Imagination boosts strategy learning in artificial and human agents

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Abstract

Despite varied studies of imagination in natural and artificial learning systems, we don't yet have an adequate understanding of the overarching computational utility of imagination. In Chapter 1 of this work I offer a structural definition of imagination based on prior literature. I then characterize imagination's relationship with the closely related process of memory replay, introducing the concept of the "replay-imagination continuum". From these characterizations follow my hypothesis that, in relation to memory replay, imagination should be most useful for learning and strategy generalization in environments that are only partially-explored. I test this hypothesis by comparing imagination-based versus replay-based training on strategy learning in impartial combinatorial games using artificial agent simulations (Chapter 2, Aim I) and human behavioral experiments (Chapter 3, Aim II). In chapter 4 I synthesize the overlapping results from the artificial agent and human experiments. In both agent types, imagination boosts performance in relation to memory replay, indicating that imagination aids strategy learning as a general computational utility.

To all of us as we work to imagine a flourishing future,

even when doing so is difficult.

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Chapter 1: Introduction

In this chapter I will describe the problem of prospective learning, explore relevant literature on the use of imagination in biological organisms, explore relevant literature on the use of imagination-like processes in artificial systems, offer a more operational definition of imagination (including its essential components) and introduce the concept of the replay-imagination continuum, and summarize the chapter and share my thesis aims.

1.1 The prospective learning problem

Most animals, including nearly all mammals, are able to generalize from one experience to the next, even if future experiences are wildly different from the past. Formally, this ability to generalize to new contexts that are dramatically different from any context previously experienced has been defined as *prospective learning*. Prospective learning is a form of learning that does not assume that past and future events arise from identical and independently sampled (*iid*) distributions (Dawid & LeCun, 2024; De Silva et al., 2023). For example, when a school teacher teaches the same course to a new class of students each year, shifts in student attitudes, group dynamics, etc., may require amended approaches as cultural trends and other social factors shift the collective prior of students over time. Prospective learning assumes that prior experiences, which are used to build predictive models, are partially correlated with future experiences and that the goal of generalization is to find the invariances that regulate these correlations while ignoring or discarding information that does not. This is in contrast to the more traditional *retrospective learning* that occurs when agents learn and subsequently operate in contexts that are iid (Vapnik, 1991). For example, the game of Solitaire is unchanging; previously learned rules and strategies will apply equivalently to future games.

Prospective learning is an area where natural intelligence (NI; i.e., living organisms) agents excel (Seligman et al., 2013), but where existing artificial intelligence (AI; e.g., computers) approaches either fail or are severely limited (De Silva et al., 2023). Many of our most challenging societal problems (e.g., drug discovery, developing novel technologies, combating global climate change) are arguably those that require the most creative and adaptive solutions, involving operating outside one's "training set" of previous history in a changing world where the future differs substantially from the past.

So how do NI agents (e.g., humans) perform such efficient generalization to new contexts? One hypothesis is that they are able to simulate novel experiences by extrapolating a composition of prior experiences, thereby aiding discovery of invariances to leverage for more efficient behavior. In essence, imagination plays a critical role in efficient generalization (Shtulman, 2023). Here I lay out an argument that imagination serves a critical role in the process of prospective learning.

1.2 Imagination as a utility for prospective learning in NI

In order to understand how imagination can facilitate generalization, let's first consider how biological agents (i.e., NIs) do it. Based on research in developmental psychology, cognitive science, and ecology, there appear to be three general areas in which the utility of imagination in NIs has been studied: reasoning, planning, and skill learning. These areas can overlap for complex use cases such as problem solving. For example, Allen et al. (2020) model how the imagination-based process of virtual trial and error learning supports human-level flexibility and creativity in their “Virtual Tools Game” problem-solving task. This game entails placing a chosen shape (“tool”) in a 2D environment such that when physics (gravity) is “turned on”, a goal object is moved to a goal area. The task necessitates reasoning, planning, and skill learning. For a good review on the utility of imagination for general problem solving, focusing on child development, see (Shtulman, 2023). Below, I map these three general areas of imagination’s study in NI.

Reasoning, Planning, and Skill learning are each broad areas which contain multiple sub-areas. In this introduction section I will go over literature regarding the sub-areas listed below.

Within reasoning, imagination has been studied in the context of *inferential reasoning* (mental rotation, logical thinking, mind-reading, etc.), *causal/counterfactual reasoning* (asking “What if something else was done?”), and *entertainment, play, and social bonding* (fictional worlds, make-believe, etc.).

Within planning, imagination has been studied in the context of *efficient search* (for discovering new ideas about the world), *navigation* (for forming and considering route plans), and *prospective memory* (remembering to perform a task at a future time).

Within skill learning, imagination has been studied in the context of *motor skill learning*, *skill learning in education*, and *memory skill training*.

I next consider each of these in turn.

Reasoning

For inferential reasoning, imagination helps us deduce hidden states of our worlds by allowing us to simulate scenarios yielding evidence toward or against competing hypotheses. Inferential reasoning is itself an expansive area, spanning modalities and cognitive domains, from spatial inference to social inference. An example of imagination for spatial inferential reasoning is the standard mental rotation task, where participants are asked to judge whether two 2D images are projections of the same 3D shape by imagining them in three dimensions and manipulating their angles ("mental rotation"). Typically the judgment response time scales linearly with the magnitude of the difference between the viewing angles (Shepard & Metzler, 1971). In other words, the greater the degree of rotation needed to align the images the longer it takes to judge their similarity. This is generally taken to mean that humans perform this task by mentally rotating a representation of one of the shapes to the other's orientation in order to compare the two. Stewart et al. (2022) found specifically that a model of mental rotation based on 2D projections of 3D shapes produced non-uniformities in performance that match human behavior. This indicates that humans simulate the changing two-dimensional views of comparison shapes during rotation rather than comparing purely three-dimensional shape representations (which would lead to more uniform performance). An example of using imagination for social inferential reasoning is inferring others' mental states (theory of mind). Based on our understanding of others' motivations and histories, we can use our imagination to deduce, with varying degrees of success, what might be going on in someone else's mind, even when they are evasive or deceitful. Goldman & Jordan (2013) examined imagination as a tool humans use for this capacity of "mindreading", which is crucial in our social world as it facilitates coming to a shared understanding. They argue based on theory and empirical findings that a simulation account (imagining others' desires and subsequent actions) better explains human mind reading abilities than a theory account (following propositional logic in absence of simulation). Collectively these, and many other findings (Badura & Kind, 2021; Battaglia et al., 2013; Hamrick, Battaglia, et al., 2016; Hegarty, 1992; Schwartenbeck et al., 2023; Spens & Burgess, 2024; Tartaglia et al., 2012) support the idea that inferential reasoning, a critical cognitive process in NIs, is supported by the flexible simulation that imagination affords.

Imagination similarly aids causal and counterfactual reasoning, the capacity of applying inference to cause-and-effect models of the world. A counterfactual is an unobservable outcome that would have resulted if past circumstances had been different. Imagination helps us predict counterfactuals (which by definition are empirically inaccessible): we can, for example, use imagination to simulate what would have happened if we had slept in an hour later today. Walker and Gopnik (2013) make the theoretical argument that children use imagination to learn from causal and counterfactual reasoning. They describe imagination as a tool for generating counterfactuals and worldly interventions to test our understanding of how things work. Humans can perform simulated hypothesis tests by referencing and manipulating causal nets (mental models of the world's causality) through imagination. In this way imagination helps us consider what might have happened in the past had we made different choices, predict what could occur in the future under specific circumstances, and contemplate outcomes of hypothetical events in general. These "thought experiments" can teach us a great deal without requiring us to change the past or explore risky real-world options in the present, interventions which can be impossible or costly. Conversely, some future predictions accorded by causal imagination can yield ideas for safe tests to perform in the real world in order to improve one's mental model of how things work. Siegel and colleagues (2021) empirically tested causal and counterfactual reasoning in children in two sets of experiments. In the first set of experiments children chose which of two boxes to open to find a target item. Children were shown beforehand which two items could be in each box. Each box could have either a target item (identical between boxes) or a separate item (different between boxes). In actuality, the experimenter put the target item in each box while hidden from view. Even though each box made the same sound when shaken by the experimenter, children reliably chose to open the box with the greater expected sound contrast, suggesting the children mentally simulated the sound each item would make when shaken in the box, using these imagined counterfactual scenarios to choose which box to open. In the second set of experiments, children were shown two amounts of marbles (one black set and one white set), either of which would be put in a box that they themselves were then permitted to shake. Deciding which set of marbles was in the box, children spent more exploration time with boxes where the black and white sets were closer in marble amount (time was independent of the actual numbers of marbles in each box). These results suggest that the children compared the evidence they perceived with possible, but absent, perceptions that they only imagined (counterfactuals) in order to make their decisions. Together, theoretical analysis and empirical findings indicate that imagination plays a useful role in causal reasoning, allowing us to simulate effects that could result from imagined causes and vice versa. Imagination thus serves as a constructive tool for causal and counterfactual reasoning, greatly benefiting our ability to make decisions that involve future consequences in our world.

Imagination also has direct utility for entertainment, play, and social bonding. We use imaginative simulation to immerse ourselves in the worlds of stories and to entertain make-believe fantasies by ourselves or with others during play. This learning from imaginative play is as useful to adults as children. Imagination allows us to step inside what is described as the “magic circle” of gameplay, where we operate according to alternate rules and personas from those of usual life (Lantz & Zimmerman, 1999; Salen & Zimmerman, 2004). This is especially clear in role-playing games like “Dungeons and Dragons” which are built mostly in the space of a shared imagination, with only limited physical aids. Tychsen et al. (2007) found that adults enjoyed a “Pen-and-Paper” version of a role-playing game over a digital version with rich computer graphics that was matched for style and rule structure. By their nature, pen-and-paper style games involve a greater use of imagination, though they also differ in other factors such as allowing for greater freedom of play. Nevertheless, this suggests that more imaginative play can be preferable to play where less is left to the imagination. The success of imaginary worlds in entertainment media generally is highlighted by Dubourg & Baumard (2022), who posit that the human affinity for the imagined results from an evolved preference for exploration of the unfamiliar. The social aspect of shared entertainment and play may be another reason for imagination’s appreciation. As an aside, imagination even can be used directly, without play, to promote prosocial attitudes: Crisp & Turner (2009) found that simply imagining a positive interaction with an outgroup member led to more positive intergroup perceptions. This even extends to sociability with imagined people. Armah and Landers-Potts (2021) found that having an imaginary companion can be associated with increased sociality and understanding of others’ minds, with greater creativity as one characteristic that persists into adulthood. In a preschool education interventional study, S. Walker et al. (2020) found that students improved executive function over the course of an imaginary playworld intervention. This playworld intervention was a teacher-led role-play with rich narrative structure, following the premise of a treehouse with 13 floors of different activities and rules. Students demonstrated significant increases in executive function measures of inhibition, shifting, and planning, respectively assessed via the

Animal Stroop task (Wright et al., 2003), an animal shifting task adapted from (van der Ven et al., 2013), and a truck loading task adapted from (Fagot & Gauvain, 1997). This study had no control group, so we must temper any inferences that we take away from the observation, but it is consistent with related findings in the literature that almost all executive function interventions that have been studied have led to similar improvements, suggesting that imaginative play is at least one effective tool for developing executive function in children.

To summarize, empirical studies on the role of imagination in reasoning, including social reasoning and bonding, vary widely methodologically yet consistently point to imagination as a powerful reasoning tool. Unfortunately, these studies are often correlational or lacking in control groups. Most research in the area is largely theoretical in nature. This makes it difficult to draw causal conclusions from this literature. Still, the evidence points to positive effects of imagination. Subjectively, many of us can identify with using our imagination when making inferences, predicting causal outcomes, and engaging with a story or game worlds. To definitively characterize imagination's role in NI reasoning, however, we will need experiments with targeted experimental designs that specifically test imaginative reasoning strategies against closely matched alternative reasoning strategies.

Planning

Reasoning can inform the process of planning, another area where imagination is of utility to NIs. Effective planning often requires efficient search, an operation which also benefits from imagination. Search can occur over real-world stimuli, e.g. finding a symbol on a map when planning a route (i.e., visual search) or across mental constructs such as concepts or ideas, for example retrieving past impressions of local activities when planning a weekend itinerary (i.e., memory search). Interestingly, imagination has been found to aid search for both external and internal items. Regarding external search, Reinhart and colleagues (2015) demonstrated that when participants imagined a given task of visual search for a target stimulus among distractor stimuli, their reaction time for reporting whether the target stimulus was present or not during the actual task was faster. This was relative to the control condition of practicing the actual search task for the same amount of time spent imagining in the imagination condition. In other words, imagining the task led to better performance than practicing the actual task. Additionally, replacing more of the actual task practice with imagined practice led to increasingly faster reaction times. However, imagined practice led to worse performance, indexed by slower reaction times, when the target stimulus was rotated relative to the imagined version, suggesting that imagined visual search causes interference when the imagined target configuration does not match the actual target configuration. Lastly, the advantage of imagined visual search disappeared when the distractor stimuli were removed from the search arrays of the task, suggesting that the imagination advantage resulted from reducing attention to the irrelevant distractor stimuli in the standard task design. Altogether, this study is a striking example of the utility of well-matched imagination for external search.

Regarding internal (mental) search, imagination can produce mental concepts that are searched over. Also, imagination can guide search through mental spaces of memory, hypotheses, goals, etc., by generating cues or ideas that are used as associative keys or search constraints. As an example, Magid and colleagues (2015) tested 4-6 year olds on a hypothesis search task, asking which of two machine controllers (one discrete vs. the other continuous) would cause demonstrated discrete vs. continuous audio and visual effects. The participants reliably identified discrete controllers for discrete effects and continuous controllers for continuous effects as their chosen explanations, even in the condition where the experimenter convinced them that neither controller was correct (in this case, they still had to choose which controller explanation was "better"). While other accounts might also explain subjects' hypothesis choice behavior, imagination seems particularly natural and likely. Mentally simulating the discrete and continuous controllers causing the discrete vs. continuous effects could allow consideration across and selection between the candidate hypotheses. Each child could converge on a choice according to which of their hypothesis simulations them made the most sense to them (e.g. rotating the wheel control back and forth to cause the visual stimulus to slide back and forth across the screen and moving the wooden magnet control between the two possible positions to make the visual stimulus disappear, reappear in the other corner, disappear, and then reappear in the original corner). This study, while not definitive, provides some empirical support for imagination as a mental search tool. In a theoretical review, Chu & Schulz (2020) argue that children's imagining of arbitrary goals and costs during play trains their ability to generate novel ideas and plans in an infinite search space. In other words, when children imagine during "pretend play", engaging in their invented situation and rules helps them practice mentally searching for new objectives in our open-ended world. Accordingly, both empirical and theoretical evidence suggests that imagination is advantageous for the broad planning operation of efficient search.

Let's now consider the planning problem of navigation, which is among the most commonplace and evolutionarily oldest problems encountered by NIs. While we often think of navigation as being a stimulus and response act, using environmental cues to plan a route, it turns out that imagination plays a vital role in the process. In navigation, imagination allows for the simulation of possible outcomes when making decisions about what action to select next.

Imagination for route planning is not unique to humans; we can record neural signatures of anticipatory simulation of choice outcomes in rodents for example. “Preplay” of hippocampal place cell activations for left vs. right turns at T-intersections have been recorded from rats (Gupta et al., 2010; Ólafsdóttir et al., 2015). Essentially, a sequence of activity patterns in place cells is observed *before* the animal experiences the sequence naturally by moving. These studies also found sequences of place cell activation that reflected novel routes that an animal had never taken before, highlighting imagination’s capacity to transcend direct experience for novel problems. Place representation activation sequences are not exclusive to novel trajectory generation; they also often recapitulate trajectories from past experience (Comrie et al., 2022; de la Prida, 2020). These sequences are often goal-directed (K. Kay et al., 2020; Mou et al., 2022; Pfeiffer & Foster, 2013; Widloski & Foster, 2022; Wikenheiser & Redish, 2015), a property useful for planning. Disruption of these activation sequences during wakeful activity interferes with task-related planning in rodents (Aleman-Zapata et al., 2022); this suggests that these neural sequences (imaginative and recollective) naturally contribute to planning, and are not merely epiphenomena observed alongside planning. Additionally, rodents can volitionally control their hippocampal space representations to navigate virtual objects toward remote goal locations, as Lai and colleagues (2023) demonstrated via brain-computer-interface experiments. In the first phases of their study, a decoder was trained to predict rat location in a VR environment from the activity of hippocampal place cells. In the last phase, rats’ treadmill activity was prevented from causing movement in the VR environment; instead location as decoded from place cell activity was used to move the rat’s egocentric view or the location of an observed object in the VR environment. In this phase, rats were able to control their place cell activity to “teleport” themselves or observed objects to goal positions in the VR environment. This suggests that prospective hippocampal place cell activations recorded in rats can indeed reflect a volitional process of anticipating possible routes while planning in navigation, in the way that we experience navigational planning (with a sense of volitional control). As humans we can sympathize with the experience of using imagination to plan a commute. We might imagine driving to work (visualizing the traffic and anticipating searching for and paying for a parking space) versus taking the bus (contemplating waiting at the stop in the elements before resting on the ride) and evaluate the two scenarios in order to make a decision on transportation. In this way our simulation of the different options aids in our selection of which action to take. By pre-playing future trajectory scenarios, we can evaluate which choices might better align with our planning goals.

