Classical Mechanics: Problem 7.2

Colton Kawamura
https://coltonkawamura.github.io/coltonkawamura/

Last updated: July 28, 2023

Part A

The Lagrangian is defined as

$$\mathcal{L} = T - U \tag{1}$$

We know that the kinetic energy T for a particle with motion in the x direction is $T = \frac{1}{2} m\dot{x}^2$. Now we determine, potential energy U from force F. Choosing some initial position $x_0 = 0$,

$$
\begin{aligned}
U &= -\int F dx \\
&= -\int (-kx) dx \\
&= \frac{kx^2}{2}.
\end{aligned}
$$

Now the Lagrangian is,

$$
\begin{aligned}
\mathcal{L} &= T - U \\
&= \frac{1}{2} m\dot{x}^2 - \frac{kx^2}{2}
\end{aligned} \tag{2}
$$

Part B

The Lagrangian equation is

$$\frac{\partial \mathcal{L}}{\partial x} = \frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{x}}.$$

Inputting our Lagrangian from equation 2,

$$\frac{\partial}{\partial x}\left(\frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2\right) = \frac{d}{dt}\frac{\partial}{\partial \dot{x}}\left(\frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2\right)$$

$$\left(0 - \frac{1}{2}k(2)x\right) = \frac{d}{dt}\left(\frac{1}{2}m(2)\dot{x} - 0\right)$$

$$-kx = m\ddot{x}$$

$$m\ddot{x} + kx = 0$$

This is the same equation of motion using other methods!