Griffiths Electrodynamics: Problem 5.11

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Last updated: July 30, 2023

Part A

In example 5.6 of the text, we found that the magnetic field a distance z above the center of a circular loop of radius R, which carries a steady current I horizontal components cancel, and the vertical components combine to give,

$$\mathbf{B}(\mathbf{z}) = \frac{\mu_0 I}{2} \frac{R^2}{(R^2+z^2)^{3/2}} \hat{z}$$

In this case, we have, essentially, *a stack* of n rings with some thickness dz. The radius is now a. The total current of this stack is nI and thickness $\int dz$. The magnetic contribution becomes,

$$\begin{aligned} \mathbf{B}(\mathbf{z}) &= \frac{\mu_0 nI}{2} \int \frac{a^2}{(a^2+z^2)^{3/2}}\, dz \hat{z} \\ &= \frac{\mu_0 nI}{2} \left. \frac{z^2}{\sqrt{a^2+z^2}} \right|_{z_1}^{z_2} \hat{z} \end{aligned} \tag{1}$$

In the problem, we were hinted to give the answer in terms of θ_1 and θ_2. From the figure in the text, we can see that $\cos\theta = z/\mathscr{r}$ where $\mathscr{r} = \sqrt{a^2+z^2}$. Equation 1 becomes,

$$\begin{aligned} \mathbf{B}(\theta) &= \frac{\mu_0 nI}{2} \left. (\cos\theta) \right|_{\theta_1}^{\theta_2} \hat{z} \\ &= \frac{\mu_0 nI}{2} (\cos\theta_2 - \cos\theta_1) \hat{z} \end{aligned} \tag{2}$$

Part B

For an infinite solenoid, $\theta_2 \to \infty$ and $\theta_1 \to \pi$ and equation 2 becomes

$$\begin{aligned}
\mathbf{B}(\theta) &= \frac{\mu_0 n I}{2} \left(1 - (-1)\right) \\
&= \mu_0 n I
\end{aligned}$$