Classical Mechanics: Problem 7.12

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Last updated: August 30, 2023

Part A

We start by summing all the forces on the particle, which is just a conservative and nonconservative force,

$$\sum F = F_{\text{cons}} + F_{\text{noncons}}.$$

We have the freedom to express a conservative force in terms of potential energy (we're only in the x direction),

$$\sum F = -\frac{\partial U}{\partial x} + F_{\text{noncons}}.$$

We're aiming to get this in terms of $\mathcal{L} = T - U$. To get there, freely add $\partial T/\partial x = 0$ because kinetic energy does not depend on position,

$$\begin{aligned}\sum F &= \frac{\partial T}{\partial x} - \frac{\partial U}{\partial x} + F_{\text{noncons}} \\ &= \frac{\partial}{\partial x}(T - U) + F_{\text{noncons}} \\ &= \frac{\partial \mathcal{L}}{\partial x} + F_{\text{noncons}}. \end{aligned} \tag{1}$$

From Newton's second law and definition of the Lagrange equations,

$$\begin{aligned} F &= \frac{d}{dt}p \\ &= \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{x}}. \end{aligned}$$

Combining this with equation 1 and given that our nonconservative force is friction,

$$\frac{\partial \mathcal{L}}{\partial x} + F_{\text{fic}} = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{x}}.$$