Term Project Report

Name: Chonghyuk Song
Student ID: 20130761

1. **Problem to be Solved:** Energy Management of Hybrid Electric Vehicles

Background

A hybrid electric vehicle (HEV), unlike the conventional internal-combustion-engine (ICE) vehicle, drives the vehicle forward using the power from both the engine and electric motor(s). HEV are superior to their ICE counterparts in terms of fuel economy **because [1]they can use regenerative braking, [2]they can shut down the ICE during idling and low-load phases, [3]and they can avoid low efficiency operating points of the ICE by driving the vehicle in electric-only mode during unfavorable driving conditions for the engine.**

However, the potential improvement of fuel economy *strongly depends on the control strategy used to optimize energy flow within the HEV.* This problem of controlling energy flow within the HEV during operation is called the "energy management problem" and can be formulated as an optimal control problem.

Before mathematical formulation let's describe the energy management problem in terms of higher level terms. The goal of the problem is to maximize fuel economy *i.e.*, minimize fuel mass consumed, while following the power demand of a given trip / driving cycle and keeping the battery charge within reasonable range. In fact, battery charge before and after the trip should be the same and should ideally be maintained on similar levels before and after the trip – otherwise, the battery's ability to store charge can be irreversibly impaired.

Problem Formulation

$$\min\ J = \alpha(SOC_f - SOC_0)^2 + m_{fuel} = \phi(SOC_f) + \int_0^{t_f} \dot{m}_{fuel}(t, \omega_{eng}, T_{eng})\, dt$$

subject to

1. $S\dot{O}C = f(SOC, \omega_{eng}, T_{eng})$ (system equations)
2. t_f given by driving cycle
3. $SOC(0)$ given

4. (*speed* operating limits)

$$\begin{cases} \omega_{eng,\min} \le \omega_{eng} \le \omega_{eng,\max} \\ \omega_{MG1,\min} \le \omega_{MG1} \le \omega_{MG1,\max} \\ \omega_{MG2,\min} \le \omega_{MG2} \le \omega_{MG2,\max} \end{cases}$$

5. (*torque* operating limits)

$$\begin{cases} T_{eng,\min}(\omega_{eng}) \le T_{eng} \le T_{eng,\max}(\omega_{eng}) \\ T_{MG1,\min}(\omega_{MG1}) \le T_{MG1} \le T_{MG1,\max}(\omega_{MG1}) \\ T_{MG2,\min}(\omega_{MG2}) \le T_{MG2} \le T_{MG2,\max}(\omega_{MG2}) \end{cases}$$

6. (*current* operating limits)

$$I_{\min}(SOC) \le I_{batt} \le I_{\max}(SOC)$$

where
m_{fuel} = (total fuel mass consumed during course of the driving cycle)
ω_{eng} = (engine speed)
T_{eng} = (torque provided by the engine)
SOC = (State of Charge: a normalized quantity that represents the amount of charge left in the battery)

The quadratic penalty term was added to the cost function as a soft constraint to enforce charge sustenance within the battery over the duration of the driving cycle. It should be noted that the operation limits (inequality constraints) are variable bound, adding a layer of difficulty to the current problem.

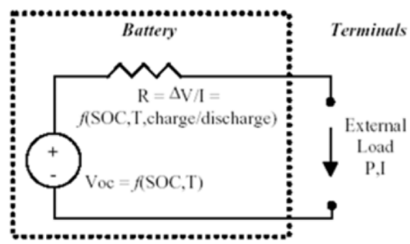
In order to complete this problem formulation, the system equations, which take into account the inner works of the vehicle, the battery, and their interactions, must be derived. **We emphasize beforehand that the description of the system largely relies on highly nonlinear, scattered empirical data.**

a) Definition of state of charge:

$$SOC \triangleq -\frac{Q(t)}{Q_{\max}} = -\frac{1}{Q_{\max}}\int_0^{t_f} I_{batt}(\tau)\,d\tau \rightarrow S\dot{O}C = -\frac{I_{batt}}{Q_{\max}}$$

b) Equivalent circuit, internal resistance model of battery:

$$P_{batt} = V_{oc}(SOC)I_{batt} - I_{batt}^{\ 2}R_{batt}(SOC)$$



where

V_{oc} = (open circuit voltage: a function of SOC)
R_{batt} = (battery resistance: also a function of SOC)

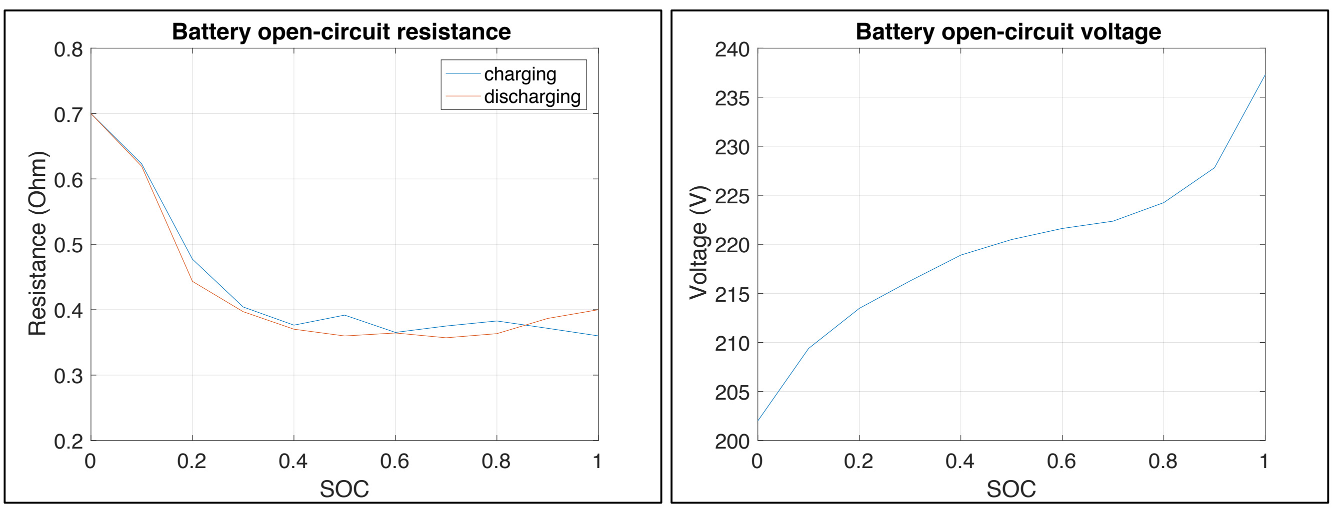


Fig. 1,2

c) Battery interaction with vehicle:

