Comp Sci program Feb 7, 2022

Finite discretization

$x \rightarrow x_i = x_0 + i\Delta x; \; i = 0, 1, \ldots n$

$$\Delta x = \frac{x_n - x_0}{n}$$

$y(x) \rightarrow y(x_i) = y_i$

Taylor expansion

$$y(x \pm \Delta x) = y_{i \pm 1} = y_i \pm \Delta x \, y_i' + \frac{\Delta x^2}{2!} y_i'' + O(\Delta x^3)$$

Euler's method

- Explicit

$$y_{i+1} = y_i + \Delta x \, y_i' + O(\Delta x^2)$$

$$\simeq y_i + \Delta x \, y_i'$$

- Implicit

$$y_{i-1} - y_i = -\Delta x \, y_i' \Rightarrow$$

$$y_i = \Delta x \, y_i' + y_{i-1}$$

Neural Networks

$$y(x) = h_1(x) + h_2(x; NN(x; \theta))$$

fully defined by initial conditions

Neural Network
Θ = parameters of NN

$$\frac{dy}{dx} = -\gamma\, y(x)$$

γ is a real constant

$$y(x) = y_0 \exp(-\gamma x)$$

Cost function

$$C(y, x; \Theta) = \left(\frac{dg}{dx} - g(x;\Theta)\right)$$

Trial function

$$g = y$$

$$g = h_1(x) + h_2(x, NN(x;\Theta))$$

initial conditions

x_0 is known

y_0 — " —

Boundary value problem

$$-g''(x) = f(x)$$

$$f(x) = (3x+x^2)e^x$$

$$x \in [0, 1]$$

$$g(0) = g(1) = 0$$

$$\frac{d^2g}{dx^2} = \frac{g_{i+1} + g_{i-1} - 2g_i}{(\Delta x)^2} + O(\Delta x^2)$$

$$g_{i\pm 1} = g(x \pm \Delta x)$$

$$g(x+\Delta x) = g(x) + g'(x)\Delta x + g''\frac{\Delta x^2}{2!} + O(\Delta x^3)$$

$$g(x-\Delta x) = g(x) - g'(x)\Delta x + g''\frac{\Delta x^2}{2!} + O(\Delta x^3)$$

$$g(x+\Delta x) + g(x-\Delta x) \simeq 2g(x) + \Delta x^2 g''(x)$$

$$-\frac{d^2g}{dx^2} = -\frac{g_{i+1} + g_{i-1} - 2g_i}{\Delta x^2} = f_i$$