# Vibration Displacement Measurement Based on Three Axes Accelerometer


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***Abstract*—This paper introduces a novel method of measuring vibration displacement using three axes accelerometer, which effectively overcomes the shortcomings of traditional measurement method based on single axis accelerometer. Firstly, the frequency domain method is used to remove the trend items and noises in the three axes acceleration signals. Then, the three axes accelerations are fused for the resultant acceleration. Secondly, the resultant acceleration is converted into the acceleration spectrum by Fourier transform, and then the acceleration spectrum is transformed into displacement spectrum. Thus, the amplitude, circular frequency and phase angle of the displacement spectrum are obtained. Furthermore, the inverse Fourier transform is adopted for changing the displacement spectrum signal into time domain signal. Finally, an experimental platform for vibration detection based on industrial Ethernet is set up to verify the effectiveness of the proposed algorithm. Experimental results demonstrate the effectiveness and practicality of the proposed algorithm.**



***Keywords*—Vibration displacement; Fourier transform; Displacement spectrum; Industrial Ethernet**


## I. Introduction

The measurement of vibration displacement is of great significance in engineering [1]. In the machine tool system, low vibration and low noise requirements of all kinds of machinery and equipment are required, also it is necessary to monitor, analyze and diagnose the running condition of the machine at any time [2]. In order to improve the vibration resistance of the mechanical structure, it is essential to carry out vibration analysis and vibration design of mechanical structures. Thus, the vibration displacement detection is particularly important. And how to effectively improve the accuracy of displacement measurement is a technical problem which needs to be solved urgently at present.

There are two main ways to detect vibration displacement in recent years: contact measurement and non-contact measurement [3]. For non-contact measurement, the vibration displacement of the object is measured directly by eddy current sensor or laser vibrometer. This method has the advantage of high precision and high frequency bandwidth. Whereas, the installation condition of the sensor is harsh. There must be a rigid foundation for sensors to be installed near the measurement point, which requires that the surface to be measured can't be too large [4]. For contact measurement, the accelerometer is directly installed on the measurement part, then the acceleration signal is obtained and the displacement signal is obtained after quadratic integral. This method overcomes the disadvantage of the inconvenient installation of sensors in non-contact measurement. However, the contact measurement method has the following disadvantages: 1) the mean of acceleration signal may not be zero and the quadratic integral may cause a deviation. 2) the installation of the accelerometer must be in the same line as the vibration direction. Otherwise, the measured displacement will be deviated [5].

Considering the simplicity of the experimental platform, the contact measurement scheme is studied in this paper. In order to overcome the disadvantages of contact measurement, the frequency conversion method is adopted to effectively overcome the deviation caused by the quadratic integral. In addition, traditional single axis accelerometer is replaced with the three axes accelerometer for improving the measurement accuracy. The platform for vibration displacement measurement is built to verify the effectiveness of the proposed algorithm. Experimental results demonstrate that the algorithm can meet the requirement of vibration displacement measurement in industrial filed.

The rest of paper is organized as follows. Section II introduces the principle of acceleration measurement. The detail algorithm design is presented in Section III. Section IV shows the experimental results and discussion. Conclusions are shown in Section V.

## II. PRINCIPLE OF ACCELERATION MEASUREMENT

Acceleration is a space vector, and the acceleration measured by a single axis accelerometer is only the component of the spatial acceleration in the measurement direction. Therefore, in order to accurately understand the motion state of the object, it is necessary to measure the components of the three coordinate axes respectively. Besides, in the case that motion direction of the object is not known in advance, only when use the three axes accelerometer can the acceleration signal be detected, and thus the direction of acceleration can be determined [6]. The gravitational acceleration is inevitable introduced when using the accelerometer for measuring the acceleration. The gravitational acceleration is expressed as a


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constant in the three axes acceleration while the vibration acceleration is a variation. It shows as Fig. 1.

