Lab 2 - Analog Modulation Techniques

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Introduction

The purpose of this lab is to illustrate and practice analog modulation techniques, namely Amplitude and Frequency Modulation. The first part in this lab will perform two types of modulations: the Double-Sideband Suppressed-Carrier (DSB-SC) modulation, and the Amplitude Modulation scheme. Part two will demonstrate the Phase Modulation (PM) and Frequency Modulation (FM). By the end of this lab, all steps of the modulation and demodulation process will be shown using a triangle wave message signal for demonstration purposes.

Part 1 - Amplitude Modulation

For this lab, I was tasked to implement some MATLAB exercises from the book. The first example is the implementation of the DSB-SM modulation and demodulation schemes on a simple triangle wave signal. The second example is similar, but with the AM modulation scheme. Code implementation can be found in the appendix. Below is the mathematical definition of the unit triangle function, which is defined in the triangl.m code provided below:

Unit Triangle Function

$$\Delta(t) = \begin{cases} 1 - 2|x| & |x| < \frac{1}{2} \\ 0 & |x| > \frac{1}{2} \end{cases}$$

1.1 Baseband Signal

Below in figures 1 and 2 are the time and frequency domain representation of our triangle wave signal. We will be modulating this signal with both DSB-SC and AM modulation schemes.

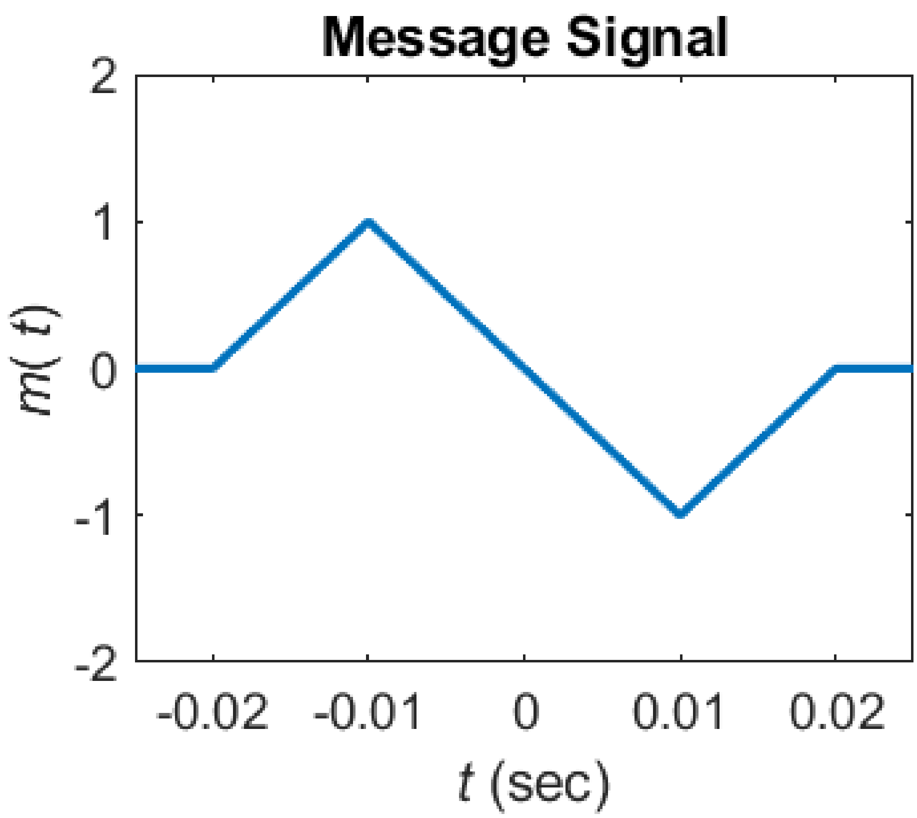

Figure 1: Plot of our triangle message signal versus time.

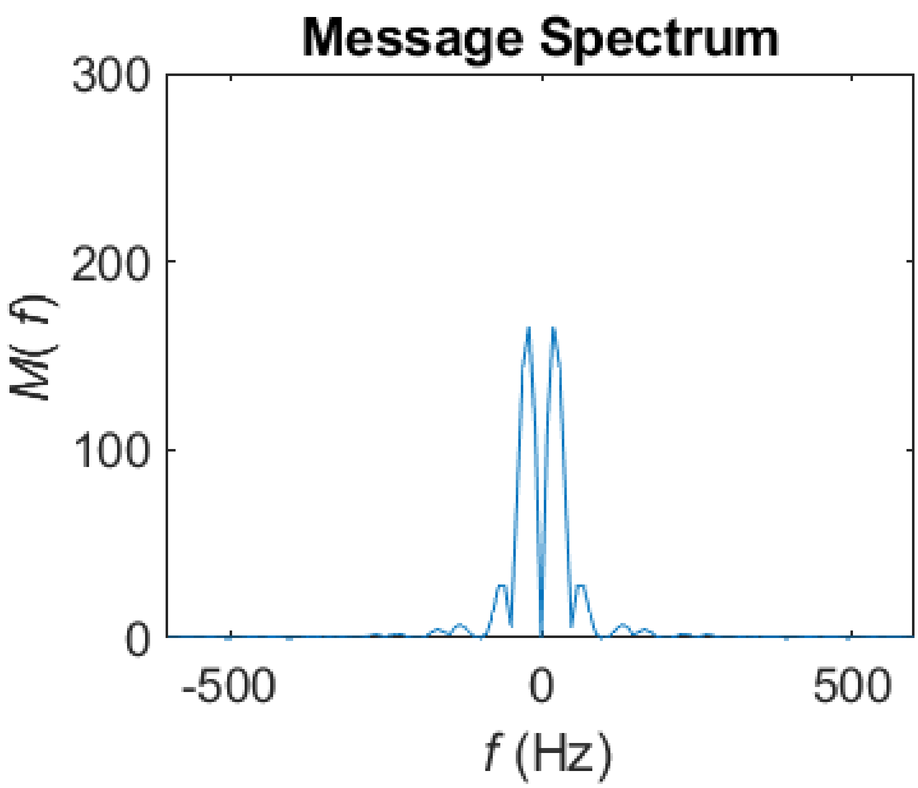

Figure 2: Plot of our baseband signal spectrum.

1.2 Amplitude Modulation - Suppressed Carrier

Figure 3 shows our triangle wave signal modulated. Note that the amplitude of the wave lines up with the modulated signal (hence amplitude modulation).

