# Sample Size Determination for Regression Models Using Monte Carlo Methods in R

A. Alexander Beaujean

# Practical Assessment, Research & Evaluation

A peer-reviewed electronic journal.

## Sample Size Determination for Regression Models Using Monte Carlo Methods in R

A. Alexander Beaujean, *Baylor University*

A common question asked by researchers using regression models is, What sample size is needed for my study? While there are formulae to estimate sample sizes, their assumptions are often not met in the collected data. A more realistic approach to sample size determination requires more information such as the model of interest, strength of the relations among the variables, and any data quirks (e.g., missing data, variable distributions, variable reliability). Such information can only be incorporated into sample size determination methods that use Monte Carlo (MC) methods. The purpose of this article is to demonstrate how to use a MC study to decide on sample size for a regression analysis using both power and parameter accuracy perspectives. Using multiple regression examples with and without data quirks, I demonstrate the MC analyses with the R statistical programming language.

A question posed in the design of many research studies is: *What sample size is needed*? Being able to answer this question is important because institutional research boards and most granting agencies require that investigators specify the size of the sample they intend to collect. In addition, most reporting guidelines for education, psychology, and health professions research require authors to state how they determined their sample size (e.g., American Educational Research Association, 2006; American Psychological Association Publications and Communications Board Working Group on Journal Article Reporting Standards, 2008; Moher et al., 2010). More practically, conducting a study with the wrong sample size can be costly–having too few participants results in the inability to find effects or precisely estimate their values, while having too many participants results in wasting the investigators' valuable resources.

Typically investigators determine the needed sample size via some table or formulae in a textbook (e.g., Murphy & Myors, 1998), or by using specifically-designed software (e.g., Faul, Erdfelder, Lang, & Buchner, 2007). While this approach can be useful for simple projects, the assumptions used in these calculations often do not hold in the actual data. An alternative approach to determining the required sample size is to use a Monte Carlo (MC) study.

MC studies use random sampling techniques, typically done through computer simulation, to build data distributions (Beasley & Rodgers, 2012). Researchers often use them as an empirical alternative to solve problems that are too difficult to solve through statistical or mathematical theory (Fan, 2012). There are a variety of uses for MC studies, ranging from understanding statistics with unknown sampling distributions to evaluating the performance of a statistical technique with data that do not meet the technique's assumptions.

Previously, Muthén and Muthén (2002) showed how MC methods can be useful to determine sample size for a structural equation model (SEM). While this approach has been praised (Barrett, 2007), it requires using specialized proprietary software and has not been readily accessible to a wide audience. Further, while regression is a specific type of

SEM (Hoyle & Smith, 1994), scholars who rely on regression analysis might not understand how they are

related. Thus, they may have ignored the MC approach to sample size determination. Consequently, there is a need to show how to use MC methods, using freely accessible software, to determine the needed sample size for use with regression models.

The purpose of this article is to demonstrate the use of a MC study to determine the required sample size for a multiple regression analysis. I demonstrate such analyses using the **R** (**R** Development Core Team, 2014) statistical programing language, which is open source, available for multiple operating systems, has extensive data simulation facilities, and has great flexibility that is unmatched by most other statistics programs (Kelley, Lai, & Wu, 2008).

## Power Analysis

Power analysis was developed concurrently with null hypothesis significance testing (NHST), although it wasn't until Jacob Cohen's work in the 1960s that it became popular (Descôteaux, 2007). NHST pits two competing hypotheses against each other: the null ($H_0$) and the alternative ($H_a$). When used for power analysis, $H_0$ is usually specified to be that the parameter of interest equals zero, while $H_a$ is specified to be that the parameter does not equal zero. The needed sample size in this scenario refers to the number of observations required to reject $H_0$.

Power analysis involves four interrelated concepts:

a)   sample size;
b)   type 1 error ($\alpha$);
c)   type 2 error ($\beta$) or statistical power ($1-\beta$); and
d)   effect size (Cohen, 1988).

The concepts are deterministically related to each other, meaning that if three are known, so is the fourth. Thus, providing values for type 1 error, type 2 error (or power), and the effect size will provide the needed sample size.

Type 1 and type 2 error values are relatively straightforward to provide, but an effect size (ES) is more difficult to specify (Cohen, 1992). Not only are there different types of ESs that use different metrics and are only useful with certain kinds of data (Grissom & Kim, 2005), but there is usually little knowledge of what constitutes a typical or clinically-relevant ES magnitude for a given field of study (Hill, Bloom, Black, & Lipsey, 2008). In regression, the ES measure is usually a regression coefficient or the amount of the variance the model explains of the outcome variable (i.e., $R^2$).

## Parameter Accuracy

Many scholars have sharply criticized NHST over the last two decades (Cumming, 2014; Wilkinson & American Psychological Association Science Directorate Task Force on Statistical Inference, 1999). More recently, scholars have begun placing the NHST-related power analysis procedure under scrutiny as well (Bacchetti, 2010, 2013). As an alternative to determining sample size through a power analysis is to determine it using accuracy in parameter estimation (AIPE; Kelley & Maxwell, 2003; Maxwell, Kelley, & Rausch, 2007). Although the two approaches are not mutually exclusive (Goodman & Berlin, 1994), their philosophies are very different. In the power analysis perspective, interest lies in having just enough accuracy so that the value of a parameter estimate is statistically different that zero (i.e., rejecting $H_0$). In the AIPE perspective, interest lies in the accuracy of a parameter's estimate, no matter if the estimate's value is zero or any other variable. Kelley and Maxwell (2003) argued that the AIPE approach leads to a better understanding of an effect than the power approach. As NHST is embedded in the power approach, the only new knowledge it provides is whether a parameter is different than zero. Obtaining sufficiently accurate parameter estimates, however, can lead to knowledge about the parameter's likely value.

The accuracy component in AIPE is defined as the discrepancy between a parameter's estimated value and its true value in the population (Hellmann & Fowler, 1999). It is measured by the mean square error of a parameter's estimator, which is comprised of two additive parts. The first is variance, the inverse of which is precision. The second part is bias. Thus, when a parameter estimator is unbiased, accuracy and precision are directly related to each other.

The square root of a parameter's variance is its standard error, which is used for creating a confidence interval (CI; Cumming & Finch, 2005). Consequently, one way to assess the accuracy of a parameter estimate is by examining the width, or half-width, of its CI. The half-width is the halved difference between the upper-bound and lower bound of the CI. The narrower the CI (i.e., the smaller the half-width), the more precise the parameter estimate and more certainty there is that

the observed parameter estimate closely approximates the corresponding population value.

### Traditional Methods for Estimating Power and Parameter Accuracy

Many have written about the methods involved in determining sample size for a regression analysis using traditional power analysis (Dupont & Plummer, 1998; Maxwell et al., 2007). It involves the following steps: (a) review similar studies to find their ES values; (b) determine the expected ES values for the current study; (c) set $\alpha$ and power at the desired values; and (d) calculate the sample size needed to find the expected effect is statistically significant at the given $\alpha$ level while retaining the desired amount of power (Cohen, 1992). This calculation can be done analytically or through computer programs designed for such analyses (for a list, see Kelley & Maxwell, 2012, p. 199).

Determining sample size for a regression using the AIPE perspective involves a similar set of steps: (a) determine the predictor and outcome variables; (b) review studies that used similar variables and find the values of the relations between the predictor and outcome variables as well as the relations among the predictor variables; (c) determine the expected variable relations for the current study; (d) set the desired half-width of the CI and confidence level (e.g., 95%, 90%); and (e) calculate the sample size needed to find the desired CI half-width for a given confidence level and set of variable relations. This calculation can be done manually or via a computer program (Kelley, 2007; Kelley & Maxwell, 2003).

