Griffiths Electrodynamics: Problem 5.24

Colton Kawamura
https://coltonkawamura.github.io/coltonkawamura/

Last updated: September 1, 2023

Part A

We can use Ampere's Law $\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$ to find the current density assuming we can find the magnetic field associated with $\mathbf{A}$, which we can,

$$\begin{aligned}\mathbf{B} &= \nabla \times \mathbf{A} \\ &= \frac{1}{s}\frac{\partial}{\partial s}(sk)\,\hat{\mathbf{z}} \\ &= \frac{k}{s}\hat{\mathbf{z}}.\end{aligned}$$

Now, using Ampere's law,

$$\begin{aligned}\mathbf{J} &= \frac{1}{\mu_0}(\nabla \times \mathbf{B}) \\ &= \frac{1}{\mu_0}\left[-\frac{\partial}{\partial s}\left(\frac{k}{s}\right)\right]\hat{\phi} \\ &= \frac{k}{\mu_0 s^2}\,\hat{\phi}.\end{aligned}$$