Lecture FYS5419, February 21, 2024

1-qubit gates

$$|\varphi\rangle \; — \; \boxed{G} \; — \; |\varphi'\rangle$$

(tikz, quantikz)

$$|\varphi'\rangle = G|\varphi\rangle$$

$\sigma_x \equiv X$ flips a qubit

$X|0\rangle = |1\rangle$

$\sigma_y \equiv Y$ $\quad Y|0\rangle = i|1\rangle$

$\sigma_z \equiv Z$ $\quad Z|0\rangle = |0\rangle$

$Z|1\rangle = -|1\rangle$

S-gate $\quad S = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$ $\quad e^{i\pi/2}$

$$T(\pi/8) = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix}$$

$$R_k = \begin{bmatrix} 1 & 0 \\ 0 & e^{i2\pi/2^k} \end{bmatrix}$$

Hadamard gate

$$|\varphi\rangle \;—\; \boxed{H} \;—\; |\varphi'\rangle$$

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$H^\dagger H =$

$H H^\dagger = \mathbb{1}$

$$HXH = Z =$$

$$\frac{1}{\sqrt{2}}(X+Z)X \qquad H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$\otimes \frac{1}{\sqrt{2}}(X+Z) \qquad X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$= \frac{1}{2}\Big(\underbrace{X^2}_{\mathbb{1}} X + X \overbrace{Z^2}^{\mathbb{1}} + \qquad Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\underbrace{Z X^2}_{\mathbb{1}} + \underbrace{Z X Z}_{-X}\Big)$$

$$= Z \qquad H^T H = \mathbb{1} \qquad H^2 = H$$

$X = HZH$

$HYH = -Y = XYX$

Project 1

$\mathcal{H} = c\mathbb{1} + dX + eZ$

Hamiltonian matrix

$$\begin{bmatrix} \langle 0|H|0\rangle & \langle 0|H|1\rangle \\ \langle 1|H|0\rangle & \langle 1|H|1\rangle \end{bmatrix}$$

$$\boxed{|\varphi\rangle = \alpha|0\rangle + \beta|1\rangle}$$

$$\langle \varphi | \mathcal{H} | \varphi \rangle$$

1) $c \langle \varphi | \mathbb{1} | \varphi \rangle$

2) $d \langle \varphi | X | \varphi \rangle$

3) $e \langle \varphi | Z | \varphi \rangle$

$$d \langle \varphi | HZH | \varphi \rangle = d \langle \varphi H | Z | H \varphi \rangle = d \langle \varphi' | Z | \varphi' \rangle$$

$$H | \varphi \rangle = | \varphi' \rangle$$

$$|\varphi\rangle \;\text{---}\; \boxed{H} \;\text{---}\; |\varphi'\rangle$$

$$|\psi\rangle \;—\; \boxed{H} \;—\; \boxed{Z} \;—\; |\psi'\rangle$$

$$\left(\left[\alpha^* \langle 0| + \beta^* \langle 1|\right] Z \left[\alpha|0\rangle + \beta|1\rangle\right] \right)$$

