Classical Mechanics: Problem 7.2

Colton Kawamura
https://coltonkawamura.github.io/coltonkawamura/

Last updated: July 29, 2023

Part A

We can create the equation for kinetic energy from the position given $T = \frac{1}{2}m\left(\dot{x}^2 + \dot{y}^2\right)$ and create the Lagrangian,

$$\begin{aligned}\mathcal{L} &= T - U \\ &= \frac{1}{2}m\left(\dot{x}^2 + \dot{y}^2\right) - \frac{1}{2}kr^3 \\ &= \frac{1}{2}m\left(\dot{x}^2 + \dot{y}^2\right) - \frac{1}{2}k\left(x^2 + y^2\right)\end{aligned} \tag{1}$$

Part B

We create the two Lagrangian equations of motion,

$$\frac{\partial \mathcal{L}}{\partial x} = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{x}}, \qquad \frac{\partial \mathcal{L}}{\partial y} = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{y}}. \tag{2}$$

For the x portion,

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial x} &= \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{x}} \\ -\frac{1}{2}k\left(2x\right) &= \frac{d}{dt}\left(\frac{1}{2}m2\dot{x}\right) \\ -kx &= m\frac{d}{dt}\left(\hat{x}\right) \\ \ddot{x} &= -\left(\frac{k}{m}\right)x.\end{aligned}$$

Looks familiar! Just like a simple harmonic oscillator. Now for the y portion, it should look the same,

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial y} &= \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{y}} \\
-\frac{1}{2}k\left(2y\right) &= \frac{d}{dt}\left(\frac{1}{2}m2\dot{y}\right) \\
-kx &= m\frac{d}{dt}(\hat{y}) \\
\ddot{y} &= -\left(\frac{k}{m}\right)y.
\end{aligned}
$$

Nice, exactly like an oscillator in two-dimensions.