Robust Motion Control For CNC Machining Centers In Cutting Process: Model Based Disturbance Attenuation

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Abstract

A disturbance attenuation method in control system, called model based disturbance attenuator (MBDA), is proposed and its properties are studied in this paper. It is very simple and easy to implement. It is shown that the MBDA is extremely robust with respect to large variation of load inertia. The MBDA is implemented in a position control system of a CNC machining center. The MBDA attenuates external disturbances significantly in the cutting process containing high frequency components as well as the frictional forces containing large dc-component. To show the effectiveness of the MBDA, several other controllers are also implemented in a position control system of a CNC machining center and the experimental results are compared with one another.

I. Introduction

Robust servo control has been widely studied in motion control systems (e.g. CNC machining centers) because plants are usually different from models no matter how the models are obtained. In addition to model uncertainty, there is an external disturbance which degrades the system performance in motion control, for example cutting forces and frictional forces. In the cutting process, the cutting force acts as an external disturbance which has a relatively high frequency component.

Many control systems have been proposed which are robust against parameter variations and insensitive to the external disturbances.

One of the most popular approach is the disturbance observer (DOB) which estimates the disturbance and compensates it [1], [2], [3], [4]. Umeno and Hori [3] proposed the DOB with a Butterworth Q-filter based on the parameterization of two-degree-of-freedom controllers [5], [6], [7]. Lee and Tomizuka [1], [4] used a binomial low-pass filter in the design of the DOB. These were basically derived from Youla-parameterization theory [8]. Larger bandwidth of the DOB results in a better performance in tracking. However, there is a limit to increase the bandwidth, and an excessive increase will destabilize the system. And the DOB allows only small variations in system parameters. If the parameter variation is large, the DOB can make the system unstable.

Recently, Yao and Tomizuka [9], [10] proposed an adaptive robust control (ARC) which combines the adaptive control and the sliding mode control (SMC)

systematically for trajectory tracking control. Yao etc. [11] applied the ARC to robust motion control of a machine tool. The ARC can handle a large parameter variation and shows a better tracking performance than the DOB[11].

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In this paper a disturbance attenuator, which we will call model based disturbance attenuator (MBDA), is introduced for motion control systems. It is as good as the ARC in the robustness and the tracking performance. It does not need the inverse dynamics of the plant like the DOB. The error signal, which is the difference between the plant output and the nominal plant output, is fed back through a compensator to the input of the plant to be attenuated.

This paper is composed as follows. In Section II, the structures of the robust feedback controllers are introduced including the DOB, the simplified ARC (SARC), and the MBDA. In Section III, the robust properties of these controllers are illustrated. In Section IV, experimental results with these robust feedback controllers are compared. Finally, conclusion follows in Section VI.

II. Structures of the Robust Control Systems.

The feed drive system of a CNC machining center can be modeled as the block diagram in Fig. 1. Here, J_a is the equivalent inertia of the motor and load, B_a is the equivalent viscous frictional damping, K_t is a known torque constant, and T_l is an external disturbance [12]. For a conventional position control

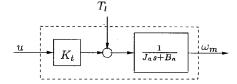


Fig. 1. Simplified model for the feed drive system.

system, a velocity controller $C_v(s)$ and a position controller $C_p(s)$ are used. The transfer function from ω_r and T_l to ω_m in the conventional control system (i.e., the control system without the \blacksquare OB in Fig. 2) is

$$\omega_m = G_v(s)\omega_r + D_p(s)T_l , \qquad (1)$$

$$G_v(s) = \frac{K_tC_v(s)}{J_as + B_a + K_tC_v(s)} , \qquad (2)$$

$$D_p(s) = \frac{1}{J_a s + B_a + K_t C_v(s)}$$

The feedback PI controller in $C_v(s)$ suppresses the disturbance T_l in this conventional control system.