Another important planning process that relies on imagination is prospective memory, which is essentially planning for future memory retrieval. The area of prospective memory concerns the study of setting intentions with the goal of enacting them in the future (Ingvar, 1985). One example of prospective memory is hearing a message and mentally preparing to remember to relay it to a family member later in the day (Kliegel et al., 2014). The messenger uses their memory prospectively: storing their planned intention to pass the message along, for later recall upon returning home. Imagination for prospective memory has been studied because one strategy in this area is to imagine the conditions under which one desires to remember their intention. For example, our messenger could thoroughly imagine the scenario of sharing their message with their partner after coming through their door and calling out “I’m home!” to help them remember to enact their plan later. In a study of subjects with neurological damage and impaired memory, self-imagination (imagining following task instructions from a personal perspective) was found to aid prospective memory performance over rote rehearsal of task instructions (Grilli & McFarland, 2011). In this experiment, the background task was to answer multiple choice questions in a knowledge test, and the prospective memory task instructions were to press the "1" key each time a target word appeared in a question. In a study with 140 undergraduate and postgraduate subjects, Abel and colleagues (2024) found that having participants visualize themselves following prospective memory task instructions (writing an "X" next to knowledge test questions about space) aided their prospective memory performance relative to writing and verbally rehearsing the prospective memory task instructions. In the second part of their study, the same task visualization condition was compared to a bizarre imagery condition (having participants imagine a UFO visiting them when they reach questions about space, prompting them to draw a picture of an alien next to the question). The original imagination condition again produced a significant positive effect on prospective task execution, but the bizarre imagination condition did not, perhaps because the prospective visualization deviated too far from reality to be useful in intention recall. Together, these studies find that imagination aids prospective memory across varied populations. Specifically, imagination shows benefits when used in a targeted manner, to simulate performing goal actions in scenarios that closely match the target conditions for intention recall (as opposed to more fantastical and/or superfluous imagination).

Planning is a very large area of cognition, encompassing domains such as efficient search, navigation, and prospective memory. And we see here that imagination's relevance to planning appears as broad as planning itself. This may not come as a surprise, considering that simulating a course of action (at any time scale) is one clear way to evaluate it as an option when making future-oriented decisions. The findings regarding imaginative training for visual search and prospective memory also suggest that imagination may further help by playing a subtler, more priming-style role in setting one up to more readily perform planned future actions (beyond simply deciding to take them). This possibility of imagination providing simulated training leads into the discussion of imagination for skill learning, which follows next.

Skill learning

Imagination has also been studied for practicing skills for general re-use in the future. In particular, mental simulation has been investigated as a means towards improving skill training in the areas of motor control, education science, and memory. These categories are not mutually exclusive; for example memorization for musical performance incorporates elements of fine motor and memory skill in music education. Iorio and colleagues (2022) found that mental rehearsal of music (imagining performing without auditory feedback or movement) combined with physical practice improved instrumental performance and long-term retention relative to physical practice alone.

Imaginative training for motor control has long been investigated, and many such studies fall under the area of sports science; see (Suinn, 1997; Toth et al., 2020) for two relevant reviews. Here, the term “mental practice” is used to refer to the imaginative simulation of a physical technique in the absence of movement. For example, Shick (1970) tested a population of college women and found that mental practice of volleyball “serves” improved a standardized measure of serving performance as compared to no practice (these women had volleyball experience but were not playing recreationally or for a class at the time). This study also found that when combined with physical practice, three minutes of daily mental practice (of the serving technique) led to greater improvements than one minute daily of the mental practice. There have been many other similar studies investigating mental practice for the training of various other motor skills; see (Desai et al., 2025; Schneider et al., 2024) for more recent reviews. The beneficial effect of mental practice on learning in this domain has held as a durable finding.

Imagination has also been explored as a pedagogical tool for teaching cognitive skills in educational settings. In one example, Rakestraw et al. (1983) taught medical students to perform pelvic exams via a standard instructional protocol vs. via instructional protocols incorporating audiotape-guided mental practice. Students in the mental practice conditions were better able to list the examination sequence and record examination findings. Similarly, Leahy & Sweller (2005) investigated the conditions under which imagination can be useful in learning complex tasks. Here primary school students were taught two tasks: reading bus timetables and reading temperature line graphs. Students were only asked to study task instructions in the study condition, and were asked to use task instructions as a prompt for imagining the task in the imagination condition. The experimenters found that among students with some prior experience, those who performed the imagined task rehearsal answered more task test questions correctly than those who simply studied the task instructions. However, among students with no prior experience in these tasks, the reverse effect was observed (studying of instructions was advantageous relative to imagining the task). This suggests that in using imagination to learn in educational settings, a baseline of familiarity with the object of learning is important. Together, imagination appears as a valuable pedagogical tool when effectively applied (when imagination is appropriately guided and when learners possess adequate prior knowledge).

The value of imagination in training also extends to mental skills like memory. In (Grilli & Glisky, 2010), imagination of detailed egocentric experiences from sentence prompts aided recognition memory of sentence prompts in memory-impaired subjects with neurological damage and in healthy controls. This suggests that richly imagining a scenario that relates a target concept to oneself promotes later recognition of those concepts, even for those with damaged memory circuitry. Additionally, imagination in the form of “memory palaces”, which involves visualizing recall cues along a route through a familiar space, is a common technique for remembering vast amounts of information. This approach is often referred to as the *method of loci*, and has been found to be more useful for individuals with stronger visualization skills (Varilias, 2019). As an example of the storage capacity this memory training tactic affords, a subject of “average intelligence” in a neuroimaging case study used the method of loci to recall more than 65,000 sequential digits of the mathematical constant pi (Raz et al., 2009). From recognition to recall, imagination serves as an effective tool for improving memory skills.

In all, imagination appears to facilitate skill learning across multiple skill types. This effect is observed when imaginative practice is compared with the alternative to no practice, and when imaginative practice is employed as a supplement to physical practice. Critical supporting conditions such as adequate prior experience arise (Leahy & Sweller, 2005). Imagination’s benefit to skill learning is not constrained to the domain of physical motor learning, as we see that imagination can be used to effectively train memory skills as well. Taking a step further, these studies of imagination for skill learning relate to the psychological area of metacognition, which concerns the process of thinking about one’s own thinking (Winne & Azevedo, 2014). Just as imagination-based skill learning can involve internal simulation of one’s body performing a specific skill, one’s expected mental states may also be simulated. In this way metacognitive skills may be supported by imagination. Burns (2022) argues that imagination is critical to metacognition and is thus imperative to actively develop for greater learning capacity, personal capacity, and democratic capacity. Whatever skill is being learned, it seems that imagination can aid in some way.

Summary of imagination's utility in NI

These main areas in which NI imagination has been studied, reasoning, planning, and skill learning, reveal a common function for imagination. They show that it critically allows for the simulation of new experiences that needn't be physically encountered for the goal of learning. This single function is expansively applicable (as evidenced by investigated utilities ranging from mental rotation to memory palaces and beyond), but the core process of internally synthesizing scenarios remains the same. By simulating possible alternatives to the past and present, imagination constitutes a mechanism invaluable in preparing for a future which differs from direct experience. This is the very definition of prospective learning. We will return to the mechanics of this process in section 1.4, but first we should look at how imagination has been used in artificial agents (i.e., AIs).

1.3 Imagination as a utility for prospective learning in AI

The insights into the utility of imagination in the psychology and education literature have inspired some attempts at implementing imagination in artificial agents. These implementations even overlap thematically with areas in the NI literature. There are five major areas where imagination (or imagination-like processes) have been deployed in the AI literature: *working with data scarcity*, *planning*, *training*, *continual learning*, and *out-of-distribution learning*. These areas are not mutually exclusive; for example the stumbler-strategist agent (Peterson et al., 2020) is trained with an imaginative process and is evaluated on an out-of-distribution learning test. In this introduction section I will evaluate each of these areas in order.

A common feature that links these areas where imagination is useful for AI is the use of internal *world models* of the environment, for the purpose of supporting flexible, robust behavior. World models are approximations of the dynamics of an environment in which an AI agent operates. They take as input hypothetical states of the world and hypothetical actions, and as output predict resultant world states (Ha & Schmidhuber, 2018; Schmidhuber, 1990; Sutton & Barto, 1981). They may be implemented in many different ways, from look-up tables to complex deep neural network algorithms. Importantly, world models are what allow these artificial agents to simulate ("imagine") their environment under different conditions. Of course, world models must effectively predict environmental dynamics to permit effective training (Kalweit & Boedecker, 2017). Imagination-like training using world models is widely useful, from filling in estimates of missing data to planning and training, and is more uniquely applicable in cases of prospective learning problems.

Working with data scarcity

When artificial agents have limited access to data, artificial imagination can allow them to generate more to work with, filling in gaps or adding variation to support robust learning. This process of producing additional simulated data is also known as synthetic sampling. There are many situations where training data may be difficult or impossible to come by; in these cases, synthetic sampling is of great use. Using synthetic sampling (Raghunathan, 2021) to fill gaps where data is sparse or nonexistent can also be useful for helping agents plan (after all, if you have no data points of the future, imagination may be the only way to generate possible outcome expectations).

For example, Aswin Nahrendra and colleagues (2023) built a quadrupedal robot that imagines terrain properties based on limited sensing input (solely proprioceptive information) to inform its walking behavior. Here, imagination was implemented with an autoencoder that takes the limited sensing data as input and predicts both the robot's internal state and the sensing data that will come in the next time step (this predicted data represents the imagined terrain properties). This method of control allowed the robot to traverse relatively long distances over a route with some features that did not match the agent's training data. Another agent, designed by Kalweit & Boedecker (2017), employs synthetic sampling to reduce the amount of training input needed by its data-hungry learning algorithm (Deep Deterministic Policy Gradient). In this design the agent's memory store is used to train a world model of the environment, which is itself used to generate artificial (synthetic) memory samples for training the agent's policy. The synthetic sampling is used to generate training data only under conditions of adequate certainty, allowing their reinforcement learning agent to learn faster with reduced reliance on real-world training data, which can be costly to acquire. By conditioning the synthetic sampling on model certainty, this approach addresses one downside of synthetic sampling: it can produce very inaccurate data in spaces of poor model performance. Taken together, we see that the artificial imagination process of synthetic sampling can effectively address the problem of data scarcity, particularly when the underlying predictive models are appropriate and when the synthetic sampling is implemented with sensitivity to estimated model performance. Imagination-like data generation can create artificial memories for training off-line when "real" memories are scarce, and can help agents fill in (infer) data that is missing or difficult-to-sense on the go. This on-line contribution of artificial imagination to agent decision making relates to the area of planning, explored next.

Planning

Much like how NI agents use imagination to plan, AI systems may use imagination in the form of internal predictive simulations for planning. In this case, artificial agents use models of their environment's dynamics to simulate outcomes of candidate actions. These predictions are then used in determining action plans. Imagination for planning is naturally of great use in the area of reinforcement learning, where artificial agents must learn to achieve rewarding goals in an environment via trial and error (Schmidhuber, 1990).

Specifically, because imagination relies on world models to function, artificial imagination is particularly useful for planning in model-based reinforcement learning agents. In discussing machine analogs of imagination, Jessica Hamrick (2019) describes imagination in the context of model-based deep reinforcement learning (MBDRL), where "deep" refers to the use of deep (multilayer) neural networks. Through training, deep networks can extract abstract patterns latent in high-dimensional data. Given their capacity to represent the often complex structure of environmental dynamics, they are excellent tools for implementing world models and simulating novel (artificial) experiences which still follow environmental regularities. Indeed there are several successful MBDRL systems which simulate outcomes of possible options to decide which action to take in their current state; two examples are (Hamrick, Pascanu, et al., 2016; Schrittwieser et al., 2020). Using artificial imagination toward more specific planning goals than solely maximizing reward, Pathak et al. (2017) and Sekar et al. (2020) designed agents that use imagination to plan where to explore in order to better gather data towards improving their world models. In all, artificial imagination allows AI agents to simulate a few (or many) steps ahead when selecting which action to take next, allowing agents to plan more effectively. Such imaginative planning can be used toward the simple goal of maximizing reward, or toward more complex goals such as gathering useful information. What about planning for a possible future situation instead of planning for an action to take in the current state? This is the domain of training, considered next.

Training

In machine learning, the process of training is distinct from the related process of planning. In essence, training involves preparing for hypothetical scenarios in advance. When scenarios are encountered later, agents can follow trained action policies that are already available instead of needing to plan on the fly (Sutton & Barto, 2018). Artificial imagination for training follows this distinction, helping agents learn in advance for situations they might encounter in the future (Ha & Schmidhuber, 2018; Hamrick, 2019). Without imagination, artificial agents can train based on scenarios recorded in memory, but artificial imagination allows agents to train based on novel simulated (synthetic) scenarios which go beyond an agent's past experience.

Of course, experiences generated by an agent's world model can be prone to varying amounts of error. Kalweit & Boedecker (2017) address this issue by designing an agent to train using synthetic experiences only when a measure of model certainty is high. Richard and colleagues (2022) describe an agent they designed with separate models for the environment and the agent's physical system (e.g. the robot and its state). By changing out its models and training a new policy using imagination, their agent learned to operate in a land environment (rover) from originally operating in a water environment (boat). On the whole, artificial imagination for training appears to serve a similar function as imagination for skill and strategy learning in humans: preparing action plans in advance through simulated practice of possible future situations. This process of employing synthesized (artificially imagined) training data which may go beyond past experience supports two problems in machine learning that we will examine in the following subsections: continual learning and out-of-distribution learning.

Continual learning

Continual learning is a subfield of machine learning research that focuses on the problem of designing agents that can continually learn from new data without forgetting previously learned information and/or tasks (L. Wang et al., 2024). In this context, artificial imagination has been harnessed to combat catastrophic forgetting, a common issue where agents lose the ability to perform well on an original task after training to perform well on a different task (Lesort et al., 2019; van de Ven et al., 2020; Z. Wang et al., 2022). Inspired by human memory and imagination, some continual learning systems have been designed with replay (training on past data together with new data) and generative replay (training on simulated data together with new data) learning processes.

To understand how such continual learning methods measure up, Lesort and colleagues (2019) compared several continual learning approaches to training generative AI systems (including combination methods). They applied these different approaches to train networks to produce images according to those in the MNIST and Fashion MNIST datasets. They found the standard generative adversarial network (GAN) combined with generative replay training to be the best-performing combination (other training strategies were rehearsal, regularization, and fine-tuning). Van de Ven et al. (2020) designed a specifically brain-inspired version of generative replay for continual learning, with feedback connections from a generator neural network module (analogized to the hippocampus) to the main model neural network module (analogized to cortex). They found their brain-inspired generative replay to outperform standard generative replay as well as other established continual learning methods, without needing an explicit store of previous experience in memory. Wang and colleagues (2022) developed another brain-inspired continual learning method that combines standard and generative replay called “Deep Retrieval and Imagination”. They showed theoretically how this method can reduce error and improve robustness, and showed empirically how this method outperformed existing state-of-the-art continual learning methods while avoiding catastrophic forgetting. Related to imagination for continual learning, and further analogizing between AI and NI learning is the “overfitted brain” hypothesis: that dreaming prevents humans from “overfitting” on new experiences to the detriment of previous learning (in essence preventing catastrophic forgetting) by hallucinating novel experiences that combine random aspects of past experiences while we sleep (Hoel, 2021). Altogether, we see that artificial imagination is an effective tool for combating catastrophic forgetting in continual learning contexts. It is interesting that many of these successful continual learning systems are specifically brain-inspired; this hints at a possible generality of imagination for maintenance of learning across AI and NI. Related to continual learning is the machine learning area of out-of-distribution learning, which drops the emphasis on alleviating catastrophic forgetting and focuses on adaptation to drifting or wholly new distributions are encountered. We will look into artificial imagination there next.

Out-of-distribution learning

The relatively new domain of out of distribution learning concerns learning for explicitly shifting or separate distributions (Ye et al., 2021). Here too, artificial imagination has been employed; there are a few instances of domain-specific AI systems that are designed to perform simulations that lie outside of their training data sets.

One example is the facial recognition system described by Churamani & Gunes (2020), which simulates novel facial expressions of individual subjects for improved expression recognition performance. Their system uses an autoencoder to take single face images from subjects and produce images with modified facial expressions, creating additional synthetic training data in a continuous learning framework. The extra data allows for better facial recognition performance. Elgammal et al. (2017) detail a process of artificial art generation in which deviation from trained style norms is maximized to improve creativity and image originality. These authors used a modified GAN architecture where they include an objective to minimize style classifiability, thus forcing the produced images to match low level features of art but defy high-level stylistic norms present in training distributions. Colas et al. (2020) built a deep reinforcement learning architecture modeled on their conception that children's curiosity-driven exploration follows the generation of out-of-distribution language goal descriptions. Their reinforcement learning agent trains goal-conditioned action policies in a 2D environment, where the language goal descriptions are generated by a construction grammar, allowing for continually novel goals. In all these systems, imagination supports performance by generating simulations that lie outside of training set distributions. The utility of artificial imagination toward generating and/or adapting to new distributions of data are evidently varied, and have only started to be studied.

