

¹ A geometric framework for capturing high-resolution
² insights into conceptual knowledge and learning in
³ classroom-like settings

⁴ Paxton C. Fitzpatrick¹, Andrew C. Heusser^{1, 2}, and Jeremy R. Manning^{1, *}

¹Department of Psychological and Brain Sciences

Dartmouth College, Hanover, NH 03755, USA

²Akili Interactive Labs

Boston, MA 02110, USA

*Corresponding author: Jeremy.R.Manning@Dartmouth.edu

⁵ **Abstract**

We develop a mathematical framework, based on natural language processing models, for tracking and characterizing the acquisition of conceptual knowledge in real-world educational contexts. Our approach embeds course content in a high-dimensional conceptual space, where nearby coordinates reflect similar or related concepts. We test our approach using behavioral data from participants who watched two lecture videos from the Khan Academy platform, interleaved between three short multiple-choice quizzes. We applied our framework to the videos' transcripts and the text of the quiz questions to quantify the conceptual content presented in each moment of video and knowledge probed by each quiz question. We used these embeddings, along with participants' quiz responses, to track how the learners' knowledge changed after watching each video. Our findings demonstrate how a small set of quiz questions may be used to obtain rich and meaningful high-resolution insights into individual students' knowledge, and how it changes over time as they learn.

19

Introduction

20 Suppose that a teacher had access to a complete “map” of everything a student knew. Defining
21 what such a map might even look like, let alone how it might be constructed or filled in, is itself a
22 non-trivial problem. But if a teacher *were* to gain access to such a map, how might it change their
23 ability to teach that student? Perhaps they might start by checking how well the student knew
24 the to-be-learned information already, or how much they knew about related concepts. For some
25 students, they could potentially optimize their teaching efforts to maximize efficiency by focusing
26 primarily on not-yet-known content. For other students (or other content areas), it might be more
27 effective to optimize for direct connections between already known content and new material.
28 Observing how the student’s knowledge changed over time, in response to their teaching, could
29 also help to guide the teacher towards the most effective strategy for that individual student.

30 Designing and building procedures and tools for mapping out knowledge touches on deep
31 questions about what it means to learn. For example, how do we acquire conceptual knowledge?
32 Memorizing course lectures or textbook chapters by rote can lead to the superficial *appearance*
33 of understanding the underlying content, but achieving true conceptual understanding seems
34 to require something deeper and richer. Does conceptual understanding entail connecting newly
35 acquired information to the scaffolding of one’s existing knowledge or experience [6, 10, 13, 14, 57]?
36 Or weaving a lecture’s atomic elements (e.g., its component words) into a structured network
37 that describes how those individual elements are related? Conceptual understanding could also
38 involve building a mental model that transcends the meanings of those individual atomic elements
39 by reflecting the deeper meaning underlying the gestalt whole [34, 38, 54].

40 The difference between “understanding” and “memorizing,” as framed by researchers in ed-
41 ucation, cognitive psychology, and cognitive neuroscience [e.g., 19, 25, 30, 38, 54] has profound
42 analogs in the fields of natural language processing and natural language understanding. For
43 example, considering the raw contents of a document (e.g., its constituent symbols, letters, and

44 words) might provide some information about what the document is about, just as memorizing a
45 passage might provide some ability to answer simple questions about it [e.g., whether it contains
46 words related to furniture versus physics; 7, 8, 37]. However, modern natural language process-
47 ing models [e.g., 9, 11, 45] also attempt to capture the deeper meaning *underlying* those atomic
48 elements. These models consider not only the co-occurrences of those elements within and across
49 documents, but also patterns in how those elements appear across different scales (e.g., sentences,
50 paragraphs, chapters, etc.), the temporal and grammatical properties of the elements, and other
51 high-level characteristics of how they are used [39, 40]. According to these models, the deep
52 conceptual meaning of a document may be captured by a feature vector in a high-dimensional
53 representation space, where nearby vectors reflect conceptually related documents. A model that
54 succeeds at capturing an analog of “understanding” is able to assign nearby feature vectors to
55 two conceptually related documents, *even when the words contained in those documents have very little*
56 *overlap*.

57 Given these insights, what form might the representation of the sum total of a person’s knowl-
58 edge take? First, we might require a means of systematically describing or representing the nearly
59 infinite set of possible things a person could know. Second, we might want to account for potential
60 associations between different concepts. For example, the concepts of “fish” and “water” might be
61 associated in the sense that fish live in water. Third, knowledge may have a critical dependency
62 structure, such that knowing about a particular concept might require first knowing about a set of
63 other concepts. For example, understanding the concept of a fish swimming in water first requires
64 understanding what fish and water *are*. Fourth, as we learn, our “current state of knowledge”
65 should change accordingly. Learning new concepts should both update our characterizations of
66 “what is known” and also unlock any now-satisfied dependencies of those newly learned concepts
67 so that they are “tagged” as available for future learning.

68 Here we develop a framework for modeling how knowledge is acquired during learning. The
69 central idea is to use text embedding models to define the coordinate systems of two maps: (a) a
70 *knowledge map* that describes the extent to which each concept is currently known and (b) a *learning*
71 *map* that describes changes in knowledge over time. Each location on these maps represents

72 a single concept, and the maps' geometries are defined such that related concepts are located
73 nearby in space. We use this framework to analyze and interpret behavioral data collected from
74 an experiment that had participants watch and answer multiple-choice questions about a series of
75 recorded course lectures.

76 Our primary research goal is to advance our understanding of what it means to acquire deep,
77 real-world conceptual knowledge. Traditional laboratory approaches to studying learning and
78 memory (e.g., list learning studies) often draw little distinction between memorization and under-
79 standing. Instead, these studies typically focus on whether information is effectively encoded or
80 retrieved, rather than whether the information is *understood*. Approaches to studying conceptual
81 learning, such as category learning experiments, can begin to investigate the distinction between
82 memorization and understanding, often by training participants to distinguish arbitrary or ran-
83 dom features in otherwise meaningless categorized stimuli. However the objective of real-world
84 training, or learning from life experiences more generally, is often to develop new knowledge
85 that may be applied in *useful* ways in the future. In this sense, the gap between modern learning
86 theories and modern pedagogical approaches and classroom learning strategies is enormous: most
87 of our theories about *how* people learn are inspired by experimental paradigms and models that
88 have only peripheral relevance to the kinds of learning that students and teachers actually seek
89 [25, 38]. To help bridge this gap, our study uses course materials from real online courses to inform,
90 fit, and test models of real-world conceptual learning. We also provide a demonstration of how
91 our models can be used to construct "maps" of what students know, and how their knowledge
92 changes with training. In addition to helping to visualize knowledge (and changes in knowledge),
93 we hope that such maps might lead to real-world tools for improving how we educate.

94 Results

95 At its core, our main modeling approach is based around a simple assumption that we sought to
96 test empirically: all else being equal, knowledge about a given concept is predictive of knowledge
97 about similar or related concepts. From a geometric perspective, this assumption implies that

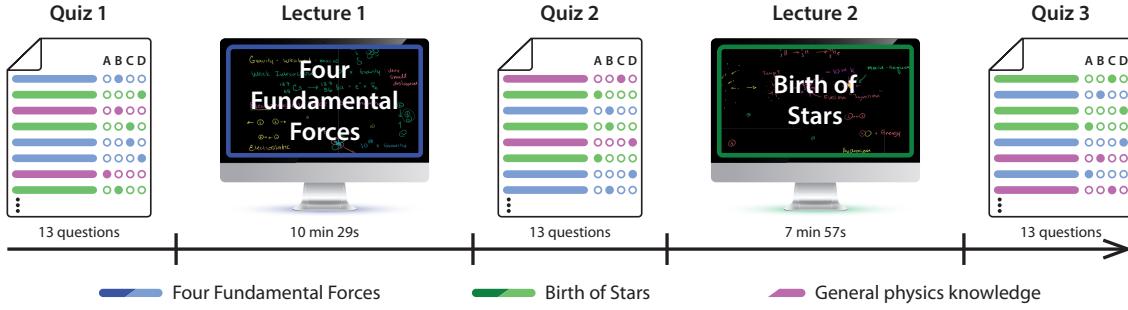


Figure 1: Experimental paradigm. Participants alternate between answering 13-question multiple-choice quizzes and watching two Khan Academy lectures. Each quiz contains a mix of 5 questions about lecture 1, 5 questions about lecture 2, and 3 questions about general physics knowledge. The specific questions reflected on each quiz, and the orders of each quiz's questions, were randomized across participants.

knowledge is fundamentally “smooth.” In other words, as one moves through a space representing an individual’s knowledge (where similar concepts occupy nearby coordinates), their “level of knowledge” should change relatively gradually throughout that space. To begin to test this smoothness assumption, we sought to track participants’ knowledge and how it changed over time in response to training.