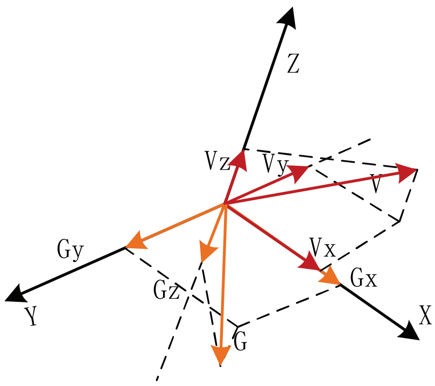


Fig. 1: Diagrammatic sketch of acceleration signals

The constant components of gravitional acceleration in the three axes of the accelerometer are respectively are $\hat{a}_{gx}$, $\hat{a}_{gy}$, and $\hat{a}_{gz}$. The components of acceleration produced by vibration respectively are $\hat{a}_{vx}$, $\hat{a}_{vy}$, and $\hat{a}_{vz}$. The acceleration measured by accelerometer are presented by $\hat{a}_{mx}$, $\hat{a}_{my}$, and $\hat{a}_{mz}$ respectively. These variables satisfy the relation shown as follows.

$$\begin{cases} \hat{a}_{mx} = \hat{a}_{gx} + \hat{a}_{vx} \\ \hat{a}_{my} = \hat{a}_{gy} + \hat{a}_{vy} \\ \hat{a}_{mz} = \hat{a}_{gz} + \hat{a}_{vz} \end{cases} \tag{1}$$

## III. Algorithm Design

### A. Preprocessing of Acceleration Signal

In the industrial field, interference exists in the acceleration measurement under the influence of noise. However, the acceleration signals collected by accelerometer are inevitably mixed with a variety of noises, such as random noise and trend item. The existence of random noise and trend error will greatly reduce the accuracy of displacement measurement [7]. Thus, it is essential to denoise the original acceleration data.

The traditional denoising method can only be used in the case where the overlap of signal and noise is very small or completely separated. And it is based on the Fourier transform, separating the signal and noise by filtering [8]. However, the useful signal of acceleration and the frequency band of noise signal are overlapped arbitrarily, thus, the traditional denoising method can't achieve the desired effect. Wavelet analysis is a time frequency analysis method for denoising signal, which has the characteristic of multi-resolution analysis and can achieve the purpose of noise removal [9]. $a_{mx}$, $a_{my}$, and $a_{mz}$ denote the denoised three axes acceleration signals respectively.

### B. spectrum conversion method

The analysis of signals in frequency domain and time domain will reflect different physical characteristics. Fourier transform is a powerful tool for converting from time domain to frequency domain [10]. A time domain signal $x(t)$ with a sample length of T formed into the discrete data $x(n)$ after data acquisition. If N data points are collected during time T, then the normalized discrete Fourier transform (DFT) of $x(t)$ can be achieved by:

$$X(k) = DFT[x(n)] = \sum_{n=0}^{N-1} x(n) e^{-(\frac{2\pi}{N})nk} \tag{2}$$

Where both the $n$ and $k$ take $0, 1, 2, 3 \ldots, (N-1)$, and DFT uses a fast discrete Fourier transform algorithm. After a discrete Fourier transform, $x(n)$ becomes a complex sequence with the length of N.

$$\begin{aligned} X(k) &= DFT\left[x(n)\right] \\ &= \left[\left(a_0 + jb_0\right), \left(a_1 + jb_1\right), \ldots, \left(a_{N-1} + jb_{N-1}\right)\right] \end{aligned} \tag{3}$$

The denoised acceleration signals of X, Y, and Z axes were transformed from time domain to frequency domain by using the above Fourier transform. The detail calculation is as below.

$$\begin{cases} A_{mx}(k) = DFT[a_{mx}(n)] \\ A_{my}(k) = DFT\left[a_{my}(n)\right] \\ A_{mz}(k) = DFT[a_{mz}(n)] \end{cases} \tag{4}$$

Through the above transformation as shown in (4), the time domain signal of acceleration is converted into harmonic component in frequency domain. Setting the amplitude of harmonic component whose frequency is lower than the useful frequency to zero, which can remove the trend item in the measured acceleration signal. The acceleration signals after removing the trend items are represented by $A_{vx}(k)$, $A_{vy}(k)$, and $A_{vz}(k)$ respectively.

