Module 1: Discrete Time and Signals
Assignment 1

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Note to the professor: this is my first time doing anything in LaTeX, but I've wanted to learn it since this is my first graduate level class. I noticed that my assignment is rather verbose; if you have any feedback on how to streamline or organize assignments let me know. Most questions will either have the answer right at the beginning or underlined towards the end. Some figures aren't with the problem, but as close as I could get them to the problem.

MATLAB Problems Section

The following problems relate to the proper installation and setup of MATLAB.

Problem 1

This class will use the latest version of MATLAB which you can obtain free on your personal computer following the instructions provided in Blackboard (Bb). Running the latest MATLAB version type "ver" in the MATLAB command window and present results in your homework submission.

Figure 1 shows my current MATLAB version along with installed toolboxes:

```
>> ver
-----------------------------------------------------------------------------------------------------
MATLAB Version: 9.8.0.1380330 (R2020a) Update 2
MATLAB License Number: 703789
Operating System: Microsoft Windows 10 Home Version 10.0 (Build 18363)
Java Version: Java 1.8.0_202-b08 with Oracle Corporation Java HotSpot(TM) 64-Bit Server VM mixed mode
-----------------------------------------------------------------------------------------------------
MATLAB                                                Version 9.8         (R2020a)
Simulink                                              Version 10.1        (R2020a)
DSP System Toolbox                                    Version 9.10        (R2020a)
Fixed-Point Designer                                  Version 7.0         (R2020a)
Signal Processing Toolbox                             Version 8.4         (R2020a)
Statistics and Machine Learning Toolbox               Version 11.7        (R2020a)
Symbolic Math Toolbox                                 Version 8.5         (R2020a)
>>
```

Figure 1: *ver* command output.

Problem 2

Answer the questions providing your reference: Can you make multiple homework submissions prior to the due date/time? Are there consequences?

In the syllabus under second section of *Student Coursework Requirements*, it is mention that blackboard will only accept one submission for homework assignments. Additional submissions must be emailed to a professor for a 5 point penalty before the due date/time. Note that the assignment must be submitted as a single PDF. MATLAB code and plots must also be included in the PDF if used, and if appropriate.

Problem 3

In Bb go to the Community tab at the top. Under My Organizations, follow the instructions to self-enroll into MATLAB Fundamentals. Once enrolled click on Course Modules. You will see 6 training modules designed by JHU-EP to introduce students to MATLAB. Students are to view the video to make sure they understand the concepts presented. If you are already very familiar with MATLAB you can speed through the lessons. However, all students are to answer the following questions as presented in the training module video.

Part A

What are the three top level operational MATLAB windows?

Upon opening MATLAB, the Command, Workspace and Current Folder windows will open up as shown in Figure 2. The command window will allow the user to type various commands and mathematical statements. The current folder window shows the working directory on the left in Figure 2. To the right there is the workspace window which shows saved variables and data.

Part B

In the Home tab what are three examples of function or operations that you can use in the top banner display?

Some functions that I have found to be useful include the "New Script", "Import Data" and the "Save Workspace" functions. I find these handy for starting or importing work from another project. Figure 3 below shows these functions with some others included in the top banner.

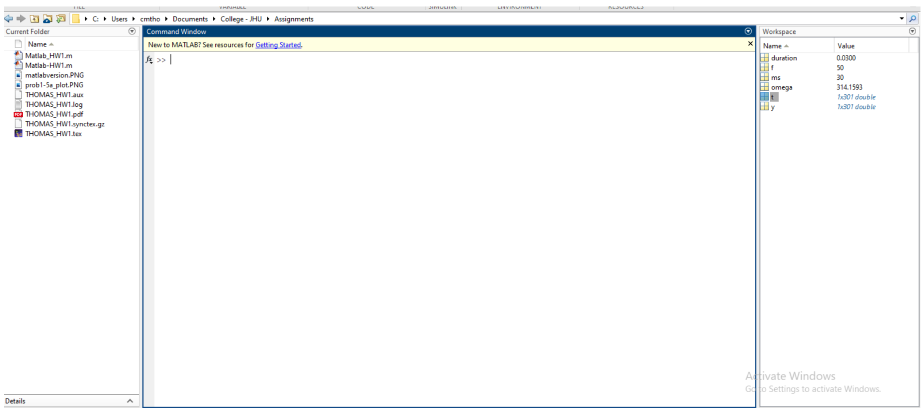

Figure 2: Initial view of MATLAB.

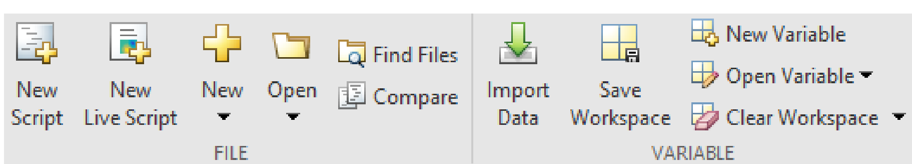

Figure 3: Part of the MATLAB toolbar.

