Classical Mechanics: Problem 7.2

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Part A

The Lagrangian is defined as

$$\mathcal{L} = T - U \tag{1}$$

We know that the kinetic energy T for a particle with motion in the x direction is $T = \frac{1}{2}\, m\dot{x}^2$. Now we determine, potential energy U from force F. Choosing some initial position $x_0 = 0$,

$$\begin{aligned} U &= -\int F dx \\ &= -\int (-kx) dx \\ &= \frac{kx^2}{2}. \end{aligned}$$

Now the Lagrangian is,

$$\begin{aligned} \mathcal{L} &= T - U \\ &= \frac{1}{2}\, m\dot{x}^2 - \frac{kx^2}{2} \end{aligned} \tag{2}$$

Part B

The Lagrangian equation is

$$\frac{\partial \mathcal{L}}{\partial x} = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{x}}.$$

Inputting our Lagrangian from equation 2,

$$
\begin{aligned}
\frac{\partial}{\partial x}\left(\frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2\right) &= \frac{d}{dt}\frac{\partial}{\partial \dot{x}}\left(\frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2\right) \\
\left(0 - \frac{1}{2}k(2)x\right) &= \frac{d}{dt}\left(\frac{1}{2}m(2)\dot{x} - 0\right) \\
-kx &= m\ddot{x} \\
m\ddot{x} + kx &= 0
\end{aligned}
$$

This is the same equation of motion using other methods!