Griffiths Electrodynamics: Problem 5.12

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Part A

Before we begin, we need to determine what the electrostatic force is. A quick Gauss's law calculation for an infinite wire and $F = qE$ shows that the force *per unit length* is on one wire from the other is

$$\begin{aligned} \frac{F}{dl} &= \left(\frac{q}{dl}\right)(E) \\ &= (\lambda)\left(\frac{\lambda}{2\pi\epsilon_0 d}\right). \end{aligned} \tag{1}$$

Now we calculate the magnetic field needed to equal this force. We determined the magnetic force between two infinite currents in equation 5.40 to be

$$f = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d}.$$

For our given information in the problem, $I_1 = I_2 = I = \lambda v$, so,

$$f = \frac{\mu_0}{2\pi}\frac{(\lambda v)^2}{d}. \tag{2}$$

Now equating equations 1 and 2 to determine the velocity needed,

$$\begin{aligned} (\lambda)\left(\frac{\lambda}{2\pi\epsilon_0 d}\right) &= \frac{\mu_0}{2\pi}\frac{(\lambda v)^2}{d} \\ v &= \frac{1}{\sqrt{\epsilon_0 \mu_0}}. \end{aligned}$$

When we crunch the numbers,

$$v = 3.00 \times 10^8 \text{ m/s}.$$

This means that the wires would have to go the speed of light in order for the magnetic force to equal the electric force. Not exactly reasonably; in the case, the electric force always wins.