Minimum Local Disk Cover Sets for Broadcasting in Heterogeneous Wireless Ad Hoc Networks *

Chih-Wei Yi
Dept. of Computer Science
National Chiao Tung University
yi@cs.nctu.edu.tw

Fang-Chun Liu
Dept. of Computer Science
National Chiao Tung University
vict.cs94g@nctu.edu.tw

Min-Te Sun
Dept. of CSSE
Auburn University
sunmint@eng.auburn.edu

Abstract

The concept of the forwarding set has been used extensively by many broadcast protocols in wireless ad hoc networks to alleviate the broadcast storm problem. In these protocols, when a node receives a broadcast, it only instructs a subset of its neighbors, a.k.a. the forwarding set, to relay the message. In this paper, we propose to use the local minimum disk cover set as the forwarding set in heterogeneous wireless ad hoc networks, where nodes may have different transmission radius. We show that the minimum local disk cover set of a node is equivalent to its skyline set, and propose a divide-and-conquer algorithm with the optimal time complexity $O(n \log n)$ to compute the skyline set locally and statelessly.

1. Introduction

A wireless ad hoc network consists of a collection of wireless devices networked together in multi-hop fashion. It has been adopted by many applications, such as environmental monitoring, battlefield surveillance, and emergency disaster relief. Network-wide broadcasting is one of the fundamental operations in wireless ad hoc networks. It is widely and frequently used to explore the network topology, discover routing paths, and monitor network integrity. Flooding, the most known broadcasting mechanism, is to let every device retransmit a broadcast when it receives the message the first time. Despite its simplicity, flooding tends to generate too many unnecessary retransmissions, which in turn causes message collisions and channel contention. This issue is known as the broadcast storm problem [3].

To address this issue, alternative broadcasting algorithms [1, 2, 4] have been proposed. In these algorithms, when a node receives a broadcast, it selects a subset of neighbors, referred as *the forwarding set* or multipoint relaying set, to relay the message. To ensure that a broadcast can reach all nodes in the network, nodes in the forwarding set of a node should cover all its 2-hop neighbors. At the same time, to better relieve the broadcast storm problem, the forwarding set should be kept as small as possible.

In [5], Sun et al. proposed constructing the forwarding set based on the coverage area of 1-hop neighbors. The idea behind it is to ensure nodes in the forwarding set of a node to cover the same area as all its 1-hop neighbors. The proposed algorithm is localized, distributed, and with the optimal time complexity $O(nlogn)$. However, the algorithm works only when all nodes in the network have the same transmission radius. In this paper, we extend the work to heterogeneous networks in which nodes may have different transmission radius. We propose a localized algorithm to compute our forwarding set and prove our algorithm is also with the optimal time complexity $O(nlogn)$.

2. Minimum Local Disk Cover Sets

To alleviate the broadcast storm problem associated with broadcast protocols, the size of the forwarding set needs to be reduced. On the other hand, to ensure a broadcast can reach all nodes in the network, the selection of the forwarding set at each node should guarantee that the message will reach all its 2-hop neighbors.

We assume that wireless nodes are distributed in the two-dimensional plane R^2. The topology of a heterogeneous ad hoc network is modeled by a disk graph. Each node u_i is associated with a transmission radius r_i. Two nodes u_i, u_j are said to be neighbors if and only if their Euclidean distance $\|u_i - u_j\|$ is not larger than $\min(r_i, r_j)$. For a node u_i, its coverage is modeled as a disk with center u_i and radius r_i, which is denoted as $B(u_i, r_i)$. A node u_j is said to be covered by u_i if $u_j \in B(u_i, r_i)$. The topological boundary of a set $S \subset R^2$ is denoted by ∂S, and

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thus, $\partial B(u_i, r_i)$ is the circle centered at u_i and with radius r_i. We define the disk cover set of a set of nodes V as $S \subseteq V$ such that $\bigcup_{u_i \in S} B(u_i, r_i) = \bigcup_{u_i \in V} B(u_i, r_i)$. For a set of nodes V, if there exists a node in V such that all other nodes in V are neighbors to that node, then V is called the local set and the corresponding disk set, $\{B(u_1, r_1), B(u_2, r_2), \ldots, B(u_n, r_n)\}$, is called the local disk set. For a local set, its cover set is called a local disk cover set. Obviously, the closed neighbor set of a relay node n_0 is a local set.

