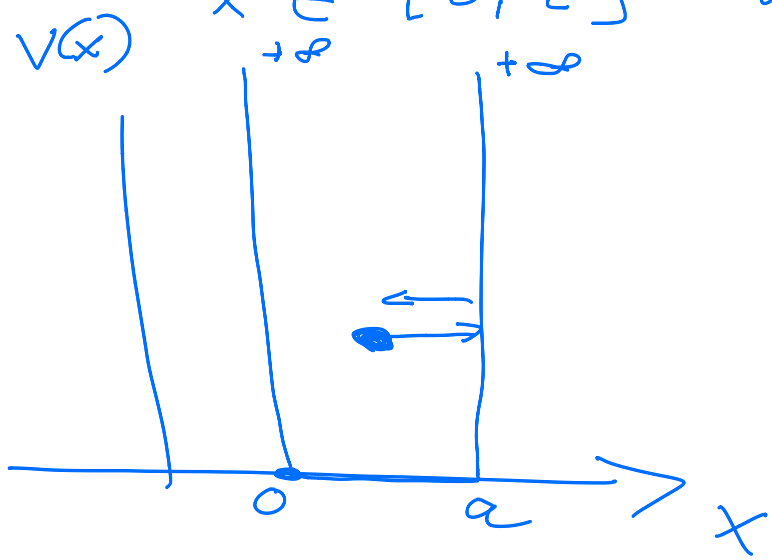
Thursday June 24

$$\gamma \frac{d^2u}{dx^2} = -Fu$$

$x \in [0, L]$ $\quad u(0) = u(L) = 0$



Quantum mech kin energy

$$\boxed{-\frac{\hbar^2}{2m}\frac{d^2u}{dx^2}} = Eu$$

$$p = -\hbar i \frac{d}{dx}$$

$$u(0) = u(a) = 0$$

Two-point ~~va~~ boundary value problem

$$\frac{d^2u}{dx^2} = -\frac{E\,2m}{\hbar^2}\,u(x)$$

$$\rho = \frac{x}{a} \qquad \rho \in [0, 1]$$

$$u(0) = u(1) = 0$$

$$\frac{1}{a^2}\frac{d^2u}{d\rho^2} = -\frac{E\,2m}{\hbar^2}\,u(\rho)$$

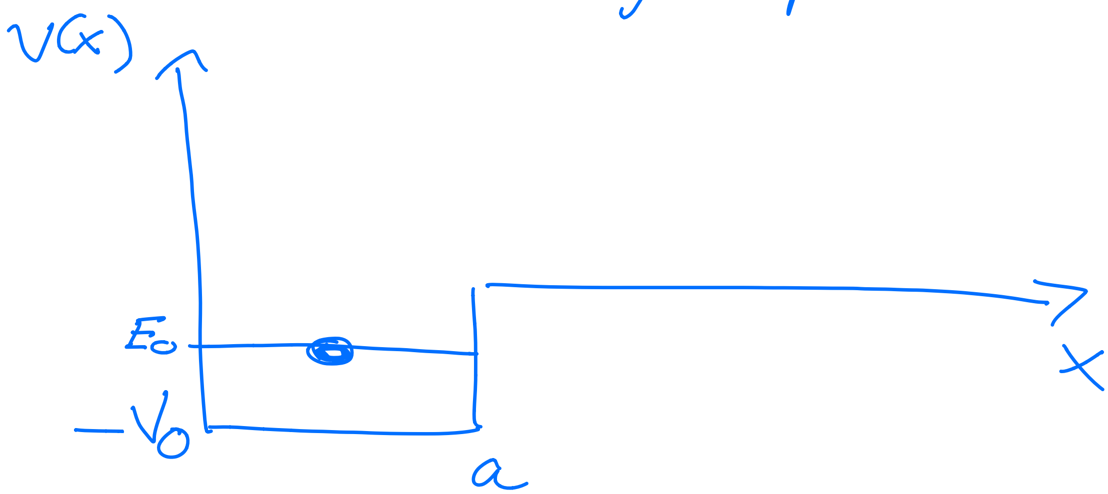
$$\boxed{\frac{d^2u}{d\rho^2} = -\lambda\, u(\rho)}$$