Fusing the acceleration data in time domain is the simplest and most effective method. $A_{vx}(k)$, $A_{vy}(k)$, and $A_{vz}(k)$ can be transformed from frequency domain into time domain through DFT.

$$\begin{cases} a_{vx}(n) = IDFT[A_{vx}(k)] \\ a_{vy}(n) = IDFT[A_{vy}(k)] \\ a_{vz}(n) = IDFT[A_{vz}(k)] \end{cases} \tag{5}$$

The acceleration signal $A_v(n)$ is obtained by fusing the three axes acceleration signals, which can be calculated by:

$$|A_v(n)| = \sqrt{a_{vx}(n)^2 + a_{vy}(n)^2 + a_{vz}(n)^2} \tag{6}$$

$$sign(A_v(n)) = sign(A_{vx}(n)) * sign(A_{vy}(n)) * sign(A_{vz}(n)) \tag{7}$$

Where the letter sign indicates the sign of the acceleration signal, and n takes $0, 1, 2, \ldots, (N-1)$, the signal fusion process can be presented by Fig. 2.

The vibration acceleration signal $A_v(n)$ is obtained by fusing the three axes acceleration signals with trend items removed.

The acceleration signal $A_v(n)$ is again transformed into a complex sequence of length N by discrete Fourier transform.

$$\begin{aligned} A_v(k) &= DFT[A_v(n)] \\ &= \left[\left(a_0 + jb_0\right), \left(a_1 + jb_1\right), \ldots, \left(a_{N-1} + jb_{N-1}\right)\right] \end{aligned} \tag{8}$$

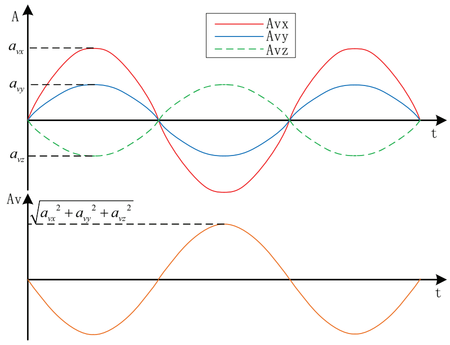

Fig. 2: Fusion process of acceleration signals

The amplitude, circle frequency and the initial phase angle of each harmonic component in $A_v(k)$ can be derived by:

$$\begin{cases} A_k = \sqrt{a_k^2 + b_k^2} \\ \omega_k = 2\pi \frac{k}{T} \\ \varphi_k = \arctan(\frac{b_k}{a_k}) \end{cases} \tag{9}$$

According to the principle of signal superposition, periodic signal is generated by simple harmonic waves [11]. The acceleration signal can be presented by (10), the corresponding displacements is represented by (11). And the relation between the magnitude and the phase is shown in (12).

$$\begin{aligned} a = Aa_0 \cos(\omega_0 t + \varphi_{a0}) + Aa_1 \cos(\omega_1 t + \varphi_{a1}) + \dots \\ + Aa_{N-1} \cos(\omega_{N-1} t + \varphi_{aN-1}) \end{aligned} \tag{10}$$

$$\begin{aligned} d = Ad_0 \cos(\omega_0 t + \varphi_{d0}) + Ad_1 \cos(\omega_1 t + \varphi_{d1}) + \dots \\ + Ad_{N-1} \cos(\omega_{N-1} t + \varphi_{dN-1}) \end{aligned} \tag{11}$$

$$\begin{cases} Ad_i = Aa_i / \omega_i^2 \\ \varphi_{di} = \varphi_{ai} - \pi \end{cases} \tag{12}$$

Through the above transformation, the harmonic component of the acceleration can be calculated by the harmonic component. The displacement harmonic component can be obtained by inverse Fourier transformation and thus the displacement can be expressed in the time domain.

### C. Algorithm summary

Compared with the quadratic integral method, the spectrum conversion method has the advantage of handling the trend item easily. In the frequency domain, setting the amplitude of harmonic component whose frequency is lower than the useful frequency to zero, which can handle the error caused by trend item.

The steps of converting three axes acceleration signals into displacement signals in frequency domain are as follows.

1) The discrete acceleration signals $a_{mx}(n)$, $a_{my}(n)$, and $a_{mz}(n)$ are implemented FFT for obtaining frequency spectrums $A_{mx}(k)$, $A_{my}(k)$, and $A_{mz}(k)$ respectively.

2) The acceleration signals with trend items removed $A_{vx}(k)$, $A_{vy}(k)$, and $A_{vz}(k)$ are obtained by setting the magnitude of harmonic component whose frequency is lower than the useful frequency to zero in $A_{mx}(k)$, $A_{my}(k)$, and $A_{mz}(k)$.

