Lecture FYS5419, March 6, 2024

FYS 5419/9419, MARCH 6, 2024

one qubit case

$$\mathcal{H} = cI + dZ + eX$$

$$\mathcal{M} = Z$$

$$X = U^{\dagger} Z U \qquad U = H$$

$$= \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & +1 \\ +1 & -1 \end{bmatrix}$$

VQE

$$|\psi(\theta, \varphi)\rangle = R_y(\theta) R_x(\chi) |0\rangle$$

$$|0\rangle \longrightarrow \boxed{R_x(\varphi)} \longrightarrow \boxed{R_y(\theta)} \longrightarrow |\psi'\rangle$$

2nd Hamiltonian

$$H_0|00\rangle = \varepsilon_{00}|00\rangle$$

$$H_0|01\rangle = \varepsilon_{01}|01\rangle$$

$$H_0|10\rangle = \varepsilon_{10}|10\rangle$$

$$H_0|11\rangle = \varepsilon_{11}|11\rangle$$

$$H_1 = H_Z\, Z\otimes Z + H_X\, X\otimes X$$

$$H_0 = \alpha \underbrace{II_{2\times 2}}_{I\otimes I} + \beta IZ + \gamma ZI + \delta ZZ$$

$$\alpha = (\varepsilon_{00} + \varepsilon_{01} + \varepsilon_{10} + \varepsilon_{11})/4$$

$$\beta = (\varepsilon_{00} - \varepsilon_{01} + \varepsilon_{10} - \varepsilon_{11})/4$$

$$\gamma = (\varepsilon_{00} + \varepsilon_{01} - \varepsilon_{10} - \varepsilon_{11})/4$$

$$\delta = (\varepsilon_{00} - \varepsilon_{01} - \varepsilon_{10} + \varepsilon_{11})/4$$

$$\mathcal{H} = \alpha\, I \otimes I + \beta\, I \otimes Z + \gamma\, Z \otimes I$$

$$+ (\delta + H_Z)\, Z \otimes Z + H_X\, X \otimes X$$

$$Z \otimes I \longrightarrow u_p^{\dagger} M u_p$$

$Z \otimes I$ has $u_p = \mathbb{1} \otimes \mathbb{1}$

$\mathbb{1} \otimes Z$ has $u_p = \mathrm{SWAP}$

$Z \otimes Z$ has $u_p = CX_{10}$

$X \otimes X$ has $u_p = CX_{10}(H \otimes H_{2\times 2})$

$$\mathbb{1} \otimes Z = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} \begin{matrix} 1 & 0 \\ 0 & -1 \end{matrix} & 0 \\ 0 & \begin{matrix} 1 & 0 \\ 0 & -1 \end{matrix} \end{bmatrix}$$

$$SWAP = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$SWAP|00\rangle = SWAP \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = |00\rangle$$

$$SWAP|11\rangle = |11\rangle = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$SWAP|01\rangle = |10\rangle$$

$$|01\rangle = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\mathrm{SWAP}|10\rangle = |01\rangle$$

$$\mathrm{SWAP}\,(1 \otimes Z)\,\mathrm{SWAP}$$

$$= \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}}_{m}$$

$$m|11\rangle = -|11\rangle$$

$$CX_{10} = \begin{bmatrix} \begin{matrix}0 & 1\\ 1 & 0\end{matrix} & 0 \\ 0 & \begin{matrix}1 & 0\\ 0 & 1\end{matrix} \end{bmatrix}$$