Part C

What are two forms of getting help regarding MATLAB functions or operations?

I typically get help from the "Help" function in the top banner (or F1 hotkey), or from the MathWorks online community. Another good resource is StackOverflow.com since there is a good support community there for programmers, scientists and engineers.

Part D

Create a vector array designated x *where* $x = [0 : 9]$. *Create a vector array designated* y *where* $y = x^2$; *Plot* x *versus* y *using MATLAB with straight lines connecting the points.*

In Figure 4 we have both the code and the plot with function $y = x^2$. Notice that by default MATLAB plots straight lines between the points.

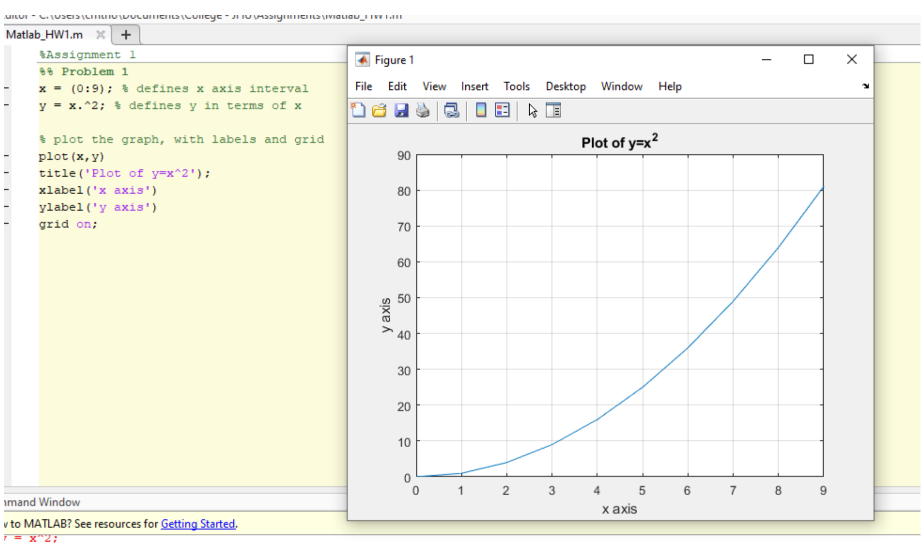

Figure 4: A quick plot of $y = x^2$, with the respective code.

Part E

repeat D creating a stem display of the x *vs* y *data.*

Changing a plot from smooth lines to a stem plot is very easy: simply change plot(x,y) to stem(x,y).

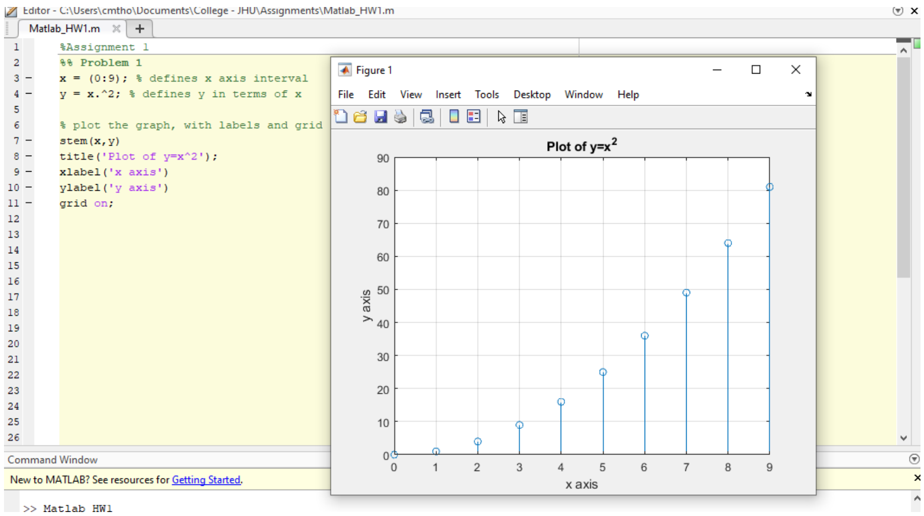

Figure 5: A stem plot of $y = x^2$, with the respective code.

Textbook Questions

Problem 4

1.1 - Classify the following signals according to whether they are (1) one- or multi-dimensional; (2) single or multichannel, (3) continuous time or discrete time, and (4) analog or digital (in amplitude). Give a brief explanation.

