

089109 - CÁLCULO 1 - TURMA C
DÉCIMA SEGUNDA LISTA DE EXERCÍCIOS

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Exercício 1. Calcule

(a) $\int_0^1 x e^{x^2} dx$ (b) $\int_{-1}^0 x(2x+1)^{50} dx$ (c) $\int_0^1 \frac{x}{(x^2+1)^5} dx$

(d) $\int_{-1}^1 x^4(x^5+3)^3 dx$ (e) $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sin x(1-\cos^2 x) dx$ (f) $\int_0^{\frac{\pi}{3}} \sin^3 x dx$

(Respostas: (a) $\frac{1}{2}e - \frac{1}{2}$ (b) $-\frac{1}{102}$ (c) $\frac{15}{128}$ (d) 12 (e) $\frac{3}{8}\sqrt{3}$ (f) $\frac{5}{24}$)

Exercício 2. Calcule

(a) $\int x^3 \cos x^4 dx$ (b) $\int \sin^5 x \cos x dx$ (c) $\int \operatorname{tg} x \sec^2 x dx$

(d) $\int \frac{\sec^2 x}{3+2 \operatorname{tg} x} dx$ (e) $\int \left(\frac{5}{x-1} + \frac{2}{x} \right) dx$; (f) $\int \frac{1}{a^2+x^2} dx$

(g) $\int \frac{1}{x \ln x} dx$ (h) $\int \frac{1}{x} \cos(\ln x) dx$

(Respostas: (a) $\frac{1}{4} \sin x^4 + k$ (b) $\frac{1}{6} \sin^6 x + k$ (c) $\frac{1}{2} \operatorname{tg}^2 x + k$ (d) $\frac{1}{2} \ln |3+2 \operatorname{tg} x| + k$ (e) $5 \ln |x-1| + 2 \ln |x| + k$ (f) $\frac{1}{a} \operatorname{arctg} \frac{x}{a} + k$ (g) $\ln |\ln x| + k$ (h) $\sin(\ln x) + k$).

Exercício 3. Suponha f contínua em $[-1, 1]$. Calcule $\int_0^1 f(2x-1) dx$ sabendo que $\int_{-1}^1 f(u) du = 10$.

Exercício 4. Seja f uma função ímpar e contínua em $[-r, r]$, $r > 0$. Mostre que

$$\int_{-r}^r f(x) dx = 0.$$

Exercício 5. Suponha f contínua em $[0, 4]$. Calcule $\int_{-2}^2 x f(x^2) dx$.

Exercício 6. Calcule

(a) $\int_0^2 \frac{x^2}{(x+1)^2} dx$ (b) $\int_{-2}^2 x^2(x^3+3)^{10} dx$ (c) $\int_0^{\frac{\pi}{6}} \sin x \cos^2 x dx$

(d) $\int_0^{\frac{\pi}{4}} \cos x \sin^5 x dx$.

(Respostas: (a) $\frac{8}{3} - 2 \ln 3$ (b) 1168 (c) $-\frac{1}{8}\sqrt{3} + \frac{1}{3}$ (d) $\frac{1}{48}$).

Exercício 7. Seja f uma função par e contínua em $[-r, r]$, com $r > 0$.

(a) Mostre que $\int_{-r}^0 f(x)dx = \int_0^r f(x)dx$.

(b) Conclua de (a) que $\int_{-r}^r f(x)dx = 2 \int_0^r f(x)dx$. Interprete graficamente.

