

<sup>1</sup> A geometric framework for capturing high-resolution  
<sup>2</sup> insights into conceptual knowledge and learning in  
<sup>3</sup> classroom-like settings

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<sup>5</sup> **Abstract**

We develop a mathematical framework, based on natural language processing models, for tracking and characterizing the acquisition of conceptual knowledge in real-world educational contexts. Our approach embeds course content in a high-dimensional conceptual space, where nearby coordinates reflect similar or related concepts. We test our approach using behavioral data from participants who watched two lecture videos from the Khan Academy platform, interleaved between three short multiple-choice quizzes. We applied our framework to the videos' transcripts and the text of the quiz questions to quantify the conceptual content presented in each moment of video and knowledge probed by each quiz question. We used these embeddings, along with participants' quiz responses, to track how the learners' knowledge changed after watching each video. Our findings demonstrate how a small set of quiz questions may be used to obtain rich and meaningful high-resolution insights into individual students' knowledge, and how it changes over time as they learn.

19

## Introduction

20 Suppose that a teacher had access to a complete “map” of everything a student knew. Defining  
21 what such a map might even look like, let alone how it might be constructed or filled in, is itself a  
22 non-trivial problem. But if a teacher *were* to gain access to such a map, how might it change their  
23 ability to teach that student? Perhaps they might start by checking how well the student knew  
24 the to-be-learned information already, or how much they knew about related concepts. For some  
25 students, they could potentially optimize their teaching efforts to maximize efficiency by focusing  
26 primarily on not-yet-known content. For other students (or other content areas), it might be more  
27 effective to optimize for direct connections between already known content and new material.  
28 Observing how the student’s knowledge changed over time, in response to their teaching, could  
29 also help to guide the teacher towards the most effective strategy for that individual student.

30 Designing and building procedures and tools for mapping out knowledge touches on deep  
31 questions about what it means to learn. For example, how do we acquire conceptual knowledge?  
32 Memorizing course lectures or textbook chapters by rote can lead to the superficial *appearance*  
33 of understanding the underlying content, but achieving true conceptual understanding seems  
34 to require something deeper and richer. Does conceptual understanding entail connecting newly  
35 acquired information to the scaffolding of one’s existing knowledge or experience [6, 10, 13, 14, 57]?  
36 Or weaving a lecture’s atomic elements (e.g., its component words) into a structured network  
37 that describes how those individual elements are related? Conceptual understanding could also  
38 involve building a mental model that transcends the meanings of those individual atomic elements  
39 by reflecting the deeper meaning underlying the gestalt whole [34, 38, 54].

40 The difference between “understanding” and “memorizing,” as framed by researchers in ed-  
41 ucation, cognitive psychology, and cognitive neuroscience [e.g., 19, 25, 30, 38, 54] has profound  
42 analogs in the fields of natural language processing and natural language understanding. For  
43 example, considering the raw contents of a document (e.g., its constituent symbols, letters, and

44 words) might provide some information about what the document is about, just as memorizing a  
45 passage might provide some ability to answer simple questions about it [e.g., whether it contains  
46 words related to furniture versus physics; 7, 8, 37]. However, modern natural language process-  
47 ing models [e.g., 9, 11, 45] also attempt to capture the deeper meaning *underlying* those atomic  
48 elements. These models consider not only the co-occurrences of those elements within and across  
49 documents, but also patterns in how those elements appear across different scales (e.g., sentences,  
50 paragraphs, chapters, etc.), the temporal and grammatical properties of the elements, and other  
51 high-level characteristics of how they are used [39, 40]. According to these models, the deep  
52 conceptual meaning of a document may be captured by a feature vector in a high-dimensional  
53 representation space, where nearby vectors reflect conceptually related documents. A model that  
54 succeeds at capturing an analog of “understanding” is able to assign nearby feature vectors to  
55 two conceptually related documents, *even when the words contained in those documents have very little*  
56 *overlap*.

57 Given these insights, what form might the representation of the sum total of a person’s knowl-  
58 edge take? First, we might require a means of systematically describing or representing the nearly  
59 infinite set of possible things a person could know. Second, we might want to account for potential  
60 associations between different concepts. For example, the concepts of “fish” and “water” might be  
61 associated in the sense that fish live in water. Third, knowledge may have a critical dependency  
62 structure, such that knowing about a particular concept might require first knowing about a set of  
63 other concepts. For example, understanding the concept of a fish swimming in water first requires  
64 understanding what fish and water *are*. Fourth, as we learn, our “current state of knowledge”  
65 should change accordingly. Learning new concepts should both update our characterizations of  
66 “what is known” and also unlock any now-satisfied dependencies of those newly learned concepts  
67 so that they are “tagged” as available for future learning.

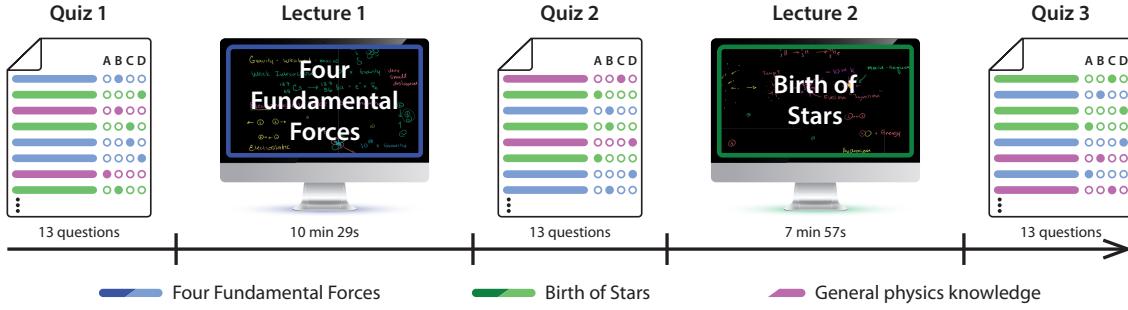
68 Here we develop a framework for modeling how knowledge is acquired during learning. The  
69 central idea is to use text embedding models to define the coordinate systems of two maps: (a) a  
70 *knowledge map* that describes the extent to which each concept is currently known and (b) a *learning*  
71 *map* that describes changes in knowledge over time. Each location on these maps represents

72 a single concept, and the maps' geometries are defined such that related concepts are located  
73 nearby in space. We use this framework to analyze and interpret behavioral data collected from  
74 an experiment that had participants watch and answer multiple-choice questions about a series of  
75 recorded course lectures.