Summary of imagination's utility in AI and synergy with NI imagination utility areas

In comparing natural imagination to artificial imagination, we see an alignment of sorts in the areas where imagination is helpful. In fact, planning is one area where NI and AI areas directly overlap. Working with data scarcity is akin to reasoning, in the sense that artificial imagination can be used to extrapolate ("infer") new data based on data of another form. Training via artificial imagination is similar to skill learning via mental practice. Prospective learning is not explicitly listed as an area of utility for imagination in NIs, though as we have already discussed, NIs are inherently prospective (De Silva et al., 2023; Seligman, 2016; Seligman et al., 2013). All of these imagination utility areas in NIs and AIs are relevant for the general purpose of decision making, and especially for making prospective decisions. This is what we will look into moving forward.

1.4 A definition of imagination

I am now ready to try to define, more operationally, what imagination is. To start I will identify its components. First and foremost, imagination in both AIs and NIs rests on the concept of a world model. While I have only briefly detailed world models above in the context of AIs, NIs also utilize what we can consider as world models. For example, the mammalian neocortex is ideally structured to be a world model: it contains mappings of the environment (Berkes et al., 2011; Rothschild & Mizrahi, 2015) that are dynamically accessible and manipulable (Fuster, 2000). Indeed, when imagining or re-imagining experiences, areas of the neocortex relevant for components of the simulated experience become active (Miry et al., 2021), suggesting that internal re-experiencing without direct sensory input takes place. Because a world model is a form of a memory, we would expect imagination to have properties of a memory system. Indeed, empirical studies find that overlapping brain structures are engaged in both memory recall and imagination (Addis et al., 2007; Comrie et al., 2022; Hassabis & Maguire, 2007; Maguire &

Hassabis, 2011). As I will describe later, imagination is in fact structurally similar to the memory process of replay.

Also, imagination appears to have a compositional structure. Here compositionality means that an imagined scenario is built up from memory elements combined in the world model. Analyses of creativity (Weisberg, 1998) and of the form of imagination observed in children and adulthood (Shtulman, 2023) find that imaginative thoughts are not generated de novo, but constructed from components of prior experience. Studies in rodents and humans have found that, in concert with other cortical regions, the hippocampus plays a key role in the formation of imagined scenarios from memory components (Gaesser et al., 2013; Schacter & Addis, 2007; Schwartenbeck et al., 2023), notably during the process of scene construction; that is, composing a mental model of a scene from spatial components (Hassabis & Maguire, 2007).

The way in which aspects of memory are composed in a world model must necessarily be constrained by the limits of the world model. Meaningful and efficient imagination should follow additional constraints, generating scenarios that follow regularities of the external world extracted from experience. As stated above, evidence from the developmental psychology literature suggests that at least some forms of imagination rely on recomposing features in a rational way based on prior experience (Atance, 2015; Bar, 2009; Irish & Piguet, 2013; Shtulman, 2023; Ward, 1994; Weisberg, 1998). For example, when asked to create novel creatures, children and adults use features of animals they already know (real or fictional) (Ward, 1994). This means that there is a rationality to how imaginative experiences are generated that forms a constraint on their dynamics. For example, imagined experiences appear to follow causal laws from the environment, by and large, suggesting that the recomposition of prior experiences is constrained by the mechanisms of causal inference (Buchsbaum et al., 2012; Phillips et al., 2019; Phillips & Knobe, 2018).

Imagination also requires a dynamic process for implementing the constrained composition of memory representations in a world model. Schwartenbeck and colleagues (2023) used fMRI and MEG to show that imagined scene representations in the prefrontal-hippocampal circuit are relational and compositional, and that these imagined scene representations are built via rapid sequences of scene element representations. Specifically, the type of sequential hippocampal activity here is referred to as a sharp wave-ripple, the same type observed in replay and during planning and navigation. Instead of previous location representations though, representations of scene elements were activated in sequence to compose imagined configurations. In other words, the group found that dynamic hippocampal mechanisms originally implicated in the replaying of past spatial navigation episodes (rapid sequential re-activation of location-specific neuronal representations) also supported visual scene imagination. This finding is corroborated in the theoretical and empirical literature on hippocampal sequential dynamics for imagination (Comrie et al., 2022; Fuster, 2000; K. Kay et al., 2020; Pezzulo et al., 2014; Pfeiffer & Foster, 2013; Widloski & Foster, 2022).

Now we have the four critical elements of imagination: a world model, compositionality, constraints, and dynamics. The process of imagination thus unfolds as follows. Memory representations within the world model are recomposed according to various implicit and learned constraints, as well as task demands, unfolding as a generative sequence of spatiotemporal events. In this way, imagination serves the purpose of bootstrapping a repertoire of experiences by simulating novel scenarios for the goal of more generalizable behavior. These simulated experiences are constrained by inductive biases learned from prior experience, or evolutionary processes.

There is another phenomenon with features very similar to our definition of imagination. This is the process of “replay” or memory recall. In replay, however, the internally simulated sequence of spatiotemporal events closely resembles the sequence of events from a prior experience. In imagination, the simulated sequence of events deviates from prior experiences, either partially or are altogether detached from any lived experience of the agent (see Figure 1). The distance by which the internal simulation in the world model deviates from prior experience characterizes a continuum between replay and imagination.

To understand this replay-imagination continuum, consider the scenario of a job candidate preparing for an interview. Here the candidate experiences this sequence of events:

(A) the interviewer asks “Why do you want this job?”

(B) the candidate replies “Honestly, I just want the money.”

(C) the interviewer responds “Oh…” with a neutral tone.

We can think of these events as occurring in a simple 2-dimensional world model where A, B, and C are distinct spaces of the world model that are experienced in an A-B-C sequential order. This is illustrated as the gray sequence of A-to-B-to-C in Figure 1. Perfect replay of these events within the context of the world model, would activate the sequence of A-to-B-to-C as veridically as possible. This is illustrated as the black sequence in the left panel of Figure 1.

Of course, replay is often imperfect. For one, our brains necessarily process and interpret high-dimensional multisensory input when experiencing the world and, as a result, experience encoding errors (Straube, 2012). This makes human recall error prone and liable to drift

(Schacter, 2022), especially with repeated reconsolidation (Schmid, 2020). Such imperfect replay of our interview scenario might go as follows:

(A) the interviewer asks “Why do you want this job?”
(B) the candidate replies “Honestly, I just want the money.”
(C') the interviewer responds “Oh…” disapprovingly.

This is illustrated in the middle panel of Figure 1 as the sequence A-to-B-to-C' where C' represents the recall in a disapproving tone, as opposed to the neutral tone of the actual experience (C). In this way the simulated sequence is closely tied to experience, but the candidate is not intending to imagine, and instead exhibits an error in their simulation.

Let us now consider a case where the candidate intentionally engages in imagination. They construct a similar context, but this time a new sequence:

(A) the interviewer asks “Why do you want this job?”
(B*) the candidate replies “Because I strongly value the company mission.”
(C*) the interviewer responds “I’m glad.” with a smile.

This is illustrated in the right panel of Figure 1 by the sequence A-to-B*-to-C*, where B* represents the candidate’s reconsidered response and C* represents a response the interviewer never produced. Through this sort of imaginative simulation after the interview, the candidate can “test out” various ways they might answer possible questions in the future and settle on responses that they expect would be received favorably. In this example, the sequence of spatiotemporal events constructed during imagination is based on the sequence recollected during replay, but differs in a direction that informs effective decision making.

Of course, imagination does not need to be anchored to any prior experience at all. For example, a simulation of playing dodgeball with aliens instead of recalling the job interview.

This would be uninformative, at least for the goal of interview preparation. The rightward arrow in Figure 1, extending from the imagination panel, is intended to illustrate the possibility of imagination diverging even farther from prior sequence of events, to the point of hardly resembling any real external experience at all.

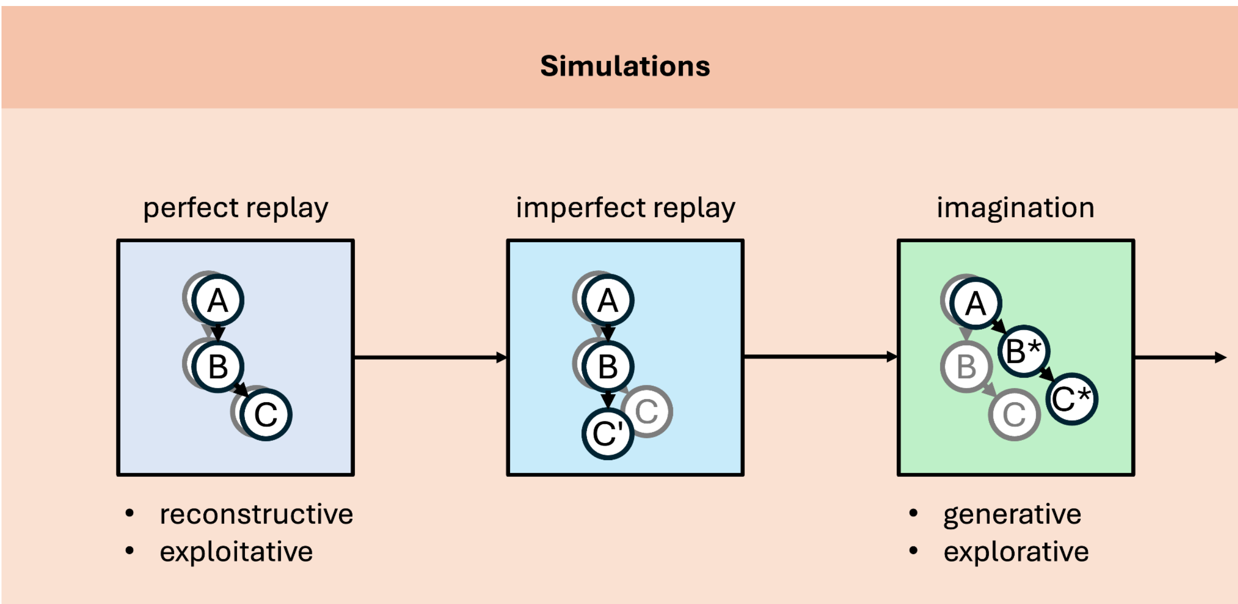

Figure 1. The replay-imagination continuum. Replay and imagination both fall under the umbrella of constructive internal simulations. In perfect replay (left), the abstract sequence of spatio-temporal events A-to-B-to-C exactly simulates a veridical sequence that was experienced in the past. In imperfect replay (center), the abstract sequence of events A-to-B-to-C' simulates the experienced sequence erroneously, ending with spatiotemporal event C' instead of C. In imagination (right), the abstract sequence of spatiotemporal events A-to-B*-to-C* is simulated. This sequence does not match experience in memory, though it may be based on memories. The rightward arrow extending from the imagination box signifies that imagination may diverge from replay to varying extremes, as in dreaming, or further, under the influence of psychedelics.

A few interesting concepts that clarify the details of the replay-imagination continuum are memory errors, replays of imagined scenarios, and false memories. A memory error, resulting from imperfect replay, can lead to a sequence of spatiotemporal events that is unintentionally farther from past experience than usual replay. Thus imperfect memory may resemble imagination, the difference being that the simulated sequence is assumed (incorrectly) to reproduce past experience. Separately, replay of a previously imagined scenario would fall under replay (as opposed to imagination) as it would involve reconstructing a previously experienced sequence of spatiotemporal events, even if the original sequence was internally generated. False memories may result from consolidated imperfect replay, or may be similar to replay of previous imagination: A sequence of spatiotemporal events that was originally only imagined to have happened, then replayed so many times that it becomes indistinguishable from externally-originating memory by the agent (Gonsalves et al., 2004; Loftus, 1979). These interrelated concepts further highlight the close association between replay and imagination.

The continuum between replay and imagination reflects their relative function. Replay is reproductive in that it internally revisits past experiences. Consequently, I propose that replay is useful for extracting or ingraining information already present in memory. By contrast, imagination is generative, forming novel internal “experiences” composed of features from a set of prior events. The behavioral utility of a Frankenstein reassembly of prior knowledge would be to learn and make decisions from this artificial experience. We can thus place the roles of replay and imagination on the exploration-exploitation continuum. The utility of the veridical, or near veridical, replay is to “exploit” prior experiences. On the other hand, I propose imagination is useful in more “explorative” contexts, where making appropriate choices requires investigating possibilities that are not locked to experience. This is precisely what makes imagination so useful for effective prospective decision making: by constructing sequences of spatiotemporal events which deviate from the remembered past in ways that the future is expected to be different, imagination helps us anticipate relevant decision factors with which agents have not had direct experience.

Imagination, as theoretically defined and compared with memory replay here, can be expected to support learning for effective decision making in some circumstances more than others. By building on elements of previous memories while deviating from direct experience in an appropriately constrained manner, imagination can simulate never-before-experienced alternative scenarios which still lie within the range of possibility. Any positive (or negative) effect of learning from this broader sampling of constructed possibilities must necessarily depend on the learning application. In applications that entail near-veridical recapitulation of prior knowledge (like memorizing a speech), learning from non-factual imaginative simulations (such as of alternative speeches) would be expected to simply add noise and possible interference. In these circumstances, replay-based learning should provide cleaner teaching signals. However, in applications that entail navigating a partially-known structured task (like playing a board game after some familiarization), imaginative simulations (such as of novel possible moves, opponent reactions, etc.) may harness new combinations of state variables from prior memory to “fill in” possible experiences. This imagination-based learning may contribute to greater understanding and more effective preparation. Under these circumstances, replay-based learning should be of little help, only serving to solidify preexisting knowledge. This kind of learning, training generalizable and/or transferable action policies, can be thought of as strategy learning. Of course, different learning applications require varying degrees of generalizable strategy, so the benefits of replay-based vs. imagination-based learning should vary continuously depending on how repetitive vs. open-ended a target task may be. These predictions for where imagination should aid learning are condensed into my specific hypothesis in the following section.

1.5 Summary and project aims

Despite the varied studies of imagination in natural and artificial learning systems, we don't yet have an adequate understanding of the overarching computational utility of imagination. *My specific hypothesis is that imagination improves performance in both learning efficiency and generalization to future scenarios that are correlated with, but not identical to, past experiences*. In this way, imagination may serve as a tool for learning structural invariances to achieve robust generalization, a necessary computation for prospective learning.

The relevant research that currently exists hints toward this proposed hypothesis only indirectly. We see imagination implicated in the generation of novel simulated constructions for wide-ranging purposes such as the entertainment of fictional worlds in NIs to the training of state-action policies in AIs. However, we currently lack the experiments to directly test whether imagination supports generalization for prospective decision making in humans and artificial agents. Secondarily, we also don’t know how other learning processes such as memory replay compare to imaginative learning for this purpose.

To this end, my dissertation answers the following two aims.

Specific Aim I: *Imagination as a computational strategy for generalization*.

Through the use of an existing neural network model with an imagination learning process (Peterson et al., 2020), I will show explicitly how imagination is crucial for generalization of invariant strategies (i.e., by testing the neural network model with imagination vs. replay-based learning processes) between impartial combinatorial games (from Wythoff's to Euclid). I hypothesize that (Hyp. I.a) the imagination networks will have increased learning efficiency in Wythoff's compared to a replay-based network or simple, model-free reinforcement learning network and (Hyp. I.b) imagination will boost generalization to Euclid compared to control networks.

Specific Aim II: *Empirical (human) imagination for generalization of invariant strategies*.

I will have neurologically healthy human participants learn to play one impartial combinatorial game (Wythoff's game) with one of three interventions in the middle: an imagination intervention, a replay intervention, and a distractor control condition. I will then test the generalization of learning to a second impartial combinatorial game (Euclid), where the optimal strategy transfers despite a different rule set. I hypothesize that (Hyp. II.a) the imagination group will boost their performance in Wythoff's compared to the other two groups, and (Hyp. II.b) better learning in Wythoff's will transfer to better performance in Euclid.

Chapter 2: Computational models of imagination and replay

In seeking a more complete understanding of imagination's value in learning, my research and theorization above suggests that imagination should support strategy learning over the closely related process of memory replay. To test this prediction, I use an artificial agent model with an imagination learning process (Peterson et al., 2020). I extend a version of this agent to learn via replay in place of imagination. I test the imagination- and replay-based agents against a control agent in a strategy learning task (Wythoff's game). I test transfer (generalization of strategy learning) to a game with a related high-level structure (Euclid). In Wythoff's, imagination is found to boost strategy learning over memory replay, and both imagination and replay agents show a learning advantage over the simpler control agent. However, pretraining in Wythoff's does not boost performance in Euclid. This pattern of results yields specific predictions for the effects of imagination and memory replay-based strategy learning in NIs, which are evaluated in Chapter 3's human experiment.