To alleviate the broadcast storm problem and ensure the reachability of broadcast messages, we propose to use the minimum local disk cover set as the forwarding set. The problem of computing minimum local disk cover set is formally stated as follows.

Minimum Local Disk Cover Set Problems
Input: A local disk set $\{B(u_0, r_0), B(u_1, r_1), ..., B(u_n, r_n)\}$ such that for all $1 \leq i \leq n$, $\|u_0 - u_i\| \leq \min(r_0, r_i)$, i.e. $u_i \in B(u_0, r_0)$ and $u_0 \in B(u_i, r_i)$. Let $V = \{u_0, u_{1,}..., u_n\}$ be the set of disk centers.
Output: A subset S of V such that $\bigcup_{u_i \in S} B(u_i, r_i) = \bigcup_{u_i \in V} B(u_i, r_i)$.
Measure: $|S|$ is minimal.

We assume that each node learns the locations and radii of its neighbors through the exchanges of beacons. In addition, we define the *skyline* for a disk set as the boundary of the union of disks in the set. Hence, the skyline of the local disk set $\{B(u_0, r_0), B(u_1, r_1), \ldots, B(u_n, r_n)\}$ is $\partial(\bigcup_{i=0}^{n} B(u_i, r_i))$. Obviously, a skyline is composed of arcs of disks. The collection of origins of disks that contribute arcs to a skyline is called the *skyline set*. In the next section, we will show that the MLDCS of a local set is the skyline set of the corresponding local disk set, and thus, we can solve the MLDCS problem for a given local disk set by computing the corresponding skyline.

3. Skyline Sets

In this section, we give properties of skylines and build the relation between the MLDCS for a given local set and the skyline set for the corresponding local disk set. We then propose a divide-and-conquer algorithm to compute the skyline set.

3.1. The Skyline of A Local Disk Set

The following geometric lemma and corollary are used to build the relation between MLDCS for a local set $V = \{u_0, u_1, \ldots, u_n\}$ and the skyline set for the corresponding local disk set $\{B(u_0, r_0), B(u_1, r_1), \ldots, B(u_n, r_n)\}$. Without loss of generality, in the following discussion we assume $n_0 = \mathbf{o}$.

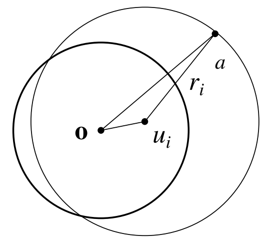

Figure 1. $\overline{\mathbf{o}a}$ is contained in $B(u_i, r_i)$.

Lemma 1 *For any point $a \in \partial B(u_i, r_i)$, the line segment $\overline{\mathbf{o}a} \subset B(u_i, r_i)$.*
Proof. *Note that $\mathbf{o} \in B(u_i, r_i)$ since $\|\mathbf{o} - u_i\| \leq r_i$. Since $B(u_i, r_i)$ is concave and $\mathbf{o}, a \in B(u_i, r_i)$, the line segment $\overline{\mathbf{o}a} \subset B(u_i, r_i)$. An example is shown in Fig. 1.*

Then, we have the following corollary.