To enhance the performance of the conventional control system, the DOB with Q-filter was proposed by Umeno and Hori [3] as shown in Fig. 2 where $G_n(s)$ denotes the nominal plant for $G_v(s)$. Umeno and Hori

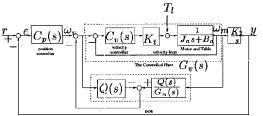


Fig. 2. The position control system with the DOB.

[3] suggested Q(s) in the following form.

$$Q(s) = \frac{1 + \sum_{k=1}^{N-r} a_k(\tau s)^k}{1 + \sum_{k=1}^{N} a_k(\tau s)^k}$$
 (2)

Lee and Tomizuka [1], [4] used a binomial low-pass filter which has the form of (2) with $\alpha_k = \frac{N!}{k!(N-k)!}$. Here, τ is a filter constant which should be chosen by taking the disturbance rejection zone in frequency domain into consideration [1]. To eliminate the effect of disturbances containing high frequency components, the design parameter τ should be small. But a very small τ can make the system unstable because of model uncertainties. The transfer function from ω_{τ} and T_l to ω_m becomes

$$\omega_{m} = G_{d}(s)\omega_{r} + D_{d}(s)D_{p}(s)T_{l} , \qquad (3)$$

$$G_{d}(s) = \frac{G_{n}(s)G_{v}(s)}{[1 - Q(s)]G_{n}(s) + Q(s)G_{v}(s)} ,
D_{d}(s) = \frac{[1 - Q(s)]G_{n}(s)}{[1 - Q(s)]G_{n}(s) + Q(s)G_{v}(s)} .$$

Recently, Yao and Tomizuka [9], [10] proposed the adaptive robust control (ARC) algorithm and applied it to robust motion control of machine tools [11]. Yao etc. [11] used only a proportional gain as a velocity controller (i.e. $C_v(s)$ has only a P gain in Fig. 3). Then, the controlled plant $G_{va}(s)$ in Fig. 3 becomes

$$G_{va}(s) = \frac{1}{Js + B} \quad ,$$

where, $J = \frac{J_a}{K_t K_{sp}}$, $B = \frac{B_a + K_t K_{sp}}{K_t K_{sp}}$, respectively. The ARC tries to find J and B adaptively. To make the analysis simple, we use the nominal values for J and B in the ARC, and call it a simplified ARC (SARC).

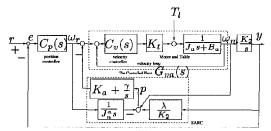


Fig. 3. The position control system with the SARC.

Its control structure is shown in Fig. 3. A switching-function-like quantity p is defined as

$$p = \frac{1}{K_2}\dot{y} + \frac{\lambda}{K_2}y - \frac{1}{J_n^a}\int_0^t \omega_r(\tau)d\tau \quad ,$$

where, $\lambda = \frac{B_n^a}{J_n^a}$ and J_n^a and B_n^a are the nominal values for J and B. Then, the transfer function from ω_r and T_l to ω_m becomes

$$\omega_{m} = G_{a}(s)\omega_{r} + D_{a}(s)D_{p}(s)T_{l} , \qquad (4)$$

$$G_{a}(s) = \frac{s^{2} + \frac{1}{J_{a}^{*}}(K_{a}s + \gamma)}{s^{2} + G_{v\bullet}(s)(K_{\bullet}s + \gamma)(s + \lambda)}G_{va}(s) ,$$

$$D_{a}(s) = \frac{s^{2}}{s^{2} + G_{va}(s)(s + \lambda)(K_{a}s + \gamma)}$$

As the gain K_a and γ increase, the performance becomes better. But an excessive increase in these parameters will destabilize the system because of model uncertainties.

