

24-Sep-2020

Q Find the limit of $f(x) = 1/x^2$ at $x = 0$ and $f(x) = 1/x^4$ at ∞

(%i1) $f(x) := 1/x^2$;

(%o1) $f(x) := \frac{1}{x^2}$

(%i2) $\text{limit}(f(x), x, 0)$;

(%o2) ∞

(%i3) $f(x) := 1/x^4$;

(%o3) $f(x) := \frac{1}{x^4}$

(%i4) $\text{limit}(f(x), x, \infty)$;

(%o4) 0

Q Find the right and left hand limit of $1/x$ at $x = 0$

(%i5) $f(x) := 1/x$;

(%o5) $f(x) := \frac{1}{x}$

(%i6) $\text{limit}(f(x), x, 0, \text{minus})$;

(%o6) $-\infty$

(%i7) $\text{limit}(f(x), x, 0, \text{plus})$;

(%o7) ∞

Q Find the right and left hand limit of $|x/x|$ at $x = 0$ and what can you conclude from it

(%i8) $f(x) := \text{abs}(x / (\text{abs}(x)))$;

(%o8) $f(x) := \left| \frac{x}{|x|} \right|$

(%i9) $\text{limit}(f(x), x, 0, \text{minus})$;

(%o9) 1

(%i10) $\text{limit}(f(x), x, 0, \text{plus})$;

(%o10) 1

(%i11) $\text{limit}(f(x), x, 0)$;

(%o11) 1

the limit of $f(x) = |x/x|$ exist at $x = 0$ and it is 1

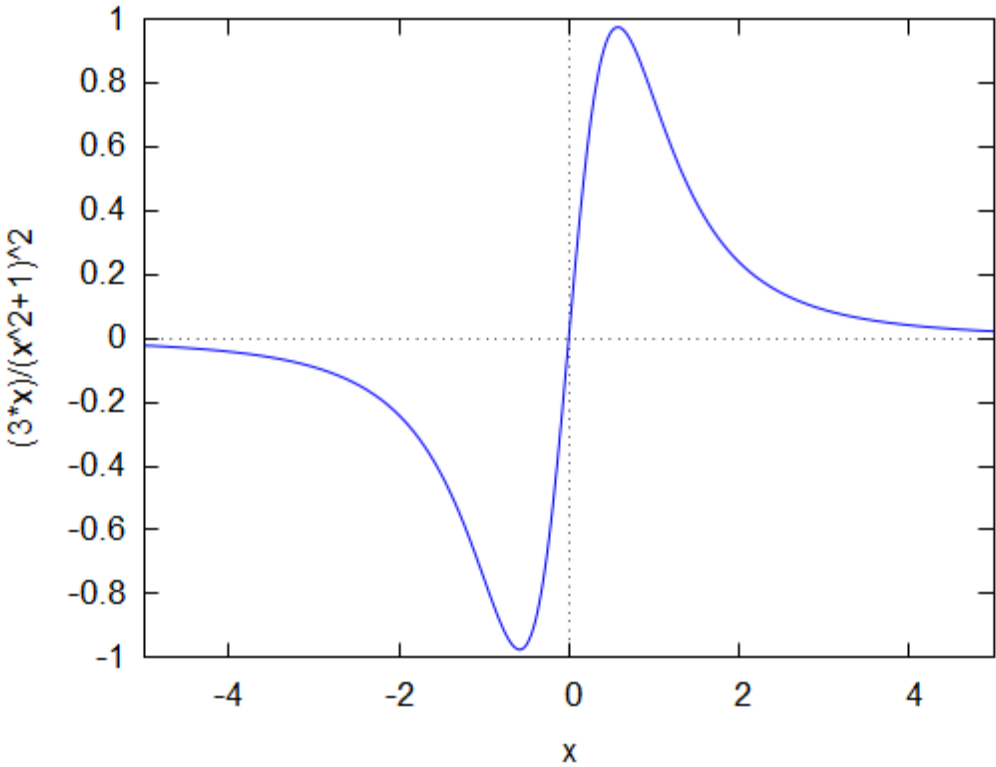
Q Find points of maxima and minima for $f(x) = 3x/(x^2+1)^2$ also find maximum and minimum value of $f(x)$ and plot the graph of $f(x)$

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(%i12) f(x) := 3 * x / ( x ^ 2 + 1 ) ^ 2 ;
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(%o12) $f(x) := \frac{3x}{(x^2 + 1)^2}$

```
(%i13) wxplot2d ( f ( x ) , [ x , - 5 , 5 ] ) ;
```

(%t13)



(%o13)

```
(%i14) solve ( diff ( f ( x ) , x ) = 0 , x ) ;
```

(%o14) $[x = -\frac{1}{\sqrt{3}}, x = \frac{1}{\sqrt{3}}]$

```
(%i15) d2f(x) := '' ( diff ( f ( x ) , x , 2 ) ) ;
```