Corollary 2 *A ray from $\mathbf{o}$ intersects the skyline at exactly one point.*
Proof. *Obviously, any ray from $\mathbf{o}$ intersects the skyline. Thus, we only need to show the uniqueness of the intersection point. We can prove this by contradiction. Assume there exists a ray that intersects the skyline at points a and b. Without loss of generality, we also assume a is farther from $\mathbf{o}$ than b. Since a is in the skyline, a is in $\partial B(u_i, r_i)$ for some i. According to Lemma 1, we have $\overline{\mathbf{o}a} \subset B(u_i, r_i)$. This implies b is inside of $B(u_i, r_i)$, and b cannot be in the skyline. We have a contradiction, and thus the corollary is proved.*

According to the corollary, we know that a skyline is composed of a sequence of arcs surrounding the origin from 0 to 2π. An arc can be represented by a quadruple $(\alpha_i, u_j, r_j, \alpha_{i+1})$ in which u_j and r_j respectively are the center and radius of the disk contributing the arc, and α_i and α_{i+1} with $\alpha_i < \alpha_{i+1}$ are two angles corresponding to two endpoints of the arc measured at $\mathbf{o}$ in counterclockwise direction. An example is shown in Fig. 2. Note that the reference point to measure α_i and α_{i+1} is $\mathbf{o}$, not u_i. For convenience, if an arc crosses the x-axis in the positive direction then it is splitting by the x-axis into two arcs. Thus, a skyline consisting n arcs can be represented as $(\alpha_0, u_{s_0}, r_{s_0}, \alpha_1, u_{s_1}, r_{s_1}, \alpha_2, ..., \alpha_n)$, where $0 = \alpha_0 < \alpha_1 < ... < \alpha_n = 2\pi$ and for any $0 \leq i \leq n-1$, $(\alpha_i, u_{s_i}, r_{s_i}, \alpha_{i+1})$ is an arc in the skyline.

Now, we give the following theorem that shows the relation between a skyline set and the corresponding MLDCS.

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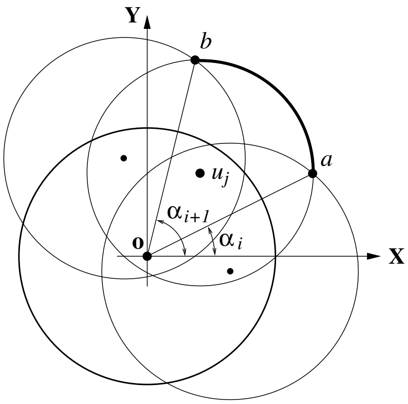

Figure 2. An arc ab is represented by 4 parameters $(\alpha_i, u_j, r_{u_j}, \alpha_{i+1})$, where u_j and r_{u_j} are the center and radius of the disc contributing the arc, and α_i and α_{i+1} are angles corresponding to a and b observed at $\mathbf{o}$.

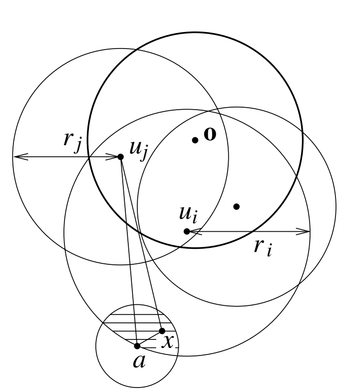

Figure 3. $B(a,r) \cap B(u_i, r_i)$ is exclusively covered by $B(u_i, r_i)$.

Theorem 3 *For a given local set $V = \{u_0, u_1, \ldots, u_n\}$, its minimum local disk cover set is the skyline set for the corresponding local disk set $\{B(u_0, r_0), B(u_1, r_1), \ldots, B(u_n, r_n)\}$.*

Proof. *To prove that the skyline set is the minimum local disk cover set of a local set, we first prove that the skyline set is a local disk cover set. Assume a skyline is composed of arcs $a_1b_1, a_2b_2, \ldots, a_kb_k$ belonging to $B(u_{i_1}, r_{i_1}), B(u_{i_2}, r_{i_2}), \ldots, B(u_{i_k}, r_{i_k})$, respectively. Let $\sphericalangle \mathbf{o}a_jb_j$ denote the sector area surrounded by line segments $\overline{\mathbf{o}a_j}$, $\overline{\mathbf{o}b_j}$, and arc a_jb_j. The covered area $\bigcup_{i=0}^{n} B(u_i, r_i)$ is equal to the union of sectors $\bigcup_{j=1}^{k} \sphericalangle \mathbf{o}a_jb_j$. According to Lemma 1, for each skyline arc a_jb_j, we have $\sphericalangle \mathbf{o}a_jb_j \subseteq B(u_{i_j}, r_{i_j})$. Thus, $\bigcup_{i=0}^{n} B(u_i, r_i) \subseteq \bigcup_{j=1}^{k} B(u_{i_j}, r_{i_j})$. This means $\{B(u_{i_1}, r_{i_1}), B(u_{i_2}, r_{i_2}), \ldots, B(u_{i_k}, r_{i_k})\}$ is a disk cover set.*