We asked participants in our study to complete brief multiple-choice quizzes before, between, and after watching two lecture videos from the Khan Academy [33] platform (Fig. 1). The first lecture video, entitled *Four Fundamental Forces*, discussed the four fundamental forces in physics: gravity, strong and weak interactions, and electromagnetism. The second, entitled *Birth of Stars*, provided an overview of our current understanding of how stars form. We selected these lessons to be (a) accessible to a broad audience, i.e., requiring minimal prerequisite knowledge to understand; (b) conceptually related to each other, i.e., covering at least *some* similar or overlapping content; and (c) largely independent of each other, i.e., focused on sufficiently different material that understanding one did not require having seen the other. The two videos we selected are introductory lectures that both belong to Khan Academy’s “Cosmology and Astronomy” course domain, but are taken from different lecture series (“Scale of the Universe” and “Stars, Black Holes, and Galaxies” for the first and second lectures, respectively).

We then created a pool of multiple-choice quiz questions that would enable us to test partici-

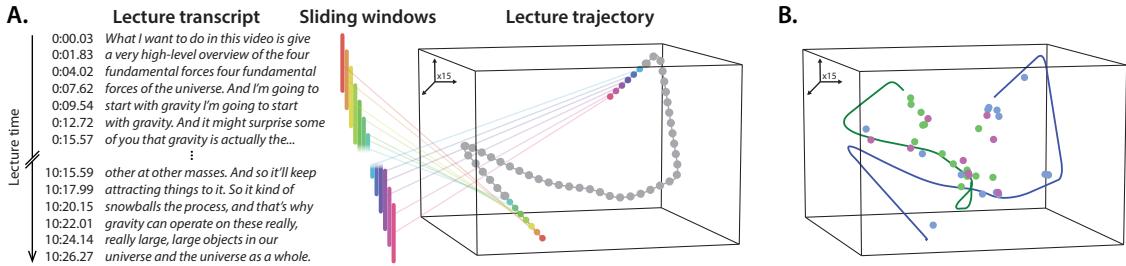


Figure 2: Constructing video content trajectories. **A. Building a document pool from sliding windows of text.** We decompose each video’s transcript into a series of overlapping sliding windows. The set of transcript snippets (across all windows) may be treated as a set of “documents” for training a text embedding model. After training a text embedding model on the two videos’ sliding windows, we construct “trajectories” through text embedding space by joining the embedding coordinates of successive sliding windows from each video. **B. Embedding multiple videos and questions.** Applying the same text embedding approach to each video, along with the text of each question in our pool (Tab. S1), results in one trajectory per video and one embedding coordinate (dot) per question (blue: *Four Fundamental Forces*; green: *Birth of Stars*; purple: general physics knowledge). Here we have projected the 15-dimensional embeddings into a 3D space using Uniform Manifold Approximation and Projection [UMAP; 41].

116 pants’ knowledge about each individual lecture, as well as related content not specifically presented
 117 in either video (see Tab. S1). Participants answered questions randomly drawn from each content
 118 area (lecture 1, lecture 2, and general physics knowledge) on each of the three quizzes. Quiz 1 was
 119 intended to assess participants’ “baseline” knowledge before training, quiz 2 assessed knowledge
 120 after watching the *Four Fundamental Forces* video (i.e., lecture 1), and quiz 3 assessed knowledge
 121 after watching the *Birth of Stars* video (i.e., lecture 2).

122 To study how participants’ conceptual knowledge changed over the course of the experiment,
 123 we first sought to characterize the abstract concepts presented to them in each of the two lectures.
 124 We adapted an approach we developed in prior work [27] to extract the latent themes from the
 125 lectures’ contents using a topic model [8]. Briefly, topic models take as input a collection of
 126 text documents and learn a set of “topics” (i.e., latent themes) from their contents. Once fit, a
 127 topic model can be used to transform arbitrary (potentially new) documents into sets of “topic
 128 proportions,” describing the weighted blend of learned topics reflected in their texts. We parsed
 129 automatically generated transcripts of the two lectures into overlapping sliding windows, and
 130 used these as documents to fit our model (Fig. 2A; see *Constructing text embeddings of multiple*
lectures and questions). The ($k=15$) topics

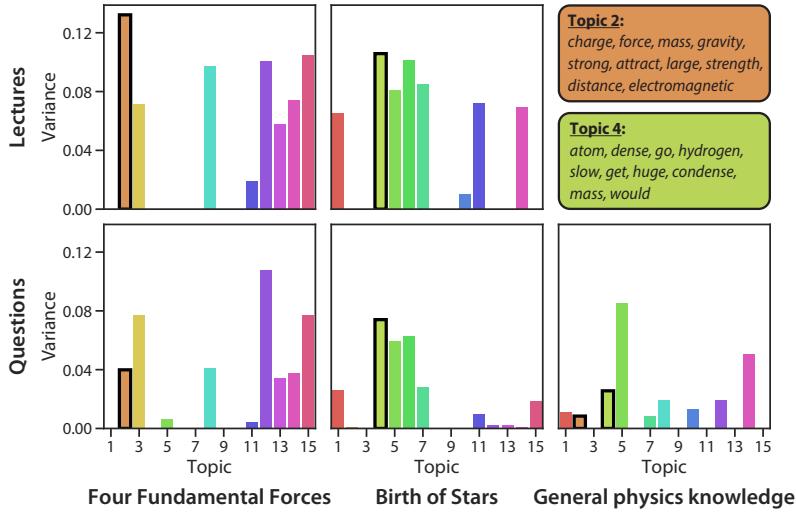


Figure 3: Lecture and question topic overlap. The bar plots display the variability in topic weights across lecture timepoints (top panels) and questions (bottom panels); colors denote topics. The top-weighted words from the most “expressive” (i.e., variable across observations) topic from each lecture are displayed in the upper right (orange: topic 2; yellow-green: topic 4). The top-weighted words from the full set of topics may be found in Table S2.

132 We trained a topic model [8] using sliding windows of text from the two lectures’ transcripts
 133 (see *Constructing text embeddings of multiple lectures and questions*). We also used the same model
 134 (i.e., trained on the videos’ transcripts) to embed the text of each question in our pool. This yielded,
 135 for each second of each video, and for each question, a single 15-dimensional topic vector—i.e.,
 136 a coordinate in a text embedding space (Fig. 2). Intuitively, each dimension of the embedding
 137 space corresponds to a “theme” or “topic” reflected in some part(s) of the videos (Tab. S2), and
 138 the coordinates in embedding space denote the blend of themes reflected by a particular excerpt
 139 of text (e.g., from part of a video’s transcript, from a question, etc.).

140 Given that we trained the text embedding model using the video transcripts, we wondered
 141 whether the questions that were (ostensibly, by design) “about” the content of each lecture would
 142 “match up” correctly with the lectures. In other words, we hoped that the text embeddings would
 143 capture something about the deeper conceptual content of the lectures, beyond surface details such
 144 as exact wording choices. If so, when we embed *new* text outside of the model’s training set, we
 145 should see a correspondance between the embeddings of the training data (i.e., snippets of text

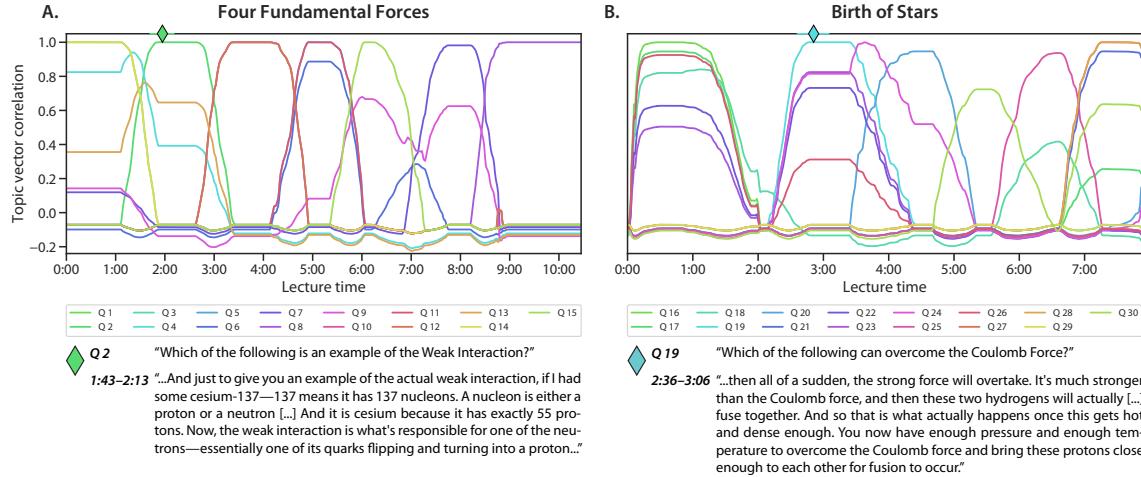


Figure 4: Which parts of each lecture are captured by each question? Each panel displays timeseries plots showing how each question’s topic vector correlates with each video timepoint’s topic vector (Panel A.: correlations for the *Four Fundamental Forces* lecture and associated questions; Panel B.: correlations for the *Birth of Stars* lecture and associated questions). The colors denote question identities. The diamonds in each panel denote the moment of peak correlation between the indicated questions, in the indicated lectures. The associated questions’ text, and snippets of the lectures’ transcripts in the best-matching sliding windows, are displayed at the bottom of the figure.

