Classical Mechanics: Problem 7.6

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Part A

For particle one,

$$\begin{aligned} F_{1x} &= m_1\ddot{x}_1 \\ F_{1y} &= m_1\ddot{y}_1 \\ F_{1x} &= m_1\ddot{z}_1. \end{aligned}$$

For particle two,

$$\begin{aligned} F_{2x} &= m_1\ddot{x}_2 \\ F_{2y} &= m_1\ddot{y}_2 \\ F_{2x} &= m_1\ddot{z}_2. \end{aligned}$$

Part B

In order to create the Lagrange equations, we need to determine kinetic energy K and potential energy U for each particle. Kinetic energy for a particle is

$$K = \frac{1}{2}m_1\dot{r}_1^2 + \frac{1}{2}m_2\dot{r}_2^2.$$

We don't know what the potential is, but that shouldn't stop us from turning U in terms of force F as the problem stated. So our Langragian is,

$$\begin{aligned} \mathcal{L} &= T - U \\ &= \frac{1}{2}m_1\dot{r}_1^2 + \frac{1}{2}m_2\dot{r}_2^2 - U(r_1, r_2) \end{aligned}$$

Our Langrange equation of motions are

$$\frac{\partial \mathcal{L}}{\partial r_1} = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{r_1}}, \qquad \frac{\partial \mathcal{L}}{\partial r_2} = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{r_2}}.$$

Looking at particle one, the right-hand side,

$$\begin{aligned} \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{r_1}} &= \frac{d}{dt}\left[\frac{1}{2}m_1(2\dot{r}_1)\right] \\ &= m\ddot{r}_1. \end{aligned} \tag{1}$$

Now the left-hand side, the only term in $\mathcal{L}$ that varies with r_1 is potential energy. Since we know that

$$\frac{\partial \mathcal{L}}{\partial r_1} = -\nabla U(r_1).$$

Now,

$$\begin{aligned} -\nabla U(r_1) &= m\ddot{r}_1 \\ -\frac{\partial U}{\partial x}\hat{x} - \frac{\partial U}{\partial y}\hat{y} - \frac{\partial U}{\partial z}\hat{z} &= m(\hat{x} + \hat{y} + \hat{z}) \end{aligned} \tag{2}$$

But we know that

$$-\nabla U(r_1) = F(r_1)$$

so by inspection, equation 2 becomes,

$$\begin{aligned} F_{1x} &= m_1\ddot{x}_1 \\ F_{1y} &= m_1\ddot{y}_1 \\ F_{1x} &= m_1\ddot{z}_1. \end{aligned}$$

The exact same steps follow for particle two.