Part A

Closing prices of utility stocks on the NYSE

This is a single-dimensional signal, multi-channel, discrete-time and discrete-valued signal. Let $\mathbf{P}[n]$ be a vector representing the price of all utility stocks on the NYSE:

$$\mathbf{P}\text{[n]}= \begin{bmatrix} P_1[n] \\ P_2[n] \\ P_3[n] \\ \dots \\ P_n[n] \end{bmatrix}$$

Where n represents the day, and $P_n[n]$ represents the closing price of the n'th utility on the NYSE. Each utility represents a channel. The signal is discrete-time since time is represented at the closing of the NYSE (a singular value) rather than during opening hours (continuous range). Dollars are a discrete representation of perceived value.

Part B

A color movie

This is a multi-dimensional, multi-channel, discrete-time signal. It can be analog or digital in values of intensity. The red, green, blue color intensities represent the 3 channels of an intensity signal $I(x, y, t)$. x and y represent the color pixel location on a 2d screen with the third dimension being t time in seconds. Depending on the filming equipment, the movie could have either analog or digital values of intensity. It would be an analog signal if the movie was being watched on a CRT with a VHS tape. It would be digital if played from a DVD and viewed from a digital TV. Regardless of analog or digital values, it is discrete time since any movie is a series of pictures.

Part C

Position of the steering wheel of a car in motion relative to the car's reference frame

Single-dimensional, single-channel, continuous-time, and analog signal. From the car's reference frame, the axial angle of the steering wheel is the only positional value that can change (assuming the wheel doesn't come off). A function $P(t) = \theta$ can give the positional angle at any time t for $-\infty < t < \infty$.

Part D

Position of the steering wheel of a car in motion relative to the ground reference frame

Single-dimensional, multi-channel, continuous-time, and analog signal. From the ground reference frame, we can have x, y, and z represent the position of the wheel in 3d space. We can have θ represent the angle of the steering wheel relative to the axle that it is attached to, and ϕ represent the direction the axle is facing on the x, y plane (also where the nose of the car is facing). We can have a vector $\mathbf{P}(t)$ containing all dimensions of the steering wheel as follows:

$$\mathbf{P}(\mathrm{t}) = \begin{bmatrix} P_x(t) \\ P_y(t) \\ P_z(t) \\ P_\theta(t) \\ P_\phi(t) \end{bmatrix}$$

Part E

Weight and height measurements of a child taken every month.

Single-dimensional, multi-channel, discrete-time, digital amplitude. The size of a child can be mathematically describes as a vector $\mathbf{S}[n]$ like so:

$$\mathbf{S}(\mathrm{t})= \begin{bmatrix} S_w[n] \\ S_h[n] \end{bmatrix}$$

with $S_w[n]$ and $S_h[n]$ representing weight and height respectively at month n. Note that the value is digital rather than analog because of the quantization that occurs to gather the height and weight.

Problem 5

1.2 - Determine which of the following sinusoids are periodic and compute their fundamental period.

Part A - $cos(0.01\pi n)$

To determine if a signal is periodic, the following condition must hold true: $x(n) = x(n+N)$ if and only if $f_o = k/N$, where N is the fundamental period and f_o is the fundamental frequency. For this signal let $x(n) = cos(0.01\pi n)$.

$$x(n) = x(n+N)$$
$$cos(0.01\pi n) = cos(0.01\pi(n+N))$$
$$x(n) = cos(0.01\pi n)cos(0.01\pi N) - sin(0.01\pi n)sin(0.01\pi N)$$

We want the right hand side of the equation to zero out, and one of the cos components to equal 1. To do this we solve: $0.01\pi N = 2\pi k$ where k is an integer.

$$0.01\pi N = 2\pi k$$
$$0.005 = k/N = 1/200$$

If we let $k = 1$, $N = 200$. When we insert N into our equation above we get the following:

$$x(n) = cos(0.01\pi n)cos(0.01\pi(200)) - sin(0.01\pi n)sin(0.01\pi(200))$$
$$x(n) = cos(0.01\pi n)(1) - sin(0.01\pi n)(0)$$
$$x(n) = cos(0.01\pi n)$$

Because $x(n) = x(n+N)$ and k/N is a rational number for every integer k, $x(n)$ is a periodic signal with fundamental period $N = 200$.

