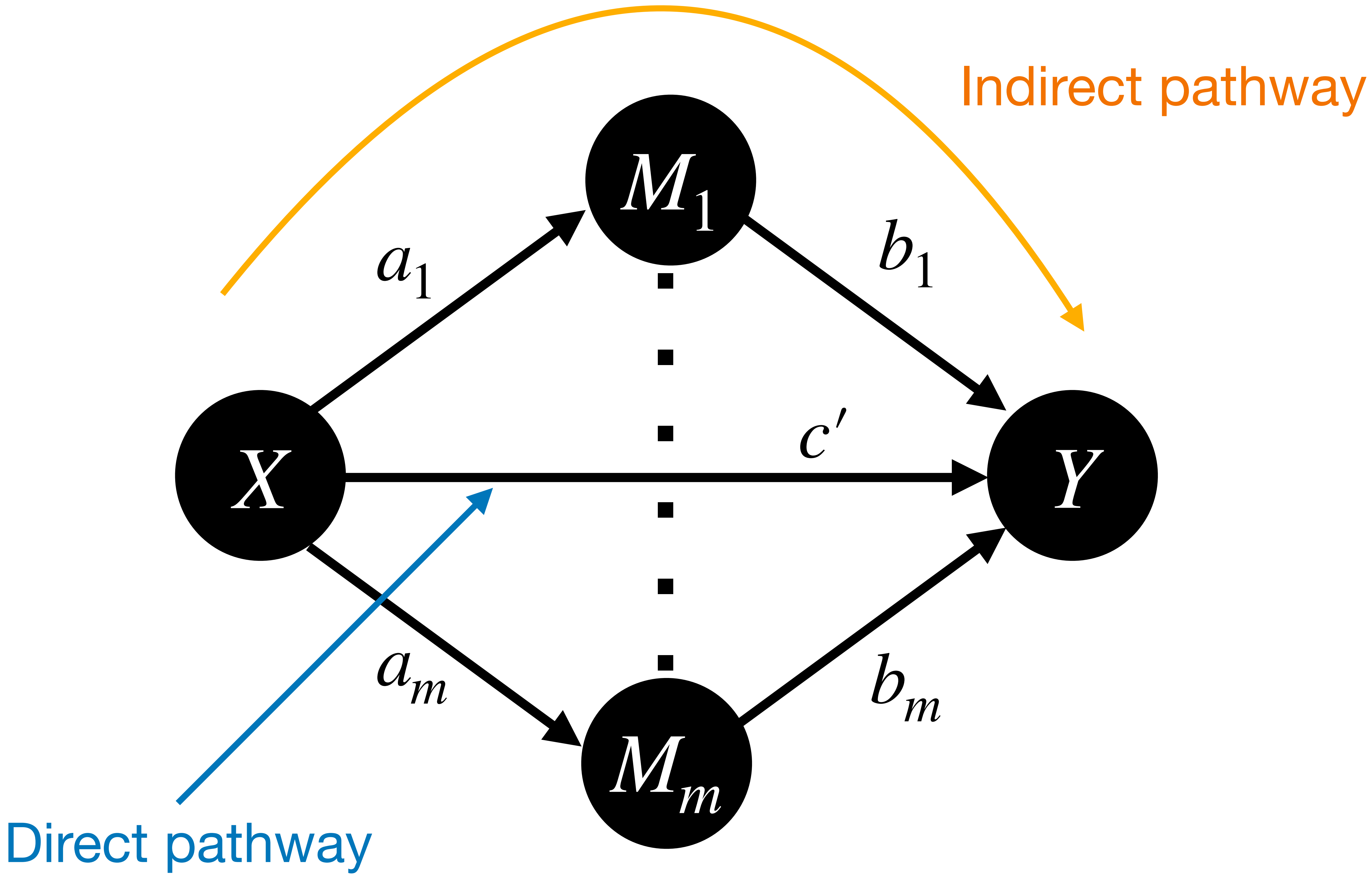


Model

$$Y_{risk} = \hat{\beta}_0 + \hat{\beta}_1 X_{CT} + \hat{\beta}_2 W_{gender} + \hat{\beta}_3 X_{CT} W_{gender}$$

