Griffiths Electrodynamics: Problem 5.17

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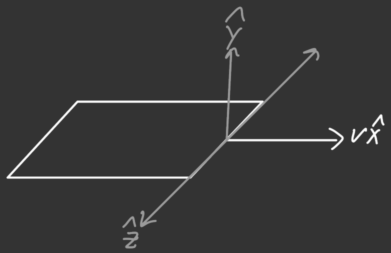
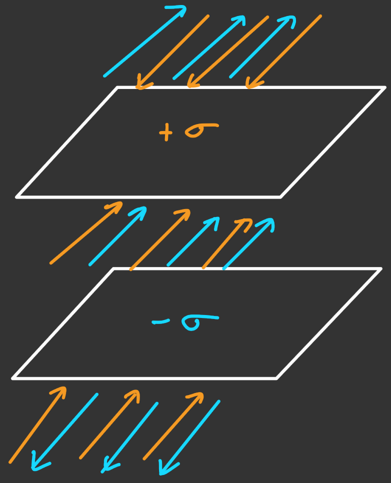
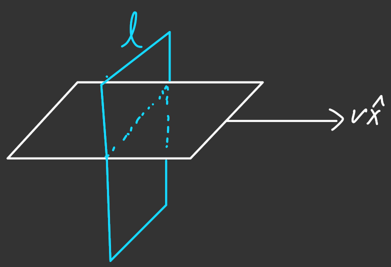
What we have here is some σ that is moving at some velocity v. Which fits the definition of a surface current, $\mathbf{K} = \sigma\mathbf{v}$. Let's arbitrarily choose the direction $\mathbf{v} = v\hat{x}$ in accordance with the figure below.



We know (from example 5.8) the direction of $\mathbf{B}$ above the plate is $\hat{z}$ and below the plate $-\hat{z}$ for plate $+\sigma$. When we add plate $-\sigma$ and super impose the magnetic fields, we see that they cancel outside the plates and are constructive between the plates.



To calculate the field between the plates, we can just calculate the contribution from one plate and double it. Let's look at the $+\sigma$ plate. We can leverage Ampere's law with an Amperage surface as shown below.



Here, out I_{enc} is

$$\begin{aligned} I_{\text{enc}} &= \int K \, dl \\ &= \int \sigma v \, dl \\ &= \sigma v l. \end{aligned}$$

Ampere's law for the region for one side of the plate becomes,

$$\begin{aligned} \oint \mathbf{B} \cdot d\mathbf{l} &= \mu_0 I_{\text{enc}} \\ B2l &= \mu_0 \sigma v l \\ B &= \frac{\mu_0 \sigma v}{2}. \end{aligned}$$

There are two equal and constructive magnetic fields in the region between the plates pointing in the $-z$ direction, thus,

$$\mathbf{B} = \mu_0 \sigma v (-\hat{z})$$

and $\mathbf{B} = 0$ elsewhere because $\mathbf{B}$ does not depend on distance for this system.

Part B

The Loretnz force law extended to surface currents is

$$\vec{F} = \int (\vec{K} \times \vec{B}) da.$$

But remember, the **B** we use is the magnetic field that is causing a force on the upper plate, not the total electric field (the plate can't exert a force on itself). So,

$$\begin{aligned}\vec{F} &= \int \left[(\mu_0 \sigma v \hat{x}) \times \left(\frac{-\mu_0 \sigma v}{2} \hat{z} \right) \right] da \\ &= \frac{\mu_0 \sigma^2 v^2}{2} \hat{y} \int da.\end{aligned}$$

The problem asked for force *per unit area* f, which is convenient because we don't know the area. Thus,

$$\vec{f} = \frac{\mu_0 \sigma^2 v^2}{2} \hat{y}$$

Part C

From Problem 5.13, we expect this to be the speed of light. But let's makes sure. First, we calculate what the electric force is. We expect the force to to be attractive (in the $-y$ direction), because we have opposite surface charges. To get the electric force per unit area f_E, we first he electrical field per unit area. Recall from Chapter 2 the electric field for some plate with surface charge density of σ is $\sigma/2\epsilon_0$. So the electric force per unit area is

$$\begin{aligned}\mathbf{f}_E &= \frac{\sigma}{2\epsilon_0}(-\sigma)\hat{y} \\ &= \frac{\sigma^2}{2\epsilon_0}(-\hat{y}).\end{aligned}$$

Now we equation our magnetic force to this and solving for velocity,

$$\begin{aligned}\left| \frac{\mu_0 \sigma^2 v^2}{2} \hat{y} \right| &= \left| \frac{\sigma^2}{2\epsilon_0}(-\hat{y}) \right| \\ v &= \frac{1}{\sqrt{\epsilon_0 \mu_0}} \\ &= c.\end{aligned}$$