The MBDA proposed in this paper is shown in Fig. 4. In this structure, the nominal plant is placed paral-

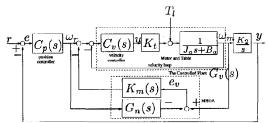


Fig. 4. The position control system with the MBDA.

lel to the controlled plant. The velocity error signal e_v is fed back to the input of the controlled plant $G_v(s)$ through a compensation $K_m(s)$. Then, the transfer function from ω_r and T_l to ω_m becomes

$$\omega_{m} = G_{m}(s)\omega_{r} + D_{m}(s)D_{p}(s)T_{l} , (5)$$

$$G_{m}(s) = \frac{1 + K_{m}(s)G_{n}(s)}{1 + K_{m}(s)G_{v}(s)}G_{v}(s) ,$$

$$D_{m}(s) = \frac{1}{1 + K_{m}(s)G_{v}(s)}$$

The response of the control system does not change much from the control system with the nominal plant if the nominal plant $G_n(s)$ is sufficiently close to the controlled plant $G_v(s)$. Note that the effect of disturbance T_l to ω_m is attenuated by $D_p(s)$ in (1). And it is further attenuated by $D_a(s)$, $D_a(s)$, and $D_m(s)$ for the DOB, the SARC, and the MBDA, respectively as can be seen in (3), (4) and (5). So, the disturbance attenuation depends on the method used to design $D_i(s)(i=d,a,m)$. The controller $K_m(s)$ in Fig. 4 may have any form, but we used a PI controller, $K_{mp} + \frac{K_{mi}}{s}$. As the gains increase, the attenuation becomes more significant. But the system becomes unstable as the gains increase too much because of model uncertainties.

The transfer functions from r and T_l to y for the conventional position control system becomes

$$y = \frac{K_2 C_p(s) G_p(s)}{s + K_2 C_p(s) G_p(s)} r + \frac{K_2 D_p(s)}{s + K_2 C_p(s) G_p(s)} T_l,$$

and the others become

$$\begin{array}{lcl} y & = & \frac{K_2C_p(s)G_i(s)}{s + K_2C_p(s)G_i(s)}r + \frac{K_2D_p(s)}{s + K_2C_p(s)G_i(s)}D_i(s)T_l \\ i & = & d,a,m \end{array}.$$

If $G_v(s) = G_n(s)$, then $G_p(s) = G_d(s) = G_m(s)$. Hence, the transfer functions from r to y are the same for the conventional control system, the control systems with the DOB and the MBDA. Also, the transfer function of the control system with the SARC will not be much different from the others when $G_{va}(s) = G_{vn}(s)$ where $G_{vn}(s)$ is the nominal plant for $G_{va}(s)$. Note that the effect of the disturbance T_l to y can be characterized by the term $D_i(s)(i = d, a, m)$ when the parameters of the plant have nominal values.

III. Robust properties of the DOB, the SARC and the MBDA

A. Robustness Against Parameter Variations

In order to study robustness against variation of J_a , the step responses of the velocity loop are examined in simulation for various values of J_a . Fig. 5 shows the step responses of the velocity control system in the conventional control system (PID), the control system with the DOB (PID+DOB), the control system with the SARC (PID+SARC), and the control system with the MBDA (PID+MBDA) as J_a varies from J_n to $5J_n$ under the condition listed in Table I. The gains of each controller are set to be sufficiently large as long as they do not destabilize the system. In the Q-filter design, N=3, r=2, and r=0.005 are used in (2). The step responses of the PID+SARC or the PID+MBDA almost do not change at all for the variation of J_a .

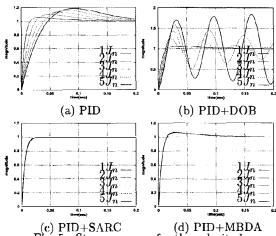


Fig. 5. Step responses for the velocity loop.