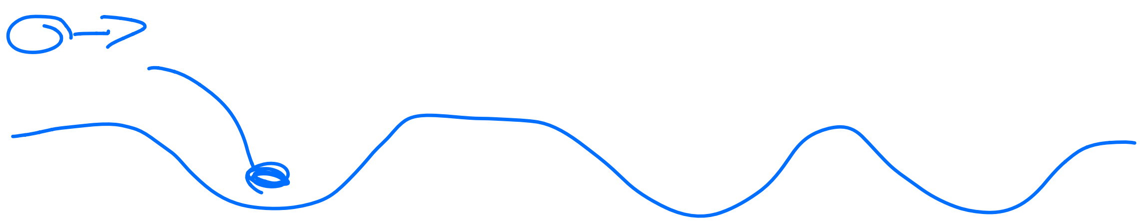
$$\lambda = \frac{E\,2m}{\hbar^2}\,a^2$$

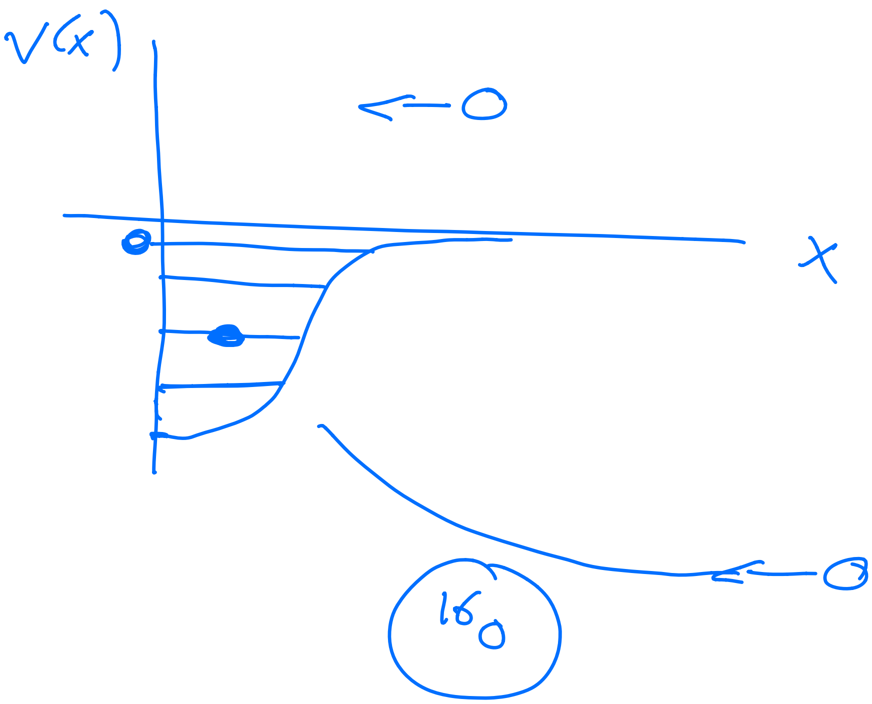
λ takes only discrete (quantized value)

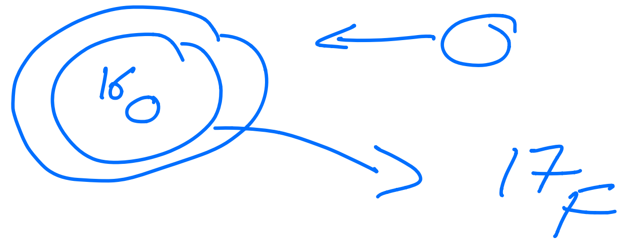
$$\lambda \Rightarrow \underline{\lambda_j} = E_j \frac{2ma^2}{\hbar^2}$$