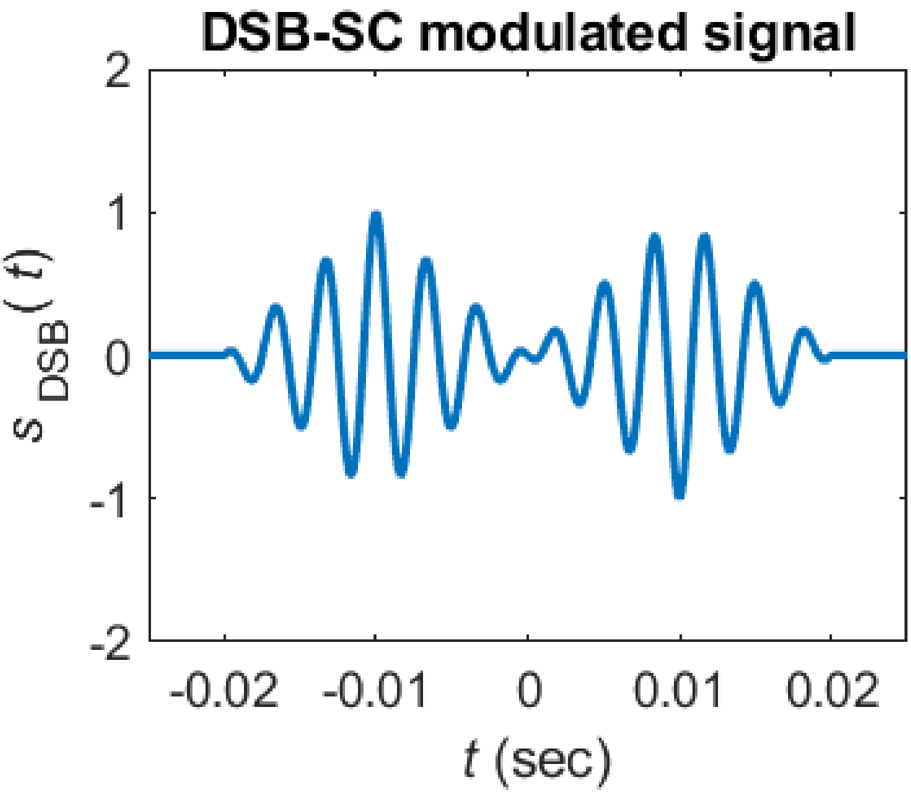

Figure 3: Plot of our SDB-SC signal versus time.

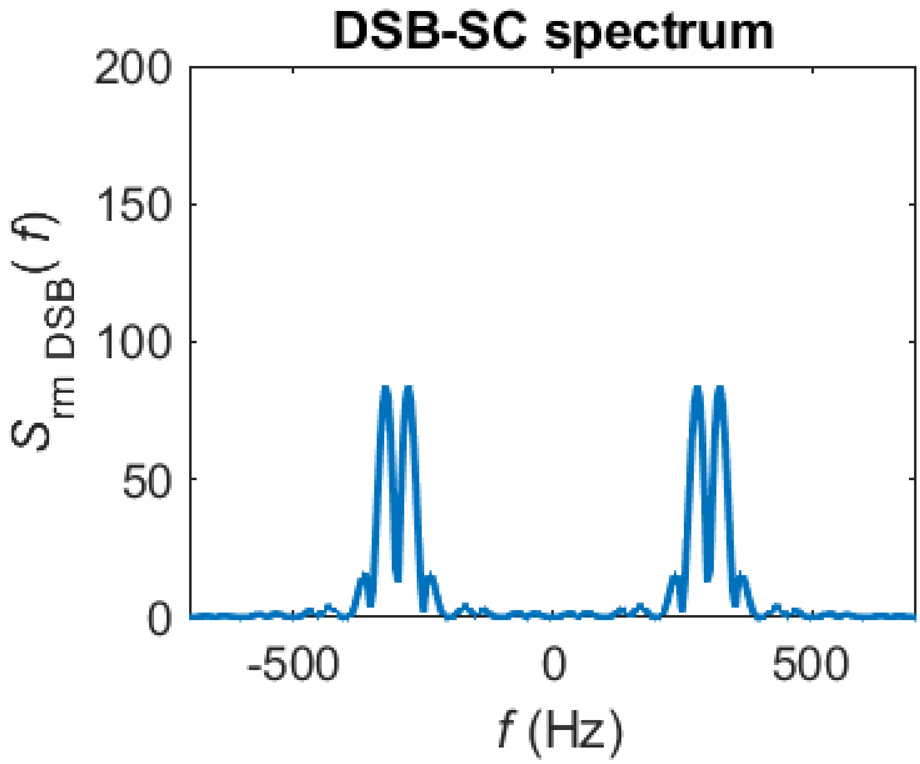

Figure 4: Plot of our SDB-SC signal spectrum.

1.3 Coherent Demodulation

Recovering the baseband signals requires a method called coherent detection. This is where the receiver needs to generate a carrier of the exact same frequency and phase of the one used for transmission. To recover our original signal, the receiver needs to synchronize with the transmitter. The receiver also needs to filter the incoming signal to cut out any noise added to the system. Figures 5 and 6 show the spectrum of the signal received before and after the filtering process. Notice how the unfiltered signal has peaks past the 500Hz marker; these are components that were not included from our initial signal from figure 2! A low-pass filter removes those undesired, higher frequency components, which ends up giving us a very clean recovered signal in figure 7.

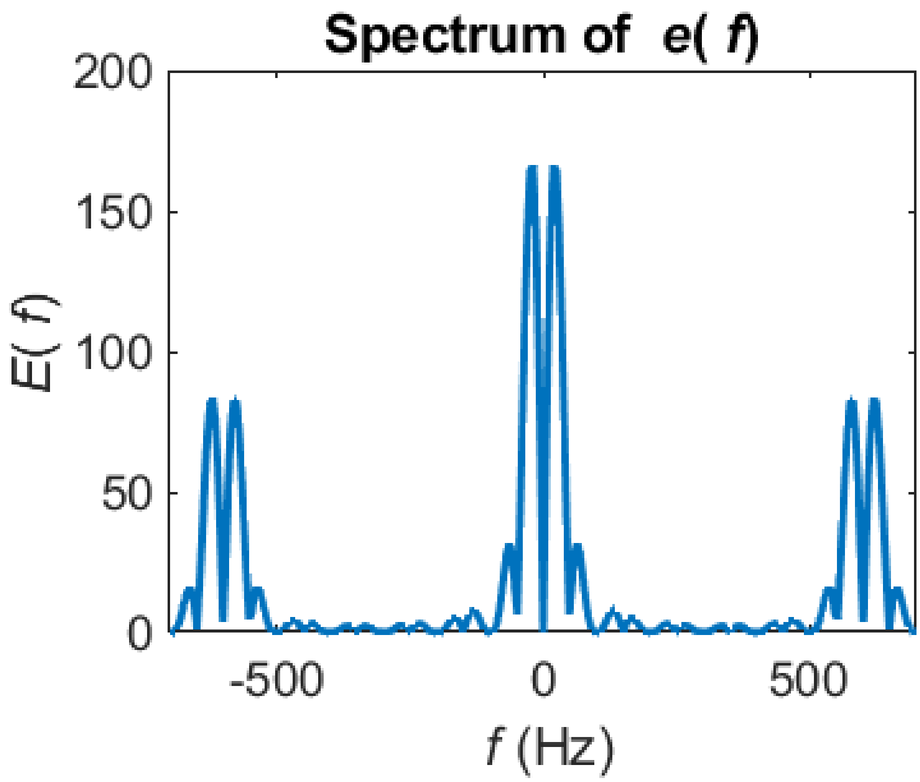

Figure 5: Plot of our pre-filtered signal spectrum.