The power from the battery flows through inverters to supply power to the motor / generators, the relationship of which interaction is governed by the following:

$$P_{batt} = T_{MG1}\omega_{MG1}\eta_{MG1} + T_{MG2}\omega_{MG2}\eta_{MG2}$$

where

T_{MG1}, T_{MG2} = (torque provided by the first and second electric motor / generator respectively)
ω_{MG1}, ω_{MG2} = (speed of the first and second electric motor / generator respectively)
η_{MG1}, η_{MG2} = (efficiency of first and second electric motor / generator respectively)

(MG is a short hand for motor/generator – the electric motor can act as a generator during regenerative braking)

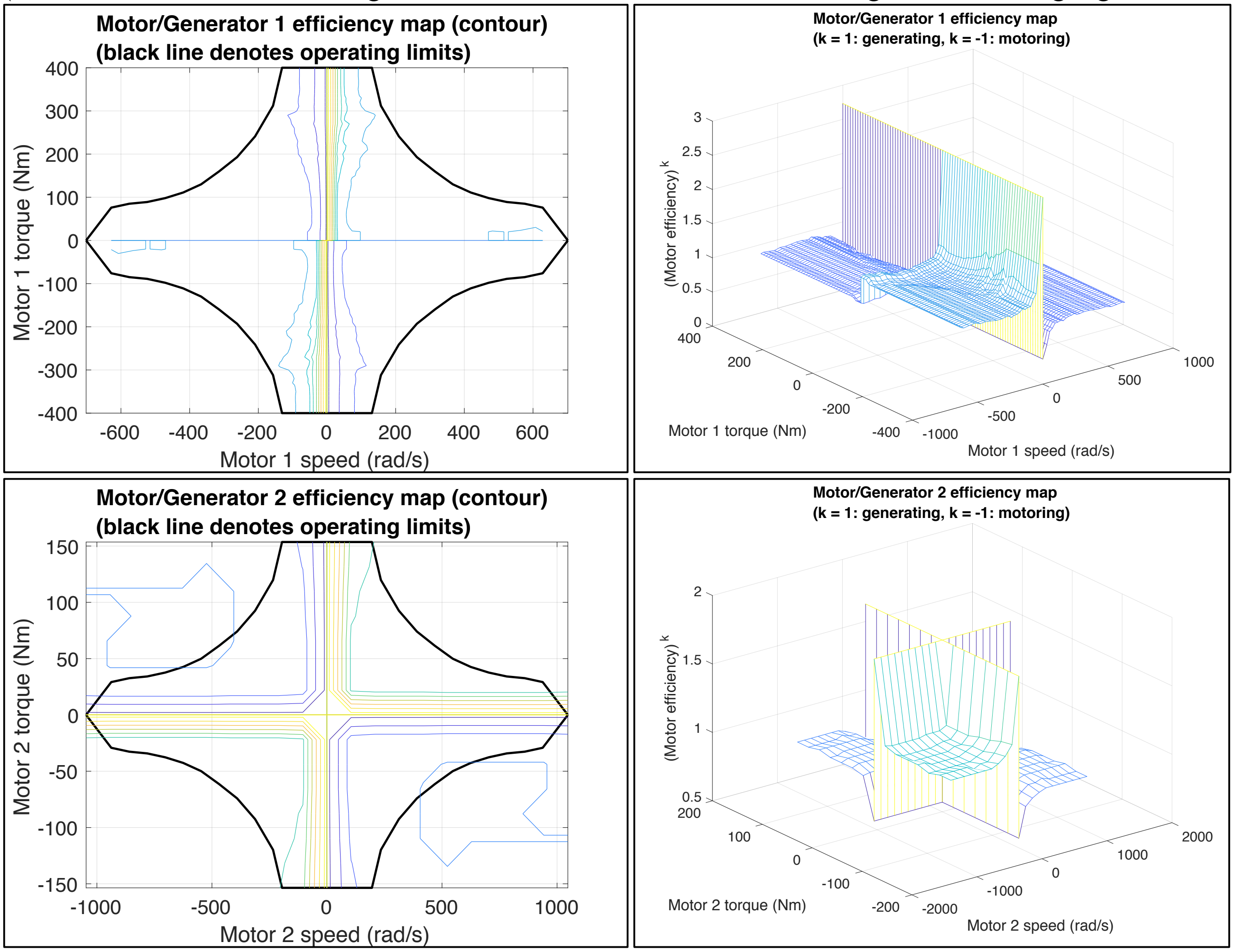


Fig. 3,4,5,6

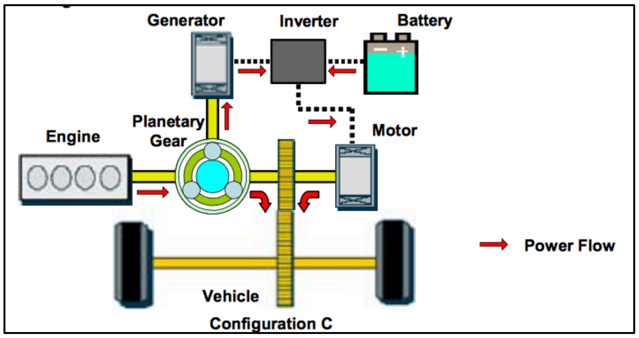
As shown in the Figures 3 to 6, the efficiency of the electric motor / generator cannot be described as a succinct analytic equation. Instead, the efficiency of both motors are given in the form of contour maps, **from which one has to interpolate the efficiency of the motor for given speed and torque.**

Combining a), b), and c) we get the following system equation:

$$S\dot{O}C = -\frac{V_{oc} - \sqrt{V_{oc}^{\ 2} - 4P_{batt}R_{batt}}}{2Q_{\max}R_{batt}} = -\frac{V_{oc} - \sqrt{V_{oc}^{\ 2} - 4(T_{MG1}\omega_{MG1}\eta_{MG1} + T_{MG2}\omega_{MG2}\eta_{MG2})R_{batt}}}{2Q_{\max}R_{batt}}$$

d) <u>Vehicle Model</u>

T_{MG1}, T_{MG2}, ω_{MG1}, ω_{MG2} can all be calculated from the vehicle model. The target vehicle for this project is the Prius 2nd Generation, which is a power-split hybrid that integrates two motor/generators and an engine through a planetary gear set *i.e.*, the power split device. A static model of the vehicle is sufficiently accurate enough for the energy management problem as additional state variables that represent the dynamic effects of the electric motor and ICE are *faster* than that of the main energy flows in the HEV, and hence are irrelevant to this problem [1]:



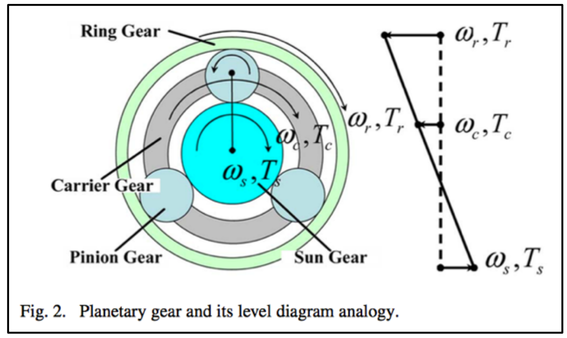
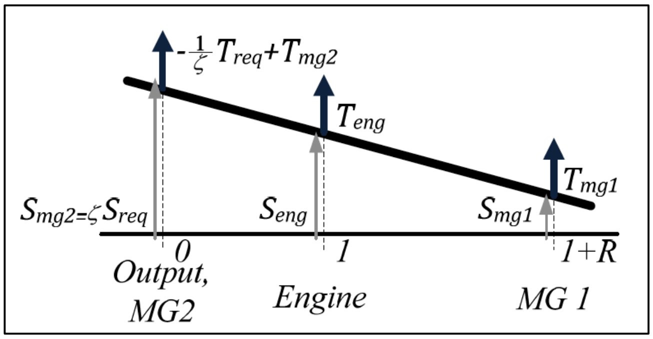


Fig. 7 Layout of a power-split hybrid [2]

Fig. 8 Lever Diagram Analogy of the Planetary Gear



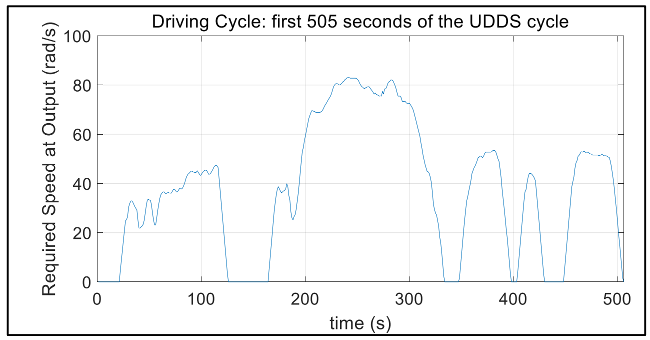


Fig. 9 [3]

Fig. 10 Driving Cycle (UDDS) [4]

In the static model of the split-hybrid system, we have two control variables T_{eng} and ω_{eng} (as the planetary gear gives the system an extra degree of freedom) and **all other variables T_{MG1}, T_{MG2}, ω_{MG1}, ω_{MG2} can be represented in terms of these two control variables and T_{req}, ω_{req} as specified by the driving cycle**, a series of datapoints representing the required speed of a vehicle versus time - T_{req} is the torque that is needs to be output by the hybrid powertrain in order to drive the vehicle mass according to the driving cycle:

$$\begin{bmatrix} T_{MG1} \\ T_{MG2} \end{bmatrix} = \frac{-1}{R+1}\begin{bmatrix} 0 & 1 \\ R+1 & R \end{bmatrix}\begin{bmatrix} T_{req}/K \\ T_{eng} \end{bmatrix}$$

$$\begin{bmatrix} \omega_{MG1} \\ \omega_{MG2} \end{bmatrix} = \begin{bmatrix} -R & R+1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} K\omega_{req} \\ \omega_{eng} \end{bmatrix}$$

where R is the gear ratio of the planetary gearset, and K is the final drive ratio.

The driving cycle shown above is a time-series of velocity data provided by the EPA (Environmental Protection Agency) as a standard for vehicle performance testing. We note that ω_{req} is simply the velocity of driving cycle (in m/s) divided by the radius of the tires; T_{req}, is the torque required to [1]overcome the aerodynamic drag on the vehicle and the rolling resistance of the tires, and to [2]accelerate the vehicle mass to follow the driving cycle:

$$\omega_{req} = v_{cycle} / R_{tire}$$

$$T_{req} = R_{tire}\left[\frac{1}{2}C_D A_{frontal}\rho_{air}v_{cycle}^2 + m_{vehicle}g(R_1 + R_2 v_{cycle} + R_3 v_{cycle}^2) + m_{vehicle}a_{cycle}\right]$$

e) Engine Model

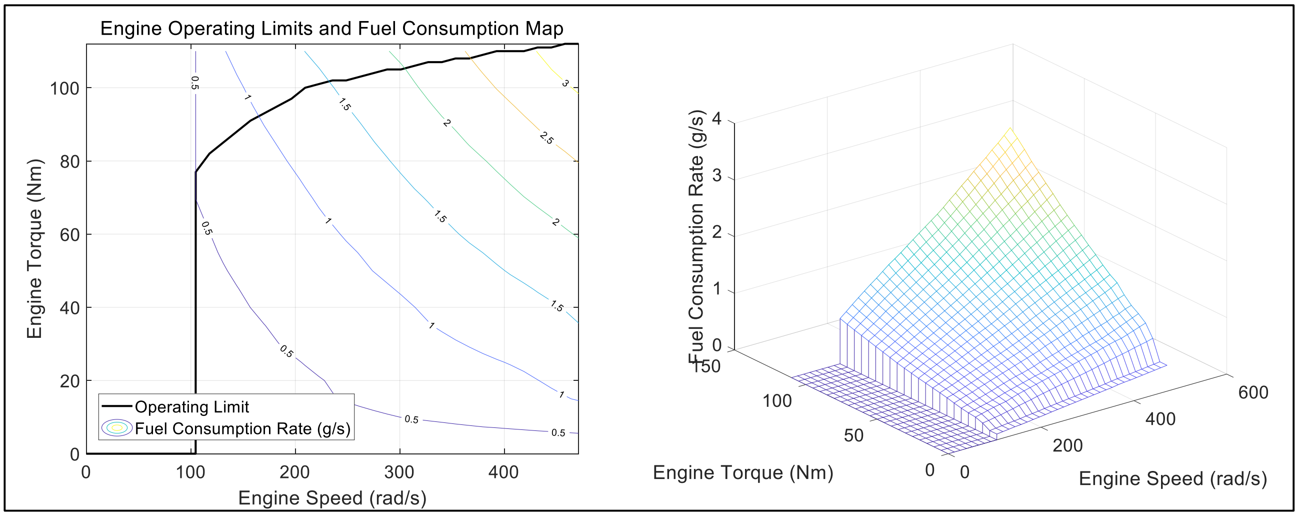


Fig. 11 Fuel Consumption Rate Map

Just like before, the fuel consumption rate of the engine is also given in the form of a contour map, from which one would have to interpolate the fuel consumption rate for given engine torque and engine speed (remember, the fuel consumption rate was previously said to be a function of the engine speed and torque). The 2D plot shows that before a certain speed, the engine does not provide any torque.