Part B - $cos(\pi\frac{30n}{105})$

We'll solve this like the previous problem. Let $x(n) = cos(\pi\frac{30n}{105})$.

$$x(n) = x(n+N)$$
$$cos(\pi\frac{30n}{105}) = cos(\pi\frac{30(n+N)}{105})$$
$$x(n) = cos(\pi\frac{30n}{105})cos(\pi\frac{30N}{105}) - sin(\pi\frac{30n}{105})sin(\pi\frac{30N}{105})$$

Like last time, we want the right hand side of the equation to zero out, and one of the cos components to equal 1. To do this we solve: $\pi\frac{30N}{105} = 2\pi k$ where k is an integer.

$$\pi\frac{30N}{105} = 2\pi k$$
$$1/7 = k/N$$

If we let $k = 1$, $N = 7$. When we insert N into our equation above we get the following:

$$x(n) = cos(\pi\frac{30n}{105})cos(\pi\frac{30(7)}{105}) - sin(\pi\frac{30n}{105})sin(\pi\frac{30(7)}{105})$$
$$x(n) = cos(\pi\frac{30n}{105})(1) - sin(\pi\frac{30n}{105})(0)$$
$$x(n) = cos(\pi\frac{30n}{105})$$

Because $x(n) = x(n+N)$ and k/N is a rational number for every integer k, $x(n)$ is a periodic signal with a fundamental period of $N = 7$

Part C - $cos(3\pi n)$

Let $x(n) = cos(3\pi n)$.

$$x(n) = x(n+N)$$
$$cos(3\pi n) = cos(3\pi(n+N))$$
$$x(n) = cos(3\pi n)cos(3\pi N) - sin(3\pi n)sin(3\pi N)$$

We want the right hand side of the equation to zero out, and one of the cos components to equal 1. Let: $3\pi N = 2\pi k$ where k is an integer.

$$3\pi N = 2\pi k$$
$$3/2 = k/N$$

$k = 3$ is the smallest non-zero integer that results in a rational k/N. Let $k = 3$, $N = 2$. When we insert N into our equation above we get the following:

$$x(n) = cos(3\pi n)cos(3\pi(2)) - sin(3\pi n)sin(3\pi(2))$$
$$x(n) = cos(3\pi n)(1) - sin(3\pi n)(0)$$
$$x(n) = cos(3\pi n)$$

Because $x(n) = x(n+N)$ and k/N is a rational number for every integer such that $N = [3, 6, 9, ...3k]$, $x(n)$ is a periodic signal with fundamental period $N = 2/3$.

Part D - $sin(3n)$

Let $x(n) = sin(3n)$.

$$x(n) = x(n + N)$$
$$sin(3n) = sin(3(n + N))$$
$$x(n) = sin(3n)cos(3N) + sin(3N)cos(3n)$$

We want the right hand side of the equation to zero out. Let: $3N = 2\pi k$ where k is an integer.

$$3N = 2\pi k$$
$$\frac{3}{2\pi} = \frac{k}{N}$$

Since k/N is irrational, x(n) is not periodic and does not have a fundamental period.

Part E - $sin(\pi \frac{62n}{10})$

Let $x(n) = sin(\pi \frac{62n}{10})$.

$$x(n) = x(n + N)$$
$$sin(\pi \frac{62n}{10}) = sin(\pi \frac{62(n + N)}{10})$$
$$x(n) = sin(\pi \frac{62n}{10})cos(\pi \frac{62N}{10}) + sin(\pi \frac{62N}{10})cos(\pi \frac{62n}{10})$$

We want the right hand side of the equation to zero out, and one of the cos components to equal 1. Let: $\pi \frac{62N}{10} = 2\pi k$ where k is an integer.

$$\pi \frac{62N}{10} = 2\pi k$$
$$31/10 = k/N$$

$k = 31$ is the smallest non-zero integer that results in a rational k/N. Let $k = 31$, $N = 10$. When we insert N into our equation above we get the following:

$$x(n) = sin(\pi \frac{62n}{10})cos(\pi \frac{62(10)}{10}) + sin(\pi \frac{62(10)}{10})cos(\pi \frac{62n}{10})$$

$$x(n) = sin(\pi \frac{62n}{10})(1) + (0)cos(\pi \frac{62n}{10})$$

$$x(n) = sin(\pi \frac{62n}{10})$$

Because $x(n) = x(n+N)$ and k/N is a rational number for every integer such that $N = [31, 62, 93, ...31k]$, $x(n)$ is a periodic signal with a fundamental period $N = 10/31$.

Problem 6

1.3 - Determine whether or not each of the following signals are periodic. In case a signal is periodic, specify its fundamental period.

Part A - $x_a(t) = 3cos(5t + \pi/6)$

$x_a(t)$ is a continuous-time signal. Such a signal is periodic when x(t) = x(t+T), where T is the fundamental period of x(t). Let $x_a(t) = x(t)$:

$$x(t) = x(t + T)$$

$$x(t) = 3cos(5(t + T) + \pi/6)$$

$$x(t) = 3cos(5t + \pi/6)cos(5T + \pi/6) - 3sin(5t + \pi/6)sin(5T + \pi/6)$$

To cancel out the right-hand side of the equation, we solve for:

$$5T + \pi/6 = 2\pi$$

$$T = \frac{11}{30}\pi$$

Since this signal is continuous-time, T does not have to be rational. Let's plug in T to our equation above to see if $x(t) = x(t + T)$:

$$x(t) = 3cos(5t + \frac{\pi}{6})cos(5(\frac{11}{30}\pi) + \frac{\pi}{6}) - 3sin(5t + \frac{\pi}{6})sin(5(\frac{11}{30}\pi) + \frac{\pi}{6})$$

$$x(t) = 3cos(5t + \frac{\pi}{6})(1) - 3sin(5t + \frac{\pi}{6})(0)$$

$$x(t) = 3cos(5t + \frac{\pi}{6})$$

Because $x(t) = x(t+T)$, $x(t)$ is periodic with a fundamental period of $T = \frac{11}{30}\pi$

