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# Bits of Architecture

— Integer Arithmetic Basics —

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# Negation

## Negation

- Two steps
  - Flip all the bits
  - Add 1

$$4 = 0100_2$$



$$-5 = 1011_2$$



$$-4 = 1100_2$$

# Negation

- Why does this work?
- We want the positive and negative of a number to add up to  $2^N$ 
  - Radix Complement
  - **Two's-Complement**
- Flipping the bits gets us one away
  - Adding 1 gets us the rest of the way

Binary	Unsigned	Signed
000	0	0
001	1	1
010	2	2
011	3	3
100	4	-4
101	5	-3
110	6	-2
111	7	-1

# Sign Extension

## Sign Extension

- How do we extend the number of bits of a signed number?
  - Simply copy the sign bit
- Why does this work?

**Positive  
Numbers**

$$4 = 0100_2$$

$$4 = 00100_2$$

**Negative  
Numbers**

$$-8 = 1000_2$$

$$-8 = 11000_2$$

# How Do We Add Binary Numbers?

# Adding Integers

$$5_{10} = 0101_2$$

$$\begin{array}{r} + \quad 1_{10} = \\ \hline 0001_2 \end{array}$$



# Adding Integers

$$\begin{array}{r} 0101_2 \\ + 0001_2 \\ \hline \end{array} \quad \rightarrow \quad \begin{array}{r} \overset{1}{0}101_2 \\ + 000\overset{1}{1}_2 \\ \hline \overset{0}{2} \end{array}$$

# Adding Integers

The diagram illustrates the addition of two binary integers,  $0101_2$  and  $0001_2$ , in three stages:

- Stage 1:** The initial addition setup:
$$\begin{array}{r} 0101_2 \\ + 0001_2 \\ \hline \end{array}$$
- Stage 2:** The first carry is generated. A '1' is placed above the third column (the 2's place):
$$\begin{array}{r} 1 \\ 0101_2 \\ + 0001_2 \\ \hline \end{array}$$
- Stage 3:** The final result is shown. The '1' in the third column and the '0' in the fourth column (the 4's place) are highlighted in yellow, representing the final sum  $10_2$ :
$$\begin{array}{r} 1 \\ 0101_2 \\ + 0001_2 \\ \hline 10_2 \end{array}$$

# Adding Integers

The diagram illustrates the addition of two binary integers,  $0101_2$  and  $0001_2$ , showing the progression of the sum as carries are propagated from right to left.

**Step 1:** Initial addition without carries.

$$\begin{array}{r} 0101_2 \\ + 0001_2 \\ \hline \end{array}$$

**Step 2:** Carry of 1 is added to the least significant bit.

$$\begin{array}{r} 1 \\ 0101_2 \\ + 0001_2 \\ \hline 0_2 \end{array}$$

**Step 3:** Carry of 1 is added to the second bit from the right.

$$\begin{array}{r} 1 \\ 0101_2 \\ + 0001_2 \\ \hline 10_2 \end{array}$$

**Step 4:** Carry of 1 is added to the third bit from the right, resulting in the final sum.

$$\begin{array}{r} 1 \\ 0101_2 \\ + 0001_2 \\ \hline 110_2 \end{array}$$

# Adding Integers

The diagram illustrates the step-by-step addition of two binary integers,  $0101_2$  and  $0001_2$ , showing the propagation of a carry bit.

Step 1: Initial addition without carry.

$$\begin{array}{r} 0101_2 \\ + 0001_2 \\ \hline \end{array}$$

Step 2: Carry of 1 is generated from the least significant bit.

$$\begin{array}{r} 1 \\ 0101_2 \\ + 0001_2 \\ \hline 0_2 \end{array}$$

Step 3: Carry of 1 is generated from the second bit.

$$\begin{array}{r} 1 \\ 0101_2 \\ + 0001_2 \\ \hline 10_2 \end{array}$$

Step 4: Carry of 1 is generated from the third bit.

