```
interpretation real-euclid: tarski-space real-euclid-C real-euclid-B
proof
  { fix a b c d t
    assume B_R a d t and B_R b d c and a ≠ d
    from real-euclid-B-def [of a d t] and ‹B_R a d t›
      obtain j where j ≥ 0 and j ≤ 1 and d − a = j *R (t − a) by auto
    from ‹d − a = j *R (t − a)› and ‹a ≠ d› have j ≠ 0 by auto
    with ‹d − a = j *R (t − a)› and rearrange-real-euclid-B-2
      have t = (1/j) *R d + (1 − 1/j) *R a by auto
    let ?x = (1/j) *R b + (1 − 1/j) *R a
    let ?y = (1/j) *R c + (1 − 1/j) *R a
    from ‹j ≠ 0› and rearrange-real-euclid-B-2 have
      b − a = j *R (?x − a) and c − a = j *R (?y − a) by auto
    with real-euclid-B-def and ‹j ≥ 0› and ‹j ≤ 1› have
      B_R a b ?x and B_R a c ?y by auto
    from real-euclid-B-def and ‹B_R b d c› obtain k where
      k ≥ 0 and k ≤ 1 and d − b = k *R (c − b) by blast
    from ‹t = (1/j) *R d + (1 − 1/j) *R a› have
      t − ?x = (1/j) *R d − (1/j) *R b by simp
    also from scaleR-right-diff-distrib [of 1/j d b] have
      ... = (1/j) *R (d − b) by simp
    also from ‹d − b = k *R (c − b)› have
      ... = k *R (1/j) *R (c − b) by simp
    also from scaleR-right-diff-distrib [of 1/j c b] have
      ... = k *R (?y − ?x) by simp
    finally have t − ?x = k *R (?y − ?x) .
    with real-euclid-B-def and ‹k ≥ 0› and ‹k ≤ 1› have B_R ?x t ?y by blast
    with ‹B_R a b ?x› and ‹B_R a c ?y› have
      ∃x y. B_R a b x ∧ B_R a c y ∧ B_R x t y by auto }
  thus ∀a b c d t. B_R a d t ∧ B_R b d c ∧ a ≠ d ⟶
          (∃x y. B_R a b x ∧ B_R a c y ∧ B_R x t y)
    by auto
qed
```