- Plan for the lectures:

Philosophy + math	• Interpretations of probability • Quick refresher on prob. theory for many-dim. probability distributions
Statistics (not ML)	• Aspects of Bayesian statistics (parameter estimation and model comparison)
ML	• Gaussian processes (formalism + applications)

Main references:

- Sivia: "Data analysis, a Bayesian tutorial"
- Rasmussen & Williams: "Gaussian Processes for Machine Learning" (Available for free online!)

A brilliant and provocative gem:

- ET Jaynes: "Probability Theory: The Logic of Science"

Probability

- Q: How many have taken a course on prob. or statistics?

- Discussion: Discuss meaning of prob. using a coin flip or dice throw

- What does the statement $P(X) = 10\%$ <u>mean</u>?
- We don't know, or at least don't agree!
- Useful reference: "Interpretations of Probability", Stanford Encyclopedia of Philosophy

- Bertrand Russel, 1929: "Probability is the most important concept in modern science, especially as nobody has the slightest notion of what it means."

- Two main interpretations:
  - Frequentist: $\boxed{P(X) \equiv \lim_{n\to\infty} \frac{n_x}{n}}$ Prob. <u>defined</u> as long-run <u>relative frequency</u>

  - Bayesian: $\boxed{P(X) \equiv \text{degree of belief/knowledge that } X \text{ is true}}$

    - Degree of <u>belief</u> $\leftrightarrow$ <u>subjective</u> Bayesian
    - Degree of <u>knowledge</u> $\leftrightarrow$ <u>objective</u> Bayesian

- Formal/deductive logic: rules for reasoning with <u>certain</u> statements (Boolean logic)

- Cox and others: Find rules for <u>plausible reasoning</u>, i.e. <u>logic under uncertainty</u>

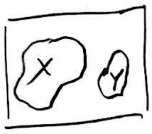
  ↳ "Rediscovered" the usual rules of prob. theory!

- Both freq. and Bayesian definitions of prob. agree with the Kolmogorov axioms that define the mathematical properties of the function $P(X)$ → Bayesians and frequentists use the same prob. rules, disagree about the interpretation.

Kolmogorov: $0 \leq P(X) \leq 1$

P is additive:

$$P(X \cup Y) = P(X) + P(Y) \quad \text{when } X \cap Y = \emptyset$$



- Frequentists: ~~P(hypothesis | data)~~ $\qquad$ P(data | hypothesis)

- Bayesians: P(hypothesis | data) $\qquad$ P(data | hypothesis)

- Subjective and objective Bayesians all happy with

$$P_{me}(X \mid I_1) \neq P_{you}(X \mid I_2)$$

- But objective Bayesians require that

$$P_{me}(X \mid I_1) = P_{you}(X \mid I_1)$$

Not required by subjective Bayesians!

- Usual rules for prob. theory does not tell us how to assign prob. in the first place, just how to <u>relate</u> probabilities in a consistent way! (Analogous with diff. eqs., which <u>relate</u> init. state to final state.)

- Objective Bayesians must introduce additional rules for assigning probabilities. Important example: "Maximum entropy"

Roughly saying: Given some information I, e.g. $X = 0.7 \pm 0.1$, choose the prob. distribution $P(x)$ that is the most uncertain but still consistent with I.

- Interpretations of prob. have important consequences:

1) Give rise to different approaches to statistics

Example: Bayesians can ask

$$P\left(\begin{smallmatrix}\text{parameter}\\ \text{value}\end{smallmatrix} \middle| \text{data}\right) = ?$$

Freq. cannot ask this, since prob. of a param. value does not make sense.

- Bayesian 95% credible interval for a parameter θ:

$$[0.1, 0.3]$$

"We have a 95% degree of belief that the true value of θ is between 0.1 and 0.3"

- Frequentist 95% confidence interval for θ:

$$[0.1, 0.3]$$

"If the experiment was repeated an infinite number of times, an interval constructed with this procedure should contain the true θ value in 95% of the repetitions"

2) Are probabilities necessarily linked to randomness?

- Is anything truly random? (Metaphysics, determinism, apparent vs. true randomness)

  Example: Is P(heads) in a coin flip necessarily 50%?

  Bayesian view: No necessary link between prob. and randomness. Can simply use prob. to quantify uncertainty. (But does not imply that randomness does not exist.)

- Probabilities in science, what do they mean?

  In particular: Interpretations of quantum mechanics.