Using either the power- or AIPE-based formulae and procedures to determine sample size can be useful for very simple situations, but has problems when it comes to more practical research situations (Bacchetti, 2013). For example, they typically assume there are no missing data and that the collected data will meet the assumptions for the statistical tests of interest—assumptions that are often not met. An alternative to the traditional formulae-based method is to use a MC study, which can estimate not only the required sample size from both the power and AIPE perspectives but also can incorporate data quirks such as missing values and assumption violations.

### Monte Carlo Methods for Determining Sample Size

Muthén and Muthén (2002) showed how MC methods can be useful for determining the sample size needed for SEMs based on a power analysis. Generally, the procedure they outlined requires simulating a large number (*m*) of samples, each of size *n*, from a population with hypothesized parameter values. The model of interest (e.g., regression) is then estimated for each of the m samples and the set of m parameter values and standard errors are then averaged. The required sample size is the smallest value of n that produces the desired power for the parameters of interest contingent on the simulated data meeting certain quality criteria, which I discuss in the subsequent section. Muthén and Muthén did not discuss parameter accuracy, but this can easily be incorporated by select the sample size based on the data having the desired CI half-width instead of having the desired power.

### Criteria to Determine Monte Carlo Study's Quality.

The following statistics can be useful to determine the quality of the simulated data in a MC study: (a) relative parameter estimate bias, (b) relative standard error bias, and (c) coverage. *Relative parameter estimate bias* is:

$$\theta_{bias} = \frac{\hat{\theta} - \theta_H}{\theta_H} \qquad (1)$$

where $\theta_H$ is the hypothesized (pre-set) value of the parameter, and $\hat{\theta}$ is the average parameter estimate from the m simulated samples. *Relative standard error* bias is:

$$\sigma_{bias} = \frac{\hat{\sigma}_\theta - \sigma_{\hat{\theta}}}{\sigma_{\hat{\theta}}} \qquad (2)$$

where $\sigma_{\hat{\theta}}$ is the standard deviation of the m parameter estimates, and $\hat{\sigma}_\theta$ is the average of the m estimated standard errors for the parameter. *Coverage* is the percent of the *m* simulated samples for which the $(1-\alpha)$% CI contains $\theta$. Table 1 contains Muthén and Muthén's (2002) suggested criteria for these statistics. Once they are met, *power* is calculated as the proportion of the m simulated samples for which $H_0$ (i.e., $\theta = 0$) is rejected using the specified $\alpha$ level.

Beaujean, Monte Carlo Sample Size Determination

Table 1. Criteria for Monte Carlo Data Quality

| Statistic | Criteria |
|---|---|
| Coverage | Between .91 and .98 |
| Relative parameter bias | Absolute value ≤ .10 for *all* model parameters |
| Relative standard error bias | Absolute value ≤ .10 for *all* model parameters |
|  | Absolute value ≤ .05 for the parameters of major interest |

*Note.* Taken from Muthén & Muthén (2002, pp. 605-606).

### Decisions to Make in a Monte Carlo Sample Size Study.

Figure 1 contains the required steps for using a MC study to determine the needed sample size for a regression analysis. Before simulating the data (Step 5), a number of decisions need to be made. First, determine the regression model to study, which includes all pertinent predictor variables as well as the nature of the associations between the predictor variables and outcome variable. Creating a detailed path diagram can greatly facilitate this step (Boker & McArdle, 2005).

Second, decide on population values for every parameter. This includes the regression coefficients, the scale and reliability of all variables, the amount of residual variance, and the covariances among the predictor variables. This step will typically be easier if the variables are standardized, as this makes the covariances become correlations, the regression coefficients become standardized, and the intercept become zero. After determining the parameter values, it is important to check that the implied covariance matrix values (i.e., the covariances based on the population parameter values) are as expected. As with the first step, path diagrams can be very helpful here as well.

Inherent in the second step is the decision on the ES value. That is, by specifying the values for the

1. Decide on regression model.
    1.1. Draw a path diagram of the model to account for all intended relationships (optional).
2  Decide on population values for *all* parameters in model, including: regression coefficients, scale and reliability of the variables, the amount of residual variance ($1 - R^2$), and covariance among the predictor variables. Standardizing the variables makes this step easier.
    2.1. Check values of the *implied* covariance matrix to make sure they are as expected.
3. Decide on any data quirks, such as missing values or assumption violations (optional).
4. Decide on the technical aspects of the MC simulations:
    4.1. Type 1 error rate ($\alpha$), which also determines the CI.
    4.2. Desired power ($1-\beta$) or confidence interval half-width.
    4.3. Number of samples to simulate (*m*).
    4.4. Sample size (*n*) or range of sample sizes.
    4.5. Random seeds (at least two).
5. Simulate the *m* samples of the regression model from *Step 2*.
6. In the simulated data, check (cf. Table 1):
    6.1. Relative parameter and standard error biases.
    6.2. Coverage.
7. If the values in *Step 6* are acceptable, examine the power or parameter accuracy of the parameters of interest. If the values are not high enough, increase *n* and repeat *Steps 5* and *6*.
8. Repeat *Steps 5 - 7* using a different random seed.
9. Compare results of simulated data from both random seeds.
    9.1. If they converge, no need for further simulations of current scenario.
    9.2. If they do not converge, repeat *Steps 5 - 8* using different random seeds or larger values of *m*.

Figure 1. Steps for sample size planning of a regression analysis using a Monte Carlo study.

regression coefficients and the predictor variables' covariance, $R^2$ (or any other regression ES measure) is already determined. I elaborate on these relations more in the first example.

The third step is to decide on any data quirks, such as having missing values or violating any assumptions. This step is optional as its usefulness depends on the variables and population from which the data will be collected. The fourth step requires decisions about values for the technical aspects of the MC simulations. This includes: $\alpha$, power, the number of samples to simulate ($m$), the sample sizes ($n$), and the random seeds. The value for m should be large, as the goal is to produce stable results and large values for m tend to produce quality simulations. Muthén and Muthén (2002) suggested setting m to 10,000, but this may be excessive for simpler regression models with no data quirks (Skrondal, 2000). The initial $n$ to use is somewhat arbitrary. If there is no reason to select one specific value, then it might be better to decrease $m$ and simulate samples for a sequence of $n$s.

The random seed is an integer used to initialize the pseudo-random number generation for the simulations (Marsaglia, 2003). A given seed value generates the same sequence of numbers, so using the same seed value will simulate the exact same data while using different seed values will simulate different data. Using different seed values is comparable to taking different independent samples from the same population. At a minimum, the MC study should be done at least twice using two different, randomly selected seed values. The results from the two different simulations should converge—that is, they should both point towards using roughly the same sample size. If that is the case, then there is no need for further MC simulations of that particular scenario. Otherwise, additional simulations using additional seeds may be needed.

### Presentation of Following Material

In what follows, I present two examples of the MC method for determining sample size for regression analysis using **R**. The first example is a typical multiple regression model. In the second example, I extend the first example by adding data quirks involving: (a) missing data, (b) the outcome variable's distribution, and (c) variable reliability. As I discuss a given analysis, I present **R** syntax to conduct the analysis in a separate text box for one random seed. The words in gray

following a pound sign (#) in the syntax are comments, so **R** ignores them. For those with no previous experience using **R**, Venables et al., (2012) provide a good introduction.

