* $\nabla E = E\nabla = \Delta$

L.H.S $= \nabla E$

$= (1-E^{-1})E$

$= E-(E)(E^{-1})$

$= E - E^{1+(-1)}$

$= E - E^{1-1}$

$= E - E^{0}$

$\Delta \quad = E - 1$

M.H.S $= E\nabla$

$= E(1-E^{-1})$

Scanned with CamScanner

$= E - (E)(E^{-1})$

$= E - E^{1+(-1)}$

$= E - E^{x-x}$

$= E - E^{0}$

$\Delta = E - 1$

R.H.S $= \Delta$

$= E - 1$

Hence proved

$\nabla E = E \nabla = \Delta$

* $E^{-1} = 1 - \nabla$

L.H.S $= E^{-1}$

$= 1 - \nabla$

$\because \nabla = 1 - E^{-1}$

$E^{-1} = 1 - \nabla$

R.H.S $= 1 - \nabla$

$= 1 - (1 - E^{-1})$

$= \cancel{1} - \cancel{1} + E^{-1}$

$= E^{-1}$

Hence proved

$E^{-1} = 1 - \nabla$ Ans

★ $EE^{-1} = 1$

L.H.S $= EE^{-1}$

$= (\Delta+1)(1-\nabla)$

$= \Delta - \Delta\nabla + 1 - \nabla$

$= (E-1)-(E-1)(1-E^{-1})$

$+1-(1-E^{-1})$

$\because \Delta = E-1$

$\Delta + 1 = E$

$\because \nabla = 1 - E^{-1}$

$E^{-1} = 1 - \nabla$

$=(E-1)-(E-1-1+E^{-1})+1-(1-E^{-1})$

$= E-1-(E-2+E^{-1})+\cancel{1}-\cancel{1}+E^{-1}$

$= \cancel{E}-1-\cancel{E}+2-\cancel{E^{-1}}+\cancel{E^{-1}}$

$= -1+2$

R.H.S $= 1$

Hence ploved

$EE^{-1} = 1$

★ $\Delta\nabla = \nabla\Delta$

L.H.S $= \Delta\nabla$

$= (E-1)(1-E^{-1})$

$= E(1-E^{-1})-1(1-E^{-1})$

$= E-(E)(E^{-1})-1+E^{-1}$

$= E-E^{1+(-1)}-1+E^{-1}$

$= E-E^{1-1}-1+E^{-1}$

$= E-E^{0}-1+E^{-1}$

$= E-1-1+E^{-1}$

$= E-2+E^{-1}.$

$= E + E^{-1} - 2$

R.H.S $= \nabla\Delta$

$= (1 - E^{-1})(E - 1)$

$= 1(E-1) - E^{-1}(E-1)$

$= E - 1 - (E^{-1})(E) + E^{-1}$

$= E - 1 - E^{-1+1} + E^{-1}$

$= E - 1 - E^{0} + E^{-1}$

$= E - 1 - 1 + E^{-1}$

$= E + E^{-1} - 2$

Hence proved

$\Delta\nabla = \nabla\Delta$

* $\mu\delta = \delta\mu$

L.H.S $= \mu\delta$

$= \frac{1}{2}\left[E^{1/2} + E^{-1/2}\right]\left(E^{1/2} - E^{-1/2}\right)$

$\because \delta = E^{1/2} - E^{-1/2}$

$\because \mu = \frac{1}{2}\left(E^{1/2} + E^{-1/2}\right)$

$= \frac{1}{2}\left(E^{1/2}\left(E^{1/2} - E^{-1/2}\right) + E^{-1/2}\left(E^{1/2} - E^{-1/2}\right)\right)$

$= \frac{1}{2}\left[(E^{1/2})(E^{1/2}) - (E^{1/2})(E^{-1/2}) + (E^{-1/2})(E^{1/2}) - (E^{-1/2})(E^{-1/2})\right]$

$= \frac{1}{2}\left[E^{\frac{2}{2}} - E^{0} + E^{0} - E^{-\frac{2}{2}}\right]$

$= \frac{1}{2}\left(E - \cancel{1} + \cancel{1} - E^{-1}\right)$

$$= \frac{1}{2}\left(E - E^{-1}\right)$$

$$= \frac{E - E^{-1}}{2}$$

R.H.S $= \delta\mu$

$$= \left(E^{1/2} - E^{-1/2}\right)\frac{1}{2}\left(E^{1/2} + E^{-1/2}\right)$$

$$= \left(E^{1/2} - E^{-1/2}\right)\left(\frac{E^{1/2} + E^{-1/2}}{2}\right)$$

$$= \frac{\left(E^{1/2} - E^{-1/2}\right)\left(E^{1/2} + E^{-1/2}\right)}{2}$$

$$= \left(E^{1/2}\right)\left(E^{1/2}\right) + \left(E^{1/2}\right)\left(E^{-1/2}\right) - \left(E^{-1/2}\right)\left(E^{1/2}\right)$$
$$- \left(E^{-1/2}\right)\left(E^{-1/2}\right)$$

$$= \frac{E^{\frac{1}{2}+\frac{1}{2}} + E^{\cancel{1/2} - \cancel{1/2}} - E^{-1/2+1/2} - E^{-1/2-1/2}}{2}$$