Mediation

Mediation models



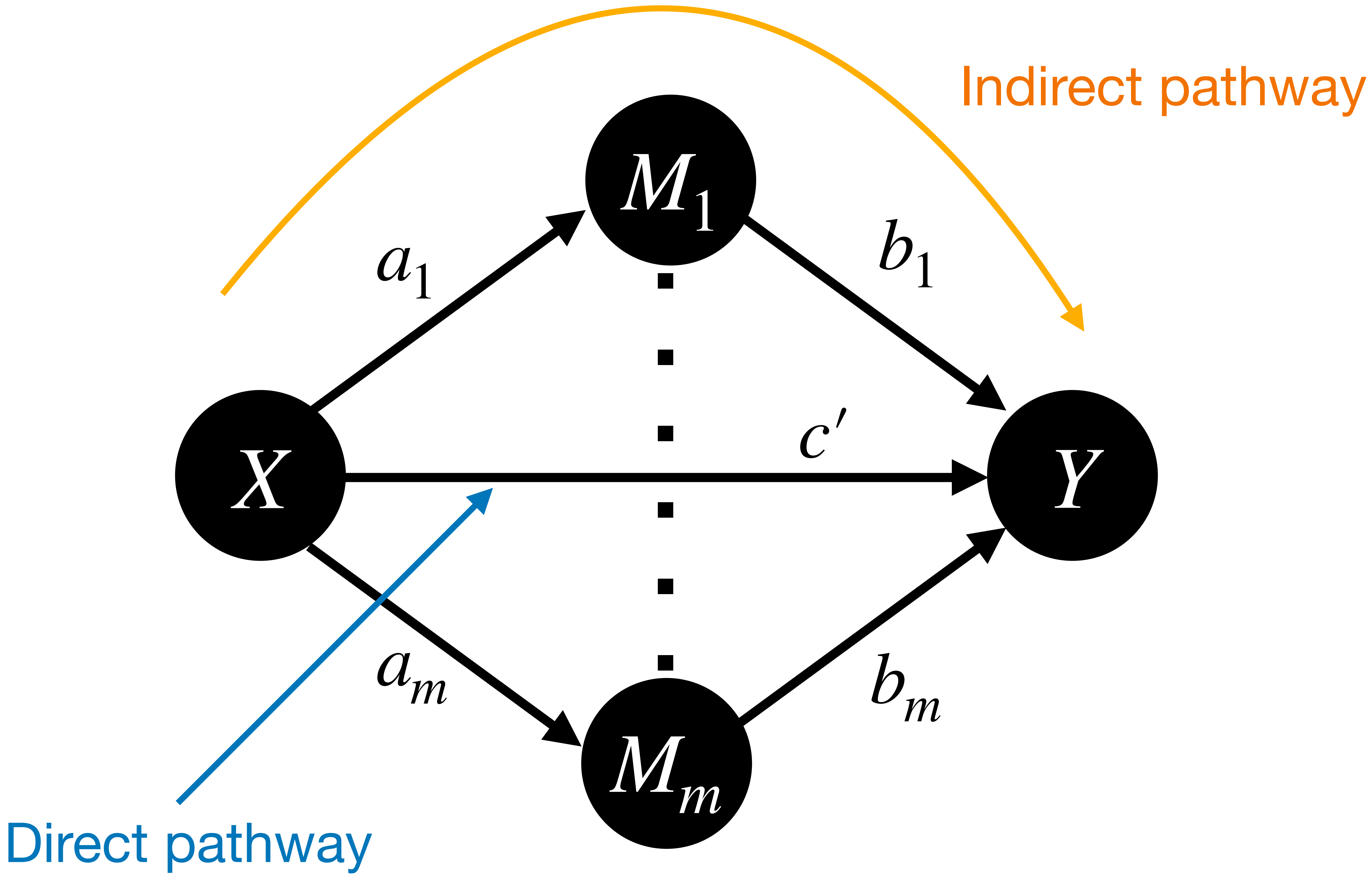
X: Predictor (independent) variable(s).

Y: Response (dependent) variable(s).

M_i: Mediating variable *i*.

(Preacher & Hayes 2008)

Mediation models



Interpretation

a_i: Influence of X on M_i.

b_i: Influence of M_i on Y.

