Comp Sci, Feb 21, 2023

$$\frac{dx}{dt} = f(x,t)$$

discretize $x \to x_i$ $t \to t_i$

$t = \{t_0, t_1, \ldots t_n\}$ $i = 0, 1, \ldots n$

$$\Delta t = \frac{t_{final} - t_{initial}}{n}$$

$$= \frac{t_n - t_0}{n}$$

$$x_{i+1} = x(t_i + \Delta t) = x_i + \Delta t \cdot \text{ODEint}$$

Euler's method

$f(x_i, t_i)$

$$x_{i+1} = x_i + \Delta t \cdot \text{ODEint}(x_i, t_i)$$

For the neural network

x_i defines input

x_{i+1} — " — output (targets)

x_0 = first input

x_1 = — " — output

x_1 = 2nd input

x_2 = — " — output

⋮

x_{n-1} final input

x_n final output

Convolution & Fourier series

$$(f*g)(x) = \int_{-\infty}^{\infty} f(x-\xi)\, g(\xi)\, d\xi$$

$$\hat{f} = \mathcal{F}(f) \wedge \hat{g} = \mathcal{F}(g)$$

$$\mathcal{F}^{-1}(\hat{f}\hat{g})(x) =$$

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} e^{i\omega x} \hat{f}(\omega)\, \hat{g}(\omega)\, d\omega$$

$$= \int_{-\infty}^{\infty} \hat{f}(\omega)\, e^{i\omega x} \left(\frac{1}{2\pi} \int_{-\infty} g(y) e^{-i\omega y} dy \right)$$

$$\times d\omega$$

$$\frac{1}{2\pi} \int_{-\infty} \int_{-\infty}^{\infty} g(y)\, \hat{f}(\omega)\, e^{i\omega(x-y)} d\omega$$

$$\cdot dy$$

$$= \int_{-\infty}^{\infty} g(y) \underbrace{\left(\frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{f}(\omega)\, e^{i\omega(x-y)} d\omega \right)}_{f(x-y)}$$

$$\times dy$$

$$= \int_{-\infty}^{\infty} g(y)\, f(x-y)\, dy =$$

$$g * f = f * g$$

Basics of CNN

Filtering (convolution)

Input (3x4 matrix)

a	b	c	d
e	f	g	h
i	j	k	l

Filter (2x2)

w	x
y	z

stride 1 $\rightarrow$ 2x3 matrix

$aw + bx + ey + fz$	$bw + cx + fy + gz$	$cw + dx + gy + hz$
$ew + fx + iy + jz$	$fw + gx + jy + kz$	$gw + hx + ky + lz$