Part B - $x(n) = 3cos(5n + \pi/6)$

This is similar to the problem 5 signals. Let $x(n) = 3cos(5n + \pi/6)$.

$$x(n) = x(n + N)$$
$$x(n) = 3cos(5(n + N) + \pi/6)$$
$$x(n) = 3cos(5n + 5N + \pi/6)$$
$$x(n) = 3cos(5n)cos(5N + \pi/6) - 3sin(5n)sin(5N + \pi/6)$$

To cancel out the right-hand side of the equation, we solve for:

$$5N + \pi/6 = 2\pi$$
$$N = \frac{11}{30}\pi$$

Since N is irrational, $x(n)$ is <u>not periodic</u>.

Part C - $x(n) = 2e^{j(n/6-\pi)}$

For $x(n)$ to be periodic, $x(n) = x(n + N)$ if and only if $f_o = k/N$. Let $x(n) = x(n + N)$:

$$x(n) = 2e^{j((n+N)/6-\pi)}$$
$$x(n) = 2e^{j(n/6+N/6-\pi)}$$
$$x(n) = 2e^{j(n/6)}e^{j(N/6)}e^{-j\pi}$$
$$x(n) = -2e^{j(n/6)}e^{j(N/6)}$$

At this point, things are a bit tricky. Let's determine if there is a rational solution for N:

$$2e^{j(n/6-\pi)} = -2e^{j(n/6)}e^{j(N/6)}$$
$$2e^{j(n/6)}e^{-j\pi} = -2e^{j(n/6)}e^{j(N/6)}$$
$$1 = e^{j(N/6)}$$

Now we solve for $N/6 = 2\pi k$, where k is a positive integer, to show that the above statement true:

$$N = \frac{\pi}{3}k$$

Because N is not rational, $x(n)$ is <u>not periodic</u>.

Part D - $x(n) = cos(n/8)cos(\pi n/8)$

This is an interesting problem because we have a periodic signal convolved with a non-periodic signal in the discrete-time frequency domain. For $x(n)$ to be periodic, $x(n) = x(n+N)$ if and only if $f_o = k/N$. Let $x(n) = x(n+N)$, but first:

$$x(n) = cos(\frac{1}{8}(n+N))cos(\frac{\pi}{8}(n+N))$$

$$x(n) = cos(\frac{1}{8}(n+N))(cos(\frac{\pi}{8}n)cos(\frac{\pi}{8}N) - sin(\frac{\pi}{8}N)sin(\frac{\pi}{8}n))$$

$$x(n) = \left(cos(\frac{n}{8})cos(\frac{N}{8}) - sin(\frac{N}{8})sin(\frac{n}{8})\right)\left(cos(\frac{\pi}{8}n)cos(\frac{\pi}{8}N) - sin(\frac{\pi}{8}N)sin(\frac{\pi}{8}n)\right)$$

This is tricky because we need an N such that $\frac{\pi}{8}N = 2\pi k$ and $\frac{N}{8} = 2\pi k$ for $x(n) = x(n+N)$. Unfortunately $\frac{N}{8} \neq \frac{N\pi}{8}$ so we can't do our usual trick, so there is no solution for $x(n) = x(n+N)$. $x(n)$ is not periodic. By accident, I found another way to solve this problem. I'm keeping it here for study reference, but feel free to skip to part e. Let's use the following trig identity:

$$x(n) = \frac{1}{2}[cos(\frac{n}{8} - \frac{\pi}{8}n) - cos(\frac{n}{8} + \frac{\pi}{8}n)]$$

$$x(n) = \frac{1}{2}\left[cos((\frac{1}{8} - \frac{\pi}{8})n) - cos((\frac{1}{8} + \frac{\pi}{8})n)\right]$$