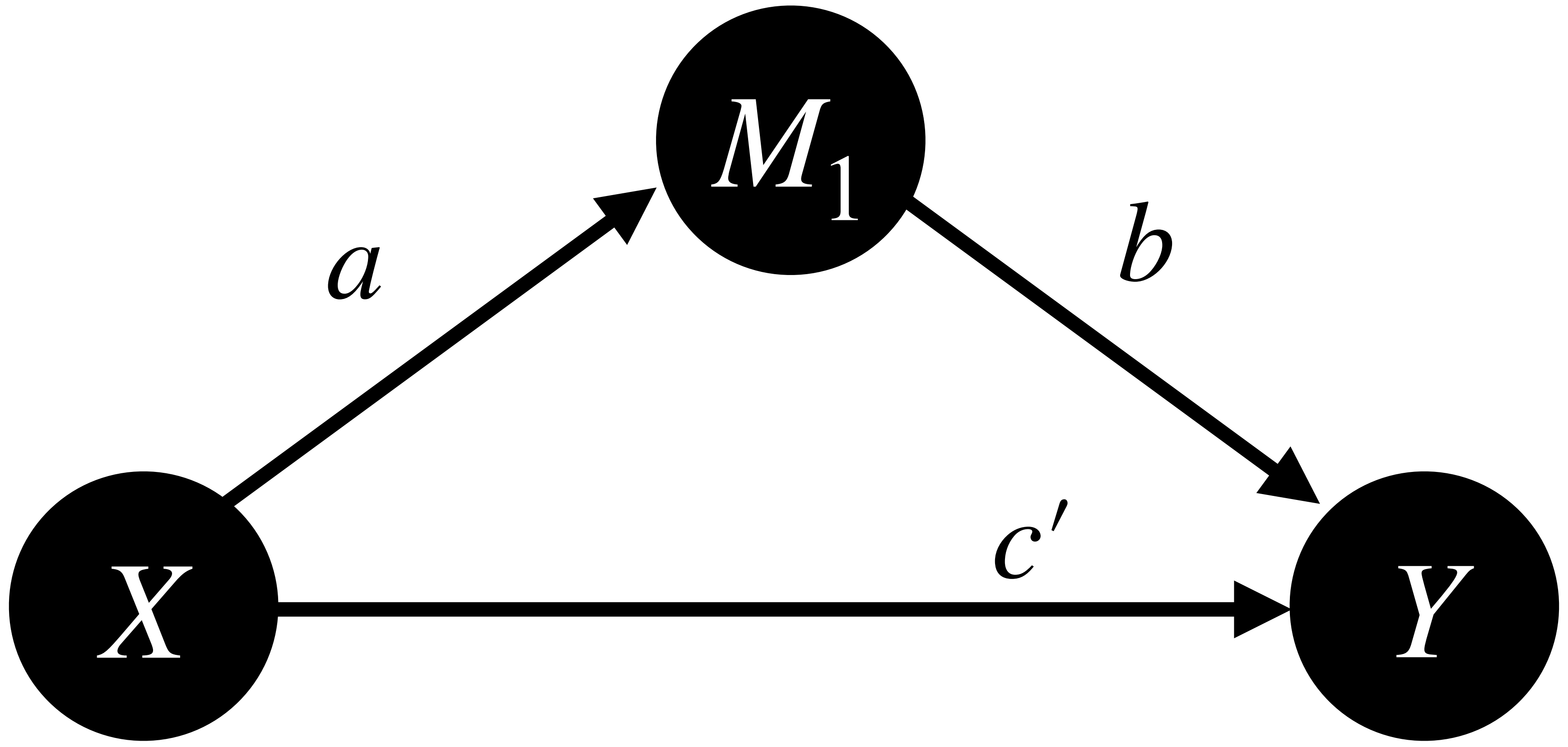
$a_i b_i$: Indirect influence of X on Y via its influence on M_i.

c': Direct influence of X on Y after accounting for all indirect effects (i.e,, all $M_1 \rightarrow M_m$).

c: Total influence of X on Y without accounting for indirect effects.

(Preacher & Hayes 2008)

Estimating mediation effects



H_0: $ab = 0$

Evaluate: Bootstrapping

$$95\,\%\,CI = E[\hat{a}\hat{b}] \pm 1.96\sigma_{bootstrap}$$

3 regression models

1. $Y = cX$
2. $M = aX$
3. $Y = bM + c'X$

$$\begin{aligned} Y &= bM + c'X \\ &= b(aX) + c'X \\ &= \underbrace{abX}_{\text{indirect}} + \underbrace{c'X}_{\text{direct}} \end{aligned}$$

(Preacher & Hayes 2008)