Exercício 8. Calcule

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| (1) $\int (x + \sqrt{x})dx$ | (2) $\int \left(\frac{3}{\sqrt{x}} - \frac{x\sqrt{x}}{4} \right) dx$ | (3) $\int \frac{x^2}{\sqrt{x}} dx$ |
| (4) $\int \left(x^2 + \frac{1}{\sqrt[3]{x}} \right)^2 dx$ | (5) $\int \operatorname{sen} ax dx$ | (6) $\int \frac{\ln x}{x} dx$ |
| (7) $\int \frac{1}{\operatorname{sen}^2 3x} dx$ | (8) $\int \frac{1}{3x-7} dx$ | (9) $\int \operatorname{tg} 2x dx$ |
| (10) $\int \operatorname{cotg} (5x-7) dx$ | (11) $\int \operatorname{cotg} \frac{x}{3} dx$ | (12) $\int \operatorname{tg} x \sec^2 x dx$ |
| (13) $\int e^x \operatorname{cotg} e^x dx$ | (14) $\int \operatorname{sen}^2 x \cos x dx$ | (15) $\int \cos^3 x \operatorname{sen} x dx$ |
| (16) $\int \frac{x}{\sqrt{2x^2+3}} dx$ | (17) $\int \frac{x^2}{\sqrt{x^3+1}} dx$ | (18) $\int \frac{\operatorname{sen} x}{\cos^3 x} dx$ |
| (19) $\int \frac{\operatorname{cotg} x}{\operatorname{sen}^2 x} dx$ | (20) $\int \frac{1}{\cos^2 x \sqrt{\operatorname{tg} x - 1}} dx$ | (21) $\int \frac{\operatorname{sen} 2x}{\sqrt{1 + \operatorname{sen}^2 x}} dx$ |
| (22) $\int \frac{\operatorname{arcsen} x}{\sqrt{1-x^2}} dx$ | (23) $\int \frac{\operatorname{arccos}^2 x}{\sqrt{1-x^2}} dx$ | (24) $\int \frac{x}{x^2+1} dx$ |
| (25) $\int \frac{x+1}{x^2+2x+3} dx$ | (26) $\int \frac{\cos x}{2 \operatorname{sen} x + 3} dx$ | (27) $\int \frac{1}{x \ln x} dx$ |
| (28) $\int 2x(x^2+1)^4 dx$ | (29) $\int \operatorname{tg}^4 x dx$ | (30) $\int \frac{1}{\cos^2 x (3 \operatorname{tg} x + 1)} dx$ |
| (31) $\int \frac{\operatorname{tg}^3 x}{\cos^2 x} dx$ | (32) $\int e^{2x} dx$ | (33) $\int x a^{x^2} dx$ |
| (34) $\int \frac{(a^x - b^x)^2}{a^x b^x} dx$ | (35) $\int \frac{e^x}{3 + 4e^x} dx$ | (36) $\int \frac{1}{1 + 2x^2} dx$ |
| (37) $\int \frac{1}{\sqrt{1-3x^2}} dx$ | (38) $\int \frac{1}{\sqrt{16-9x^2}} dx$ | (39) $\int \frac{1}{9x^2+4} dx$ |
| (40) $\int \frac{1}{4-9x^2} dx$ | (41) $\int \frac{1}{\sqrt{x^2+9}} dx$ | (42) $\int \frac{1}{\sqrt{b^2 x^2 - a^2}} dx$ |
| (43) $\int \frac{x^2}{5-x^6} dx$ | (44) $\int \frac{x}{\sqrt{1-x^4}} dx$ | (45) $\int \frac{x}{x^4+a^4} dx$ |

