Module 10 - Homework 10

Colt Thomas

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9.1-1 In a baseband binary transmission, binary digits are transmitted by using...

$$A\,p(t) \quad 0 < t < T_b \quad \text{sending } 1$$
$$-A\,p(t) \quad 0 < t < T_b \quad \text{sending } 0$$

The bit duration is T_b seconds, and the pulse shape is

$$p(t) = 1 - \left(\frac{2t}{T_b} - 1\right)^2, \quad 0 \le t \le T_b$$

Here data bits 0 and 1 are equally likely. The channel noise is AWGN with power spectrum $N/2$.

a.) Find the optimum receiver filter $h(t)$ for sampling instant $t_m = T_b$ and sketch $h(t)$ in the time domain

Section 9.1.2 steps through an optimum receiver filter that maximizes SNR, given by equation 9.10b

$$H(f) = \lambda \frac{P^*(f)e^{-j2\pi f t_m}}{S_n(f)}$$

where λ is the gain at the receiver and t_m is the sampling constant. $S_n(f)$ is the PSD of the noise.
Note that λ and $S_n(f)$ are constant, and that we actually have a time shift property in the expression above; we would get the following in the time domain.

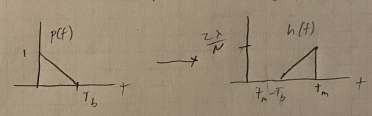
$$H(f) = \frac{2\lambda}{N} P(f) e^{-j2\pi f t_m}$$

pulse is real valued

$$h(t) = \frac{2\lambda}{N} p(t_m - t)$$

$$\boxed{h(t) = \frac{2\lambda}{N}\left[1 - \left(\frac{2(t_m - t)}{T_b} - 1\right)\right]^2}$$

ignore, since this doesn't affect SNR



b.) Determine the probability of error as a function of the E_b/N ratio

Probability of Error is also known as Bit Error Rate

$$BER = Q\left(\sqrt{\frac{E_b}{\eta}}\right)$$

amplitude of white gaussian noise PDF

bit energy

$$E_b = \int_0^{T_b} p^2(t)\,dt$$

The Q function is a gaussian pdf

$$= \int_0^{T_b} \left(1-\left(\frac{2t}{T_b}-1\right)^2\right)^2 dt = \int_0^{T_b} \frac{16}{T_b^4} t^2 (t-T_b)^2\,dt$$

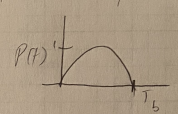
$$E_b = \frac{8}{15} T_b$$

$$\boxed{BER = Q\left(\sqrt{\frac{8T_b}{15N}}\right)}$$

c.) Compare the results from a) and b) for a pulse shape of

$$p(t) = \sin\frac{\pi t}{T_b} \quad 0 \le t \le T_b$$

A plot of our pulse would look like



And our BER would be

$$E_b = \int_0^{T_b} \sin^2\left(\frac{\pi t}{T_b}\right) dt \rightarrow \boxed{BER = Q\left(\sqrt{\frac{T_b}{2N}}\right)}$$

$$= \frac{T_b}{2}$$

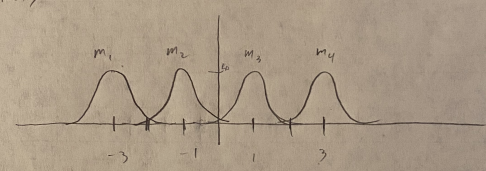
slightly smaller SNR

9.2-6 For 4-ary comms, messages are chosen from symbols

$m_1 = 00 \qquad m_2 = 01 \qquad m_3 = 10 \qquad m_4 = 11$

which are transmitted by pulses $\pm p(t)$ and $\pm 3p(t)$. Denote the energy of $p(t)$ as E_p. The channel noise is AWGN with spectrum $N/2$.

a.) if r is the matched filter output at t_m, plot $P_r(r|m_i)$ for the four message symbols and if all messages are equally likely.

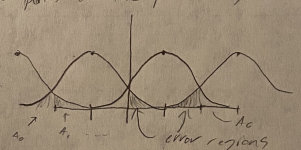


b.) To minimize probability of error, determine the optimum decision thresholds and the corresponding P_e as a function of the average symbol energy to noise ratio

The optimal thresholds are going to split the distributions evenly such that

$$a_1 < -2, \quad -2 \le a_2 < 0, \quad 0 \le a_3 < 2, \quad a_4 \ge 2$$

The probability of error will account for any overlap between the pdf's of the possible signals.



we get 6 overlapping areas we need to worry about for 4-ary

$$P_e = \sum_i P(E|m_i)P(m_i) = \text{average of the overlapping areas}$$

σ_n = AWGN std. dev
$\gamma = 0$ for AWGN

all $P(m_i)$ are equal so it can be reduced

$$= \frac{1}{4}\left[Q\left(\frac{\gamma+3}{\sigma_n}\right) + 2Q\left(\frac{\gamma+1}{\sigma_n}\right) + 2Q\left(\frac{\gamma-1}{\sigma_n}\right) + Q\left(\frac{\gamma-3}{\sigma_n}\right)\right]$$

$$= \boxed{\frac{3}{2}\left[Q\left(\frac{A_p}{\sigma_n}\right)\right]} \qquad A_p = A_0 = A_1 \cdots A_6$$

Matlab Code