Classical Mechanics: Problem 7.4

Colton Kawamura
https://coltonkawamura.github.io/coltonkawamura/

Last updated: July 30, 2023

Part A

We can create the equation for kinetic energy from the position given $T = \frac{1}{2}m\left(\dot{x}^2 + \dot{y}^2\right)$. To finish the Lagrangian, we need to create an expression for potential energy U. We know gravitational potential energy is $U(y) = mgh$ where h is distance from the starting position y_0 and its current position $y \sin\alpha$ so potential energy becomes,

$$U(y) = mg(y_0 - y\sin\alpha)$$

We can create the Lagrangian,

$$\begin{aligned}\mathcal{L} &= T - U \\ &= \frac{1}{2}m\left(\dot{x}^2 + \dot{y}^2\right) - mg(y_0 - y\sin\alpha) \qquad (1)\end{aligned}$$

Part B

We create the two Lagrangian equations of motion,

$$\frac{\partial\mathcal{L}}{\partial x} = \frac{d}{dt}\frac{\partial\mathcal{L}}{\partial\dot{x}}, \qquad \frac{\partial\mathcal{L}}{\partial y} = \frac{d}{dt}\frac{\partial\mathcal{L}}{\partial\dot{y}}. \qquad (2)$$

For the x portion,

$$\begin{aligned}\frac{\partial\mathcal{L}}{\partial x} &= \frac{d}{dt}\frac{\partial\mathcal{L}}{\partial\dot{x}} \\ 0 &= \frac{d}{dt}\left(\frac{1}{2}m2\dot{x}\right) \\ 0 &= m\frac{d}{dt}(\dot{x}) \\ \ddot{x} &= 0.\end{aligned}$$

As expected, gravity is only accelerating in the y-direction. Now for the y-portion,

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial y} &= \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{y}} \\
mg\sin\alpha &= \frac{d}{dt}\left(\frac{1}{2}m2\dot{y}\right) \\
mg\sin\alpha &= m\frac{d}{dt}(\dot{y}) \\
\ddot{y} &= g\sin\alpha.
\end{aligned}$$

Just as before without using Lagrangian's!