Module 7 - Homework 7

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August 18, 2021

Problem 6.2-1

All problems worked by hand

(A)

A random binary data sequence 01001010110... is transmitted by means of a Manchester (split phase) line code with the pulse p(t) shown in Fig 6.7a. Sketch the waveform y(t)

(B)

Using the derived PSD results of polar signals, determine the PSD $S_y(t)$ of this Manchester signal in Part a, assuming 1 and 0 equally likely.

(C)

Sketch this PSD and find its bandwidth.

The bandwidth in this case would be $\frac{4\pi}{T_b}$ for the sketch below:

Problem 6.2-4

For baseband modulation, each bit duration is T_b. If the pulse shape is:

$$p_2(t) = \Pi(\frac{t}{T_b})$$

(A)

Find PSDs for the polar, on-off, and bipolar signaling.

6.2-4 Given pulse shape

$$P_2(t) = \Pi\left(\frac{t}{T_b}\right)$$

a) Find the PSD for the polar, on-off, and bipolar signaling

- The general PSD is $S_Y(f) = |P(f)|^2 S_X(f)$

$$P_2(f) = T_b \operatorname{sinc}(T_b \pi f)$$

First null at $1/T_b$ ... n/T_b

$$S_X(f) = \mathcal{F}[R_X(\tau)] = \frac{1}{T_b}\sum_{n=-\infty}^{\infty} R_n e^{-j2\pi f_0 T_b}$$

deltas

the binary symbol $(0, \pm 1)$

Correlation values

$$S_{\hat{X}}(f) = \mathcal{F}[R_{\hat{X}}(\tau)] = \frac{1}{T_b}\left[R_0 + 2\sum_{n=1}^{\infty} R_n \cos(2\pi n f_0 T_b)\right]$$

Find autocorrelation value R_0 and R_n

Polar: $R_0 = E[a_k a_k] = \frac{(-1)^2 + (1)^2}{2} = 1$

$R_n = E[a_k a_{k+1}] = \frac{1}{4}\left[(1)(1) + (1)(-1) + (-1)(1) + (-1)(-1)\right] = 0$

On-off: $R_0 = E[a_k a_k] = \frac{1^2 + 0^2}{2} = 1/2$

$R_n = E[a_k a_{k+1}] = \frac{1}{4}\left[(1)(0) + (1)(1) + (0)(1) + (0)(0)\right] = 1/4$

bipolar: $R_0 = E[a_k a_k] = \frac{(1)^2 + (-1)^2}{2} = 1$

* R_n for bipolar isn't so trivial. See the table on pg 380 of the book. you would get:

$$R_n = \lim_{N\to\infty} \frac{1}{N}\sum_k a_k a_{k+1} = 0 \quad n > 1$$

The Resulting PSD for the given pulse shape is:

polar: $S_y(f) = \frac{|P_2(f)|^2}{T_b}\left[(1) + 2\sum^{\infty}(0)\cos(\sim)\right]$

$$S_y(f) = \frac{|T_b \operatorname{sinc}(\pi f T_b)|^2}{T_b} = \boxed{T_b \operatorname{sinc}^2(T_b \pi f)}$$

On-off: $S_y(f) = T_b \operatorname{sinc}^2(T_b \pi f)\left[\frac{1}{2} + 2\sum_{n=1}^{\infty}\frac{1}{4}\cos(2\pi n f T_b)\right]$

$$= (\quad '' \quad)\left[\frac{1}{2} + \frac{1}{4}\sum_{n=-\infty}^{\infty} e^{-j2\pi f T_b}\right]$$

use substitution shown on pg 377 btwn 6.18b and 6.19a

$$= T_b \operatorname{sinc}^2(T_b \pi f)\left[\frac{1}{T_b}\sum_{n=-\infty}^{\infty}\delta\left(f - \frac{n}{T_b}\right)\right]$$

$$\boxed{S_y(f) = \operatorname{sinc}^2(T_b \pi f)\left[\sum_{n=-\infty}^{\infty}\delta\left(f - \frac{n}{T_b}\right)\right]}$$

bipolar: $S_y(f) = \frac{|P_2(f)|^2}{T_b}\left[\left(\frac{1}{2}\right) + 2\sum^{\infty} R_n \cos(\sim)\right]$

$$= T_b \operatorname{sinc}^2(T_b \pi f)\left[\frac{1}{2} + 2\left(-\frac{1}{4}\cos(2\pi f T_b)\right)\right]$$

$$= \frac{T_b}{2}\operatorname{sinc}^2(T_b \pi f)\sin^2(\pi f T_b)$$

(B)

Sketch these PSDs and find their bandwidths. For each case, compare the bandwidth to the PSD obtained in Prob. 6.2-3.

b) Sketch of PSDs with bandwidths:

Bandwidths are

bipolar: $\frac{\pi}{T_b}$ polar: $\frac{2\pi}{T_b}$ on-off: $\frac{2\pi}{T_b}$