Next, we prove that if $B(u_i, r_i)$ is in the skyline set, $B(u_i, r_i)$ must be in any disk cover set. To prove this, we are going to show there exists some area belonging to $B(u_i, r_i)$ but not belonging to any other disk. Assume a is a point in the internal of the skyline arc contributed by $B(u_i, r_i)$. See Fig. 3. For any $j \neq i$, we have $\|u_j - a\| > r_j$. Let $r = \frac{1}{2}(\min_{j \neq i} \|u_j - a\| - r_j)$. For any $x \in B(a, r)$ and $j \neq i$,

$$\begin{aligned}\|u_j - x\| &\geq \|u_j - a\| - \|x - a\| \\ &\geq \|u_j - a\| - \frac{1}{2}\left(\min_{j \neq i} \|u_j - a\| - r_j\right) \\ &> \|u_j - a\| - (\|u_j - a\| - r_j) \\ &= r_j.\end{aligned}$$

Thus, for any $j \neq i$, $B(a, r) \cap B(u_j, r_j) = \varnothing$. This implies that $B(a, r) \cap B(u_i, r_i)$ belongs to $B(u_i, r_i)$ but does not belong to any other disk. So the theorem is proved.

3.2. A Divide-and-Conquer Algorithm

According to Theorem 3, computing the MLDCS of a local set is the same as finding the skyline set of the corresponding local disk set. In this subsection, a divide-and-conquer algorithm is proposed to find the skyline set. Recursively, the disk set is divided into two subsets of disks. After skylines of both subsets are computed by recursive calls, they are merged to find the skyline of all disks. Note that the position of n_0 (i.e., $\mathbf{o}$) and the value r_0 are stored by global variables used in the *Merge* procedure.

procedure $Skyline\left(DS = \{(u_1, r_1), ..., (u_n, r_n)\}\right)$
// (u_i, r_i) represents the center and radius of a disk.
begin
if $|DS| = 1$ **return** the skyline of $\{B(u_1, r_1)\}$
if $|DS| > 1$
  begin
  $DS1 = \left\{(u_1, r_{u_1}), ..., \left(u_{\lfloor \frac{n}{2} \rfloor}, r_{u_{\lfloor \frac{n}{2} \rfloor}}\right)\right\}$
  $DS2 = \left\{\left(u_{\lfloor \frac{n}{2} \rfloor + 1}, r_{u_{\lfloor \frac{n}{2} \rfloor + 1}}\right), ..., (u_n, r_{u_n})\right\}$
  $Skyline1 = Skyline(DS1)$
  $Skyline2 = Skyline(DS2)$
  return $Merge(Skyline1, Skyline2)$
  end
end

procedure $Merge(SL1, SL2)$
// $SL1$ and $SL2$ are skylines.
begin

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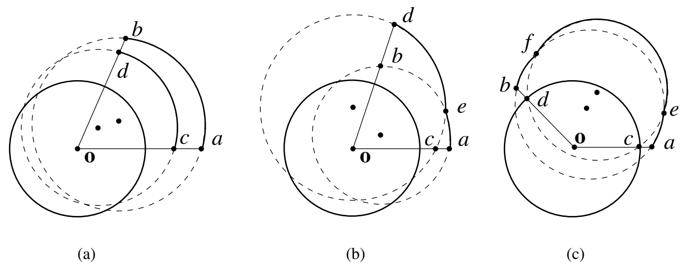

Figure 4. Merging two arcs with the same corresponding angles. Two arcs have (a) no intersection, (b)one intersection point, (c) two intersection points.

