Module 11 - Homework 11

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Problem 10.1-2

Consider a fast hopping binary ASK system. The AWGN spectrum equals $S_n(f) = 10^{-6}$, and the binary signal amplitudes are 0 and 2V respectively. The ASK uses a data rate of 100kbit/s and is detected noncoherently. The ASK requires 100kHz bandwidth for transmission. However, the frequency hopping is over 12 equal ASK bands with bandwidth totaling 1.2 MHz. The partial band jammer can generate a strong Gaussian noise-like interference with total power of 26 dBW.

(a)

If a partial band jammer randomly jams one of the 12 FH channels, derive the BER of the FH-ASK if the ASK signal hops 6 bands per bit period.

First we need to calculate the BER under normal circumstances We are given $S_n(f) = 10^{-6}$ which converts to $10log_10(10^{-6}) = -60dBW$ Thus...

Out of time. See Example 10.1 for details on how to calculate this. Use:

$$\frac{E_b}{I} * L$$

Where L is the number of channels being jammed.

(b)

If a partial band jammer randomly jams two of the 12 FH channels, derive the BER of the FH-ASK if the ASK signal hops 6 bands per bit period.

See the last problem with $L = 2$ instead.

(c)

If a partial band jammer jams all 12 FH channels, derive the BER of the FH-ASK if the ASK signal hops 6 bands per bit period.

Could either try $L = 12$ or just treat the jamming power as noise.

Problem 12.1-1

A message source generates one of five messages randomly every 0.2 microsecond. The probabilities of these messages are 0.4, 0.3,0.2, 0.05 and 0.05. Each emitted message is independent of the other messages in the sequence.

(a)

What is the source entropy?

$$H(m) = \sum_{i=1}^{5} P_i log_2(1/P_i)$$

$$H(m) = 0.4log_2(1/0.4) + 0.3log_2(1/0.3) + ...$$

$$H(m) = 0.5288 + 0.5211 + 0.4644 + 0.2161 + 0.2161$$

$$\boxed{H(m) = 1.9465}$$

(b)

What is the rate of information generated by this source (in bits per second)?

A message is sent with a period of 0.2 microseconds, which means we are sending 5 million messages per minute. With our entropy calculated, that would be an estimated $\boxed{9.73MB/s}$

Problem 12.2-1

A source emits eight messages with probabilities 1/2,1/4,1/8,1/16,1/32,1/64,1/128, and 1/128, respectively. Find the entropy of the source. Obtain the compact binary code and find the average length of the codeword. Determine the efficiency and the redundancy of the code.

The entropy of the source can be calculated as follows:

$$H(m) = \sum_{i=1}^{8} P_i log_2(1/P_i)$$

$$H(m) = 1/2log_2(2) + 1/4log_2(4) + ... + 1/128log_2(128)$$

$$H(m) = 1/2 + 2/4 + 3/8 + 4/16 + 5/32 + 6/64 + 7/128 + 7/128$$

$$\boxed{H(m) \approxeq 2.00}$$

The compact code (Huffman Code) is below in the figure. The average length of the a codeword is:

$$L = \sum_{n}^{i=1} P_i L_i = 1/2(1)+1/4(2)+1/8(3)+1/16(4)+1/32(5)+1/64(6)+1/128(7)+1/128(7)$$

$$\boxed{L \approxeq 2.00}$$

Code efficiency is calculated as:

$$\eta = \frac{H(m)}{L} \approxeq \boxed{1.00}$$

And the redundancy is calculated as:

$$\gamma = 1 - \eta = \boxed{0.00}$$

Problem 13.2-3

Confirm the possibility of a (18,7) binary code that can correct any error pattern of up to three errors. Can this code correct up to four errors?

For our case we have 18 total bits being sent with 7 information bits. We can tell how many errors this coding method can correct by computing the Hamming Bound:

$$2^{n-k} \geq \sum_{j=0}^{t} \binom{n}{j}$$

If we have $t = 3$ errors, we have a minimum distance of $D_{min} = 2t + 1 = 7$ needed to correct any errors. Let's find our Hamming bound:

$$2^{18-7} \geq \binom{18}{0} + \binom{18}{1} + \binom{18}{2} + \binom{18}{3}$$
$$2048 \geq 988$$

The inequality holds, so we can indeed correct up to three errors. But can we correct $t = 4$ errors?

$$2^{11} \geq \binom{18}{0} + \binom{18}{1} + \binom{18}{2} + \binom{18}{3} + \binom{18}{4}$$
$$2048 \geq 4048$$

The inequality is false, so no, we cannot correct 4 errors.

Problem 13.3-4

A generator matrix

$$G = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

generates a (4,2) code.

(a)

Is this a systematic code?

Yes, you can tell because the left end of the matrix is an identity matrix.

(b)

What is the parity check matrix of this code?

Remember that when we send a codeword c, constructed such that c=dG, we need a parity check matrix on the receiver end. Let H be the parity check matrix such that $H = [P^T I_{n-k}]$. We get:

$$H = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

If r is a vector of a possible received codeword we can solve rH^T and we should get a zero matrix.

(c)

Find the codewords for all possible input bits.

For $d = [00]$

$$c = dG = [0000]$$

For $d = [01]$

$$c = dG = [0111]$$

For $d = [10]$

$$c = dG = [1010]$$

For $d = [11]$

$$c = dG = [1101]$$

(d)

Determine the minimum distance of the code and the number of bit errors this code can correct.

The minimum distance is $D_{min} = 2t - 1$. We can find t in the same way we did for the last problem where $n = 4$ and $k = 2$.

$$2^{4-2} \geq \sum_{j=1}^{t} \binom{4}{j}$$

$$4 \geq \binom{4}{0}$$

$$4 \geq 1$$

We get $\boxed{t = 1}$ to satisfy the conditional, which is the number of bit errors we can correct. Our minimum distance is $\boxed{D_{min} = 3}$.