76 Our primary research goal is to advance our understanding of what it means to acquire deep,  
77 real-world conceptual knowledge. Traditional laboratory approaches to studying learning and  
78 memory (e.g., list learning studies) often draw little distinction between memorization and under-  
79 standing. Instead, these studies typically focus on whether information is effectively encoded or  
80 retrieved, rather than whether the information is *understood*. Approaches to studying conceptual  
81 learning, such as category learning experiments, can begin to investigate the distinction between  
82 memorization and understanding, often by training participants to distinguish arbitrary or ran-  
83 dom features in otherwise meaningless categorized stimuli. However the objective of real-world  
84 training, or learning from life experiences more generally, is often to develop new knowledge  
85 that may be applied in *useful* ways in the future. In this sense, the gap between modern learning  
86 theories and modern pedagogical approaches and classroom learning strategies is enormous: most  
87 of our theories about *how* people learn are inspired by experimental paradigms and models that  
88 have only peripheral relevance to the kinds of learning that students and teachers actually seek  
89 [25, 38]. To help bridge this gap, our study uses course materials from real online courses to inform,  
90 fit, and test models of real-world conceptual learning. We also provide a demonstration of how  
91 our models can be used to construct "maps" of what students know, and how their knowledge  
92 changes with training. In addition to helping to visualize knowledge (and changes in knowledge),  
93 we hope that such maps might lead to real-world tools for improving how we educate.

## 94 Results

95 At its core, our main modeling approach is based around a simple assumption that we sought to  
96 test empirically: all else being equal, knowledge about a given concept is predictive of knowledge  
97 about similar or related concepts. From a geometric perspective, this assumption implies that

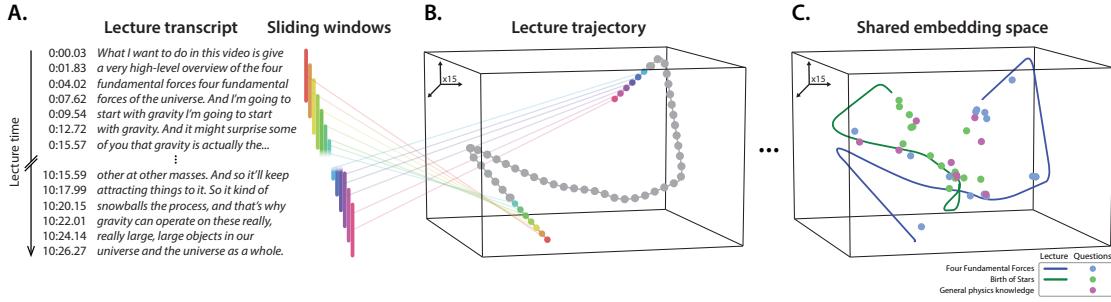


**Figure 1: Experimental paradigm.** Participants alternate between answering 13-question multiple-choice quizzes and watching two Khan Academy lectures. Each quiz contains a mix of 5 questions about lecture 1, 5 questions about lecture 2, and 3 questions about general physics knowledge. The specific questions reflected on each quiz, and the orders of each quiz’s questions, were randomized across participants.

knowledge is fundamentally “smooth.” In other words, as one moves through a space representing an individual’s knowledge (where similar concepts occupy nearby coordinates), their “level of knowledge” should change relatively gradually throughout that space. To begin to test this smoothness assumption, we sought to track participants’ knowledge and how it changed over time in response to training.

We asked participants in our study to complete brief multiple-choice quizzes before, between, and after watching two lecture videos from the Khan Academy [33] platform (Fig. 1). The first lecture video, entitled *Four Fundamental Forces*, discussed the four fundamental forces in physics: gravity, strong and weak interactions, and electromagnetism. The second, entitled *Birth of Stars*, provided an overview of our current understanding of how stars form. We selected these lessons to be (a) accessible to a broad audience, i.e., requiring minimal prerequisite knowledge to understand; (b) conceptually related to each other, i.e., covering at least *some* similar or overlapping content; and (c) largely independent of each other, i.e., focused on sufficiently different material that understanding one did not require having seen the other. The two videos we selected are introductory lectures that both belong to Khan Academy’s “Cosmology and Astronomy” course domain, but are taken from different lecture series (“Scale of the Universe” and “Stars, Black Holes, and Galaxies” for the first and second lectures, respectively).

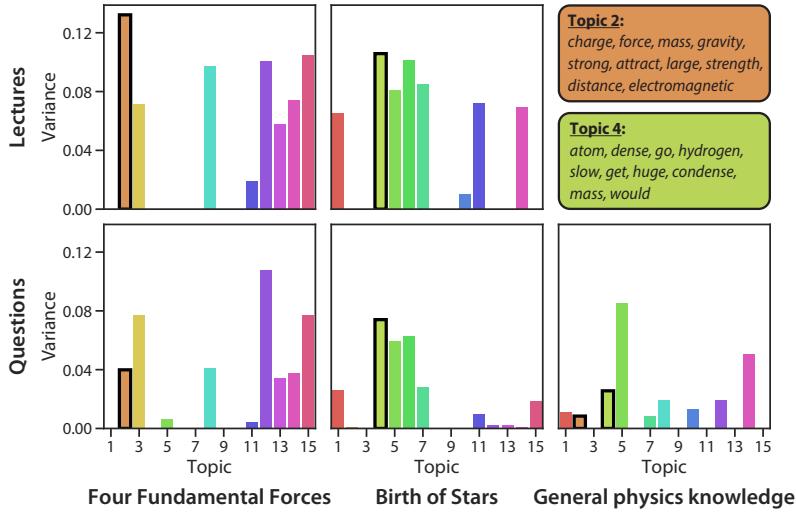
We then created a pool of multiple-choice quiz questions that would enable us to test partici-



**Figure 2: Modeling course content.** **A. Building a document pool from sliding windows of text.** We decompose each lecture’s transcript into a series of overlapping sliding windows. The set of transcript snippets (across all windows) may be treated as a set of “documents” for training a text embedding model. **B. Constructing lecture content trajectories.** After training our model on the sliding windows from both lectures, we transform each lecture into a “trajectory” through text embedding space by joining the embedding coordinates of successive sliding windows parsed from its transcript. **C. Embedding multiple lectures and questions in a shared space.** We apply the same model (trained on the two lectures’ windows) to both lectures, along with the text of each question in our pool (Tab. S1), to project them into a shared text embedding space. This results in one trajectory per lecture and one coordinate for each question. Here we have projected the 15-dimensional embeddings onto their first 3 principal components for visualization.