Step 1: Refine $SL1$ and $SL2$ to align arcs in skylines. Then, we may assume $SL1 = (\alpha_0, u_1, r_{u_1}, \alpha_1, ..., \alpha_m)$ and $SL2 = (\alpha_0, v_1, r_{v_1}, \alpha_1, ..., \alpha_m)$.
Step 2: For each $0 \leq i \leq m$, decide new skyline arcs from $(\alpha_i, u_i, r_{u_i}, \alpha_{i+1})$ and $(\alpha_i, v_i, r_{v_i}, \alpha_{i+1})$.
Step 3: Combine neighboring skyline arcs that are from the same disk.
return the new skyline
end

$Skyline$ is a divide-and-conquer algorithm, and most works are in the procedure $Merge$. There are three steps in $Merge$. In the first step, two skylines are aligned by splitting arcs such that two skylines have the same angle sequences. For example, assume $SL1 = \left(\beta_0, u_1', r_{u_1}', \beta_1, u_1', r_{u_1}', \beta_2, ..., \beta_k\right)$ and $SL2 = \left(\gamma_0, v_0', r_{v_0}', \gamma_1, v_1', r_{v_1}', \gamma_2, ..., \gamma_l\right)$ are two skylines. Let $(\alpha_0, \alpha_1, \cdots, \alpha_m)$ be a monotonic sequence of angles such that $\{\alpha_0, \alpha_1, \cdots, \alpha_m\} = \{\beta_0, \beta_1, ..., \beta_k\} \cup \{\gamma_0, \gamma_1, ..., \gamma_l\}$. Then, $SL1$ and $SL2$ are refined according to angles $\alpha_0, \alpha_1, \cdots, \alpha_m$. After that, both lists should have the same angle sequences and numbers of arcs, and we may assume $SL1 = (\alpha_0, u_1, r_{u_1}, \alpha_1, ..., \alpha_m)$ and $SL2 = (\alpha_0, v_1, r_{v_1}, \alpha_1, ..., \alpha_m)$. In the second step, for each $0 \leq i \leq m$, new skyline arcs are decided from $(\alpha_i, u_i, r_{u_i}, \alpha_{i+1})$ and $(\alpha_i, v_i, r_{v_i}, \alpha_{i+1})$. Given two arcs (α, u, r_u, β) and (α, v, r_v, β), we have following three cases to decide the new skyline arc.

Case 1 *Arcs (α, u, r_u, β) and (α, v, r_v, β) have no intersection. One arc is closer to* $\mathbf{o}$ *than the other, and the arc closer to* $\mathbf{o}$ *can not be in the new skyline. For instance, in Fig. 4 (a), arc ab are the new skyline arc of arcs ab and cd.*

Case 2 *Arcs (α, u, r_u, β) and (α, v, r_v, β) intersect at one point e. Let γ be the angle corresponding to the intersection point. Applying the principle used in Case 1, new skyline arcs can be decided from arcs (α, u, r_u, γ) and (α, v, r_v, γ), and arcs (γ, u, r_u, β) and (γ, v, r_v, β). For instance, in Fig. 4 (b), arcs ae and ed are the new skyline arcs of arcs ab and cd.*

Case 3 *Arcs (α, u, r_u, β) and (α, v, r_v, β) intersect at two points e, f. Let γ_1 and γ_2 with $\gamma_1 < \gamma_2$ be angles corresponding to intersection points. Applying the principle used in Case 1, new skyline arcs can be decided from arcs $(\alpha, u, r_u, \gamma_1)$ and $(\alpha, v, r_v, \gamma_1)$; arcs $(\gamma_1, u, r_u, \gamma_2)$ and $(\gamma_1, v, r_v, \gamma_2)$; and arcs $(\gamma_2, u, r_u, \beta)$ and $(\gamma_2, v, r_v, \beta)$. For instance, in Fig. 4 (c), arcs ae, ef and fb are the new skyline arcs of arcs ab and cd.*

In the first and second steps, one arc may be split into several pieces. So, in the last step, before returning the new skyline, we try to combine neighboring skyline arcs if they are from the same disk.