## Example 1: Multiple Regression

### Background

A typical regression power analysis involves examining a model's $R^2$ value or a change in $R^2$ from one model to another.[1] The MC method requires more information as it needs values for the relations between the outcome variable and all the predictor variables as well as the relations among all the predictors. Once those are specified, then the model's $R^2$ can be calculated using Equation 3. If the p predictor variables and single outcome variable are mean-centered, then

$$R^2 = \frac{\mathbf{b}'_{YX}\mathbf{V}_{XX}\mathbf{b}_{YX}}{\sigma_Y^2} = \boldsymbol{\rho}'_{YX}\mathbf{C}_{XX}^{-1}\boldsymbol{\rho}_{YX} \tag{3}$$

where $\boldsymbol{\rho}_{YX}$ is the $p \times 1$ column vector of correlations between each of the predictor variables and the outcome, $\mathbf{b}_{YX}$ is the $p \times 1$ column vector of regression coefficients of each of the predictor variables, $\mathbf{C}_{XX}$ and $\mathbf{V}_{XX}$ are the $p \times p$ correlation and covariance matrices, respectively, of the predictor variables, and $\sigma_Y^2$ is the variance of the outcome (Christensen, 2002). A little manipulation of Equation 3 reveals that the set of standardized regression coefficients, $\mathbf{b}^*$, can be estimated by

$$\mathbf{b}^* = \mathbf{C}_{XX}^{-1}\boldsymbol{\rho}_{XX} \tag{4}$$

### Regression Model

Kelley and Maxwell (2003) presented an example of a sample size study for a regression model with three predictor variables. The predictor variables' correlations with each other, $\mathbf{R}_{XX}$, as well as the correlations between each of the predictor variables and the outcome, $\boldsymbol{\rho}_{YX}$, are:

$$\mathrm{R_{XX}} = \begin{bmatrix} 1.00 & .40 & .60 \\ .40 & 1.00 & .05 \\ .60 & .05 & 1.00 \end{bmatrix} \text{ and } \rho_{YX} = \begin{bmatrix} .50 \\ .30 \\ .10 \end{bmatrix}$$

To calculate the $R^2$ value, plug the values into Equation 3; likewise, to calculate the standardized regression coefficients, plug in the known values into

---

[1] Cohen (1988) used the $f^2$ value, but it is just a transformation of $R^2$

Equation 4. The values for the three regression coefficients, respectively, are 0.66, 0.05, and –0.30.

```
# correlations between predictors and outcome
xy <- c(0.5, 0.3, 0.1)
# correlation matrix among predictors
C <- matrix(c(1, 0.4, 0.6, 0.4, 1, 0.05, 0.6, 0.05, 1),
    ncol = 3)
# R2
R2 <- t(xy) %*% solve(C) %*% xy
# standardized regression coefficients
b <- solve(C) %*% xy
```

Simulating data for regression models with multiple predictors can be tricky, as it has to account for the relationships among all the variables. Using path diagrams eases this process, as proper diagrams show all the model parameters. A path diagram for Kelley and Maxwell's (2003) example with the parameter values is in Figure 2.
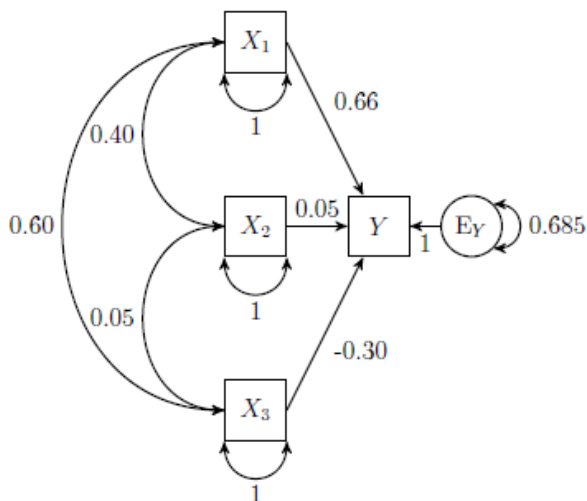


Figure 2. Path model of multiple regression for Example 1.

Another advantage of using path diagrams is that they can facilitate specifying the regression model in **R**, as the *lavaan* (Rosseel, 2012) package uses path models for input. The *lavaan* operators for specifying path models are given in Table 2. Beaujean (2014) contains some worked examples of regression models using *lavaan*.

The following syntax specifies the regression model (i.e., Figure 2) in *lavaan* using the known values.

```
# load lavaan
library( lavaan)
# specify regression model with population values
```

```
pop.model<-'
# regression model
y ~  0.66*x1 + 0.05*x2 + -0.30*x3
# predictor variable correlations
x1~~0.40*x2 + 0.60*x3
x2~~0.05*x3
# residual variance
y ~~ 0.6854*y
'
```

Table 2. *lavaan* Operators for Specifying Path Models.

| Syntax | Command | Example |
|---|---|---|
| ~ | Regress onto | Regress B onto A: B ~A |
| ~~ | (Co)varaince | Variance of A: A ~~A |
|  |  | Covariance of A and B: A ~~B |
| ~1 | Constant/mean/ intercept | Regress B onto A, and include the intercept in the model: B ~A B ~1 or B ~1 + A |
| =~ | Define reflective latent variable | Define Factor 1 by A-D: F1 =~A+B+C+D |
| * | Label or constrain parameters (the label/constraint has to be pre-multiplied) | Label the regression of Z onto X as *b*: Z ~b*X Make the regression coefficient 0.30: Z ~.30*X |

After specifying the regression model and deciding on the population values, Step 2.1. requires checking to make sure the specified values produce the correct results. This can be done in **R** by estimating the model using the specified parameter values and examining the results. To do this in *lavaan*, use the sem() function with the fixed.x=FALSE argument. The fixed.x=FALSE argument is required when fixing a predictor variable's variance or covariance. The fitted() function returns the model-based means and covariances (i.e., those implied by using the fixed parameter values), while the cov2cor() converts a covariance matrix to a correlation matrix. As I used standardized values for the parameters, the covariance and correlation matrices are identical for this example.

Beaujean, Monte Carlo Sample Size Determination

```
# check model parameters
pop.fit <- sem(pop.model, fixed.x = FALSE)
summary(pop.fit, standardized = TRUE, rsquare =
    TRUE)
# model implied covariances
pop.cov <- fitted(pop.fit)$cov
# model implied correlations
Cov2cor(pop.cov)
```

The resulting model-implied correlations are the same as the values given by Kelley and Maxwell (2003), indicating that the regression parameters are specified correctly for the MC simulations. As I do not have any data quirks in this initial example, the next step is to decide on the technical aspects of the MC simulations. Kelley and Maxwell (2003) wrote that with $n = 237$, all the 95% CI half-widths will be $\leq 0.15$. Thus, I set the following: (a) $\alpha$ : .05; (b) CI half-width : $\leq 0.15$; (c) $n$ : 237; (d) random seeds : 565 and 54447; and (e) $m$ : 500. I selected a relatively small number for m as this model is not very complex.

Because *lavaan* cannot run the MC study directly, I use the *simsem* package (Pornprasertmanit, Miller, & Schoemann, 2012) for the simulations. This package is designed for MC studies of sample size and accepts *lavaan* model specification. All subsequent **R** syntax is for *simsem* functions.

