

<sup>1</sup> Text embedding models yield high resolution insights  
<sup>2</sup> into conceptual knowledge from short multiple choice  
<sup>3</sup> quizzes

<sup>4</sup> Paxton C. Fitzpatrick<sup>1</sup>, Andrew C. Heusser<sup>1, 2</sup>, and Jeremy R. Manning<sup>1,\*</sup>

<sup>1</sup>Department of Psychological and Brain Sciences

Dartmouth College, Hanover, NH 03755, USA

<sup>2</sup>Akili Interactive Labs

Boston, MA 02110, USA

\*Corresponding author: Jeremy.R.Manning@Dartmouth.edu

<sup>5</sup>

### Abstract

<sup>6</sup> We develop a mathematical framework, based on natural language processing models, for track-  
<sup>7</sup> ing and characterizing the acquisition of conceptual knowledge. Our approach embeds each  
<sup>8</sup> concept in a high dimensional representation space, where nearby coordinates reflect similar or  
<sup>9</sup> related concepts. We test our approach using behavioral data from participants who answered  
<sup>10</sup> small sets of multiple choice quiz questions interleaved between watching two course videos  
<sup>11</sup> from the Khan Academy platform. We applied our framework to the videos' transcripts, and  
<sup>12</sup> to text of the quiz questions, to quantify the content of each moment of video and each quiz  
<sup>13</sup> question. We used these embeddings, along with participants' quiz responses, to track how the  
<sup>14</sup> learners' knowledge changed after watching each video. Our findings show how a small set of  
<sup>15</sup> quiz questions may be used to obtain rich and meaningful high resolution insights into what  
<sup>16</sup> each learner knows, and how their knowledge changes over time as they learn.

<sup>17</sup>

**Keywords:** education, learning, knowledge, concepts, natural language processing

<sup>18</sup> **Introduction**

<sup>19</sup> Suppose that a teacher had access to a complete tangible “map” of everything a student knew.  
<sup>20</sup> Defining what such a map might even look like, let alone how it might be constructed or filled in, is  
<sup>21</sup> itself a non-trivial problem. But if a teacher *were* to gain access to such a map, how might it change  
<sup>22</sup> their ability to teach that student? Perhaps they might start by checking how well the student knew  
<sup>23</sup> the to-be-learned information already, or how much they knew about related concepts. For some  
<sup>24</sup> students, they could potentially optimize their teaching efforts to maximize efficiency by focusing  
<sup>25</sup> primarily on not-yet-known content. For other students (or other content areas), it might be more  
<sup>26</sup> effective to optimize for direct connections between already known content and new material.  
<sup>27</sup> Observing how the student’s knowledge changed over time, in response to their teaching, could  
<sup>28</sup> also help to guide the teacher towards the most effective strategy for that individual student.

<sup>29</sup> A common approach to assessing students’ learning is to compile a set of quiz questions, tally  
<sup>30</sup> up the proportion of correctly answered questions, and provide the student with feedback in the  
<sup>31</sup> form of a letter grade. Although numerical or letter grades give *some* indication about whether  
<sup>32</sup> the student has mastered the to-be-learned material, any univariate measure of performance on a  
<sup>33</sup> complex task is at risk of conflating underlying factors, losing relevant information, and so on. For  
<sup>34</sup> example, consider the relative utility of the imaginary map described above that characterizes in  
<sup>35</sup> detail a student’s knowledge, versus a single annotation saying that the student answered 85% of  
<sup>36</sup> their quiz questions correctly, or that they received a ‘B’. Here, we show that the same quiz data  
<sup>37</sup> required to compute proportion correct scores or letter grades can be used to instead obtain much  
<sup>38</sup> more detailed insights into what the student knows at the time they took the quiz.

<sup>39</sup> Designing and building procedures and tools for mapping out knowledge touches on deep  
<sup>40</sup> questions about what it means to learn. For example, how do we acquire conceptual knowledge?  
<sup>41</sup> Memorizing course lectures or textbook chapters by rote can lead to the superficial *appearance*  
<sup>42</sup> of understanding the underlying content, but achieving true conceptual understanding seems to  
<sup>43</sup> require something deeper and richer. Does conceptual understanding entail connecting newly  
<sup>44</sup> acquired information to the scaffolding of one’s existing knowledge or experience [2, 6, 8, 9, 42]?

45 Or weaving a lecture’s atomic elements (e.g., its component words) into a structured network  
46 that describes how those individual elements are related? Conceptual understanding could also  
47 involve building a mental model that transcends the meanings of those individual atomic elements  
48 by reflecting the deeper meaning underlying the gestalt whole [23, 26, 39].

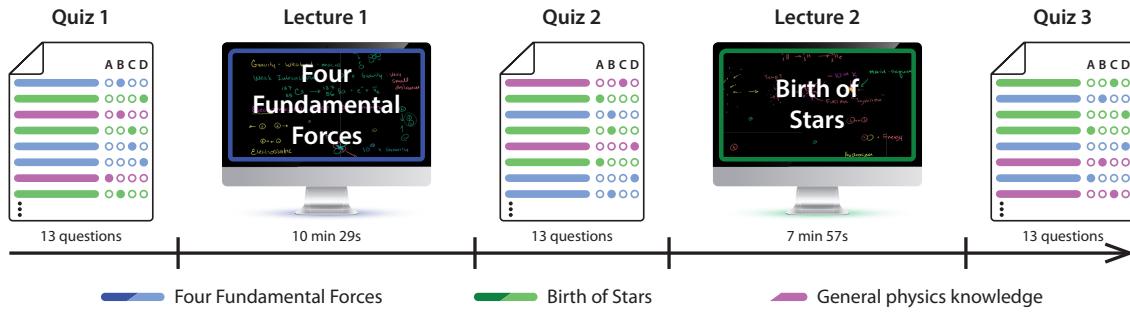
49 The difference between “understanding” and “memorizing,” as framed by researchers in ed-  
50 ucation, cognitive psychology, and cognitive neuroscience [e.g., 14, 16, 19, 26, 39] has profound  
51 analogs in the fields of natural language processing and natural language understanding. For  
52 example, considering the raw contents of a document (e.g., its constituent symbols, letters, and  
53 words) might provide some information about what the document is about, just as memorizing a  
54 passage might provide some ability to answer simple questions about it. However, text embedding  
55 models [e.g., 3–5, 7, 10, 25, 32] also attempt to capture the deeper meaning *underlying* those atomic  
56 elements. These models consider not only the co-occurrences of those elements within and across  
57 documents, but also patterns in how those elements appear across different scales (e.g., sentences,  
58 paragraphs, chapters, etc.), the temporal and grammatical properties of the elements, and other  
59 high-level characteristics of how they are used [27, 28]. According to these models, the deep  
60 conceptual meaning of a document may be captured by a feature vector in a high-dimensional  
61 representation space, where nearby vectors reflect conceptually related documents. A model that  
62 succeeds at capturing an analog of “understanding” is able to assign nearby feature vectors to  
63 two conceptually related documents, *even when the words contained in those documents have very little*  
64 *overlap*.

65 Given these insights, what form might the representation of the sum total of a person’s knowl-  
66 edge take? First, we might require a means of systematically describing or representing the nearly  
67 infinite set of possible things a person could know. Second, we might want to account for potential  
68 associations between different concepts. For example, the concepts of “fish” and “water” might be  
69 associated in the sense that fish live in water. Third, knowledge may have a critical dependency  
70 structure, such that knowing about a particular concept might require first knowing about a set of  
71 other concepts. For example, understanding the concept of a fish swimming in water first requires  
72 understanding what fish and water *are*. Fourth, as we learn, our “current state of knowledge”

73 should change accordingly. Learning new concepts should both update our characterizations of  
74 “what is known” and also unlock any now-satisfied dependencies of those newly learned concepts  
75 so that they are “tagged” as available for future learning.

76 Here we develop a framework for modeling how knowledge is acquired during learning. The  
77 central idea is to use text embedding models to define the coordinate systems of two maps: (a) a  
78 *knowledge map* that describes the extent to which each concept is currently known and (b) a *learning*  
79 *map* that describes changes in knowledge over time. Each location on these maps represents  
80 a single concept, and the maps’ geometries are defined such that related concepts are located  
81 nearby in space. We use this framework to analyze and interpret behavioral data collected from  
82 an experiment that had participants watch and answer multiple-choice questions about a series of  
83 recorded course lectures.