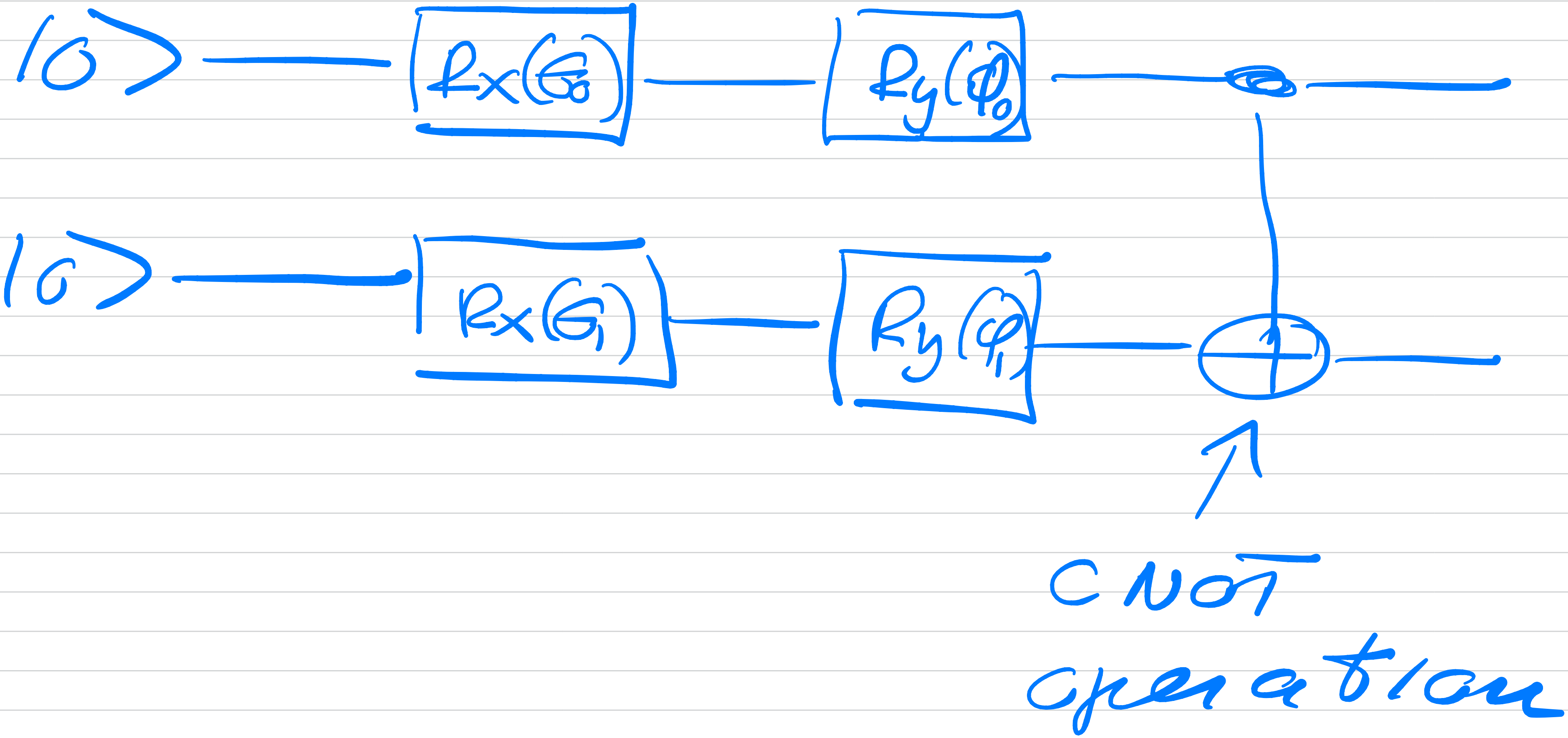
$$CNOT = CX = CX_{01} = \begin{bmatrix} \begin{matrix}1 & 0\\ 0 & 1\end{matrix} & 0 \\ 0 & \begin{matrix}0 & 1\\ 1 & 0\end{matrix} \end{bmatrix}$$