Conducting a MC study in *simsem* requires specifying two models. The first generates the samples, and is the one I previously specified. The second model estimates parameters from the simulated samples. Typically, the second model will be the same as the first except it will not contain values for the parameters.

```
# multiple regression data analysis model
analysis.model <- ' y ~ x1 + x2 + x3
'
```

To simulate the data, use the sim() function. Its main arguments are: (a) the number of samples, m (nRep); (b) the data generating model (generate); (c) the model to analyze the data (model); (d) the sample size (n); (e) the *lavaan* function to use for the analysis (lavaanfun); and (f) the random seed (seed). The multicore argument is optional, but if set to TRUE then **R** will use multiple processors for the simulation. This can considerably lessen the time required to create the data.

```
# load simsem package
library(simsem)
# simulate data
analysis.237 <- sim(nRep = 500,
    model=analysis.model, n = 237,
    generate=pop.model, lavaanfun = "sem",
    seed=565, multicore=TRUE)
```

The summaryParam() function, using the detail=TRUE argument, returns the averaged values of interest from the simulated samples. Using the alpha = 0.05 argument makes all CIs set at 95%. In Table 3, I explain each of the output values.

Table 3 Returned values from *simsem*'s summaryParam() function.

| Name | Statistic |
| --- | --- |
| *Estimate.Average* | Average parameter estimate across all samples. |
| *Estimate.SD* | Standard Deviation of parameter estimates across all samples. |
| *Average..SE* | Average of parameter standard errors across all samples. |
| *Power..Not.equal.0.* | Power of parameter at given $\alpha$.[a] |
| *Std.Est* | Average standardized parameter estimate across all samples. |
| *Std.Est.SD* | Standard deviation of standardized parameter estimates across all samples. |
| *Average.Param* | Specified parameter value. |
| *Average.Bias* | The difference between average parameter estimate and specified parameter value. |
| *Coverage* | Coverage of parameter using $(1-\alpha)\%$ confidence intervals.[a] |
| *Rel.Bias* | Relative parameter bias. |
| *Std.Bias* | Standardized parameter bias $(\hat{\theta} - \theta_H)/\sigma_{\hat{\theta}}$ |
| *Rel.SE.Bias* | Relative standard error bias. |
| *Average.CI.Width* | Average $(1-\alpha)\%$ confidence interval width (*not* half-width).[a] |
| *SD.CI.Width* | Standard deviation of $(1-\alpha)\%$ confidence interval width.[a] |

Note. To produce all the statistics requires using the detail=TRUE argument.

[a] $\alpha$ = .05 by default, but can be changed using the alpha argument.

```
# return averaged results from simulated data
summaryParam(analysis.237, detail = TRUE, alpha
    = 0.05)
```

The values from the MC study are in the top of Table 4. The relative biases and coverage are within specified values. As expected, the 95% CI half-widths are all $\leq 0.15$. Power is $\geq .80$ for X1 and X3's regression coefficients, but for X2 it is only .12. This illustrates the difference between the power and AIPE approaches as parameter estimates can be accurate but not powerful, especially when they are very close to zero.

### Unknown Sample Size

If the sample size to use is unknown, then instead of giving a single value for the n argument give a range of values using the sequence function, seq(). For example, to examine power and accuracy for values from $n = 200$ to $n = 400$, increasing by increments of 25, use seq(200,400,25) for the n argument. This produces one simulation with $n = 200$, one with $n = 225$, and so forth. To increase *m* at each *n*, wrap the seq() function inside the replicate function, rep(). For example, rep(seq(200,400,25), 50) repeats the 200-400 sequence 50 times (i.e., $m = 50$). The goal here is not to meet the criteria in Table 1, but to hone in on plausible values of *n* using smaller values of *m*. After finding some possible values for *n*, complete the MC study with a single sample size and a much larger *m*.

```
# simulate data with sample sizes from 200-400
    increasing by 25 (m=50)
analysis.n <- sim(nRep = NULL,
    model=analysis.model,
n = rep(seq(200,400,25), 50), generate=pop.model,
    lavaanfun = "sem", seed=565,
    multicore=TRUE)
```

Saving the results from the multiple sample size simulations allows for the creation of both a power curve and an accuracy curve, which is a graph of the $(1-\alpha)\%$ CI width as a function of sample size. To create the the former, use the plotPower() function with the parameter of interest as the value for the powerParam argument and the $\alpha$ value as the value for the alpha argument. To crate the latter, use the plotCIwidth() function with the parameter of interest as the value for the targetParam argument and $1-\alpha$ as the

value for the **assurance** argument.[2] Power and accuracy curves using sample sizes spanning 200-400 for the $X_2 - Y$ relation are shown in Figures 3a and 3b.
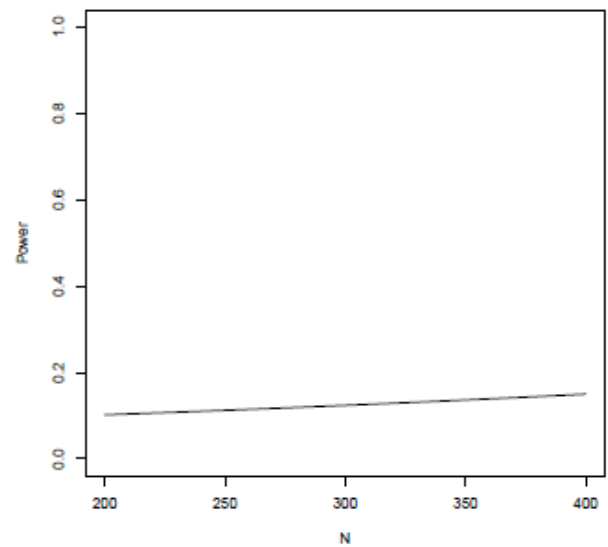


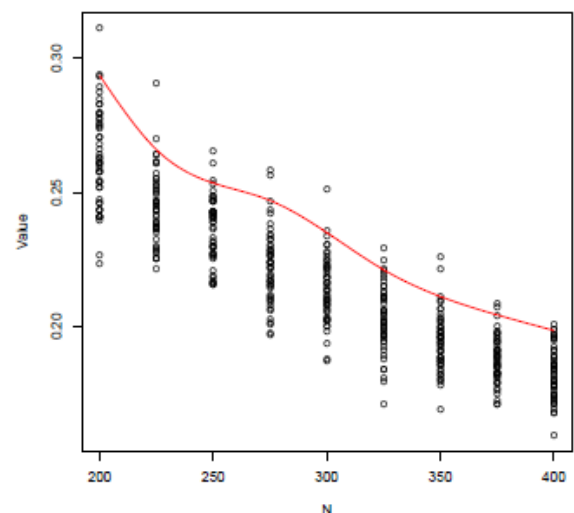Figure 3a. Power curve for Example 1 using $\alpha = .05$.



Figure 3b. Accuracy curve for Example 1 using $\alpha = .05$.

---

[2] Incorporating a level of *assurance* (i.e., probability) of the CI's half-width study is a different form of the AIPE perspective than I discuss in the current article. For more information about it, see Kelley and Maxwell (2008).

```
# power curve of the X2-Y relation
plotPower(analysis.n, powerParam = "y~x2", alpha
    = 0.05)
# accuracy curve of the X2-Y relation
plotCIwidth(analysis.n, c("y~x2"), assurance = 0.95)
```

An alternative to graphically displaying the results is to use the getPower(), findPower(), and getCIwidth() functions. The first and third functions return the power and CI widths, respectively, for each parameter at the specified sample sizes (nVal). The second function uses a getPower() object to find the sample size for a given level of power. If the findPower() function returns the values *Inf* or *NA*, it means the sample size values are too large or too small, respectively, for that parameter at the specified power level.

```
# find n for power of .80
power.n <- getPower(analysis.n, alpha=.05,
    nVal=200:300)
findPower(power.n, iv="N", power=0.80)
# find CI half-widths when n=200
getCIwidth(analysis.n, assurance = 0.95,
    nVal=200)/2
```

### Example 2: Multiple Regression With Data Quirks

For this second example, I add some quirks to the data from Example 2. I only present models that include one quirk, but combining multiple quirks in a single model is a simple extension.