84 Our primary research goal is to advance our understanding of what it means to acquire deep,  
85 real-world conceptual knowledge. Traditional laboratory approaches to studying learning and  
86 memory (e.g., list learning studies) often draw little distinction between memorization and under-  
87 standing. Instead, these studies typically focus on whether information is effectively encoded or  
88 retrieved, rather than whether the information is *understood*. Approaches to studying conceptual  
89 learning, such as category learning experiments, can begin to investigate the distinction between  
90 memorization and understanding, often by training participants to distinguish arbitrary or ran-  
91 dom features in otherwise meaningless categorized stimuli. However the objective of real-world  
92 training, or learning from life experiences more generally, is often to develop new knowledge that  
93 may be applied in *useful* ways in the future. In this sense, the gap between modern learning theo-  
94 ries and modern pedagogical approaches and classroom learning strategies is enormous: most of  
95 our theories about *how* people learn are inspired by experimental paradigms and models that have  
96 only peripheral relevance to the kinds of learning that students and teachers actually seek [16, 26].  
97 To help bridge this gap, our study uses course materials from real online courses to inform, fit,  
98 and test models of real-world conceptual learning. We also provide a demonstration of how our  
99 models can be used to construct “maps” of what students know, and how their knowledge changes  
100 with training. In addition to helping to visualize knowledge (and changes in knowledge), we hope



**Figure 1: Experimental paradigm.** Participants alternate between answering 13-question multiple-choice quizzes and watching two Khan Academy lectures. Each quiz contains a mix of 5 questions about lecture 1, 5 questions about lecture 2, and 3 questions about general physics knowledge. The specific questions reflected on each quiz, and the orders of each quiz's questions, were randomized across participants.

101 that such maps might lead to real-world tools for improving how we educate.

## Results

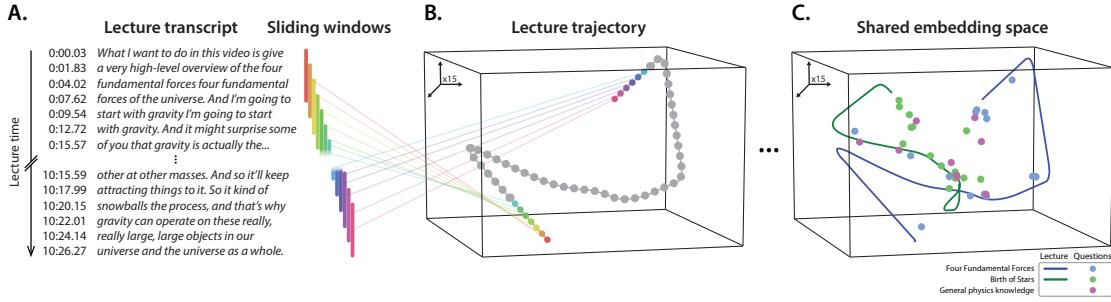
102 At its core, our main modeling approach is based around a simple assumption that we sought to  
 103 test empirically: all else being equal, knowledge about a given concept is predictive of knowledge  
 104 about similar or related concepts. From a geometric perspective, this assumption implies that  
 105 knowledge is fundamentally “smooth.” In other words, as one moves through a space representing  
 106 an individual’s knowledge (where similar concepts occupy nearby coordinates), their “level of  
 107 knowledge” should change relatively gradually throughout that space. To begin to test this  
 108 smoothness assumption, we sought to track participants’ knowledge and how it changed over  
 109 time in response to training. Two overarching goals guide our approach. First, we want to gain  
 110 detailed insights into what learners know, at different points in their training. For example, rather  
 111 than simply reporting on the proportions of questions participants answer correctly (i.e., their  
 112 overall performance), we seek estimates of their knowledge about a variety of specific concepts.  
 113 Second, we want our approach to be potentially scalable to large numbers of concepts, courses,  
 114 and students. This requires the conceptual content of interest to be discovered *automatically*, rather  
 115 than relying on manually produced ratings or labels.

116 We asked participants in our study to complete brief multiple-choice quizzes before, between,

118 and after watching two lecture videos from the Khan Academy [22] platform (Fig. 1). The first  
119 lecture video, entitled *Four Fundamental Forces*, discussed the four fundamental forces in physics:  
120 gravity, strong and weak interactions, and electromagnetism. The second, entitled *Birth of Stars*,  
121 provided an overview of our current understanding of how stars form. We selected these particular  
122 lectures to satisfy three general criteria. First, we wanted both lectures to be accessible to a broad  
123 audience (i.e., with minimal prerequisite knowledge) so as to limit the impact of prior training on  
124 our participants' abilities to learn from the lectures. To this end, we selected two introductory videos  
125 that were intended to be viewed at the start of students' training in their respective content areas.  
126 Second, we wanted both lectures to have some related content, so that we could test our approach's  
127 ability to distinguish similar conceptual content. To this end, we chose two videos from the same  
128 (per instructor annotations) Khan Academy course domain, "Cosmology and Astronomy." Third,  
129 we sought to minimize dependencies and specific overlap between the videos. For example, we  
130 did not want participants' abilities to understand one video to (directly) influence their abilities to  
131 understand the other. To satisfy this last criterion, we chose videos from two different lecture series  
132 (lectures 1 and 2 were from the "Scale of the Universe" and "Stars, Black Holes, and Galaxies"  
133 series, respectively).

134 We also wrote a set of multiple-choice quiz questions that we hoped would enable us to  
135 evaluate participants' knowledge about each individual lecture, along with related knowledge  
136 about physics not specifically presented in either video (see Tab. S1 for the full list of questions  
137 in our stimulus pool). Participants answered questions randomly drawn from each content area  
138 (lecture 1, lecture 2, and general physics knowledge) on each of the three quizzes. Quiz 1 was  
139 intended to assess participants' "baseline" knowledge before training, quiz 2 assessed knowledge  
140 after watching the *Four Fundamental Forces* video (i.e., lecture 1), and quiz 3 assessed knowledge  
141 after watching the *Birth of Stars* video (i.e., lecture 2).

142 To study in detail how participants' conceptual knowledge changed over the course of the  
143 experiment, we first sought to model the conceptual content presented to them at each moment  
144 throughout each of the two lectures. We adapted an approach we developed in prior work [17]  
145 to identify the latent themes in the lectures using a topic model [4]. Briefly, topic models take



**Figure 2: Modeling course content.** **A. Building a document pool from sliding windows of text.** We decompose each lecture’s transcript into a series of overlapping sliding windows. The set of transcript snippets (across all windows) may be treated as a set of “documents” for training a text embedding model. **B. Constructing lecture content trajectories.** After training our model on the sliding windows from both lectures, we transform each lecture into a “trajectory” through text embedding space by joining the embedding coordinates of successive sliding windows parsed from its transcript. **C. Embedding multiple lectures and questions in a shared space.** We apply the same model (trained on the two lectures’ windows) to both lectures, along with the text of each question in our pool (Tab. S1), to project them into a shared text embedding space. This results in one trajectory per lecture and one coordinate for each question. Here we have projected the 15-dimensional embeddings onto their first 3 principal components for visualization.

as input a collection of text documents and learn a set of “topics” (i.e., latent themes) from their contents. Once fit, a topic model can be used to transform arbitrary (potentially new) documents into sets of “topic proportions,” describing the weighted blend of learned topics reflected in their texts. We parsed automatically generated transcripts of the two lectures into overlapping sliding windows, where each window contained the text of the lecture transcript from a particular time range. We treated the set of text snippets (across all of these windows) as documents to fit our model (Fig. 2A; see Constructing text embeddings of multiple lectures and questions). Transforming the text from every sliding window with our model yielded a number-of-windows by number-of-topics (15) topic-proportions matrix that described the unique mixture of broad themes from both lectures reflected in each window’s text. Each window’s “topic vector” (i.e., column of the topic-proportions matrix) is a coordinate in a 15-dimensional space whose axes are topics discovered by the model. Within this space, each lecture’s sequence of topic vectors (i.e., corresponding to its transcript’s overlapping text snippets across sliding windows) forms a *trajectory* that captures how its conceptual content unfolds over time (Fig. 2B). We resampled these trajectories to a resolution of one topic vector for each second of video (i.e., 1 Hz).

161 We hypothesized that a topic model trained on transcripts of the two lectures should also  
162 capture the conceptual knowledge probed by each quiz question. If indeed the topic model could  
163 capture information about the deeper conceptual content of the lectures (i.e., beyond surface-  
164 level details such as particular word choices) then we should be able to recover a correspondence  
165 between each lecture and questions *about* each lecture. Importantly, such a correspondence could  
166 not solely arise from superficial text matching between lecture transcripts and questions, since  
167 the lectures and questions used different words. Simply comparing the average topic weights  
168 from each lecture and question sets (averaging across time and questions, respectively) reveals a  
169 striking correspondence (Fig. S1). Specifically, the average topic weights from lecture 1 are strongly  
170 correlated with the average topic weights from lecture 1 questions ( $r(13) = 0.809, p < 0.001, 95\%$   
171 confidence interval (CI) = [0.633, 0.962]), and the average topic weights from lecture 2 are strongly  
172 correlated with the average topic weights from lecture 2 questions ( $r(13) = 0.728, p = 0.002, CI =$   
173 [0.456, 0.920]). At the same time, the average topics from two lectures are *negatively* correlated  
174 ( $r(13) = -0.634, p = 0.011, CI = [-0.924, -0.237]$ ), indicating that the topic model also exhibits some  
175 degree of specificity. The full set of pairwise comparisons between topic vectors for the lectures  
176 and each question set is reported in Figure S1.