(%o15) $d2f(x) := \frac{72x^3}{(x^2 + 1)^4} - \frac{36x}{(x^2 + 1)^3}$

```
(%i16) d2f( - 1 / ( sqrt ( 3 ) ) ) ;
```

(%o16) $\frac{3^{\frac{9}{2}}}{32}$

```
(%i17) d2f( 1 / ( sqrt ( 3 ) ) ) ;
```

(%o17) $-\frac{3^{\frac{9}{2}}}{32}$

$f(x)$ is minimum at $x = -1/\sqrt{3}$ as $f'(x)$ at $x = -1/\sqrt{3}$ is +ive and $f(x)$ is maximum at $x = 1/\sqrt{3}$ as $f'(x)$ at $x = 1/\sqrt{3}$ is -

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(%i18) f(-1/sqrt(3));
```

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(%o18) -3^(5/2)/16
```

```
(%i19) f(1/sqrt(3));
```

```
(%o19) 3^(5/2)/16
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maximum value of $f(x)$ is $3^{5/2}/16$ and minimum value of $f(x)$ is $-3^{5/2}/16$

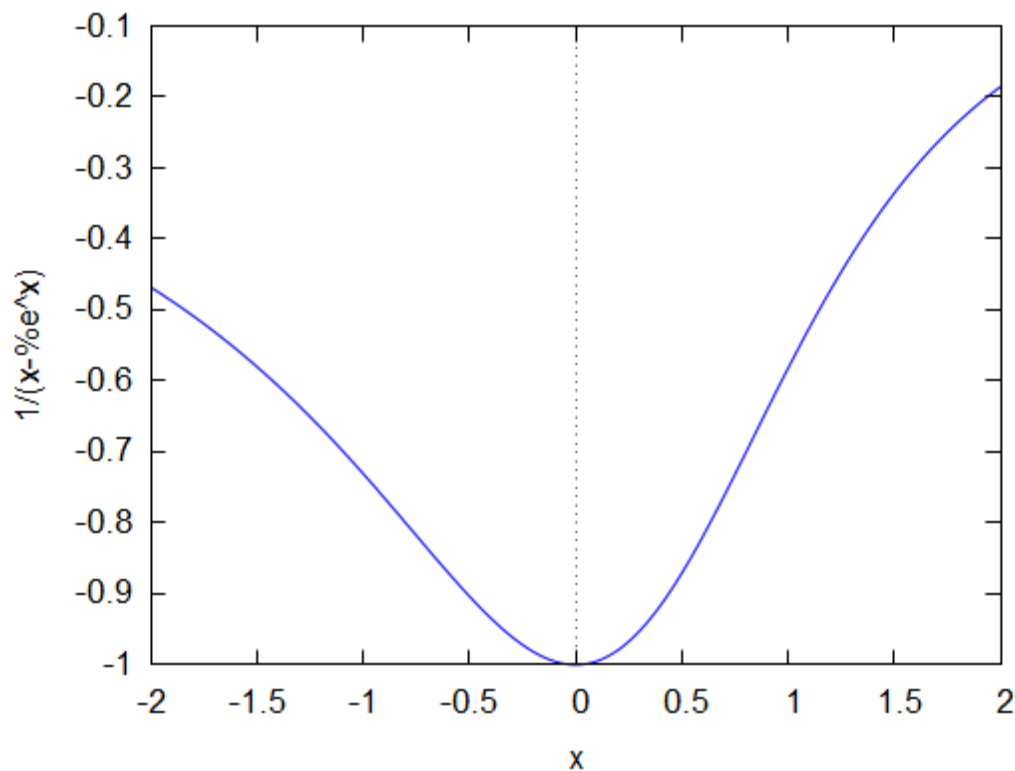
Q Find points of maxima and minima for $f(x) = 1/(x - e^x)$ also find maximum and minimum value of $f(x)$ and plot the graph of $f(x)$

```
(%i20) f(x) := 1/(x - %e^x);
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```
(%o20) f(x) := 1/(x - %e^x)
```

```
(%i21) wxplot2d(f(x), [x, -2, 2]);
```

```
(%t21)
```



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(%o21)
```

```
(%i22) solve(diff(f(x), x) = 0, x);
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```
(%o22) [x = 0]
```

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(%i23) d2f(x) := '(diff(f(x), x, 2));
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(%o23)
$$d^2f(x) := \frac{e^x}{(x - e^x)^2} + \frac{2(1 - e^x)^2}{(x - e^x)^3}$$

(%i24)
$$d^2f(-\log(\log(e))/\log(e));$$

(%o24) 1

f(x) is minimum at x = 1 as f'(x)
at x = 0 is 1 and there is no
maximum.

(%i25)
$$f(0);$$

(%o25) -1

minimum value of f(x) is -1 at x = 0.