#### Data Quirk 1: Missing Data

Missing data is often a problem in research, so conducting a sample size analysis without accounting for missing data is often unrealistic (Graham, 2009). For the current example, I made 20% of $X_2$'s data missing completely at random (MCAR), but $X_1$ missing values dependent on values of $X_3$. Specifically, when $X_3 = 0$, 15% of $X_1$'s data is missing; for each unit increase and decrease in $X_3$, the amount of missing data increases and decreases, respectively. As long as I include $X_3$ in the regression model, the missing values for $X_1$ are missing at random (MAR).

**simsem** has a variety of ways to include missing data in the simulations, most of which use the miss() function. I use the logit method because it has the ability to graph the amount of missing data. The logit method requires a *lavaan*-like script that specifies how

much data should be missing for a given variable. Each line of the script begins with a variable, then the regression symbol (~), and then values for the amount of missing data.

The values after the ~ are input for the inverse logit function:

$$\frac{\exp{(a + b_1 X_1 + b_2 X_2 + \ldots)}}{1 + \exp{(a + b_1 X_1 + b_2 X_2 + \ldots)}} \quad (5)$$

where *a* is the intercept, the *b*s are slope values, and the *X*s are predictor variables in the regression. For example, if $a = -1.38$ and there are no predictors, then the inverse logit value is 0.20, so approximately 20% of the values should be missing. Likewise, if $a = -1.73$ and $b = 0.25$, then approximately 15% of the values are missing when $X = 0$, 19% of the values when $X = 1$, and so forth.

```
# specify amount of missing data using logit
    method
pcnt.missing <- '
# 20% of data missing
x2 ~ -1.38
# 15% of X1 data is missing when X3 is zero
x1 ~ -1.73 + 0.25*x3
'
```

To plot the amount of missing data specified in the logit equations, use the plotLogitMiss() function. The plot for the current example is shown in Figure 4.
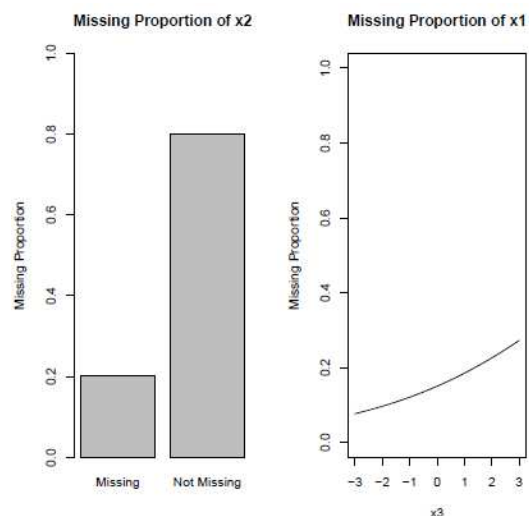


Figure 4 . Plot of missing data to include in the simulated datasets for $X_2$ and $X_1$.

```
# plot amount of missing data specified in the logit
    equations
plotLogitMiss(pcnt.missing)
```

Having missing values requires determining how to estimate the parameters in the presence of this missingness. Only full information maximum likelihood (FIML) and multiple imputation (MI) are available estimation options in *simsem*, which can be used with or without auxiliary variables. For details on FIML, MI and auxiliary variables, see Enders (2011). By default, *simsem* uses FIML when there are missing values. To use MI requires two additional arguments: the number of imputations for each data set (*m*) and the **R** package to conduct the imputation (package)[3]. Currently, only function from the *mice* (van Buuren & Groothuis-Oudshoorn, 2011) and *Amelia II* (Honaker, King, & Blackwell, 2011) packages can be used for the imputation.

```
# FIML
missing.model.fiml <- miss(logit = pcnt.missing)
# MI
missing.model.mi <- miss(logit = pcnt.missing, m =
    10, package = "mice")
# simulate regression data with missing data using
    FIML
analysis.mis.237 <- sim(nRep=750,
    model=analysis.model, n=237,
    generate=pop.model, lavaanfun = "sem",
    miss=missing.model.fiml, seed=565,
    multicore=TRUE)
```

For the current example, I simulated the data with *n* = 237 using FIML to handle the missing data. As I included missing values, I increased *m* to 750. The results are given in Table 4. The relative bias and coverage values are within specified limits. Compared to initial model (Example 1), power decreases slightly for $X_2$'s and $X_3$'s regression coefficients and CI half-widths for all thee predictors increase.

$X_2$'s regression coefficient is the smallest in value. Thus, finding the sample size needed for it to be estimated with power of 0.80, would mean that all other regression coefficients would have at least a power of 0.80. To find the sample size needed for $X_2$'s regression coefficient to be estimated with power of 0.80, I use the same procedures described in finding an unknown sample size for Example 1. As data are missing, I specified the search to go from *n* = 200 to

---

[3] The m argument in the miss() function is not related to the number of simulated samples in the MC study, *m*.

Table 4. Values From Monte Carlo Sample Size Studies.

| Pre-dictor | Model Value | Relative Bias | | Cover-age | Power | 95% CI |
| | | Parameter | SE | | | Half-Width |
|---|---|---|---|---|---|---|
| *No Data Quirks (Example 1)* | | | | | | |
| X1 | 0.66 | 0.01 | 0.04 | 0.96 | 1.00 | 0.15 |
| X2 | 0.05 | -0.09 | 0.00 | 0.95 | 0.12 | 0.12 |
| X3 | -0.30 | 0.01 | 0.02 | 0.95 | 0.99 | 0.14 |
| *Missing Data with Full Information Maximum Likelihood Estimation* | | | | | | |
| X1 | 0.66 | -0.00 | 0.03 | 0.96 | 1.00 | 0.18 |
| X2 | 0.05 | -0.06 | 0.02 | 0.95 | 0.09 | 0.15 |
| X3 | -0.30 | 0.01 | 0.01 | 0.95 | 0.94 | 0.17 |
| *Y's Distribution with Excess Skew and Kurtosis* | | | | | | |
| X1 | 0.66 | -0.18 | -0.19 | 0.67 | 1.00 | 0.16 |
| X2 | 0.05 | -0.15 | -0.02 | 0.95 | 0.12 | 0.13 |
| X3 | -0.30 | -0.16 | -0.09 | 0.87 | 0.89 | 0.15 |
| *Unreliability of $X_1$ and $X_2$* | | | | | | |
| X1 | 0.66 | 0.12 | -0.03 | 0.99 | 0.48 | 0.90 |
| X2 | 0.05 | -1.00 | -0.04 | 0.99 | 0.04 | 0.59 |
| X3 | -0.30 | 0.16 | -0.03 | 0.97 | 0.25 | 0.54 |

*n* = 5000, increasing by 200, and using *m* = 50 simulations per sample size. The results indicate that when *n* = 3820 power is .80. I already showed that with *n* = 237 the CI half-with is 0.15; increasing it to 3820 makes the CI half-width approximately 0.06.

```
# sample size study for X2 using FIML to handle
    missing data
# simulate data from n=200 to n=5000 by 25
missing.n <- sim(nRep=NULL,
    model=analysis.model,
    n=rep(seq(200,5000,200), 50),
    generate=pop.model, lavaanfun = "sem",
    miss=missing.model.fiml, multicore=TRUE)
# power curve
plotPower(missing.n, powerParam="y~x2",
    alpha=.05)
# accuracy curve
plotCIwidth(missing.n, c("y~x2"), assurance = 0.95)

# find n for power of .80
power.mis <- getPower(missing.n, alpha=.05)
findPower(power.mis, iv="N", power=0.80)
# find CI half-widths
getCIwidth(missing.n, assurance = 0.95,
    nVal=3820)/2
```

As a point of comparison, I examined the sample size required for a power of .80 using traditional methods. Specifically, I used the *G*Power* program

(Faul, Erdfelder, Buchner, & Lang, 2009) and followed the steps the authors outlined for a power analysis of a single regression coefficient in a multiple regression (what they call *Deviation of a Single Linear Regression Coefficient From Zero*). The values I used for the *G\*Power* program and its output are in Figure 5, which shows that the sample size needed is 2057.