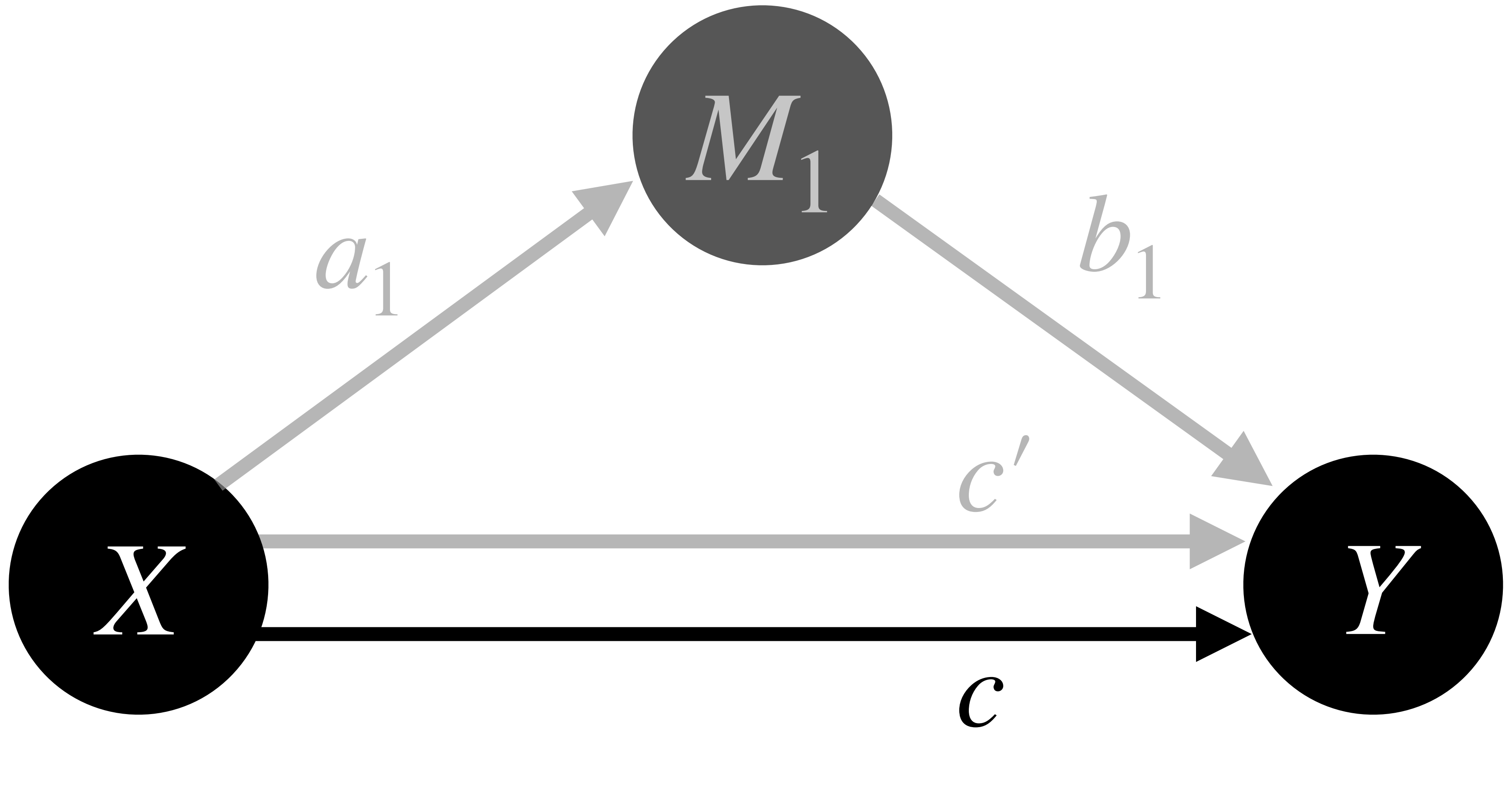
Assumptions

- Both $\hat{a}$ & $\hat{b}$ edges have to be $\neq 0$ for the indirect pathway to be evaluated (i.e., cannot infer indirect pathway effects if only one link is non-zero).
- Very sensitive to low statistical power ($\downarrow$ power $=$ $\uparrow$ false positive rate).

(Preacher & Hayes 2008)

Finding hidden relationships

Sometimes indirect pathways can hide total (c) pathway effects.



$$
\begin{aligned}
Y &= cX \\
&= b(aX) + c'X \\
&= \underbrace{(ab + c')}_{c} X
\end{aligned}
$$

<u>Hidden total path</u>

$$c = (ab + c') = 0$$

$$\frac{ab}{c'} = -1$$

When direct and indirect pathways have equally opposing influences.