Adding a potential







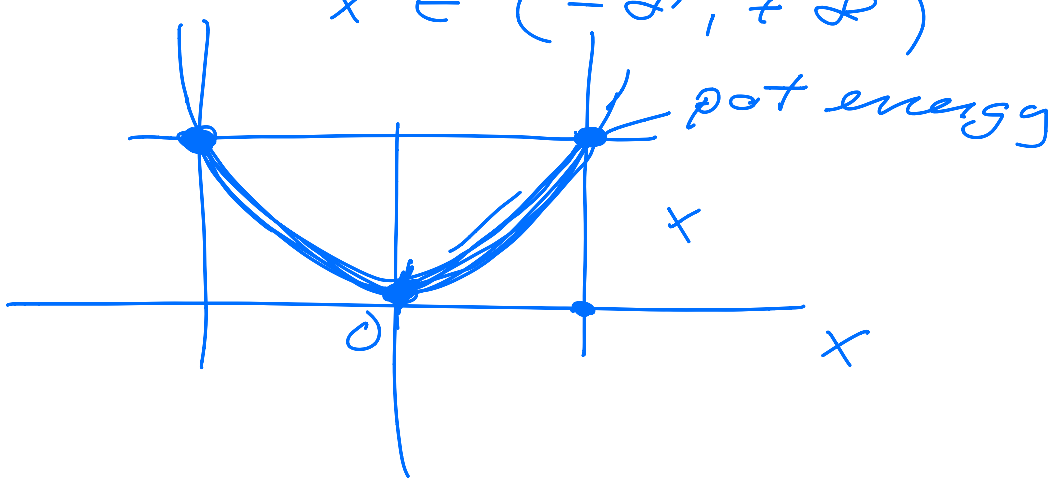


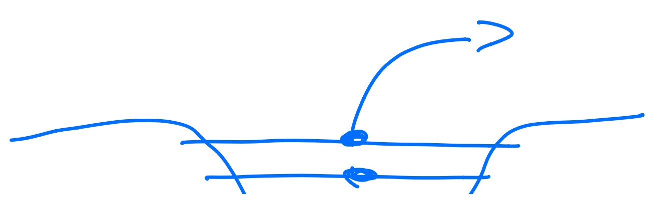
$$V(x) = \begin{cases} 0 \le x \le a & -V_0 \\ 0 & +\infty \\ x > a & 0 \end{cases}$$

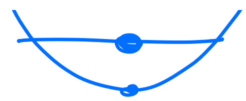
$$u(0) = u(\infty) = 0$$

$$V(x) = \frac{1}{2} m \omega^2 x^2 = \frac{1}{2} k x^2$$

$$x \in (-\infty, +\infty)$$







Adding $V(x)$ to Schrödinger's equation

$$\boxed{-\frac{\hbar^2}{2m}\frac{d^2u}{dx^2} + V(x)u(x) = Eu(x)}$$

$$V(x) = \frac{1}{2}kx^2$$

Scale the equations

$$\rho = \frac{x}{\alpha}$$

α has dimension length, its value is not specified.

$$V(x) = V(\rho) = \frac{1}{2}k\rho^2\alpha^2 - V_0$$

$$-\frac{\hbar^2}{2m}\frac{d^2u}{dx^2} = -\frac{\hbar^2}{2m\alpha^2}\frac{d^2u}{d\rho^2}$$

$$\times \frac{m\alpha^2}{\hbar^2} : \quad -\frac{\hbar^2}{2m\alpha^2}\frac{d^2u}{d\xi^2}$$

$$+ \frac{1}{2}k\xi^2\alpha^2 u = Eu$$

$$-\frac{1}{2}\frac{d^2u}{d^2\xi^2} + \frac{1}{2}\boxed{\frac{km\alpha^4}{\hbar^2}}\,\underline{\xi}^2 u$$

$$= \lambda u \qquad \frac{1}{\xi}$$

$$\lambda = \frac{Em\alpha^2}{\hbar^2}$$

$$\frac{km\alpha^4}{\hbar^2} = 1$$

$$\alpha^4 = \frac{\hbar^2}{km}$$

$$\alpha = \left(\frac{\hbar^2}{km}\right)^{1/4}$$

$\hookrightarrow$ natural length scale

$$-\frac{1}{2}\frac{d^2u}{d\rho^2} + \frac{1}{2}\underline{\rho}^2 u = \lambda u$$

Exercise $\underline{1}$ $\quad V(x) = \begin{cases} -V_0 & 0 < x \leq a \\ 0 & x > a \\ +\infty & x = 0 \end{cases}$

Discretize :

$$\frac{d^2u}{d\rho^2} \simeq \frac{u_{i+1} + u_{i-1} - 2u_i}{\Delta\rho^2}$$

$$\frac{1}{2}\rho^2 u \longrightarrow \frac{1}{2}\rho_i^2 u_i$$

Discritized eq :

$$-\frac{1}{2}\frac{u_{i+1} + u_{i-1} - 2u_i}{\Delta\rho^2} + \frac{1}{2}u_i\rho_i^2$$

$$= \lambda u_i$$

Only change : Diagonal

mtx elements

$$+ \frac{u_i}{\Delta\rho^2} + \frac{1}{2}\rho_i^2 u_i \Rightarrow$$

Diag mtx element

$$\frac{1}{\Delta\rho^2} + \frac{1}{2}\rho_i^2$$

$$- Z/\rho_i$$