$$\begin{array}{r} 1 \\ 0101_2 \\ + 0001_2 \\ \hline 110_2 \end{array}$$

Step 5: Final result with carry of 1.

$$\begin{array}{r} 1 \\ 0101_2 \\ + 0001_2 \\ \hline 0110_2 \end{array}$$

# Can We Add Positives and Negatives?

# Adding Integers

$$5_{10} = 0101_2$$

$$+ -1_{10} = 1111_2$$

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# Adding Integers

$$\begin{array}{r} 0101_2 \\ + 1111_2 \\ \hline \end{array} \quad \rightarrow \quad \begin{array}{r} \phantom{0}1 \\ 0101_2 \\ + 1111_2 \\ \hline 0_2 \end{array}$$

# Adding Integers

The diagram illustrates the addition of two 4-bit binary integers,  $0101_2$  and  $1111_2$ , showing the progression of carry bits.

**Step 1:** Initial addition setup.

$$\begin{array}{r} 0101_2 \\ + 1111_2 \\ \hline \end{array}$$

**Step 2:** First carry (1) is generated from the least significant bit (LSB) addition.

$$\begin{array}{r} 1 \\ 0101_2 \\ + 1111_2 \\ \hline 0_2 \end{array}$$

**Step 3:** Second carry (11) is generated from the next bit position.

$$\begin{array}{r} 11 \\ 0101_2 \\ + 1111_2 \\ \hline 00_2 \end{array}$$



# Adding Integers

The diagram illustrates the addition of two 4-bit binary integers,  $0101_2$  and  $1111_2$ , showing the progression of carry bits from right to left.

**Step 1:** Initial addition. The least significant bit (rightmost) of the second number is highlighted in yellow.

$$\begin{array}{r} 0101_2 \\ + 1111_2 \\ \hline \end{array}$$

**Step 2:** A carry of 1 is generated from the first column and is added to the second column. The carry bit '1' and the updated least significant bit '1' are highlighted in yellow.

$$\begin{array}{r} 1 \\ 0101_2 \\ + 1111_2 \\ \hline \end{array}$$

**Step 3:** A carry of 11 is generated from the second column and is added to the third column. The carry bits '11' and the updated second bit '1' are highlighted in yellow.

$$\begin{array}{r} 11 \\ 0101_2 \\ + 1111_2 \\ \hline \end{array}$$

**Step 4:** A carry of 111 is generated from the third column and is added to the fourth column. The carry bits '111' and the updated third bit '1' are highlighted in yellow.

$$\begin{array}{r} 111 \\ 0101_2 \\ + 1111_2 \\ \hline \end{array}$$

The final result of the addition is  $100_2$ , with the result '100' highlighted in yellow.

# Adding Integers

The diagram illustrates the step-by-step process of adding two 4-bit binary integers,  $0101_2$  and  $1111_2$ , showing the propagation of a carry from right to left.

**Step 1:** Initial addition. The carry-in is 0.

$$\begin{array}{r} 0101_2 \\ + 1111_2 \\ \hline \end{array}$$

**Step 2:** The first carry (1) is generated from the least significant bit.

$$\begin{array}{r} 1 \\ 0101_2 \\ + 1111_2 \\ \hline 0_2 \end{array}$$

**Step 3:** The carry (11) propagates to the next bit position.

$$\begin{array}{r} 11 \\ 0101_2 \\ + 1111_2 \\ \hline 00_2 \end{array}$$

**Step 4:** The carry (111) propagates to the next bit position.

$$\begin{array}{r} 111 \\ 0101_2 \\ + 1111_2 \\ \hline 100_2 \end{array}$$

**Step 5:** The final carry (1111) propagates to the next bit position, resulting in the final sum.

$$\begin{array}{r} 1111 \\ 0101_2 \\ + 1111_2 \\ \hline 0100_2 \end{array}$$

# Important Considerations

# Bit Limitations

- Can we add -4 and -4 and get the desired result?
  - In 4 bits - yes!
  - In 3 bits - no!
- Can the product of two 4-bit numbers fit in 4-bits?
  - Often times not