Lecture FYS5419, February 7, 2024

$$h_0(x)\,\varphi_\alpha(x) = \varepsilon_\alpha\,\varphi_\alpha(x)$$

$$\psi(x_1, x_2) = \varphi_\alpha(x_1) \otimes \varphi_\beta(x_2)$$

$$H_0 = \sum_{i=1}^{2} h_0(x_i)$$

$$H_0\,\psi(x_1, x_2) = E_0\,\psi(x_1, x_2)$$

$$P(x_1, x_2) = p(x_1)\,p(x_2)$$

i.i.d

$$\psi(x_1, x_2) = \frac{1}{\sqrt{2}}\Big(\varphi_\alpha(x_1)\,\varphi_\beta(x_2) - \varphi_\alpha(x_2)\,\varphi_\beta(x_1)\Big)$$

$p(x_1, x_2)$

marginal probability

$$\underline{p}(x_1) = \int_{x_2 \in D} p(x_1, x_2)\, dx_2$$

$$p(x_1) = \sum_{x_2 \in D} p(x_1, x_2)$$

$$\rho_0 = |\psi_0\rangle\langle\psi_0|$$

$$|\psi_0\rangle = \alpha_{00}|00\rangle + \alpha_{01}|01\rangle + \alpha_{10}|10\rangle + \alpha_{11}|11\rangle$$

$$|ij\rangle = |i\rangle_A \otimes |j\rangle_B$$

Example

$$|\phi^+\rangle_{AB} = \frac{1}{\sqrt{2}}\left[|0\rangle_A \otimes |0\rangle_B + |1\rangle_A \otimes |1\rangle_B \right]$$

$$= \frac{1}{\sqrt{2}}\left[|00\rangle + |11\rangle \right]$$

$d = 2$ $\qquad \lambda_1 = \lambda_2 = \frac{1}{\sqrt{2}}$

$$\lambda_1^2 + \lambda_2^2 = 1$$

$$\rho = |\phi^+\rangle_{AB}\langle\phi^+|_{AB}$$

$$\frac{1}{2}\Big[\, |00\rangle\langle 00| + |00\rangle\langle 11| + |11\rangle\langle 00| + |11\rangle\langle 11| \,\Big]$$

$$= \frac{1}{2}\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

$$Tr_B(\rho)$$

we want to trace out qubit 0 and 1 of subsystem A

$$I_A \otimes \langle i|_B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

qubit 0

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \qquad \left(|0\rangle\langle 0| = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \right)$$

qubit 1

$$\vdots \qquad \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \qquad \left(= |1\rangle\langle 1| \right)$$

$$\rho_A = \mathrm{tr}_B(\rho_{AB}) = \mathrm{tr}_B(\rho)$$

$$= \frac{1}{2}\mathbb{1}$$

$$\mathrm{tr}\,\rho_A = \frac{1}{2} + \frac{1}{2} = 1$$

$$\mathrm{tr}(\rho_A^2) = \frac{1}{2} < 1 \text{, mixed state}$$

$$\mathrm{tr}(\rho_A^2) = 1 \quad \text{pure state}$$

show $\rho_B = \rho_A = \frac{1}{2}\mathbb{1}$

$$\rho_{AB} \neq \rho_A \otimes \rho_B$$

The joint state of 2 qubits (entangled or not) is a pure state. It is known exactly

$$|\phi^+\rangle_{AB} = \frac{1}{\sqrt{2}}\left(|00\rangle_{AB} + |11\rangle_{AB}\right)$$

However, looking at individual qubits for $|\phi^+\rangle_{AB}$, we find that they are mixed. We do not have the full knowledge of their states.

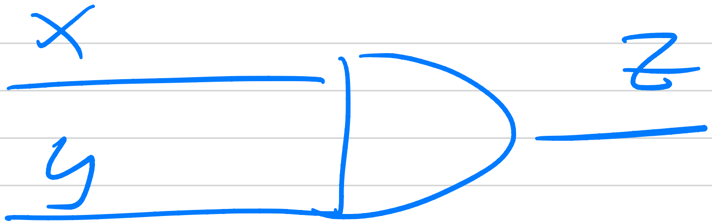
Classical (logic) gates & circuits

NOT GATE

reversible

x	y
0	1
1	0

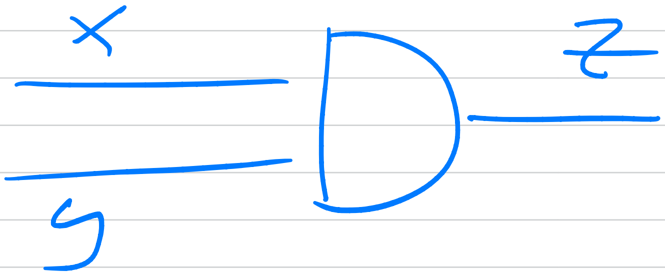
XOR GATE



irreversible

x	y	z
0	0	0
0	1	1
1	0	1
1	1	0

NAND



x	y	z
0	0	1
0	1	1
1	0	1
1	1	0

Quantum computing

- input and outputs are n-qubit states
  (2-qubit $|0\rangle_A \otimes |1\rangle_B$)
- 1-qubit gates
  2-qubit — " — ...

initial state $|\psi\rangle$
output state $|\varphi\rangle$

$$|\varphi\rangle = G|\psi\rangle$$

unitary transformations

- one-qubit gates
initial state $|\psi\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle$
output state $|\varphi\rangle = \alpha_0'|0\rangle + \alpha_1'|1\rangle$

$$G = \begin{bmatrix} g_{00} & g_{01} \\ g_{10} & g_{11} \end{bmatrix}$$

$$GG^{+} = G^{+}G = \underline{\mathbb{1}}$$

$$|\varphi\rangle = G|\psi\rangle$$

$$\begin{bmatrix} \alpha_0' \\ \alpha_1' \end{bmatrix} = \begin{bmatrix} g_{00} & g_{01} \\ g_{10} & g_{11} \end{bmatrix} \begin{bmatrix} \alpha_0 \\ \alpha_1 \end{bmatrix}$$