This concludes the problem formulation of the **energy management problem, which can be represented as an optimal control problem with one state variable (*SOC*) and two control variables (T_{eng}, ω_{eng}).**

2. Approach / Methodology

The energy management problem can be solved via two ways: an indirect method such as Pontryagin's Minimum Principle (PMP) combined with the shooting method, and a direct transcription method that uses the Trapezoidal method.

a) Indirect method: PMP + shooting method

$$J = \alpha(SOC_f - SOC_0)^2 + \int_0^{t_f} \dot{m}_{fuel}(\omega_{eng}, T_{eng})\, dt$$

$$H \triangleq \dot{m}_{fuel}(\omega_{eng}, T_{eng}) + \lambda f(\omega_{eng}, T_{eng}, SOC)$$

$$state\ eqn: \qquad S\dot{O}C = \frac{\partial H}{\partial \lambda} = f(\omega_{eng}, T_{eng}, SOC)$$

$$\cos tate\ eqn: \qquad \dot{\lambda} = -\frac{\partial H}{\partial (SOC)} = -\lambda\ \frac{\partial f}{\partial (SOC)}$$

$$control\ condition: \qquad H(\omega^*_{eng}, T^*_{eng}, \lambda^*, SOC^*, t) \le H(\omega_{eng}, T_{eng}, \lambda^*, SOC^*, t) \qquad (\forall u = [\omega_{eng}, T_{eng}] \in U_{feasible})$$

$$natural\ boundary: \qquad \lambda(t_f) = \frac{\partial \phi}{\partial SOC}(SOC(t_f)) = 2\alpha(SOC_f - SOC_0)$$

$$boundary\ conditions: \quad t_f\ given,\ SOC_0\ given$$

Pontryagin's Minimum Principle is a generalized form of the control equation that can be used to find the optimal control. In a problem where the control variables have bounds (in this case variable bounds), the PMP is the perfect tool for solving for the optimal control as the Hamiltonian in the current problem is neither given as an analytic nor explicit function due to the empirical nature of the system model.

In other words, we *cannot* express the optimal control in terms of the optimal state and co-state trajectory so as to eliminate the control terms from the Euler-Lagrange equations to create the standard boundary value problem. What we *can* do is the following: we discretize a feasible region of the control plane (T_{eng}, ω_{eng}) into a grid of datapoints with a resolution of 1 rad/s for speed and 1Nm for torque.
Then, for each (T_{eng}, ω_{eng}) pair we check whether the corresponding Motor1 speed and torque, Motor 2 speed and torque, and the battery current **are also within their respective operating limits** (shown in black in Fig.). Note that since the Motor 1, Motor 2 speed and torque and the battery current are all dependent on the engine speed and torque, **the range of feasible operation for the engine is not only bound by its own operating limits, but also on that of Motor 1, Motor 2, and the battery.**

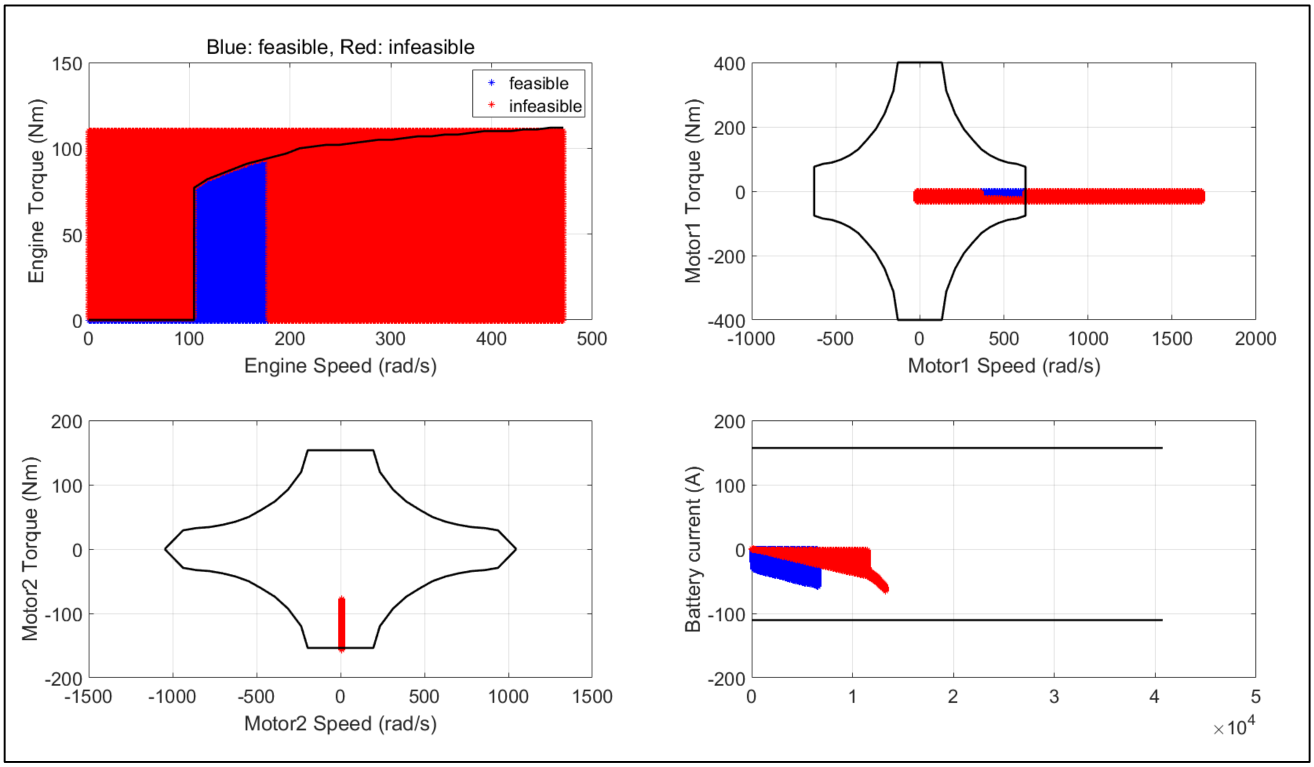


Fig. 12 Blue points: satisfy *all* operating limits; Red points: violates at least *one* operating limit

We are now left with a set of control grid points that are candidates for the optimal control. The Hamiltonian for each of these candidate points are calculated based on the current values of costate λ and state *SOC* – the control pair (T_{eng}, ω_{eng}) that results in the smallest Hamiltonian can be regarded as the optimal control for that time step. The optimal control at that time step is then substituted into the Euler-Lagrange equations for forward integration. We now write the pseudocode for the indirect method below:

Pseudocode for PMP + shooting method

- Make an initial guess for initial costate (at time step 0): $\lambda^{(0)}(t_0)$
- Initial SOC is given: 0.6

While (True):

for $k = 0{:}1{:}T\text{-}1$ % final time step T

given $\lambda(k)$ and SOC(k):