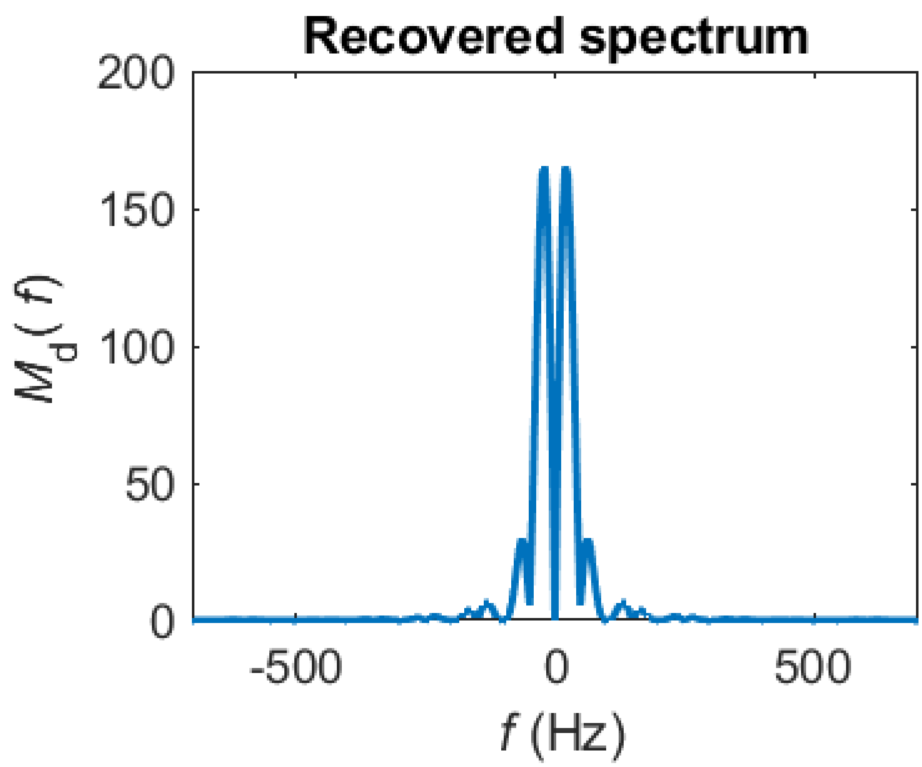

Figure 6: Plot of our post-filtered signal spectrum.

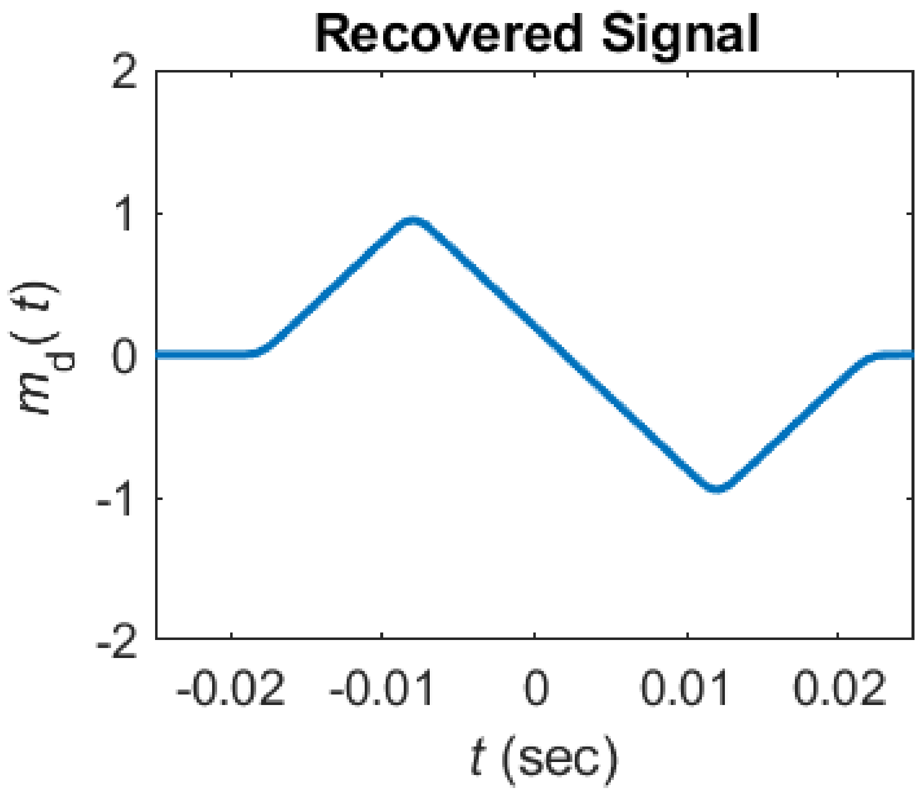

Figure 7: Plot of our demodulated signal versus time.

1.4 Amplitude Modulation

One of the downsides to the DSB-wC is that it is prone to error when traveling long distances due to Doppler shifts. AM Modulation is slightly different where it will send a carrier with the modulated signal. Note the difference in amplitude in figure 8; the original signal had an amplitude of 1 but we see an amplitude of 2. This is because we are adding that carrier with the same amplitude. The spectrum in figure 9 also looks different; it's hard to tell but between the left and right lobes that you see from the original spectrum there is a carrier signal precisely at 500Hz.

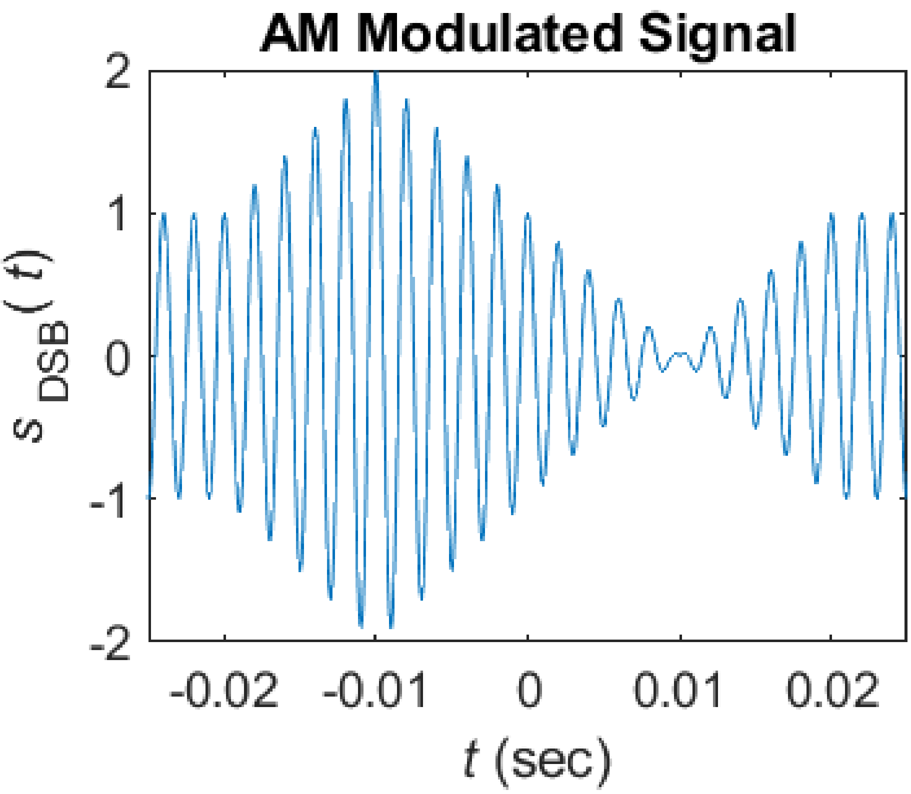

Figure 8: Plot of our DSB-wC signal versus time.