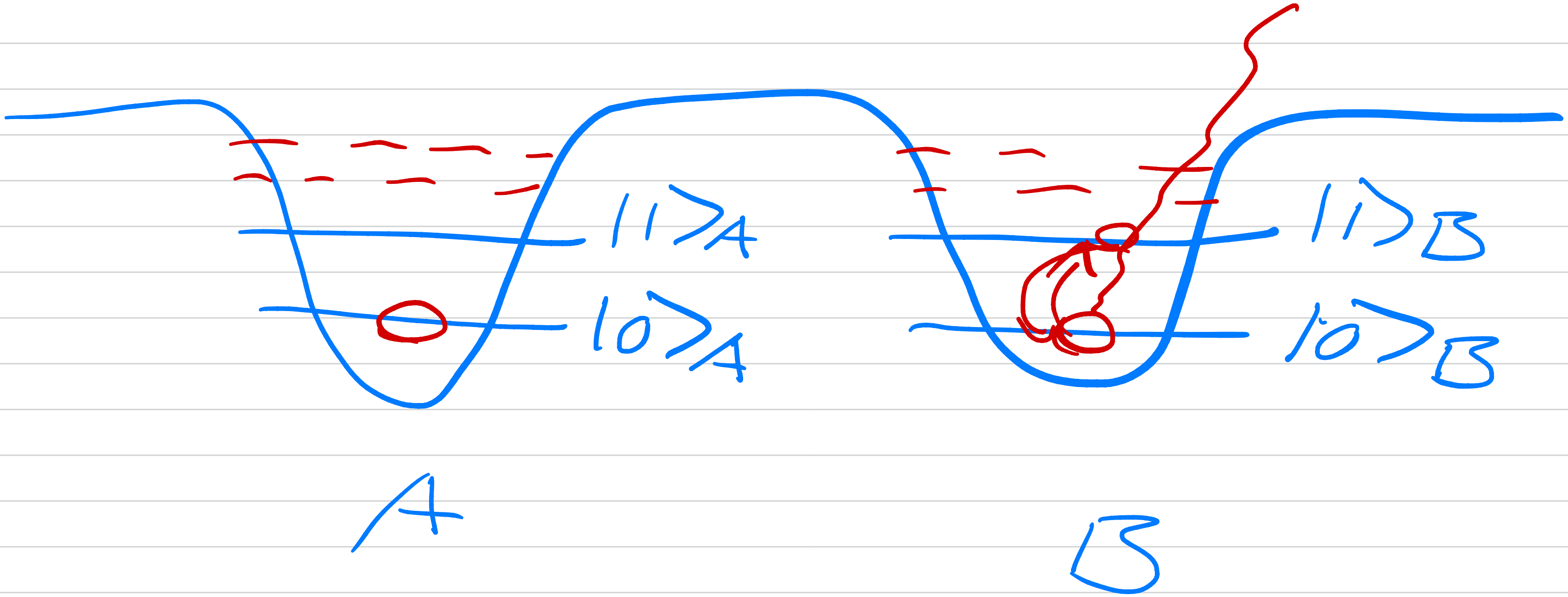
VQE-ansatz 2-qubits

$|\psi(\theta_0, \varphi_0, \theta_1, \varphi_1)\rangle$

with N-qubits we have $2N$ variational parameters

preparing the state





$$|0\rangle_A \otimes |1\rangle_B = |01\rangle$$

$$|\psi\rangle = \alpha |10\rangle + \beta |01\rangle$$

$$(\alpha |0\rangle_A + \beta |1\rangle_A) \otimes (\gamma |0\rangle_B + \delta |1\rangle_B)$$

Lipkin model

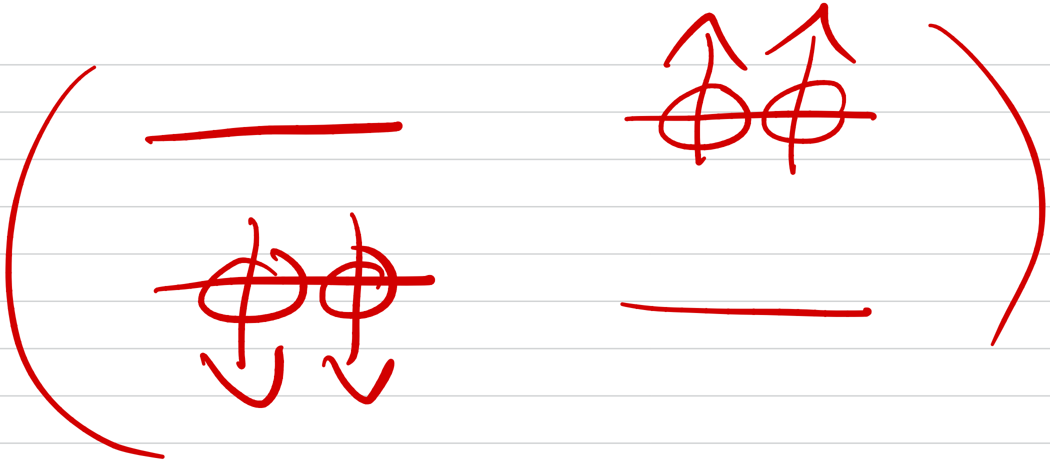
$N=2$ case

$\boxed{1}\boxed{2}$ $\quad \uparrow \quad m_J = +1/2 \qquad +\varepsilon/2$

$p=1,2$ $\quad \boxed{1}\boxed{2}$ $\quad \downarrow \quad m_J = -1/2 \qquad -\varepsilon/2$

