# Classical Mechanics: Problem 7.11

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## Part A

To get the position in terms of the single variable $\phi$, we first look from the reference frame within the car, the bob has a position

$$\mathbf{r}_{\text{bob}} \equiv (x, y) = (l \sin\phi, l\cos\phi).$$

The car has a position

$$\mathbf{r}_{\text{car}} \equiv (x, y) = (A\cos\omega t, 0).$$

From the reference frame outside the car, the bob has as a a position that combines the two reference frame positions,

$$\mathbf{r}_{\text{bob}} \equiv (x, y) = (l\sin\phi + A\cos\omega t, l\cos\phi). \tag{1}$$

## Part B

Now we need to get this in terms of $x$ and $y$. Let's start with $x$ and solve for $\phi$,

$$\begin{aligned} x &= l\sin\phi + A\cos(\omega t) \\ x - d\cos(\omega t) &= l\sin\phi, \end{aligned}$$

because $y = \phi, l\cos\phi$,

$$\begin{aligned} \frac{x - d\cos(\omega t)}{y} &= \frac{l\sin\phi}{l\cos\phi} \\ &= \tan\phi, \end{aligned}$$

thus,

$$\phi = \arctan\left(\frac{x - d\cos(\omega t)}{y}\right).$$

To finish, we simply replace this expression in equation 1,

$$\mathbf{r}_{\text{bob}} \equiv (x, y) = (l\sin\left[\arctan\left(\frac{x - d\cos(\omega t)}{y}\right)\right] + A\cos\omega t, , l\cos\left[\arctan\left(\frac{x - d\cos(\omega t)}{y}\right)\right]).$$

As you can see, one coordinate system is much less complicated than the other.