3) The vibration acceleration signal $A_v(n)$ is obtained by fusing the $a_{vx}(n)$, $a_{vy}(n)$, and $a_{vz}(n)$.

4) The $A_v(k)$ is obtained by transforming the $A_v(n)$ into the frequency through Fourier transform.

5) The amplitude, the circle frequency and the initial phase angle of each acceleration harmonic component are calculated by (9).

6) The amplitude and initial phase angle of each displacement harmonic component are calculated by (12).

7) The vibration displacement signal is obtained by (11).

## IV. EXPERIMENT AND DISCUSSION

### A. The construction of experimental platform

The experimental test is completed by motion control platform. The real mechanical vibration is simulated by the reciprocating motion of the single axle platform. EtherCAT bus is used to collect and record acceleration signals and displacement signals in real time.

The structure of the experimental platform is shown in Fig. 3.

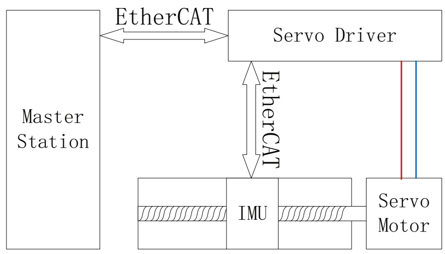

Fig. 3: Structure of experimental platform

The master uses the electronic cam of motion control module to realize mechanical reciprocating motion of the slider on the single axle platform and simulates the mechanical vibration. The master station is built by TwinCAT, and the servo driver adopts AC servo driver in the experimental testThe servo motor and the slider are connected by lead screw, and the inertial measurement unit (IMU) forms rigid connection with the slider of the single axle platform. As illustrated in Fig. 3, EtherCAT bus communication is used between the TwinCAT master station and the servo driver as well as between the servo driver and the IMU. The master station records the acceleration value of the three axes accelerometer and the encoder value of the servo driver in real time.

Industrial Ethernet is used for communication between the IMU and the master station. The schematic diagram of IMU connection is shown in Fig. 4.

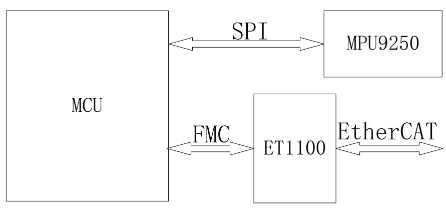

Fig. 4: Schematic diagram of IMU connection

The MPU9250 consists of a three axes accelerometer, a three axes gyroscope and a three axes magnetometer. Only the data from three axes accelerometer is used here. Micro control unit (MCU) and MPU9250 communicate with SPI bus to improve communication speed and ensure real-time. The communication between IMU and master station is designed by using ET1100, a slave station dedicated chip.

### B. Experimental test

With the platform mentioned above, the vibration is simulated by mechanical movement of the single axle platform, and the standard sinusoidal vibration signal and the non-standard sinusoidal vibration signal are generated respectively. The master station records the position information feedback from servo motor encoder and the acceleration information obtained by IMU. According to the spectrum conversion method, the acceleration data obtained by IMU are converted into the displacement data. The three axes acceleration signals are processed by wavelet denoising in real time, and the comparison of original and denoised signals are plotted in Fig. 5 (the blue line represents the original signal of accelerometer while the red line represents the denoised signal). The resultant acceleration with the trend item removed is shown in Fig. 6. The comparison of measured and real displacement is demonstrated in Fig. 7 (the green line represents the real displacement, the blue line represents the measured displacement, and the red line represents the error of displacement measurement).