$$= \frac{E^{\frac{1+1}{2}} + E^{0} - E^{0} - E^{\frac{-1-1}{2}}}{2}$$

$$= \frac{E^{\cancel{2}/\cancel{2}} + \cancel{1} - \cancel{1} - E^{-\cancel{2}/\cancel{2}}}{2}$$

$$= \frac{E - E^{-1}}{2}$$

Hence proved

$\mu\delta = \delta\mu$

* $\delta^2 = \Delta^2 (1+\Delta)^{-1}$

L.H.S $= \delta^2$

$= (E^{1/2} - E^{-1/2})^2$

$= (E^{1/2})^2 + (E^{-1/2})^2 - 2(E^{1/2})(E^{-1/2})$

$= E + E^{-1} - 2\, E^{1/2 - 1/2}$

$= E + E^{-1} - 2E^0$

$= E + E^{-1} - 2(1)$

$= E + E^{-1} - 2$

R.H.S $= \Delta^2 (1+\Delta)^{-1}$

$= \dfrac{\Delta^2}{(1+\Delta)}$

$= \dfrac{(E-1)^2}{(\cancel{1} + E - \cancel{1})}$

$= \dfrac{E^2 + 1 - 2E}{E}$

$= E^{-1}(E^2 + 1 - 2E)$

$= E^{-1+2} + E^{-1} - 2EE^{-1}$

$= E + E^{-1} - 2E^{\cancel{1}-\cancel{1}}$

$= E + E^{-1} - 2E^0$

$= E + E^{-1} - 2(1)$

$= E + E^{-1} - 2$

Hence proved

$\delta^2 = \Delta^2 (1+\Delta)^{-1}$

$$\delta^2 = \nabla^2(1-\nabla)^{-1}$$

$$L.H.S = \delta^2$$

$$= \left(E^{1/2} - E^{-1/2}\right)^2$$

$$= \left(E^{1/2}\right)^2 + \left(E^{-1/2}\right)^2 - 2\left(E^{1/2}\right)\left(E^{-1/2}\right)$$

$$= E + E^{-1} - 2\left(E^{1/2}\right)\left(E^{-1/2}\right)$$

$$= E + E^{-1} - 2\left(E^{1/2-1/2}\right)$$

$$= E + E^{-1} - 2E^{0}$$

$$= E + E^{-1} - 2(1)$$

$$= E + E^{-1} - 2$$

$$R.H.S = \nabla^2(1-\nabla)^{-1}$$

$$= \frac{\nabla^2}{(1-\nabla)}$$

$$= \frac{(1-E^{-1})^2}{1-(1-E^{-1})}$$

$$= \frac{1+E^{-2}-2E^{-1}}{\cancel{1}-\cancel{1}+E^{-1}}$$

$$= \frac{1+E^{-2}-2E^{-1}}{E^{-1}}$$

$$= E^{1}(1+E^{-2}-2E^{-1})$$

$$= E+EE^{-2}-2E^{-1}E$$

$$= E+E^{1-2}-2E^{-\cancel{1}+\cancel{1}}$$

$$= E+E^{-1}-2E^{0}$$

$$= E+E^{-1}-2(1)$$

$$= E+E^{-1}-2$$

Hence proved

$$\delta^2 = \nabla^2(1-\nabla)^{-1}$$

$$\mu^2 = \left(1+\frac{\Delta}{2}\right)^2(1+\Delta)^{-1}$$

$$\text{L.H.S} = \mu^2$$

$$= \left[\frac{1}{2}(E^{1/2}+E^{-1/2})\right]^2$$

$$= \frac{1}{4}\left(E^{1/2}+E^{-1/2}\right)^2$$

$$= \frac{1}{4}\left[(E^{1/2})^2+(E^{-1/2})^2+2(E^{1/2})(E^{-1/2})\right]$$

$$= \frac{1}{4}\left(E + E^{-1} + 2\, E^{1/2 - 1/2}\right)$$

$$= \frac{1}{4}\left(E + E^{-1} + 2(1)\right)$$

$$= \frac{1}{4}\left(E + E^{-1} + 2\right)$$

$$= \frac{E + E^{-1} + 2}{4}$$

$$R.H.S = \left(1 + \frac{\Delta}{2}\right)^2 (1 + \Delta)^{-1}$$

$$\left(1 + \frac{\Delta}{2}\right)^2 = \left(\frac{2 + \Delta}{2}\right)^2$$

$$= \frac{(2 + \Delta)^2}{(2)^2}$$

$$= \frac{(2 + E - 1)^2}{4}$$

$$= \frac{(E + 1)^2}{4}$$

$$= \frac{E^2 + 1 + 2E}{4}$$

$$= \left(\frac{E^2 + 1 + 2E}{4}\right)(\cancel{1} + E - \cancel{1})^{-1}$$

$$= \left(\frac{E^2 + 2E + 1}{4}\right)(E^{-1})$$

$$= \frac{(E^2)(E^{-1}) + 2EE^{-1} + E^{-1}}{4}$$

$$= \frac{E^{2-1} + 2E^{x-x} + E^{-1}}{4}$$

$$= \frac{E^1 + 2E^0 + E^{-1}}{4}$$

$$= \frac{E + 2(1) + E^{-1}}{4}$$

$$= \frac{E + 2 + E^{-1}}{4}$$

$$= \frac{E + E^{-1} + 2}{4}$$

Hence proved

$$\mu^2 = \left(1 + \frac{\Delta}{2}\right)^2 (1+\Delta)^{-1}$$