Let's express $x(n)$ in terms of u and v with $u = \frac{1}{8} - \frac{\pi}{8}$ and $v = \frac{1}{8} + \frac{\pi}{8}$:

$$x(n) = \frac{1}{2}\left[cos(un) - cos(vn)\right]$$

Likewise, $x(n+N)$ can be expressed as:

$$x(n+N) = \frac{1}{2}\left[cos((\frac{1}{8} - \frac{\pi}{8})(n+N)) - cos((\frac{1}{8} + \frac{\pi}{8})(n+N))\right]$$

$$x(n+N) = \frac{1}{2}\left[cos(un+uN) - cos(vn+vN)\right]$$

$$x(n+N) = \frac{1}{2}\left[(cos(un)cos(uN) - sin(un)sin(uN)) - (cos(vn)cos(vN) - sin(vn)sin(vN))\right]$$

Now let's simplify the expression $x(n) = x(n+N)$

$$\frac{1}{2}\left[cos(un) - cos(vn)\right] = \frac{1}{2}\left[(cos(un)cos(uN) - sin(un)sin(uN)) - (cos(vn)cos(vN) - sin(vn)sin(vN))\right]$$

Notice that this statement will turn out true if and only if $uN = 2\pi k$ and $vN = 2\pi k$. Because $uN \neq vn$, this signal is not periodic.

Part E - $x(n) = cos(\pi n/2) - sin(\pi n/8) + 3cos(\pi n/4 + \pi/3)$

We can break this down a bit by proving each of the components are periodic. If $x(n) = x(n+N)$ and $x(n) = u(n)+v(n)+w(n)$, then $x(n+N) = u(n+N)+v(n+N)+w(n+N)$ if $u(n), v(n), w(n)$ are periodic. Let $u(n) = cos(\pi n/2)$:

$$u(n) = cos(\frac{\pi}{2}(n+N))$$

$$u(n) = cos(\frac{\pi}{2}n)cos(\frac{\pi}{2}N) - sin(\frac{\pi}{2}n)sin(\frac{\pi}{2}N)$$

We now solve for the fundamental period N, $\frac{\pi}{2}N = 2\pi k$ where k is an integer.

$$N = 4k$$

$u(n)$ is periodic with fundamental period $N = 4$. Notice that for a sinusoid, the frequency component is paired with the n such that $cos(\omega n)$ and ω is in radians/sample. Also, $\omega = 2\pi f_o$, where f_o is the fundamental frequency. In this previous example we can also find the fundamental period by finding f_o with $\omega = \pi/2$:

$$\pi/2 = 2\pi f_o$$

$$f_o = 1/4 = 1/N$$

With this trick, let $v(n) = -sin(\pi n/8)$, and $\omega = \pi/8$:

$$\pi/8 = 2\pi f_o$$

$$f_o = 1/16 = 1/N$$

The fundamental period of $v(n)$ is $N = 16$. Now let $w(n) = 3cos(\pi n/4 + \pi/3)$, and $\omega = \pi/4$:

$$\pi/4 = 2\pi f_o$$

$$f_o = 1/8 = 1/N$$

The fundamental period of $v(n)$ is $N = 8$. Because the components $u(n), v(n), w(n)$ are periodic, $x(n)$ is also <u>periodic</u>. The fundamental period of $x(n)$ will be the least common multiple between the sub-components, which is $N = 16$. Therefore, $x(n)$ has a <u>fundamental period $N = 16$</u>.

Problem 7

1.5 - Consider the following analog sinusoidal signal:

$$x_a(t) = 3sin(100\pi t)$$

Part A

Sketch the signal $x_a(t)$ for $0 <= t <= 30ms$.

I created a plot in MATLAB using a line to connect the points. The signal in MATLAB is technically discrete-time, but with enough points it begins to appear like an analog signal. See Figure 6.

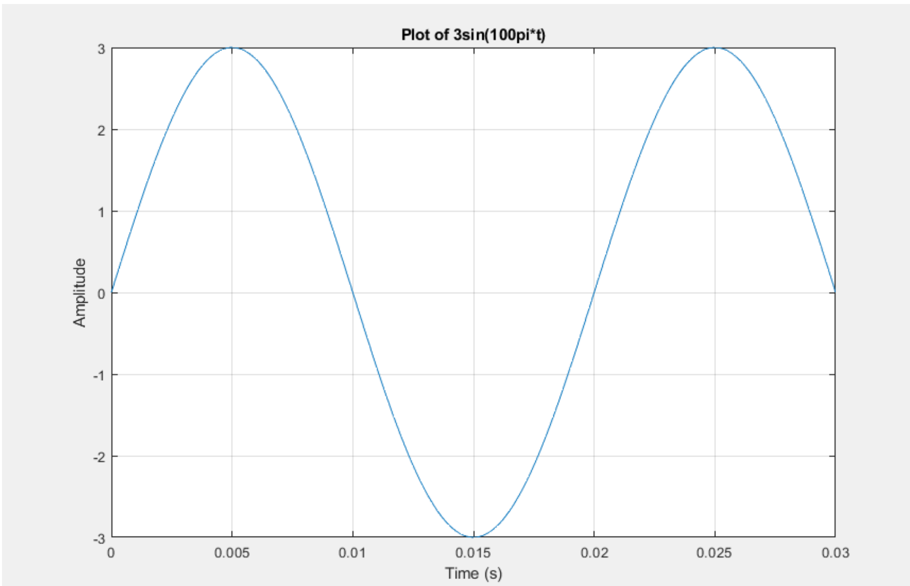

Figure 6: Plot of signal $x_a(t)$ in MATLAB

Part B

The signal $x_a(t)$ is sampled with a sampling rate $F_s = 300$ samples/s. Determine the frequency of the discrete-time signal $x(n) = x_a(nT), T = 1/F_S$, and show that it is periodic.