One way to handle missing data in this situation is to divide the sample size required for a complete dataset by the proportion of observations thought to be without missing values. Thus, if 20% of the observations had missing values, then divide *n* by .80 to find the final sample size estimate. Assuming that between 20-30% of the data are missing makes the required sample size between 2571-2939, likely making the study underpowered for $X_2$'s effects.
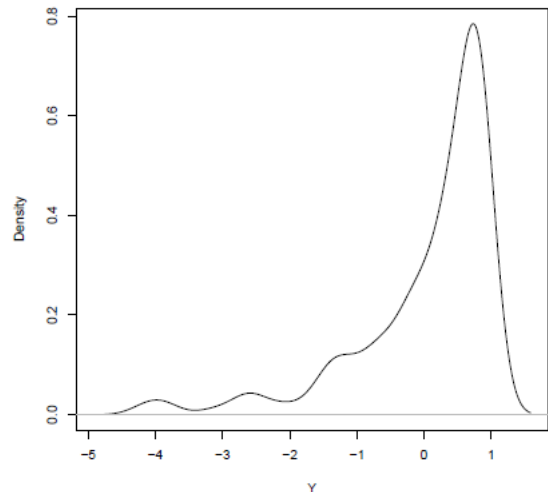


Figure 6 . Kernel density plot of Y with skew = −4 and kurtosis = 7.



Figure 5. G\*Power value specification for example with missing data.

## Data Quirk 2: Non-Normality

One assumption in multiple regression is that the residuals are normally distributed (Williams, Grajales, & Kurkiewicz, 2013). There are a variety of ways for the residuals to fail to meet this assumption, but a common one is for the outcome variable to have a non-normal distribution, such as when it has excessive skew or kurtosis. For the current example, I made *Y*'s skew equal to negative four and its kurtosis equal to seven. A plot of such a variable is in Figure 6.

*simsem*'s bindDist() function makes any of the variables in the simulated data have the desired amount of skew and kurtosis using the **skewness** and **kurtosis** arguments, respectively. Skew and kurtosis values need to be included for each variable in the data. By setting the **indDist** argument equal to the **bindDist()** object, the **sim()** function uses the specified skew and kurtosis values.

```
# add skew and kurtosis to only to Y
distrib <- bindDist(skewness = c(-4,0,0,0), kurtosis
    = c(7,0,0,0))
```

```
# simulate data with non-normal Y
analysis.nn.237 <- sim(nRep=10000,
    model=analysis.model, n=237,
    generate=pop.model, lavaanfun = "sem",
    indDist=distrib, seed=565, multicore=TRUE)
```

The results of the MC simulations with a non-normal $Y$ variable are in Table 4. With $m = 750$, the relative parameter bias values are outside of the specified limits for all three variables, as is the relative SE bias and coverage for $X_1$, and the coverage $X_3$. The aberrant bias values decrease minimally with a $m$ of 10,000, indicating that using a typical regression model for this data will produce biased regression coefficients. Compared to the results from Example 1, the power decreased for $X_3$ and the 95% CI half-widths increased for all three predictors. As $X_1$'s relative SE bias is somewhat large, its CI will likely be inaccurate so the half-width should be interpreted cautiously.

### Data Quirk 3: Reliability

Another assumption of multiple regression is that the variables are measured without error. While ideal, this is seldom the case for measures of psychological constructs. Not accounting for measurement unreliability in the model results in biased parameter estimates and a decrease in power (Cole & Preacher, 2014).

One way to account for variables measured without perfect reliability is to use single-indicator latent variables (Keith, 2006). Single-indicator latent variables explicitly model a variable's variance, which is what is affected with unreliable measures. If $s_X^2$ is a variable's variance and $r_{XX'}$ is the reliability of a variable's scores, then single-indicator latent variables fix the variable's error variance to $(1-r_{XX'})\,s_X^2$, the true variance to $(r_{XX'})\,s_X^2$, and the path coefficients to one.

To make single-indicator latent variables more concrete, say the reliability of the scores for $X_1$ and $X_2$ are both .70. Figure 7 contains a path diagram of the regression model with these imperfectly-measured variables. Note the addition of the error and true score components for $X_1$ and $X_2$. In addition, while the correlation and regression coefficients are the same as those from Example 1, the residual variance has increased from 0.685 to 0.816 because the $R^2$ decreased after modeling their unreliability.
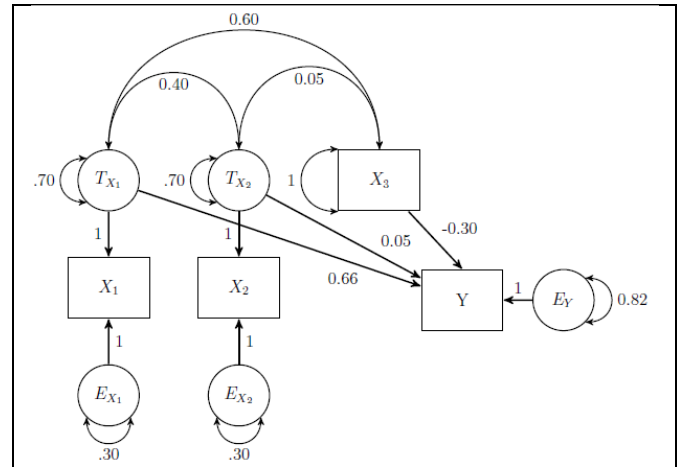


Figure 7 . Path model using single-indicator latent variables to account for the imperfect reliability of $X_1$ and $X_2$, which is .70 for both variables. $T$ represents the true score and $E$ represents measurement error. The new $R^2$ value can be calculated via:
.66 × .7 × .66 + .05 × .70 × .05 + −.30 × 1 × −.30 + (.66 × .40 × .05) × 2 + (.66 × .60 × −.30) × 2 + (.05 × .05 × −.30) × 2 = .184.

```
# data generating model with measurement error
pop.rel.model <-'
# measurement model
x1.true =~ 1*x1
x2.true =~ 1*x2
# for reliabilities of .70
# constrain error variances of X1 and X2 to be .30
x1 ~~ .3*x1
x2 ~~ .3*x2
# constrain true score variances of X1 and X2 to be
    .70
x1.true ~~ .7*x1.true
x2.true ~~ .7*x2.true
# regression
y ~  0.66*x1.true + 0.05*x2.true + -0.30*x3
# predictor variables covariance
x1.true ~~ 0.40*x2.true + 0.60*x3
x2.true ~~ 0.05*x3
# residual variance
y~~ 0.816*y
'
```

```
# data analysis model accounting for measurement
    error
analysis.rel.model <-'
```

```
# measurement model
x1.true =~ 1*x1
x2.true =~ 1*x2
# constrain error variances
x1 ~~ 0.3*x1
x2 ~~ 0.3*x2
# constrain true score variances
x1.true ~~ 0.7*x1.true
x2.true ~~ 0.7*x2.true
# predictor variables covariance
x1.true ~~ x2.true + x3 x2.true ~~ x3
# regression
y ~  x1.true + x2.true + x3
'
# simulate data with unreliable variables
analysis.rel.237 <- sim(nRep = 750,
    model=analysis.rel.model, n = 237,
    generate=pop.rel.model, lavaanfun = "sem",
    seed=565, multicore=TRUE)
```

The results of the MC analysis are shown in Table 4. The relative parameter bias is a little high for $X_1$ and $X_3$, and very high for $X^2$. When I set $m = 5000$, the amount of relative parameter bias decreased to –.75 for $X_2$, indicating that even higher values of $m$ might produce better results for the MC study. The relative SE bias and coverage are all within specified limits. When compared to the values from Example 1, the power for all three regression coefficients substantially decreased and the 95% CI half-widths substantially increased.