from the lectures’ transcripts) and other text that reflects related concepts (e.g., questions *about* each lecture). Further, although the content from any given moment from a lecture might stray from the average content (across all timepoints), we hoped that *variability* in each topic’s expression over timepoints within a lecture would match up with the variability in topic expressions for questions about that lecture. Intuitively, the variability in the expression of a given topic relates to how much “information” [18] the lecture (or questions) reflect about that topic. When we compared the variability in topic weights across each lecture’s timepoints with the variability in topic weights across each question set, we found a strong correspondence (Fig. 3). The most variable topics from the *Four Fundamental Forces* lecture, and questions about that lecture, are 2, 3, 8, 12, 13, 14, and 15. The most variable topics from the *Birth of Stars* lecture, and questions about that lecture, are 1, 4, 5, 6, and 7. This strong overlap between the lectures and questions specifically about each lecture indicates that the topic model captures some of the underlying conceptual content.

Although a single lecture may be organized around a single broad theme at a coarse scale, at a

159 finer scale each moment of a lecture typically covers a narrower range of content. We wondered
160 whether a text embedding model trained on the lectures' transcripts might capture some of this
161 finer scale content. For example, if a particular question asks about the content from one small
162 part of a lecture, we wondered whether our text embedding model could be used to automatically
163 identify the "matching" moment(s) in the lecture. When we correlated each question's topic vector
164 with the topic vectors for each second of the lectures, we found some evidence that each question is
165 temporally specific (Fig. 4). In particular, most questions' topic vectors were maximally correlated
166 with a well-defined (and relatively narrow) range of timepoints from their corresponding lectures,
167 and the correlations fell off sharply outside of that range. We also examined the best-matching
168 intervals for each question qualitatively by comparing the text of the question to the text of the most-
169 correlated parts of the lectures. Despite that the questions were excluded from the text embedding
170 model's training set, in general we found (through manual inspection) a close correspondence
171 between the conceptual content that each question covered and the content covered by the best-
172 matching moments of the lectures. Two representative examples are shown at the bottom of
173 Figure 4.

174 The ability to quantify how much each question is "asking about" the content from each moment
175 of the lectures could enable high-resolution insights into participants' knowledge. Traditional
176 approaches to estimating how much a student "knows" about the content of a given lecture entail
177 computing the proportion of correctly answered questions. But if two students receive identical
178 scores on an exam, might our modeling framework help us to gain more nuanced insights into the
179 *specific* content that each student has mastered (or failed to master)? For example, a student who
180 misses three questions that were all about the same concept (e.g., concept A) will have gotten the
181 same *proportion* of questions correct as another student who missed three questions about three
182 *different* concepts (e.g., A, B, and C). But if we wanted to fill in the "gaps" in the two students'
183 understandings, we might do well to focus on concept A for the first student, but to also add in
184 materials pertaining to concepts B and C for the second student.

185 We developed a simple formula (Eqn. 1) for using a participant's responses to a small set
186 of multiple-choice questions to estimate how much the participant "knows" about the concept

reflected by any arbitrary coordinate, x , in text embedding space (e.g., the content reflected by any moment in a lecture they had watched; see *Estimating dynamic knowledge traces*). Essentially, the estimated knowledge at the coordinate is given by the weighted average proportion of quiz questions the participant answered correctly, where the weights reflect how much each question is “about” the content at x . When we apply this approach to estimate the participant’s knowledge about the content presented in each moment of each lecture, we can obtain a detailed timecourse describing how much “knowledge” the participant has about any part of the lecture. As shown in Figure 5, we can also apply this approach separately for the questions from each quiz the participants took throughout the experiment. From just 13 questions per quiz, we obtain a high-resolution snapshot (at the time each quiz was taken) of what the participants knew about any moment’s content, from either of the two lectures they watched (comprising a total of 1106 samples across the two lectures).

Of course, even though the timecourses in Figure 5A and C provide detailed *estimates* about participants’ knowlege, those estimates are only *useful* to the extent that they accurately reflect what participants actually know. As one sanity check, we anticipated that the knowledge estimates should show a content-specific “boost” in participants’ knowledge after watching each lecture. In other words, if participants learn about each lecture’s content when they watch each lecture, the knowledge estimates should reflect that. After watching the *Four Fundamental Forces* lecture, participants should show more knowledge for the content of that lecture than they had before, and that knowledge should persist for the remainder of the experiment. Specifically, knowledge about that lecture’s content should be relatively low when estimated using Quiz 1 responses, but should increase when estimated using Quiz 2 or 3 responses (Fig. 5B). Indeed, we found that participants’ estimated knowledge about the content of the *Four Fundamental Forces* was substantially higher on Quiz 2 versus Quiz 1 ($t(49) = 8.764, p < 0.001$) and on Quiz 3 versus Quiz 1 ($t(49) = 10.519, p < 0.001$). We found no reliable differences in estimated knowledge about that lecture’s content on Quiz 2 versus 3 ($t(49) = 0.160, p = 0.874$). Similarly, we hypothesized (and subsequently confirmed) that participants should show more estimated knowledge about the content of the *Birth of Stars* lecture after (versus before) watching it (Fig. 5D). Specifically, since

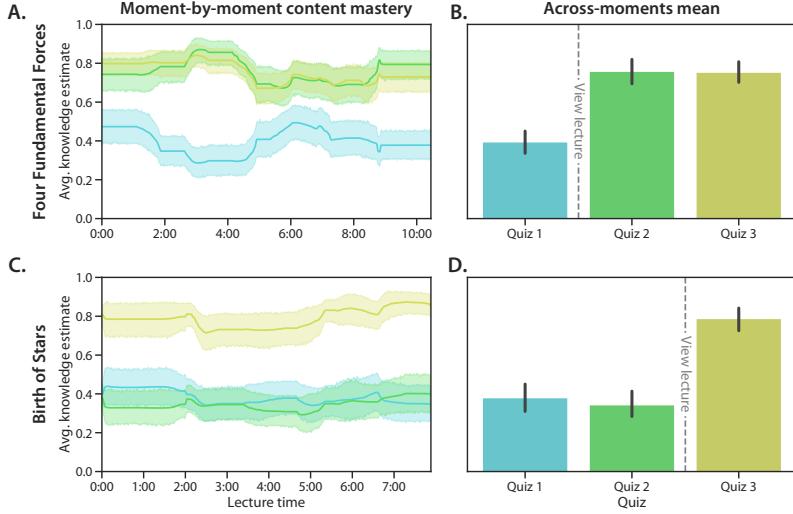


Figure 5: Estimating moment-by-moment knowledge acquisition. **A. Moment-by-moment knowledge about the *Four Fundamental Forces*.** Each trace displays the weighted proportion of correctly answered questions about the content reflected in each moment of the lecture (see *Estimating dynamic knowledge traces*), using responses from one quiz (color). The traces are averaged across participants. **B. Average estimated knowledge about the *Four Fundamental Forces*.** Each bar displays the across-timepoint average knowledge, estimated using the responses to one quiz's questions. **C. Moment-by-moment knowledge about the *Birth of Stars*.** The panel is in the same format as Panel A, but here the knowledge estimates are for the moment-by-moment content of the *Birth of Stars* lecture. **D. Average estimated knowledge about the *Birth of Stars*.** The panel is in the same format as Panel B, but here the knowledge estimates are for the content of the *Birth of Stars* lecture. All panels: error ribbons and error bars denote 95% confidence intervals, estimated across participants.

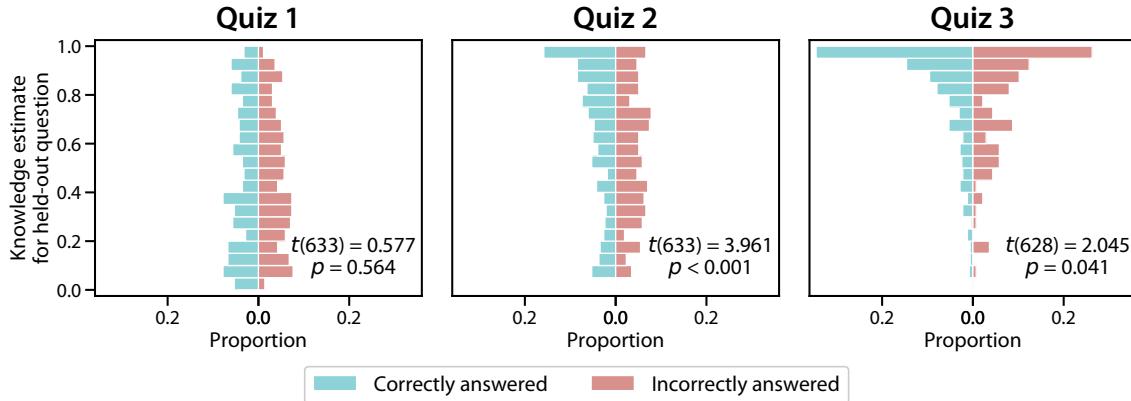


Figure 6: Estimating knowledge at the embedding coordinates of held-out questions. Separately for each quiz (panel), we plot the distributions of predicted knowledge at the embedding coordinates of each held out correctly (blue) or incorrectly (red) answered question. The t -tests reported in each panel are between the distributions of estimated knowledge at the coordinates of correctly versus incorrectly answered held-out questions.