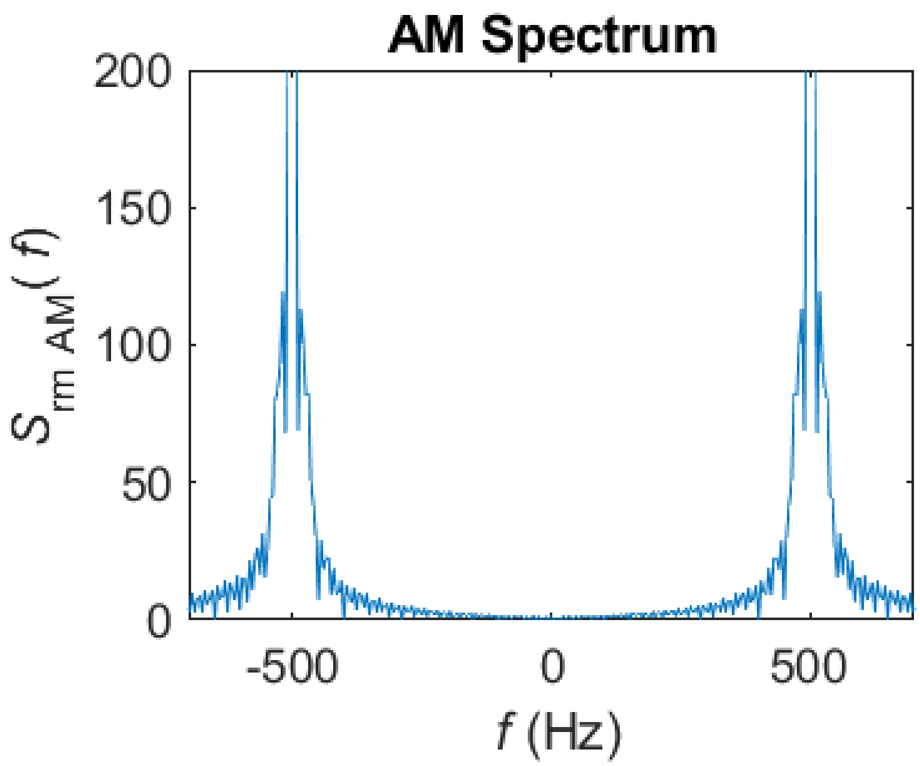

Figure 9: Plot of our DSB-wC signal spectrum.

1.5 Rectifier Demodulator

A rectifier can be used as a non-coherent way to demodulate an AM signal. This will cut out all negative amplitudes of the modulated signal and give us the results in figure 10. The output of the rectifier can be passed through a lowpass filter, which will produce the results in figure 11. Note that the received signal is no longer centered about 0, but we still get a DC and amplitude shifted variation of the original signal which we can recover.

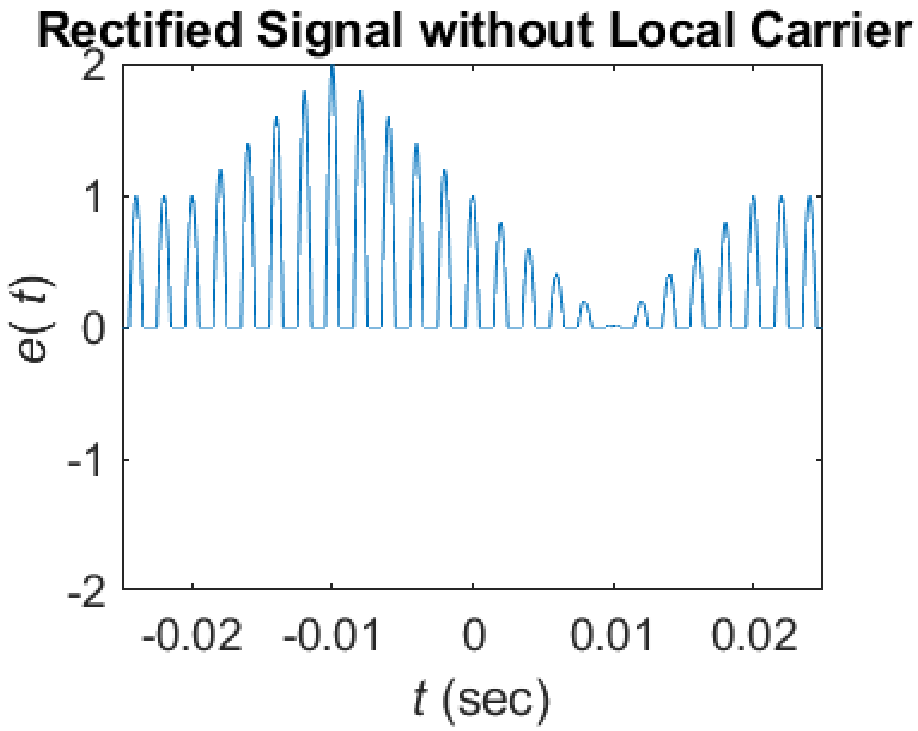

Figure 10: Plot of our rectified signal versus time.

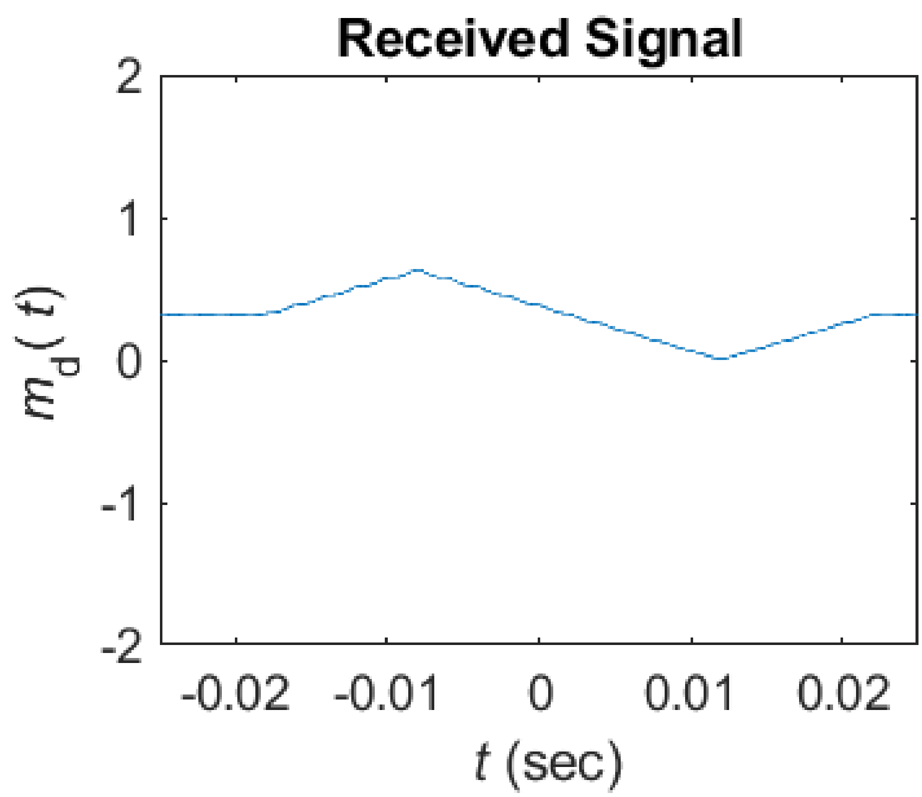

Figure 11: Plot of our received signal versus time.

1.6 AM Questions

This portion of the lab is a question/answer section. Questions are listed below with appropriate responses:

a) What is the modulation index of the AM signal in Part 1.4? The modulation index is the ratio of the message amplitude over the carrier amplitude. Since they are equal, $\boxed{\mu = 1}$

b) Could you set the modulation index to 0.5 and successfully demodulate this signal with an envelope detector? Could you set it to 1.5? Why or why not?

