Griffiths Electrodynamics: Problem 5.16

Colton Kawamura
https://coltonkawamura.github.io/coltonkawamura/

Last updated: August 10, 2023

This isn't too different from Example 5.9, expect our I_{enc} is going to be different for each region. First, we create an Amperian loop outside of both solenoids just like in Figure 3.7 of the text. Under the same logic as explained in the example, there is no radial or circumferential components. Outside, the solenoids, is no current enclosed. Therefore, Ampere's law becomes,

$$\oint \mathbf{B} \cdot d\ell = B_\phi(2\pi s) = \mu_0 I_{\text{enc}} = 0. \text{ (for } s > b\text{)}$$

For the region between both solenoids, we shift that Amperian loop so that one leg of the loop, parallel to the axis of the solenoid so its distance from the axis s is $(a < s < b)$.

Here, the current enclosed is the current of outer solenoid. Choosing the direction to the right of the page relative to Figure 5.42 in the text to be $+\hat{x}$, Ampere's law becomes,

$$\begin{aligned} \oint \vec{B} \cdot dl &= \mu_0 I_{\text{enc}} \\ (B)(l) &= \mu_0 I n_2 l \\ \vec{B} &= \mu_0 I n_2 \hat{x}. \text{ (for } a < s < b\text{)} \end{aligned}$$

For the final region inside the inner solenoid, $I_{\text{enc}} = In_2 - In_1$, since they are in opposite direction (and arbitrarily choosing In_2 to be *positive*,

$$\begin{aligned} \oint \vec{B} \cdot dl &= \mu_0 I_{\text{enc}} \\ (B)(l) &= \mu_0 (In_2 - In_1) l \\ \vec{B} &= \mu_0 I (n_2 - n_1) \hat{x}. \text{ (for } s < a\text{)} \end{aligned}$$