215 participants watched that lecture after taking Quiz 2 (but before Quiz 3), we hypothesized that their
 216 knowledge estimates should be relatively low on Quizzes 1 and 2, but should show a “boost” on
 217 Quiz 3. Consistent with this prediction, we found no reliable differences in estimated knowledge
 218 about the *Birth of Stars* lecture content on Quizzes 1 versus 2 ($t(49) = 1.013, p = 0.316$), but the
 219 estimated knowledge was substantially higher on Quiz 3 versus 2 ($t(49) = 10.561, p < 0.001$) and
 220 Quiz 3 versus 1 ($t(49) = 8.969, p < 0.001$).

221 If we are able to accurately estimate a participant’s knowledge about the content tested by a
 222 given question, the estimated knowledge should have some predictive information about whether
 223 the participant is likely to answer the question correctly or incorrectly. For each question in turn, for
 224 each participant, we used Equation 1 to estimate (using all *other* questions from the same quiz, from
 225 the same participant) the participant’s knowledge at the held-out question’s embedding coordinate.
 226 For each quiz, we grouped these estimates into two distributions: one for the estimated knowledge
 227 at the coordinates of each *correctly* answered question, and another for the estimated knowledge at
 228 the coordinates of each *incorrectly* answered question (Fig. 6). We then used independent samples
 229 t -tests to compare the means of these distributions of estimated knowledge.

230 For the initial quizzes participants took (prior to watching either lecture), participants’ estimated

knowledge tended to be low overall, and relatively unstructured (Fig. 6, left panel). When we held out individual questions and estimated their knowledge at the held-out questions' embedding coordinates, we found no reliable differences in the estimates when the held-out question had been correctly versus incorrectly answered ($t(633) = 0.577, p = 0.564$). After watching the first video, estimated knowledge for held-out correctly answered questions (from the second quiz; Fig. 6, middle panel) exhibited a positive shift relative to held-out incorrectly answered questions ($t(633) = 3.961, p < 0.001$). After watching the second video, estimated knowledge (from the third quiz; Fig. 6, right panel) for *all* questions exhibited a positive shift. However, the increase in estimated knowledge for held-out correctly answered questions was larger than for held-out incorrectly answered questions ($t(628) = 2.045, p = 0.041$).

Knowledge estimates need not be limited to the content of the lectures. As illustrated in Figure 7, our general approach to estimating knowledge from a small number of quiz questions may be applied to *any* content, given its text embedding coordinate. To visualize how knowledge "spreads" through text embedding space to content beyond the lectures participants watched, we first fit a new topic model to the lectures' sliding windows with $k = 100$ topics. We hoped that increasing the number of topics from 15 to 100 might help us to generalize the knowledge predictions. (Aside from increasing the number of topics from 15 to 100, all other procedures and model parameters were carried over from the preceding analyses.) As in our other analyses, we resampled each lecture's topic trajectory to 1 Hz and also projected each question into a shared text embedding space.

We projected the resulting 100-dimensional topic vectors (for each second of video and for each question) into a shared 2-dimensional space (see *Creating knowledge and learning map visualizations*). Next, we sampled points evenly from a 100×100 grid of coordinates that evenly tiled a rectangle enclosing the 2D projections of the videos and questions. We used Equation 4 to estimate participants' knowledge at each of these 10K sampled locations, and we averaged these estimates across participants to obtain an estimated average *knowledge map* (Fig. 7). Intuitively, the knowledge map constructed from a given quiz's responses provides a visualization of how "much" participants know about any content expressible by the fitted text embedding model.

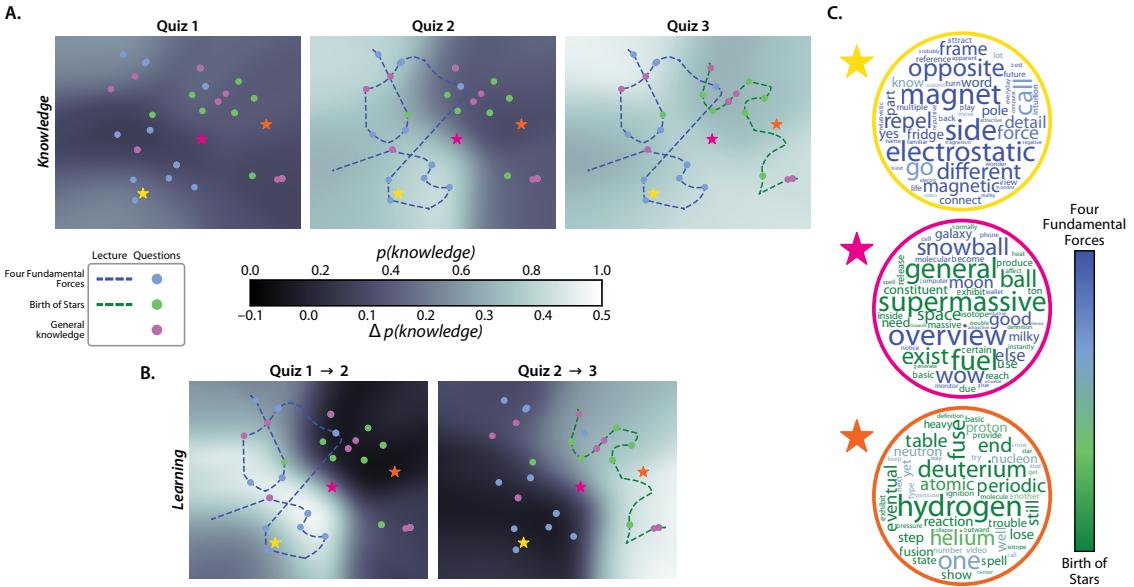


Figure 7: Mapping out the geometry of knowledge and learning. **A.** Average “knowledge maps” estimated using each quiz. Each map displays a 2D projection of the estimated knowledge about the content reflected by *all* regions of topic space (see *Creating knowledge and learning map visualizations*). The topic trajectories of each lecture and the coordinates of each question are indicated by dotted lines and dots. Each map reflects an average across all participants. For individual participants’ maps, see Figures S1, S2, and S3. **B.** Average “learning maps” estimated between each successive pair of quizzes. The learning maps are in the same general format as the knowledge maps in Panel A, but each coordinate in the learning maps indicates the *difference* between the corresponding coordinates in the indicated *pair* of knowledge maps—i.e., how much the estimated knowledge “changed” across the two quizzes. Each map reflects an average across all participants. For individual participants’ maps, see Figures S4 and S5. **C.** Word clouds for sampled points in topic space. Each word cloud displays the relative weights of each word reflected by the blend of topics represented at the locations of the stars in the maps. The words’ colors indicate how much each word is weighted on average across all timepoints’ topic vectors in the *Four Fundamental Forces* (blue) and *Birth of Stars* (green) videos, respectively.

Several features of the resulting knowledge maps are worth noting. The average knowledge map estimated from Quiz 1 responses (Fig. 7, leftmost map) shows that participants tended to have relatively little knowledge about any parts of the text embedding space (i.e., the shading is relatively dark everywhere). The knowledge map estimated from Quiz 2 responses shows a marked increase in knowledge on the left side of the map (around roughly the same range of coordinates covered by the *Four Fundamental Forces* lecture, indicated by the dotted blue line). In other words, participants' estimated increase in knowledge is localized to conceptual content that is nearby (i.e., related to) the content from the lecture they watched prior to taking Quiz 2. This localization is non-trivial: the knowledge estimates are informed only by the embedded coordinates of the *quiz questions*, not by the embeddings of either lecture (Eqn. 4). Finally, the knowledge map estimated from Quiz 3 responses shows a second increase in knowledge, localized to the region surrounding the embedding of the *Birth of Stars* lecture participants watched prior to taking Quiz 3.

Another way of visualizing these content-specific increases in knowledge (apparently driven by watching each lecture) is displayed in Figure 7B. Taking the point-by-point difference between the knowledge maps estimated from responses to a successive pair of quizzes yields a *learning map* that describes the *change* in knowledge estimates from one quiz to the next. These learning maps highlight that the estimated knowledge increases we observed across maps were specific to the regions around the embeddings of each lecture in turn.