The condition for envelope detection is $A + m(t) \geq 0$. If we set $A = 2$, then $\mu = \frac{m_p}{A} = 0.5$ This upholds the condition stated, and we can sucessfully demodulate the signal. If we set $A = 2/3$ we will have the modulation index of 1.5, but then the condition will no longer hold. See figure 12 for an illustration.

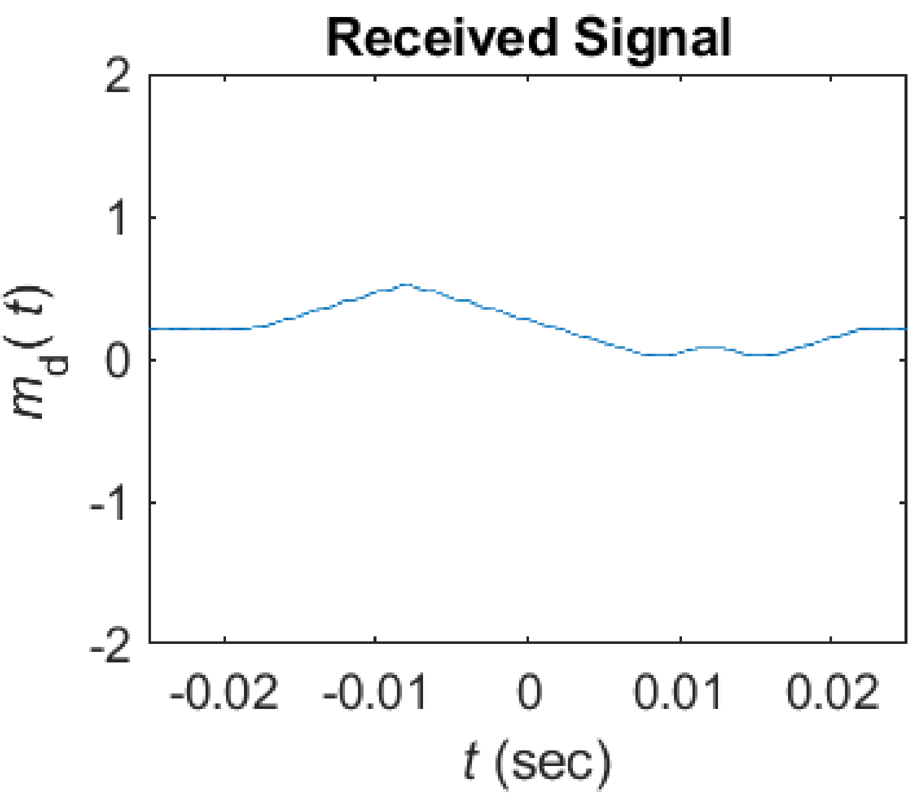

Figure 12: Demodulation with modulation index set to 1.5. Note that because $A + m(t) < 1$, there is distortion as the signal dips below DC.

c) How is the envelope detector different from the coherent filter? What would happen, if you demodulated the AM-sC signal with an envelope detector?

An envelope detector utilizes a discharging capacitor to "track" the incoming AM signal peaks and will discharge as the signal drops below the max peak m_p. This will create a sawtooth texture to the output signal, which will ripple at the carrier frequency. If we demodulate the DSB-SC we won't have the ripple. Otherwise, the coherent filter will remove the carrier from the rectified incoming signal to reproduce the original signal.

Part 2 - Frequency Modulation

This portion of the lab will illustrate the differences between PM and FM modulation techniques, using our original triangle wave signal for the message.

2.1 Frequency Modulated Signal

FM will have a carrier frequency, and will shift in frequency over time as the message signal is modulated. The amplitude of the message signal will shift the frequency by a factor, which give the modulated signal an "accordian" appearance. Figure 13 shows the signal in the time domain. Note that from $-0.2 \leq t \leq 0$ the cosine wave is more compressed and then it decompresses from $0 \leq t \leq 0.02$; this corresponds to the positive and negative amplitudes of the message signal at those respective times. Also notice the spectrum in Figure 14 contains a carrier frequency component at 300Hz with the message signal components being manifest in the left and right lobes.

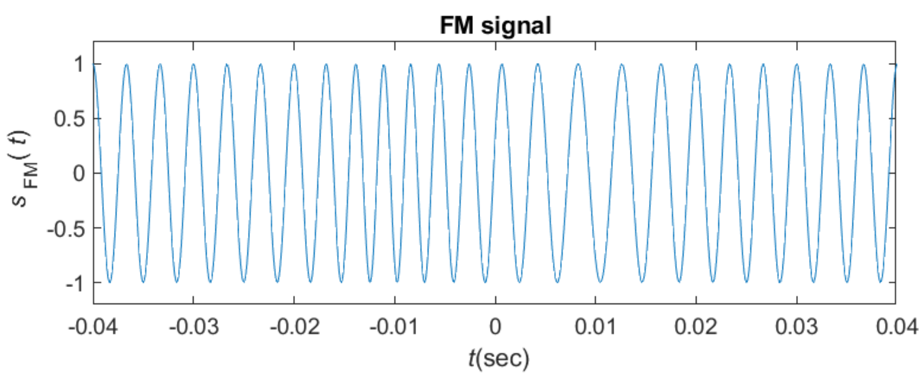

Figure 13: Plot of our FM signal versus time.

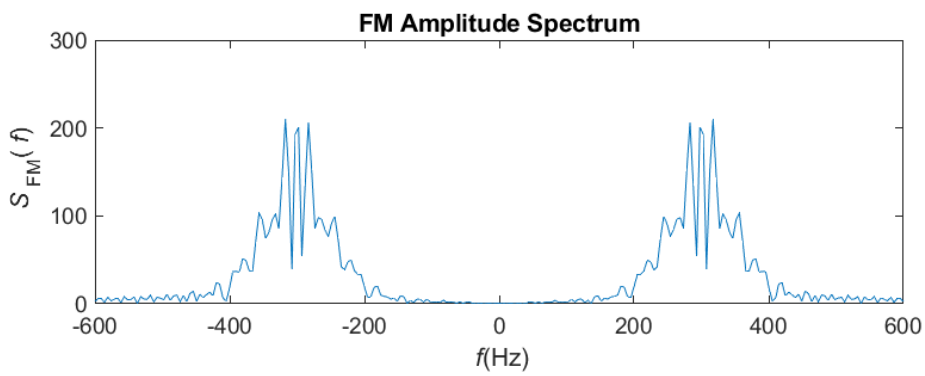

Figure 14: Plot of our FM signal spectrum.

2.2 Phase Modulated Signal

PM modulation is different from FM, in the sense that signal message amplitude creates variation in the phase of the carrier rather than frequency. It is harder to notice in the time domain for our case in Figure 15, but we can see a spectrum difference in Figure 16.

