Griffiths Electrodynamics: Problem 5.20

Colton Kawamura
https://coltonkawamura.github.io/coltonkawamura/

Last updated: August 29, 2023

Part A

We know that charge density is charge per unit volume $\rho = q/V$. We are given that each *atom* of copper gives one electron, so our $q = 1.6 \times 10^{-19}\,\text{C}$. The volume of one atom of copper is given by

$$\begin{aligned} V &= \frac{\text{density}}{\text{atomic mass}}(\text{Avogadro's Number}) \\ &= \frac{9.0\,\text{g/cm}^3}{64\,\text{g/mole}}(6.0 \times 10^{23}\,\text{mole}^{-1}). \end{aligned}$$

For the record, I had to look all this up in a chemistry textbook. Anyways, our total charge density becomes,

$$\rho = (1.6 \times 10^{-19})(6.0 \times 10^{23})\left(\frac{9.0}{64}\right) = 1.4 \times 10^4\ \text{C/cm}^3.$$

Why did we need to calculate this? To use in Part B.

Part B

Remember that current density is defined as a charge density moving at some velocity $\mathbf{J} = \rho\mathbf{v}$. Another definition for current density is the ratio of some current to the area that current is passing through. Assuming a cylindrical wire with a diameter d, $\mathbf{J} = \mathbf{I}/\pi(d/2)^2$. The velocity is,

$$\begin{aligned} \frac{I}{\pi s^2} &= \rho v \\ v &= \frac{I}{\pi s^2 \rho} \\ &= \frac{1}{\pi(2.5 \times 10^{-3})(1.4 \times 10^4)} \\ &= 9.1 \times 10^{-3}\ \text{cm/s},. \end{aligned}$$

This is comparable to the speed of glacial movements and rates of crystal growth - very, very slow. Electrons in a current do indeed move relatively slowly through a conductor like a wire. This phenomenon is often referred to as the "drift velocity." Despite the slow movement of individual electrons, electrical signals and communication can still happen quickly due to the way electromagnetic fields propagate through the conductor.

In a simple analogy, consider a line of people holding hands and waving them in a stadium. Even though each person moves their hand up and down slowly, the wave travels quickly around the stadium due to the hand-to-hand interaction. Similarly, in a conductor, when a voltage is applied at one end, it creates an electric field that pushes electrons forward. When an electron moves due to the applied electric field, it creates an electromagnetic disturbance that propagates along the conductor at close to the speed of light. This electromagnetic disturbance is what we refer to as the electrical signal.

Part C

The force between two steady state currents was derived in equation 5.40. For a distance $d = 1$ cm apart,

$$\begin{aligned} f_m &= \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d} \\ &= \frac{\mu_0}{2\pi}\frac{2I}{d} \\ &= 2 \times 10^{-7}\,\text{N/cm}. \end{aligned} \tag{1}$$

Why did we just calculate this? Well...

Part D

We can use our result from Part C for this part. The electric field per unit length at some distance r of an infinite line segment is

$$E = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{r}.$$

To get the electric force per unit length, we simply multiply by the linear charge density of the other line λ_2,

$$f_e = \frac{1}{2\pi\epsilon_0}\left(\frac{\lambda_1\lambda_2}{r}\right).$$

We don't have the value λ, but do have the values for current and velocity. We can leverage the relation $I = \lambda v$,

$$f_e = \frac{1}{2\pi\epsilon_0}\left(\frac{1}{r}\right)\left(\frac{I_1 I_2}{v^2}\right).$$

This is really close to equation 1, let's change the ϵ_0 to μ_0 through $c \equiv 1/\sqrt{\epsilon_0\mu_0}$,

$$\begin{aligned} f_e &= \left(\frac{c^2}{v^2}\right)\frac{\mu_0}{2\pi}\frac{I_1 I_2}{r} \\ &= \frac{c^2}{v^2} f_m. \end{aligned}$$

Finally, we can see that using our velocity from Part B,

$$\begin{aligned} \frac{f_e}{f_m} &= \frac{c^2}{v^2} \\ &= 1.1 \times 10^{25}. \end{aligned}$$

The electric for is significantly stronger than the magnetic force, which is

$$\begin{aligned} f_e &= f_m(1.1 \times 10^{25}) \\ &= 2 \times 10^{18}\ \mathrm{N/cm}. \end{aligned}$$