$$(46) \int \frac{\cos x}{a^2 + \sin^2 x} dx$$

$$(47) \int \frac{1}{x\sqrt{1 - \ln^2 x}} dx$$

$$(48) \int \frac{\arccos x - x}{\sqrt{1 - x^2}} dx$$

$$(49) \int \frac{x - \operatorname{arctg} x}{1 + x^2} dx$$

$$(50) \int \frac{\sqrt{1 + \sqrt{x}}}{\sqrt{x}} dx$$

$$(51) \int \frac{1}{2 \sin^2 x + 3 \cos^2 x} dx$$

$$(52) \int \sqrt{1 + 3 \cos^2 x} \sin 2x dx$$

$$(53) \int \frac{\cos^3 x}{\sin^4 x} dx$$

(Respostas: (1) $\frac{x^2}{2} + \frac{2x\sqrt{x}}{3} + C$ (2) $-6\sqrt{x} - \frac{1}{10}x^2\sqrt{x} + C$ (3) $\frac{2}{5}x^2\sqrt{x} + C$ (4) $\frac{x^5}{5} + \frac{3}{4}x^2\sqrt[3]{x^2} + 3\sqrt[3]{x} + C$
 $-\frac{\cos ax}{a} + C$ (6) $\frac{\ln^2 x}{2} + C$ (7) $-\frac{\cotg 3x}{3} + C$ $\frac{1}{3} \ln |3x - 7| + C$ $-\frac{1}{2} \ln |\cos 2x| + C$ (10) $\frac{1}{5} \ln |\sin (5x - 7)| + C$ $3 \ln |\sin x| + C$ $\frac{1}{2} \operatorname{tg}^2 x + C$ (13) $\ln |\sin e^x| + C$ $\frac{\sin^3 x}{3} + C$ (15) $-\frac{\cos 4x}{4} + C$ (16)
 $\frac{1}{2}\sqrt{2x^2 + 3} + C$ (17) $\frac{2}{3}\sqrt{x^3 + 1} + C$ (18) $\frac{1}{2\cos^2 x} + C$ (19) $-\frac{\cotg^2 x}{2} + C$ (20) $2\sqrt{\operatorname{tg} x - 1} + C$ (21)
 $2\sqrt{1 + \sin^2 x} + C$ (22) $\frac{\operatorname{arcsen}^2 x}{2} + C$ (23) $-\frac{\arccos^3 x}{3} + C$ (24) $\frac{1}{2} \ln(1 + x^2) + C$ (25) $\frac{1}{2} \ln(x^2 + 2x + 3) + C$
(26) $\frac{1}{2} \ln(2 \sin x + 3) + C$ (27) $\ln |\ln x| + C$ (28) $\frac{(x^2+1)^5}{5} + C$ (29) $\frac{\operatorname{tg}^3 x}{3} - \operatorname{tg} x + x + C$ Sugestão.
Mostre que $\operatorname{tg}^4 x = \operatorname{tg}^2 x \cdot \operatorname{tg}^2 x = \sec^2 x \cdot \operatorname{tg}^2 x - \sec^2 x + 1$ (30) $\frac{1}{3} \ln |3 \operatorname{tg} x + 1| + C$ (31) $\frac{\operatorname{tg}^4 x}{4} + C$
(32) $\frac{1}{2} e^{2x} + C$ (33) $\frac{a^{x^2}}{2 \ln a} + C$ (34) $\frac{(\frac{a}{b})^x - (\frac{b}{a})^x}{\ln a - \ln b} - 2x + C$ (35) $\frac{1}{4} \ln(3 + 4e^x) + C$ (36) $\frac{1}{\sqrt{2}} \operatorname{arctg}(\sqrt{2x}) + C$
(37) $\frac{1}{\sqrt{3}} \operatorname{arcsen}(\sqrt{3x}) + C$ (38) $\frac{1}{3} \operatorname{arcsen} \frac{3x}{4} + C$ (39) $\frac{1}{6} \operatorname{arctg} \frac{3x}{2} + C$ (40) $\frac{1}{12} \ln \left| \frac{2+3x}{2-3x} \right| + C$ (41)
 $\ln(x + \sqrt{x^2 + 9}) + C$ (42) $\frac{1}{b} \ln |bx + \sqrt{b^2 x^2 - a^2}| + C$ (43) $\frac{1}{6\sqrt{5}} \ln \left| \frac{x^3 + \sqrt{5}}{x^3 - \sqrt{5}} \right| + C$ (44) $\frac{1}{2} \operatorname{arcsen} x^2 + C$
Sugestão. $u = x^2$. (45) $\frac{1}{2a^2} \operatorname{arctg} \frac{x^2}{a^2} + C$ (46) $\frac{1}{a} \operatorname{arctg} \left(\frac{\sin x}{a} \right) + C$ (47) $\operatorname{arcsen}(\ln x) + C$ (48)
 $-\frac{1}{2}(\arccos x)^2 + \sqrt{1 - x^2} + C$ (49) $\frac{1}{2} \ln(1 + x^2) - \frac{1}{2}(\operatorname{arctg} x)^2 + C$ (50) $\frac{4}{3} \sqrt{(1 + \sqrt{x})^3} + C$ (51)
 $\frac{\sqrt{3}}{3\sqrt{2}} \operatorname{arctg} \left(\frac{\sqrt{2}}{\sqrt{3}} \operatorname{tg} x \right) + C$ (52) $-\frac{2}{9} \sqrt{(1 - 3 \cos^2 x)^3} + C$ (53) $\frac{1}{\sin x} - \frac{1}{3 \sin^3 x} + C$)