$$= |\alpha|^2 - |\beta|^2$$

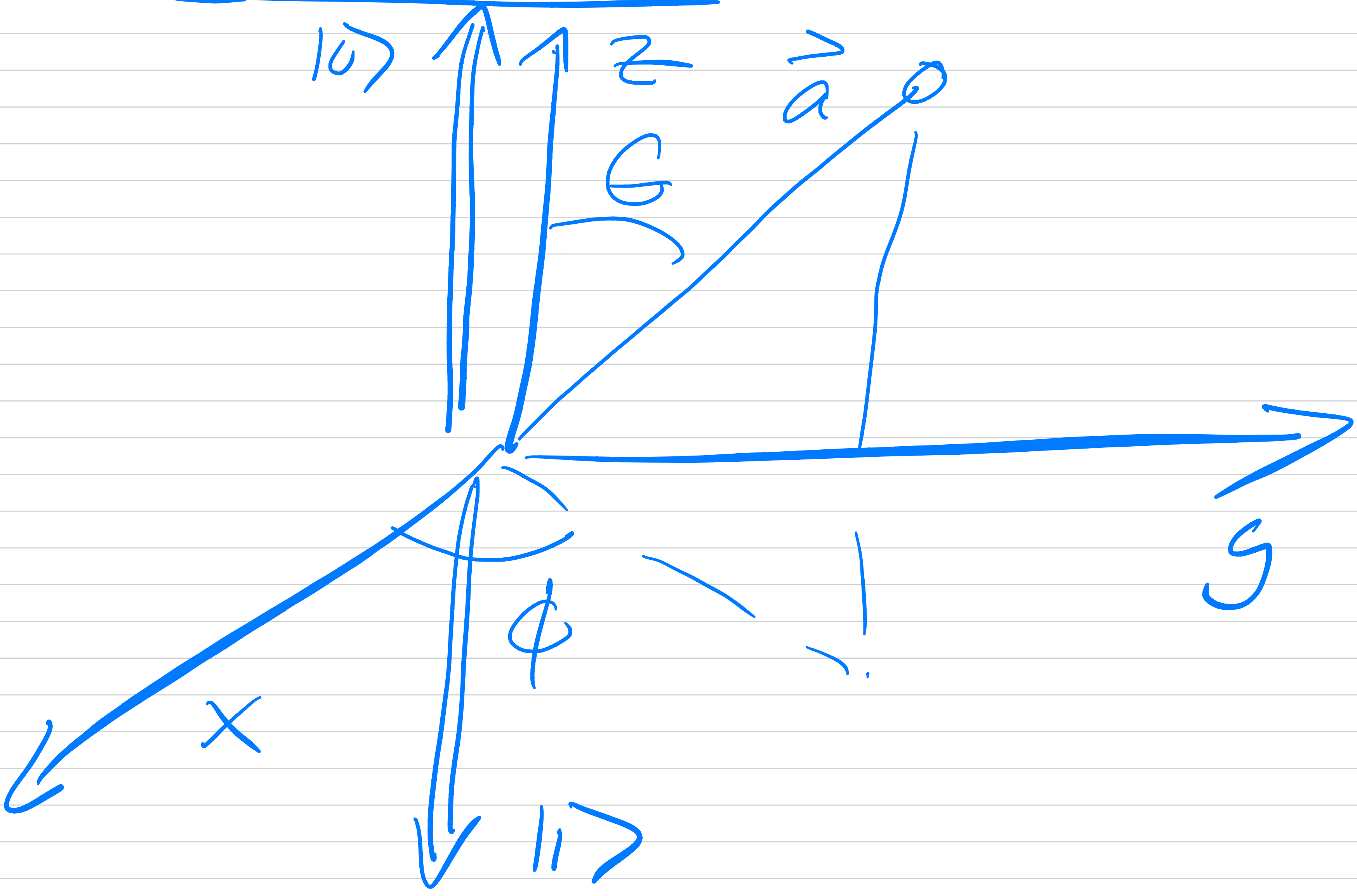
N expt, n_0 times $|0\rangle$

n_1 times $|1\rangle$

$$|\alpha|^2 \simeq \frac{n_0}{N} \qquad N = n_0 + n_1$$

$$|\beta|^2 \simeq \frac{n_1}{N}$$

Bloch Sphere



Every qubit system can be visualized as a point on a 3-Dim sphere with radius — $r = 1$ — and angles ϕ, φ

$$|\psi\rangle = \cos\left(\frac{\theta}{2}\right)|0\rangle + e^{i\phi}\sin\left(\frac{\theta}{2}\right)|1\rangle$$

Rotation matrix

$$R_x(\theta) = e^{i\theta/2 X}$$

$$= \cos(\theta/2)\mathbb{1} - (i\sin\theta/2)X$$

$$R_y(\theta) = \cos\left(\frac{\theta}{2}\right)\mathbb{1} - i\sin(\theta/2)\,Y$$

$$R_z(\theta) = \begin{bmatrix} e^{-i\theta/2} & 0 \\ 0 & e^{i\theta/2} \end{bmatrix}$$

Exercise

$$X R_z(\theta) X = R_z(-\theta)$$

initial state $|\psi\rangle =$

$$R_y(\theta) R_x(\varphi) |0\rangle = |\psi(\theta,\varphi)\rangle$$

ansatz for the wave function

$$\langle \varphi(\theta,\varphi) | \mathcal{H} | \varphi(\theta,\varphi) \rangle = E(\theta,\varphi)$$

we will need for the VQE to optimize $E(\theta,\varphi)$

$$\frac{\partial E}{\partial \theta} = 0 \wedge \frac{\partial E}{\partial \varphi} = 0$$

Variational principle

$$\mathcal{H} |\psi\rangle = E |\psi\rangle$$

$$\mathcal{H} |\psi_n\rangle = E_n |\psi_n\rangle$$

$$\langle \psi_n | \psi_m \rangle = \delta_{nm}$$

ansatz $|\varphi\rangle = \sum_n c_n |\psi_n\rangle$

$$\langle \varphi | \mathcal{H} | \varphi \rangle = \frac{\sum_{nm} c_n^* c_m \langle \psi_n | \mathcal{H} | \psi_m \rangle}{\sum_n |c_n|^2}$$

$$\frac{\sum_n |c_n|^2 E_n}{\sum_n |c_n|^2}$$

$$E_0 < E_1 < E_2 < \ldots < E_\infty$$

$$\langle \varphi | \mathcal{H} | \varphi \rangle \geq E_0$$

Gradient descent

$$W = \{\sigma, \varphi\}$$

$$w \Leftarrow w - \eta \nabla_w E(w)$$

$$\left(f(x) = 0 \quad ; \quad x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)} \right)$$