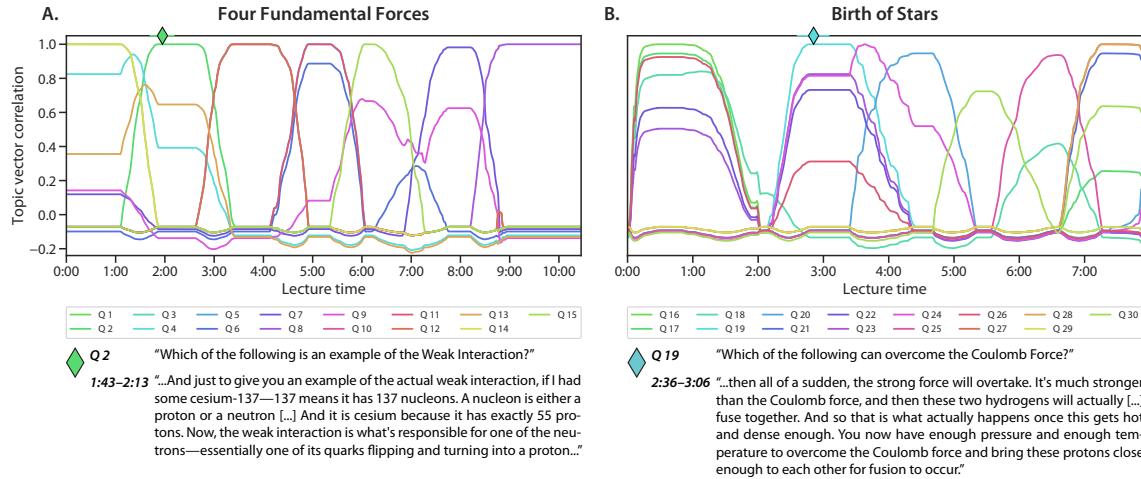
177 Another, more sensitive, way of summarizing the conceptual content of the lectures and ques-  
178 tions is to look at *variability* in how topics are weighted over time and across different questions  
179 (Fig. 3). Intuitively, the variability in the expression of a given topic relates to how much “infor-  
180 mation” [13] the lecture (or questions) reflect about that topic. For example, suppose a given topic  
181 is weighted on heavily throughout a lecture. That topic might be characteristic of some aspect or  
182 property of the lecture *overall* (conceptual or otherwise), but unless the topic’s weights changed in  
183 meaningful ways over time, the topic would be a poor indicator of any *specific* conceptual content  
184 in the lecture. We therefore also compared the variances in topic weights (across time or questions)  
185 between the lectures and questions. The variability in topic expression (over time and across ques-  
186 tions) was similar for the lecture 1 video and questions ( $r(13) = 0.824, p < 0.001, CI = [0.696, 0.973]$ )  
187 and the lecture 2 video and questions ( $r(13) = 0.801, p < 0.001, 95\% CI = [0.539, 0.958]$ ). However,  
188 as reported in Figure 3B, the variability in topic expressions across *different* videos and lecture-



**Figure 3: Lecture and question topic overlap.** **A. Topic weight variability.** The bar plots display the variance of each topic's weight across lecture timepoints (top row) and questions (bottom row); colors denote topics. The top-weighted words from the most “expressive” (i.e., variable across observations) topic from each lecture are displayed in the upper right (orange: topic 2; yellow-green: topic 4). The top-weighted words from the full set of topics may be found in Table S2. **B. Relationships between topic weight variability.** Pairwise correlations between the distributions of topic weight variance for each lecture and question category. Each row and column corresponds to a bar plot in Panel A.

189 specific questions (i.e., lecture 1 video versus lecture 2 questions; lecture 2 video versus lecture 1  
190 questions) were negatively correlated, and neither video’s topic variability was reliably correlated  
191 with the topic variability across general physics knowledge questions. Taken together, the analyses  
192 reported in Figures 3 and S1 indicate that a topic model fit to the videos’ transcripts can also reveal  
193 correspondances (at a coarse scale) between the lectures and (held-out) questions.

194 Although a single lecture may be organized around a single broad theme at a coarse scale, at a  
195 finer scale each moment of a lecture typically covers a narrower range of content. We wondered  
196 whether a text embedding model trained on the lectures’ transcripts might capture some of this  
197 finer scale content. For example, if a particular question asks about the content from one small part  
198 of a lecture, we wondered whether the text embeddings could be used to automatically identify  
199 the “matching” moment(s) in the lecture. When we correlated each question’s topic vector with  
200 the topic vectors from each second of the lectures, we found some evidence that each question is  
201 temporally specific (Fig. 4). In particular, most questions’ topic vectors were maximally correlated  
202 with a well-defined (and relatively narrow) range of timepoints from their corresponding lectures,



**Figure 4: Which parts of each lecture are captured by each question?** Each panel displays timeseries plots showing how each question’s topic vector correlates with each video timepoint’s topic vector (Panel A.: correlations for the *Four Fundamental Forces* lecture and associated questions; Panel B.: correlations for the *Birth of Stars* lecture and associated questions). The colors denote question identities. The diamonds in each panel denote the moment of peak correlation between the indicated question and the lecture trajectory. The associated questions’ text and snippets of the lectures’ transcripts from the surrounding 30 seconds, are displayed at the bottom of the figure.

and the correlations fell off sharply outside of that range. We also examined the best-matching intervals for each question qualitatively by comparing the text of the question to the text of the most-correlated parts of the lectures. Despite that the questions were excluded from the text embedding model’s training set, in general we found (through manual inspection) a close correspondence between the conceptual content that each question covered and the content covered by the best-matching moments of the lectures. Two representative examples are shown at the bottom of Figure 4.

The ability to quantify how much each question is “asking about” the content from each moment of the lectures could enable high-resolution insights into participants’ knowledge. Traditional approaches to estimating how much a student “knows” about the content of a given lecture entail computing the proportion of correctly answered questions. But if two students receive identical scores on an exam, might our modeling framework help us to gain more nuanced insights into the *specific* content that each student has mastered (or failed to master)? For example, a student

216 who misses three questions that were all about the same concept (e.g., concept *A*) will have gotten  
217 the same *proportion* of questions correct as another student who missed three questions about  
218 three *different* concepts (e.g., *A*, *B*, and *C*). But if we wanted to fill in the “gaps” in the two  
219 students’ understandings, we might do well to focus on concept *A* for the first student, but to  
220 also add in materials pertaining to concepts *B* and *C* for the second student. In other words, raw  
221 “proportion-correct” measures may capture *how much* a student knows, but not *what* they know.  
222 We wondered whether our modeling framework might enable us to (formally and automatically)  
223 infer participants’ knowledge at the scale of individual concepts (e.g., as captured by a single  
224 question).

225 We developed a simple formula (Eqn. 1) for using a participant’s responses to a small set  
226 of multiple-choice questions to estimate how much the participant “knows” about the concept  
227 reflected by any arbitrary coordinate,  $x$ , in text embedding space (e.g., the content reflected by any  
228 moment in a lecture they had watched; see Estimating dynamic knowledge traces). Essentially,  
229 the estimated knowledge at the coordinate is given by the weighted average proportion of quiz  
230 questions the participant answered correctly, where the weights reflect how much each question  
231 is “about” the content at  $x$ . When we apply this approach to estimate the participant’s knowledge  
232 about the content presented in each moment of each lecture, we can obtain a detailed timecourse  
233 describing how much “knowledge” the participant has about any part of the lecture. As shown  
234 in Figure 5, we can also apply this approach separately for the questions from each quiz the  
235 participants took throughout the experiment. From just 13 questions per quiz, we obtain a high-  
236 resolution snapshot (at the time each quiz was taken) of what the participants knew about any  
237 moment’s content, from either of the two lectures they watched (comprising a total of 1106 samples  
238 across the two lectures).