116 pants’ knowledge about each individual lecture, as well as related content not specifically presented  
 117 in either video ( see Tab. S1). Participants answered questions randomly drawn from each content  
 118 area (lecture 1, lecture 2, and general physics knowledge) on each of the three quizzes. Quiz 1 was  
 119 intended to assess participants’ “baseline” knowledge before training, quiz 2 assessed knowledge  
 120 after watching the *Four Fundamental Forces* video (i.e., lecture 1), and quiz 3 assessed knowledge  
 121 after watching the *Birth of Stars* video (i.e., lecture 2).

122 To study how participants’ conceptual knowledge changed over the course of the experiment,  
 123 we first sought to characterize the abstract concepts presented to them in each of the two lectures.  
 124 We adapted an approach we developed in prior work [27] to extract the latent themes from the  
 125 lectures’ contents using a topic model [8]. Briefly, topic models take as input a collection of text doc-  
 126 uments and learn a set of “topics” (i.e., latent themes) from their contents. Once fit, a topic model  
 127 can be used to transform arbitrary (potentially new) documents into sets of “topic proportions,” de-  
 128 scribing the weighted blend of learned topics reflected in their texts. We parsed automatically gen-  
 129 erated transcripts of the two lectures into overlapping sliding windows, which we treated as doc-  
 130 uments to fit our model (Fig. 2A; see *Constructing text embeddings of multiple lectures and questions*).



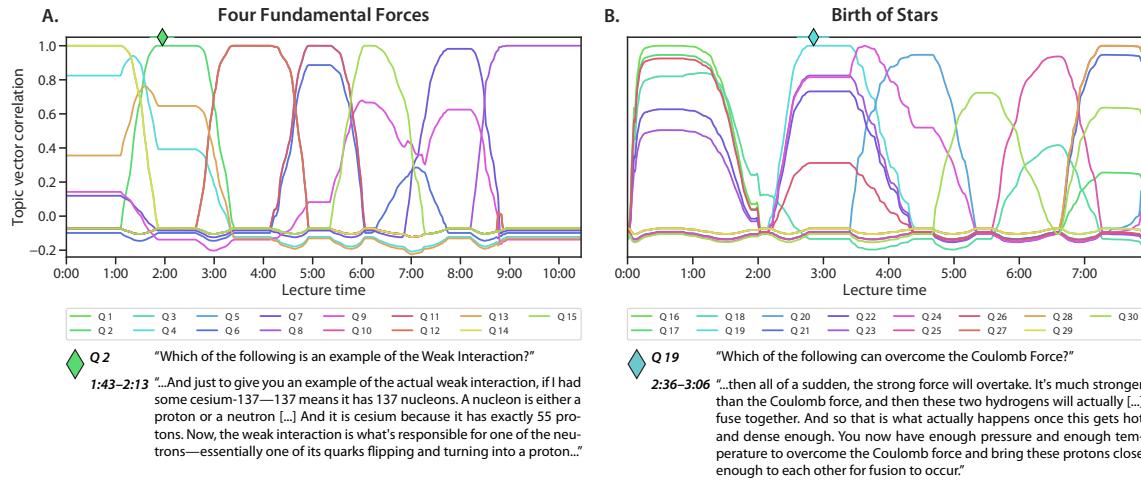
**Figure 3: Lecture and question topic overlap.** The bar plots display the variability in topic weights across lecture timepoints (top panels) and questions (bottom panels); colors denote topics. The top-weighted words from the most “expressive” (i.e., variable across observations) topic from each lecture are displayed in the upper right (orange: topic 2; yellow-green: topic 4). The top-weighted words from the full set of topics may be found in Table S2.

131 Transforming these windows with our model resulted in a number-of-windows by number-of-  
 132 topics (15) topic-proportions matrix, denoting the unique mixture of broad themes from both  
 133 lectures reflected in each window’s content. Intuitively, each window’s “topic vector” (i.e., col-  
 134 umn of the topic-proportions matrix) is analogous to a coordinate in a 15-dimensional space (whose  
 135 axes are topics learned by the model). Within this space, each lecture’s series of topic vectors (i.e.,  
 136 corresponding to sliding windows parsed from its transcript) forms a *trajectory* that captures how  
 137 its conceptual content unfolds over time (Fig. 2A). We resampled these trajectories to a resolution  
 138 of 1 topic vector for each second of video.

139 Next, wondered whether we could use this same model to capture the  
 140 Given that we trained the text embedding model using the video transcripts, we wondered  
 141 whether the questions that were (ostensibly, by design) “about” the content of each lecture would  
 142 “match up” correctly with the lectures. In other words, we hoped that the text embeddings would  
 143 capture something about the deeper conceptual content of the lectures, beyond surface details such  
 144 as exact wording choices. If so, when we embed *new* text outside of the model’s training set, we

145 should see a correspondance between the embeddings of the training data (i.e., snippets of text  
146 from the lectures' transcripts) and other text that reflects related concepts (e.g., questions *about* each  
147 lecture). Further, although the content from any given moment from a lecture might stray from the  
148 average content (across all timepoints), we hoped that *variability* in each topic's expression over  
149 timepoints within a lecture would match up with the variability in topic expressions for questions  
150 about that lecture. Intuitively, the *variability* in the expression of a given topic relates to how  
151 much "information" [18] the lecture (or questions) reflect about that topic. When we compared the  
152 variability in topic weights across each lecture's timepoints with the variability in topic weights  
153 across each question set, we found a strong correspondence (Fig. 3). The most variable topics from  
154 the *Four Fundamental Forces* lecture, and questions about that lecture, are 2, 3, 8, 12, 13, 14, and 15.  
155 The most variable topics from the *Birth of Stars* lecture, and questions about that lecture, are 1, 4,  
156 5, 6, and 7. This strong overlap between the lectures and questions specifically about each lecture  
157 indicates that the topic model captures some of the underlying conceptual content.