4. Time Complexity Analysis

In this section, we show that the time complexity of the proposed algorithm is $O(n \ln n)$. The time complexity can be formulated by a recursive equation

$$\begin{cases} T(n) = O(1) \text{ if } n = 1, \\ T(n) = 2T\left(\frac{n}{2}\right) + f(n) \text{ otherwise.} \end{cases}$$

Here $f(n)$ is the time complexity time of $Merge$. Since $f(n)$ is linear with respect to the number of arcs, and in Lemma 8, we will show that the number of arcs in a skyline is upper bounded by $2n$, we may have $f(n) = O(n)$. Hence, according to the master theorem, $T(n) = O(n \log n)$. To avoid being distracted by long geometric arguments, we present the theorem of time complexity first, and then give related lemmas to support the proof in the following subsection.

Theorem 4 *The time complexity of $Skyline$ is $O(n \ln n)$, where n is the number of disks.*

In [5] it has been shown that the time complexity of the algorithm that computes the minimum local disk cover set for homogeneous networks (i.e., all nodes have the same radius) is bounded by $O(n \ln n)$. Since homogeneous networks are special cases of heterogeneous networks, the time complexity for the algorithm that computes the minimum local disk cover set for homogeneous networks is at least $O(n \ln n)$. Hence, the proposed algorithm is with the optimal time complexity.

4.1. Geometry of Disks

We would like to show that the number of arcs in a skyline can not be more than $2n$. This can be proved if we can show that adding a disk into a disk set increases the number of skyline arcs by at most two. Unfortunately, this is

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not true in general. However, if the disks are added into the disk set follows the descending order of radius, the claim will be true. Since the order to add disks into the disk set should not change the final skyline, in what follows, we assume disks are added to the disk set in the decreasing order of the radius. Due to the limitation on the paper length, we give the following three lemmas without proof.

Lemma 5 *In the skyline of* $\{B_1, B_2, \ldots, B_n\}$*, if* B_n *contributes at least three arcs, then we can pick* B_i, B_j, B_k *from* $B_1, B_2, \ldots, B_{n-1}$ *such that* B_n *contributes three arcs in the skyline of* B_i, B_j, B_k, B_n*.*

Lemma 6 *Assume two circle* ∂B_1 *and* ∂B_2 *have two intersection points* a *and* d*. Let* ac' *(and* ab'*, respectively) be a diameters of* B_1 *(and* B_2*, respectively), and* c *(and* b*, respectively) is a point in arc* $c'd$ *(*$b'd$ *respectively). See Fig. 5. If the angle* $\angle cab$ *is obtuse, we have*

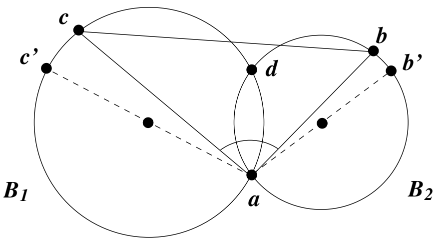

Figure 5. The structure in Lemma 5.

$$\|b - c\| > 2 \min (r_1, r_2) .$$

Lemma 7 *Given an acute or right triangle, for each edge of the triangle, draw a circle with the edge as a chord whose center is outside the triangle and radius is larger than the circumradius. Then, three circles have no intersection.*

The following is the lemma used in the proof of Theorem 3.

Lemma 8 *The number of arcs in a skyline of* n *disks is at most* $2n$*.*

Proof. We prove this by mathematical induction on n. Without loss of generality, we may assume all n nodes have arcs in the skyline and $B_1, B_2, \cdots, B_n$ have been sorted according to their radii in decreasing ordering.