TABLE I THE PARAMETERS OF THE FEED DRIVE SYSTEM

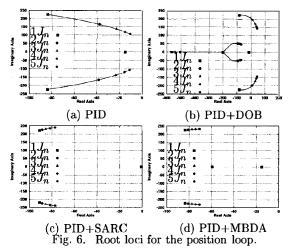
THE TAKAMETERS OF THE TEED DIGVE STSTEM					
J_n	0.008597 Kg m ²	J_a	$J_n,,5J_n$		
K_{sp}	$1.3003 \text{ A}(r/s)^{-1}$	K_{si}	19.5045 A/r		
B_n, B_a	$0.0 \text{ Nt-m}(r/s)^{-1}$	K_{mp}	50.0		
K_{mi}	3000.0 1/s	K_a	32.9097		
λ	182.32 1/s	γ	27.4247 1/s		
K_t	1.2054 Nt-m/A	au	0.00 5 s		
K_1, K_{pp}	0.0512,1.6	K_2	3819.7		

The response of the system with the DOB is the most sensitive to the variation of J_a .

Another method for investigating the robustness against parameter variation is the root locus. The root loci of the position loop for the above-mentioned systems are plotted in Fig. 6 as J_a varies. To show it in detail, only the dominant poles are plotted. Only the P controller is used for the position loop control, i.e. $C_p(s) = K_1 K_{pp}$ where K_1 is another unit conversion constant and K_{pp} is a proportional gain. The position control system with the DOB is already unstable when J_a is greater than twice the nominal inertia as shown in Fig. 6 (b). The PID+DOB is more sensitive to the load inertia than the PID. The PID+MBDA and the PID+SARC are very robust against the variation of J_a .

B. Performance Analysis In Frequency Domain

Performance criteria of a controller should include set-point tracking, disturbance rejection, and suppression of measurement noise [13]. The performances of a SISO system can be analyzed by its sensitivity and complementary sensitivity function. To compare the sensitivity functions of the systems mentioned-above, the transfer function G_{yr} , from the position input rto the output y is considered. The sensitivity function [1], [13] shows how sensitive the transfer function is to the variations of plant when $G_v = G_n$. The sensitivity



functions of the four control systems are as follows.

$$S_p(s) = \frac{s}{s + K_2 C_p(s) G_n(s)} \tag{6}$$

$$S_d(s) = [1 - Q(s)]S_p(s) \tag{7}$$

$$S_{d}(s) = [1 - Q(s)]S_{p}(s)$$
 (7)

$$S_{a}(s) = \frac{1}{1 + \frac{1}{J_{n}^{a}s^{2}}(K_{a}s + \gamma)}S_{pa}(s)$$
 (8)

$$S_m(s) = \frac{1}{1 + K_m(s)G_n(s)} S_p(s)$$
 (9)

where, $S_{pa}(s) = \frac{s}{s+K_2C_n(s)G_{nn}(s)}$. Note that

$$\frac{S_d(s)}{S_p(s)} = D_d(s)|_{G_v = G_n}, \quad \frac{S_a(s)}{S_{pa}(s)} = D_a(s)|_{G_{va} = G_{vn}},
\frac{S_m(s)}{S_n(s)} = \mathbf{P}_m(s)|_{G_v = G_n}, \quad ,$$

and these terms are closely related to the disturbance attenuation as shown in the previous section. The complementary sensitivity functions are defined as

$$T_i(s) = 1 - S_i(s)$$
 $i = p, d, a, m,$ (10)

and it is closely related to noise suppression. The sensitivity and complementary sensitivity functions for these robust control systems are plotted in Fig. 7. The sensitivity function for the PID has the biggest gain in the frequency range up to 100 Hz. Thus, the property for disturbance attenuation up to 100 Hz is the worst of all. The PID+DOB has the smallest gain in the low frequency range up to 2.5 Hz. This means the performance of dc-disturbance attenuation is the best for the PID+DOB. But its gain is bigger than the PID+SARC or the PID+MBDA in the frequency region from 10 Hz to 100 Hz. As a result, the performance for the disturbance attenuation in the cutting process whose main frequency lies in this frequency region is worse than the other two methods, which will be shown later in the experiment.

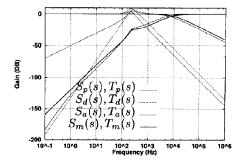


Fig. 7. (complementary) sensitivity functions.