239 Of course, even though the timecourses in Figure 5A and C provide detailed *estimates* about  
240 participants’ knowlege, those estimates are only *useful* to the extent that they accurately reflect what  
241 participants actually know. As one sanity check, we anticipated that the knowledge estimates  
242 should show a content-specific “boost” in participants’ knowledge after watching each lecture.  
243 In other words, if participants learn about each lecture’s content when they watch each lecture,

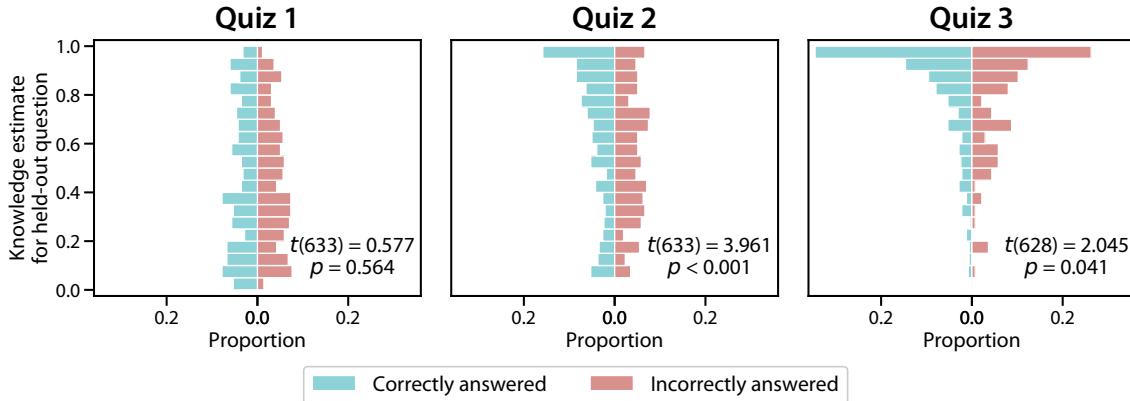


**Figure 5: Estimating moment-by-moment knowledge acquisition.** **A. Moment-by-moment knowledge about the *Four Fundamental Forces*.** Each trace displays the weighted proportion of correctly answered questions about the content reflected in each moment of the lecture (see Estimating dynamic knowledge traces), using responses from one quiz (color). The traces are averaged across participants. **B. Average estimated knowledge about the *Four Fundamental Forces*.** Each bar displays the across-timepoint average knowledge, estimated using the responses to one quiz's questions. **C. Moment-by-moment knowledge about the *Birth of Stars*.** The panel is in the same format as Panel A, but here the knowledge estimates are for the moment-by-moment content of the *Birth of Stars* lecture. **D. Average estimated knowledge about the *Birth of Stars*.** The panel is in the same format as Panel B, but here the knowledge estimates are for the content of the *Birth of Stars* lecture. All panels: error ribbons and error bars denote 95% confidence intervals, estimated across participants.

the knowledge estimates should reflect that. After watching the *Four Fundamental Forces* lecture, participants should show more knowledge for the content of that lecture than they had before, and that knowledge should persist for the remainder of the experiment. Specifically, knowledge about that lecture's content should be relatively low when estimated using Quiz 1 responses, but should increase when estimated using Quiz 2 or 3 responses (Fig. 5B). Indeed, we found that participants' estimated knowledge about the content of the *Four Fundamental Forces* was substantially higher on Quiz 2 versus Quiz 1 ( $t(49) = 8.764, p < 0.001$ ) and on Quiz 3 versus Quiz 1 ( $t(49) = 10.519, p < 0.001$ ). We found no reliable differences in estimated knowledge about that lecture's content on Quiz 2 versus 3 ( $t(49) = 0.160, p = 0.874$ ). Similarly, we hypothesized (and subsequently confirmed) that participants should show more estimated knowledge about the content of the *Birth of Stars* lecture after (versus before) watching it (Fig. 5D). Specifically, since participants watched that lecture after taking Quiz 2 (but before Quiz 3), we hypothesized that their knowledge estimates should be relatively low on Quizzes 1 and 2, but should show a "boost" on Quiz 3. Consistent with this prediction, we found no reliable differences in estimated knowledge about the *Birth of Stars* lecture content on Quizzes 1 versus 2 ( $t(49) = 1.013, p = 0.316$ ), but the estimated knowledge was substantially higher on Quiz 3 versus 2 ( $t(49) = 10.561, p < 0.001$ ) and Quiz 3 versus 1 ( $t(49) = 8.969, p < 0.001$ ).

If we are able to accurately estimate a participant's knowledge about the content tested by a given question, the estimated knowledge should have some predictive information about whether the participant is likely to answer the question correctly or incorrectly. For each question in turn, for each participant, we used Equation 1 to estimate (using all *other* questions from the same quiz, from the same participant) the participant's knowledge at the held-out question's embedding coordinate. For each quiz, we grouped these estimates into two distributions: one for the estimated knowledge at the coordinates of each *correctly* answered question, and another for the estimated knowledge at the coordinates of each *incorrectly* answered question (Fig. 6). We then used independent samples  $t$ -tests to compare the means of these distributions of estimated knowledge.

For the initial quizzes participants took (prior to watching either lecture), participants' estimated knowledge tended to be low overall, and relatively unstructured (Fig. 6, left panel). When we held



**Figure 6: Estimating knowledge at the embedding coordinates of held-out questions.** Separately for each quiz (panel), we plot the distributions of predicted knowledge at the embedding coordinates of each held out correctly (blue) or incorrectly (red) answered question. The  $t$ -tests reported in each panel are between the distributions of estimated knowledge at the coordinates of correctly versus incorrectly answered held-out questions.

out individual questions and estimated their knowledge at the held-out questions' embedding coordinates, we found no reliable differences in the estimates when the held-out question had been correctly versus incorrectly answered ( $t(633) = 0.577, p = 0.564$ ). After watching the first video, estimated knowledge for held-out correctly answered questions (from the second quiz; Fig. 6, middle panel) exhibited a positive shift relative to held-out incorrectly answered questions ( $t(633) = 3.961, p < 0.001$ ). After watching the second video, estimated knowledge (from the third quiz; Fig. 6, right panel) for *all* questions exhibited a positive shift. However, the increase in estimated knowledge for held-out correctly answered questions was larger than for held-out incorrectly answered questions ( $t(628) = 2.045, p = 0.041$ ).

Knowledge estimates need not be limited to the content of the lectures. As illustrated in Figure 7, our general approach to estimating knowledge from a small number of quiz questions may be applied to *any* content, given its text embedding coordinate. To visualize how knowledge "spreads" through text embedding space to content beyond the lectures participants watched, we first fit a new topic model to the lectures' sliding windows with  $k = 100$  topics. We hoped that increasing the number of topics from 15 to 100 might help us to generalize the knowledge predictions. (Aside from increasing the number of topics from 15 to 100, all other procedures and

288 model parameters were carried over from the preceding analyses.) As in our other analyses, we  
289 resampled each lecture’s topic trajectory to 1 Hz and also projected each question into a shared  
290 text embedding space.

291 We projected the resulting 100-dimensional topic vectors (for each second of video and for  
292 each question) into a shared 2-dimensional space (see Creating knowledge and learning map  
293 visualizations). Next, we sampled points from a  $100 \times 100$  grid of coordinates that evenly tiled  
294 a rectangle enclosing the 2D projections of the videos and questions. We used Equation 4 to  
295 estimate participants’ knowledge at each of these 10,000 sampled locations, and we averaged these  
296 estimates across participants to obtain an estimated average *knowledge map* (Fig. 7). Intuitively,  
297 the knowledge map constructed from a given quiz’s responses provides a visualization of how  
298 “much” participants know about any content expressible by the fitted text embedding model.

299 Several features of the resulting knowledge maps are worth noting. The average knowledge  
300 map estimated from Quiz 1 responses (Fig. 7, leftmost map) shows that participants tended to  
301 have relatively little knowledge about any parts of the text embedding space (i.e., the shading  
302 is relatively dark everywhere). The knowledge map estimated from Quiz 2 responses shows a  
303 marked increase in knowledge on the left side of the map (around roughly the same range of  
304 coordinates covered by the *Four Fundamental Forces* lecture, indicated by the dotted blue line).  
305 In other words, participants’ estimated increase in knowledge is localized to conceptual content  
306 that is nearby (i.e., related to) the content from the lecture they watched prior to taking Quiz  
307 2. This localization is non-trivial: the knowledge estimates are informed only by the embedded  
308 coordinates of the *quiz questions*, not by the embeddings of either lecture (Eqn. 4). Finally, the  
309 knowledge map estimated from Quiz 3 responses shows a second increase in knowledge, localized  
310 to the region surrounding the embedding of the *Birth of Stars* lecture participants watched prior to  
311 taking Quiz 3.