Because the 2D projection we used to construct the knowledge and learning maps is (partially) invertible, we may gain additional insights into the estimates by reconstructing the original high-dimensional topic vectors for any point(s) in the maps we are interested in. For example, this could serve as a useful tool for an instructor looking to better understand which content areas a student (or a group of students) knows well (or poorly). As a demonstration, we show the top-weighted words from the blends of topics reconstructed from three example locations on the maps (Fig. 7C): one point near the *Four Fundamental Forces* embedding (yellow); a second point near the *Birth of Stars* embedding (orange), and a third point somewhere in between the two lectures' embeddings (pink). As shown in the word clouds in the Panel, the top-weighted words

287 at the example coordinate near the *Four Fundamental Forces* embedding also tended to be weighted
288 heavily by the topics expressed in that lecture. Similarly, the top-weighted words at the example
289 coordinate near the *Birth of Stars* embedding tended to be weighted most heavily by the topics
290 expressed in *that* lecture. And the top-weighted words at the example coordinate between the
291 two lectures' embeddings show a roughly even mix of words most strongly associated with each
292 lecture.

293 Discussion

294 Teaching, like effective writing and speaking, is fundamentally about empathy [1, 44, 59]. Great
295 teachers consider students' interests [12, 60], backgrounds [15, 47, 53], and working memory capac-
296 ities [2], and flexibly optimize their teaching strategies within those constraints [4, 22, 28]. In the
297 classroom, empathizing with students also means maintaining open lines of communication [65] by
298 fostering an environment in which all students feel comfortable speaking up if they have an excit-
299 ing new idea, or if they are having trouble understanding something [20, 61]. In-person instruction
300 also often entails dynamic student-teacher and student-student interactions. These in-person in-
301 teractions can provide the instructor with valuable information about students' understanding of
302 the course material, beyond what they can glean solely from exams or assignments [17, 24, 62].
303 In turn, this can allow the instructor to adapt their teaching approaches on-the-fly according to
304 students' questions and behaviors. But what does great teaching look like in asynchronous online
305 courses, when the instructor typically prepares course lectures and materials without knowing
306 who will ultimately be learning from them? Can the empathetic side of teaching be automated
307 and scaled?

308 The notion of empathy also related to "theory of mind" of other individuals [21, 29, 42].
309 Considering others' unique perspectives, prior experiences, knowledge, goals, etc., can help us
310 to more effectively interact and communicate [51, 55, 58]. The knowledge and learning maps
311 we estimate in our study (Fig. 7) hint at one potential form that an automated "empathetic"
312 teacher might take. We imagine automated content delivery systems that adapt lessons on the

313 fly according to continually updated estimates of what students know and how quickly they are
314 learning different conceptual content [e.g., building on ideas such as 3, 23, 36, 64, and others].

315 Over the past several years, the global pandemic has forced many educators to teach re-
316 motely [32, 46, 56, 63]. This change in world circumstances is happening alongside (and perhaps
317 accelerating) geometric growth in the availability of high quality online courses on platforms such
318 as Khan Academy [33], Coursera [66], EdX [35], and others [52]. Continued expansion of the global
319 internet backbone and improvements in computing hardware have also facilitated improvements
320 in video streaming, enabling videos to be easily downloaded and shared by large segments of the
321 world's population. This exciting time for online course instruction provides an opportunity to
322 re-evaluate how we, as a global community, educate ourselves and each other. For example, we
323 can ask: what makes an effective course or training program? Which aspects of teaching might be
324 optimized or automated? How and why do learning needs and goals vary across people? How
325 might we lower barriers to achieving a high quality education?

326 Alongside these questions, there is a growing desire to extend existing theories beyond the
327 domain of lab testing rooms and into real classrooms [31]. In part, this has led to a recent
328 resurgence of "naturalistic" or "observational" experimental paradigms that attempt to better
329 reflect more ethologically valid phenomena that are more directly relevant to real-world situations
330 and behaviors [48]. In turn, this has brought new challenges in data analysis and interpretation. A
331 key step towards solving these challenges will be to build explicit models of real-world scenarios
332 and how people behave in them (e.g., models of how people learn conceptual content from real-
333 world courses, as in our current study). A second key step will be to understand which sorts
334 of signals derived from behaviors and/or other measurements [e.g., neurophysiological data; 5,
335 16, 43, 49, 50] might help to inform these models. A third major step will be to develop and
336 employ reliable ways of evaluating the complex models and data that are a hallmark of naturalistic
337 paradigms.

338 Ultimately, our work suggests a new line of questions regarding the future of education:
339 which aspects of teaching can be optimized and/or automated? The social benefits of face-to-face
340 instruction, such as social interactions, friendships, and emotional support, cannot (and perhaps

341 should not) be fully replaced by an automated computer-based system. Nor can modern computer
342 systems experience emotional empathy in the human sense of the word. On the other hand,
343 perhaps it is possible to separate out the social aspects of classroom instruction from the purely
344 learning-related aspects. Our study shows that text embedding models can uncover detailed
345 insights into students' knowledge and how it changes over time during learning. We hope that
346 these advances might help pave the way for new ways of teaching or delivering educational content
347 that are tailored to individual students' learning needs and goals.

348 Materials and methods

349 Participants

350 We enrolled a total of 50 Dartmouth undergraduate students in our study. Participants received
351 course credit for enrolling. We asked each participant to fill out a demographic survey that included
352 questions about their age, gender, native spoken language, ethnicity, race, hearing, color vision,
353 sleep, coffee consumption, level of alertness, and several aspects of their educational background
354 and prior coursework.

355 Participants' ages ranged from 18 to 22 years (mean: 19.52 years; standard deviation: 1.09
356 years). A total of 15 participants reported their gender as male and 35 participants reported their
357 gender as female. A total of 49 participants reported their native language as "English" and 1
358 reported having another native language. A total of 47 participants reported their ethnicity as
359 "Not Hispanic or Latino" and three reported their ethnicity as "Hispanic or Latino." Participants
360 reported their races as White (32 participants), Asian (14 participants), Black or African American
361 (5 participants), American Indian or Alaska Native (1 participant), and Native Hawaiian or Other
362 Pacific Islander (1 participant). (Note that some participants selected multiple racial categories.)

363 A total of 49 participants reporting having normal hearing and 1 participant reported having
364 some hearing impairment. A total of 49 participants reported having normal color vision and 1
365 participant reported being color blind. Participants reported having had, on the night prior to

366 testing, 2–4 hours of sleep (1 participant), 4–6 hours of sleep (9 participants), 6–8 hours of sleep (35
367 participants), or 8+ hours of sleep (5 participants). They reported having consumed, on the same
368 day and leading up to their testing session, 0 cups of coffee (38 participants), 1 cup of coffee (10
369 participants), 3 cups of coffee (1 participant), or 4+ cups of coffee (1 participant).

370 No participants reported that their focus was currently impaired (e.g., by drugs or alcohol).
371 Participants reported their current level of alertness, and we converted their responses to numerical
372 scores as follows: “very sluggish” (-2), “a little sluggish” (-1), “neutral” (0), “fairly alert” (1), and
373 “very alert” (2). Across all participants, a range of alertness levels were reported (range: -2 – 1;
374 mean: -0.10; standard deviation: 0.84).

375 Participants reported their undergraduate major(s) as “social sciences” (28 participants), “nat-
376 ural sciences” (16 participants), “professional” (e.g., pre-med or pre-law; 8 participants), “mathe-
377 matics and engineering” (7 participants), “humanities” (4 participants), or “undecided” (3 partici-
378 pants). Note that some participants selected multiple categories for their undergraduate major. We
379 also asked participants about the courses they had taken. In total, 45 participants reported having
380 taken at least one Khan Academy course in the past, and 5 reported not having taken any Khan
381 Academy courses. Of those who reported having watched at least one Khan Academy course,
382 7 participants reported having watched 1–2 courses, 11 reported having watched 3–5 courses, 8
383 reported having watched 5–10 courses, and 19 reported having watched 10 or more courses. We
384 also asked participants about the specific courses they had watched, categorized under different
385 subject areas. In the “Mathematics” area, participants reported having watched videos on AP
386 Calculus AB (21 participants), Precalculus (17 participants), Algebra 2 (14 participants), AP Cal-
387 culus BC (12 participants), Trigonometry (11 participants), Algebra 1 (10 participants), Geometry
388 (8 participants), Pre-algebra (7 participants), Multivariable Calculus (5 participants), Differential
389 Equations (5 participants), Statistics and Probability (4 participants), AP Statistics (2 participants),
390 Linear Algebra (2 participants), Early Math (1 participant), Arithmetic (1 participant), and other
391 videos not listed in our survey (5 participants). In the “Science and engineering” area, participants
392 reported having watched videos on Chemistry, AP Chemistry, or Organic Chemistry (21 partic-
393 ipants); Physics, AP Physics I, or AP Physics II (15 participants); Biology, AP Biology; or High