## Discussion

In this article, I demonstrated the use of a Monte Carlo (MC) study for the purpose of deciding on sample size in regression models based on power and accuracy in parameter estimation (AIPE). In the examples, I used a multiple regression model with three predictors and examined the sample size needed for data without any quirks as well as data with missing values, a non-normal outcome, and less-than-perfect reliability. For the examples with no data quirks, the results mapped directly onto the results from traditional formula-based sample size determination methods. When there are quirks in the data, however, there are no simple formulae to determine sample size. The results from the example MC studies showed that ignoring these data quirks could result in underpowered parameter estimation, inaccurate parameter estimates, or both.

Unlike previous articles that showed how to use MC studies to determine sample size, I focused on regression models, since they are one of the most common ways to analyze data (Troncoso Skidmore & Thompson, 2010). In addition, I used the **R** statistical language for all example analyses. As **R** is free and available on many computer operating systems, the procedures and **R** syntax in this article should be readily usable by investigators for their own data analysis.

### *Drawback of Using Monte Carlo Studies*

While one of the purposes of this article was to show the flexibility and benefits of the MC approach to determining sample size, there is a drawback: it requires users to know more about their studies' variables than traditional methods. Investigators have to specify not only $\alpha$ and power, but they also have to specify values for all the variables' relations. Thus, Cohen's (1992) concern about researchers not knowing appropriate effect size values for their particular field is amplified if they have to know how all the variables relate to each other. In the best situation, scholars would select the model's values from theory or previous research. In the complete absence of any theoretical expectations, Maxwell (2000) suggested starting with the assumption that all zero-order correlations are .30, then changing the values to see how it influences the required sample size (i.e., sensitivity analysis). Using values of .30 gives $R^2$ values around 0.14 (2 predictors) to 0.24 (10 predictors), which may or may not be appropriate for a study.

Using the AIPE perspective adds yet one more piece of information for the investigator to know: an appropriate size for the CI half-width. There are currently not any guidelines to determine the appropriate CI half-width for a regression, but values between 0.10 and 0.20 for standardized regression coefficients are commonly used in the AIPE literature, so are probably a good place to start. Of course, this somewhat depends on the hypothesized value of the regression coefficient. For example, narrower CIs are likely better with coefficient values expected to be close to zero in order to determine if the direction of the effect is positive or negative. Likewise, if the coefficient is expected to be large and well beyond some threshold set for usefulness (e.g., a clinically-relevant effect), then

having a wider CI may be acceptable as long as its bounds do not cross the threshold.

### Monte Carlo Study Extensions

In the current article, I only focused on determining the sample size for specific regression coefficients. An alternative is to focus on the entire model (i.e., omnibus) and base the sample size on the $R^2$ value. This is easy to do using the simsem package as the amount of error variance (i.e., $1-R^2$) is already included as an estimated parameter in the output.

Another limitation of the models I used in this article is that the outcome variable was continuous. The same procedures could be used with variations of this model, such as having categorical or count outcomes. While this would require more complex models and different effect sizes, the same basic procedures still apply. Likewise, regression models with nested data could also use this approach (e.g., Meuleman & Billiet, 2009).

### References

American Educational Research Association. (2006). Standards for reporting on empirical social science research in AERA publications. *Educational Researcher*, *35*, 33-40.

Bacchetti, P. (2010). Current sample size conventions: Flaws, harms, and alternatives. *BMC Medicine*, *8*, 17.

Bacchetti, P. (2013). Small sample size is not the real problem. *Nature Reviews Neuroscience*, *14*, 585-585. doi: 10.1038/nrn3475-c3

Barrett, P. (2007). Structural equation modelling: Adjudging model fit. *Personality and Individual Differences*, *42*, 815-824. doi: 10.1016/j.paid.2006.09.018

Beasley, W. H., & Rodgers, J. L. (2012). Bootstrapping and Monte Carlo methods. In H. Cooper, P. M. Camic, D. L. Long, A. T. Panter, D. Rindskopf, & K. J. Sher (Eds.), *APA handbook of research methods in psychology, Vol 2: Research designs: Quantitative, qualitative, neuropsychological, and biological* (pp. 407–425). Washington, DC: American Psychological Association.

Beaujean, A. A. (2014). *Latent variable modeling using **R**: A step-by-step guide*. New York, NY: Routledge/Taylor and Francis.

American Psychological Association Publications and Communications Board Working Group on Journal Article Reporting Standards. (2008). Reporting standards for research in psychology: Why do we need

them? What might they be? *American Psychologist*, *63*, 839-851. doi: 10.1037/0003-066X.63.9.839

**R** Development Core Team. (2014). ***R**: A language and environment for statistical computing* [Computer program]. Vienna, Austria: **R** Foundation for Statistical Computing.

Boker, S. M., & McArdle, J. J. (2005). Path analysis and path diagrams. In B. Everitt & D. C. Howell (Eds.), *Encyclopedia of behavioral statistics* (Vol. 3, pp. 1529–1531). West Sussex, England: Wiley.

Christensen, R. (2002). *Plane answers to complex questions: The theory of linear models* (3rd ed.). New York, NY: Springer-Verlag.

Cohen, J. (1988). *Statistical power analysis for the behavioral sciences* (2nd ed.). Mahwah, NJ: Erlbaum.

Cohen, J. (1992). A power primer. *Psychological Bulletin*, *112*, 155-159. doi: 10.1037/0033-2909.112.1.155

Cole, D. A., & Preacher, K. J. (2014). Manifest variable path analysis: Potentially serious and misleading consequences due to uncorrected measurement error. *Psychological Methods, 19,* 300-315. doi: 10.1037/a0033805

Cumming, G. (2014). The new statistics: Why and how. *Psychological Science*, *25*, 7-29. doi: 10.1177/0956797613504966

Cumming, G., & Finch, S. (2005). Inference by eye: Confidence intervals and how to read pictures of data. *American Psychologist*, *60*, 170-180. doi: 10.1037/0003-066X.60.2.170

Descôteaux, J. (2007). Statistical power: An historical introduction. *Tutorials in Quantitative Methods for Psychology*, *3* (2), 28-34. Retrieved from http://www.tqmp.org/Content/vol03-2/p028/p028.pdf

Dupont, W. D., & Plummer, W. D., Jr.. (1998). Power and sample size calculations for studies involving linear regression. *Controlled Clinical Trials*, *19*, 589-601. doi: 10.1016/S0197-2456(98)00037-3

Enders, C. K. (2011). *Applied missing data analysis*. New York, NY: Guilford.