158 Although a single lecture may be organized around a single broad theme at a coarse scale, at a  
159 finer scale each moment of a lecture typically covers a narrower range of content. We wondered  
160 whether a text embedding model trained on the lectures' transcripts might capture some of this  
161 finer scale content. For example, if a particular question asks about the content from one small  
162 part of a lecture, we wondered whether our text embedding model could be used to automatically  
163 identify the "matching" moment(s) in the lecture. When we correlated each question's topic vector  
164 with the topic vectors for each second of the lectures, we found some evidence that each question is  
165 temporally specific (Fig. 4). In particular, most questions' topic vectors were maximally correlated  
166 with a well-defined (and relatively narrow) range of timepoints from their corresponding lectures,  
167 and the correlations fell off sharply outside of that range. We also examined the best-matching  
168 intervals for each question qualitatively by comparing the text of the question to the text of the most-  
169 correlated parts of the lectures. Despite that the questions were excluded from the text embedding  
170 model's training set, in general we found (through manual inspection) a close correspondence  
171 between the conceptual content that each question covered and the content covered by the best-  
172 matching moments of the lectures. Two representative examples are shown at the bottom of



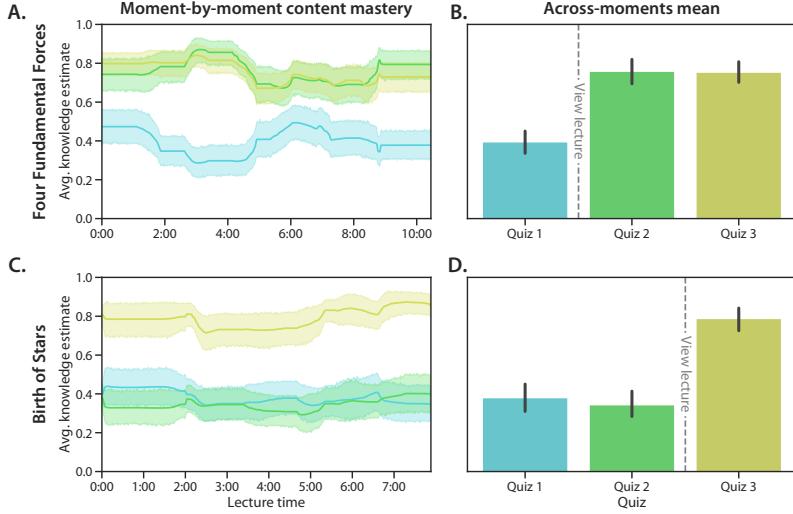
**Figure 4: Which parts of each lecture are captured by each question?** Each panel displays timeseries plots showing how each question’s topic vector correlates with each video timepoint’s topic vector (Panel A.: correlations for the *Four Fundamental Forces* lecture and associated questions; Panel B.: correlations for the *Birth of Stars* lecture and associated questions). The colors denote question identities. The diamonds in each panel denote the moment of peak correlation between the indicated questions, in the indicated lectures. The associated questions’ text, and snippets of the lectures’ transcripts in the best-matching sliding windows, are displayed at the bottom of the figure.

173 Figure 4.

174 The ability to quantify how much each question is “asking about” the content from each moment  
175 of the lectures could enable high-resolution insights into participants’ knowledge. Traditional  
176 approaches to estimating how much a student “knows” about the content of a given lecture entail  
177 computing the proportion of correctly answered questions. But if two students receive identical  
178 scores on an exam, might our modeling framework help us to gain more nuanced insights into the  
179 *specific* content that each student has mastered (or failed to master)? For example, a student who  
180 misses three questions that were all about the same concept (e.g., concept *A*) will have gotten the  
181 same *proportion* of questions correct as another student who missed three questions about three  
182 *different* concepts (e.g., *A*, *B*, and *C*). But if we wanted to fill in the “gaps” in the two students’  
183 understandings, we might do well to focus on concept *A* for the first student, but to also add in  
184 materials pertaining to concepts *B* and *C* for the second student.

185 We developed a simple formula (Eqn. 1) for using a participant’s responses to a small set  
186 of multiple-choice questions to estimate how much the participant “knows” about the concept  
187 reflected by any arbitrary coordinate,  $x$ , in text embedding space (e.g., the content reflected by  
188 any moment in a lecture they had watched; see *Estimating dynamic knowledge traces*). Essentially,  
189 the estimated knowledge at the coordinate is given by the weighted average proportion of quiz  
190 questions the participant answered correctly, where the weights reflect how much each question  
191 is “about” the content at  $x$ . When we apply this approach to estimate the participant’s knowledge  
192 about the content presented in each moment of each lecture, we can obtain a detailed timecourse  
193 describing how much “knowledge” the participant has about any part of the lecture. As shown  
194 in Figure 5, we can also apply this approach separately for the questions from each quiz the  
195 participants took throughout the experiment. From just 13 questions per quiz, we obtain a high-  
196 resolution snapshot (at the time each quiz was taken) of what the participants knew about any  
197 moment’s content, from either of the two lectures they watched (comprising a total of 1106 samples  
198 across the two lectures).