(Preacher & Hayes 2008)

Multiple mediator models

<u>Q:</u> Is the effect of childhood trauma on risk aversion mediated by parental income, psychiatric risk, & social network size.

X: childhood trauma

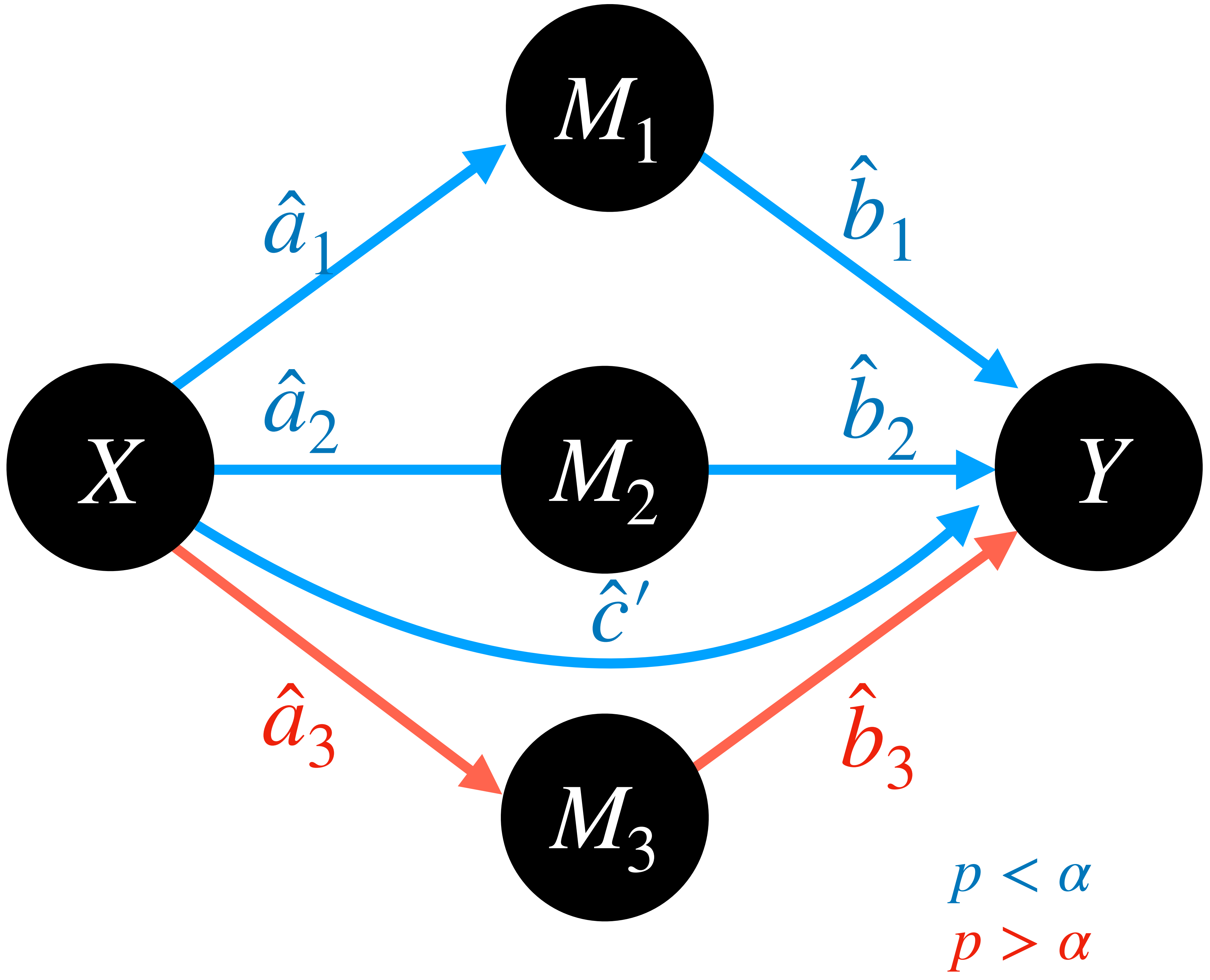
Y: risk aversion

M_1: parental income

M_2: psychiatric risk

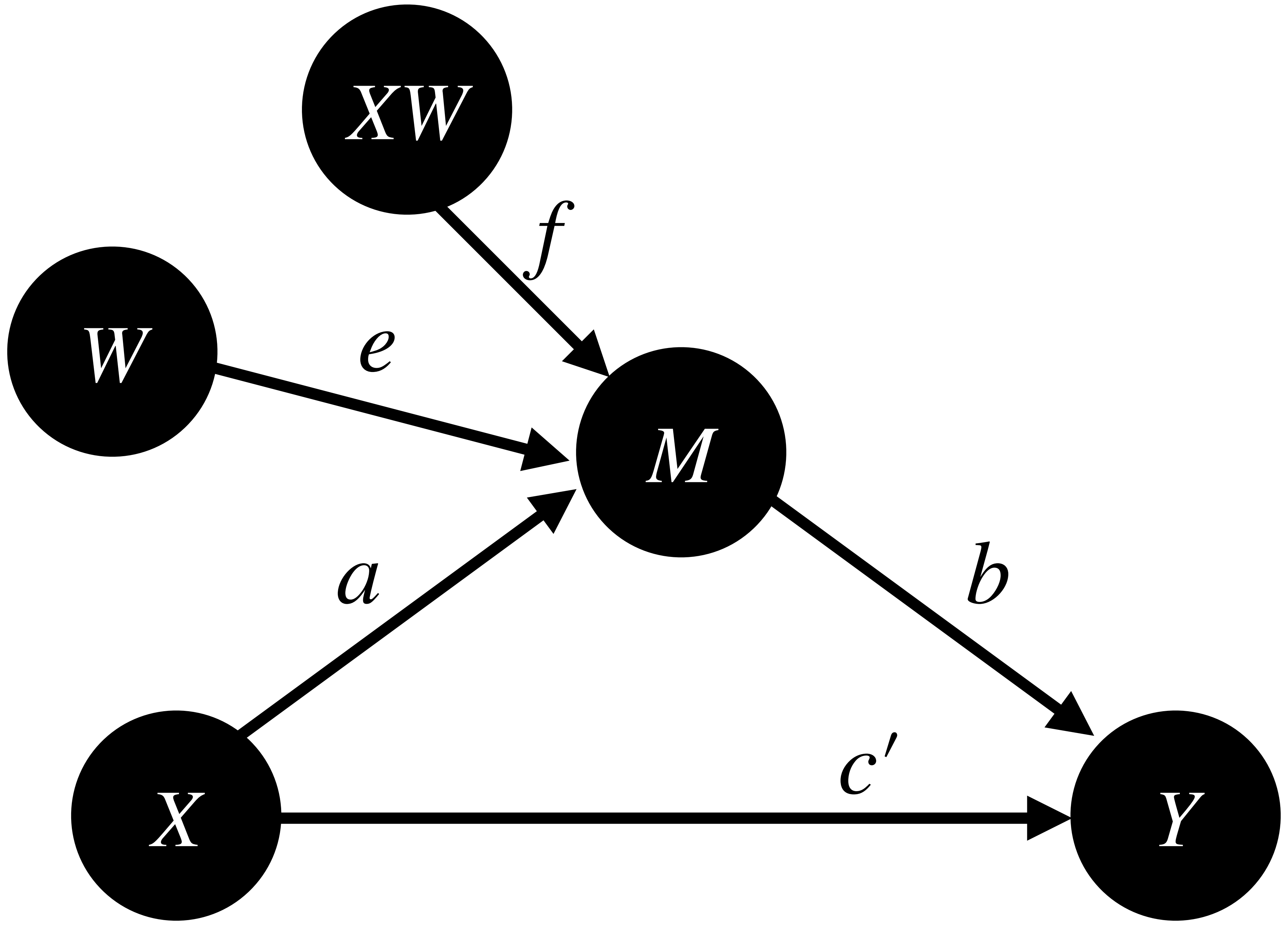
M_3: social network size

<u>Model:</u> $Y = \sum_{i=1}^{p} \hat{a}_i \hat{b}_i X_i + \hat{c}' X$



(Preacher & Hayes 2008)

Moderated mediation models



Does W moderate the indirect relationship between X and Y via M?

<u>Moderated mediation</u>

$$M = aX + eW + fXW$$

a, e: main effects

f: interaction

<u>Full Model</u>

$$Y = \hat{b}M + \hat{c}'X$$
$$= b(aX + eW + fXW) + c'X$$

(Preacher & Hayes 2008)

Take home message

- Representing relations as graphs provides an intuitive understanding of complex relationships.
- Moderation and mediation models allow for capturing relationships beyond first-order associations, even revealing hidden relationships in your data.