$$H = \frac{\varepsilon}{2} \sum_{m_J p} m_J \, a^{+}_{p m_J} a_{p m_J}$$

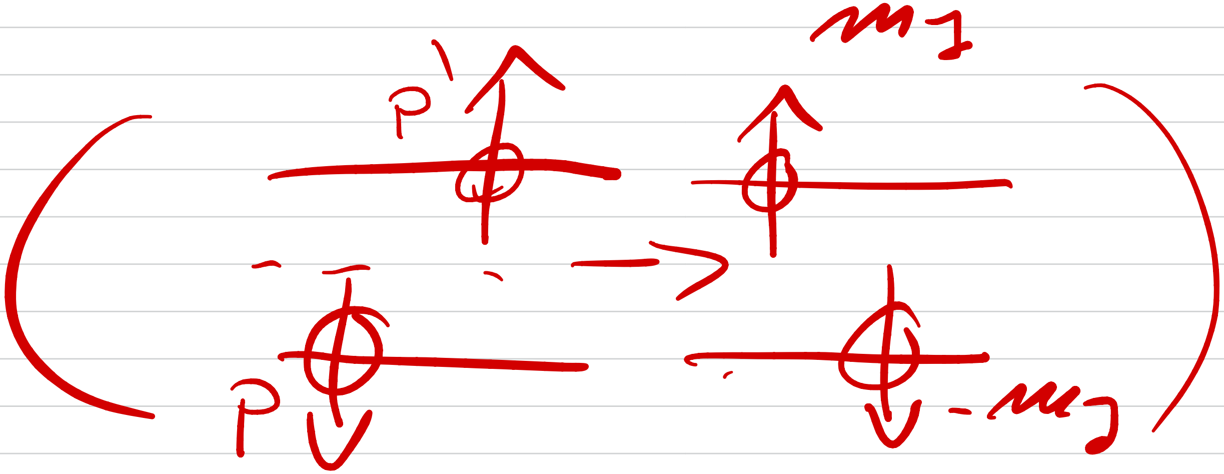
$$- \frac{V}{2} \sum_{m_J p p'} a^{+}_{p m_J} a^{+}_{p' m_J} a_{p' -m_J} a_{p -m_J}$$



$\langle \alpha\beta | v | \gamma\delta \rangle$

$a^\dagger_\alpha a^\dagger_\beta a_\delta a_\gamma$

$$- \frac{W}{2} \sum_{m_j p p'} a^\dagger_{p' m_j} a^\dagger_{p' -m_j} a_{p' m_j} a_{p-m_j}$$



$$H = \frac{\varepsilon}{2} J_z - \frac{V}{2}\left(J_+^2 + J_-^2\right)$$

$$- \frac{W}{2}\left(J_+J_- + J_-J_+ - N\right)$$

$$J_z = \frac{1}{2} \sum_{i=1}^{N} z_i$$

$$J_\pm = \sum_{i=1}^{N} \frac{\left(x_i \pm i y_i\right)}{\sqrt{2}}$$

$$H := \frac{\varepsilon}{2} \sum_i z_i - \frac{V}{2} \sum_{i<j} \left(X_i X_j - Y_i Y_j \right) - \frac{W}{2} \sum_{i<j} \left(X_i X_j + Y_i Y_j \right)$$