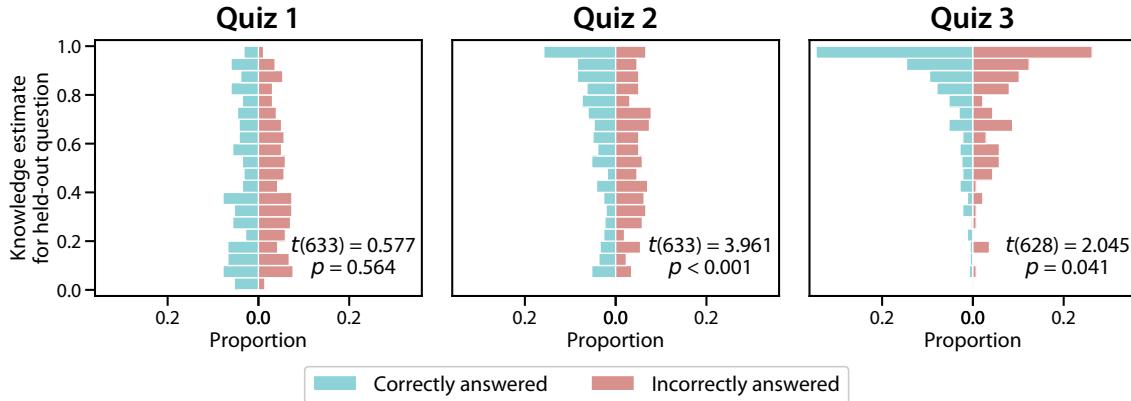
199 Of course, even though the timecourses in Figure 5A and C provide detailed *estimates* about  
200 participants’ knowledge, those estimates are only *useful* to the extent that they accurately reflect what



**Figure 5: Estimating moment-by-moment knowledge acquisition.** **A. Moment-by-moment knowledge about the *Four Fundamental Forces*.** Each trace displays the weighted proportion of correctly answered questions about the content reflected in each moment of the lecture (see *Estimating dynamic knowledge traces*), using responses from one quiz (color). The traces are averaged across participants. **B. Average estimated knowledge about the *Four Fundamental Forces*.** Each bar displays the across-timepoint average knowledge, estimated using the responses to one quiz's questions. **C. Moment-by-moment knowledge about the *Birth of Stars*.** The panel is in the same format as Panel A, but here the knowledge estimates are for the moment-by-moment content of the *Birth of Stars* lecture. **D. Average estimated knowledge about the *Birth of Stars*.** The panel is in the same format as Panel B, but here the knowledge estimates are for the content of the *Birth of Stars* lecture. All panels: error ribbons and error bars denote 95% confidence intervals, estimated across participants.

201 participants actually know. As one sanity check, we anticipated that the knowledge estimates  
202 should show a content-specific “boost” in participants’ knowledge after watching each lecture.  
203 In other words, if participants learn about each lecture’s content when they watch each lecture,  
204 the knowledge estimates should reflect that. After watching the *Four Fundamental Forces* lecture,  
205 participants should show more knowledge for the content of that lecture than they had before,  
206 and that knowledge should persist for the remainder of the experiment. Specifically, knowledge  
207 about that lecture’s content should be relatively low when estimated using Quiz 1 responses,  
208 but should increase when estimated using Quiz 2 or 3 responses (Fig. 5B). Indeed, we found  
209 that participants’ estimated knowledge about the content of the *Four Fundamental Forces* was  
210 substantially higher on Quiz 2 versus Quiz 1 ( $t(49) = 8.764, p < 0.001$ ) and on Quiz 3 versus Quiz  
211 1 ( $t(49) = 10.519, p < 0.001$ ). We found no reliable differences in estimated knowledge about  
212 that lecture’s content on Quiz 2 versus 3 ( $t(49) = 0.160, p = 0.874$ ). Similarly, we hypothesized  
213 (and subsequently confirmed) that participants should show more estimated knowledge about the  
214 content of the *Birth of Stars* lecture after (versus before) watching it (Fig. 5D). Specifically, since  
215 participants watched that lecture after taking Quiz 2 (but before Quiz 3), we hypothesized that their  
216 knowledge estimates should be relatively low on Quizzes 1 and 2, but should show a “boost” on  
217 Quiz 3. Consistent with this prediction, we found no reliable differences in estimated knowledge  
218 about the *Birth of Stars* lecture content on Quizzes 1 versus 2 ( $t(49) = 1.013, p = 0.316$ ), but the  
219 estimated knowledge was substantially higher on Quiz 3 versus 2 ( $t(49) = 10.561, p < 0.001$ ) and  
220 Quiz 3 versus 1 ( $t(49) = 8.969, p < 0.001$ ).

221 If we are able to accurately estimate a participant’s knowledge about the content tested by a  
222 given question, the estimated knowledge should have some predictive information about whether  
223 the participant is likely to answer the question correctly or incorrectly. For each question in turn, for  
224 each participant, we used Equation 1 to estimate (using all *other* questions from the same quiz, from  
225 the same participant) the participant’s knowledge at the held-out question’s embedding coordinate.  
226 For each quiz, we grouped these estimates into two distributions: one for the estimated knowledge  
227 at the coordinates of each *correctly* answered question, and another for the estimated knowledge at  
228 the coordinates of each *incorrectly* answered question (Fig. 6). We then used independent samples



**Figure 6: Estimating knowledge at the embedding coordinates of held-out questions.** Separately for each quiz (panel), we plot the distributions of predicted knowledge at the embedding coordinates of each held out correctly (blue) or incorrectly (red) answered question. The  $t$ -tests reported in each panel are between the distributions of estimated knowledge at the coordinates of correctly versus incorrectly answered held-out questions.