312 Another way of visualizing these content-specific increases in knowledge (apparently driven  
313 by watching each lecture) is displayed in Figure 7B. Taking the point-by-point difference between  
314 the knowledge maps estimated from responses to a successive pair of quizzes yields a *learning map*  
315 that describes the *change* in knowledge estimates from one quiz to the next. These learning maps



**Figure 7: Mapping out the geometry of knowledge and learning.** **A.** Average “knowledge maps” estimated using each quiz. Each map displays a 2D projection of the estimated knowledge about the content reflected by all regions of topic space (see Creating knowledge and learning map visualizations). The topic trajectories of each lecture and the coordinates of each question are indicated by dotted lines and dots. Each map reflects an average across all participants. For individual participants’ maps, see Figures S2, S3, and S4. **B.** Average “learning maps” estimated between each successive pair of quizzes. The learning maps are in the same general format as the knowledge maps in Panel A, but each coordinate in the learning maps indicates the difference between the corresponding coordinates in the indicated pair of knowledge maps—i.e., how much the estimated knowledge “changed” across the two quizzes. Each map reflects an average across all participants. For individual participants’ maps, see Figures S5 and S6. **C.** Word clouds for sampled points in topic space. Each word cloud displays the relative weights of each word (via their relative sizes) reflected by the blend of topics represented at the locations of the stars in the maps. The words’ colors indicate how much each word is weighted, on average, across all timepoints’ topic vectors in the *Four Fundamental Forces* (blue) and *Birth of Stars* (green) videos, respectively.

316 highlight that the estimated knowledge increases we observed across maps were specific to the  
317 regions around the embeddings of each lecture in turn.

318 Because the 2D projection we used to construct the knowledge and learning maps is invertible,  
319 we may gain additional insights into the estimates by reconstructing the original high-dimensional  
320 topic vectors for any point(s) in the maps we are interested in. For example, this could serve as  
321 a useful tool for an instructor looking to better understand which content areas a student (or a  
322 group of students) knows well (or poorly). As a demonstration, we show the top-weighted words  
323 from the blends of topics reconstructed from three example locations on the maps (Fig. 7C): one  
324 point near the *Four Fundamental Forces* embedding (yellow); a second point near the *Birth of Stars*  
325 embedding (orange), and a third point somewhere in between the two lectures' embeddings (pink).  
326 As shown in the word clouds in the Panel, the top-weighted words at the example coordinate near  
327 the *Four Fundamental Forces* embedding also tended to be weighted heavily by the topics expressed  
328 in that lecture. Similarly, the top-weighted words at the example coordinate near the *Birth of Stars*  
329 embedding tended to be weighted most heavily by the topics expressed in *that* lecture. And the  
330 top-weighted words at the example coordinate between the two lectures' embeddings show a  
331 roughly even mix of words most strongly associated with each lecture.

## 332 Discussion

333 We developed a computational framework that uses short multiple choice quizzes to provide  
334 nuanced insights into what learners know and how their knowledge changes with training. First,  
335 we show that our approach can automatically match up the conceptual content of individual  
336 questions with the corresponding moments in lecture videos when those concepts were presented  
337 (Fig. 4). Next, we demonstrate how we can estimate moment-by-moment “knowledge traces”  
338 that reflect how much knowledge participants have about each video’s content before and after  
339 watching each lecture video (Fig. 5). We also show that these knowledge estimates can generalize  
340 to held-out questions (Fig. 6). Finally, we use our framework to construct visual maps that provide  
341 snapshot estimates of how much participants know about any concept within the scope of our text

342 embedding model, and how much their knowledge changes with training (Fig. 7).

343 Over the past several years, the global pandemic has forced many educators to teach re-  
344 motely [21, 33, 41, 44]. This change in world circumstances is happening alongside (and perhaps  
345 accelerating) geometric growth in the availability of high quality online courses on platforms such  
346 as Khan Academy [22], Coursera [45], EdX [24], and others [38]. Continued expansion of the global  
347 internet backbone and improvements in computing hardware have also facilitated improvements  
348 in video streaming, enabling videos to be easily downloaded and shared by large segments of the  
349 world’s population. This exciting time for online course instruction provides an opportunity to  
350 re-evaluate how we, as a global community, educate ourselves and each other. For example, we  
351 can ask: what makes an effective course or training program? Which aspects of teaching might be  
352 optimized or automated? How and why do learning needs and goals vary across people? How  
353 might we lower barriers to achieving a high quality education?

354 Alongside these questions, there is a growing desire to extend existing theories beyond the  
355 domain of lab testing rooms and into real classrooms [20]. In part, this has led to a recent  
356 resurgence of “naturalistic” or “observational” experimental paradigms that attempt to better  
357 reflect more ethologically valid phenomena that are more directly relevant to real-world situations  
358 and behaviors [34]. In turn, this has brought new challenges in data analysis and interpretation. A  
359 key step towards solving these challenges will be to build explicit models of real-world scenarios  
360 and how people behave in them (e.g., models of how people learn conceptual content from real-  
361 world courses, as in our current study). A second key step will be to understand which sorts  
362 of signals derived from behaviors and/or other measurements [e.g., neurophysiological data; 1,  
363 12, 31, 35, 36] might help to inform these models. A third major step will be to develop and  
364 employ reliable ways of evaluating the complex models and data that are a hallmark of naturalistic  
365 paradigms.

366 Beyond specifically predicting what people *know*, the fundamental ideas we develop here also  
367 relate to the notion of “theory of mind” of other individuals [15, 18, 30]. Considering others’ unique  
368 perspectives, prior experiences, knowledge, goals, etc., can help us to more effectively interact and  
369 communicate [37, 40, 43]. One could imagine future extensions of our work (e.g., analogous to

370 the knowledge and learning maps shown in Fig. 7), that attempt to characterize how well-aligned  
371 different people’s knowledge bases or backgrounds are. In turn, this might be used to model how  
372 knowledge (or other forms of communicable information) flows not just between teachers and  
373 students, but between friends having a conversation, individuals out on a first date, participants at  
374 a business meeting, doctors and patients, experts and non-experts, political allies or adversaries,  
375 and more. For example, the extent to which two people’s knowledge maps “match” or “align” in  
376 a given region of text embedding space might serve as a predictor of how effectively the people  
377 will communicate about the corresponding conceptual content.

378 Ultimately, our work suggests a rich new line of questions about the geometric “form” of  
379 knowledge, how knowledge changes over time, and how we might map out the full space of  
380 what an individual knows. Our finding that detailed estimates about knowledge may be obtained  
381 from short quizzes shows one way that traditional approaches to evaluation in education might  
382 be extended. We hope that these advances might help pave the way for new ways of teaching or  
383 delivering educational content that are tailored to individual students’ learning needs and goals.

## 384 Materials and methods

### 385 Participants

386 We enrolled a total of 50 Dartmouth undergraduate students in our study. Participants received  
387 course credit for enrolling. We asked each participant to fill out a demographic survey that included  
388 questions about their age, gender, native spoken language, ethnicity, race, hearing, color vision,  
389 sleep, coffee consumption, level of alertness, and several aspects of their educational background  
390 and prior coursework.

391 Participants’ ages ranged from 18 to 22 years (mean: 19.52 years; standard deviation: 1.09  
392 years). A total of 15 participants reported their gender as male and 35 participants reported their  
393 gender as female. A total of 49 participants reported their native language as “English” and 1  
394 reported having another native language. A total of 47 participants reported their ethnicity as

395 "Not Hispanic or Latino" and three reported their ethnicity as "Hispanic or Latino." Participants  
396 reported their races as White (32 participants), Asian (14 participants), Black or African American  
397 (5 participants), American Indian or Alaska Native (1 participant), and Native Hawaiian or Other  
398 Pacific Islander (1 participant). (Note that some participants selected multiple racial categories.)

399 A total of 49 participants reporting having normal hearing and 1 participant reported having  
400 some hearing impairment. A total of 49 participants reported having normal color vision and 1  
401 participant reported being color blind. Participants reported having had, on the night prior to  
402 testing, 2–4 hours of sleep (1 participant), 4–6 hours of sleep (9 participants), 6–8 hours of sleep (35  
403 participants), or 8+ hours of sleep (5 participants). They reported having consumed, on the same  
404 day and leading up to their testing session, 0 cups of coffee (38 participants), 1 cup of coffee (10  
405 participants), 3 cups of coffee (1 participant), or 4+ cups of coffee (1 participant).

406 No participants reported that their focus was currently impaired (e.g., by drugs or alcohol).  
407 Participants reported their current level of alertness, and we converted their responses to numerical  
408 scores as follows: "very sluggish" (-2), "a little sluggish" (-1), "neutral" (0), "fairly alert" (1), and  
409 "very alert" (2). Across all participants, a range of alertness levels were reported (range: -2 – 1;  
410 mean: -0.10; standard deviation: 0.84).