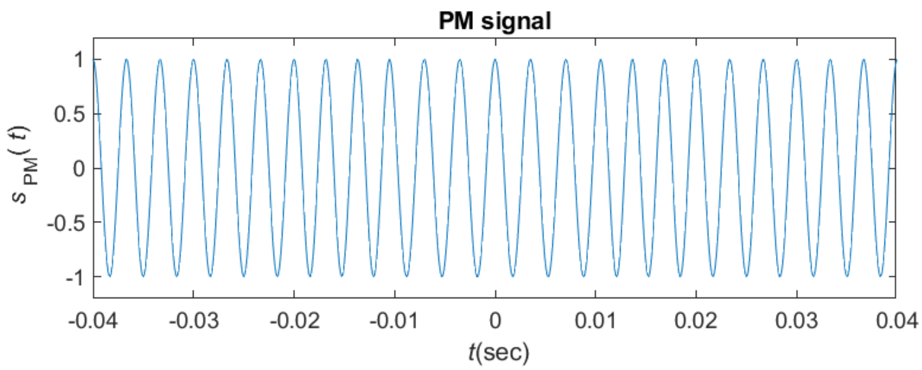

Figure 15: Plot of our PM signal versus time.

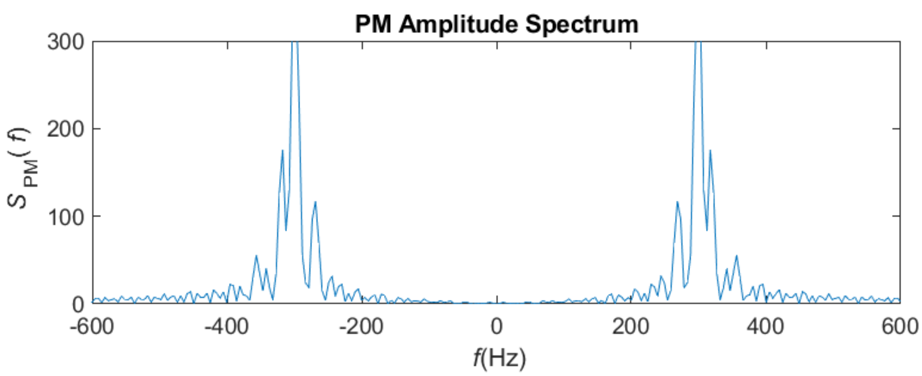

Figure 16: Plot of our PM Signal Spectrum.

2.3 Demodulated FM Signal

When we demodulate an FM signal we won't get the exact replica of the original message, but we can extract it. Notice that the peak of the message signal has a magnitude of 1.5 and is slightly offset to the right from $t = -0.01$ where the peak originally was. This is due to time delay from the filter in the demodulation process. Also note that there is some distortion at $t = 0$; we will discuss this in a following section.

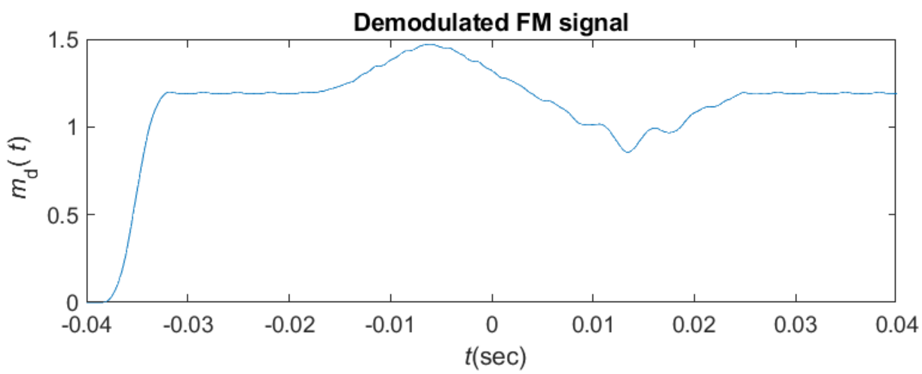

Figure 17: Plot of our demodulated FM signal versus time, with distortion starting at $t = 0$.

2.4 FM Questions

a) Judging from the spectral plots of the FM and PM signals, what would you say is the bandwidth of each? Does this compare to the theoretical bandwidth estimates determined by $B_y = 2(\Delta F + B_x)$? (Use 25Hz as an estimate of Bx).

The PM signal has a much narrower bandwidth compared to FM that we see in figure 14 and 16. Recall that the max frequency deviation for the PM and FM signals are calculated respectively as:

$$\Delta F_{PM} = \frac{1}{2\pi} k_p max[\frac{d}{dt}x(t)]$$

$$\Delta F_{PM} = \frac{1}{2\pi}\pi 100 = \boxed{\text{200Hz}}$$

$$\Delta F_{FM} = \frac{1}{2\pi} k_f max[x(t)]$$

$$\Delta F_{FM} = \frac{1}{2\pi} 80(1) = \boxed{\text{50.9Hz}}$$

Note that $x(t)$ is our triangle wave message signal. The max value of it is 1, and the max derivative of the value is 100. This does not line up with our theoretical values, meaning we have an issue with our FM modulation.

b) What is happening in the FM demodulator? (Explain this demodulator.)

We are using a high order lowpass filter which has a longer response time and delay. Also notice that our message signal goes negative at $t = 0$, where the distortion is of our demodulated FM signal. If you look at the time domain FM modulated signal you will notice a large stretch right when the message goes negative ($0 \leq t \leq 0.02$). That huge stretch is distorting the received signal and adding extra bandwidth, and is a visible way of seeing bandwidth deviation in the time domain. This can be fixed by increasing the FM carrier frequency to a higher value.

Part 3 - Try It Yourself

From the last section we noted that the FM modulation experienced distortion. This section discusses the effects of changing the FM coefficient k_f by a factor of two, and the resulting modulation/demodulation.

3.1 Increasing FM Coefficient

a) What happens to the modulated signal in the time domain? Doubling the coefficient caused even more distortion in the time domain at the zero crossing timestamp of our original message signal, specifically from $0 \leq t \leq 0.02$. Note the distortion that you see in figure 18

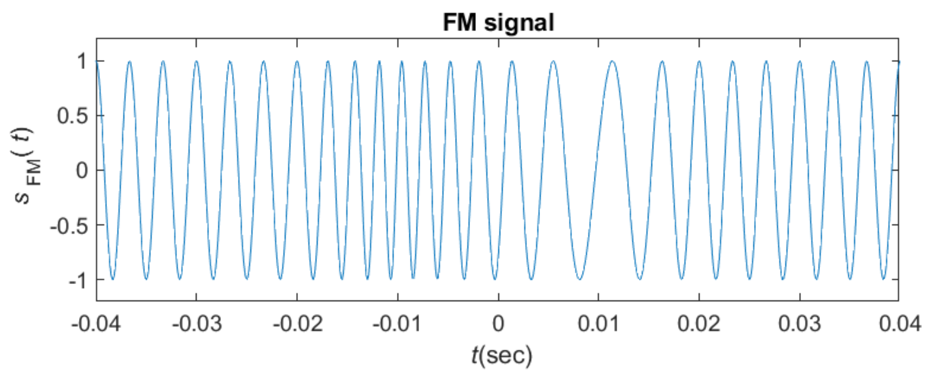

Figure 18: Plot of our modulated FM signal versus time, with k_f doubled. Note the more pronounced distortion after t=0.

b) What happens to the signal spectrum? In part two, the bandwidth calculations were outlined. If we double k_f, we will also end up doubling our bandwidth of the modulated FM signal. Figure 19 illustrates the even more pronounced distortion from performing this operation.

3.2 Demodulating New Signals

Creating additional distortion does not make it any easier to demodulate the signal. if we reduce the FM coefficient by 2 we are able to get a better replica of the original figure. I changed the coefficient to $k_f = 0.5$ and was able to produce the following demodulated message in figure 20. Note that the distortion is no longer as pronounced as the previous demodulated FM signals. There is still a bit of distortion with this coefficient, but it mitigates the demodulation issues.