IV. Experimental Results

Linear interpolations and circular interpolations are performed both in air-cutting and in steel-cutting. The feedrate is given much higher in the air-cutting. In the steel-cutting, cutting force acts as an external disturbance in addition to the frictional force. Therefore, the performance in the attenuation of the disturbance containing high frequency component as well as the low frequency component can be seen in the steel-cutting. To measure the performance of each controller, the absolute maximum tracking error (e_M^x, e_M^y) , the absolute maximum contour error (c_M) , the standard deviation of the tracking error for each axis (e_S^x, e_S^y) , and the standard deviation of the contour error (c_S) are used. The values e_M^x , e_M^y , and c_M measure the attenuation of the frictional forces because the maximum values occurs when the sign of the velocity of the table changes. Because of the cutting forces, the tracking or contouring errors are highly oscillatory and the degree for oscillations are roughly estimated by e_S^x , e_S^y , and c_S .

A. Air-Cutting

In linear interpolation, three parameters, X (mm), Y (mm), and FR (mm/min) are usually used. They denote the relative lengths for X-axis to go, for Yaxis to go, and the feedrate of the table, respectively. In circular interpolations, two parameters, R (mm) and FR (mm/min) which denote the radius and the feedrate respectively, are usually used. The trapezoidal velocity profile was used in the linear interpolation and the trapezoidal angular velocity profile was used in the circular interpolation. X = 10, Y = 10,and FR = 3000 for the linear interpolation and R=10, FR=3000 for the circular interpolation. The maximum absolute contour error and the mean value of the absolute contour error m_{ce} are shown in Table II. In the linear interpolation, the PID+DOB had the biggest c_M . The the PID+MBDA shows the best perfermance. The PID+SARC also has good performance. In the circular interpolation, the PID had big quadrant glitches. PID+MBDA had the smallest c_M

TABLE II PERFORMANCE MEASURES IN AIR-CUTTING.

Linear Interpolation (μm)

ĺ	measure	PID	DOB	SARC	MBDA
	c_{M}	7.37	10.31	5.30	5.00
	m_{ce}	1.30	1.40	0.86	0.71

Circular Interpolation (μm)

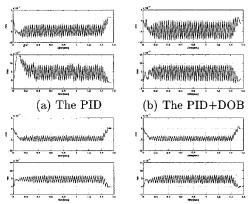
measure	PID	DOB	SARC	MBDA
c_{M}	27.64	15.36	10.85	5.88
m_{ce}	5.14	1.60	3.32	1.45

and m_{ce} both in the linear and the circular interpolation.

B. Steel-Cutting

Experiments for the linear and the circular motions were performed for the CNC machining center under cutting process. In the cutting process, an end-mill of 8 mm in diameter was used as a cutting tool and the workpiece was the steel SS41C. The spindle speed and the depth of cut are denoted by S (rpm) and D(mm) respectively. If the end-mill has N blades, the disturbance due to the cutting process has the main frequency component at f = NS/60 Hz. Since the end-mill used in the experiment had two blades, the disturbance due to cutting had the main frequency component at f = 33.3 Hz when S = 1000, and f = 50Hz when S = 1500.

Fig. 8 shows tracking errors of X-axis and Y-axis in the cutting process with X = -2, Y = 2, FR =2, S = 1000, and D = 3. The linear interpolation under the same condition except S = 1500 was also performed. The main frequency components of the ex-



The PID+SARC (d) The PID+MBDA Tracking errors, linear motion (X = -2, Y = -22, FR = 120, D = 3, S = 1000.

ternal disturbances in these two experiments are different. The bias in tracking error for the linear motion is due to system delay and the feedforward controllers are usually added to compensate it [1], [11]. In order

TABLE III PERFORMANCE MEASURES (LINEAR MOTION).