411 Participants reported their undergraduate major(s) as "social sciences" (28 participants), "nat-  
412 ural sciences" (16 participants), "professional" (e.g., pre-med or pre-law; 8 participants), "mathe-  
413 matics and engineering" (7 participants), "humanities" (4 participants), or "undecided" (3 partici-  
414 pants). Note that some participants selected multiple categories for their undergraduate major. We  
415 also asked participants about the courses they had taken. In total, 45 participants reported having  
416 taken at least one Khan Academy course in the past, and 5 reported not having taken any Khan  
417 Academy courses. Of those who reported having watched at least one Khan Academy course,  
418 7 participants reported having watched 1–2 courses, 11 reported having watched 3–5 courses, 8  
419 reported having watched 5–10 courses, and 19 reported having watched 10 or more courses. We  
420 also asked participants about the specific courses they had watched, categorized under different  
421 subject areas. In the "Mathematics" area, participants reported having watched videos on AP  
422 Calculus AB (21 participants), Precalculus (17 participants), Algebra 2 (14 participants), AP Cal-

423 calculus BC (12 participants), Trigonometry (11 participants), Algebra 1 (10 participants), Geometry  
424 (8 participants), Pre-algebra (7 participants), Multivariable Calculus (5 participants), Differential  
425 Equations (5 participants), Statistics and Probability (4 participants), AP Statistics (2 participants),  
426 Linear Algebra (2 participants), Early Math (1 participant), Arithmetic (1 participant), and other  
427 videos not listed in our survey (5 participants). In the “Science and engineering” area, participants  
428 reported having watched videos on Chemistry, AP Chemistry, or Organic Chemistry (21 partic-  
429 ipants); Physics, AP Physics I, or AP Physics II (15 participants); Biology, AP Biology; or High  
430 school Biology (15 participants); Health and Medicine (1 participant); or other videos not listed  
431 in our survey (19 participants). We also asked participants whether they had specifically seen the  
432 videos used in our experiment. Of the 45 participants who reported having taken at least  
433 one Khan Academy course in the past, 44 participants reported that they had not watched the *Four*  
434 *Fundamental Forces* video, and 1 participant reported that they were not sure whether they had  
435 watched it. All participants reported that they had not watched the *Birth of Stars* video. When  
436 we asked participants about non-Khan Academy online courses, they reported having watched  
437 or taken courses on Mathematics (15 participants), Science and engineering (11 participants), Test  
438 preparation (9 participants), Economics and finance (3 participants), Arts and humanities (2 partic-  
439 ipants), Computing (2 participants), and other categories not listed in our survey (18 participants).  
440 Finally, we asked participants about in-person courses they had taken in different subject areas.  
441 They reported taking courses in Mathematics (39 participants), Science and engineering (38 par-  
442 ticipants), Arts and humanities (35 participants), Test preparation (27 participants), Economics  
443 and finance (26 participants), Computing (15 participants), College and careers (7 participants), or  
444 other courses not listed in our survey (6 participants).

## 445 Experiment

446 We hand-selected two course videos from the Khan Academy platform: *Four Fundamental Forces*  
447 (an introduction to gravity, electromagnetism, the weak nuclear force, and the strong nuclear force;  
448 duration: 10 minutes and 29 seconds) and *Birth of Stars* (an introduction to how stars are formed;  
449 duration: 7 minutes and 57 seconds). We then hand-created 39 multiple-choice questions: 15 about

450 the conceptual content of *Four Fundamental Forces* (i.e., lecture 1), 15 about the conceptual content  
451 of *Birth of Stars* (i.e., lecture 2), and 9 questions that tested for general conceptual knowledge about  
452 basic physics (covering material that was not presented in either video). The full set of questions  
453 and answer choices may be found in Table S1.

454 Over the course of the experiment, participants completed three 13-question multiple-choice  
455 quizzes: the first before viewing lecture 1, the second between lectures 1 and 2, and the third  
456 after viewing lecture 2 (Fig. 1). The questions appearing on each quiz, for each participant, were  
457 randomly chosen from the full set of 39, with the constraints that (a) each quiz contain 5 questions  
458 about lecture 1, 5 questions about lecture 2, and 3 questions about general physics knowledge, and  
459 (b) each question appear exactly once for each participant. The orders of questions on each quiz,  
460 and the orders of answer options for each question, were also randomized. Our experimental  
461 protocol was approved by the Committee for the Protection of Human Subjects at Dartmouth  
462 College. We used the experiment to develop and test our computational framework for estimating  
463 knowledge and learning.

## 464 Analysis

### 465 Constructing text embeddings of multiple lectures and questions

466 We adapted an approach we developed in prior work [17] to embed each moment of the two  
467 lectures and each question in our pool in a common representational space. Briefly, our approach  
468 uses a topic model (Latent Dirichlet Allocation; 4), trained on a set of documents, to discover a set  
469 of  $k$  “topics” or “themes.” Formally, each topic is defined as a set of weights over each word in  
470 the model’s vocabulary (i.e., the union of all unique words, across all documents, excluding “stop  
471 words.”). Conceptually, each topic is intended to give larger weights to words that are semantically  
472 related or that tend to co-occur in the same documents. After fitting a topic model, each document  
473 in the training set, or any *new* document that contains at least some of the words in the model’s  
474 vocabulary, may be represented as a  $k$ -dimensional vector describing how much the document  
475 (most probably) reflects each topic. (Unless, otherwise noted, we used  $k = 15$  topics.)

476 As illustrated in Figure 2A, we start by building up a corpus of documents using overlapping  
477 sliding windows that span each video’s transcript. Khan Academy provides professionally created,  
478 manual transcriptions of all videos for closed captioning. However, such transcripts would not  
479 be readily available in all contexts to which our framework could potentially be applied. Khan  
480 Academy videos are hosted on the YouTube platform, which additionally provides automated  
481 captions. We opted to use these automated transcripts (which, in prior work, we have found are  
482 sufficiently near-human quality yield reliable data in behavioral studies; 46) when developing our  
483 framework in order to make it more directly extensible and adaptable by others in the future.

484 We fetched these automated transcripts using the `youtube-transcript-api` Python pack-  
485 age [11]. The transcripts consisted of one timestamped line of text for every few seconds (mean:  
486 2.34 s; standard deviation: 0.83 s) of spoken content in the video (i.e., corresponding to each indi-  
487 vidual caption that would appear on-screen if viewing the lecture via YouTube, and when those  
488 lines would appear). We defined a sliding window length of (up to)  $w = 30$  transcript lines, and  
489 assigned each window a timestamp corresponding to the midpoint between its first and last lines’  
490 timestamps. These sliding windows ramped up and down in length at the very beginning and  
491 end of the transcript, respectively. In other words, the first sliding window covered only the first  
492 line from the transcript; the second sliding window covered the first two lines; and so on. This  
493 insured that each line of the transcript appeared in the same number ( $w$ ) of sliding windows. After  
494 performing various standard text preprocessing (e.g., normalizing case, lemmatizing, removing  
495 punctuation and stop-words), we treated the text from each sliding window as a single “doc-  
496 ument,” and we combined these documents across the two videos’ windows to create a single  
497 training corpus for the topic model. The top words from each of the 15 discovered topics may be  
498 found in Table S2.

499 After fitting a topic model to each videos’ transcripts, we could use the trained model to  
500 transform arbitrary (potentially new) documents into  $k$ -dimensional topic vectors. A convenient  
501 property of these topic vectors is that documents that reflect similar blends of topics (i.e., documents  
502 that reflect similar themes, according to the model) will yield similar coordinates (in terms of  
503 Euclidean distance, correlation, or other geometric measures). In general, the similarity between

504 different documents' topic vectors may be used to characterize the similarity in conceptual content  
505 between the documents.

506 We transformed each sliding window's text into a topic vector, and then used linear interpolation  
507 (independently for each topic dimension) to resample the resulting timeseries to one vector  
508 per second. We also used the fitted model to obtain topic vectors for each question in our pool  
509 (Tab. S1). Taken together, we obtained a *trajectory* for each video, describing its path through topic  
510 space, and a single coordinate for each question (Fig. 2C). Embedding both videos and all of the  
511 questions using a common model enables us to compare the content from different moments of  
512 videos, compare the content across videos, and estimate potential associations between specific  
513 questions and specific moments of video.