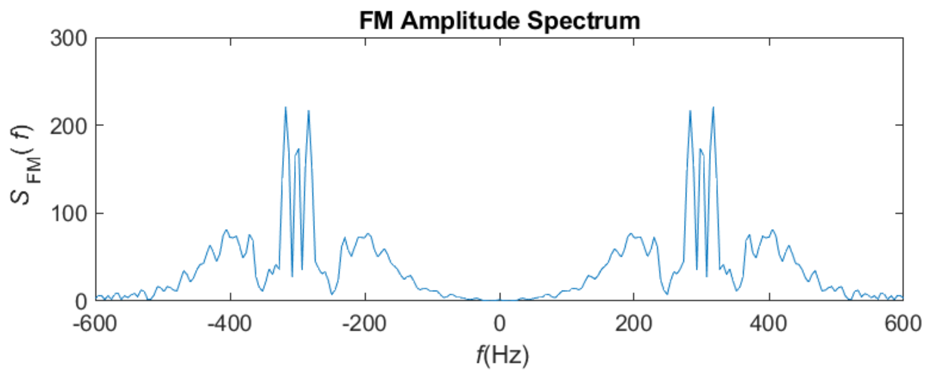

Figure 19: Plot of our demodulated FM signal spectrum, with a doubled coefficient k_f. Notice the bandwidth also doubles.

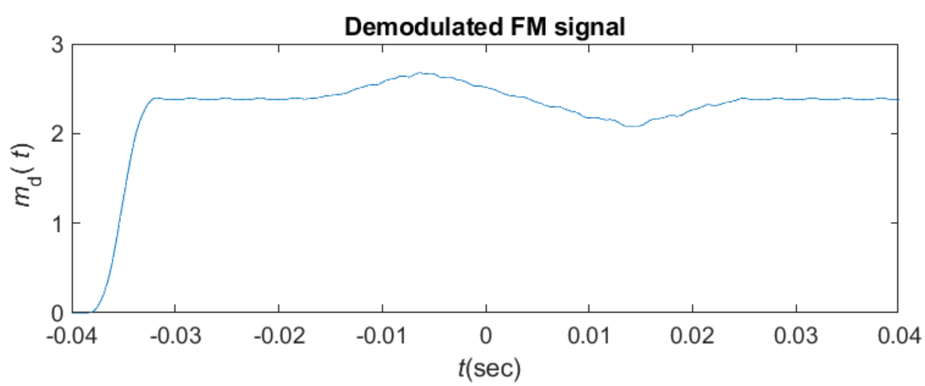

Figure 20: Plot of our demodulated FM signal versus time, with a halved coefficient $k_f = 0.5$. Notice the amplitude of the signal reaches 3 rather than 1.5 like the original demodulated signal.

Conclusion

This lab covered the basics of analog amplitude and frequency modulation techniques. For analog modulation, one of the primary differences that were illustrated was the carrier frequency that was included in AM. The demodulating techniques for amplitude modulation vary depending on the carrier that is attached, and differences were discussed. Examples of frequency modulation were done to illustrate differences in timing and bandwidth of our sent signal, and distortion was intentionally added to show the importance of the PM and FM coefficients for proper demodulation.

Matlab Code

triangl.m

```
% (file name: triangl.m)
% The triangle function is a function of time 't'.
%
% triangl(t) = 1-|t|, if |t| < 1
% triangl(t) = 0    , if |t| > 1
%
% Usage y = triangl(t)
% t - must be real-valued and can be a vector or a matrix

function y=triangl(t)
    y = (1-abs(t)).*(t>=-1).*(t<1);
end
```

ExampleDSBdemfilt.m

```
% (ExampleDSBdemfilt.m)
% This program uses triangl.m to illustrate DSB modulation
% and demodulation

ts = 1.e-4;

t=-0.04:ts:0.04;
Ta = 0.01;
m_sig=triangl((t+0.01)/0.01)-triangl((t-0.01)/0.01);
Lm_sig=length(m_sig);
Lfft=length(t); Lfft=2^ceil(log2(Lfft));
M_fre=fftshift(fft(m_sig,Lfft));
freqm=(-Lfft/2:Lfft/2-1)/(Lfft*ts);
B_m=150; %Bandwidth of the signal is B_m Hz.
h=fir1(40,[B_m*ts]);

t=-0.04:ts:0.04;
Ta=0.01;fc=300;
s_dsb=m_sig.*cos(2*pi*fc*t);
Lfft=length(t); Lfft=2^ceil(log2(Lfft)+1);
S_dsb=fftshift(fft(s_dsb,Lfft));
freqs=(-Lfft/2:Lfft/2-1)/(Lfft*ts);

% Demodulation begins by multiplying with carrier
s_dem=s_dsb.*cos(2*pi*fc*t)*2;
S_dem=fftshift(fft(s_dem,Lfft));
```

```
% Using an ideal LPF with bandwidth 150 Hz
s_rec=filter(h,1,s_dem);
S_rec=fftshift(fft(s_rec,Lfft));

Trange=[-0.025 0.025 -2 2];

% Plots
figure(1)
subplot(221);td1=plot(t,m_sig);
axis(Trange);set(td1,'Linewidth',1.5);
xlabel('{\it t} (sec)'); ylabel('{\it m}({\it t})');
title('message signal');

subplot(222); td2=plot(t,s_dsb);
axis(Trange); set(td2,'Linewidth',1.5);
xlabel('{\it t} (sec)'); ylabel('{\it s}_{\rm DSB}({\it t
    })')
title('DSB-SC modulated signal');

subplot(223); td3=plot(t,s_dem);
axis(Trange); set(td3,'Linewidth',1.5);
xlabel('{\it t} (sec)'); ylabel('{\it e}({\it t})')
title('{\it e}({\it t})');

subplot(224); td4=plot(t,s_rec);
axis(Trange); set(td4,'Linewidth',1.5);
xlabel('{\it t} (sec)'); ylabel('{\it m}_d({\it t})')
title('Recovered Signal');

Frange=[-700 700 0 200];
figure(2)
subplot(221);fd1=plot(freqm,abs(M_fre));
axis(Frange); set(fd1,'Linewidth',1.5);
xlabel('{\it f} (Hz)'); ylabel('{\it M}({\it f})')
title('Message Spectrum');

subplot(222);fd2=plot(freqs,abs(S_dsb));
axis(Frange); set(fd2,'Linewidth',1.5);
xlabel('{\it f} (Hz)'); ylabel('{\it S}_{rm DSB}({\it f})
    ')
title('DSB-SC spectrum');

subplot(223);fd3=plot(freqs,abs(S_dem));
axis(Frange); set(fd3,'Linewidth',1.5);
xlabel('{\it f} (Hz)'); ylabel('{\it E}({\it f})')
```

```
title('Spectrum of {\it e}({\it f})');

subplot(224);fd4=plot(freqs,abs(S_rec));
axis(Frange); set(fd4,'Linewidth',1.5);
xlabel('{\it f} (Hz)'); ylabel('{\it M}_d({\it f})')
title('Recovered spectrum');
% m_sig=triplesinc(t,Ta);
% Lfft=length(t);   Lfft=2^ceil(log2(Lfft));
% M_fre=fftshift(fft(m_sig,Lfft));
% freqm(-Lfft
```