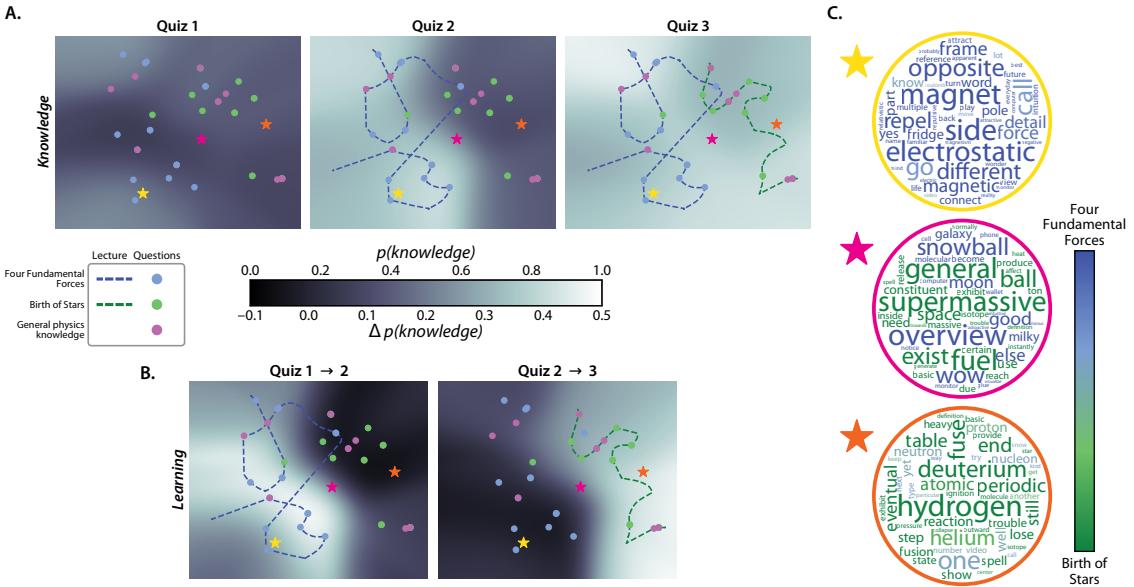
229  $t$ -tests to compare the means of these distributions of estimated knowledge.  
 230 For the initial quizzes participants took (prior to watching either lecture), participants' estimated  
 231 knowledge tended to be low overall, and relatively unstructured (Fig. 6, left panel). When we held  
 232 out individual questions and estimated their knowledge at the held-out questions' embedding  
 233 coordinates, we found no reliable differences in the estimates when the held-out question had  
 234 been correctly versus incorrectly answered ( $t(633) = 0.577, p = 0.564$ ). After watching the first  
 235 video, estimated knowledge for held-out correctly answered questions (from the second quiz;  
 236 Fig. 6, middle panel) exhibited a positive shift relative to held-out incorrectly answered questions  
 237 ( $t(633) = 3.961, p < 0.001$ ). After watching the second video, estimated knowledge (from the  
 238 third quiz; Fig. 6, right panel) for *all* questions exhibited a positive shift. However, the increase  
 239 in estimated knowledge for held-out correctly answered questions was larger than for held-out  
 240 incorrectly answered questions ( $t(628) = 2.045, p = 0.041$ ).  
 241 Knowledge estimates need not be limited to the content of the lectures. As illustrated in  
 242 Figure 7, our general approach to estimating knowledge from a small number of quiz questions  
 243 may be applied to *any* content, given its text embedding coordinate. To visualize how knowledge  
 244 "spreads" through text embedding space to content beyond the lectures participants watched,

245 we first fit a new topic model to the lectures' sliding windows with  $k = 100$  topics. We hoped  
246 that increasing the number of topics from 15 to 100 might help us to generalize the knowledge  
247 predictions. (Aside from increasing the number of topics from 15 to 100, all other procedures and  
248 model parameters were carried over from the preceding analyses.) As in our other analyses, we  
249 resampled each lecture's topic trajectory to 1 Hz and also projected each question into a shared  
250 text embedding space.

251 We projected the resulting 100-dimensional topic vectors (for each second of video and for each  
252 question) into a shared 2-dimensional space (see *Creating knowledge and learning map visualizations*).  
253 Next, we sampled points evenly from a  $100 \times 100$  grid of coordinates that evenly tiled a rectangle  
254 enclosing the 2D projections of the videos and questions. We used Equation 4 to estimate partici-  
255 pants' knowledge at each of these 10K sampled locations, and we averaged these estimates across  
256 participants to obtain an estimated average *knowledge map* (Fig. 7). Intuitively, the knowledge map  
257 constructed from a given quiz's responses provides a visualization of how "much" participants  
258 know about any content expressible by the fitted text embedding model.

259 Several features of the resulting knowledge maps are worth noting. The average knowledge  
260 map estimated from Quiz 1 responses (Fig. 7, leftmost map) shows that participants tended to  
261 have relatively little knowledge about any parts of the text embedding space (i.e., the shading  
262 is relatively dark everywhere). The knowledge map estimated from Quiz 2 responses shows a  
263 marked increase in knowledge on the left side of the map (around roughly the same range of  
264 coordinates covered by the *Four Fundamental Forces* lecture, indicated by the dotted blue line).  
265 In other words, participants' estimated increase in knowledge is localized to conceptual content  
266 that is nearby (i.e., related to) the content from the lecture they watched prior to taking Quiz  
267 2. This localization is non-trivial: the knowledge estimates are informed only by the embedded  
268 coordinates of the *quiz questions*, not by the embeddings of either lecture (Eqn. 4). Finally, the  
269 knowledge map estimated from Quiz 3 responses shows a second increase in knowledge, localized  
270 to the region surrounding the embedding of the *Birth of Stars* lecture participants watched prior to  
271 taking Quiz 3.

272 Another way of visualizing these content-specific increases in knowledge (apparently driven



**Figure 7: Mapping out the geometry of knowledge and learning.** **A.** Average “knowledge maps” estimated using each quiz. Each map displays a 2D projection of the estimated knowledge about the content reflected by *all* regions of topic space (see *Creating knowledge and learning map visualizations*). The topic trajectories of each lecture and the coordinates of each question are indicated by dotted lines and dots. Each map reflects an average across all participants. For individual participants’ maps, see Figures S1, S2, and S3. **B.** Average “learning maps” estimated between each successive pair of quizzes. The learning maps are in the same general format as the knowledge maps in Panel A, but each coordinate in the learning maps indicates the *difference* between the corresponding coordinates in the indicated *pair* of knowledge maps—i.e., how much the estimated knowledge “changed” across the two quizzes. Each map reflects an average across all participants. For individual participants’ maps, see Figures S4 and S5. **C.** Word clouds for sampled points in topic space. Each word cloud displays the relative weights of each word reflected by the blend of topics represented at the locations of the stars in the maps. The words’ colors indicate how much each word is weighted on average across all timepoints’ topic vectors in the *Four Fundamental Forces* (blue) and *Birth of Stars* (green) videos, respectively.