514 **Estimating dynamic knowledge traces**

515 We used the following equation to estimate each participant's knowledge about timepoint  $t$  of a  
516 given lecture,  $\hat{k}(t)$ :

$$\hat{k}(f(t, L)) = \frac{\sum_{i \in \text{correct}} \text{ncorr}(f(t, L), f(i, Q))}{\sum_{j=1}^N \text{ncorr}(f(t, L), f(j, Q))}, \quad (1)$$

517 where

$$\text{ncorr}(x, y) = \frac{\text{corr}(x, y) - \text{mincorr}}{\text{maxcorr} - \text{mincorr}}, \quad (2)$$

518 and where mincorr and maxcorr are the minimum and maximum correlations between any lecture  
519 timepoint and question, taken over all timepoints and questions across both lectures and all five  
520 question used to estimate the knowledge trace. We also define  $f(s, \Omega)$  as the  $s^{\text{th}}$  topic vector from  
521 the set of topic vectors  $\Omega$ . Here  $t$  indexes the set of lecture topic vectors,  $L$ , and  $i$  and  $j$  index the  
522 topic vectors of questions used to estimate the knowledge trace,  $Q$ . Note that "correct" denotes  
523 the set of indices of the questions the participant answered correctly on the given quiz.

524 Intuitively,  $\text{ncorr}(x, y)$  is the correlation between two topic vectors (e.g., the topic vector from one  
525 timepoint in a lecture,  $x$ , and the topic vector for one question,  $y$ ), normalized by the minimum and  
526 maximum correlations (across all timepoints  $t$  and questions  $Q$ ) to range between 0 and 1, inclusive.

527 Equation 1 then computes the weighted average proportion of correctly answered questions about  
528 the content presented at timepoint  $t$ , where the weights are given by the normalized correlations  
529 between timepoint  $t$ 's topic vector and the topic vectors for each question. The normalization  
530 step (i.e., using `ncorr` instead of the raw correlations) insures that every question contributes some  
531 non-zero amount to the knowledge estimate.

532 **Creating knowledge and learning map visualizations**

533 An important feature of our approach is that, given a trained text embedding model and partic-  
534 ipants' quiz performance on each question, we can estimate their knowledge about *any* content  
535 expressible by the embedding model—not solely the content explicitly probed by the quiz ques-  
536 tions or even appearing in the lectures. To visualize these estimates (Figs. 7, S2, S3, S4, S5, and S6),  
537 we used Uniform Manifold Approximation and Projection (UMAP; 29) to construct a 2D projection  
538 of the text embedding space. Sampling the original 100-dimensional space at high resolution to  
539 obtain an adequate set of topic vectors spanning the embedding space would be computationally  
540 intractable. However, sampling a 2D grid is trivial.

541 At a high level, the UMAP algorithm obtains low-dimensional embeddings by minimizing  
542 the cross entropy between the pairwise (clustered) distances between the observations in their  
543 original (e.g., 100-dimensional) space and the pairwise (clustered) distances in the low-dimensional  
544 embedding space (in our approach, the embedding space is 2D). In our implementation, pairwise  
545 distances in the original high-dimensional space were defined as 1 minus the correlation between  
546 the pair of coordinates, and pairwise distances in the low-dimensional embedding space were  
547 defined as the Euclidean distance between the pair of coordinates.

548 In our application, all of the coordinates we embedded were topic vectors, whose elements  
549 are always non-negative. Although UMAP is an invertible transformation at the embedding  
550 locations of the original data, other locations in the embedding space will not necessarily follow  
551 the same implicit “rules” as the original high-dimensional data. For example, inverting an arbitrary  
552 coordinate in the embedding space might result in negative-valued vectors, which are incompatible  
553 with the topic modeling framework. To protect against this issue, we log-transformed the topic

554 vectors prior to embedding them in the 2D space. When we inverted the embedded vectors (e.g.,  
555 to estimate topic vectors or word clouds, as in Fig. 7C), we passed the inverted (log-transformed)  
556 values through the exponential function to obtain a vector of non-negative values.

557 After embedding both lectures' topic trajectories and the topic vectors of every question, we  
558 defined a rectangle enclosing the 2D projections of the lectures' and quizzes' embeddings. We then  
559 sampled points from a regular  $100 \times 100$  grid of coordinates that evenly tiled this enclosing rectangle.  
560 We sought to estimate participants' knowledge (and learning, i.e., changes in knowledge) at each  
561 of the resulting 10,000 coordinates.

562 To generate our estimates, we placed a set of 39 radial basis functions (RBFs) throughout the  
563 embedding space, centered on the 2D projections for each question (i.e., we included one RBF for  
564 each question). At coordinate  $x$ , the value of an RBF centered on a question's coordinate  $\mu$ , is given  
565 by:

$$\text{RBF}(x, \mu, \lambda) = \exp \left\{ -\frac{\|x - \mu\|^2}{\lambda} \right\}. \quad (3)$$

566 The  $\lambda$  term in the RBF equation controls the "smoothness" of the function, where larger values  
567 of  $\lambda$  result in smoother maps. In our implementation we used  $\lambda = 50$ . Next, we estimated the  
568 "knowledge" at each coordinate,  $x$ , using:

$$\hat{k}(x) = \frac{\sum_{i \in \text{correct}} \text{RBF}(x, q_i, \lambda)}{\sum_{j=1}^N \text{RBF}(x, q_j, \lambda)}. \quad (4)$$

569 Intuitively, Equation 4 computes the weighted proportion of correctly answered questions, where  
570 the weights are given by how nearby (in the 2D space) each question is to the  $x$ . We also defined  
571 *learning maps* as the coordinate-by-coordinate differences between any pair of knowledge maps.  
572 Intuitively, learning maps reflect the *change* in knowledge across two maps.

## 573 Author contributions

574 Conceptualization: PCF, ACH, and JRM. Methodology: PCF, ACH, and JRM. Software: PCF.  
575 Validation: PCF. Formal analysis: PCF. Resources: PCF, ACH, and JRM. Data curation: PCF.

576 Writing (original draft): JRM. Writing (review and editing): PCF, ACH, and JRM. Visualization:  
577 PCF and JRM. Supervision: JRM. Project administration: PCF. Funding acquisition: JRM.

## 578 Data and code availability

579 All of the data analyzed in this manuscript, along with all of the code for running our experiment  
580 and carrying out the analyses may be found at <https://github.com/ContextLab/efficient-learning-khan>.  
581

## 582 Acknowledgements

583 We acknowledge useful discussions, assistance in setting up an earlier (unpublished) version of  
584 this study, and assistance with some of the data collection efforts from Will Baxley, Max Bluestone,  
585 Daniel Carstensen, Kunal Jha, Caroline Lee, Lucy Owen, Xinming Xu, and Kirsten Ziman. Our  
586 work was supported in part by NSF CAREER Award Number 2145172 to JRM. The content is  
587 solely the responsibility of the authors and does not necessarily represent the official views of our  
588 supporting organizations. The funders had no role in study design, data collection and analysis,  
589 decision to publish, or preparation of the manuscript.

## 590 References

- 591 [1] Bevilacque, D., Davidesco, I., Wan, L., and Chaloner, K. (2019). Brain-to-brain synchrony and  
592 learning outcomes vary by student-teacher dynamics: evidence from a real-world classroom  
593 electroencephalography study. *Journal of Cognitive Neuroscience*, 31(3):401–411.
- 594 [2] Blaye, A., Bernard-Peyron, V., Paour, J.-L., and Bonthoux, F. (2006). Category flexibility in chil-  
595 dren: distinguishing response flexibility from conceptual flexibility; the protracted development  
596 of taxonomic representations. *European Journal of Developmental Psychology*, 3(2):163–188.

- 597 [3] Blei, D. M. and Lafferty, J. D. (2006). Dynamic topic models. In *Proceedings of the International*  
598 *Conference on Machine Learning*, pages 113–120, New York, NY. Association for Computing  
599 Machinery.
- 600 [4] Blei, D. M., Ng, A. Y., and Jordan, M. I. (2003). Latent dirichlet allocation. *Journal of Machine*  
601 *Learning Research*, 3:993–1022.
- 602 [5] Brown, T. B., Mann, B., Ryder, N., Subbiah, M., Kaplan, J., Dhariwal, P., Neelakantan, A.,  
603 Shyam, P., Sastry, G., Askell, A., Agarwal, S., Herbert-Voss, A., Krueger, G., Henighan, T., Child,  
604 R., Ramesh, A., Ziegler, D. M., Wu, J., Winter, C., Hesse, C., Chen, M., Sigler, E., Litwin, M.,  
605 Gray, S., Chess, B., Clark, J., Berner, C., McCandlish, S., Radford, A., Sutskever, I., and Amodei,  
606 D. (2020). Language models are few-shot learners. *arXiv*, 2005.14165.
- 607 [6] Caramazza, A. and Mahon, B. Z. (2003). The organization of conceptual knowledge: the  
608 evidence from category-specific semantic deficits. *Trends in Cognitive Sciences*, 7(8):354–361.
- 609 [7] Cer, D., Yang, Y., Kong, S. Y., Hua, N., Limtiaco, N., John, R. S., Constant, N., Guajardo-  
610 Cespedes, M., Yuan, S., Tar, C., Sung, Y.-H., Strope, B., and Kurzweil, R. (2018). Universal  
611 sentence encoder. *arXiv*, 1803.11175.
- 612 [8] Constantinescu, A. O., O'Reilly, J. X., and Behrens, T. E. J. (2016). Organizing conceptual  
613 knowledge in humans with a gridlike code. *Science*, 352(6292):1464–1468.
- 614 [9] Deacon, D., Grose-Fifer, J., Yang, C. M., Stanick, V., Hewitt, S., and Dynowska, A. (2004).  
615 Evidence for a new conceptualization of semantic representation in the left and right cerebral  
616 hemispheres. *Cortex*, 40(3):467–478.
- 617 [10] Deerwester, S., Dumais, S. T., Furnas, G. W., Landauer, T. K., and Harshman, R. (1990).  
618 Indexing by latent semantic analysis. *Journal of the American Society for Information Science*,  
619 41(6):391–407.
- 620 [11] Depoix, J. (2019). YouTube transcript/subtitle API. <https://github.com/jdepoix/youtube-transcript-api>.

