089109 - Cálculo 1 - Turma C

Décima Terceira lista de exercícios

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Exercício 1. Calcule:

(a) $\int xe^{-x}\,dx$ (b) $\int x\cos\,x\,dx$ (c) $\int [x^2e^{x^3} - x^3\ln x]\,dx$

(d) $\int \text{arcsen}\,2x\,dx$ (e) $\int x^2e^x dx$ (f) $\int x^2\ln x dx$

(g) $\int (\ln x)^2 dx$ (h) $\int e^x\cos x dx$ (i) $\int x^2\,\text{sen}\,x dx$

(*Respostas:* (a) $-e^{-x}(x+1)+k$ (b) $x\,\text{sen}\,x+\cos x+k$ (c) $\frac{e^{x^3}}{3}-\frac{x^4\ln x}{2}+k$ (d) $x\,\text{arcsen}\,2x+\frac{1}{2}\sqrt{1-4x^2}+k$ (e) $e^x(x^2-2x+2)+k$ (f) $\frac{1}{3}x^3(\ln x-\frac{1}{3})+k$ (g) $x(\ln x)^2-2x(\ln x-1)+k$ (h) $\frac{1}{2}e^x(\,\text{sen}\,x+\cos x)+k$ (i) $-x^2\cos x+2x\,\text{sen}\,x+2\cos x+k$)

Exercício 2. Calcule $\int e^{-st}\,\text{sen}\,t dt$, onde $s>0$ é constante.

(*Respostas:* $-\frac{e^{-st}}{1+s^2}(\cos t+s\,\text{sen}\,t)+k$

Exercício 3. Para todo $n\geq 1$ e todo $s>0$, verifique que

$$\int t^ne^{-st}dt = -\frac{1}{s}\,t^ne^{-st}+\frac{n}{s}\int t^{n-1}e^{-st}dt.$$

Exercício 4. Calcule

(a) $\int_0^1 xe^x dx$ (b) $\int_1^2 \ln x dx$

(c) $\int_0^{\frac{\pi}{2}} e^x\cos x dx$ (d) $\int_0^x t^2e^{-st}dt$ $(s\neq 0)$

(*Respostas:* (a) 1 (b) $2\ln 2-1$ (c) $\frac{1}{2}(e^{\frac{\pi}{2}}-1)$ (d) $-\frac{1}{s}x^2e^{-sx}-\frac{2}{s^2}xe^{-sx}-\frac{2}{s^3}e^{-sx}+\frac{2}{s^3}$)

Exercício 5. Suponha que f'' seja contínua em $[a,b]$. Verifique que

$$f(b)=f(a)+f'(a)(b-a)+\int_a^b (b-t)f''(t)dt.$$

Exercício 6. Sejam m e n naturais não nulos. Verifique que:

(a) $\int_0^1 x^n(1-x)^m dx = \frac{m}{n+1}\int_0^1 x^{n+1}(1-x)^{m-1}dx$

(b) $\int_0^1 x^n(1-x)^m dx = \frac{n!m!}{(m+n+1)!}$

Exercício 7. Calcule $\int \sqrt{a^2+x^2}\,dx$, $a>0$.

(*Respostas:* $\frac{1}{2}\left[\frac{x}{a^2}\sqrt{a^2+x^2}+\ln\left|\frac{x+\sqrt{a^2+x^2}}{a}\right|\right]+k$)

Exercício 8. Deduza a área do círculo de raio r, $r > 0$.

(*Respostas:* πr^2)

Exercício 9. Calcule

(a) $\displaystyle\int \sqrt{-x^2+2x+3}\,dx$ (b) $\displaystyle\int \sqrt{6-3x^2}\,dx$ (c) $\displaystyle\int \frac{1}{x\sqrt{1+x^2}}\,dx$

(*Respostas:* (a) $2\,\operatorname{arcsen}\dfrac{x-1}{2}+\dfrac{x-1}{2}\sqrt{4-(x-1)^2}+k$ (b) $\dfrac{1}{2}x\sqrt{(6-3x^2)}+\sqrt{3}\operatorname{arcsen}\dfrac{1}{2}\sqrt{2}x+k$ (c) $\ln\left|\dfrac{x}{1+\sqrt{1+x^2}}\right|+k$).

Exercício 10. Calcule a área da elipse descrita pela equação $9x^2+y^2\leq 3$.
(*Respostas:* π).

