Quantum Mechanics (Griffiths): Problem 2.1

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Last updated: August 30, 2023

Part A

Let's write out a time-dependent wave function as the instructions say with $E_0 + \Gamma$,

$$\begin{aligned}\Psi(x,t) &= \psi(x)e^{-i(E_0+i\Gamma)t/\hbar}\\ &= \psi(x)e^{\Gamma t/\hbar}e^{-iE_0t/\hbar}\end{aligned}$$

and normalize it,

$$\begin{aligned}\int_{-\infty}^{\infty}|\Psi(x,t)|^2\,dx &= |\psi|^2\,e^{2\Gamma t/\hbar}\\ &= e^{2\Gamma t/\hbar}\int_{-\infty}^{\infty}|\psi|^2dx.\end{aligned}$$

In order for the probability of finding the particle to be 1 for all of time, we have a restriction that $\Gamma = 0$, thus, E must be real.

Part B

Since the time-independent Schrodinger equation is a linear homogeneous differential equation, if there are two solutions to it, the linear combination of those solutions is also a solution. Lets say ψ_1 and ψ_2 are solutions, let's show their combination $\psi_3 = c_1\psi_1 + c_2\psi_2$ is also a solution,

$$\begin{aligned}-\frac{\hbar^2}{2m}\frac{\partial^2\psi_3}{dx^2} + V\psi_3 &= -\frac{\hbar^2}{2m}\left(c_1\frac{\partial^2\psi_1}{dx^2} + c_2\frac{\partial^2\psi_2}{\partial x^2}\right) + V(c_1\psi_1 + c_2\psi_2)\\ &= c_1\left[-\frac{\hbar^2}{2m}\frac{d^2\psi_1}{dx^2} + V\psi_1\right] + c_2\left[-\frac{\hbar^2}{2m}\frac{d^2\psi_2}{dx^2} + V\psi_2\right]\\ &= c_1(E\psi_1) + c_2(E\psi_2)\\ &= E(c_1\psi_1 + c_2\psi_2)\\ &= E\psi_3.\end{aligned}$$

So, if we have two real solutions, $\psi_1 = (\psi + \psi^*)$ and $\psi_2 = i(\psi - \psi^*)$, then their linear combinations will satisfy the time-independent Schrodinger equation.

Part C

Let's plug in $\psi(-x)$ to the time-independent Schrodinger equation and see if anything changes,

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \psi(-x)}{dx^2} + V(-x)\psi(-x) = E\psi(-x). \tag{1}$$

Since $\partial^2/\partial(-x)^2 = \partial^2/\partial x^2$ and given $V(-x) = V(x)$, equation 1 can be expressed as

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \psi(+x)}{dx^2} + V(+x)\psi(+x) = E\psi(+x).$$

As mentioned in Part B, any liner combination of solutions to the time-independent Schrodinger equation is also a solution. Therefore, linear combinations of $\psi(x) \pm \psi\,(-x)$ are solutions.