- 622 [12] Dikker, S., Wan, L., Davidesco, I., Kaggen, L., Oostrik, M., McClintock, J., Rowland, J.,  
623 Michalareas, G., van Bavel, J. J., Ding, M., and Poeppel, D. (2017). Brain-to-brain synchrony  
624 tracks real-world dynamic group interactions in the classroom. *Current Biology*, 27(9):1375–1380.
- 625 [13] Fisher, R. A. (1922). On the mathematical foundations of theoretical statistics. *Philosophical  
626 Transactions of the Royal Society A*, 222(602):309–368.
- 627 [14] Gallagher, J. J. (2000). Teaching for understanding and application of science knowledge.  
628 *School Science and Mathematics*, 100(6):310–318.
- 629 [15] Goldstein, T. R. and Winner, E. (2012). Enhancing empathy and theory of mind. *Journal of  
630 Cognition and Development*, 13(1):19–37.
- 631 [16] Hall, R. and Greeno, J. (2008). *21st century education: A reference handbook*, chapter Conceptual  
632 learning, pages 212–221. Sage Publications.
- 633 [17] Heusser, A. C., Fitzpatrick, P. C., and Manning, J. R. (2021). Geometric models reveal be-  
634 havioral and neural signatures of transforming naturalistic experiences into episodic memories.  
635 *Nature Human Behavior*, 5:905–919.
- 636 [18] Kanske, P., Böckler, A., and Singer, T. (2015). Models, mechanisms and moderators dissociating  
637 empathy and theory of mind. In *Social Behavior from Rodents to Humans*, pages 193–206. Springer.
- 638 [19] Katona, G. (1940). *Organizing and memorizing: studies in the psychology of learning and teaching*.  
639 Columbia University Press.
- 640 [20] Kaufman, D. M. (2003). Applying educational theory in practice. *British Medical Journal*,  
641 326(7382):213–216.
- 642 [21] Kawasaki, H., Yamasaki, S., Masuoka, Y., Iwasa, M., Fukita, S., and Matsuyama, R. (2021).  
643 Remote teaching due to COVID-19: an exploration of its effectiveness and issues. *International  
644 Journal of Environmental Research and Public Health*, 18(5):2672.
- 645 [22] Khan, S. (2004). *The Khan Academy*. Salman Khan.

- 646 [23] Kintsch (1970). *Learning, memory, and conceptual processes*. Wiley.
- 647 [24] Kolowich, S. (2013). How EdX plans to earn, and share, revenue from its free online courses.
- 648 *The Chronicle of Higher Education*, 21:1–5.
- 649 [25] Landauer, T. K. and Dumais, S. T. (1997). A solution to Plato's problem: the latent semantic  
650 analysis theory of acquisition, induction, and representation of knowledge. *Psychological Review*,  
651 104:211–240.
- 652 [26] Maclellan, E. (2005). Conceptual learning: the priority for higher education. *British Journal of  
653 Educational Studies*, 53(2):129–147.
- 654 [27] Manning, J. R. (2020). Context reinstatement. In Kahana, M. J. and Wagner, A. D., editors,  
655 *Handbook of Human Memory*. Oxford University Press.
- 656 [28] Manning, J. R. (2021). Episodic memory: mental time travel or a quantum "memory wave"  
657 function? *Psychological Review*, 128(4):711–725.
- 658 [29] McInnes, L., Healy, J., and Melville, J. (2018). UMAP: Uniform manifold approximation and  
659 projection for dimension reduction. *arXiv*, 1802(03426).
- 660 [30] Meltzoff, A. N. (2011). Social cognition and the origins of imitation, empathy, and theory of  
661 mind. In *The Wiley-Blackwell handbook of childhood cognitive development*. Wiley-Blackwell.
- 662 [31] Meshulam, M., Hasenfratz, L., Hillman, H., Liu, Y. F., Nguyen, M., Norman, K. A., and Hasson,  
663 U. (2020). Neural alignment predicts learning outcomes in students taking an introduction to  
664 computer science course. *Nature Communications*, 12(1922):doi.org/10.1038/s41467–021–22202–3.
- 665 [32] Mikolov, T., Chen, K., Corrado, G., and Dean, J. (2013). Efficient estimation of word represen-  
666 tations in vector space. *arXiv*, 1301.3781.
- 667 [33] Moser, K. M., Wei, T., and Brenner, D. (2021). Remote teaching during COVID-19: implications  
668 from a national survey of language educators. *System*, 97:102431.

- 669 [34] Nastase, S. A., Goldstein, A., and Hasson, U. (2020). Keep it real: rethinking the primacy of  
670 experimental control in cognitive neuroscience. *NeuroImage*, 15(222):117254–117261.
- 671 [35] Nguyen, M., Chang, A., Micciche, E., Meshulam, M., Nastase, S. A., and Hasson, U. (2022).  
672 Teacher-student neural coupling during teaching and learning. *Social Cognitive and Affective*  
673 *Neuroscience*, 17(4):367–376.
- 674 [36] Poulsen, A. T., Kamronn, S., Dmochowski, J., Parra, L. C., and Hansen, L. K. (2017). EEG  
675 in the classroom: synchronised neural recordings during video presentation. *Scientific Reports*,  
676 7:43916.
- 677 [37] Ratka, A. (2018). Empathy and the development of affective skills. *American Journal of*  
678 *Pharmaceutical Education*, 82(10):doi.org/10.5688/ajpe7192.
- 679 [38] Rhoads, R. A., Berdan, J., and Toven-Lindsey, B. (2013). The open courseware movement in  
680 higher education: unmasking power and raising questions about the movement’s democratic  
681 potential. *Educational Theory*, 63(1):87–110.
- 682 [39] Scott, P., Asoko, H., and Leach, J. (2007). *Handbook of research on science education*, chapter  
683 Student conceptions and conceptual learning in science. Routledge.
- 684 [40] Shao, Y. N., Sun, H. M., Huang, J. W., Li, M. L., Huang, R. R., and Li, N. (2018). Simulation-  
685 based empathy training improves the communication skills of neonatal nurses. *Clinical Simula-*  
686 *tion in Nursing*, 22:32–42.
- 687 [41] Shim, T. E. and Lee, S. Y. (2020). College students’ experience of emergency remote teaching  
688 during COVID-19. *Children and Youth Services Review*, 119:105578.
- 689 [42] Simon, M. A., Tzur, R., Heinz, K., and Kinzel, M. (2004). Explicating a mechanism for  
690 conceptual learning: elaborating the construct of reflective abstraction. *Journal for Research in*  
691 *Mathematics Education*, 35(5):305–329.
- 692 [43] Stepien, K. A. and Baernstein, A. (2006). Education for empathy. *Journal of General Internal*  
693 *Medicine*, 21:524–530.

- 694 [44] Whalen, J. (2020). Should teachers be trained in emergency remote teaching? Lessons learned  
695 from the COVID-19 pandemic. *Journal of Technology and Teacher Education*, 28(2):189–199.
- 696 [45] Young, J. R. (2012). Inside the Coursera contract: how an upstart company might profit from  
697 free courses. *The Chronicle of Higher Education*, 19(7):1–4.
- 698 [46] Ziman, K., Heusser, A. C., Fitzpatrick, P. C., Field, C. E., and Manning, J. R. (2018). Is  
699 automatic speech-to-text transcription ready for use in psychological experiments? *Behavior*  
700 *Research Methods*, 50:2597–2605.