Griffiths Electrodynamics: Problem 5.15

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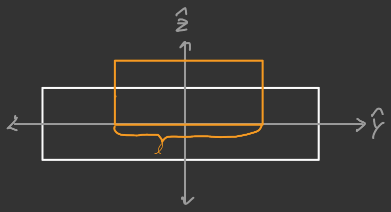
We have a problem with symmetry that allows us to pull B outside of the integral of $\oint \mathbf{B} \cdot d\mathbf{l}$, we can leverage Ampere's Law. Looking down the x-axis, we our Amperian loop to calculate $\mathbf{B}$ on the top of the slab as in the figure below.



We know from the right-hand rule that the direction of $\mathbf{B}$ is in $-y$ for the top and $+y$ for the bottom. To calculate I_{enc},

$$\begin{aligned} I_{\text{enc}} &= \int \vec{J} \cdot d\vec{a} \\ &= Jla. \end{aligned}$$

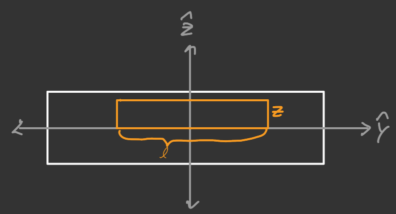
Now leveraging Ampere's Law, since there is no magnetic field contribution on the portions of the in the $\hat{z}$ or $y = 0$ portions of the Amperian loop,

$$\begin{aligned} &\oint \vec{B} \cdot d\vec{a} = \mu_0 I_{enc} \\ &Bl = \mu_0 Jla \\ &\vec{B} = \mu_0 Ja(-\hat{y}). \, (\text{for } z > a) \end{aligned}$$

By symmetry, the portion below the slab is,

$$\vec{B} = \mu_0 Ja(+\hat{y}). \, (\text{for } z < -a)$$

Now, looking inside the slab, our Amperian loop is:



This changes our I_{enc},

$$\oint \vec{B} \cdot d\vec{a} = \mu_0 I_{enc}$$
$$Bl = \mu_0 J l z$$
$$\vec{B} = \mu_0 J z(-\hat{y}). \text{ (for } z > 0)$$

Correspondingly, below the y-axis,

$$\vec{B} = \mu_0 J z(+\hat{y}). \text{ (for } z < 0)$$