2.1 Introduction

To investigate the utility of imagination in learning, specifically contrasted with memory replay, we can start with computational modeling. By testing well-defined artificial agents that are as simple as possible (to minimize extraneous detail), yet complex enough to model imagination and memory replay learning processes, we can better understand imagination's value in AI and generate predictions regarding imagination's value in NI. In line with this reasoning, I extended and tested the previously-published stumbler-strategist agent (Peterson et al., 2020). This agent has a relatively simple structure (a model-free RL component connected to a shallow two-layer neural network) that is biologically-inspired by the hierarchical organization of the prefrontal cortex in humans. Critically, the stumbler-strategist agent employs an imagination-based learning process. This agent has previously been tested on impartial combinatorial games, which are suitable environments for evaluating learning (see below).

Impartial combinatorial games

The impartial combinatorial games of Wythoff's (Wythoff, 1907) and Euclid (Cole & Davie, 1969) have single ground truth optimal strategies which generalize spatially (see Figure 3). This makes them tasks that are well-suited for testing the strategy learning of simple imaginative neural network agents. These games have been used to test the generalization capabilities of the stumbler-strategist network of Peterson et al. (2020), whose study specifically trained this AI agent on Wythoff's game (see Figure 2) and investigated its generalizability to Euclid (see Figure 4). Originally, these games were introduced as objects of mathematical study.

The objective of Wythoff's is to reach the top left corner of the grid board. A player and an opponent alternate plays, moving the game piece leftwards, upwards, or diagonally up/leftwards, by any number of spaces within the board (Figure 2a). The structure of the rules and game space make it such that there is an optimal strategy to try and position yourself at specific spaces ("cold" spaces, blue in Figure 2b). Landing on these spaces forces the opponent to make moves that increase the likelihood of you winning by reaching the end point first. The position of these optimal cold spaces generalizes out along the grid according to the golden ratio.

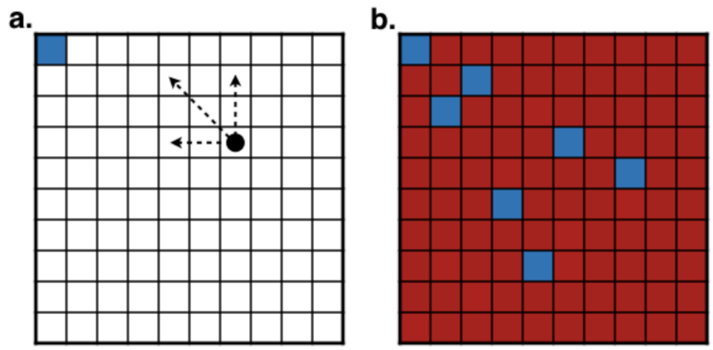

Figure 2. Wythoff's game play. Reproduced from (Peterson et al., 2020) with permission. **a.** Play in Wythoff's game. A player and opponent take turns moving a game piece across the board. Moving any number of spaces leftwards, upwards, or diagonally up-left within the board is permitted. The goal is to reach the upper left space (thus preventing the opponent from reaching the goal). **b.** Wythoff's optimal strategy. The game rules result in a partition of optimal "cold" moves (blue) and suboptimal "hot" moves (red). The optimal strategy is to move to a cold position, which forces the opponent to move to a hot position. This strategy can be employed until the goal position is reached. Cold positions are diagonally-symmetrical and lie dotted along slopes following the golden ratio.

The game of Euclid is identical to Wythoff's, with the exception of having further constrained move rules: diagonal moves are disallowed, and the number of spaces moved toward one edge (the top or left edge) must be a multiple of the number of spaces from the other edge (if already on an edge, any number of spaces are permitted, including moving all the way to the goal). For example, if a player is at (row 2, column 7), they may move to (2, 5), (2, 3), or (2, 1), because they can move any multiple of 2 spaces leftward within the game board. Euclid's rules lead to an optimal strategy that also follows the golden ratio (related to but distinct from the optimal strategy in Wythoff's). In Euclid, the optimal strategy is to move to within the slopes following the golden ratio (see Figure 3b).

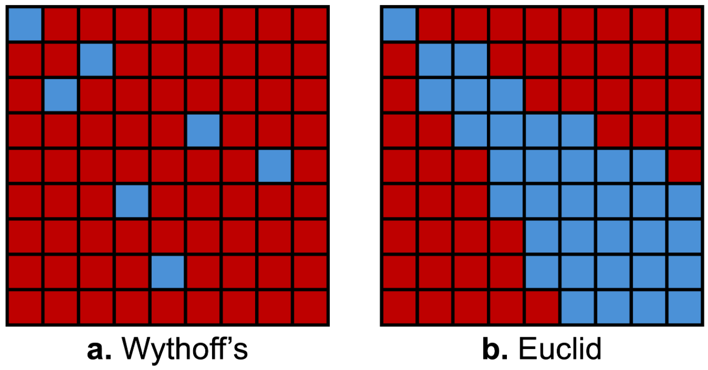

Figure 3. Optimal strategies of Wythoff's and Euclid. In each of these games, there is a single optimal strategy: moving to a cold (blue) position if possible, until the goal space (top left) is reached. Depicted are optimal strategies for **a.** Wythoff's game and **b.** Euclid. These games may be played on boards of any size (their hot/cold structure is size invariant, generalizing out according to the golden ratio slopes).

2.2 Methods

The imagination stumbler-strategist agent

The stumbler-strategist architecture has two main parts (see Figure 4): a standard model-free Q-learning component which is trained directly from experience (the "stumber"), and a deep neural network component which trains on a heuristic of the stumbler's state-value estimates (the "strategist"). The influence (bias) of the strategist over the stumbler's actions is determined by an imaginative gameplay process (detailed below).

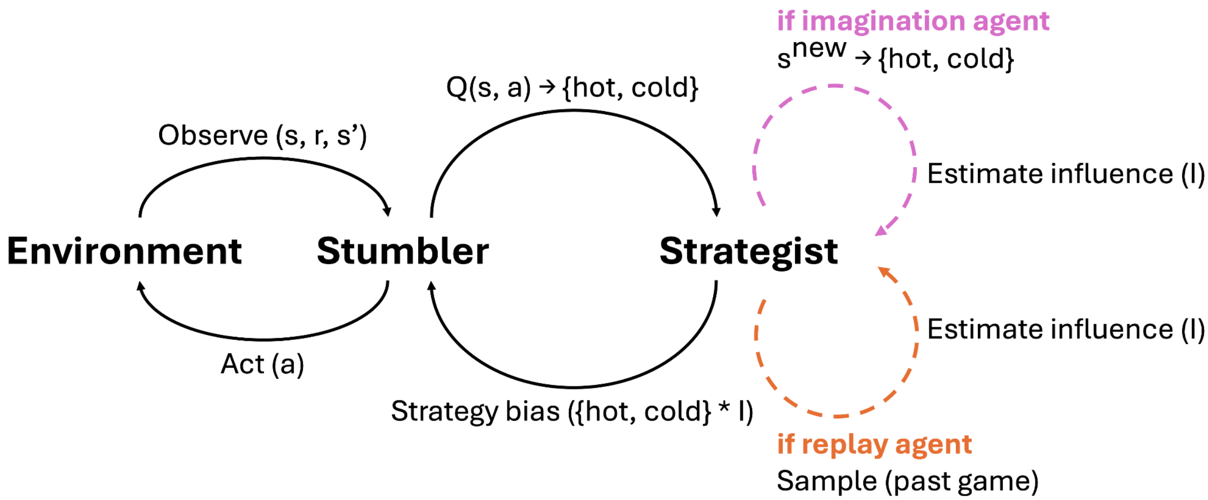

Figure 4. Diagram of the stumbler-strategist architecture. Adapted from (Peterson et al., 2020) with permission. The purple loop (upper right) shows the *original* method of tuning the strategist influence parameter with an imagination learning process. The orange loop (lower right) shows my adapted version of tuning the strategist influence parameter with a *replay* learning process.

The stumbler-strategist learns over many iterations of a three-stage sequence (illustrated via the consecutive loops of Figure 4). Stage 1 (stumbler Q-learning): the stumbler operates in the standard 15x15 grid game environment, performing traditional Q-learning via a lookup table. Stage 2 (strategist training on binned stumbler Q-values): the state-action value estimates are copied from the stumbler, a binary good/bad ("hot/cold") heuristic is applied, and then the strategist trains on this simplified data via stochastic gradient descent. Stage 3 (imaginative play): the stumbler and strategist engage in imaginative head-to-head play (Algorithm 1): the strategist's policy is pitted against the stumbler's policy on a larger 50x50 game board simulated with perfect game rules. The outcome of the simulated play determines whether the strategist's influence over the stumbler is increased or decreased (within the parameter range of 0.0 to 1.0). Learning in the stumbler's Q-table and strategist's network is turned off during this stage so that the strategist influence parameter is the only aspect of the agent that is updated. Finally the agent returns to learning stage 1 (now with an updated strategist bias influencing the stumbler's moves), repeating the learning sequence until a specified number of iterations have been reached.

Algorithm 1: The stumbler-strategist agent imagination learning process (Peterson et al., 2020)

```
procedure INFLUENCE(Stumbler, Strategist)
    α_I ← Influence learning rate
    I ← Influence
    win ← Strategist score
    G ← Initialize Wythoff's game
    s ← A position in G
    while G continues do
        action ← greedy(Stumbler(s))
        do action on G, update s
        if G ends then
            win ← 0
        action ← greedy(Strategist(s))
        do action on G, update s
        if G ends then
            win ← 1
    if win > 0 then
        I ← I + α_I
    else
        I ← I − α_I
    I ← clip(I , 0, 1)
    return I
```

The replay stumbler-strategist agent

Peterson et al. (2020) found that the imaginative stumbler-strategist architecture permitted effective strategy learning by the agent, as compared to other state-of-the art reinforcement learning algorithms and a stumbler-only agent. This supports my Hypothesis 1a. However, it could be that any internal simulation on a world model would boost learning. For example, memory replay is similar to imagination except that it involves simulating specific spatiotemporal sequences of experienced events, as opposed to novel imagined sequences of events. Perhaps memory replay is an equally useful strategy for improved learning.

To test this I built a variant of the stumbler-strategist architecture from Peterson et al. 2020, and replaced the third stage of its learning procedure (imaginative play, Algorithm 1) with a replay-based learning process that does not rely on generative simulation (replay learning, Algorithm 2). Instead of pitting the strategist against the stumbler in a simulated game on a larger board, a previous game is sampled from the last batch of gameplay (Figure 4, purple loop). The sampled game is then “replayed”: the strategist takes the role of the player and the stumbler takes the role of the strategist. The strategist influence parameter is updated according to the outcome of the replayed game: if the stumbler (taking the role of the player) won, the influence is increased and if the stumbler (taking the role of the opponent) won, the influence is decreased. This corresponds to increasing the strategist influence if the sampled game was a winning game and decreasing the influence if the sampled game was a losing game; this is how I implement the replay learning process in the code (again maintaining a minimum influence of 0.0 and a maximum influence of 1.0). This replay learning method was chosen to be as close to the original imagination method as possible while relying on memory replay rather than generative simulation. Comparing outcomes across these two agent types allows for a more specific test of the value of imagination as opposed to the computationally related process of replay.

Algorithm 2: The stumbler-strategist agent replay learning process

```
procedure INFLUENCE(Stumbler, Strategist)
    α_I ← Influence learning rate
    I ← Influence
    win ← Strategist score
    G_sample ← sample(games_in_last_batch)
    for action in G_sample do
        if action was by Opponent
            Stumbler replays action
            if G_sample ends then
                win ← 0
        if action was by Player
            Strategist replays action
            if G_sample ends then
                win ← 1
    if win > 0 then
        I ← I + α_I
    else
        I ← I − α_I
    I ← clip(I , 0, 1)
    return I
```

To increase the interpretability of the results I also made two basic changes to the stumbler-strategist agent simulations. First, I modified the opponent to be of fixed-difficulty (in the original simulations the opponent was a separate stumbler that also learned across gameplay).

For each move the fixed-difficulty opponent makes, it chooses among all legal actions following weighted selection probabilities determined by a softmax function (1). This function computes the probability of selecting an action i based on its value a_i:

$$softmax(a_i) = exp\left(a_i/\tau\right) / \sum_{i=1}^{n} exp\left(a_i/\tau\right) \tag{1}$$

Optimal actions are given value 1.0, other actions are given value 0.0, and an inverse temperature parameter τ setting of 0.55 is used to adjust the exploitativeness of the opponent (at this setting the opponent operates at medium difficulty, not always acting optimally). With the fixed-difficulty opponent, the behavior of the player agent no longer influences the learning of the opponent, as it did in the original Peterson et al. (2020) paper. This means we can more easily compare the performance of different agent types when they all play against this identical opponent. The second basic change I implemented was to have the starting player be random (in the original simulations, the player would always start first meaning the opponent would always take the second move). This more evenly distributes the starting-player advantage across the player and opponent. Removing this bias puts performance results into a space that we can more intuitively understand.

To test the effectiveness of the imagination-based versus the replay-based learning process, 200 simulations of each agent were run. Each simulation began with the same initialization weights but proceeded with a different random seed. For a control condition, 200 simulations of a stumbler without a strategist were run (each also with identical initializations but different random seeds for progression).

Performance was measured according to win rate (a metric of strategy effectiveness) and optimal moves per game (a metric of strategy optimality) (Figure 5). Win rate is calculated as the proportion of games that the player wins out of all games played (2). Optimal moves per game is calculated as the number of optimal moves in each game played (3).

$$win\ rate\ =\ total\ games\ won\ /\ total\ games\ played \tag{2}$$

$$optimal\ moves\ per\ game\ =\ total\ optimal\ moves\ /\ total\ games\ played \tag{3}$$

The training procedures and settings were adopted from the original study (Peterson et al., 2020) with the exception of increasing the number of independent simulations for each agent and ending simulations earlier during plateau. 200 independent simulations were run (original: 20), each with 40 episodes (original: 150). Each episode consisted of 500 iterations of stage 1 stumbler learning with a stumbler learning rate of 0.4, 500 iterations of stage 2 strategist learning with a strategist learning rate of 0.025, and 1 iteration of stage 3 imaginative (or memory-based) stumbler-vs-strategist influence parameter tuning with an influence learning rate of 0.2. As in the original study, a comparison with a stumbler on its own was included: 200 independent simulations (original: 20) of 20,000 episodes each (original: 75,000). GNU Parallel (Tange, 2018) was used for parallelizing independent simulations.

Within-game transfer (strategy generalization to larger board sizes of Wythoff's) for the imagination and replay based agents was qualitatively displayed by visualizing average strategist value heatmaps for spaces outside of the standard game play, up to a 50x50 board size (Figure 6a-b). To more qualitatively compare between imagination and replay strategist transfer beyond the standard game board, the distribution (mean and SE) of strategist values across optimal positions, by board size, were plotted (Figure 7).

Between-game transfer (strategy generalization from pretraining on Wythoff's to retraining on Euclid) was originally tested in Peterson et al. (2020). There, strategists from stumbler-strategists trained on Wythoff's were applied to newly-initialized stumblers and trained in the novel game Euclid. When the agent pre-trained on Wythoff's was meant to be tested on Euclid, it was erroneously tested on the simpler game of Nim (Bouton, 1901) instead. Because Nim is easier for the agent to learn, the agent had a higher win rate than expected, appearing to transfer from Wythoff's to Euclid (Figure 8a). When the original simulation code was corrected to actually test the pre-trained agent on Euclid, no transfer effect was observed (Figure 8b).

An updated test of between-game transfer was performed by comparing performance across four stumbler-strategist agent variants (Figure 9). These agent variants and simulations followed the updated structure used in each of the present simulations. This test was an extension of the original transfer experiment (above), which compared a naive imagination-based stumbler-strategist (naive-to-imagination) to an imagination-based stumbler-strategist composed of a naive stumbler and a strategist pre-trained on Wythoff's (imagination-to-imagination). For testing replay as well, two new agent types were added to these simulations: a replay-based stumbler-strategist composed of a naive stumbler and a strategist pretrained on Wythoff's using imagination (imagination-to-replay) and a replay-based stumbler-strategist composed of a naive stumbler and a strategist pretrained on Wythoff's using replay (replay-to-replay). The pretrained strategists were copied from the Wythoff's simulations described above (either from the imagination- or replay-trained agents as specified). Otherwise, simulations followed the same parameters as the training procedures described above, with the exception of following the rules of Euclid in place of Wythoff's. Cumulative average reward (4) performance at the end of Euclid retraining was compared between all four agent groups using a standard ANOVA.

$$cumulative\ average\ reward\ = \frac{total\ games\ won - total\ games\ lost}{total\ games\ played} \tag{4}$$

2.3 Results

All agents that I tested were able to learn to play Wythoff's game effectively. As seen in Figure 5, performance for each agent type rises quickly before approaching a plateau after around 500 games played. For win rate (i.e., games won out of games played; Figure 5a), at the end of training I found significant differences between the performance of all three agents ($F(2, 597) = 317.461$, $p < 0.001$). Here the imagination agent (μ [mean] = 0.861, σ [SD] = 0.010) outperformed the replay agent ($t(398) = 3.90$, $p < 0.001$) and the control agent ($\mu = 0.831$, $\sigma = 0.004$; $t(398) = 40.80$, $p < 0.001$). The replay agent, however, substantially outperformed the stumbler-alone control agent ($t(398) = 17.37$, $p < 0.001$), showing that the process of internal replay does boost performance generally, although imagination seems to amplify this effect.

