Part (a): Getting Your Hands Dirty.

1 (a) $P(\text{positive}) = \frac{3}{6} = \frac{1}{2}$ (Without smoothing).

(b) $P(\text{negative}) = \frac{3}{6} = \frac{1}{2}$

(c) $P(I|\text{positive}) = ?$ Recall, $P(X|Y) = \frac{P(X,Y)}{P(Y)}$

$P(I|\text{positive}) = \frac{0.5}{0.5} = 1$

(d) $P(\text{hated}|\text{positive}) = \frac{(1/6)}{(1/2)} = \frac{1}{6} \times \frac{2}{1} = \frac{1}{3}$

(e) $P(\text{that}|\text{positive}) = \frac{(2/6)}{(1/2)} = \frac{2}{\cancel{6}3} \times \frac{\cancel{2}1}{1} = \frac{2}{3}$

(f) $P(\text{movie}|\text{positive}) = \frac{(1/6)}{(1/2)} = \frac{1}{6} \times \frac{2}{1} = \frac{1}{3}$

(g) $P(\text{loved}|\text{positive}) = \frac{(3/6)}{(1/2)} = \frac{3}{6} \times \frac{2}{1} = 1$

(h) $P(\text{it}|\text{positive}) = \frac{(2/6)}{(1/2)} = \frac{\cancel{2}1}{\cancel{6}3} \times \frac{2}{1} = \frac{2}{3}$

(i) $P(I|\text{negative}) = \frac{(3/6)}{(1/2)} = \frac{0.5}{0.5} = 1$

(j) $P(\text{hated}|\text{negative}) = \frac{(3/6)}{(1/2)} = 1$

(k) $P(\text{that}|\text{negative}) = \frac{(2/6)}{(1/2)} = \frac{2}{3}$

(l) $P(\text{movie}|\text{negative}) = \frac{(1/6)}{(1/2)} = \frac{1}{6} \times \frac{2}{1} = \frac{1}{3}$

(m) $P(\text{loved}|\text{negative}) = \frac{(1/6)}{(1/2)} = \frac{1}{3}$

(n) $P(\text{it}|\text{negative}) = \frac{(2/6)}{(1/2)} = \frac{2}{3}$

1 (b)

(b) Calculate manually what the probability "score" (i.e. $p(\texttt{features}|\texttt{label})p(\texttt{label})$) of the following sentence would be for both the positive class and the negative class using a) the "correct" approach that includes all features and b) the positive features-only approach:

`I loved it`

(You should have four probabilities, i.e., $p(features, label)$ for the two labels for the two approaches.)

i) The positive-features-only approach:

P(features | label) p(label)

Recall $P(x|y) = p(y) \prod_{j=1}^{m} p(x_i|y)$

- $P(x_1 = I, x_2 = loved, x_3 = it \mid y = positive) = P(positive) P(x_1 = I \mid positive) P(x_2 = loved \mid positive) P(x_3 = it \mid positive)$

$P(x|y) = \frac{1}{\cancel{2}_1} \times 1 \times 1 \times \frac{\cancel{2}^1}{3} = \frac{1}{3}$

- $P(x_1 = I, x_2 = loved, x_3 = it \mid y = negative) = P(negative) P(x_1 = I \mid negative) P(x_2 = loved \mid negative) P(x_3 = it \mid negative)$

$P(x|y) = \frac{1}{\cancel{2}} \times 1 \times \frac{1}{3} \times \frac{\cancel{2}}{3} = \frac{1}{9}$

ii) The "correct" approach that includes all features:

All features = {I, hated, that, movie, it, loved}

$P(x \mid positive) = \frac{1}{\cancel{2}} \left(1 \times \frac{1}{3} \times \frac{\cancel{2}}{3} \times \frac{1}{3} \times \frac{2}{3} \times 1\right) = \frac{\cancel{2}}{3^4} = \frac{2}{81}$

$P(x \mid negative) = \frac{1}{\cancel{2}} \left(1 \times 1 \times \frac{\cancel{2}}{3} \times \frac{1}{3} \times \frac{2}{3} \times \frac{1}{3}\right) = \frac{2}{81}$

$$P(x_i|y) = \frac{\text{Count}(x_i, y) + \lambda}{\text{Count}(y) + \text{possibleValuesOf}\,x_i * \lambda}$$

2.

2. Probabilities with smoothing

Repeat all of the calculations for Problem 1 with $\lambda = 1$.

i) $P(\text{positive}) = \frac{1}{2}$

ii) $P(\text{negative}) = \frac{1}{2}$

iii) $P(I|\text{positive}) = \frac{3+1}{3+2*1} = \frac{4}{5}$

iv) $P(\text{hated}|\text{positive}) = \frac{1+1}{5} = \frac{2}{5}$

v) $P(\text{that}|\text{positive}) = \frac{2+1}{5} = \frac{3}{5}$

vi) $P(\text{movie}|\text{positive}) = \frac{1}{5}$

vii) $P(\text{loved}|\text{positive}) = \frac{3+1}{5} = \frac{4}{5}$

viii) $P(\text{it}|\text{positive}) = \frac{2+1}{5} = \frac{3}{5}$

ix) $P(I|\text{negative}) = \frac{3+1}{3+2*1} = \frac{4}{5}$

x) $P(\text{hated}|\text{negative}) = \frac{3+1}{5} = \frac{4}{5}$

xi) $P(\text{that}|\text{negative}) = \frac{2+1}{5} = \frac{3}{5}$

xii) $P(\text{movie}|\text{negative}) = \frac{1+1}{5} = \frac{2}{5}$

xiii) $P(\text{loved}|\text{negative}) = \frac{1+1}{5} = \frac{2}{5}$

xiv) $P(\text{it}|\text{negative}) = \frac{2+1}{5} = \frac{3}{5}$

* All features approach:

* Recall $P(x|y) = p(y) \prod_{j=1}^{m} p(x_i|y)$

All features = {I, hated, that, movie, it, loved}

* $P(x|positive) = \frac{1}{\cancel{2}}\left(\frac{4}{5} \times \frac{\cancel{2}}{5} \times \frac{3}{5} \times \frac{1}{5} \times \frac{3}{5} \times \frac{4}{5}\right) = \frac{4^2 \times 3^2}{5^6} = \frac{16 \times 9}{5^6}$

$5 \times 5 \times 5 \times 5 \times 5 \times 5$

$25 \times 25 \times 5$

$= \frac{144}{3125} = 0.04608$

* $P(x|negative) = \frac{1}{\cancel{2}}\left(\frac{4}{5} \times \frac{4}{5} \times \frac{3}{5} \times \frac{\cancel{2}}{5} \times \frac{3}{5} \times \frac{2}{5}\right) = \frac{4^2 \times 3^2 \times 2}{5^6} =$

$= \frac{288}{3125} = 0.09216$

* Features-positive-only approach:

$\rightarrow P(x|positive) = \frac{1}{\cancel{2}}\left(\frac{\cancel{4}^{2}}{5} \times \frac{4}{5} \times \frac{3}{5}\right) = \frac{24}{5^3} = \frac{24}{125} = 0.192$

$25 \times 5 = 125$

$\rightarrow P(x|negative) = \frac{1}{\cancel{2}}\left(\frac{4}{5} \times \frac{\cancel{2}}{5} \times \frac{3}{5}\right) = \frac{12}{125} = 0.096$