If $n = 1$, there is only one disk, and thus, the skyline consists of one arc. If $n = 2$, two disks intersect at 2 points. There are at most 2 arcs in the skyline. See Fig. 6(a). If $n = 3$, the topology can be categorized into 2 configurations, like Fig. 6(b) and Fig. 6(c). Fig. 6(b) illustrates one configuration in which each disk contains one intersection point of the other two disks, and the skyline is composed of 3 arcs. Fig. 6(c) illustrates the other configuration in which

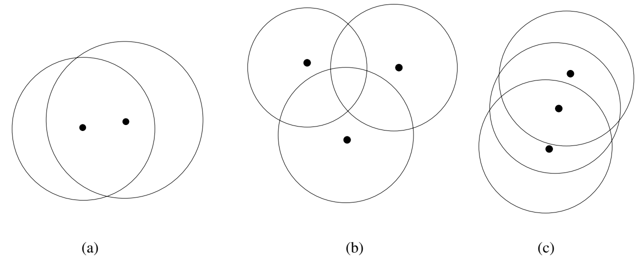

Figure 6. $n \leq 3$, the skyline contains $2n$ arcs at most. (a) Two disks, (b) and (c) three disks.

one disk contains two intersection points of the other two disks. In addition, 3 disks is allowed to have a common intersection point like Fig. 7. The skyline is composed of

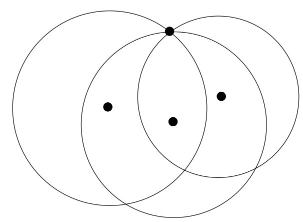

Figure 7. 3 disks have a common intersection point.

either 3 or 4 arcs. No matter how, the number of arcs is no more than $2n$.

Now, assume that as $n = k$, the skyline contains at most $2k$ arcs. If we can show that after a disk B_{k+1} is added into the set, the number of arcs in the skyline increases at most by two, then the new skyline contains no more than $2(k+1)$ arcs, and the proof is complete. Without loss of generality, we may assume B_{k+1} is the disk with the smallest radius among $B_0, B_1, B_2, \ldots, B_{k+1}$ since it doesn't change the skyline. We denote the number of arcs in the skyline of $B_1, B_2, \ldots, B_k$ as $Sky(B_1, B_2, \ldots, B_k)$. Now, we are going to prove this by contradiction. Assume B_{k+1} can contribute at least 3 arcs. According to Lemma 5, without loss of generality, we may assume that B_{k+1} contributes 3 arcs in the skyline of B_1, B_2, B_3, B_{k+1}. According to the two possible configurations of B_1, B_2, B_3 like Fig. 6(b) or Fig. 6(c), we discuss the problem in the two cases.

First, we consider the configuration like Fig. 6(b). Let a be the intersection point of B_1 and B_2 not in B_3; b be the intersection point of B_1 and B_3 not in B_2; and c be the intersecting point of B_2 and B_3 not in B_1. In order to contribute 3 arcs, B_{k+1} must intersect with 3 disks and contain a, b, c. Now, the problem is discussed by following two cases: (1) Δabc is an acute or right triangle; and (2) Δabc is an obtuse triangle.

Case 1: Δabc is an acute or right triangle. Let r_c be the circumradius of Δabc. Since Δabc is an acute or right triangle and B_{k+1} contains a, b, c, we have r_{k+1} is larger than r_c. In addition, since B_{k+1} is the smallest among $B_0, B_1, B_2, \ldots, B_{k+1}$. So, we have $r_c < r_{k+1} \leq r_1, r_2, r_3$. But according to Lemma 7, if r_1, r_2, r_3 are larger than r_c, B_1, B_2, B_3 have no intersection. This is a contradict to the fact that the intersection of B_1, B_2, B_3 is not empty.

