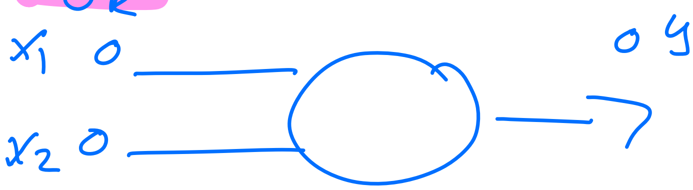
Lecture Jan 31, 2022

XOR, OR, AND

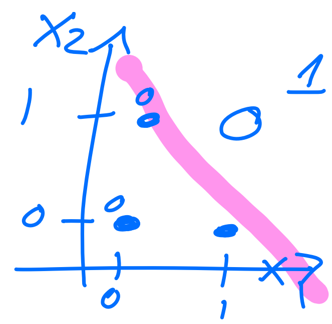
OR



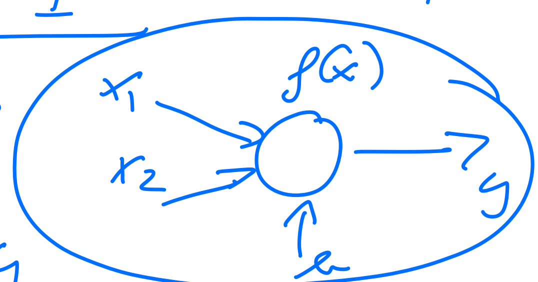
x_1	x_2	y
0	0	0
0	1	1
1	0	1
1	1	1

AND

x_1	x_2	y
0	0	0
0	1	0
1	0	0
1	1	1



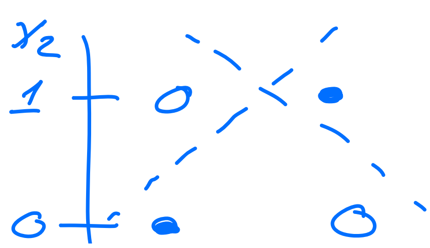
single perceptron

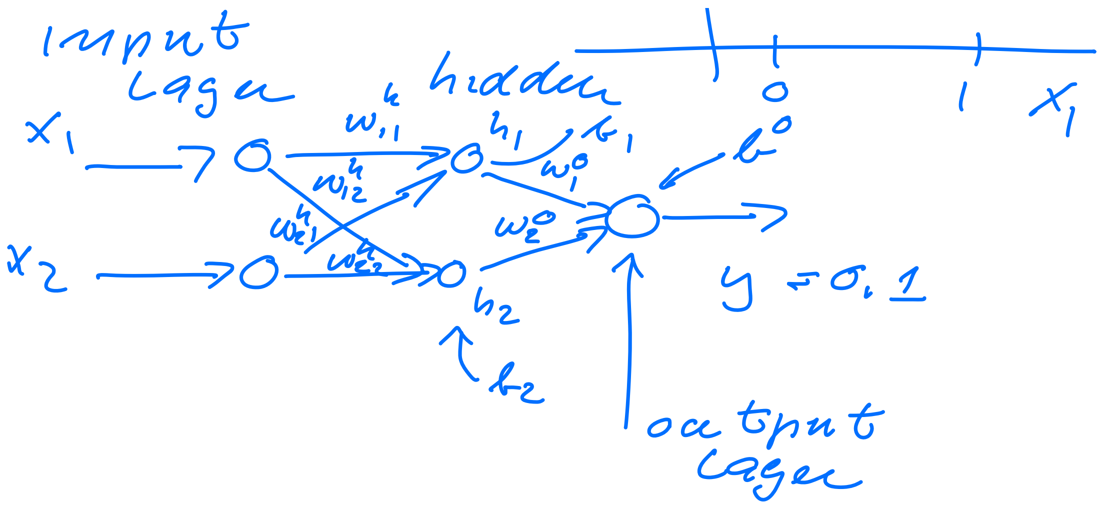


$y = f(x) + b$

XOR

x_1	x_2	y
0	0	0
0	1	1
1	0	1
1	1	0





Solving ODEs and PDEs numerically

1st-order ODE

$$\frac{dy}{dt} = f(y, y', t)$$

$$\frac{dy}{dt} - f(y, y', t) = 0$$

$$= F\left(y, \frac{dy}{dt}, \frac{d^2y}{dt^2}, t\right)$$

2nd-order ODE

$$\frac{d^2y}{dt^2} = \boxed{g(y, y', t)}$$ known

rewrite in terms

v of 2 coupled
1st order ODEs

$$a(y, y'; t) = \frac{d^2y}{dt^2}$$

$$v(y, y'; t) = \frac{dy}{dt}$$

$$a = \frac{dv}{dt} \quad \wedge \quad v = \frac{dy}{dt}$$

Discretize

$y_i \in D \qquad t \in [0, t_{final}]$

$\uparrow$ t_0

$$\Delta t = \frac{t_{final}}{n}$$

$$t \to t_i = t_0 + i \cdot \Delta t$$

$i = 0, 1, 2, \ldots n$

$y \to y_i$, $i = 0, 1, 2, \ldots n$

$a \to a_i$

$v \to v_i$

Taylor expansion

$$v(t \pm \Delta t) = v(t) \pm \frac{\Delta t}{1!} v'(t) + \frac{\Delta t^2}{2!} v''(t) \ldots$$

$$+ O(\Delta t^{2})$$

$$y(t \pm \Delta t) = y(t) \pm \Delta t\, y'(t) + \frac{\Delta t^2}{2!} y''(t) + O(\Delta t^3)$$

Discretize and leave out $O(\Delta t^2)$ => Euler's method

$$y(t \pm \Delta t) = y_{i \pm 1} = y(t_i \pm \Delta t)$$

$$y_{i+1} = y_i + \Delta t\, y_i'$$
$$= y_i + \Delta t\, v_i$$

$$v_{i+1} = v_i + \Delta t\, v_i'$$
$$= v_i + \Delta t\, a_i$$

y_0 and v_0 initial conditions

g_i

Forward-Euler (Explicit scheme)

Discretized domain (1st order ODE)

$$y \in \{ y_0, y_1 \ldots y_n \}$$

$t \in \{ t_0, t_1, \dots t_n \}$

$y' \in \{ y_0', y_1', \dots y_n' \}$

$$\frac{dy}{dt} = f(y, y', t) \longrightarrow$$

$$\frac{dy}{dt} - f(y, y', t) = F(y, y', t) = 0$$

Discretized version

$$F(y_i, y_i', t_i) = 0$$

Construct a trial solution

$$y_t(t) = \underbrace{h_1(t)}_{\text{satisfies initial conditions}} + h_2(t, N(t, \epsilon))$$

no adjustable parameters

Neural network which depends on t as input and parameters ϵ to be optimized.

Cost/loss function depends on $\frac{dy}{dt} - f(y, y', t)$

$$= C(y, y', t)$$

$$y \to y_t$$

$C(y_t, y_t', t, \theta)$ weights + biases of neural networks