1. check which of the gridpoints on the control plane (T_{eng}, ω_{eng}) do not violate the operating limits of the system (inequality constraints)
2. find the optimal control at time step k by performing the "argmin H" operation on the set of feasible controls
3. substitute optimal controls T_{eng}*, ω_{eng}* into the algebraic difference equations (forward integration):

$$SOC(k+1) = SOC(k) + f\left[SOC(k), T_e^*(k), \omega_e^*(k)\right]$$

$$\lambda(k+1) = \lambda(k) + \left[-\lambda(k)\frac{\partial f}{\partial SOC}\bigg|_{SOC(k),T_e^*(k),\omega_e^*(k)}\right]$$

if ($\lambda(T)$ - φ_{xf}) < 0.5 % small enough convergence limit as $\lambda \sim$ - 300

update the initial costate (Newton's method):

$$\lambda^{(1)}(t_0) \triangleq \lambda^{(0)}(t_0) - \gamma\left[\frac{d\lambda(t_f)}{d\lambda(t_0)}\right]^{-1}_{\lambda^{(0)}(t_0)}\left[\lambda^{(0)}(t_f) - \phi_{SOC_f}\right]$$

$$\approx \lambda^{(0)}(t_0) - \gamma\left[\frac{\delta\lambda^{(0)}(t_f)}{\delta\lambda^{(0)}(t_0)}\right]^{-1}_{\lambda^{(0)}(t_0)}\left[\lambda^{(0)}(t_f) - 2\alpha(SOC^{(0)}(t_f) - SOC(t_0))\right]$$

$(learning\ rate\ \gamma = 0.05)$

else

break % shooting method has converged

* The slope can be approximated by perturbing the value of the initial costate and evaluating the resulting perturbation at the final costate calculated by forward integration – **although this method of approximating the slope is less accurate than the transition matrix method, it is the only means available as it is *extremely* difficult to evaluate F_z without an analytic form of the reduced state and costate equations.**

b) Direct method: Euler Method + NLP

The optimal control problem above is now transcribed into a parameter optimization problem:

"Find vector p that minimizes the objective function (evaluated by the trapezoidal method)

$$J = m_{fuel} = \int_{t_0}^{t_f} \dot{m}_{fuel}(\omega_{eng}, T_{eng})dt$$

subject to the dynamic constraints (and the inequality constraints mentioned above)

$$S\dot{O}C = f(SOC, \omega_{eng}, T_{eng}) \quad \Rightarrow \quad SOC_k = SOC_{k-1} + \left(\frac{t_f}{N}\right) f(SOC_{k-1}, \omega_{eng,k-1}, T_{eng,k-1})$$

$$(k = 1, 2, \cdots, N)$$

where p is a vector of parameters consisting of (N-1) states and 2N controls (N for each control); N = 505 denotes the number of time steps in the driving cycle:

$$p = \left[SOC_1, SOC_2, \cdots, SOC_{N-1}, \quad \omega_{eng,0}, \omega_{eng,1}, \cdots, \omega_{eng,N-1}, \quad T_{eng,0}, T_{eng,1}, \cdots, T_{eng,N-1} \right]$$

The MATLAB function "fmincon", a nonlinear programming (NLP) solver that solves nonlinear optimization problems using the interior point method, was used to solve this parameter optimization problem.
Since nonlinear programming methods generally do not guarantee global optimum and instead can be sensitive to the choice of the initial guess of the parameters, especially for problems with many optimization parameters like this one, **the initial guess will be taken from the results of the indirect method above.**

3. Results

a) Indirect method: PMP + shooting method

To demonstrate convergence of this method, the total cost, initial costate, and final costate error were plotted against the iteration number:

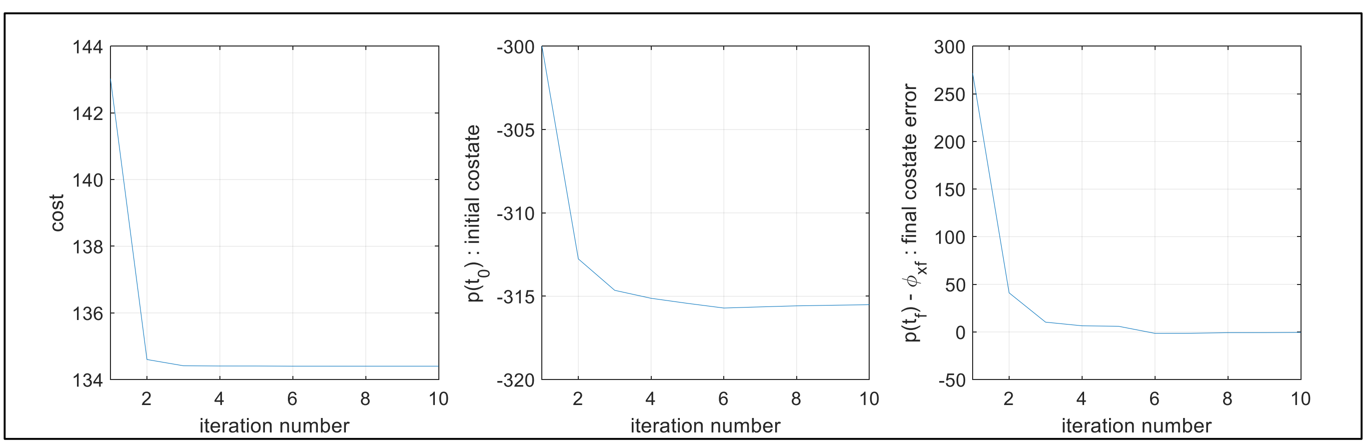


Fig. 13 initial guess: λ_0 = -300

The results for other initial guesses for the initial co-state were obtained as well. It was found that **the method converged to the *same initial costate value and total cost* for a range of different initial guesses, providing some evidence that the solution that was found was indeed the global optimum:**

