

# Notes FYS5419, January 17, 2024

# Harmonic oscillator (1-dim)

$$H_0(x) = -\frac{1}{2} \frac{d^2}{dx^2} + \frac{1}{2} k x^2 \quad x \in (-\infty, +\infty)$$

$$H_0(x) \psi_n(x) = E_n \psi_n(x)$$

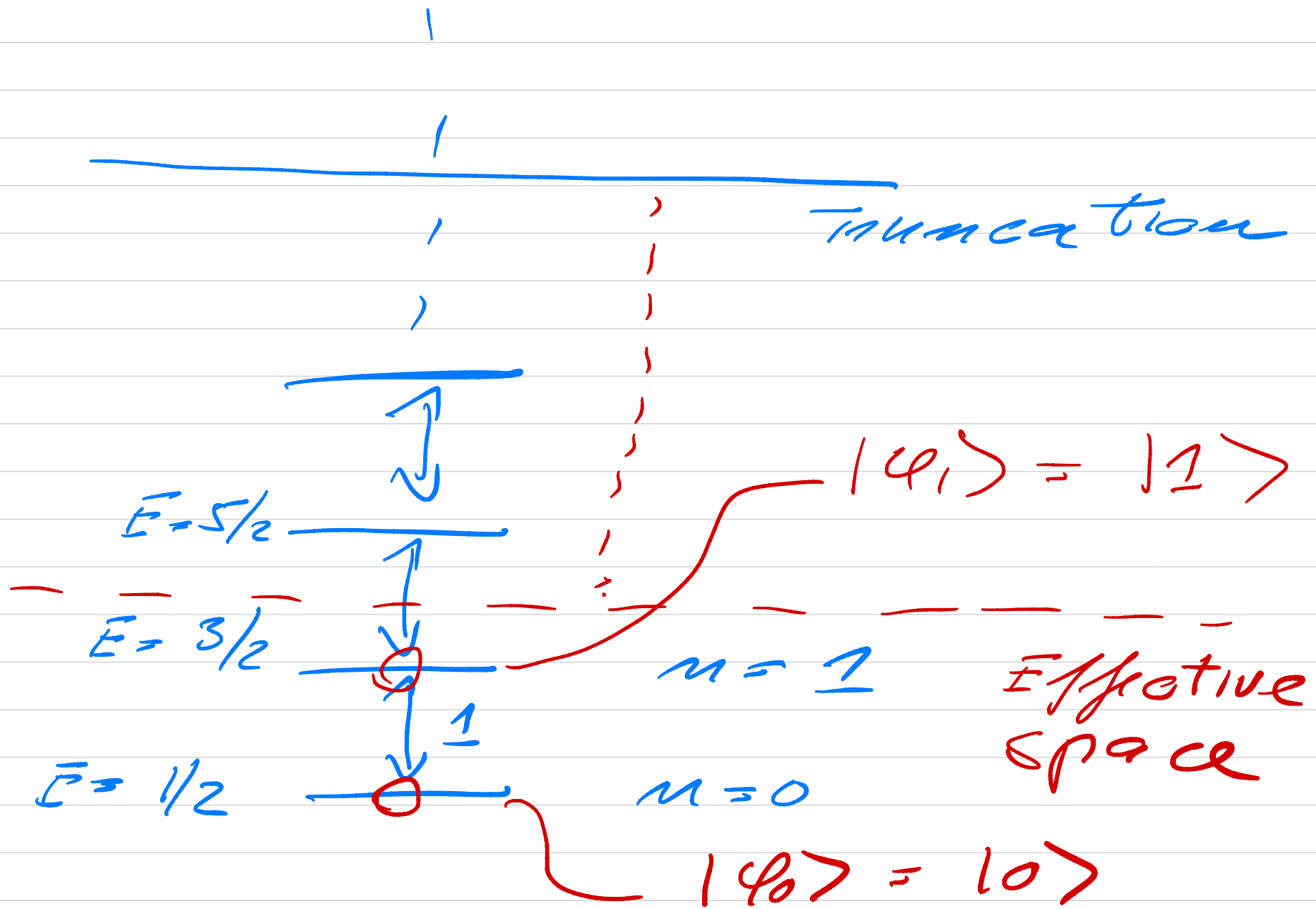
$$E_n = n + 1/2$$

$$\psi_n(x) = H_n(x) e^{-\frac{1}{2} x^2}$$

$$\langle \psi_n | \psi_m \rangle = \int_{-\infty}^{\infty} dx \psi_n^*(x) \psi_m(x) = \delta_{nm}$$

