Module 1 - Homework 1

Colt Thomas

June 8, 2021

Problem 2.1-3

Find the average power of the signals in Fig. P2.1-3

The average power can be computed over a period by finding the square of the amplitude:

$$P = \frac{1}{T}\int_0^T s^2(t)dt$$

The first signal $\rho(t)$ has an average power of:

$$P_\rho = \frac{1}{\pi}\int_0^\pi (e^{-t/2})^2 dt \approx \boxed{3.064 \text{ units}}$$

The second signal $w_o(t)$ has an average power of:

$$P_{w_o} = \frac{1}{T_o}\int_0^{T_o} w_o^2(t)dt$$

$$P_{w_o} = \frac{1}{T_o}\int_0^{T_o} (u(t) - 2u(t - \frac{T_o}{4}) + 2u(t - \frac{3T_o}{4}))^2 dt$$

*note: the amplitude goes to -1 hence the -2u(), and then back to 1 hence the 2u. If we calculate this out we will get the average power of the signal, but we can use a trick... notice that if we adjust the period we can instead calculate:

$$P_{w_o} = \frac{1}{T_o}\int_{-T_o/4}^{3T_o/2} (u(t + \frac{T_o}{4}) - 2u(t - \frac{T_o}{4}))^2 dt$$

Then we can use symmetry of the signal to simplify this to:

$$P_{w_o} = \frac{2}{T_o}\int_{-T_o/4}^{T_o/4} (u(t + \frac{T_o}{4}))^2 dt$$

$$P_{w_o} = \frac{2}{T_o}\int_{-T_o/4}^{T_o/4} (1)dt = \boxed{1 \text{ unit}}$$

Problem 2.2-2

Part A

Determine whether signal t^2 is a power signal. A signal is a power signal if it has a finite power for some period T:

$$P = \frac{1}{T}\int_{-T/2}^{T/2} (t^2)^2 dt$$

$$P = \frac{1}{T}\left[t^5/5\right]_{-T/2}^{T/2}$$

$$P = \frac{1}{T}(\frac{T^5}{80})$$

$$P = \frac{T^4}{80}$$

Notice that as the limit of T approaches infinity, the power also approaches infinity. This signal is non-periodic so it is $\boxed{\text{not a power signal.}}$

Part B

Determine whether signal $|t|$ is an energy signal. We need to determine if this has finite energy:

$$E = \int_{-\infty}^{\infty} |t|^2 dt$$

$$E = \int_{-\infty}^{0} (-t)^2 dt + \int_{0}^{\infty} t^2 dt = \infty$$

Since the energy is not finite, the signal is $\boxed{\text{not an energy signal}}$

Problem 2.4-1

Simplify the following expressions:

* Note: When multiplying by a delta, evaluate the multiplier function at the zero of the delta. Keep the delta function!

Part A

$$\left(\frac{tan 3t}{2t^2+1}\right)\delta(t-\pi/4)$$

$$\left(\frac{tan(3\pi/4)}{2(\pi/4)^2+1}\right)\delta(t-\pi/4)$$

$$\left(\frac{-1}{\frac{\pi^2}{8}+1}\right)\delta(t-\pi/4)$$

$$\boxed{\frac{-8}{\pi^2+8}\delta(t-\pi/4)}$$

Part D

$$\left(\frac{sin 0.5\pi(t+2)}{t^2-4}\delta(t-1)\right)$$

$$\left(\frac{sin 0.5\pi((1)+2)}{(1)^2-4}\right)\delta(t-1)$$

$$\boxed{\frac{1}{3}\delta(t-1)}$$

Part E

$$\left(\frac{cos(\pi t)}{t+2}\delta(2t+3)\right)$$

* Solve for $2t+3=0$ for the offset

$$\left(\frac{cos(\pi(-3/2))}{(-3/2)+2}\right)\delta(2t+3)$$

$$\boxed{0}$$

Problem 2.4-4

Solve the following:

Part A

$$\int_{-\infty}^{\infty} g(3\tau+a)\delta(t-\tau)d\tau$$

This problem illustrates the sampling property of the unit impulse function. Note that we are integrating with respect to τ and not t:

$$\boxed{g(3(t)+a)}$$

Part H

$$\int_{-\infty}^{\infty} cos\frac{\pi}{2}(x-5)\delta(3x-1)dx$$

$$cos(\frac{\pi}{2}(1/3)-5)$$

$$\boxed{cos(\frac{\pi}{6}-5)\approx -0.234}$$

Problem 2.6-2

Find the correlation coefficient between the signal $g_1(t) = u(t) - u(t-2)$ *and the signal* $g_2(t) = exp(-0.5t)u(t)$ Here is one way to calculate the correlation coefficient between two signals:

$$\rho = cos\theta = \frac{< g, x >}{||g||||x||}$$

Let's solve the various components first; note that the bounds are established by the overlap between $g_1(t)$ and $g_2(t)$ for the inner product:

$$< g_1, g_2 >= \int_0^2 g_1(t)g_2(t)dt$$

$$< g_1, g_2 >= \int_0^2 (1)e^{-0.5t}dt$$

$$< g_1, g_2 >= 2 - 2e^{-1} \approx 1.264$$

Now we need to find the norm of both signals:

$$||g_1(t)|| = \sqrt{< g_1(t), g_1(t) >}$$

$$||g_1(t)|| = \sqrt{\int_0^2 (1)(1)dt}$$

$$||g_1(t)|| = \sqrt{2}$$

$$||g_2(t)|| = \sqrt{< g_2(t), g_2(t) >}$$

$$||g_2(t)|| = \sqrt{\int_0^\infty (e^{-0.5t})(e^{-0.5t})dt}$$

$$||g_2(t)|| = 1$$

Pull it all together to get:

$$\rho = \frac{1.264}{\sqrt{2}} \approx \boxed{0.894}$$

Notice that this value is less than one; this is a good check to see if you have a reasonably correct computation.