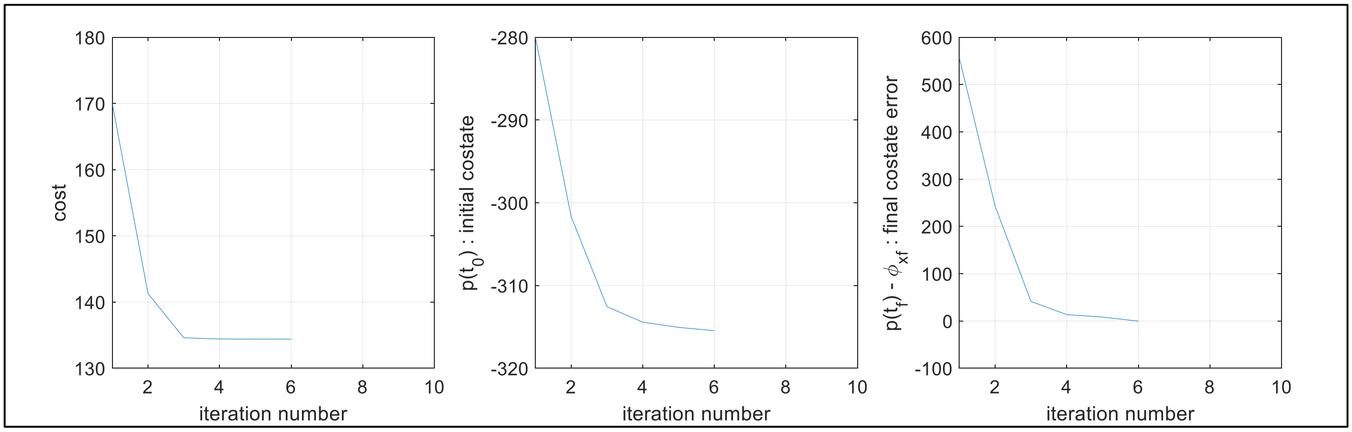


Fig. 14 initial guess: λ_0 = - 280

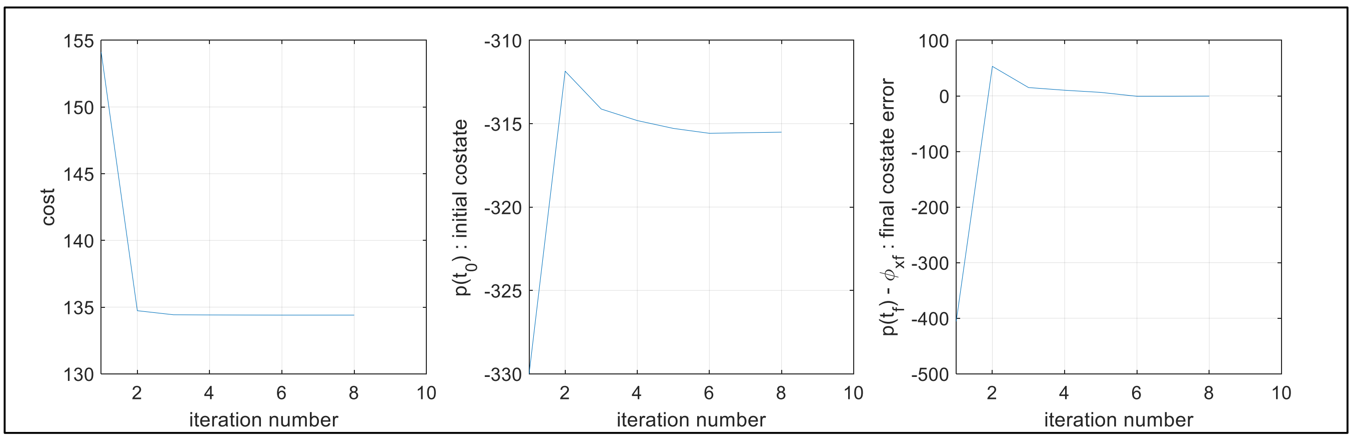


Fig. 15 initial guess: λ_0 = - 330

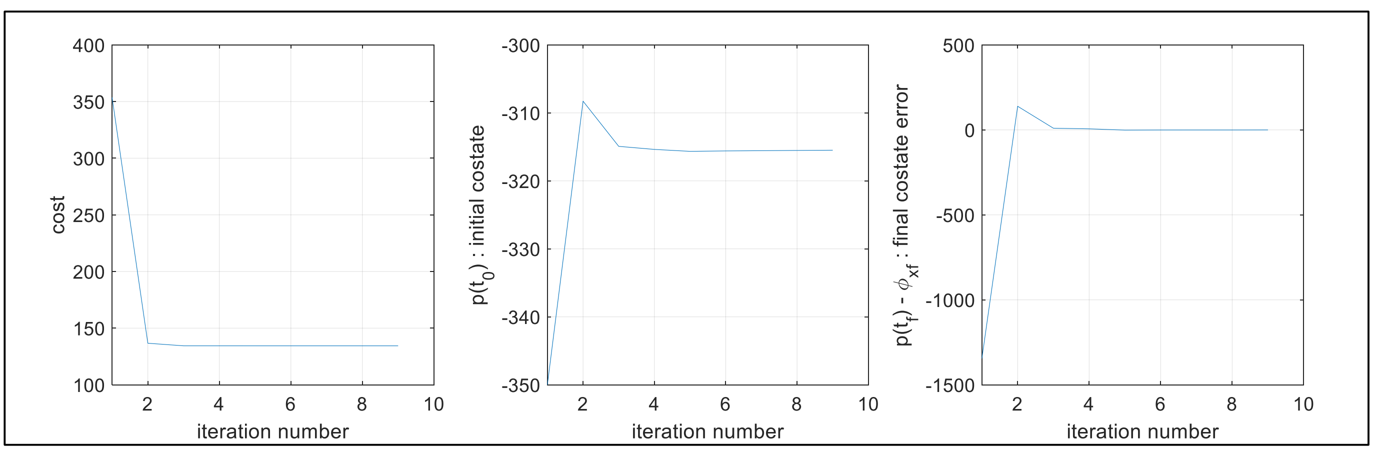


Fig. 16 initial guess: λ_0 = - 350

Finally, it was necessary to check whether the results *made sense in the context of HEV vehicles*. It was verified that the engine turned on only during high-load phases of the driving *i.e.*, when the power required to track the driving cycle was large. During low-load phases, the 2nd electric motor acted as the sole power source to the vehicle:

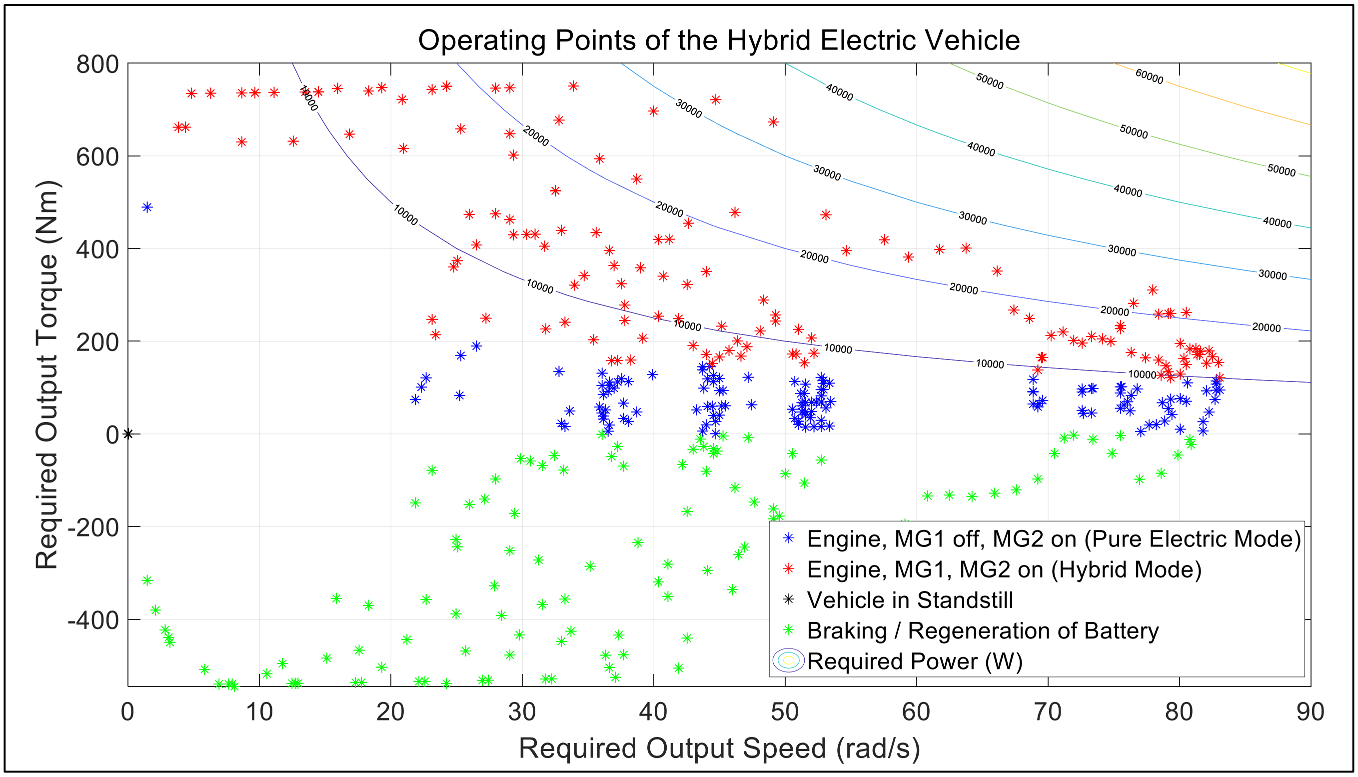


Fig. 17 Operating points of the hybrid electric vehicle are distributed as expected

It was also verified by the following fuel consumption map that the optimal control rendered the engine to operate in its most fuel-efficient region *i.e.*, region that consumes the *least* mass of fuel per unit power required, per unit time:

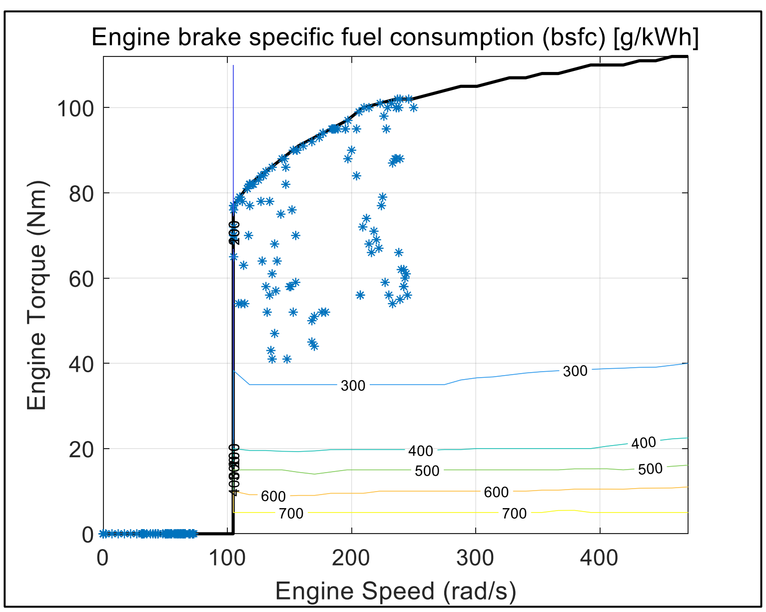


Fig. 18 (Black line denotes the operating limit, and the colored lines are equi-fuel-consuming contours)

The previous two features are characteristic of the operation of the HEV as mentioned in the Background section of this report, and hence it is deemed that the results of the indirect method (PMP + shooting method) are in fact valid.