273 by watching each lecture) is displayed in Figure 7B. Taking the point-by-point difference between  
274 the knowledge maps estimated from responses to a successive pair of quizzes yields a *learning map*  
275 that describes the *change* in knowledge estimates from one quiz to the next. These learning maps  
276 highlight that the estimated knowledge increases we observed across maps were specific to the  
277 regions around the embeddings of each lecture in turn.

278 Because the 2D projection we used to construct the knowledge and learning maps is (partially)  
279 invertible, we may gain additional insights into the estimates by reconstructing the original high-  
280 dimensional topic vectors for any point(s) in the maps we are interested in. For example, this  
281 could serve as a useful tool for an instructor looking to better understand which content areas  
282 a student (or a group of students) knows well (or poorly). As a demonstration, we show the  
283 top-weighted words from the blends of topics reconstructed from three example locations on the  
284 maps (Fig. 7C): one point near the *Four Fundamental Forces* embedding (yellow); a second point  
285 near the *Birth of Stars* embedding (orange), and a third point somewhere in between the two  
286 lectures' embeddings (pink). As shown in the word clouds in the Panel, the top-weighted words  
287 at the example coordinate near the *Four Fundamental Forces* embedding also tended to be weighted  
288 heavily by the topics expressed in that lecture. Similarly, the top-weighted words at the example  
289 coordinate near the *Birth of Stars* embedding tended to be weighted most heavily by the topics  
290 expressed in *that* lecture. And the top-weighted words at the example coordinate between the  
291 two lectures' embeddings show a roughly even mix of words most strongly associated with each  
292 lecture.

## 293 Discussion

294 Teaching, like effective writing and speaking, is fundamentally about empathy [1, 44, 59]. Great  
295 teachers consider students' interests [12, 60], backgrounds [15, 47, 53], and working memory capac-  
296 ities [2], and flexibly optimize their teaching strategies within those constraints [4, 22, 28]. In the  
297 classroom, empathizing with students also means maintaining open lines of communication [65] by  
298 fostering an environment in which all students feel comfortable speaking up if they have an excit-

299 ing new idea, or if they are having trouble understanding something [20, 61]. In-person instruction  
300 also often entails dynamic student-teacher and student-student interactions. These in-person in-  
301 teractions can provide the instructor with valuable information about students' understanding of  
302 the course material, beyond what they can glean solely from exams or assignments [17, 24, 62].  
303 In turn, this can allow the instructor to adapt their teaching approaches on-the-fly according to  
304 students' questions and behaviors. But what does great teaching look like in asynchronous online  
305 courses, when the instructor typically prepares course lectures and materials without knowing  
306 who will ultimately be learning from them? Can the empathetic side of teaching be automated  
307 and scaled?

308 The notion of empathy also related to "theory of mind" of other individuals [21, 29, 42].  
309 Considering others' unique perspectives, prior experiences, knowledge, goals, etc., can help us  
310 to more effectively interact and communicate [51, 55, 58]. The knowledge and learning maps  
311 we estimate in our study (Fig. 7) hint at one potential form that an automated "empathetic"  
312 teacher might take. We imagine automated content delivery systems that adapt lessons on the  
313 fly according to continually updated estimates of what students know and how quickly they are  
314 learning different conceptual content [e.g., building on ideas such as 3, 23, 36, 64, and others].

315 Over the past several years, the global pandemic has forced many educators to teach re-  
316 motely [32, 46, 56, 63]. This change in world circumstances is happening alongside (and perhaps  
317 accelerating) geometric growth in the availability of high quality online courses on platforms such  
318 as Khan Academy [33], Coursera [66], EdX [35], and others [52]. Continued expansion of the global  
319 internet backbone and improvements in computing hardware have also facilitated improvements  
320 in video streaming, enabling videos to be easily downloaded and shared by large segments of the  
321 world's population. This exciting time for online course instruction provides an opportunity to  
322 re-evaluate how we, as a global community, educate ourselves and each other. For example, we  
323 can ask: what makes an effective course or training program? Which aspects of teaching might be  
324 optimized or automated? How and why do learning needs and goals vary across people? How  
325 might we lower barriers to achieving a high quality education?

326 Alongside these questions, there is a growing desire to extend existing theories beyond the

327 domain of lab testing rooms and into real classrooms [31]. In part, this has led to a recent  
328 resurgence of “naturalistic” or “observational” experimental paradigms that attempt to better  
329 reflect more ethologically valid phenomena that are more directly relevant to real-world situations  
330 and behaviors [48]. In turn, this has brought new challenges in data analysis and interpretation. A  
331 key step towards solving these challenges will be to build explicit models of real-world scenarios  
332 and how people behave in them (e.g., models of how people learn conceptual content from real-  
333 world courses, as in our current study). A second key step will be to understand which sorts  
334 of signals derived from behaviors and/or other measurements [e.g., neurophysiological data; 5,  
335 16, 43, 49, 50] might help to inform these models. A third major step will be to develop and  
336 employ reliable ways of evaluating the complex models and data that are a hallmark of naturalistic  
337 paradigms.

338 Ultimately, our work suggests a new line of questions regarding the future of education:  
339 which aspects of teaching can be optimized and/or automated? The social benefits of face-to-face  
340 instruction, such as social interactions, friendships, and emotional support, cannot (and perhaps  
341 should not) be fully replaced by an automated computer-based system. Nor can modern computer  
342 systems experience emotional empathy in the human sense of the word. On the other hand,  
343 perhaps it is possible to separate out the social aspects of classroom instruction from the purely  
344 learning-related aspects. Our study shows that text embedding models can uncover detailed  
345 insights into students’ knowledge and how it changes over time during learning. We hope that  
346 these advances might help pave the way for new ways of teaching or delivering educational content  
347 that are tailored to individual students’ learning needs and goals.

## 348 Materials and methods

### 349 Participants

350 We enrolled a total of 50 Dartmouth undergraduate students in our study. Participants received  
351 course credit for enrolling. We asked each participant to fill out a demographic survey that included

352 questions about their age, gender, native spoken language, ethnicity, race, hearing, color vision,  
353 sleep, coffee consumption, level of alertness, and several aspects of their educational background  
354 and prior coursework.