A nearly identical pattern is found when looking at optimal moves per game (Figure 5b), a measure of detecting use of the optimal strategy. Here again we see a significant difference across all agent types, ($F(2, 597) = 2786.647$, $p < 0.001$), with the imagination agent ($\mu = 1.833$, $\sigma = 0.045$) using slightly more optimal moves than the replay agent ($\mu = 1.813$, $\sigma = 0.075$); $t(398) = 3.227$, $p = 0.001$) and substantially more than the control agent ($\mu = 1.489$, $\sigma = 0.019$); $t(398) = 98.776$, $p < 0.001$). The replay agent still outperformed the stumbler alone control agent in terms of making optimal moves ($t(398) = 59.321$, $p < 0.001$). Note that it only takes 1 or 2 optimal moves to win at Wythoff's game, so anything over 1 is considered expert level performance.

In short, for both win rate and optimal moves per game, the imagination-based learning process led to statistically significantly greater performance than the replay-based learning process. Also for each measure, both the imagination and replay agents performed substantially better than the stumbler-alone control agent.

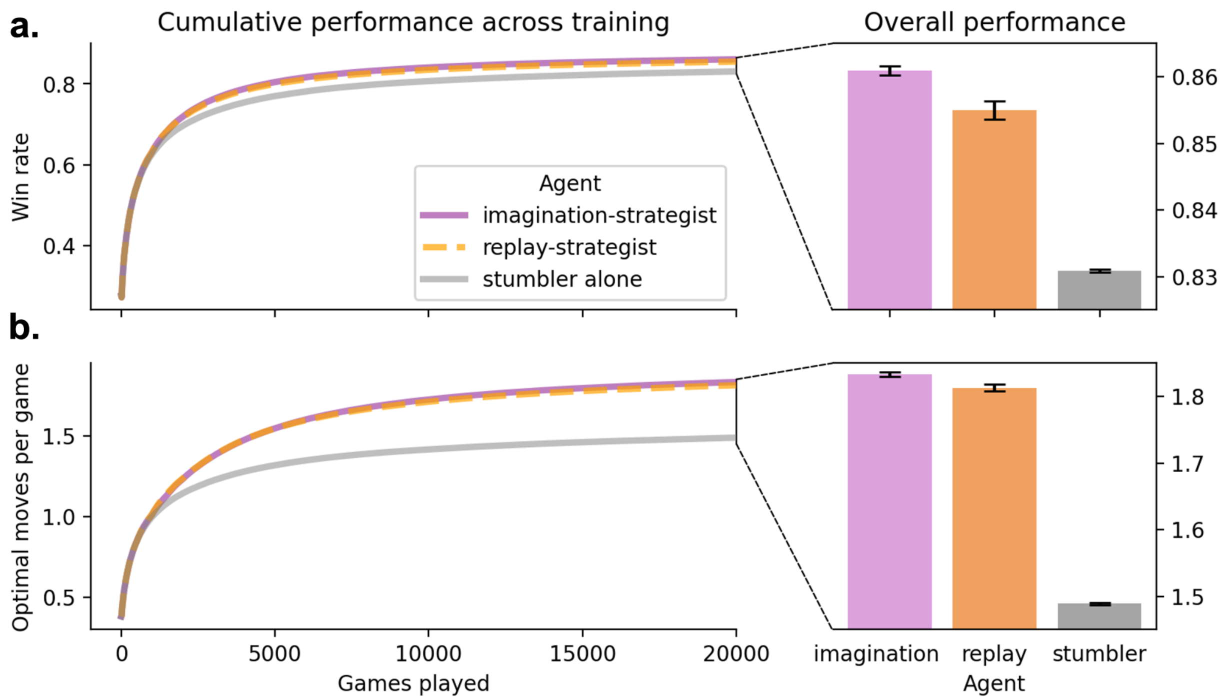

Figure 5. RL agent strategy learning with imagination vs. memory replay. a. Win rate and **b.** number of optimal moves per game performance of the imagination, replay, and control agents across learning (n=200 separate runs of each agent type). Error bars represent standard error.

To see what strategies the agents learned, I visualized their average position value for the influence maps for each board space after learning (Figure 6). Average strategist value heatmaps are plotted for the imagination (Figure 6a) and replay (Figure 6b) agents. In these agents, the strategist becomes the dominant decision maker during learning. The average stumbler heatmap is plotted for the control agent, the stumbler on its own (Figure 6c). Because of the strategists' neural network structure, they can generate value estimates for move positions outside of their training data. These strategist neural networks take a scalar (x, y) coordinate input representation (rather than a binary-type or board image representation of position). Consider that a simple neural network with only two scalar inputs weighted respectively by 1 and -1 in connection to a zero-thresholded output node can signal which input is greater, regardless of magnitude. This is the same as signaling whether an (x, y) input coordinate lies above or below the identity function (x = y) regardless of distance out in the x y plane. Similarly the strategist networks, with (x, y) input representations and many more nodes, can signal (extrapolate) output values for board positions arbitrarily far from the origin. Note also that both the imagination and replay-based agent strategist networks learn only from stumbler value estimates of the standard 15x15 board. Thus any generalization to larger board sizes is due to a strategist network's weights capturing patterns such that when untrained positions are propagated through the network, values for these untrained moves are effectively estimated. The imagination-based agent does play on the extended board, but this play only updates the strategist's overall influence parameter, not any individual move valuations. A stumbler's Q-table (state-action value lookup table) does not support extrapolation. For this reason, the imagination and replay strategist value heatmaps are expanded in Figure 6 while the stumbler control heatmap is limited to the size of the game board (15x15). We see that the stumbler-only control agent, which is never guided by a strategist, does not learn more than the first few optimal ("cold") moves closest to the goal (Figure 6c). The imagination and replay agents both learn the invariant golden ratio structure of the optimal game strategy. However, the imagination agent appears to learn this invariant strategy more strongly, as evidenced by its higher heatmap values along the golden ratio slopes farther out from the goal space (Figure 6a-b). This visual qualitative pattern is more qualitatively visualized in Figure 7.

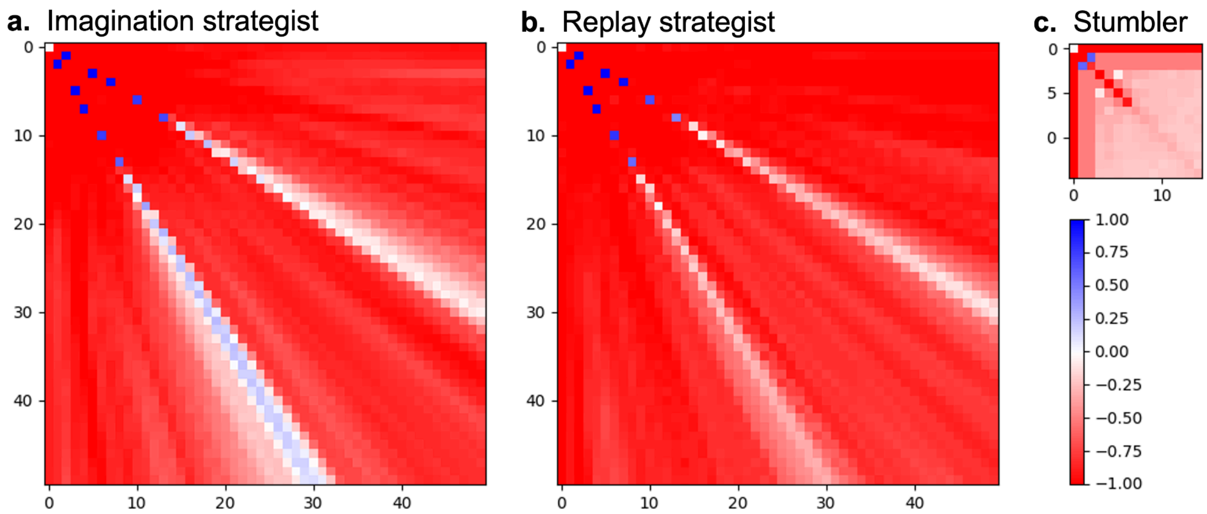

Figure 6. Agent component strategy maps. a. Imagination agent strategist value heatmap, **b.** replay agent strategist value heatmap, and **c.** stumbler-alone agent stumbler value heatmap (averages over n=200 separate runs of each agent type). Value maps are from after 20,000 games of learning. Imagination and replay agent heatmaps are extended out to a 50x50 board size to show strategy generalization of each strategist (though the replay agent never plays in a game that size). Each plot shares the same value-color range mapping (color bar in the lower right).

When comparing the distribution (mean and standard error) of strategist value estimates across optimal positions by board size (Figure 7), we see that for most board sizes, the imagination agent strategists estimate higher values for optimal moves. This begins around board sizes of 10x10 (within the standard board size), and extends to much larger board sizes (standard error intervals around means begin to overlap around board sizes of 40x40). This shows stronger learning of optimal moves within the standard 15x15 board by the imagination agent, as well as stronger generalization to optimal moves at larger board sizes.

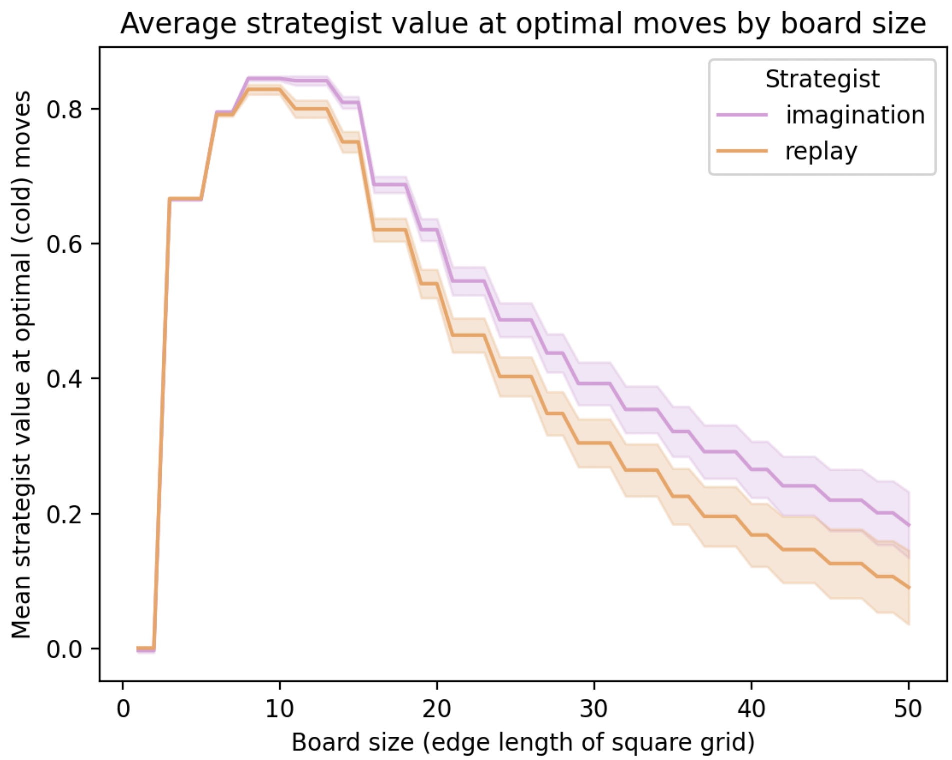

Figure 7. Board size strategy transfer in Wythoff's. Average strategist valuation of optimal (cold) moves by board size. Board size is measured by edge length ("50" means a 50x50 grid). Means are from n=200 separate runs of each agent type. Shading represents standard error.

One surprising finding from the original study of the stumbler-strategist (Peterson et al., 2020) was its game transfer result: when pre-trained on Wythoff's, the agent appeared to learn Euclid substantially more effectively. Unfortunately, I found that this was due to a copy-paste error; this effect does not actually exist (the agent was run on the easier game of Nim instead of Euclid). This strategy transfer was erroneously found to aid performance (wins against a traditional Q-learning agent) relative to newly-initialized stumbler-strategists (see Figure 8). The co-authors of the original paper and I will be submitting an erratum to the journal where the original paper was published.

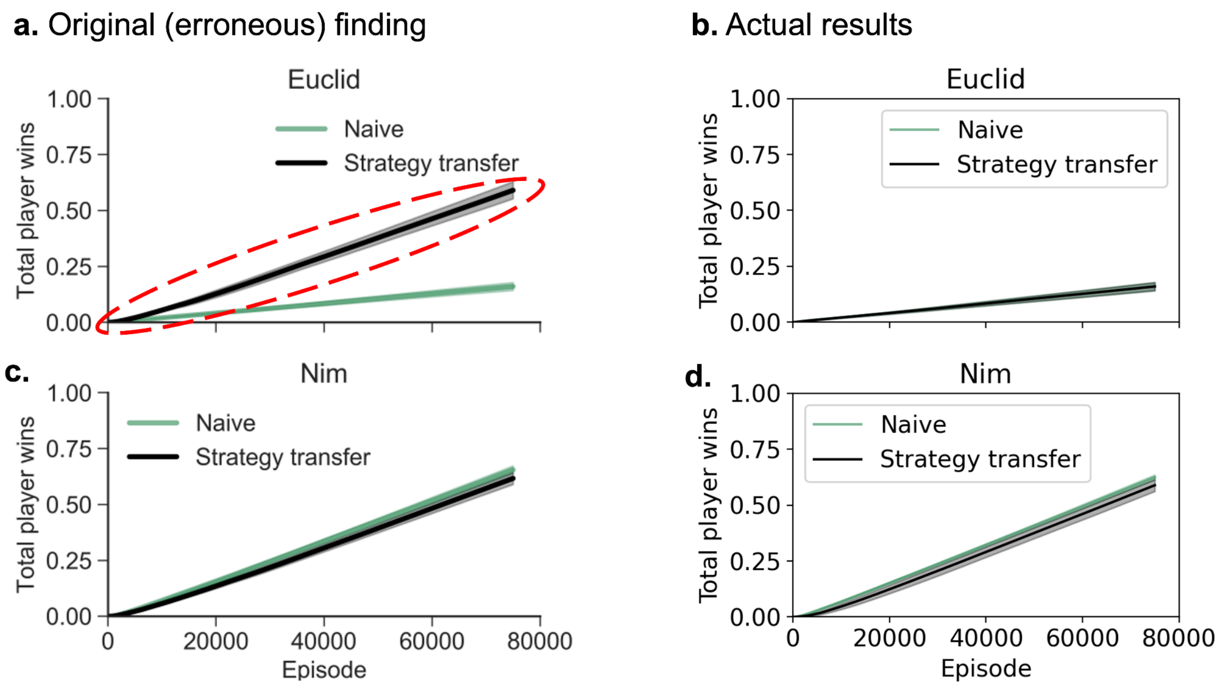

Figure 8. Original erroneous pre-trained strategist transfer result and correction. a. Erroneous original finding of player wins in Euclid's game, with and without strategy transfer (plotted data circled in red were in actuality from a pre-trained agent tested on Nim). **b.** My corrected finding of player wins in Euclid's game. **c.** Original finding of player wins in Nim. **d.** My reproduced finding of player wins in Nim. The opponent in these simulations was a traditional Q-learner. Adapted from (Peterson et al., 2020) with permission.

To test for effects of imagination vs. replay based learning strategies on transfer (strategy generalization) from Wythoff's-pretraining to Euclid, cumulative average reward in Euclid was compared between 4 agent types (Figure 9): a naive, imagination-based stumbler-strategist (naive-to-imagination), an imagination-based stumbler-strategist pretrained on Wythoff's (imagination-to-imagination), a replay-based stumbler-strategist pretrained on Wythoff's using imagination (imagination-to-replay), and a replay-based stumbler-strategist pretrained on Wythoff's using replay (replay-to-replay). No significant differences in total average reward between agent types was observed (F(3, 896) = 0.061, p = 0.980). Because no agent exhibited an advantage over no pretraining (the naive agent), there is no evidence of stumbler-strategist transfer from Wythoff's to Euclid. If anything, there appears to be a slight negative trend, with all pre-trained agents performing slightly worse, especially with greater degrees of replay learning (though this pattern is not significant).