394 school Biology (15 participants); Health and Medicine (1 participant); or other videos not listed
395 in our survey (19 participants). We also asked participants whether they had specifically seen the
396 videos used in our experiment. Of the 45 participants who reported having taken at least
397 one Khan Academy course in the past, 44 participants reported that they had not watched the *Four*
398 *Fundamental Forces* video, and 1 participant reported that they were not sure whether they had
399 watched it. All participants reported that they had not watched the *Birth of Stars* video. When
400 we asked participants about non-Khan Academy online courses, they reported having watched
401 or taken courses on Mathematics (15 participants), Science and engineering (11 participants), Test
402 preparation (9 participants), Economics and finance (3 participants), Arts and humanities (2 partic-
403 ipants), Computing (2 participants), and other categories not listed in our survey (18 participants).
404 Finally, we asked participants about in-person courses they had taken in different subject areas.
405 They reported taking courses in Mathematics (39 participants), Science and engineering (38 par-
406 ticipants), Arts and humanities (35 participants), Test preparation (27 participants), Economics
407 and finance (26 participants), Computing (15 participants), College and careers (7 participants), or
408 other courses not listed in our survey (6 participants).

409 **Experiment**

410 We hand-selected two course videos from the Khan Academy platform: *Four Fundamental Forces*
411 (an introduction to gravity, electromagnetism, the weak nuclear force, and the strong nuclear force;
412 duration: 10 minutes and 29 seconds) and *Birth of Stars* (an introduction to how stars are formed;
413 duration: 7 minutes and 57 seconds). We then hand-created 39 multiple-choice questions: 15 about
414 the conceptual content of *Four Fundamental Forces* (i.e., lecture 1), 15 about the conceptual content
415 of *Birth of Stars* (i.e., lecture 2), and 9 questions that tested for general conceptual knowledge about
416 basic physics (covering material that was not presented in either video). The full set of questions
417 and answer choices may be found in Table S1.

418 Over the course of the experiment, participants completed three 13-question multiple-choice
419 quizzes: the first before viewing lecture 1, the second between lectures 1 and 2, and the third
420 after viewing lecture 2 (Fig. 1). The questions appearing on each quiz, for each participant, were

421 randomly chosen from the full set of 39, with the constraints that (a) each quiz contain 5 questions
422 about lecture 1, 5 questions about lecture 2, and 3 questions, and (b) each question appear exactly
423 once. The order of questions on each quiz, and the order of answer options for each question, were
424 also randomized. Our experimental protocol was approved by the Committee for the Protection
425 of Human Subjects at Dartmouth College. We used the experiment to develop and test our
426 computational framework for estimating knowledge and learning.

427 **Analysis**

428 **Constructing text embeddings of multiple lectures and questions**

429 We adapted an approach we developed in prior work [27] to embed each moment of the two
430 lectures and each question in our pool in a common representational space. Briefly, our approach
431 uses a topic model [Latent Dirichlet Allocation; 8], fit to a corpus of documents to discover a set of
432 k “topics” or “themes” from their contents. Formally, each topic is defined as a set of weights over
433 each word in the model’s vocabulary (i.e., the union of all unique words, across all documents,
434 excluding “stop words.”).

435 We adapted an approach we developed in prior work [27] to embed each moment of the two
436 lectures and each question in our pool in a common representational space. Briefly, our approach
437 uses a topic model [Latent Dirichlet Allocation; 8], trained on a set of documents, to discover a
438 set of k “topics” or “themes.” Formally, each topic is defined as a set of weights over each word
439 in the model’s vocabulary (i.e., the union of all unique words, across all documents, excluding
440 “stop words.”). Conceptually, each topic is intended to give larger weights to words that are
441 conceptually related or that tend to co-occur in the same documents. After fitting a topic model,
442 each document in the training set, or any *new* document that contains at least some of the words in
443 the model’s vocabulary, may be represented as a k -dimensional vector describing how much the
444 document (most probably) reflects each topic. (Unless, otherwise noted, we used $k = 15$ topics.)

445 As illustrated in Figure 2A, we first

446 As illustrated in Figure 2A, we start by building up a corpus of documents using overlapping

447 sliding windows that span each video’s transcript. Khan Academy provides professionally created,
448 manual transcriptions of all videos for closed captioning. However, such transcripts would not
449 be readily available in all contexts to which our framework could potentially be applied. Khan
450 Academy videos are hosted on the YouTube platform, which additionally provides automated
451 transcripts

452 Khan Academy videos are hosted on the YouTube platform, and all YouTube videos are run
453 through Google’s speech-to-text API [26] to derive a timestamped transcript of any detected speech
454 in the video. The resulting transcripts contain one timestamped row per line, and each line
455 generally corresponds to a few seconds of spoken content from the video. We defined a sliding
456 window length of (up to) $w = 30$ transcript lines, and we assigned each window a timestamp
457 according to the midpoint between its first and last lines’ timestamps. These sliding windows
458 ramped up and down in length at the very beginning and end of the transcript, respectively. In
459 other words, the first sliding window covered only the first line from the transcript; the second
460 sliding window covered the first two lines; and so on. This insured that each line of the transcript
461 appeared in the same number (w) of sliding windows. We treated the text from each sliding window
462 as a single “document,” and we combined these documents across the two videos’ windows to
463 create a single training corpus for the topic model. The top words from each of the 15 discovered
464 topics may be found in Table S2.

465 After fitting a topic model to each videos’ transcripts, we could use the trained model to
466 transform arbitrary (potentially new) documents into k -dimensional topic vectors. A convenient
467 property of these topic vectors is that documents that reflect similar blends of topics (i.e., documents
468 that reflect similar themes, according to the model) will yield similar (in terms of Euclidean distance,
469 correlation, etc.) topic vectors. In general, the similarity between different documents’ topic vectors
470 may be used to characterize the similarity in conceptual content between the documents.

471 We transformed each sliding window’s text into a topic vector, and then used linear interplo-
472 lation (independently for each topic dimension) to resample the resulting timeseries to once per
473 second. This yielded a single topic vector for each second of each video. We also used the fitted
474 model to obtain topic vectors for each question in our pool (Tab. S1). Taken together, we obtained

475 a *trajectory* for each video, describing its path through topic space, and a single coordinate for each
 476 question (Fig. 2B). Embedding both videos and all of the questions using a common model enables
 477 us to compare the content from different moments of videos, compare the content across videos,
 478 and estimate potential associations between specific questions and specific moments of video.

479 **Estimating dynamic knowledge traces**

480 We used the following equation to estimate each participant’s knowledge about timepoint t of a
 481 given lecture, $\hat{k}(t)$:

$$\hat{k}(f(t, L)) = \frac{\sum_{i \in \text{correct}} \text{ncorr}(f(t, L), f(i, Q))}{\sum_{j=1}^N \text{ncorr}(f(t, L), f(j, Q))}, \quad (1)$$

482 where

$$\text{ncorr}(x, y) = \frac{\text{corr}(x, y) - \text{mincorr}}{\text{maxcorr} - \text{mincorr}}, \quad (2)$$

483 and where mincorr and maxcorr are the minimum and maximum correlations between any lecture
 484 timepoint and question, taken over all timepoints and questions across both lectures and all three
 485 question sets. We also define $f(s, \Omega)$ as the s^{th} topic vector from the set of topic vectors Ω . Here
 486 t indexes the set of lecture topic vectors, L , and i and j index the topic vectors of questions in the
 487 quiz’s question set, Q . Note that “correct” denotes the set of indices of the questions the participant
 488 answered correctly on the given quiz.

489 Intuitively, $\text{ncorr}(x, y)$ is the correlation between two topic vectors (e.g., the topic vector from one
 490 timepoint in a lecture, x , and the topic vector for one question, y), normalized by the minimum and
 491 maximum correlations (across all timepoints and questions) to range between 0 and 1, inclusive.
 492 Equation 1 then computes the weighted average proportion of correctly answered questions about
 493 the content presented at timepoint t , where the weights are given by the normalized correlations
 494 between timepoint t ’s topic vector and the topic vectors for each question. The normalization
 495 step (i.e., using ncorr instead of the raw correlations) insures that every question (except the
 496 least-relevant question) contributes some non-zero amount to the knowledge estimate.