Fan, X. (2012). Designing simulation studies. In H. Cooper (Ed.), *APA handbook of research methods in psychology (Vol. 3.): Data analysis and research publication* (p. 427-444). Washington, DC: American Psychological Association.

Faul, F., Erdfelder, E., Buchner, A., & Lang, A.-G. (2009). Statistical power analyses using G*Power 3.1: Tests for correlation and regression analyses. *Behavior Research Methods*, *41*, 1149-1160. doi: 10.3758/brm.41.4.1149

Faul, F., Erdfelder, E., Lang, A.-G., & Buchner, A. (2007). G*Power 3: A flexible statistical power analysis

Beaujean, Monte Carlo Sample Size Determination

program for the social, behavioral, and biomedical sciences. *Behavior Research Methods*, *39* , 175-191. doi: 10.3758/bf03193146

Goodman, S. N., & Berlin, J. A. (1994). The use of predicted confidence Intervals when planning experiments and the misuse of power when interpreting results. *Annals of Internal Medicine*, *121* , 200-206. doi: 10.7326/0003-4819-121-3-199408010-00008

Graham, J. W. (2009). Missing data analysis: Making it work in the real world. *Annual Review of Psychology*, *60* , 549-576. doi: 10.1146/annurev.psych.58.110405.085530

Grissom, R. J., & Kim, J. J. (2005). *Effect sizes for research: A broad practical approach*. Mahwah, NJ: Lawrence Erlbaum.

Hellmann, J. J., & Fowler, G. W. (1999). Bias, precision, and accuracy of four measures of species richness. *Ecological Applications*, *9* , 824-834. doi: 10.1890/1051-0761(1999)009[0824:BPAAOF]2.0.CO;2

Hill, C. J., Bloom, H. S., Black, A. R., & Lipsey, M. W. (2008). Empirical benchmarks for interpreting effect sizes in research. *Child Development Perspectives*, *2* , 172-177. doi: 10.1111/j.1750-8606.2008.00061.x

Honaker, J., King, G., & Blackwell, M. (2011). Amelia II: A program for missing data. *Journal of Statistical Software, 45*(7), 1-47. Retrieved from http://www.jstatsoft.org/v45/i07

Hoyle, R. H., & Smith, G. T. (1994). Formulating clinical research hypotheses as structural equation models: A conceptual overview. *Journal of Consulting and Clinical Psychology*, *62* , 429-440. doi: 10.1037/0022-006X.62.3.429

Keith, T. Z. (2006). *Multiple regression and beyond*. Boston: Pearson.

Kelley, K. (2007). Methods for the Behavioral, Educational, and Social Sciences: An **R** package. *Behavior Research Methods*, *39* , 979-984. doi: 10.3758/BF03192993

Kelley, K., Lai, K., & Wu, P.-J. (2008). Using **R** for data analysis: A best practice for research. In J. W. Osborne (Ed.), *Best practices in quantitative methods* (pp. 535-572). Thousand Oaks, CA: Sage.

Kelley, K., & Maxwell, S. E. (2003). Sample size for multiple regression: Obtaining regression coefficients that are accurate, not simply significant. *Psychological Methods*, *8* , 305-321. doi: 10.1037/1082-989X.8.3.305

Kelley, K., & Maxwell, S. E. (2008). Sample size planning with applications to multiple regression: Power and accuracy for omnibus and targeted effects. In P. Alasuutari, L. B. Bickman, & J. Brannen (Eds.), *The Sage handbook of social research methods* (pp. 166-192). Thousand Oaks, CA: Sage.

Kelley, K., & Maxwell, S. E. (2012). Sample size planning. In H. Cooper, P. M. Camic, D. L. Long, A. T. Panter, D. Rindskopf, & K. J. Sher (Eds.), *APA handbook of research methods in psychology, Vol 1: Foundations, planning, measures, and psychometrics* (pp. 181–202). Washington, DC: American Psychological Association.

Marsaglia, G. (2003). Seeds for random number generators. *Communications of the ACM* , *46* , 90-93. doi: 10.1145/769800.769827

Maxwell, S. E. (2000). Sample size and multiple regression analysis. *Psychological Methods*, *5* , 434-458. doi: 10.1037/1082-989X.5.4.434

Maxwell, S. E., Kelley, K., & Rausch, J. R. (2007). Sample size planning for statistical power and accuracy in parameter estimation. *Annual Review of Psychology*, *59* , 537-563. doi: 10.1146/annurev.psych.59.103006.093735

Meuleman, B., & Billiet, J. (2009). A Monte Carlo sample size study: How many countries are needed for accurate multilevel SEM? *Survey Research Methods*, *3* (1), 45-58. Retrieved from http://ojs.ub.uni-konstanz.de/srm/article/view/666.

Moher, D., Hopewell, S., Schulz, K. F., Montori, V., Gøtzsche, P. C., Devereaux, P. J., Altman, D. G. (2010). CONSORT 2010 Explanation and Elaboration: updated guidelines for reporting parallel group randomised trials. *Journal of Clinical Epidemiology*, *63* , e1-e37. doi: 10.1016/j.jclinepi.2010.03.004

Murphy, K. R., & Myors, B. (1998). *Statistical power analysis: A simple and general model for traditional and modern hypothesis tests*. Mahwah, NJ: Erlbaum.

Muthén, L. K., & Muthén, B. O. (2002). How to use a Monte Carlo study to decide on sample size and determine power. *Structural Equation Modeling: A Multidisciplinary Journal* , *9* , 599-620. doi: 0.1207/S15328007SEM0904_8

Pornprasertmanit, S., Miller, P., & Schoemann, A. (2012). *simsem: SIMulated Structural Equation Modeling* [Computer software]. Retrieved from http://CRAN.R-project.org/package=simsem

Rosseel, Y. (2012). *lavaan*: An **R** package for structural equation modeling. *Journal of Statistical Software, 48* (2), 1-36. Retrieved from http://www.jstatsoft.org/v48/i02

Skrondal, A. (2000). Design and analysis of Monte Carlo experiments: Attacking the conventional wisdom. *Multivariate Behavioral Research, 35* , 137-167. doi: 10.1207/S15327906MBR3502_1

Troncoso Skidmore, S., & Thompson, B. (2010). Statistical techniques used in published articles: A historical review of reviews. *Educational and Psychological*

Beaujean, Monte Carlo Sample Size Determination

*Measurement, 70* , 777-795. doi: 10.1177/0013164410379320

van Buuren, S., & Groothuis-Oudshoorn, K. (2011). Multivariate imputation by chained equations in **R**. *Journal of Statistical Software, 45* (3). Retrieved from http://www.jstatsoft.org/v45/i03

Venables, W. N., Smith, D. M., & R Development Core Team. (2012). An introduction to **R**. **R** Development Core Team. Retrieved from http://cran.r-project.org/doc/manuals/R-intro.pdf

Wilkinson, L., & American Psychological Association Science Directorate Task Force on Statistical Inference. (1999). Statistical methods in psychology journals: Guidelines and explanations. *American Psychologist, 54* , 594-604. doi:10.1037/0003-066X.54.8.594

Williams, M. N., Grajales, C. A. G., & Kurkiewicz, D. (2013). Assumptions of multiple regression: Correcting two misconceptions. *Practical Assessment Research and Evaluation, 18* (11), 1-14. Retrieved from http://pareonline.net/getvn.asp?v=18&n=11

## Citation:

## Author:

A.  Alexander Beaujean, Director
Baylor Psychometric Laboratory
Department of Educational Psychology
One Bear Place #97301
Waco, TX 76798-7301
Alex_Beaujean [at] Baylor.edu
http://blogs.baylor.edu/alex_beaujean