Bits of Architecture

Integer Arithmetic Basics

Negation

Negation

- Two steps
  - Flip all the bits
  - Add 1

$$4 = 0100_2$$

$$\downarrow$$

$$-5 = 1011_2$$

$$\downarrow$$

$$-4 = 1100_2$$

Negation

- Why does this work?
- We want the positive and negative of a number to add up to 2^N
    - Radix Complement
    - **Two's-Complement**
- Flipping the bits gets us one away
    - Adding 1 gets us the rest of the way

Binary	Unsigned	Signed
000	0	0
001	1	1
010	2	2
011	3	3
100	4	-4
101	5	-3
110	6	-2
111	7	-1

Sign Extension

Sign Extension

- How do we extend the number of bits of a signed number?
  - Simply copy the sign bit
- Why does this work?

Positive Numbers	**Negative Numbers**
$4 = 0100_2$	$-8 = 1000_2$
$4 = 00100_2$	$-8 = 11000_2$

How Do We Add Binary Numbers?

Adding Integers

$5_{10} = 0101_2$

$+ \quad 1_{10} = 0001_2$

Adding Integers

$$\begin{array}{rr} & 0101_2 \\ + & 0001_2 \\ \hline & \end{array} \rightarrow \begin{array}{rr} & \overset{1}{} \\ & 0101_2 \\ + & 0001_2 \\ \hline & 0_2 \end{array}$$

Adding Integers

$$\begin{array}{r} \\ 0101_2 \\ +\ 0001_2 \\ \hline \\ \end{array} \rightarrow \begin{array}{r} 1 \\ 0101_2 \\ +\ 0001_2 \\ \hline 0_2 \end{array} \rightarrow \begin{array}{r} 1 \\ 0101_2 \\ +\ 0001_2 \\ \hline 10_2 \end{array}$$

Adding Integers

$$
\begin{array}{r} \\ 0101_2 \\ +\ \ 0001_2 \\ \hline \\ \end{array}
\rightarrow
\begin{array}{r} 1 \\ 0101_2 \\ +\ \ 0001_2 \\ \hline 0_2 \end{array}
\rightarrow
\begin{array}{r} 1 \\ 0101_2 \\ +\ \ 0001_2 \\ \hline 10_2 \end{array}
\rightarrow
\begin{array}{r} 1 \\ 0101_2 \\ +\ \ 0001_2 \\ \hline 110_2 \end{array}
$$

Adding Integers

$$
\begin{array}{ccccc} & & & & \\ & 0 & 1 & 0 & 1_2 \\ + & 0 & 0 & 0 & 1_2 \\ \hline & & & & \end{array}
\rightarrow
\begin{array}{ccccc} & & & 1 & \\ & 0 & 1 & 0 & 1_2 \\ + & 0 & 0 & 0 & 1_2 \\ \hline & & & & 0_2 \end{array}
\rightarrow
\begin{array}{ccccc} & & & 1 & \\ & 0 & 1 & 0 & 1_2 \\ + & 0 & 0 & 0 & 1_2 \\ \hline & & & 1 & 0_2 \end{array}
\rightarrow
\begin{array}{ccccc} & & & 1 & \\ & 0 & 1 & 0 & 1_2 \\ + & 0 & 0 & 0 & 1_2 \\ \hline & & 1 & 1 & 0_2 \end{array}
\rightarrow
\begin{array}{ccccc} & & & 1 & \\ & 0 & 1 & 0 & 1_2 \\ + & 0 & 0 & 0 & 1_2 \\ \hline & 0 & 1 & 1 & 0_2 \end{array}
$$

Can We Add Positives and Negatives?

Adding Integers

$$
\begin{array}{r}
5_{10} = 0101_2 \\
+\ \ -1_{10} = 1111_2 \\
\hline
\end{array}
$$

Adding Integers

$$\begin{array}{r} 0101_2 \\ +\ \ 1111_2 \\ \hline \end{array} \rightarrow \begin{array}{r} 1 \\ 0101_2 \\ +\ \ 1111_2 \\ \hline 0_2 \end{array}$$

Adding Integers

$$
\begin{array}{rr}
 & \\
 & 0101_2 \\
+ & 1111_2 \\
\hline
 & \\
\end{array}
\rightarrow
\begin{array}{rr}
 & 1 \\
 & 0101_2 \\
+ & 1111_2 \\
\hline
 & 0_2 \\
\end{array}
\rightarrow
\begin{array}{rr}
 & 11 \\
 & 0101_2 \\
+ & 1111_2 \\
\hline
 & 00_2 \\
\end{array}
$$

Adding Integers

$$\begin{array}{r} \\ 0101_2 \\ +\ \ 1111_2 \\ \hline \\ \end{array} \rightarrow \begin{array}{r} 1 \\ 0101_2 \\ +\ \ 1111_2 \\ \hline 0_2 \end{array} \rightarrow \begin{array}{r} 11 \\ 0101_2 \\ +\ \ 1111_2 \\ \hline 00_2 \end{array} \rightarrow \begin{array}{r} 111 \\ 0101_2 \\ +\ \ 1111_2 \\ \hline 100_2 \end{array}$$

Adding Integers

$$
\begin{array}{r}
\\
0101_2 \\
+\quad 1111_2 \\
\hline
\\
\end{array}
\rightarrow
\begin{array}{r}
1 \\
0101_2 \\
+\quad 1111_2 \\
\hline
0_2
\end{array}
\rightarrow
\begin{array}{r}
11 \\
0101_2 \\
+\quad 1111_2 \\
\hline
00_2
\end{array}
\rightarrow
\begin{array}{r}
111 \\
0101_2 \\
+\quad 1111_2 \\
\hline
100_2
\end{array}
\rightarrow
\begin{array}{r}
1111 \\
0101_2 \\
+\quad 1111_2 \\
\hline
0100_2
\end{array}
$$

Important Considerations

Bit Limitations

- Can we add -4 and -4 and get the desired result?
  - In 4 bits - yes!
  - In 3 bits - no!
- Can the product of two 4-bit numbers fit in 4-bits?
  - Often times not