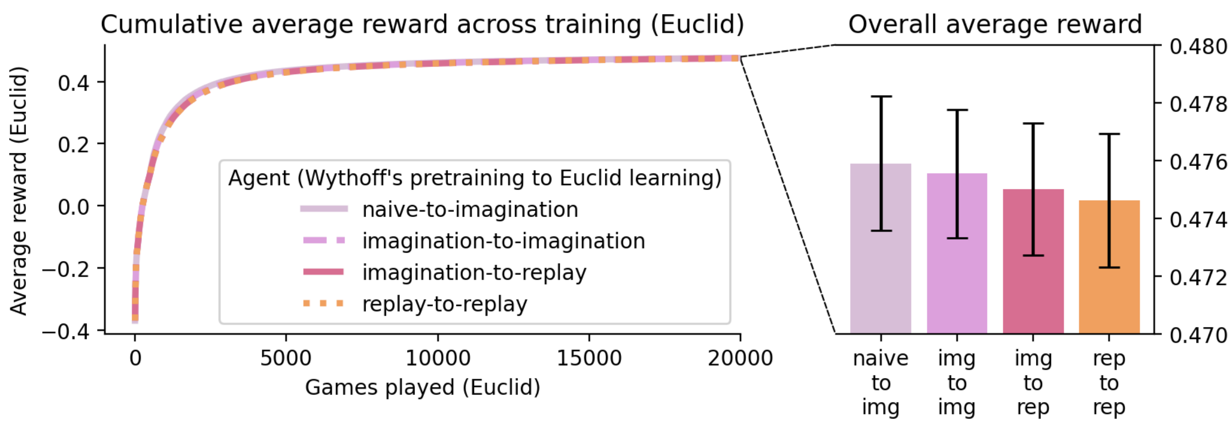

Figure 9. Lack of transfer from Wythoff's pretraining to Euclid learning. Average reward achieved by different agent types across learning in Euclid (n=200 separate runs of each agent type). Agents differ by Wythoff's-pretraining method and Euclid learning method: (1) no pretraining on Wythoff's (naive) and learning Euclid with imagination, (2) pretraining on Wythoff's using imagination and learning Euclid with imagination, (3) pretraining on Wythoff's using imagination and learning Euclid with replay, and (4) pretraining on Wythoff's using replay and learning Euclid with replay. Error bars represent standard error.

2.4 Discussion

Hypothesis I.a predicted that the imagination agents should have increased learning efficiency in Wythoff's compared to a replay-based network or a simple, model-free reinforcement learning network. We do, in fact, see that the imagination-based learning method affords an advantage over the replay-based learning method and a basic RL control condition (Figure 5). Why would this be the case? Let us remember that the only difference between the imagination and replay agents are their separate methods for tuning the strategist's influence parameter. There are three possible reasons why the simulation-based tuning process may afford an advantage over the replay-based tuning process. First, the outcome of a simulated game with the strategist vs. the stumbler may be a better signal of whether the overall agent would benefit from relying more on the strategist or the stumbler (a replay sampled from the last batch of games may be a nosier and/or less direct signal of this). By simulating a game on a larger board, the imaginative simulation can take into account a greater sampling of the space of possible learning scenarios. This may support a more informed signaling of whether strategist influence should be incremented or decremented. Second, the imagination method employs the most up-to-date version of the agent, from after learning phase 2 (strategist neural network training). The replay method samples a game enacted during phase 1 (stumbler learning) after which some amount of stumbler learning and strategist training can update the agent. Imaginative simulation allows an agent to make a decision to tune strategist influence up or down based on the current version of the agent, while replay is necessarily based on a past experience after which some amount of change may have occurred in the agent. Third, these primary differences between the imagination-based and replay-based learning methods may also have subtle downstream effects on exploratory behavior which play into performance. For example, the simulation-based strategist influence tuning may guide the stumbler towards moves which are more informative for improving strategy.

These three possible reasons for imagination's advantage (more directly signaling future possibilities and predicting based on an agent's most current status) may be general. In humans, for example, it is not unlikely that imagining anticipated future scenarios would support better preparation for upcoming events than retrieving past memories which necessarily may be less directly relevant and out-of-date with the current situation in a changing world (and self). Shtulman (2023) brings up this trade-off between knowledge and imagination. This principle relates closely to imagination as previously studied for reasoning, planning, and prospective memory, as these are all use cases which require explicit predictions based on specific scenarios. For example, human 3D rotational reasoning (Shepard & Metzler, 1971) operates on the trial at hand, where dynamic simulation of an object is more directly relevant to the task than recalling a similar trial from the past. In planning, simulation can allow for more goal-directed control than searching over limited memory stores. The artificial agents developed by Pathak et al. (2017) and Sekar et al. (2020) use imaginative simulation to plan exploration that will better address gaps in agent knowledge. In this way, imagination allows for the most current status of an agent to factor into future-oriented decision making. We also see that in studies of rodent navigation, many place cell activation sequences are goal-directed (K. Kay et al., 2020; Mou et al., 2022; Pfeiffer & Foster, 2013; Widloski & Foster, 2022; Wikenheiser & Redish, 2015), and that mental representations of goal positions can be invoked volitionally (Lai et al., 2023), suggesting that simulation similarly affords NIs mental control based on the current situation. Prospective memory, which is aided by imagining one's future self enacting a prepared intention (Abel et al., 2024; Grilli & McFarland, 2011), may also benefit from imagination's often more direct signaling of future possibilities than replays of past selves enacting past intentions. Across all these examples, a pattern emerges that task-relevant memories to replay may not always be available, in which case imagination is even more valuable. The stumbler-strategist simulations in this work were specifically designed to compare imagination against replay, but do not model conditions where no pertinent replays are possible, in which we would likely see an even greater advantage for imagination.

Another question is why the imagination and replay agents do so much better than the control agent. This is expected, given that the control agent (a stumbler on its own) is much simpler than the imagination and replay agents (stumbler-strategist agents). Peterson et al. (2020) has already investigated the effect of testing a stumbler-alone vs the original (imagination-based) stumbler-strategist. Basically the strategist's neural network, trained on hot/cold-categorized state-action value data from the stumbler, is able to extract meaningful patterns that guide behavior more strategically. Without a strategist, a stumbler can make no informed estimates of the value of spaces it has never visited; it can only learn the value of spaces it does visit slowly, one game at a time, through trial and error. Thus we see a trade-off between the simpler model-free stumbler agent and the more complex model-based stumbler-strategist agents which are more effective picking up variance, but at a greater computational cost.

I also predicted that imagination would boost generalization to Euclid compared to replay and control networks (Hypothesis I.b). This hypothesis was premised on the original transfer effect that I found to be erroneous (Figure 8), and was not confirmed. Pretraining on Wythoff's game with imagination did not support better performance in Euclid, and neither did pretraining and/or retraining with replay-based learning methods (Figure 9). However, optimal strategy in Euclid bears relation to optimal strategy in Wythoff's (Figure 3), so why do we find no "far" (between-game) transfer effect at all? One reason is that the similarity between Wythoff's and Euclid may be too high-level for the fine-grained strategy learned in Wythoff's to benefit move decisions in Euclid. Additionally, the pre-training may cause interference in terms of perseverating along old ingrained modes of action and having to unlearn old behaviors instead of learning new behaviors from scratch. Perhaps, if the imagination process was not constrained to Wythoff's and was instead extended to follow Euclid move rules, agents would be nudged toward generalizing to Euclid.

While no far transfer results were observed, it is relevant to note that Peterson et al. (2020) did find that pre-trained stumbler-strategists achieved "near" (within-game) transfer. When tested with frozen learning on larger board sizes of Wythoff's, strategist move optimality remained high while stumbler move optimality dropped. This is expected considering the size-invariant strategy learned by strategists (Figure 6) and the fact that the stumbler has no way of estimating move values outside of its original board size. One possible future test of generalization could be to evaluate pre-trained imagination-based vs. pre-trained replay-based stumbler-strategists on larger board sizes (a test of within-game board size transfer). My prediction is that we would see a pattern similar to the current Wythoff's performance results (Figure 5), with the imagination agent showing a small but significant advantage. After all, we see both imagination and replay agents learn size-invariant Wythoff's strategies, though the imagination agent's strategy appears stronger (Figure 6-7).

The pattern of when we see generalization in these simulations (within variations of the same task but not between separate tasks) may be general. Ultimately, it makes sense that imagination should be most useful for tasks that more closely resemble the simulations it produces. Imagining one task may not provide an advantage for a task that doesn't match the imagined task closely enough, and may even interfere with performance if inappropriate behaviors are overtrained through misplaced imagination. Across the literature we see examples of imagination being useful when tailored closely enough to the task at hand. One case is the prospective memory task of Abel and colleagues (2024), where the task was to write an “X” next to knowledge test questions about space. They found that imagining a salient but less directly task-relevant scenario (being visited by UFO at one’s desk) was less effective at preparing subjects to perform the task than imagining the perhaps less engaging but more directly relevant scenario of the task itself. In Reinhart et al. (2015), imagined rehearsal of a specific visual search task yielded better search performance during the task itself than spending the same amount of time actually practicing the task. Similar to interference caused by inadequately relevant replay, actual practice may have interfered with future performance in a way that targeted imagination did not. On a related note, Leahy and Sweller (2005) found that among students with prior experience with a task (reading bus timetables and temperature time graphs), imagining the task increased performance relative to studying task instructions, but having no prior knowledge reversed the effect. This highlights that being able to form simulations that can adequately match the relevant task (by having prior experience) is necessary for imagination to be advantageous. The problem of needing imagined scenarios to align closely enough with future experiences to help (and not harm) future performance appears critical to any application of imagination for preparation.

Conclusion

Toward Aim I, my results show that imagination, more than replay, boosts the performance of the stumbler-strategist agent model. Imagination may help by providing more relevant and up-to-date signals for decision making. Additionally, simulation with a world model (in both imagination and replay strategist learning) boosts learning over model-free RL (in the stumbler-only agent). No agent exhibits far (between game) transfer, though imagination supports near (within-game) board-size invariant strategy learning (as does replay, to a lesser extent). These findings hint at possible generalities for the utility of imagination in contrast to memory replay. However, the findings arise only from tests of stumbler and stumbler-strategist agent variants. The advantages and limitations of testing with a computational model can be balanced by empirical testing with humans. In the next chapter, I report the results of a similar experiment for my human empirical aim to see if the patterns identified here extend to NIs.

Chapter 3: Human empirical tests of imagination and replay

The simulation results in Chapter 2 suggest that both imagination and memory replay have advantages for strategy learning in Wythoff's game, but with imagination boosting performance over replay. To determine if these patterns extend beyond the AI models, I test their predictions in a human behavioral experiment. The empirical results validate the predicted advantage of imagination over replay but find no performance boost from replay relative to control. As in the simulations, far generalization (between games, to Euclid) is not observed. Thus in humans imagination appears to boost strategy learning within an imagined task but not necessarily beyond it.

3.1 Introduction

Imagination interventions in the literature

As described in the introduction, many studies have investigated the effect of imaginative practice (mental rehearsal) on the learning of skills, particularly in the area of sports performance. Most if not all existing experiments have tested imaginative practice for defined unchanging tasks (retrospective learning) such as making volleyball serves (Shick, 1970) or performing pelvic exams (Rakestraw et al., 1983). Existing experimental methodologies which appear best suited to test game learning and generalization appear to be those utilized by Leahy & Sweller. These authors examined the “imagination effect” in learning complex cognitive skills for tasks in which the learned procedure stays the same but the object/data on which the procedure is applied may change, such as reading a bus timetable (Leahy & Sweller, 2005), a temperature time graph (Leahy & Sweller, 2004, 2005), or calculating elevation gradients from a topographic map (Leahy & Sweller, 2005). Their setup entails comparing conventional studying of task instructions (control) to imagining the task while going over the same instructions. Importantly, Leahy and Sweller (2005) found imagination to be beneficial for task learning (relative to studying) only if the task was complex enough and only after participants had some initial task experience. It thus appears that in human tests of imagination for strategy learning, participants should have adequate experience with a task of appropriate complexity to derive effective learning from task imagination.

It is tricky to design appropriate control conditions for imagination experiments. For most empirical studies of imagination in the literature, if there is a control condition for the imagination intervention, it is often “physical practice”, “no practice”, or some variant of a “study” condition (like mentioned above). Imagination in the form of mental practice of athletic skills is frequently compared against physical practice (of the skills of interest) or against no practice (neither imagined nor physical practice); for a review of the effects see Toth et al. (2020). Studying of task instructions is the last control condition commonly employed in the literature. Outside of the pedagogical imagination studies by Leahy and Sweller, prospective memory studies of imagination also take variants of this approach: Grilli & McFarland (2011) test imagination of a prospective memory task against verbal rehearsal of the prospective memory instructions, and Abel et al. (2024) test imagination of a prospective memory task against verbalizing an intention to implement the prospective memory task instructions. All the control conditions discussed here are clever choices that can help isolate benefits of imagination in relation to other common learning approaches in each respective application area (sports, pedagogy, prospective memory).

However, these controls are limited for more general utilization. Physical practice of a behavior may come at a high cost or may be unavailable (e.g., play-fighting can be energy-intensive and/or dangerous, and inaccessible without a partner). Also, we rarely have instructions to study or rehearse for the behaviors that we learn and practice in our daily lives (and instructions of the sort familiar to us are not relevant for nonhuman NI learning). In my human experiment based on Chapter 2's model situations, the replay comparison takes care of many of these issues. Replay is much more comparable to imagination in terms of cost and accessibility, and can be used similarly to imagination as long as relevant memories are available (as is often the case when there is at least some experience with a desired task). And as discussed in Chapter 1, memory replay is closely related to imagination, even at the neural level, making it a widely appropriate control. Replay however is more than a control; it is its own learning condition. As a control for the imagination and replay conditions, and to match the model situations, I include a third condition which does not require imagination or memory (though it does require attention from participants, like the imagination and replay conditions).

Based on the literature introduced here and the results of my model simulations above, I designed an intervention version of Chapter 2's simulation experiments for testing in humans. This behavioral experiment tests for the specific outcomes across conditions predicted by the model simulations. I use an intervention-style design with a pre/post intervention structure and a generalization test. The intervention is introduced only after participants have some game experience. Because the open-ended games of Wythoff's and Euclid are nontrivial and require strategy learning to play well, they should be adequately complex for imagination to affect participant learning.

3.2 Methods

Participants

Participants were recruited from the Carnegie Mellon University Psychology Department for-credit participant pool. In total, there were 274 participants. Data from 19 participants were not usable due to fire drills, technical issues, and experimenter error (running participants on the wrong version of the task). Of participants with usable data, there were 255 (85 in each condition: imagination, replay, and attentional control). The duration of each experimental session from participant arrival to discharge was on average approximately one hour. All participants provided informed consent under a procedure approved by Carnegie Mellon University's institutional review board.

Tasks

Participants' phones were stowed and any smartwatches were turned to do-not-disturb for the duration of each study time slot. The task was built and run using the PsychoPy builder and coder interface (Peirce et al., 2022). The experiment structure mimicked what was done with the computational models in theme. Each participant played 3 tutorial rounds followed by 30 initial rounds of Wythoff's game against the same computer opponent as the simulations. The opponent's fixed level (τ=0.55) is high enough to push participants to engage in strategy learning without being so high as to push participants to give up. Random starting positions (and the first move if the opponent starts) are fixed across subjects to reduce variability. No starting position is within one move of the goal, to prevent trivial games.

After the initial game rounds, participants completed 30 trials of an intervention task that differs depending on the participant's assigned condition group. The mouse pointer was removed during intervention trials for all groups to minimize the degree of noise from possible interactions. Participants in the imagination condition (N=85) were shown starting positions and asked to imagine finishing the game playing both their role and the opponent's role. For each starting position, participants were given a delay during which to imagine, and were then prompted to report the winner after each imagined game. The duration of each delay was 75% of the duration of the original game played from each prompted starting position; starting positions were sampled from the starting positions of the initial gameplay rounds. These were shuffled in a random order that was fixed across participants to reduce variability and keep participants from being aware of this link to the pre-intervention trials. Keeping the same pool of starting positions as in the pre-intervention trials reduces non-critical differences between the imagination condition and the replay condition (see below), which shows the same starting positions due to the fact that replays are being shown. Pilot runs confirmed that starting positions were not recognized by the participants.

Participants in the replay group (N=85) were shown move-by-move replays of their previous games in a fixed random order (the same order as in the imagination condition). The duration of each replay was determined by the duration of the original game (compressed by 25%), just as the duration of the imagination delay is determined by the duration of the original game in the imagination condition. The ordering of the replay trials was shuffled from their order during the pre-intervention trials (the same shuffling order as in the imagination intervention was used). At the end of each replay, participants were prompted to report the winner of the reviewed game.

Participants in an attentional control group (N=85) were shown starting positions of previous games in the same random order as in the imagination and replay conditions, but were only asked to report whether the game piece changes color briefly in a “blink” (500ms duration). Each trial had a 50% chance of having the stimulus change color. Thus, this condition required attention and responses like the imagination and replay conditions, but did not require subjects to engage in any form of internal simulations or reasoning.

After the intervention trials, all participants played 30 more rounds of Wythoff’s with new fixed random starting positions and fixed first moves when the opponent starts. This allowed us to compare performance changes from the pre-intervention section to the post-intervention section across the conditions.

Finally, possible generalization to Euclid was tested with a fourth section (see Chapter 2 for a description of Euclid’s rules). Participants were introduced to the new game Euclid, of which they played 3 tutorial games followed by 30 standard games. The fixed-difficulty opponent followed the same algorithm as before, but selected among legal Euclid moves based on an approximation of Euclid move optimality. As in the previous sections, the starting positions (and first moves for opponent starts) followed a predetermined random sequence that was fixed across subjects.

Analysis

The same performance measures from the artificial agent analyses (win rate and optimal moves per game) were used for the human experiment analyses. For the human participants, the difference in performance from the pre-intervention section to the post-intervention trials was specifically considered (post intervention performance - pre intervention performance).