$J = 2, \quad N = 4$

$\uparrow \quad \varepsilon/2$ ✓

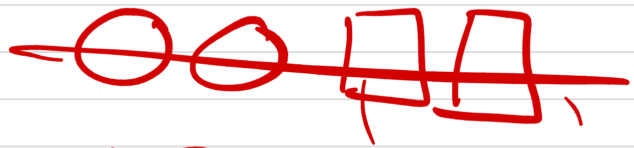
$\downarrow \quad -\varepsilon/2$

p 1 2 3 4

$M_J = -2 \quad \Rightarrow \quad M_J = 0$

2p2h

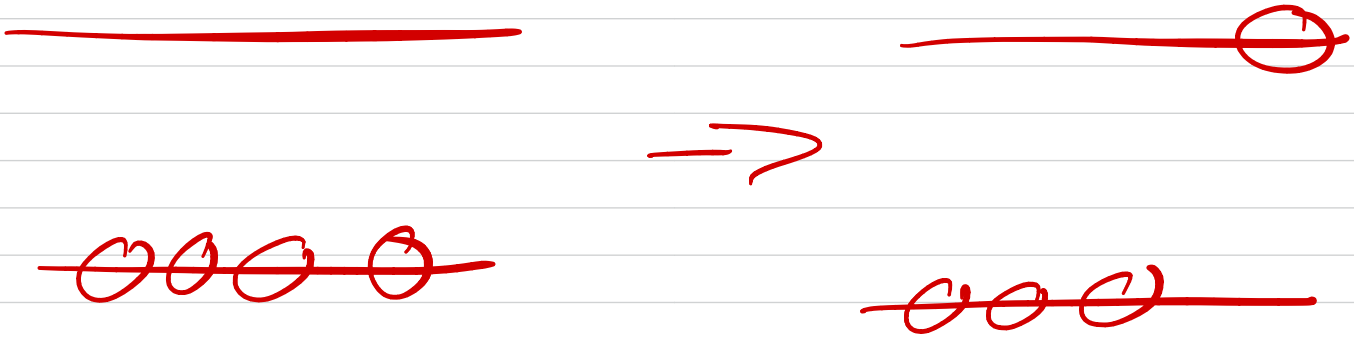
$M_J = +2$



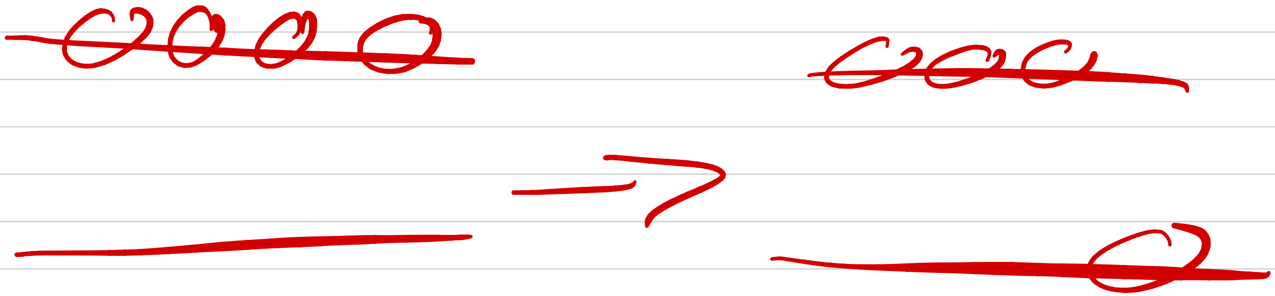
1 2

$M_J = -J, -J+1 \,..\, 0, +1, \,...\, J-1, J$

W



$M_J = -1$



$M_J = +1$

$$N=2 \quad , \quad W=0 \quad , \quad J=1$$

$$M_J = -1, 0, 1$$

$$H(J=1) = \begin{bmatrix} -\varepsilon & 0 & -V \\ 0 & 0 & 0 \\ -V & 0 & \varepsilon \end{bmatrix}$$

$$H|J M_J\rangle \qquad |1 -1\rangle, \ |1, 0\rangle, \ |1 +1\rangle$$

$$H|1 -1\rangle = -\varepsilon|1 -1\rangle - V|11\rangle$$

$$H|10\rangle = 0$$

$$H|1 +1\rangle = \varepsilon|11\rangle - V|1 -1\rangle$$

$$|1,-1\rangle \rightarrow |10\rangle$$

$$|11\rangle \rightarrow |11\rangle$$

$$H(J=1) = -\varepsilon Z - VX$$