497 **Creating knowledge and learning map visualizations**

498 An important feature of our approach is that, given a trained text embedding model and partic-
499 ipants’ quiz performance on each question, we can estimate their knowledge about *any* content
500 expressible by the embedding model— not solely the content explicitly probed by the quiz ques-
501 tions. To visualize these estimates (Figs. 7, S1, S2, S3, S4, and S5), we used UMAP [41] to define a
502 2D projection of the text embedding space. Sampling the original 100-dimensional space at high
503 resolution to obtain an adequate set of topic vectors spanning the embedding space would be
504 computationally intractable. However, sampling a 2D grid is much more feasible. We defined a
505 rectangle enclosing the 2D projections of the lectures’ and quizzes’ embeddings, and we sampled
506 points from a regular 100×100 grid of coordinates that evenly tiled the enclosing rectangle. We
507 sought to estimate participants’ knowledge (and learning—i.e., changes in knowledge) at each of
508 the resulting 10000 coordinates.

509 To generate our estimates, we placed a set of 39 radial basis functions (RBFs) throughout the
510 embedding space, centered on the 2D projections for each question (i.e., we included one RBF for
511 each question). At coordinate x , the value of an RBF centered on a question’s coordinate μ , is given
512 by:

$$\text{RBF}(x, \mu, \lambda) = \exp\left\{-\frac{\|x - \mu\|^2}{\lambda}\right\}. \quad (3)$$

513 The λ term in the RBF equation controls the “smoothness” of the function, where larger values
514 of λ result in smoother maps. In our implementation we used $\lambda = 50$. Next, we estimated the
515 “knowledge” at each coordinate, x , using:

$$\hat{k}(x) = \frac{\sum_{i \in \text{correct}} \text{RBF}(x, q_i, \lambda)}{\sum_{j=1}^N \text{RBF}(x, q_j, \lambda)}. \quad (4)$$

516 Intuitively, Equation 4 computes the weighted proportion of correctly answered questions, where
517 the weights are given by how nearby (in the 2D space) each question is to the x . We also defined
518 *learning maps* as the coordinate-by-coordinate differences between any pair of knowledge maps.
519 Intuitively, learning maps reflect the *change* in knowledge across two maps.

520 **Author contributions**

521 Conceptualization: PCF, ACH, and JRM. Methodology: PCF, ACH, and JRM. Software: PCF.
522 Validation: PCF. Formal analysis: PCF. Resources: PCF, ACH, and JRM. Data curation: PCF.
523 Writing (original draft): JRM. Writing (review and editing): PCF, ACH, and JRM. Visualization:
524 PCF and JRM. Supervision: JRM. Project administration: PCF. Funding acquisition: JRM.

525 **Data and code availability**

526 All of the data analyzed in this manuscript, along with all of the code for running our experiment
527 and carrying out the analyses may be found at [https://github.com/ContextLab/efficient-learning-](https://github.com/ContextLab/efficient-learning-khan)
528 [khan](#).

529 **Acknowledgements**

530 We acknowledge useful discussions, assistance in setting up an earlier (unpublished) version of
531 this study, and assistance with some of the data collection efforts from Will Baxley, Max Bluestone,
532 Daniel Carstensen, Kunal Jha, Caroline Lee, Lucy Owen, Xinming Xu, and Kirsten Ziman. Our
533 work was supported in part by NSF CAREER Award Number 2145172 to JRM. The content is
534 solely the responsibility of the authors and does not necessarily represent the official views of our
535 supporting organizations. The funders had no role in study design, data collection and analysis,
536 decision to publish, or preparation of the manuscript.

537 **References**

- 538 [1] Aldrup, K., Carstensen, B., and Klusmann, U. (2022). Is empathy the key to effective teaching?
539 A systematic review of its association with teacher-student interactions and student outcomes.
540 *Educational Psychology Review*, 34:1177001216.

- 541 [2] Alloway, T. P. (2012). Teachers' perceptions of classroom behaviour and working memory.
- 542 *Educational Research and Review*, 7(6):138–142.
- 543 [3] Anderson, J. R. and Skwarecki, E. (1986). The automated tutoring of introductory computer
- 544 programming. *Communications of the ACM*, 29(9):842–849.
- 545 [4] Anderton, R. S., Vitali, J., Blackmore, C., and Bakeberg, M. C. (2021). Flexible teaching and learn-
- 546 ing modalities in undergraduate science amid the COVID-19 pandemic. *Frontiers in Education*,
- 547 5:doi.org/10.3389/feduc.2020.609703.
- 548 [5] Bevilacque, D., Davidesco, I., Wan, L., and Chaloner, K. (2019). Brain-to-brain synchrony and
- 549 learning outcomes vary by student-teacher dynamics: evidence from a real-world classroom
- 550 electroencephalography study. *Journal of Cognitive Neuroscience*, 31(3):401–411.
- 551 [6] Blaye, A., Bernard-Peyron, V., Paour, J.-L., and Bonthoux, F. (2006). Category flexibility in chil-
- 552 dren: distinguishing response flexibility from conceptual flexibility; the protracted development
- 553 of taxonomic representations. *European Journal of Developmental Psychology*, 3(2):163–188.
- 554 [7] Blei, D. M. and Lafferty, J. D. (2006). Dynamic topic models. In *Proceedings of the International*
- 555 *Conference on Machine Learning*, pages 113–120, New York, NY. Association for Computing
- 556 Machinery.
- 557 [8] Blei, D. M., Ng, A. Y., and Jordan, M. I. (2003). Latent dirichlet allocation. *Journal of Machine*
- 558 *Learning Research*, 3:993–1022.
- 559 [9] Brown, T. B., Mann, B., Ryder, N., Subbiah, M., Kaplan, J., Dhariwal, P., Neelakantan, A.,
- 560 Shyam, P., Sastry, G., Askell, A., Agarwal, S., Herbert-Voss, A., Krueger, G., Henighan, T., Child,
- 561 R., Ramesh, A., Ziegler, D. M., Wu, J., Winter, C., Hesse, C., Chen, M., Sigler, E., Litwin, M.,
- 562 Gray, S., Chess, B., Clark, J., Berner, C., McCandlish, S., Radford, A., Sutskever, I., and Amodei,
- 563 D. (2020). Language models are few-shot learners. *arXiv*, 2005.14165.
- 564 [10] Caramazza, A. and Mahon, B. Z. (2003). The organization of conceptual knowledge: the
- 565 evidence from category-specific semantic deficits. *Trends in Cognitive Sciences*, 7(8):354–361.

- 566 [11] Cer, D., Yang, Y., Kong, S. Y., Hua, N., Limtiaco, N., John, R. S., Constant, N., Guajardo-
567 Cespedes, M., Yuan, S., Tar, C., Sung, Y.-H., Strope, B., and Kurzweil, R. (2018). Universal
568 sentence encoder. *arXiv*, 1803.11175.
- 569 [12] Clark, J. (2010). Powerpoint and pedagogy: maintaining student interest in university lectures.
570 *College Teaching*, 56(1):39–44.
- 571 [13] Constantinescu, A. O., O'Reilly, J. X., and Behrens, T. E. J. (2016). Organizing conceptual
572 knowledge in humans with a gridlike code. *Science*, 352(6292):1464–1468.
- 573 [14] Deacon, D., Grose-Fifer, J., Yang, C. M., Stanick, V., Hewitt, S., and Dynowska, A. (2004).
574 Evidence for a new conceptualization of semantic representation in the left and right cerebral
575 hemispheres. *Cortex*, 40(3):467–478.
- 576 [15] den Brok, P., van Tartwijk, J., Wubbels, T., and Veldman, I. (2010). The differential effect of
577 the teacher-student interpersonal relationship on student outcomes for students with different
578 ethnic backgrounds. *British Journal of Educational Psychology*, 80(2):199–221.
- 579 [16] Dikker, S., Wan, L., Davidesco, I., Kaggen, L., Oostrik, M., McClintock, J., Rowland, J.,
580 Michalareas, G., van Bavel, J. J., Ding, M., and Poeppel, D. (2017). Brain-to-brain synchrony
581 tracks real-world dynamic group interactions in the classroom. *Current Biology*, 27(9):1375–1380.
- 582 [17] Englehart, J. M. (2009). Teacher-student interaction. In Saha, L. J. and Dworkin, A. G., editors,
583 *International Handbook of Research on Teachers and Teaching*. Springer International Handbooks of
584 Education.
- 585 [18] Fisher, R. A. (1922). On the mathematical foundations of theoretical statistics. *Philosophical
586 Transactions of the Royal Society A*, 222(602):309–368.
- 587 [19] Gallagher, J. J. (2000). Teaching for understanding and application of science knowledge.
588 *School Science and Mathematics*, 100(6):310–318.
- 589 [20] Garran, A. M. and Rasmussen, B. M. (2014). Safety in the classroom: reconsidered. *Journal of
590 Teaching in Social Work*, 34(4):401–412.