The first step was to confirm that there were no significant differences in performance measures between the human groups during the pre-intervention section, when the experiment is the same for each participant. This was done using standard ANOVAs. Statistical analysis of human performance deltas consisted of outlier removal followed by the same tests used on the artificial agent measures (ANOVAs followed by two-sample t-tests if appropriate). Grubbs' test at an alpha of 0.05 was used for outlier exclusion (python function outliers.smirnov_grubbs.test). For pre-intervention win rate performance, one outlier in the replay group was excluded (no outliers were excluded for pre-intervention optimal moves per game performance).

For both difference measures (post intervention performance - pre intervention performance, for win rate and optimal moves per game), one outlier from the replay group and one outlier from the control group were excluded. For the generalization test (Euclid play), no outliers were excluded for either performance measure (Euclid win rate and Euclid-optimal moves per game).

3.3 Results

As with my simulations, I looked at both the overall win rate and optimal moves per game as the primary performance measures. One-way ANOVAs, after outlier detection, confirm a lack of significant differences across groups during the pre-intervention sections. For both win rate ($F(2, 251) = 1.457$, $p = 0.235$) and number of optimal moves per game ($F(2, 252) = 0.967$, $p = 0.382$). Thus baseline performance was considered equivalent for all three groups.

All three groups showed a small but reliable increase in win percentages after the intervention period (Figure 10a-b). This reflected an improvement in 1-2 games (3-7%) out of 30 across groups. Among these group-level improvements, there were significant group differences ($F(2, 250) = 5.118$, $p = 0.006$). In general it appears that the imagination group ($\mu = 0.065$, $\sigma = 0.096$) had a greater improvement in terms of number of games won compared to the replay group ($\mu = 0.018$, $\sigma = 0.116$); $t(167) = 2.888$, $p = 0.004$), and control group ($\mu = 0.025$, $\sigma = 0.096$); $t(167) = 2.720$, $p = 0.007$). As a difference from our simulations, the replay and control groups were not significantly different in their win rate change during the second testing phase ($t(166) = -0.431$, $p = 0.667$).

This pattern was largely the same for improvement in optimal moves per game. All groups once again showed a small but reliable increase in optimal moves per game after the intervention period (Figure 10c-d). This reflected an increase of 1-4 optimal moves, in aggregate, over the 30 games between groups. As a reminder, it usually only takes 1-2 optimal moves to win in Wythoff's game. Significant group level differences were again found between the three groups (one-way ANOVA $F(2, 250) = 4.274$, $p = 0.015$). As with win rate, the imagination group ($\mu = 0.140$, $\sigma = 0.221$) showed significantly more optimal moves during the second phase of training compared to the replay group ($\mu = 0.050$, $\sigma = 0.254$); $t(167) = 2.457$, $p = 0.015$) and the control group ($\mu = 0.045$, $\sigma = 0.238$); $t(167) = 2.691$, $p = 0.008$). The replay and control groups did not differ in terms of change in optimal moves t ($t(166) = 0.135$, $p = 0.893$).

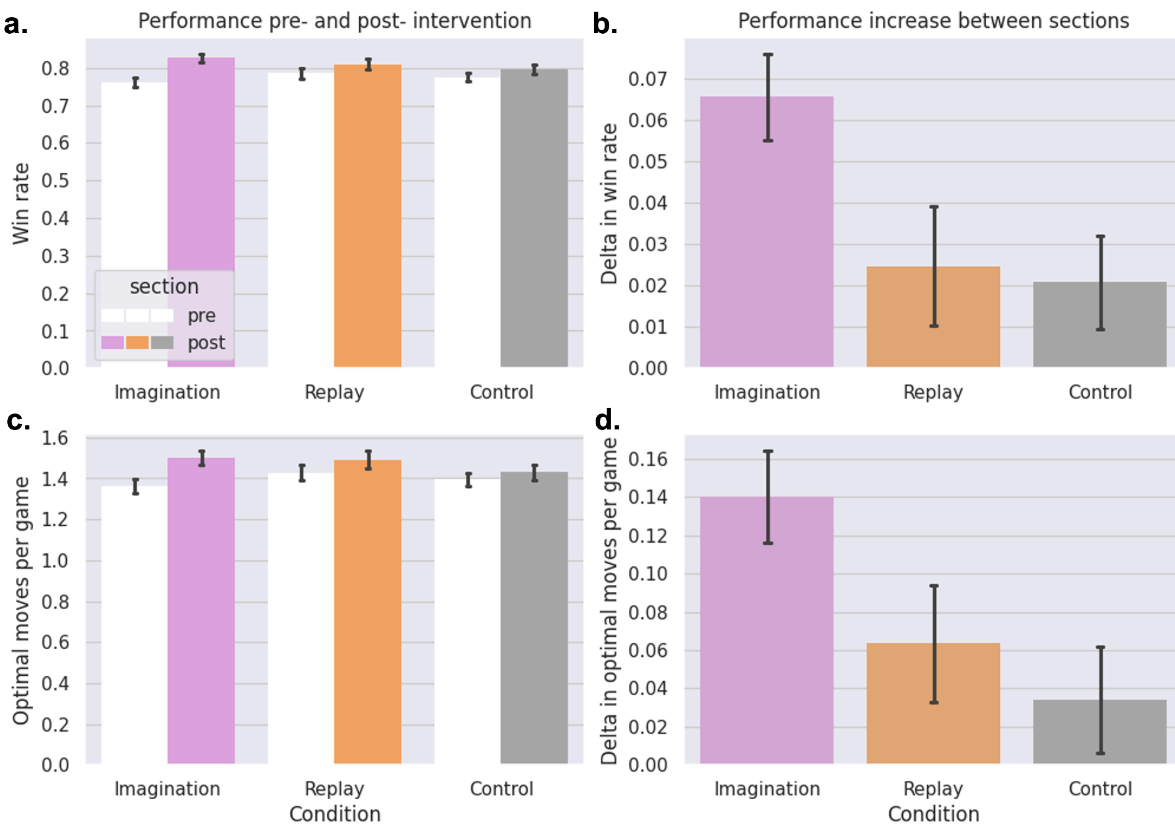

Figure 10. Human strategy learning with imagination vs. memory replay. Performance of the imagination, replay, and control groups in the human experiment: **a.** Win rate before and after the intervention. **b.** Change in win rate from before to after intervention. **c.** Optimal moves per game before and after the intervention. **d.** Change in optimal moves per game from before to after intervention. Error bars represent standard error.

As in the simulations, possible generalization to the game of Euclid was evaluated. Here, win rate was again measured, along with optimal moves per game (this time according to Euclid's ground truth solution). During this final experiment section of Euclid game play, there was no clear effect of any intervention (Figure 11). There were no significant group-level differences in Euclid win rate ($F(2, 252) = 0.548$, $p = 0.579$). There were also no significant differences in Euclid optimal moves per game between the groups ($F(2, 252) = 0.194$, $p = 0.824$). As there were no group-level differences, pairwise t-tests were not performed.

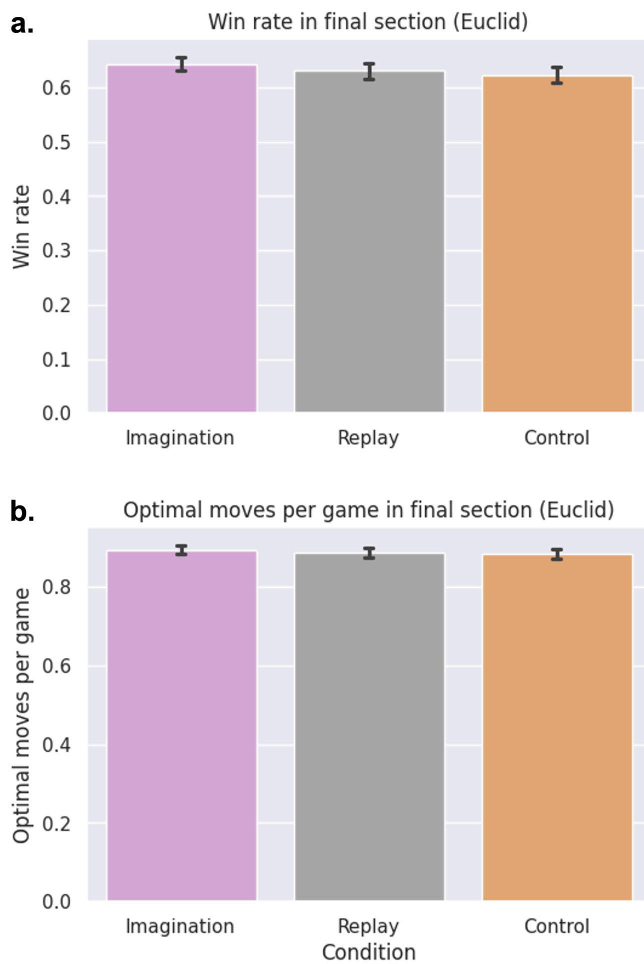

Figure 11. Win rate and optimal moves per game in Euclid. a. Win rate for human participants across all 30 games of section 4 (the final section). **b.** Optimal moves per game for human participants across all 30 games of section 4. Error bars represent standard error.

3.4 Discussion

Hypothesis II.a predicted that the imagination group should boost their performance in Wythoff's compared to the other two groups. Indeed, this was the case in our human experiment: the imagination intervention group exhibited significantly greater increases in both measures of performance, win rate and optimal moves per game, relative to the replay and control groups (Figure 10). Interestingly, the replay group performance increases were not significantly different from the control group changes.

Regarding the relative advantage of the imagination intervention, there are a few possible explanations in addition to those discussed concerning imagination's advantage in the model simulations. In relation to watching replays, imagining games may give participants experience with new play sequences that they might encounter later, while replay can only cover plays that are not new. Also, through engaging in the more exploratory imagined play, participants may be more likely to discover patterns of game optimality structure and strategy than when reviewing past plays. It is relevant to note that we see this learning advantage of imagination over replay despite the fact that the replay condition is much more visually rich. With greater sensory stimulation and precise recapitulation of the task, the replay condition could have drawn more participant attention and provided cleaner training signals. Still, the imagination intervention with its static prompt (starting player & position) screens led to greater learning than the replay intervention. Of course, imagination participants may have produced their own sort of comparable sensory stimuli by focusing their gaze on one board position at a time or by hovering their fingers over the monitor screen (see "Limitations and extensions" below). One last possibility for replay's relative disadvantage as seen here is that the replay in this task was externally produced rather than internally produced (maybe if the replay had been self-generated through the standard process of memory recall, replay group performance would have been higher). After all, the imagination intervention entailed (internally) self-generating games. This said, game replay videos were used to mimic perfect replay, and because accurately recalling each of the 30 initial games may be impossible (as mentioned in the methods, pilot participants did not even recognize the starting positions). Clever follow-up experimental designs may be able to compare self-generated imagination against self-generated replay in case externally-generated is that much less effective (also see “Limitations and extensions”). As for the imagination group’s learning advantage over the attentional control group, it makes sense that imagining games should provide more “training” and strategy learning opportunity than watching for game piece color changes, which does not engage imagination or replay.

It is surprising that contrary to Chapter 2’s simulation results, there are no significant differences between replay and control group learning performance measures. After all, it seems that replay would constitute some form of useful practice. There may be several factors at play. For one, it is possible that replay does not actually aid learning in situations like this experiment’s task. It could be that any additional learning from reviewing a past game was negligible compared to the learning that took place at the time of the game itself (during the memory encoding). In this vein, replay may only help if repeated several times, helping to overtrain on tasks with minimal branching structure, where a replay will correspond to future scenarios. On the other hand, replay might indeed support strategy learning beyond control conditions in situations like this experiment’s task, though not in a way capturable by the present experimental design. For example, a replay advantage may have such a small effect size that to be uncovered, many more participants would need to be run. It could also be that replay could show some advantage if there were more games in the initial Wythoff’s section, in case a greater pool of replays would be more beneficial. Separately, replay may aid learning on a different timescale than tested here, helping to consolidate the learning from the initial games for diminished forgetting long-term (if tested later). Replay after a longer delay between the initial gameplay section and the intervention section might also help refresh the initial learning via spaced repetition (C. A. Mace, 1932; Ebbinghaus, 1913). Alternatively, backward replay as opposed to the forward replay emulated in this design could have boosted learning; backward replay is a natural phenomenon which has been found to increase in frequency following increases in reward (Ambrose et al., 2016). Finally, it may be that replay would have shown a relative advantage if participants had attended more strongly during the intervention, or it may be that internal replay (rather than this task's external replay) would show a relative advantage. These last two possibilities are considered in the "Limitations and extensions" section.

The learning advantage of imagination over replay and control groups observed here matches the effects seen in the artificial agent simulations (Chapter 2), supporting the idea that a computational utility of imagination is strategy learning (regardless of whether the imagination is artificial or natural). That replay does not show a learning advantage over the control group is somewhat unexpected and does not match the artificial agent simulations, though this lack of an effect may be due to limitations of the experimental design. Also, as noted in the discussion of Chapter 2, the control agent was much simpler than the imagination and replay agents, whereas the control task in this human experiment was more matched to the imagination and replay interventions. In any case, the contrast results involving the imagination intervention were the primary interest of this experiment; the replay vs. control intervention results do not take away from these findings.

I also predicted that better learning in Wythoff's should transfer to better performance in Euclid, i.e., that the intervention groups' relative learning performances in Wythoff's would be reflected in their relative performances in Euclid (Hypothesis II.b). In actuality, we see no differences across groups during Euclid for either measure of performance. This is perhaps unsurprising given that this section of this experiment was designed to test for the far (between game) transfer effect in the original stumbler-strategist simulations (Peterson et al., 2020) that turned out to be erroneous. Regardless, the likely reason why don't we see any difference between groups during the Euclid section is because Euclid, with its much more complex move rules and separate optimality structure, was just too different from Wythoff's for any differences in Wythoff's learning to affect Euclid results. Participants in the imagination and control conditions trained specifically on Wythoff's, and were not prompted to consider in a more open-minded manner, how they might play in games with different rules. This relates to the discussion of imagination likely being most useful for future scenarios closely resembling imagined scenarios in Chapter 2. Perhaps, if imagination had been specifically targeted towards play in Euclid after the new rules had been presented, imagination would have boosted Euclid performance. An effect of this type might be possible without Euclid practice, or after only a few game plays.

Limitations and extensions

While we tried to keep the experimental design as tight as possible, there are inevitable limitations of empirical testing that are difficult to avoid. One standard limitation is that this is just one test of one type of an imagination intervention vs. a replay intervention. This limitation is not unique to this experiment's design, and results of variations may of course strengthen and/or isolate the patterns identified here. For example, additional experiments could test imagination vs. replay immediately after a game to determine the effects of closely time-locked counterfactual imagination (what might have happened if different moves were chosen) vs. instant replay on strategy learning. Different games or non-game strategy learning tasks could also be tested with imagination and replay interventions to identify which types of strategies are most learnable using imagination and consequently which problems benefit the most from imagination.

There are a few limitations which are unique to this experiment's specific experimental design. Once concern could be that the imagination intervention task may not be a perfect test of purely internal simulation. Participants might externalize some internal decisions to assist their imagination, looking at specific board positions and/or tracing games with their fingers over the screen (we did not monitor participants during the experiment). I would argue however that this is still a relevant form of imagination to test. After all, imagination can be used in conjunction with environmental interaction under normal circumstances (for instance, when taking notes during brainstorming). Thus this "assisted" imagination, if it does occur, would still be ecological. Furthermore, this would not give imagination participants an unfair advantage, as those in the replay condition likely perform similar eye movements when following replays, and may trace replays with their fingers as well. As such, the human results here may still be interpreted as valid. To investigate the specific contributions of possible externalized assistance during imagination and/or replay, a tightly controlled experiment could be conducted, with eye tracking and EMG to track movement. Certain brain imaging setups may be conducive to holding subjects in relatively fixed positions, with the added benefit of measuring internal correlates of task behaviors. With an appropriate choice of task, neuroimaging paradigms could possibly even identify whether participants always follow instructions to imagine vs. replay, and when imagining, what the object of imagination is.

The replay condition also has its limitations. As noted earlier, watching replays of prior games may not be the most appropriate stand-in for standard memory replay. This is because standard memory replay in NIs is internally generated, while the replays in this intervention are externally generated (displayed on the computer screen). There may also be associated effects on task effort (e.g., attending to externally-generated replays may be easier than standard memory replay, or vice versa). Also mentioned earlier, these limitations were accepted for this study because presenting replays allows for cleaner experimental control (after all, participants cannot remember and replay each of their 30 initial games). More fine-grained investigation of memory replay could come from tightly designed experiments, for example comparing imagination vs. replay at the level of individual moves. With the right setup, it could be possible to have participants either report what move they remember happening during a specific past game scenario (replay) or report a move they imagine performing next (imagination). With a design like this, replay and imagination would be more constrained, but it would be possible to compare imagination and replay events at an even more comparable level. An added benefit would be the ability to measure participant replay accuracy and the distance between participant imagination and their past experience. As mentioned before, the issues discussed in this paragraph highlight that another possible advantage of imagination may be that it can be employed even when clear isolated replay memories are unavailable (even if participants cannot remember the plays of 30 games, they can still imagine 30 games of plays).