For analog signal $x_a(t)$, we get the following discrete signal $x_n(n)$:

$$x(n) = 3sin\left(\frac{100\pi}{300}n\right)$$

This gives us a (fundamental) frequency of $\pi/3 = 2\pi f_o$, or $f_o = 1/6hz$. To prove the signal is periodic, we show that $x(n) = x(n+N)$:

$$x(n) = 3sin\left(\frac{\pi}{3}(n+N)\right)$$

$$x(n) = 3\left(sin(\frac{\pi}{3}n)cos(\frac{\pi}{3}N) + cos(\frac{\pi}{3}n)sin(\frac{\pi}{3}N)\right)$$

With $N = 1/f_o = 6$ we can show that $x(n)$ is a periodic function:

$$x(n) = 3\left(sin(\frac{\pi}{3}n)(1) + cos(\frac{\pi}{3}n)(0)\right)$$

$$x(n) = 3sin(\frac{\pi}{3}n)$$

Part C

Compute the sample values in one period of $x(n)$. Sketch $x(n)$ on the same diagram with $x_a(t)$. What is the period of the discrete-time signal in milliseconds?

We can visually tell in Figure 7 that the period of the discrete-time signal is $\underline{20ms}$, which is the same as $x_a(t)$:

$$\omega = 100\pi = 2\pi f$$

$$f = 50, T = 1/f = 0.02s = 20ms$$

Notice that even though the period is the same, $x(n)$ does not accurately represent the signal $x_s(t)$.

Part D

Can you find a sampling rate F_s such that the signal $x(n)$ reaches its peak value of 3? What is the minimum F_s suitable for this task?

The minimum suitable F_s for the signal is the Nyquist frequency. From part C, we calculated the frequency $f = 50hz$ for $x_s(t)$. Let f_n be the Nyquist frequency:

$$f_n = 2f$$
$$f_n = 100Hz$$

A good rule of thumb is to sample at least twice the highest frequency of a signal. But bear in mind this is a *minimal* requirement. Figure 8 shows a signal being sampled at $100hz$. Notice how a phase shift could keep you from getting a signal, but at just the right phase you can perfectly recreate the sampled signal. A higher sample rate can make it easier to recreate a signal as shown in Figure 9.

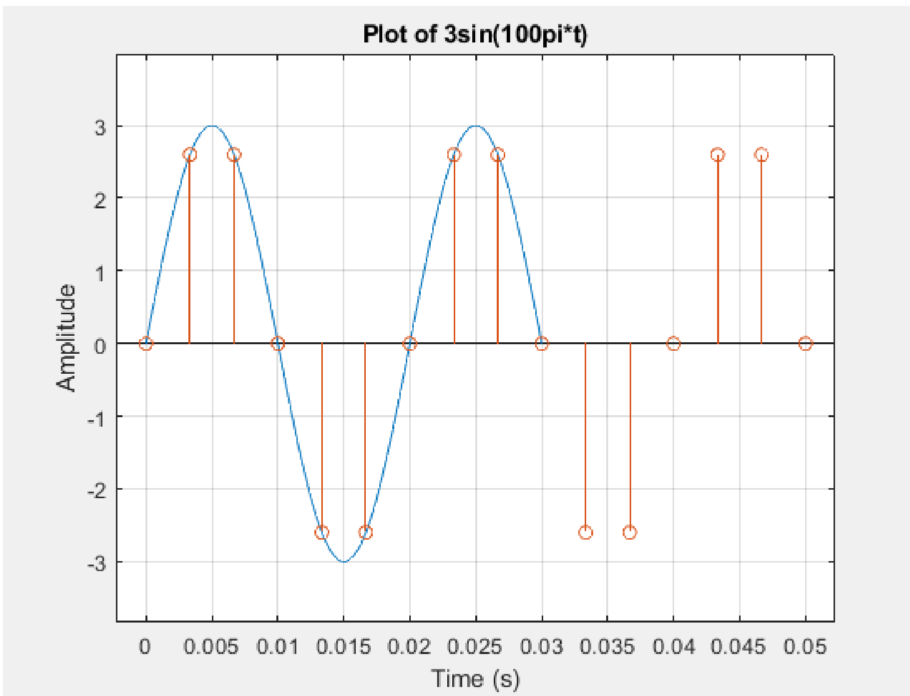

Figure 7: A visual of sampled $x_s(t)$ as 300 samples/s. Notice that the sampled signal has the same period as the original.