 $S=1000 (\mu m)$

	PID.	DOB	SARC	MBDA
e_M^x, e_M^y	7.81, 21.9	9.10, 9.19	6.11, 7.17	6.10, 7.62
e_S^x, e_S^y	2.27, 4.40	2.90, 3.18	1.41, 1.75	1.35, 1.94
c_{M}	11.79	2.36	2.06	2.06
$c_{\mathcal{S}}$	2.46	0.90	0.86	0.97

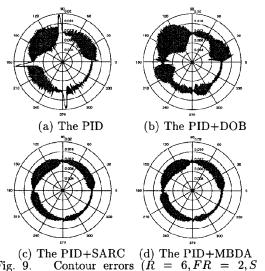
 $S=1500 \ (\mu m)$

	PID	DOB	SARC	MBDA
e_M^x, e_M^y	7.28, 22.3	7.53, 7.50	6.36, 6.53	5.92, 6.64
e_S^x, e_S^y	1.84, 3.68	2.12, 2.03	1.48, 1.56	1.38, 1.54
c_M	11.79	2.65	2.36	2.06
c_S	2.19	1.28	0.47	0.47

to see the only effects of the controllers, no feedforward controller was used. The PID+SARC or the PID+MBDA attenuated the disturbance much better than the PID or the PID+DOB. The performance measures are listed in Table III.

It shows that both the MBDA and the SARC are suitable in attenuating the external disturbances.

Fig. 9 shows the contour errors of the circular interpolation with R = 6, FR = 2, D = 3, and S = 1000. In Fig. 9, 99.5 % of the circle core is abbreviated to show the results in detail and the target radius is $0.012 \ mm$. The circular interpolation under the same condition except S = 1500 was also performed. For



Contour errors (R = 6, FR = 2, S =1000, D = 3).

each control system, the general trend of the contour errors with S = 1000 and S = 1500 were similar. The fourth quadrant had the smallest oscillation than the other quadrants because the cutting tool came back to the starting position where much of the material

TABLE IV PERFORMANCE MEASURES (CIRCULAR MOTION).

 $S=1000 (\mu m)$

	PID	DOB	SARC	MBDA
e_{M}^{x}, e_{M}^{y}	10.7, 23.5	12.8, 13.1	8.75, 9.20	8.40, 9.57
e_S^x, e_S^y	4.94, 5.36	5.31, 5.25	4.58, 4.62	4.57, 4.66
c_M	7.89	6.79	2.58	3.15
c_{S}	1.97	1.88	0.82	0.88
S-1500 (um)				

 $S=1500 \ (\mu m)$

	PID	DOB	SARC	MBDA
e_M^x, e_M^y	9.44, 24.6	11.0, 9.56	8.58, 8.94	8.41, 8.90
e_M^x, e_M^y	4.72, 4.97	4.93, 4.69	4.60, 4.59	4.58, 4.59
c_M	7.89	5.43	2.74	2.51
$\overline{c_S}$	1.51	1.21	0.65	0.62

had already been removed. The contour errors for the conventional control system had the biggest quadrant glitches as well as the biggest oscillation. The PID+DOB did not attenuate this oscillation much but cancels out the quadrant glitches very effectively. The PID+MBDA or the PID+SARC cancelled out the quadrant glitches and attenuated the oscillation significantly due to the cutting. The performance measures for the circular motion in the experiment are listed in Table IV. The PID+SARC and the PID+MBDA give similar performances, which are better than the other two algorithms.

V. Conclusion

The MBDA is proposed to attenuate the disturbances such as the frictional force, force due to cutting, and load variations. The MBDA first tries to make the plant perform similarly to the nominal plant, as much as possible, by using a compensator. Then, a controller, which is designed for the nominal plant, is added in the control system. The philosophy of the design is simple and intuitive. The control system with the MBDA and several other controllers were analyzed and implemented in position control system of a CNC machining center. It is shown that the control systems with the MBDA or the SARC are extremely robust to the variations of inertia, and they reduce the contour error significantly in the steel-cutting process. These two perform better than the DOB, which is one of the most popular method in disturbance attenuation. The structures of the two controllers are different, but the properties of these two controllers are very similar. These two control structure need to be studied further so that they can be applied to the control of more general system3.

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