Exercício 11. Sejam m e n constantes não nulas Mostre que:

$$\int \frac{mx+n}{1+x^2}\,dx = \frac{m}{2}\ln(1+x^2)+n\operatorname{arctg}x+k$$

Exercício 12. Calcule

(a) $\displaystyle\int \frac{x^3+x+1}{x^2-4x+3}\,dx$ (b) $\displaystyle\int \frac{3}{x^2+3}\,dx$

(c) $\displaystyle\int \frac{x^2+3}{x^2-9}\,dx$ (d) $\displaystyle\int \frac{x^2+1}{(x-3)^2}\,dx$

(*Respostas:* (a) $\dfrac{x^2}{2}+4x-\dfrac{3}{2}\ln|x-1|+\dfrac{31}{2}\ln|x-3|+k$ (c) $x+2\ln|x-3|-2\ln|x+3|+k$)

Exercício 13. Calcule e verifique o resultado por derivação.

(a) $\displaystyle\int \frac{x+3}{x(x-3)(x-4)}\,dx$ (b) $\displaystyle\int \frac{3}{x^3-16x}\,dx$

(c) $\displaystyle\int \frac{x^3+1}{x^3-x^2-2x}\,dx$ (d) $\displaystyle\int \frac{5}{(x^2-1)(x^2-9)}\,dx$

Exercício 14. Siga as instruções:

1. Determine A, B, C e D tais que
$$\frac{x-3}{(x-1)^2(x+2)^2}=\frac{A}{x-1}+\frac{B}{(x-1)^2}+\frac{C}{x+2}+\frac{D}{(x+2)^2}.$$
2. Calcule $\displaystyle\int \frac{x-3}{(x-1)^2(x+2)^2}\,dx$.

(*Respostas:* $\dfrac{7}{27}\ln|x-1|+\dfrac{6}{27(x-1)}-\dfrac{7}{27}\ln|x+2|+\dfrac{15}{27(x+2)}+k$.)

Exercício 15. Calcule as integrais:

(a) $\displaystyle\int \frac{2x-1}{(x-1)(x-2)}\,dx$ (b) $\displaystyle\int \frac{x}{(x+1)(x+3)(x+5)}\,dx$

(c) $\displaystyle\int \frac{x^4}{(x^2-1)(x+2)}\,dx$ (d) $\displaystyle\int \frac{dx}{(x-1)^2(x-2)}$

(e) $\displaystyle\int \frac{x-8}{x^3-4x^2+4x}\,dx$ (f) $\displaystyle\int \frac{dx}{x(x^2+1)}\,dx$

(g) $\displaystyle\int \frac{dx}{x^3+1}$ (h) $\displaystyle\int \frac{4x^2-8x}{(x-1)^2(x^2+1)^2}\,dx$.

(*Respostas:* (a) $\ln\left|\dfrac{(x-2)^3}{x-1}\right|+k$ (b) $\dfrac{1}{8}\ln\left|\dfrac{(x+3)^6}{(x+5)^5(x+1)}\right|+k$

(c) $\dfrac{x^2}{2}-2x+\dfrac{1}{6}\left|\dfrac{x-1}{(x+1)^3}\right|+\dfrac{16}{3}\ln|x+2|+k$ (d) $\dfrac{1}{x-1}+\ln\left|\dfrac{x-2}{x-1}\right|+k$

(e) $\dfrac{3}{x-8}+\ln\dfrac{(x-2)^2}{x^2}+k$ (f) $\ln\dfrac{|x|}{\sqrt{x^2+1}}+k$ (g) $\dfrac{1}{6}\ln\dfrac{(x+1)^2}{x^2-x+1}+\dfrac{1}{\sqrt{3}}\operatorname{arctg}\dfrac{2x-1}{\sqrt{3}}+k$

(h) $\dfrac{3x^2-1}{(x-1)(x^2+1)}+\ln\dfrac{(x-1)^2}{x^2+1}+\operatorname{arctg}x+k$)

Exercício 16. Calcule as integrais:

(a) $\displaystyle\int\frac{\sqrt{a^2-x^2}}{x^2}dx$ (b) $\displaystyle\int\frac{dx}{x^2\sqrt{1+x^2}}dx$

(c) $\displaystyle\int\frac{\sqrt{x^2-a^2}}{x}dx$ (d) $\displaystyle\int\frac{1}{\sqrt{(a^2+x^2)^3}}dx.$

(*Respostas:*

(a) $-\dfrac{\sqrt{a^2-x^2}}{x}-\operatorname{arcsen}\dfrac{x}{a}+k$ (b) $-\dfrac{\sqrt{1+x^2}}{x}+k$ (c) $\sqrt{x^2-a^2}-a\arccos\dfrac{a}{x}+k$

(d) $\dfrac{x}{a^2\sqrt{a^2+x^2}}+k$)