Physics Immersion Week, Day 1
June 21, 2021

The Classical Two-Body Problem

Velocity-Verlet method

Need the Discretize

$$v(t) = \int_{t_0}^{t} a\,dt = v_0 + at$$

↳ analytical solution
↳ continuous

Numerical methods
↳ discretize

time $\quad t =$ any number

$$t = \{t_0,\ t_0 + \Delta t,\ t_0 + 2\Delta t,\ \dots\}$$

time step
$\Delta t \ll 1$

$$= \{t_0,\ t_1,\ t_2,\ t_3,\ \dots\}$$

↳ t_N

give F

$$\rightarrow F = ma \Rightarrow a = F/m$$

$$v = \int a\,dt$$

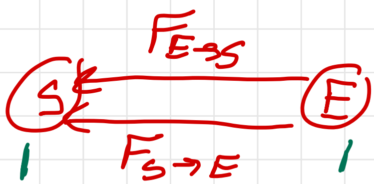
$$x = \iint a\,dt = \int v\,dt$$

Velocity - Verlet method

$$x_{i+1} = x_i + v_i \Delta t + \frac{(\Delta t)^2}{2} a_i$$

$$v_{i+1} = v_i + \frac{\Delta t}{2}(a_{i+1} + a_i)$$

Classical Two-Body



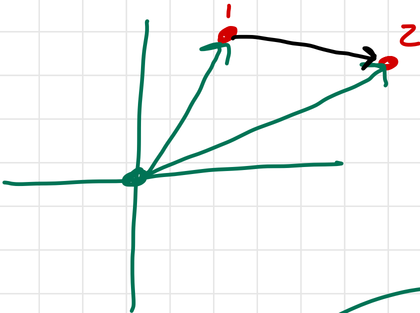
Newton's 3rd Law

$$F_G \sim 1/r^2$$

Relative and Center-of-mass Coordinates

Given a system
Object 1: $\vec{r}_1$, m_1
Object 2: $\vec{r}_2$, m_2

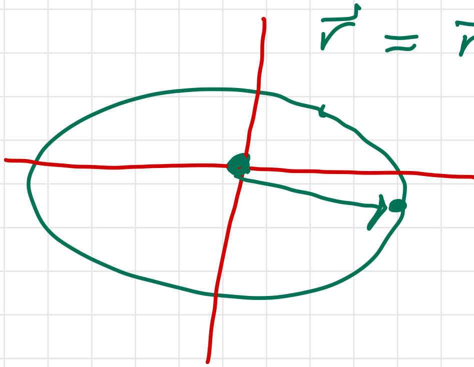
Relative Coordinates



$\vec{r}_1, \vec{r}_2$

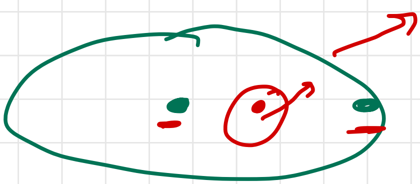
$\Downarrow$

$$\vec{r} = \vec{r}_1 - \vec{r}_2$$



Center-of-mass Coordinates

$$\vec{R} = \frac{m_1 r_1 + m_2 r_2}{m_1 + m_2}$$ average

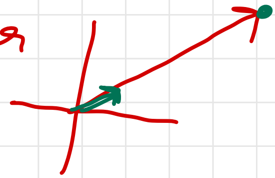


Earth-Sun Problem

$$\vec{F}_G(\vec{r}) = -G\frac{m_1 m_2}{|\vec{r}_1-\vec{r}_2|^2}\hat{r}_{12}$$

$$\hat{r} = \frac{\vec{r}}{|r|}$$

$$\vec{r} = \langle x, y, z \rangle$$



$$|r| = \sqrt{x^2+y^2+z^2}$$

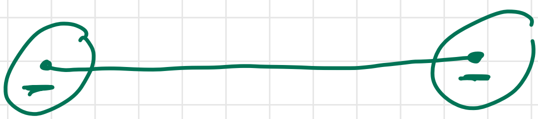
$$\hat{r} = \frac{\vec{r}}{|r|} \Rightarrow \text{direction}$$

$$F_G(\vec{r}) = -G\frac{m_1 m_2}{|\vec{r}_1-\vec{r}_2|^3}\vec{r}_{12}$$

$$F_G(\vec{r}) = -G\frac{m_\odot m_E}{|\vec{r}_{E\odot}|^3}\vec{r}_{E\odot}$$

Sun is origin

$g = 9.81\, m/s^2$



Sun Earth

Accleration of Earth

$$F_{G,E} = -G \frac{m_\odot}{|\vec{r}|^2} m_E \hat{r}$$

$$F = ma \Rightarrow a = F/m$$

$$\frac{F_{G,E}}{m_E} = -G \frac{m_\odot}{|\vec{r}|^2} \hat{r}$$

$$F_g = -mg$$