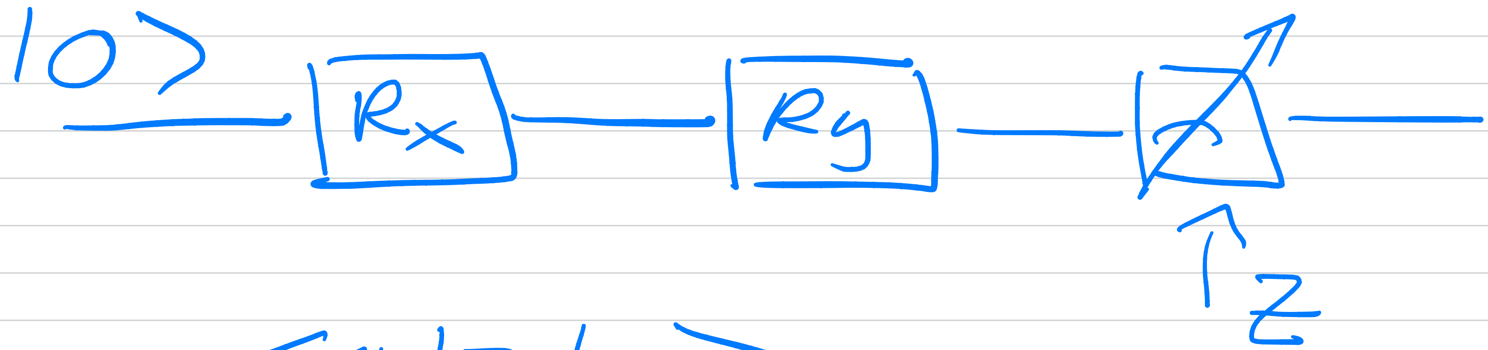
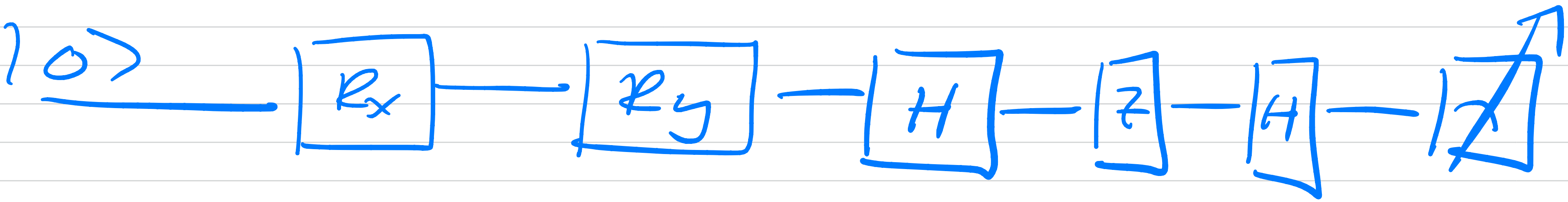
Lecture FYS5419, February 28, 2024

FYS5419, FEB 28, 2024



$$\langle\psi|Z|\psi\rangle$$



$$\langle\psi|X|\psi\rangle$$

$$X = HZH \qquad H^2 = \mathbb{1}$$

$$R_x(\theta) = \cos\left(\frac{\theta}{2}\right)\mathbb{1} - i\sin\frac{\theta}{2}X$$

General situation

$$P = \bigotimes_{i=0}^{N-1} P_i$$

string of Pauli X, Y, Z and $\mathbb{1}$

$N = \#$ qubits

$N = 2$ case

$$H_I = H_Z Z \otimes Z + H_X X \otimes X$$

$$H_0 = \alpha \cdot I^2 + \beta I \otimes Z + \gamma Z \otimes \mathbb{1} + \delta Z \otimes Z$$

$$H = \begin{bmatrix} \mathcal{E}_{00}+H_z & 0 & 0 & H_x \\ 0 & \mathcal{E}_{01}-H_z & H_x & 0 \\ 0 & H_x & \mathcal{E}_{10}+H_z & 0 \\ H_x & 0 & 0 & \mathcal{E}_{11}-H_z \end{bmatrix}$$

$$\alpha = (\mathcal{E}_{00}+\mathcal{E}_{01}+\mathcal{E}_{10}+\mathcal{E}_{11})/4$$

$$\beta = (\mathcal{E}_{00}-\mathcal{E}_{01}+\mathcal{E}_{10}-\mathcal{E}_{11})/4$$

$$\gamma = (\mathcal{E}_{00}+\mathcal{E}_{01}-\mathcal{E}_{10}-\mathcal{E}_{11})/4$$

$$\delta = (\mathcal{E}_{00}-\mathcal{E}_{01}-\mathcal{E}_{10}+\mathcal{E}_{11})/4$$

$$H_0|00\rangle = \mathcal{E}_{00}|00\rangle \quad \text{etc}$$

$$P = U^{+} M U$$

$$X = H Z H$$

$\underset{2\times 2}{I} \otimes Z$ has U as swap gate

$$\text{SWAP} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\text{SWAP}|01\rangle = |10\rangle$$
$$\text{SWAP}|10\rangle = |01\rangle$$

$$SWAP\,(1 \otimes Z)\,SWAP$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

Gradient methods (Newtons)

$f(x) = 0$ Our case $f(x) \Rightarrow \nabla_G E$

$$x_{k+1} = x_k - f(x_k) / f'(x_k)$$

Taylor expand $f(x_{k+1})$

$$x_{k+1} = x_k + \Delta x$$

$$\theta_{k+1} = \theta_k - \frac{1}{\nabla_\theta^2 E(\theta_k)} \nabla_\theta E(\theta_k)$$

$$\theta_{k+1} = \theta_k - \underbrace{H^{-1}}_{\text{Hessian}} \cdot \nabla_\theta E(\theta_k)$$

1) simplest way $H^{-1} \Rightarrow$ const
Gradient descent

2) quasi-Newton Methods
- Broyden
- Broyden-Fletcher-...
- Powell ...