- 591 [21] Goldstein, T. R. and Winner, E. (2012). Enhancing empathy and theory of mind. *Journal of*
592 *Cognition and Development*, 13(1):19–37.
- 593 [22] Goode, S., Willis, R. A., Wolf, J. R., and Harris, A. L. (2007). Enhancing IS education with
594 flexible teaching and learning. *Journal of Information Systems Education*, 18(3):297–302.
- 595 [23] Halff, H. M. (1988). Curriculum and instruction in automated tutors. *Foundations of intelligent*
596 *tutoring systems*, pages 79–108.
- 597 [24] Hall, J. K. and Walsh, M. (2002). Teacher-student interaction and language learning. *Annual*
598 *Review of Applied Linguistics*, 22:186–203.
- 599 [25] Hall, R. and Greeno, J. (2008). *21st century education: A reference handbook*, chapter Conceptual
600 learning, pages 212–221. Sage Publications.
- 601 [26] Halpern, Y., Hall, K. B., Schogol, V., Riley, M., Roark, B., Skobeltsyn, G., and Bäuml, M. (2016).
602 Contextual prediction models for speech recognition. In *Interspeech*, pages 2338–2342.
- 603 [27] Heusser, A. C., Fitzpatrick, P. C., and Manning, J. R. (2021). Geometric models reveal be-
604 havioral and neural signatures of transforming naturalistic experiences into episodic memories.
605 *Nature Human Behavior*, 5:905–919.
- 606 [28] Johnston, S. (2002). Introducing and supporting change towards more flexible teaching ap-
607 proaches. In *The convergence of distance and conventional education*. Routledge.
- 608 [29] Kanske, P., Böckler, A., and Singer, T. (2015). Models, mechanisms and moderators dissociating
609 empathy and theory of mind. In *Social Behavior from Rodents to Humans*, pages 193–206. Springer.
- 610 [30] Katona, G. (1940). *Organizing and memorizing: studies in the psychology of learning and teaching*.
611 Columbia University Press.
- 612 [31] Kaufman, D. M. (2003). Applying educational theory in practice. *British Medical Journal*,
613 326(7382):213–216.

- 614 [32] Kawasaki, H., Yamasaki, S., Masuoka, Y., Iwasa, M., Fukita, S., and Matsuyama, R. (2021).
615 Remote teaching due to COVID-19: an exploration of its effectiveness and issues. *International*
616 *Journal of Environmental Research and Public Health*, 18(5):2672.
- 617 [33] Khan, S. (2004). *The Khan Academy*. Salman Khan.
- 618 [34] Kintsch (1970). *Learning, memory, and conceptual processes*. Wiley.
- 619 [35] Kolowich, S. (2013). How EdX plans to earn, and share, revenue from its free online courses.
620 *The Chronicle of Higher Education*, 21:1–5.
- 621 [36] Kumar, A. N. (2005). Generation of problems, answers, grade, and feedback—case study of a
622 fully automated tutor. *Journal on Educational Resources in Computing*, 5(3):1–25.
- 623 [37] Landauer, T. K. and Dumais, S. T. (1997). A solution to Plato’s problem: the latent semantic
624 analysis theory of acquisition, induction, and representation of knowledge. *Psychological Review*,
625 104:211–240.
- 626 [38] Macellan, E. (2005). Conceptual learning: the priority for higher education. *British Journal of*
627 *Educational Studies*, 53(2):129–147.
- 628 [39] Manning, J. R. (2020). Context reinstatement. In Kahana, M. J. and Wagner, A. D., editors,
629 *Handbook of Human Memory*. Oxford University Press.
- 630 [40] Manning, J. R. (2021). Episodic memory: mental time travel or a quantum “memory wave”
631 function? *Psychological Review*, 128(4):711–725.
- 632 [41] McInnes, L., Healy, J., and Melville, J. (2018). UMAP: Uniform manifold approximation and
633 projection for dimension reduction. *arXiv*, 1802(03426).
- 634 [42] Meltzoff, A. N. (2011). Social cognition and the origins of imitation, empathy, and theory of
635 mind. In *The Wiley-Blackwell handbook of childhood cognitive development*. Wiley-Blackwell.
- 636 [43] Meshulam, M., Hasenfratz, L., Hillman, H., Liu, Y. F., Nguyen, M., Norman, K. A., and Hasson,
637 U. (2020). Neural alignment predicts learning outcomes in students taking an introduction to
638 computer science course. *Nature Communications*, 12(1922):doi.org/10.1038/s41467–021–22202–3.

- 639 [44] Meyers, S., Rowell, K., Wells, M., and Smith, B. C. (2019). Teacher empathy: a model of
640 empathy for teaching for student success. *College Teaching*, 67(3):160–168.
- 641 [45] Mikolov, T., Chen, K., Corrado, G., and Dean, J. (2013). Efficient estimation of word represen-
642 tations in vector space. *arXiv*, 1301.3781.
- 643 [46] Moser, K. M., Wei, T., and Brenner, D. (2021). Remote teaching during COVID-19: implications
644 from a national survey of language educators. *System*, 97:102431.
- 645 [47] Muijs, D. and Reynolds, D. (2003). Student background and teacher effects on achievement and
646 attainment in mathematics: a longitudinal study. *Educational Research and Evaluation*, 9(3):289–
647 314.
- 648 [48] Nastase, S. A., Goldstein, A., and Hasson, U. (2020). Keep it real: rethinking the primacy of
649 experimental control in cognitive neuroscience. *NeuroImage*, 15(222):117254–117261.
- 650 [49] Nguyen, M., Chang, A., Micciche, E., Meshulam, M., Nastase, S. A., and Hasson, U. (2022).
651 Teacher-student neural coupling during teaching and learning. *Social Cognitive and Affective*
652 *Neuroscience*, 17(4):367–376.
- 653 [50] Poulsen, A. T., Kamronn, S., Dmochowski, J., Parra, L. C., and Hansen, L. K. (2017). EEG
654 in the classroom: synchronised neural recordings during video presentation. *Scientific Reports*,
655 7:43916.
- 656 [51] Ratka, A. (2018). Empathy and the development of affective skills. *American Journal of*
657 *Pharmaceutical Education*, 82(10):doi.org/10.5688/ajpe7192.
- 658 [52] Rhoads, R. A., Berdan, J., and Toven-Lindsey, B. (2013). The open courseware movement in
659 higher education: unmasking power and raising questions about the movement’s democratic
660 potential. *Educational Theory*, 63(1):87–110.
- 661 [53] Rosenshine, B. (1976). Recent research on teaching behaviors and student achievement. *Journal*
662 *of Teacher Education*, 27(1):61–64.

- 663 [54] Scott, P., Asoko, H., and Leach, J. (2007). *Handbook of research on science education*, chapter
664 Student conceptions and conceptual learning in science. Routledge.
- 665 [55] Shao, Y. N., Sun, H. M., Huang, J. W., Li, M. L., Huang, R. R., and Li, N. (2018). Simulation-
666 based empathy training improves the communication skills of neonatal nurses. *Clinical Simula-*
667 *tion in Nursing*, 22:32–42.
- 668 [56] Shim, T. E. and Lee, S. Y. (2020). College students' experience of emergency remote teaching
669 during COVID-19. *Children and Youth Services Review*, 119:105578.
- 670 [57] Simon, M. A., Tzur, R., Heinz, K., and Kinzel, M. (2004). Explicating a mechanism for
671 conceptual learning: elaborating the construct of reflective abstraction. *Journal for Research in*
672 *Mathematics Education*, 35(5):305–329.
- 673 [58] Stepien, K. A. and Baernstein, A. (2006). Education for empathy. *Journal of General Internal*
674 *Medicine*, 21:524–530.
- 675 [59] Stojiljković, S., Djigić, G., and Zlatković, B. (2012). Empathy and teachers' roles. *Procedia –*
676 *Social and Behavioral Sciences*, 69:960–966.
- 677 [60] Swarat, S., Ortony, A., and Revelle, W. (2012). Activity matters: understanding student interest
678 in school science. *Journal of Research in Science Teaching*, 49(4):515–537.
- 679 [61] Turner, S. and Braine, M. (2015). Unravelling the 'safe' concept in teaching: what can we learn
680 from teachers' understanding? *Pastoral Care in Education*, 33(1):47–62.
- 681 [62] van de Pol, J., Volman, M., and Beishuizen, J. (2010). Scaffolding in teacher-student interaction:
682 a decade of research. *Educational Psychology Review*, 22:271–296.
- 683 [63] Whalen, J. (2020). Should teachers be trained in emergency remote teaching? Lessons learned
684 from the COVID-19 pandemic. *Journal of Technology and Teacher Education*, 28(2):189–199.
- 685 [64] Wolz, U., McKeown, K., and Kaiser, G. E. (1988). Automated tutoring in interactive environ-
686 ments: a task centered approach. Technical report, Columbia University.

- 687 [65] Wulff, S. S. and Wulff, D. H. (2004). "of course i'm communicating; I lecture every day":
688 enhancing teaching and learning in introductory statistics. *Communication Education*, 53(1):92–
689 103.
- 690 [66] Young, J. R. (2012). Inside the Coursera contract: how an upstart company might profit from
691 free courses. *The Chronicle of Higher Education*, 19(7):1–4.