

CS158 - Assignment 7a Solutions

1. Probabilities by hand

- (a) $p(\text{positive}) = 3/6 = 0.5$
 $p(\text{negative}) = 3/6 = 0.5$
 $p(I \mid \text{positive}) = 3/3 = 1.0$
 $p(\text{hated} \mid \text{positive}) = 1/3 = 0.333$
 $p(\text{that} \mid \text{positive}) = 2/3 = 0.667$
 $p(\text{movie} \mid \text{positive}) = 1/3 = 0.333$
 $p(\text{loved} \mid \text{positive}) = 3/3 = 1.0$
 $p(\text{it} \mid \text{positive}) = 2/3 = 0.667$
 $p(I \mid \text{negative}) = 3/3 = 1.0$
 $p(\text{hated} \mid \text{negative}) = 3/3 = 1.0$
 $p(\text{that} \mid \text{negative}) = 2/3 = 0.667$
 $p(\text{movie} \mid \text{negative}) = 1/3 = 0.333$
 $p(\text{loved} \mid \text{negative}) = 1/3 = 0.333$
 $p(\text{it} \mid \text{negative}) = 2/3 = 0.667$

(b) **All features:**

$$\begin{aligned} p(I \text{ loved it} \mid \text{pos}) &= p(\text{pos}) * p(I \mid \text{pos}) * p(\neg \text{hated} \mid \text{pos}) * p(\neg \text{that} \mid \text{pos}) * p(\neg \text{movie} \mid \text{pos}) * p(\text{loved} \mid \text{pos}) * p(\text{it} \mid \text{pos}) \\ &= 0.5 \quad * 1.0 \quad * 0.667 \quad * 0.333 \quad * 0.667 \quad * 1.0 \quad * 0.667 \\ &= 0.0494 \text{ (give or take depending on precision)} \end{aligned}$$

$$\begin{aligned} p(I \text{ loved it} \mid \text{neg}) &= p(\text{neg}) * p(I \mid \text{neg}) * p(\neg \text{hated} \mid \text{neg}) * p(\neg \text{that} \mid \text{neg}) * p(\neg \text{movie} \mid \text{neg}) * p(\text{loved} \mid \text{neg}) * p(\text{it} \mid \text{neg}) \\ &= 0.5 \quad * 1.0 \quad * 0.0 \quad * 0.333 \quad * 0.667 \quad * 0.333 \quad * 0.667 \\ &= 0.0 \end{aligned}$$

(This is why smoothing is important!)

Positive only features:

$$\begin{aligned} p(I \text{ loved it} \mid \text{pos}) &= p(\text{pos}) * p(I \mid \text{pos}) * p(\text{loved} \mid \text{pos}) * p(\text{it} \mid \text{pos}) \\ &= 0.5 \quad * 1.0 \quad * 1.0 \quad * 0.667 \\ &= 0.3335 \end{aligned}$$

$$\begin{aligned}
p(\text{I loved it}|\text{neg}) &= p(\text{neg}) * p(\text{I}|\text{neg}) * p(\text{loved}|\text{neg}) * p(\text{it}|\text{neg}) \\
&= 0.5 \quad * 1.0 \quad * 0.333 \quad * 0.667 \\
&= 0.1111
\end{aligned}$$

2. Probabilities with smoothing

(a) $p(\text{positive}) = 3/6 = 0.5$
 $p(\text{negative}) = 3/6 = 0.5$
 $p(\text{I} \mid \text{positive}) = 4/5 = 0.8$
 $p(\text{hated} \mid \text{positive}) = 2/5 = 0.4$
 $p(\text{that} \mid \text{positive}) = 3/5 = 0.6$
 $p(\text{movie} \mid \text{positive}) = 2/5 = 0.4$
 $p(\text{loved} \mid \text{positive}) = 4/5 = 0.8$
 $p(\text{it} \mid \text{positive}) = 3/5 = 0.6$
 $p(\text{I} \mid \text{negative}) = 4/5 = 0.8$
 $p(\text{hated} \mid \text{negative}) = 4/5 = 0.8$
 $p(\text{that} \mid \text{negative}) = 3/5 = 0.6$
 $p(\text{movie} \mid \text{negative}) = 2/5 = 0.4$
 $p(\text{loved} \mid \text{negative}) = 2/5 = 0.4$
 $p(\text{it} \mid \text{negative}) = 3/5 = 0.6$

(b) **All features:**

$$\begin{aligned}
p(\text{I loved it}|\text{pos}) &= p(\text{pos}) * p(\text{I}|\text{pos}) * p(\neg\text{hated}|\text{pos}) * p(\neg\text{that}|\text{pos}) * p(\neg\text{movie}|\text{pos}) * p(\text{loved}|\text{pos}) * p(\text{it}|\text{pos}) \\
&= 0.5 \quad * 0.8 \quad * 0.6 \quad * 0.4 \quad * 0.6 \quad * 0.8 \quad * 0.6 \\
&= 0.0276 \quad (\text{give or take depending on precision})
\end{aligned}$$

$$\begin{aligned}
p(\text{I loved it}|\text{neg}) &= p(\text{neg}) * p(\text{I}|\text{neg}) * p(\neg\text{hated}|\text{neg}) * p(\neg\text{that}|\text{neg}) * p(\neg\text{movie}|\text{neg}) * p(\text{loved}|\text{neg}) * p(\text{it}|\text{neg}) \\
&= 0.5 \quad * 0.8 \quad * 0.2 \quad * 0.4 \quad * 0.6 \quad * 0.4 \quad * 0.6 \\
&= 0.004608
\end{aligned}$$

(Much better....this is why smoothing is important!)

Positive only features:

$$\begin{aligned}
p(\text{I loved it}|\text{pos}) &= p(\text{pos}) * p(\text{I}|\text{pos}) * p(\text{loved}|\text{pos}) * p(\text{it}|\text{pos}) \\
&= 0.5 \quad * 0.8 \quad * 0.8 \quad * 0.6 \\
&= 0.192
\end{aligned}$$

$$\begin{aligned}
 p(\text{I loved it}|\text{neg}) &= p(\text{neg}) * p(\text{I}|\text{neg}) * p(\text{loved}|\text{neg}) * p(\text{it}|\text{neg}) \\
 &= 0.5 \quad * 0.8 \quad * 0.4 \quad * 0.6 \\
 &= 0.096
 \end{aligned}$$