Module 6 - Homework 6

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Problem 5.1-2

Figure p5.1-2 shows Fourier spectra of signals $g_1(t)$ and $g_2(t)$. Determine the Nyquist sampling rate for signals $g_1(t), g_1^2(t), g_2^m(t),$ and $g_1(t)g_2(t)$.

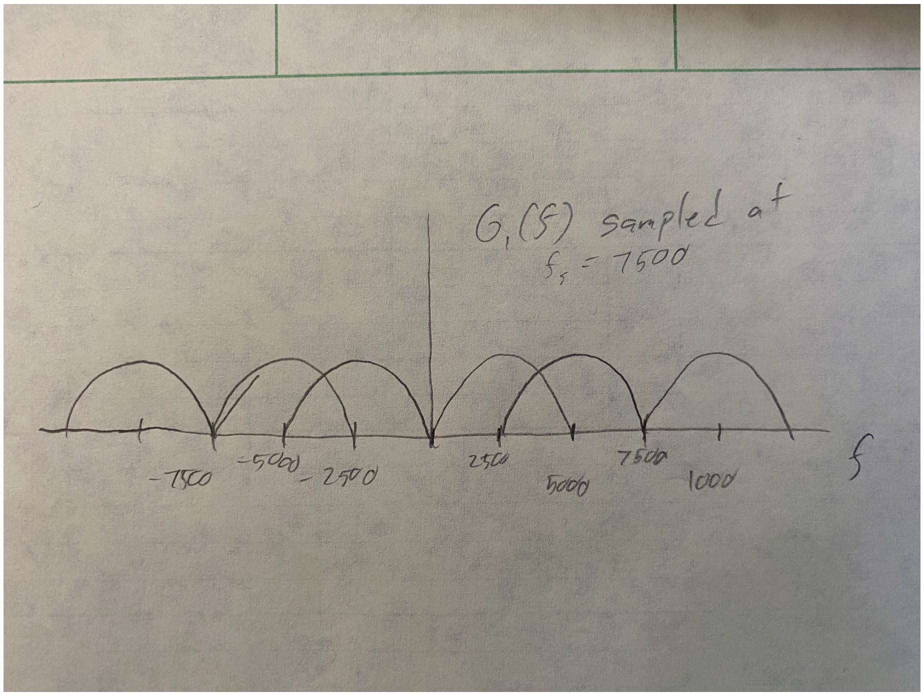
$G_1(f)$ is a baseband signal with bandwidth of 5000Hz, and $G_2(f)$ is also baseband with a bandwidth of 12000Hz. Remember that the convolution property is when you convolve two signals, the resulting signal is the sum of their individual bandwidths. Also note that we need to sample at twice the bandwidth to be able to reconstruct the signal. With that info, we get the following bandwidths:

1. $g_1(t) \rightarrow$ Bandwidth: $5,000Hz \rightarrow$ Nyquist: $\boxed{10,000Hz}$
2. $g_1^2(t) \rightarrow$ Bandwidth: $10,000Hz \rightarrow$ Nyquist: $\boxed{20,000Hz}$
3. $g_2^m(t) \rightarrow$ Bandwidth: $m * 12,000Hz \rightarrow$ Nyquist: $\boxed{m * 24,000Hz}$
4. $g_1(t)g_2(t) \rightarrow$ Bandwidth: $17,000Hz \rightarrow$ Nyquist: $\boxed{34,000Hz}$

Problem 5.1-3a

For the $G_1(f)$ in figure P5.1-2, find and sketch the spectrum of its ideally and uniformly sampled signals at the sampling rate of $f_s = 7500$

Recall that the bandwidth of this signal is 5000Hz. Since we are not sampling at the Nyquist rate we will get some aliasing that looks like this:



Notice the overlap between 2500-5000. This is what aliasing looks like on the spectrum when we undersample.

Problem 5.2-3

In a satellite radio system, 500 stations of stereo quality are to be multiplexed in one data stream. For each station, two (left and right) signal channels each of bandwidth 15,000Hz are sampled, quantized and binary-coded into PCM signals.

(a)

If the maximum acceptable quantization error in sample amplitudes is 1% of the peak signal voltage, find the minimum number of bits needed for a uniform quantizer.

In order to keep the quantization error under 1% of the peak voltage m_p, the following condition must hold:

$$\Delta v = \frac{2m_p}{L} \leq \frac{1}{100} m_p$$

This condition is satisfied when $L \geq 100$, so rounding this up to the nearest power of two would give us $\boxed{L = 128}$

(b)

If the sampling rate must be 8% higher than the Nyquist rate, find the minimum bit rate of the multiplexed data stream based on the quantizer of part (a)

Nyquist rate would be at 30KHz, so 8% higher would put our sample rate at 32400 samples/second. We are quantizing our samples by $L = 128$, which requires 8 bits per sample. Our bit rate would be:

$$\frac{32400 \text{ samples}}{\text{second}} \frac{8 \text{ bits}}{\text{sample}} = \boxed{\frac{259200 \text{ bits}}{\text{second}}}$$

(c)

If 2% more bits are added to the multiplexed data for error protection and synchronization determine the minimum bandwidth needed to transmit the final data stream to receivers.

Adding 2% more bits to the data rate would then be $1.02 \times 259200 = 264384$ bits/s Our multiplexed signal will require a total bitrate of $500 \times 264384 = 132.192$ Mbits/s. The minimum bandwidth needed for these stations would be half the bitrate: $132.192 \times 10^6/2 = \boxed{66.096 \text{ MHz}}$

Matlab Code