Another limitation of this experiment is that the observed effects, while significant, are not particularly large. There are plenty of reasons why this may be the case, and there are follow up experiments which could be run to investigate each of these possibilities. For one, participants may near a plateau in learning during the first 30 games of Wythoff's, which may dilute the delta measures of change in win rate and optimal moves per game. New experiments with varying numbers of games during the initial section could identify effects of imagination vs. replay depending on how much initial experience a participant has. Also, as previously touched on, imagination and memory replay may afford different advantages over different time scales. In the present experiment, the intervention condition follows almost immediately from the initial gameplay section, and so on (participants see section completion screens which they may skip as soon as they like). Both imagination and replay may afford variable effects on strategy learning depending on how soon imagination or replay occurs after initial experience and/or before evaluation. Overnight rest (sleep) may even interact heavily with these effects; for example, Son et al. (2024) found that overnight replay explains increased efficiency in an empirical test of human social network navigation. Of course, adding time before and after an intervention section permits more uncontrollable activity (participants may engage in extra replay and/or imagination), likely adding noise to results. Nevertheless, successful experiment variations conducted over a longer timescale would help to isolate the temporal pattern of imagination's utility.

Conclusion

The human experiment results validate the imagination learning advantage effects (relative to replay and control) observed in the artificial agent simulations of Chapter 2, but not the boost that replay was predicted to have over control. While there are limitations to both experiments (artificial and human), these limitations are largely necessary, and not so severe as to invalidate these main findings. Further, many of the limitations of the agent simulation experiment are balanced by the human experiment, and vice versa. Together, the concurring imagination advantage findings from both AI and NI experiments lend strong support to the idea that imagination boosts strategy learning as a general computation-level utility.

Chapter 4: Synthesis of artificial agent and human findings

This work revolved around the question “What is the computational value of imagination in learning?”. Towards this question I defined a form of imagination and tested it in simulated and empirical settings, validating specific hypotheses. My overarching hypothesis was that imagination improves performance in both learning efficiency and generalization to future scenarios that are correlated with, but not identical to, past experiences. In essence I hypothesized that imagination supports strategy learning, as learned strategies are used to act in such future scenarios.

In the context of my simulation work, I first hypothesized (Hyp I.a) that imagination based learning would provide an advantage to the performance of artificial models relative to memory replay based learning and simple, model-free reinforcement learning. Secondarily I hypothesized (Hyp I.b) that imagination based learning would boost generalization between the (abstractly related) game tasks used for strategy learning evaluation. The first hypothesis here was borne out in the simulation results, with the imagination agent outperforming the replay agent and model-free control agent, both in terms of strategy effectiveness (win rate) and strategy optimality (optimal moves per game). Here, the replay agent still substantially outperformed the model free agent. However, the second hypothesis was not confirmed, as pretraining on the first game (Wythoff’s) did not transfer to better performance on the second (Euclid). This was regardless of agent learning method (imagination vs. replay) during pre-training or adaptation. While this lack of a far (between-game) transfer result was observed, near (within-game) transfer was evident in the Wythoff’s strategy extrapolation analyses of the imagination agents and (to a lesser extent) replay agents. As such it appears that simulated imagination can improve learning efficiency and generalization to future scenarios that are *closely* correlated with past experiences, but not necessarily to future scenarios which are more weakly correlated with past experiences.

In the context of my empirical experiments in humans, I hypothesized (Hyp II.a) that an imagination-based learning intervention would boost performance over replay-based and control interventions. I also hypothesized (Hyp II.b) that the degree of transfer from Wythoff's to Euclid would follow the relative effectiveness of each intervention on learning in Wythoff's. As in the model simulations, the imagination-based intervention boosted performance over the replay and control interventions. Unlike the model simulations however, the replay intervention did not boost performance significantly above control group performance. The results relating to the second empirical hypothesis also reflected the model simulation findings; no differences in Euclid performance were observed between the intervention groups (therefore like Hyp I.b, Hyp. II.b was not confirmed). Thus in both artificial model simulations and human empirical validations, imagination was found to improve learning performance in the task it was used to simulate, but was not found to improve generalization to more distantly related tasks.

Lessons learned

There are two main, high-level lessons regarding imagination and replay that we can take away from this work. First is that imagination can be an effective tool for strategy learning when used in a targeted manner (directly simulating a desired task). This is in relation to the closely affiliated process of memory replay, and in relation to not using imagination or replay. This finding appears general in that it holds regardless of whether a learner is artificial or natural. As discussed, imagination may effectively boost strategy learning by recombining task memory representations in a world model following implicit and learned constraints to simulate possible future scenarios. Imagination may better support learning over replay because targeted imaginative simulations may be more directly relevant to future scenarios than replay simulations of potentially outdated and/or impertinent memories. As previously noted, essential qualities of imagination such as its generativity and loose reliance on the exact details of previous experience may make it even more uniquely valuable in certain situations. Examples of such include encountering a nearly entirely novel situation (to which previous memories hardly apply), or constructing a coherent story from memories that have blurred together.

Second, and related, is that imagining (or replaying) a task does not necessarily support generalization of strategy to situations that are not close enough to the original task. Even when novel situations bear profound albeit abstract similarities (as in this work), strategy transfer may not be advantaged by imagination or replay based learning of the original task. This is seen across both AI and NI, suggesting it may also be a general pattern. That replay of an original task did not boost strategy generalization is relatively unsurprising. It was however initially unexpected that imagination did not boost generalization, considering the original model transfer result (only discovered later to be erroneous), and considering that imagination's flexibility should in theory allow for wider-ranging simulations that can prepare an agent for even remote task variations. In light of the work here, a more qualified interpretation arises: imagination may be powerfully flexible, but if its flexibility isn't harnessed to entertain alternate possibilities that overlap with possible generalization tasks, it likely will not boost transfer. My present experiments do not test such broader imagination, only simulation that is very near to the original task. Of course, much of the time, this "narrow" imagination may be perfectly useful. In life after all, future situations often hew close to the past, so imagination that is largely past-aligned is likely to regularly benefit us. As Shtulman (2023) notes, human imagination is indeed tightly locked to past experience most of the time, and this is often a practical way to use imagination. Presumably, if the goal is broader generalization, more effortful open-ended imagination should be employed. A learner might attempt to construct scenarios where various constraints are relaxed, expectations challenged, and alternatives explored. It would likely be particularly productive to account for signals predicting possible future change, so as to guide imagined scenarios in the direction of anticipated novel situations. The possibility of alternatively-targeted imagination benefitting far generalization beyond closely-targeted imagination's confirmed contribution to near generalization is addressed with proposed experiments further along in the next section.

Moving forward

Several questions regarding imagination remain unanswered. One largely theoretical question is how imagination relates to reasoning. Imagination, while useful for reasoning in many cases as discussed in the introduction, may not be necessary for reasoning. In other words, reasoning may be possible through solely non-imaginative capacities. This possibility is supported by L. Kay et al. (2024), who found that participants with aphantasia (individuals who cannot form mental images) exhibited slower but more accurate responses in a mental rotation task. Aphantasic participants reported employing strategies based solely on logical criteria instead of using imagination, unlike the control (non-aphantasic) participants. It is unclear to what degree the cognitive procedures involved in reasoning (processing propositional relations, performing operations such as mental math, etc.) fall under the definition of imagination. This question might better be answered using thought experiments. Let us recall that the definition of imagination offered in this work is the following: memory representations within a world model are recomposed according to various implicit and learned constraints, as well as task demands, unfolding as a generative sequence of spatiotemporal events. How do cognitive reasoning processes not traditionally considered as imagination fit with this definition? It could be argued that when solving a simple math problem mentally, for example, one recomposes working memory representations of specific numeric values according to a world model of mathematical relations in a sequence of (abstract) spatio-temporal events. However, mental math does not seem to match our colloquial understanding of imagination. This form of processing seems to follow a definite algorithm, leading to a prescribed outcome, whereas imagination seems more open-ended, often supporting more narrative structure. This said, mental math as described does appear to fit the definition of imagination as offered. It might make the most sense to understand such prescribed reasoning procedures as heavily constrained operations that are supported by the same working memory, executive control, and memory recall processes that underlie imaginative faculties. Said differently, that they exist as an extreme, rule-based form of imagination that does not engage its characteristic flexibility. Or perhaps, an improved definition of imagination would clarify that memory representations are *flexibly* recomposed. Regardless, it makes sense to regard imagination as separate from these constrained processes in designing imagination experiments, as studying solely inflexible reasoning processes cannot inform us on the fuller effects of our prototypical broader, more flexible imagination.

To better understand the contributions of imagination to learning, it will also be important to more thoroughly investigate memory replay. This process, with which imagination has been compared in this work, merits deeper consideration: stronger replay conditions will be needed to determine when replay can boost learning in humans. As previously discussed, the memory replay condition in Chapter 3's human experiment (watching video replays of prior games) was designed to emulate perfect replay in a setup where accurate replay is practically impossible. However, this condition differed from naturalistic replay (and from the imagination condition) in two important ways. First, the video replays were visualized on-screen instead of occurring internally, as in naturalistic replay. Second, participants passively received the video replays, while naturalistic replay is self-generated. Together, internal self-recall and generation of memory replay simulations may entail greater levels of effort and cognitive engagement, producing a memory boost as was not seen with the video replay. As discussed in Chapter 3, a stronger replay condition could be implemented with an alternatively-structured experiment where imagination and replay trials occur immediately following played games. With replay immediately following a game, at least semi-accurate naturalistic replay should be possible. With this experimental design, participants could report their imagined or replayed move sequences, allowing for the analysis of possible effects of imagination "quality" and replay accuracy. Questions such as whether imperfect replay boosts learning as much as imagination, for example, could be asked. To account for possible differences in learning from forwards versus backwards replay, an additional condition could be included where participants would report replays of previous games in reverse order. An analogous "backwards imagination" condition could also be included to somewhat match the backward replay, in case the act of considering games in reverse has a profound impact on learning. And as in the original experiment, a non-imaginative and non-replay distractor control task could be used as comparison for the imagination and replay conditions. I anticipate that the naturalistic-style replay conditions would produce learning boosts beyond that of the control condition due to the greater engagement required, but short of those of the imagination conditions which should benefit more from introducing novel game training examples. It is possible that the backwards simulation conditions would boost learning over the forwards simulation conditions, given that changes in reward (an important learning signal) have been found to modulate the frequency backwards replay (Ambrose et al., 2016). Also, consideration of games in reverse may trigger deeper contemplation of game structure, informing more effective strategy. Altogether, stronger, more naturalistic tests of replay will bring us a more definitive understanding of memory replay's contribution to learning, especially in comparison to imagination.

It also remains to be known exactly which forms of strategy learning benefit most from imaginative training. Of course, we see many individual examples of where imagination-based learning boosts strategy learning in Chapter 1: social strategy (Armah & Landers-Potts, 2021), visual search (Reinhart et al., 2015), interpreting tables and plots (Leahy & Sweller, 2005), training AI action policies using simulation (Ha & Schmidhuber, 2018; Hamrick, 2019; Kalweit & Boedecker, 2017; Richard et al., 2022), etc. And while imagination was found to aid strategy learning in this work, only two related types of strategy learning were tested: move choice strategy learning in the impartial combinatorial games of Wythoff's and Euclid. It is likely however that imagination (and memory replay) based training differs in effectiveness depending on the structure of tasks for which strategy is learned. As learnable strategies may be as arbitrary as the near-infinite range of possible strategy-learning tasks, it would be impossible to test imagination and replay based learning methods for each one. We can however test along representative categories or dimensions of strategy learning tasks; see (Ott et al., 2025) for one suggested taxonomy of sequential decision making tasks for humans that could be followed. I hypothesize that imagination should boost strategy learning over replay specifically in 1) more open-ended tasks (tasks with a wider branching structure of outcomes based on in-task decisions), and 2) partially-explored tasks with invariant structure (including tasks which change into the future, following certain regularities). Here, the exploratory and generative qualities of imagination may privilege imagination-based learning methods through 1) producing simulations which cover unexplored areas of the task space and 2) simulating alternative (anticipated) scenarios constrained by patterns of past experience. Of course, imagination would likely require proper application, specifically targeting appropriate deviations from task experience, for maximal learning benefit. Conversely, I predict that memory replay would boost strategy learning over imagination in more narrow tasks (with relatively few branching outcomes) and in tasks which are more static in nature. Here, the more memorization-style learning necessitated may be advantaged by the exploitative and reproductive qualities of memory replay. It could make sense to test these predictions first using relatively simple multi-level (tree) bandit tasks like in (Baheri, 2025) with specifically varied properties (number of arms at each decision point, decision tree depth, and degree of arm value shift over time). This way, task parameters of open-endedness (arm number and tree depth) and novelty (arm value change) can be finely manipulated, with effects on performance observed with respect to interventions of imagined or replayed task decision paths. Degree of regularity across decision tree nodes and in patterns of change over time may specifically be varied to tease out how levels of invariance may affect imagination and replay based strategy learning. An ideal such experiment might be structured with alternating trials of a) sequential bandit tree navigation (root to leaf) and b) intervention learning (imagining an upcoming sequence of bandit choices, internally replaying the previous sequence of bandit choices, or a non-simulative control activity). Imagined and replayed sequences could also be reported by participants in real time so that their quality could later be investigated as part of the data analysis. Such experiments will help to fill in our understanding of which types of problem structures can be better addressed through imagination based or memory replay based strategy learning.

Following the previous paragraph's hypotheses, it appears that imagination has the potential to support broader, more prospective generalization. This is an exciting possibility, as it could help explain how NIs excel at the critical problem of prospective learning (Seligman et al., 2013), a problem which represents the future of AI learning (Dawid & LeCun, 2024; De Silva et al., 2023). As seen in this work however, imagination of Wythoff's gameplay was not found to aid in far transfer to the abstractly related game of Euclid. My hypothesis here is that properly targeted imagination, applied to simulate alternative versions of an original task in the direction of a novel future task, will boost generalization to the novel future task. Model simulation and human empirical tests of this could be achieved using adapted versions of the original experiments in this work (detailed in Chapter 2 and Chapter 3). For an updated computational modeling experiment, move selection could be constrained to the subset of Euclid-valid moves during learning phase 3 (imagined game simulation). For an updated human experiment, participants could similarly be asked to constrain their imagined moves during the intervention task. More specifically, the same ABAC experiment structure could be followed, while imagination group participants in the B (intervention) section would be asked to only imagine Euclid-allowed moves in their imagined games. This more Euclid-targeted condition would give participants Euclid-like simulated experience and could stimulate participant reasoning on invarances between tasks, boosting far transfer. Wythoff's-targeted imagination (the original intervention condition) could be compared against the more Euclid-targeted imagination. In these ways, past-targeted vs. future-targeted imagination could be evaluated for prospective generalization.

Stronger prospective generalization tests of imagination may be accomplished through measuring transfer following changes to the structural nature of a task. Such "farther transfer" may be achievable in task setups which better take advantage of imagination's powerful compositionality, a core aspect identified in Chapter 1 and investigated in theoretical work and empirical studies in rodents and humans (Gaesser et al., 2013; Hassabis & Maguire, 2007; Schacter & Addis, 2007; Schwartenbeck et al., 2023; Shtulman, 2023; Weisberg, 1998). Toward measuring farther transfer via strongly compositional imagination, we see that the Wythoff's to Euclid transfer test described above is not fully appropriate. For one, while there is some structural change between games, the difference is not dramatic. Euclid moves are a restricted subset of Wythoff's moves and the games' optimal strategies are profoundly related at a high level. Furthermore, Wythoff's and Euclid may be overly constrained to benefit from imagination-based strategy learning, as compared to tasks which could make greater use of imagination's compositionality. While imagining games of Wythoff's relies on compositionality to some degree (composing representations of player vs. opponent with board positions and moves), this compositionality is fairly limited. Imagination for more distant prospective generalization could be more effectively investigated by testing transfer to more structurally different tasks that allow for more compositional imagination. For example, participants could be asked to imagine and then play a version of Wythoff's with two game pieces which may interact (imagining two interacting game pieces would require greater compositionality of imagination). Here, I would again predict that imagination targeted toward the new task (with two game pieces) would aid generalization though simulation-based preparatory strategy learning. I predict that replay and control conditions, not able to simulate scenarios relating to the novel task, would not aid transfer. Together, this and similar experiments should help to elucidate where imagination's full capacities can be engaged to boost more distant prospective learning.

As discussed above, we are only just beginning to understand imagination's value regarding strategy learning; we require additional experimentation to chart the conditions and relevant dimensions that determine where imagination-based strategy learning has the most to offer. Is it safe for us to say, however, that strategy learning is the base computational utility of imagination in learning? I would argue that strategy learning is simply another name for learning generalizable behaviors. Therefore, because it is through behaviors that agents operate in the world, I would argue that strategy learning can be considered as at least one foundational computational utility of imagination in learning. At its core this work represents one example of how imagination can boost strategy learning. This does not complete our understanding of imagination's value in learning, but it does show that strategy learning is a worthy candidate for imagination's base learning utility.

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