Module 9 - Homework 9

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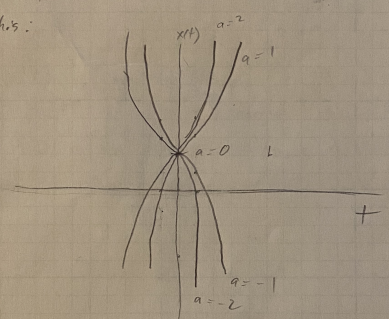
8.2-4

a.) Sketch the ensemble of the random process

$$X(t) = a t^2 + b$$

where b is a constant and a is an RV uniformly distributed in the range $(-2, 2)$

This is a parabolic function that can resemble something like this:



b.) This is a non-stationary process because the distribution of $x(t)$ is different depending on a. For example, if $a=-2$, you get $x(t) \in [-\infty, b]$ but with $a=2$, $x(t) \in [b, \infty]$ and $a=0$, $x(t) \in [0]$.

8.3-2 Recall that the autocorrelation function of a stationary process must satisfy $R_X^*(\tau) = R_X(-\tau)$. Determine which of the following functions can be a valid autocorrelation function of a stationary random process.

a.) $(\tau^2 + 9)^{-1}$ $\boxed{\text{valid}}$ because $\tau^2 = (-\tau)^2$

b.) $(\tau^2 - 1)^{-1}$ $\boxed{\text{valid}}$ because $\tau^2 = (-\tau)^2$

c.) $e^{-\tau} u(\tau)$ $\boxed{\text{invalid}}$ because $e^{-\tau} \neq e^{\tau}$

d.) $N\delta(\tau) + (\tau^2 + 4)^{-1}$ $\boxed{\text{invalid}}$ because $\delta(\tau) \neq \delta(-\tau)$

e.) $\sin(\omega_0 \tau)$ $\boxed{\text{invalid}}$ since $\sin(\omega_0 \tau) \neq \sin(-\omega_0 \tau)$

f.) $\cos(\omega_0 \tau)$ $\boxed{\text{valid}}$ since $\cos(-\omega_0 \tau) = \cos(\omega_0 \tau)$

8.4-2) Two random processes $x(t)$ and $y(t)$ are

$$x(t) = A\cos(\omega_0 t - \varphi) \quad \text{and} \quad y(t) = B\sin(n\omega_0 t + n\varphi + \psi)$$

where n = integer $\neq 0$ and A, B, ψ and ω_0 are constants, and φ is an RV uniformly distributed in the range $(0, 2\pi)$.

Show that the two processes are incoherent.

We need to show that the cross correlation between the two is:

$$R_{xy}(\tau) = 0$$

Remember that a cross correlation is zero when they are non-overlapping in time

$$R_{xy}(\tau) \int_{-\infty}^{\infty} A\cos(\omega_0 t - \varphi)\, B\sin(n\omega_0(t+\tau) + n\varphi + \psi)\, dt$$

~~Using Euler's, we get~~

$$\cancel{AB \int_{-\infty}^{\infty} \left(\frac{1}{2}e^{(\omega_0 t - \varphi)jt} + \frac{1}{2}e^{-(\omega_0 t - \varphi)jt}\right)\left(\frac{1}{2j}e^{(\ldots)jt} - \frac{1}{2j}e^{-(\ldots)jt}\right) dt}$$

~~let $a = \omega_0 t - \varphi$ and $b = n\omega_0(t+\tau) + n\varphi + \psi$~~
~~$= n\omega_0 t + n\varphi + n\omega_0\tau + \psi$~~
~~$= an + n\omega_0\tau + \psi$~~

$$\cancel{R_{xy} = \frac{1}{4j}\int \left(e^{aj t} + e^{-aj t}\right)\left(e^{bj t} - e^{-bj t}\right) dt}$$

$$\cancel{= \frac{1}{4}\int e^{aj t}e^{bj t} - e^{aj t}e^{-bj t} + e^{-aj t}e^{bj t} - e^{-aj t}e^{-bj t}\, dt}$$

$$\cancel{= \frac{1}{4}\int e^{(a+b)jt} - e^{(a-b)jt} - e^{(-a-b)jt} + e^{(b-a)jt}\, dt}$$

~~Solve the exponential components:~~

$$\cancel{a+b = a(n+1) + n\omega_0\tau + \psi \qquad -a-b = a(n+1) - n\omega_0\tau - \psi}$$

$$\cancel{a-b = a(1-n) - n\omega_0\tau - \psi \qquad b-a = a(n-1) + n\omega_0\tau - \psi}$$

~~note that $a+b+(-a-b) = 0$, and $(a-b)+(b-a) = 0$~~

We actually can find the cross-correlation with the product of means

$$E[x(t)] = A\,\overline{\cos(\omega_0 t + \varphi)}$$

density function

$$= A\int_0^{2\pi} \cos(\omega_c t + \theta)\, P_\varphi(\theta)\, d\theta$$

$$P_\varphi(\theta) = \begin{cases} 1/2\pi, & 0 < \theta < 2\pi \\ 0 & \text{else} \end{cases}$$

$$= A\int_0^{2\pi} \cos(\omega_c t + \theta)\, \frac{1}{2\pi}\, d\theta$$

$$= \frac{A}{2\pi}\int_0^{2\pi} \cos(\omega_c t + \theta)\, d\theta = 0$$

$$E[y(t+\tau)] = B\,\overline{\sin(n\omega_0(t+\tau) + n\varphi + \psi)}$$

$$= B\int_0^{2\pi} \sin(n\omega_0(t+\tau) + n\theta + \psi)\, P_\varphi(\theta)\, d\theta$$

$$= \frac{B}{2\pi}\int_0^{2\pi} \sin(n\omega_0(t+\tau) + n\theta + \psi)\, d\theta = 0$$

$$\therefore \boxed{R_{xy}(\tau) = \overline{x(t)}\;\overline{y(t+\tau)} = 0}$$

Matlab Code