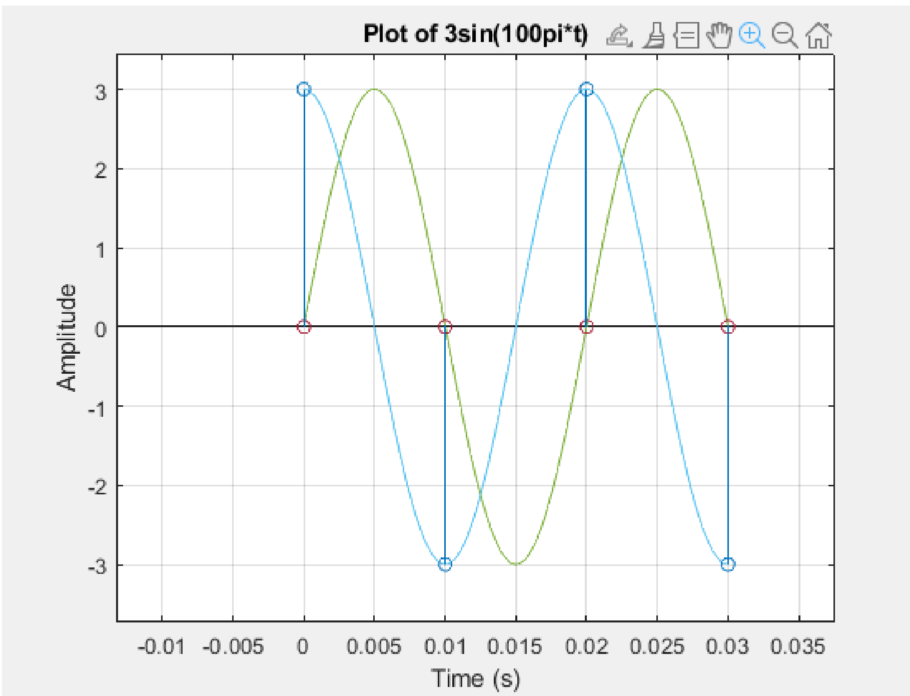

Figure 8: A visual of sampled $x_s(t)$ at 100 samples/s. Notice that this is the minimum frequency needed to recreate the signal, but a phase shift can alter the amplitude.

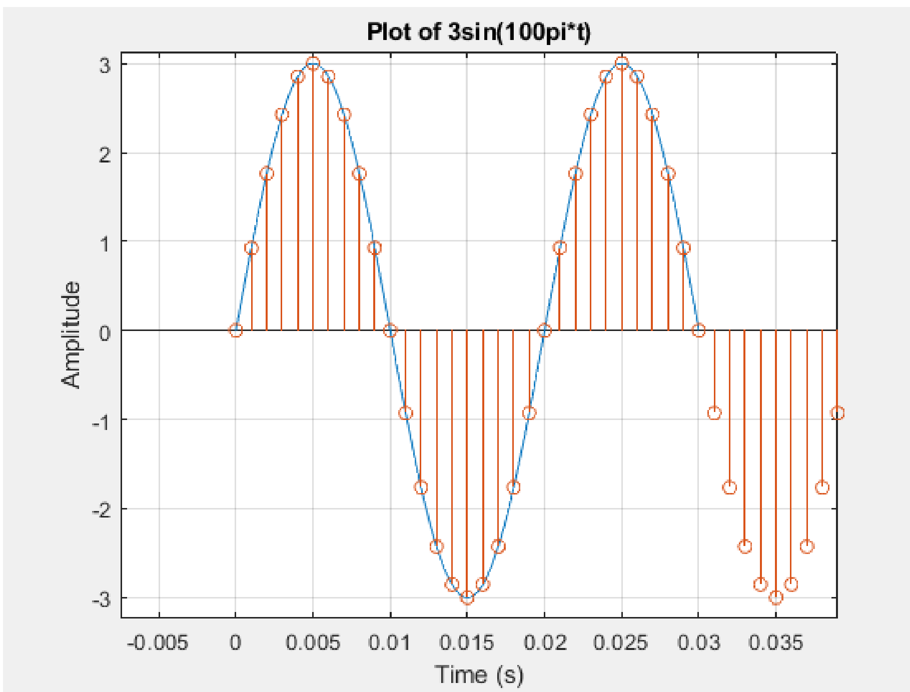

Figure 9: A visual of sampled $x_s(t)$ at 1000 samples/s. Signals are more easily represented as the sampling rate is increased.