Case 2: Δabc is an obtuse triangle. It doesn't affect the correctness of following argument if we assume $\angle cab$ is obtuse. Since B_{k+1} contribute three arcs in the skyline of B_1, B_2, B_3, B_{k+1} and $r_{k+1} \leq r_1, r_2, r_3$, degree of each arc in the skyline of B_1, B_2, B_3 must be larger than π. If ac' is a diameter of B_2 and ab' is a diameter of B_3, c' and b' are in the skyline of B_1, B_2, B_3. c is in the arc ac' intersected by arc ab' and not contained in B_1 and b is in the arc ab' intersected by arc ac' and not contained in B_2. According to Lemma 6, if $\angle cab$ is obtuse, we have $\|b - c\| > 2\min(r_1, r_2)$. On the other hand, since B_{k+1} contains Δabc, we have $r_{k+1} \geq \frac{1}{2}\|b - c\|$. Thus, we have a contradiction.

Next, we consider the configuration like Fig. 6(c). Without affecting the correctness of following argument, we assume B_3 is the one containing two intersection point of the boundary of other two disk. Let b, e denote intersection points of B_1 and B_3, and c, f denote intersection points of B_2 and B_3. To contribute three arcs to the skyline of B_1, B_2, B_3, B_{k+1}, B_{k+1} should cover at least 3 intersection points. According to the number of covered intersection points, there are two cases.

Case 3: If B_{k+1} cover exactly 3 intersection points, like Fig. 8(a), then B_{k+1} must have 3 intersection points with B_3. This is not possible to happen.

Case 4: If B_{k+1} cover exactly 4 intersection points, like Fig. 8(b), let e, f denote the two intersection points covered by the same arc of B_{k+1} and a is a intersection point of B_1 and B_2 that is closer to the arc covering e, f. Since B_{k+1} is smaller than B_1, B_2, B_3, arcs bc of B_3 outside B_1, B_2, arc be of B_1 outside B_3, and arc ef of B_2 outside B_3 are larger than π. So, the angle $\measuredangle bac > \pi/2$, the diameter of B_1 with one endpoint at a is outside of B_2, and the diameter of B_2 with one endpoint at a is outside of B_1. According to Lemma 6, just like **Case 2**, we have $2r_{k+1} \geq \|b - c\| > 2\min(r_1, r_2)$. This is a contradiction.

According to previous discussion, B_{k+1} can not contribute three arcs to the skyline of $B_1, B_2, \cdots, B_{k+1}$, and therefore, the number of arcs in the skyline $B_1, B_2, \cdots, B_{k+1}$ is at most $2(k+1)$. By mathematical induction, we conclude that the number of arcs in the skyline of n disks is upper bounded by $2n$.

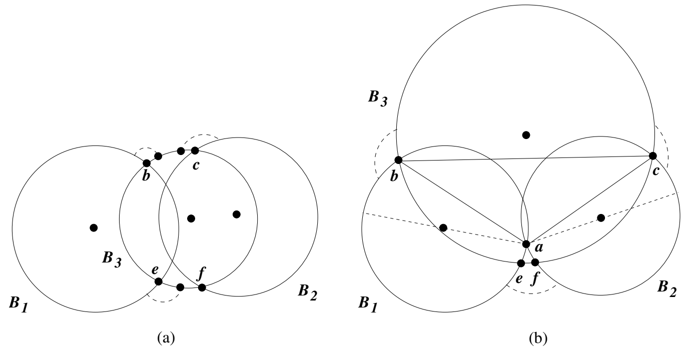

Figure 8. In this configuration, B_{k+1} doesn't contribute 3 arcs. (a)B_{k+1} covers 3 intersection points, and (b)B_{k+1} covers 4 intersection points.

5. Conclusions

The minimum local disk cover set can be used as the forwarding set in MANET broadcast protocols to alleviate the broadcast storm problem without sacrificing the functionality of the broadcasting. In this paper, we have established the equivalence of the MLDCS for a closed neighbor set in heterogeneous ad hoc networks and the corresponding skyline set. We propose a divide-and-conquer algorithm to compute the MLDCS, and show that the proposed algorithm is with the optimal time complexity $O(nlogn)$.

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