In the following problems, when you are asked to plot some point, **you need to grab a screenshot of a plot, and print it out and include it with your solutions**.

I provide a utility file `trolley.py`.

1. Plot the following points using the `plot` procedure of the `plotting` module. (These points are listed in the utility file `trolley.py`.) Print the plot and include it with your hand-in.

| | | | | | |
|---|---|---|---|---|---|
| (1.55, 0.9) | (-0.87, -2.42) | (0.61, -1.84) | (0.72, -0.1) | (0.16, 0.48) | (0.46, 0.81) |
| (-0.71, -1.69) | (1.26, 1.11) | (-2.14, -5.34) | (-1.09, -4.2) | (-0.61, 0.14) | (-0.33, -0.59) |
| (-0.64, -0.76) | (1.73, -0.67) | (2.38, 1.38) | (-0.34, -0.97) | (-2.17, -2.51) | (0.42, -0.93) |
| (2.43, -0.43) | (1.9, 1.22) | (2.24, 1.98) | (-1.83, -1.6) | (-0.17, 1.22) | (0.64, -0.61) |
| (-2.07, -3.11) | (0.59, 1.77) | (1.96, -0.38) | (-0.05, -2.64) | (0.49, 1.6) | (-0.23, 2.43) |
| (1.62, 1.06) | (2.06, -0.03) | (1.6, 0.41) | (1.38, 2.29) | (-0.96, -0.8) | (-0.4, -2.42) |
| (2.97, 1.56) | (-0.03, 1.46) | (-0.1, 0.94) | (1.29, -2.39) | | |

   Recall that you can give `plot` a list of $(x, y)$ pairs:

```
>>> from plotting import plot
>>> plot([(1,2),(2,3),(3,4)])
```

2. You're familiar with linear regression, in which, given some points, the goal is to find the numbers $m, b$ so that the line defined by $y = mx + b$ is the best line in that it minimizes the sum of squares of distances between the points $(x_i, y_i)$ and the points on the line with the same $x$-value.

   In this problem, you will do something even simpler. You will find the number $m$ so that the line defined by $y = mx + 0$ is the best line through the origin in the same sense: it minimizes the sum of squares of distances between the points $(x_i, y_i)$given in Problem 1 and the points on the line with the same $x$-value. (You might call this "vertical error".) This is the same as linear regression except $b$ is required to be zero.

   (a) Your goal in this part is to calculate the value of $m$. First, write down algebra you use to compute it. If you used Python to compute it, provide that too. Then give the actual value. (You are graded on the algebra and the value.)

   (b) Form a list of eighty or so $(x, y)$ pairs that are on the line $y = mx + 0$ (but don't use values of $x$ or $y$ with absolute value greater than, say, two). Combine it with the list of $(x, y)$ points given in Problem 1. Finally, plot the combined list. You should see the points and the line. Print the plot and include it in your hand-in.

3. In this problem, you will solve the trolley-line-location problem for the points given in Problem 1.

   (a) The utility file `trolley.py` includes a procedure `first_right_singular_vector(A)`. It finds a rough approximation to the first right singular vector of a Mat $A$. We'll later see why this works. For now, use it to find the first right singular vector vector $\boldsymbol{v}_1$ for the given points.

   (b) Form a list of eighty or so $(x, y)$ pairs that are on the line Span $\boldsymbol{v}_1$ (but don't use values of $x$ or $y$ with absolute value greater than, say, two). Combine it with the list of $(x, y)$ points given in Proble 1. Finally, plot the combined list. You should see the points and the line. Print the plot and include it in your hand-in.

4. Now you will do both:

   (a) Make a list consisting of the points from Problem 1, the points forming the line in Problem 2b, and the points forming the line in Problem 3b. Plot all these points together. Print the plot and include it in your hand-in.

   (b) Note that the two lines are different. Which has greater slope? Give an explanation for why that one has greater slope by considering those points from Problem 1 that lie close to the $y$ axis.

5. Plot the following points and print the plot and include it:

| | | | | | |
|---|---|---|---|---|---|
| (1.38, 0.44) | (-0.59, -0.26) | (-1.24, 0.07) | (-1.42, 1.99) | (1.2, 0.01) | (-0.88, -2.63) |
| (-1.35, -0.83) | (-1.09, 2.34) | (-0.22, 1.21) | (0.77, -0.32) | (-0.15, 1.88) | (-0.86, -0.38) |
| (0.91, -2.46) | (0.71, 1.06) | (-1.98, -0.26) | (-0.71, 1.71) | (0.0, 0.15) | (-0.94, 1.25) |
| (0.55, 0.85) | (-1.06, -0.58) | (-1.06, -1.55) | (-0.49, 0.65) | (-0.37, 1.2) | (-0.78, 1.02) |
| (0.5, 0.48) | (-0.38, -0.28) | (-2.29, 0.2) | (-1.27, 1.14) | (-0.47, 1.68) | (-0.65, 0.15) |
| (2.24, -0.33) | (0.6, -0.95) | (-0.2, 1.85) | (-0.53, -0.7) | (0.23, -1.68) | (0.58, 0.05) |
| (0.81, 0.38) | (-1.87, -0.1) | (0.08, 0.63) | (0.63, 0.05) | | |

6. (a) Do linear regression on the above points. That includes finding both $m$ and $b$. Show how you got them (including algebra and code) and give us the values you obtained.

   (b) Plot the points together with representative points from the line $y = mx + b$.

7. (a) Find the one-dimensional affine space that is closest to the above points. You can express that affine space as $\boldsymbol{c} + \{\alpha \boldsymbol{b} \ : \ \alpha \in \mathbb{R}\}$. Report the vectors $\boldsymbol{c}$ and $\boldsymbol{b}$.

   (b) Plot the points together with representative points from the line comprised by the closest one-dimensional affine space.