ExampleAMdemfilt.m

```
% (ExampleAMdemfilt.m)
% This program uses triangl.m to illustrate AM modulation and demodulation

ts=1.e-4;
t=-0.04:ts:0.04;
Ta=0.01;fc=500;
m_sig=triangl((t+0.01)/0.01)-triangl((t-0.01)/0.01);
Lm_sig=length(m_sig);
Lfft=length(t); Lfft=2^ceil(log2(Lfft));
M_sig=fftshift(fft(m_sig,Lfft));
freqm=(-Lfft/2:Lfft/2-1)/(Lfft*ts);
B_m=150;%Bandwidth of the signal is B_m Hz.
h=fir1(40,[B_m*ts]); % defining a FIR filter of order 40 and cutoff frequency B_m*ts

% AM signal generated by adding a carrier to DSB-SC
s_am=(2/3+m_sig).*cos(2*pi*fc*t);
Lfft=length(t);
Lfft=2^ceil(log2(Lfft)+1);
S_am=fftshift(fft(s_am,Lfft));
freqs=(-Lfft/2:Lfft/2-1)/(Lfft*ts);


% AM Demodulation begins by using a rectifier
s_dem=s_am.*(s_am>0);
S_dem=fftshift(fft(s_dem,Lfft));

% Using an ideal low pass filter with bandwidth 150 Hz
s_rec=filter(h,1,s_dem);
S_rec=fftshift(fft(s_rec,Lfft)); % Demodulatede signal in frequency domain

```

```
Trange=[-0.025 0.025 -2 2];

figure(1);
subplot(221); td1=plot(t,m_sig);
axis(Trange); set(td1, 'Linewidth',1.5);
xlabel('{\it t} (sec)'); ylabel('{\it m}({\it t})');
title('Message Signal');

subplot(222); td2=plot(t,s_am);
axis(Trange); set(td1, 'Linewidth',1.5);
xlabel('{\it t} (sec)'); ylabel('{\it s}_{\rm DSB}({\it t})');
title('AM Modulated Signal')

subplot(223); td3=plot(t, s_dem);
axis(Trange); set(td1, 'Linewidth',1.5);
xlabel('{\it t} (sec)'); ylabel('{\it e}({\it t})');
title('Rectified Signal without Local Carrier');

subplot(224); td4=plot(t,s_rec);
axis(Trange); set(td1, 'Linewidth',1.5);
xlabel('{\it t} (sec)'); ylabel('{\it m}_d({\it t})');
title('Received Signal');


figure(2);
subplot(221); plot(freqm,abs(M_sig))
axis(Frange);
xlabel('{\it f} (Hz)'); ylabel('{\it M}({\it f})');
title('Message Spectrum');

subplot(222); plot(freqs,abs(S_am))
axis(Frange)
xlabel('{\it f} (Hz)'); ylabel('{\it S}_{rm AM}({\it f})');
title('AM Spectrum')

subplot(223); plot(freqs, abs(S_dem))
axis(Frange)
xlabel('{\it f} (Hz)'); ylabel('{\it E}({\it f})');
title('Rectified Spectrum')

subplot(224); plot(freqs,abs(S_rec))
axis(Frange)
```

```
xlabel('{\it f} (Hz)'); ylabel('{\it M}_d({\it f})');
title('Recovered Spectrum')
```

ExampleFM.m

```
% (ExampleFM.m)
% This program uses triangl.m to illustrate frequency modulation
% and demodulation

ts=1e-4; % sampling interval
t=-0.04:ts:0.04;
Ta=0.01;

% Use triangl function to generate message signal
m_sig=triangl((t+0.01)/Ta)-triangl((t-0.01)/Ta);

%Lm_sig=length(m_sig1);
Lfft=length(t);   % defining DFT (or FFT) size
Lfft=2^ceil(log2(Lfft));  % making Lfft a power of 2 since this makes the fft algorithm work fast
M_fre=fftshift(fft(m_sig,Lfft));  % i.e. calculating the frequency domain message signal,
                                  % fft algorithm calculates points from ) to Lfft-1, hence we use fftshift on this
                                  % result to order samples from -Lfft/2 to Lfft/2 -1
freqm=(-Lfft/2:Lfft/2-1)/(Lfft*ts); % Defining the frequency axis for the frequency domain DSB modulated signal
B_m=100; % bandwidth of the signal is B_m Hz
h=fir1(80,[B_m*ts]); % defining a simple lowpass filter with bandwidth B_m Hz

kf=160*pi/2;
m_intg=kf*ts*cumsum(m_sig);
s_fm=cos(2*pi*300*t+m_intg);
s_pm=cos(2*pi*300*t+m_sig);
Lfft=length(t);   % defining DFT (or FFT) size
Lfft=2^ceil(log2(Lfft)+1);  % increasing Lfft by factor of 2

```

```
S_fm=fftshift(fft(s_fm,Lfft)); % obtaining frequency domain modulated signal
S_pm=fftshift(fft(s_pm,Lfft)); % obtaining frequency domain modulated signal
freqs=(-Lfft/2:Lfft/2-1)/(Lfft*ts); % Defining the frequency axis for the frequency domain DSB modulated signal


%%% Demodulation
% Using an ideal low pass filter with bandwidth 200 Hz
s_fmdem=diff([s_fm(1) s_fm])/ts/kf;
s_fmrec=s_fmdem.*(s_fmdem>0);
s_dec=filter(h,1,s_fmrec);


Trange=[-0.04 0.04 -1.2 1.2]; % axis ranges for signal, this specifies the range of axis for the plot, the first two parameters are range limits for x-axis, and last two parameters are for y-axis

Frange=[-600 600 0 300]; % axis range for frequency domain plots

figure(1)
subplot(211); m1=plot(t,m_sig);
axis(Trange) % set x-axis and y-axis limits
xlabel('{\it t}(sec)'); ylabel('{\it m}({\it t})')
title('Message signal')

subplot(212); m2=plot(t,s_dec);
xlabel('{\it t}(sec)'); ylabel('{\it m}_d({\it t})')
title('Demodulated FM signal')

figure(2)
subplot(211); td1=plot(t,s_fm);
axis(Trange) % set x-axis and y-axis limits
title('FM signal')
xlabel('{\it t}(sec)'); ylabel('{\it s}_{\rm FM}({\it t})')


subplot(212); td1=plot(t,s_pm);
axis(Trange) % set x-axis and y-axis limits
title('PM signal')
xlabel('{\it t}(sec)'); ylabel('{\it s}_{\rm PM}({\it t})
```

```
')




figure(3)
subplot(211); fp1=plot(t,s_fmdem);
%axis(Trange) % set x-axis and y-axis limits
xlabel('{\it t}(sec)'); ylabel('{\it d s}_{\rm FM}({\it t})')
title('FM Derivative')

subplot(212); fp2=plot(t,s_fmrec);
%axis(Trange) % set x-axis and y-axis limits
xlabel('{\it t}(sec)'); %ylabel('{\it d}_{\rm PM}({\it t})')
title('rectified FM Derivative')



figure(4)

subplot(211); fd1=plot(freqs,abs(S_fm));
axis(Frange) % set x-axis and y-axis limits
xlabel('{\it f}(Hz)'); ylabel('{\it S}_{\rm FM}({\it f})')
title('FM Amplitude Spectrum')

subplot(212); fd2=plot(freqs,abs(S_pm));
axis(Frange) % set x-axis and y-axis limits
xlabel('{\it f}(Hz)'); ylabel('{\it S}_{\rm PM}({\it f})')
title('PM Amplitude Spectrum')
```