$\psi_n$  is an ONB

(orthogonal, normalized basis)



our computational basis  
is  $|0\rangle$  and  $|1\rangle$

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \wedge \quad |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\langle 1|0\rangle = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = 0$$

$$\langle 0|0\rangle = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = 1$$

$$\langle 1|1\rangle = 1$$

with an ONB

$$|\psi_i\rangle = \sum_j c_{ij} |\varphi_j\rangle$$

$$|\psi_0\rangle = c_{00}|0\rangle + c_{01}|1\rangle$$

( $|\psi_0\rangle$ )

$$P = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$P|\psi_0\rangle = P(c_{00}|0\rangle + c_{01}|1\rangle)$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \left( c_{00} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + c_{01} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right)$$

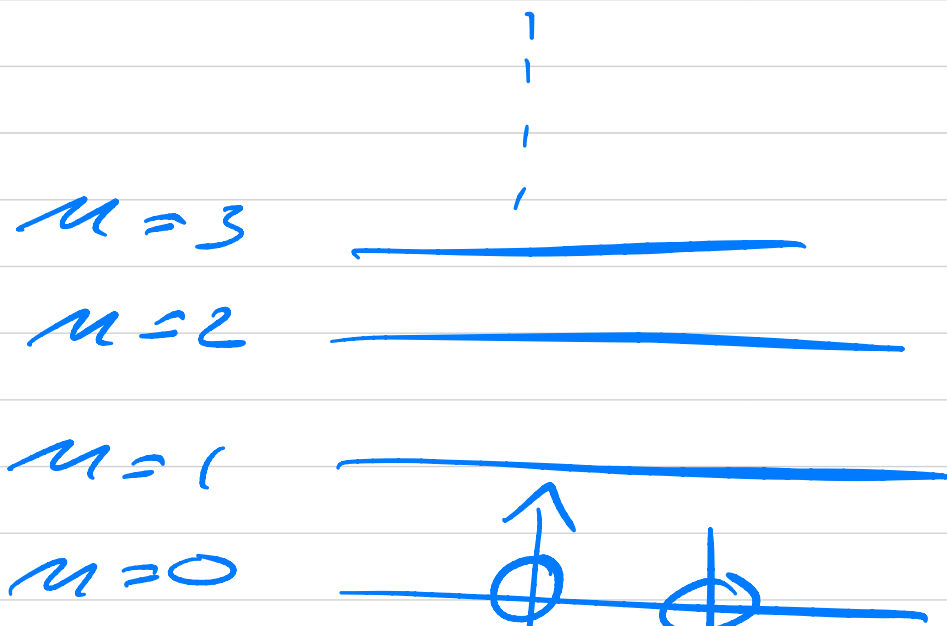
$$= c_{00} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = c_{00}|0\rangle$$

$P$  projects out a specific component of  $|\psi_0\rangle$  given by the computational basis  $|0\rangle$  and  $|1\rangle$

$$C_{00} = \langle \psi_0 | 0 \rangle \quad \left( \int dx \psi_0^*(x) \psi_0(x) \right)$$

$$C_{01} = \langle \psi_0 | 1 \rangle$$

Harmonic oscillator



$$\psi_0(x) = |\psi_0\rangle$$

$$S = 1/2$$

$$m_s = +1/2$$

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$m_s = -1/2$$

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} \psi_0(x_0) \\ \psi_0(x_1) \\ \vdots \\ \psi_0(x_{n-1}) \end{bmatrix}$$

$$n=0 \quad s=1/2 \quad m_s=1/2$$

$$|\psi_0(x)\rangle = |n=0 \ s=1/2 \ m_s=1/2\rangle \\ = |0 \ 1/2 \ 1/2\rangle$$

$$|\psi_n(x)\rangle = |n \ s \ m_s\rangle$$

$$\begin{bmatrix} \psi_0(x_0) \\ \psi_0(x_1) \\ \vdots \\ \psi_0(x_{n-1}) \end{bmatrix} \otimes \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$