b) <u>Direct method:</u> Euler Method + NLP

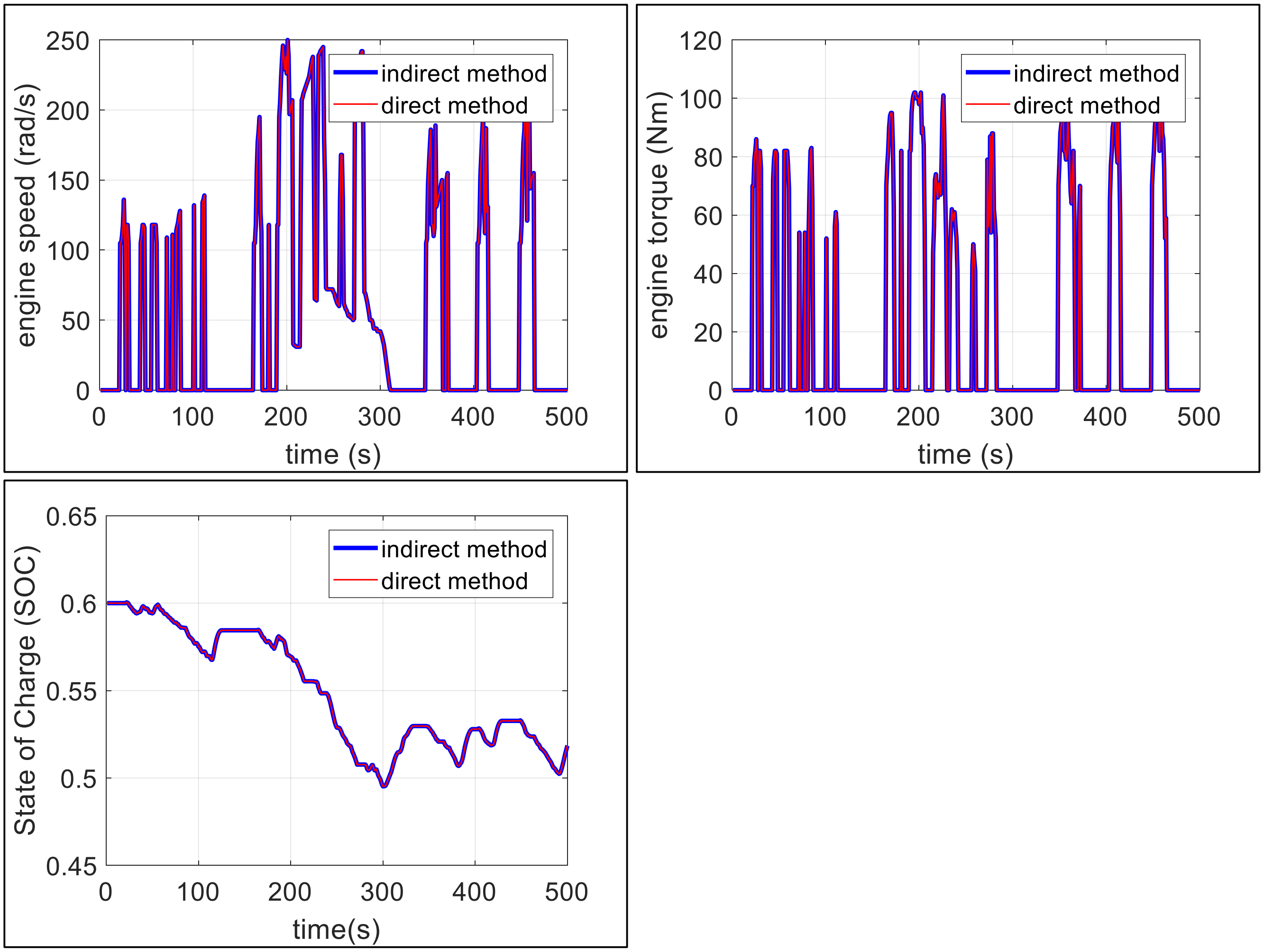


Fig. 18~20 Comparison of results between indirect method and direct method

```
 Iter  Func-count            Fval   Feasibility   Step Length       Norm of   First-order
                                                                       step    optimality
    0        1516    1.344024e+02     5.529e-17     1.000e+00     0.000e+00     3.127e+02
Objective function returned NaN; trying a new point...
    1        1532    1.344024e+02     5.529e-17     3.323e-03     1.986e-04     1.398e-02

Local minimum possible. Constraints satisfied.

fmincon stopped because the size of the current step is less than
the default value of the step size tolerance and constraints are
satisfied to within the default value of the constraint tolerance.

<stopping criteria details>
```

Having taken the results of the indirect method as an initial guess for the nonlinear programming algorithm fmincon, the local optimum was reached in a single iteration, corroborating the results of the indirect method. As a result, the state and control trajectories of the two methods were nearly identical as above.

4. Discussion

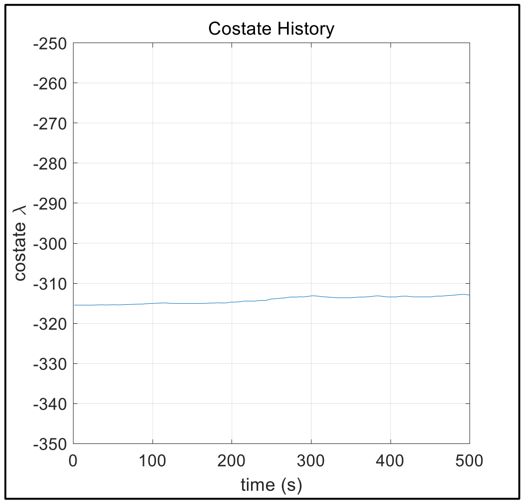


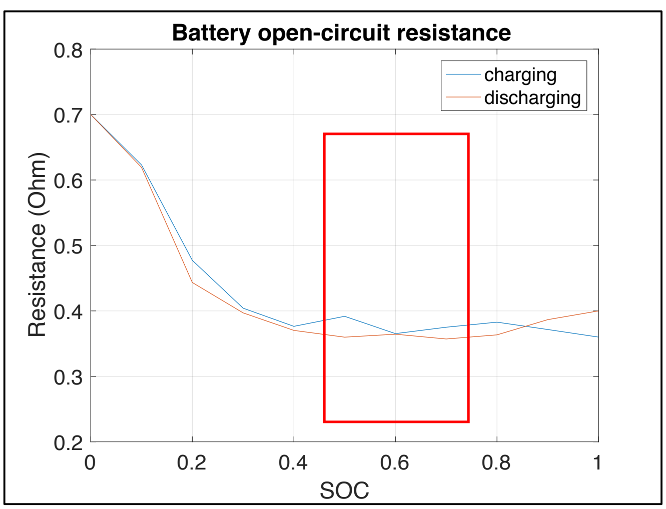
Fig. 21 Costate History

We can see in Fig. 21 that the change in costate values are minimal compared to their absolute magnitude, meaning that we can regard them as constant for the duration of the driving cycle. **This is an interesting characteristic of the HEV energy management problem!** To understand why the costate remains more or less the same throughout the duration of the driving cycle, we go back to the costate equation:

$$\dot{\lambda} = -\lambda \frac{\partial f}{\partial SOC}$$

$$f(SOC, T_{eng}, \omega_{eng}) = \frac{\sqrt{V_{OC}^2(SOC) - 4R_{OC}(SOC)P_{batt}(T_{eng}, \omega_{eng})}}{2Q_{\max} R_{OC}(SOC)}$$

We see that by virtue of the quotient rule, the partial derivative of f with respect to SOC can be expressed in terms of the derivatives of V_{OC} and R_{OC} with respect to SOC. The figures below show how the open-circuit resistance and open-circuit voltage changes with respect to SOC:



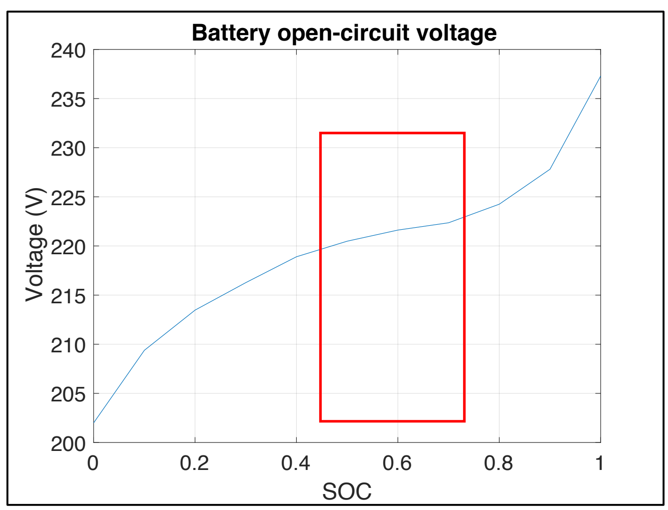


Fig. 22,23 The battery operates inside red box

What is interesting is that these variables have more or less the same values in the region of SOC commonly dealt with in this problem, that is, between 0.5 and 0.7. Therefore, the derivatives of V_{OC} and R_{OC} are close to zero and hence so is the partial derivative of f with respect to SOC. **In other words, since the right-hand side of the costate equation is close to zero, the costate remains constant throughout vehicle operation!** For future reference, if we can ensure that the SOC during the course of a driving cycle stays within the above-mentioned bounds, then we can use the constant costate assumption to greatly *simplify* the energy management problem.

5. Bibliography

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