Griffiths Electrodynamics: Problem 5.22

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Last updated: August 31, 2023

Part A

If we had magnetic monopoles, we would have magnetic field lines that begin or end somewhere, so that would require a nonzero divergence. We'd expect it to look like Gauss's law, with some constant C like μ_0 or ϵ_0,

$$\nabla \cdot \mathbf{B} = C\rho_m$$

where ρ_m is the magnetic charge density.

Ampere's law wouldn't change and Gauss's law is safe. However, the curl of the electric field would change as now we are able to have a moving magnetic charge causing some magnetic current density $\mathbf{J}_m$,

$$\nabla \times \mathbf{E} = D\mathbf{J}_m,$$

again, where D is some analogous constant.

Now lets look at the force law. We'd have to add another force law for a magnetic charge Q_m moving through an electric field, but also subject to a magnetic field,

$$\mathbf{F}_m = Q_m[\mathbf{B} + (\mathbf{v} \times \mathbf{E})],$$

but we need to fudge the units, so I'm going to multiply by another analogous constant Z,

$$\mathbf{F}_m = Z\left(Q_m[\mathbf{B} + (\mathbf{v} \times \mathbf{E})]\right).$$

Now we would have two force laws, one for magnetic and one for electric forces that we can combine via superposition for a system of both magnetic and electric particles.