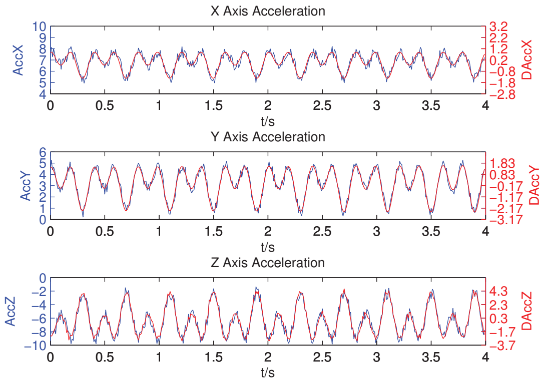


Fig. 5: Comparison of original and denoised acceleration signals

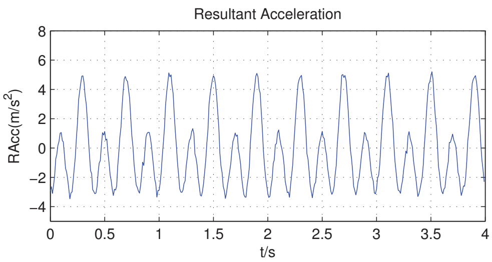


Fig. 6: Resultant acceleration

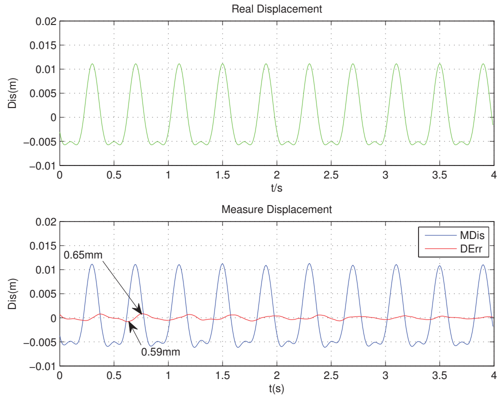


Fig. 7: Comparison of measured and real displacement

### C. Discussion

As shown in Fig. 5, after denoising, the burr of acceleration signal is less, and the curve becomes smoother, indicating that the random noise in the original signal has been largely filtered out. It can be seen from the Fig. 6 that the resultant acceleration. By analysing the Fig. 7, it is demonstrated that the measured displacement is roughly the same as the real displacement only with a small deviation. In order to analyse the measurement error, the error cure is also plotted in Fig. 7. And the maximum of the measurement error is 0.65 millimeter, which can meet the actual project requirements. It can further proves that the proposed algorithm can eliminate the noise and trend item in the acceleration signal effectively, and greatly improve the accuracy of displacement measurement.

## V. Conclusion

A novel vibration displacement measurement method based on three axes accelerometer is proposed in this paper. The random noise and trend item are effectively removed by wavelet denoising method firstly, then the spectrum conversion method is used for obtaining the accurate displacement measurement. Finally, the experimental test based on the motion control platform is carried out for evaluating the effectiveness of the proposed algorithm. Experimental results demonstrate that the maximum of the measurement error is 0.65 millimeter, which indicates that the proposed approach can effectively eliminate noise and trend item, and obviously improve the accuracy of displacement measurement.

Considering the efficiency and low consumption of the proposed algorithm, its can be concluded that the algorithm has certain application value in many fields.

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## References

[1] Carrera E, Pagani A. Free vibration analysis of civil engineering structures by component-wise models[J]. Journal of Sound and Vibration, 2014, 333(19): 4597-4620.

[2] Wan S, Li D, Gao J, et al. Process and knowledge management in a collaborative maintenance planning system for high value machine tools[J]. Computers in Industry, 2017, 84: 14-24.
[3] Matsubara A, Yamazaki T, Ikenaga S. Non-contact measurement of spindle stiffness by using magnetic loading device[J]. International Journal of Machine Tools and Manufacture, 2013, 71: 20-25.
[4] Atwell P C, York F J, Briggs C H. Non-contact measurement device: U.S. Patent Application 14/149,888[P]. 2014-1-8.
[5] Beck B S, Cunefare K A, Collet M, et al. Active vibration control of a stiffened panel through application of negative capacitance shunts[C]//SPIE Smart Structures and Materials+ Nondestructive Evaluation and Health Monitoring. International Society for Optics and Photonics, 2011: 79770C-79770C-12.
[6] Garcła Y R, Corres J M, Goicoechea J. Vibration detection using optical fiber sensors[J]. Journal of Sensors, 2010, 2010.
[7] Lotfi B, Huang L. An approach for velocity and position estimation through acceleration measurements[J]. Measurement, 2016, 90: 242-249.
[8] Weller H. Fourier analysis[J]. 2015.
[9] Brmaud P. Mathematical principles of signal processing: Fourier and wavelet analysis[M]. Springer Science & Business Media, 2013.
[10] Jones G W. High efficiency and long life optical spectrum conversion device and process: U.S. Patent 8,724,054[P]. 2014-5-13.
[11] Jicheng Liu, Qinghua Xu, Jianxin Zha. Method for measuring vibration displacement by acceleration sensor[J]. Modern radar, 2007, 29(5): 69-71.