355 Participants' ages ranged from 18 to 22 years (mean: 19.52 years; standard deviation: 1.09  
356 years). A total of 15 participants reported their gender as male and 35 participants reported their  
357 gender as female. A total of 49 participants reported their native language as "English" and 1  
358 reported having another native language. A total of 47 participants reported their ethnicity as  
359 "Not Hispanic or Latino" and three reported their ethnicity as "Hispanic or Latino." Participants  
360 reported their races as White (32 participants), Asian (14 participants), Black or African American  
361 (5 participants), American Indian or Alaska Native (1 participant), and Native Hawaiian or Other  
362 Pacific Islander (1 participant). (Note that some participants selected multiple racial categories.)

363 A total of 49 participants reporting having normal hearing and 1 participant reported having  
364 some hearing impairment. A total of 49 participants reported having normal color vision and 1  
365 participant reported being color blind. Participants reported having had, on the night prior to  
366 testing, 2–4 hours of sleep (1 participant), 4–6 hours of sleep (9 participants), 6–8 hours of sleep (35  
367 participants), or 8+ hours of sleep (5 participants). They reported having consumed, on the same  
368 day and leading up to their testing session, 0 cups of coffee (38 participants), 1 cup of coffee (10  
369 participants), 3 cups of coffee (1 participant), or 4+ cups of coffee (1 participant).

370 No participants reported that their focus was currently impaired (e.g., by drugs or alcohol).  
371 Participants reported their current level of alertness, and we converted their responses to numerical  
372 scores as follows: "very sluggish" (-2), "a little sluggish" (-1), "neutral" (0), "fairly alert" (1), and  
373 "very alert" (2). Across all participants, a range of alertness levels were reported (range: -2 – 1;  
374 mean: -0.10; standard deviation: 0.84).

375 Participants reported their undergraduate major(s) as "social sciences" (28 participants), "nat-  
376 ural sciences" (16 participants), "professional" (e.g., pre-med or pre-law; 8 participants), "mathe-  
377 matics and engineering" (7 participants), "humanities" (4 participants), or "undecided" (3 partici-  
378 pants). Note that some participants selected multiple categories for their undergraduate major. We  
379 also asked participants about the courses they had taken. In total, 45 participants reported having

380 taken at least one Khan Academy course in the past, and 5 reported not having taken any Khan  
381 Academy courses. Of those who reported having watched at least one Khan Academy course,  
382 7 participants reported having watched 1–2 courses, 11 reported having watched 3–5 courses, 8  
383 reported having watched 5–10 courses, and 19 reported having watched 10 or more courses. We  
384 also asked participants about the specific courses they had watched, categorized under different  
385 subject areas. In the “Mathematics” area, participants reported having watched videos on AP  
386 Calculus AB (21 participants), Precalculus (17 participants), Algebra 2 (14 participants), AP Cal-  
387 culus BC (12 participants), Trigonometry (11 participants), Algebra 1 (10 participants), Geometry  
388 (8 participants), Pre-algebra (7 participants), Multivariable Calculus (5 participants), Differential  
389 Equations (5 participants), Statistics and Probability (4 participants), AP Statistics (2 participants),  
390 Linear Algebra (2 participants), Early Math (1 participant), Arithmetic (1 participant), and other  
391 videos not listed in our survey (5 participants). In the “Science and engineering” area, participants  
392 reported having watched videos on Chemistry, AP Chemistry, or Organic Chemistry (21 partic-  
393 ipants); Physics, AP Physics I, or AP Physics II (15 participants); Biology, AP Biology; or High  
394 school Biology (15 participants); Health and Medicine (1 participant); or other videos not listed  
395 in our survey (19 participants). We also asked participants whether they had specifically seen the  
396 videos used in our experiment. Of the 45 participants who reported having having taken at least  
397 one Khan Academy course in the past, 44 participants reported that they had not watched the *Four*  
398 *Fundamental Forces* video, and 1 participant reported that they were not sure whether they had  
399 watched it. All participants reported that they had not watched the *Birth of Stars* video. When  
400 we asked participants about non-Khan Academy online courses, they reported having watched  
401 or taken courses on Mathematics (15 participants), Science and engineering (11 participants), Test  
402 preparation (9 participants), Economics and finance (3 participants), Arts and humanities (2 partic-  
403 ipants), Computing (2 participants), and other categories not listed in our survey (18 participants).  
404 Finally, we asked participants about in-person courses they had taken in different subject areas.  
405 They reported taking courses in Mathematics (39 participants), Science and engineering (38 par-  
406 ticipants), Arts and humanities (35 participants), Test preparation (27 participants), Economics  
407 and finance (26 participants), Computing (15 participants), College and careers (7 participants), or

<sup>408</sup> other courses not listed in our survey (6 participants).

## <sup>409</sup> **Experiment**

<sup>410</sup> We hand-selected two course videos from the Khan Academy platform: *Four Fundamental Forces*  
<sup>411</sup> (an introduction to gravity, electromagnetism, the weak nuclear force, and the strong nuclear force;  
<sup>412</sup> duration: 10 minutes and 29 seconds) and *Birth of Stars* (an introduction to how stars are formed;  
<sup>413</sup> duration: 7 minutes and 57 seconds). We then hand-created 39 multiple-choice questions: 15 about  
<sup>414</sup> the conceptual content of *Four Fundamental Forces* (i.e., lecture 1), 15 about the conceptual content  
<sup>415</sup> of *Birth of Stars* (i.e., lecture 2), and 9 questions that tested for general conceptual knowledge about  
<sup>416</sup> basic physics (covering material that was not presented in either video). The full set of questions  
<sup>417</sup> and answer choices may be found in Table S1.

<sup>418</sup> Over the course of the experiment, participants completed three 13-question multiple-choice  
<sup>419</sup> quizzes: the first before viewing lecture 1, the second between lectures 1 and 2, and the third  
<sup>420</sup> after viewing lecture 2 (Fig. 1). The questions appearing on each quiz, for each participant, were  
<sup>421</sup> randomly chosen from the full set of 39, with the constraints that (a) each quiz contain 5 questions  
<sup>422</sup> about lecture 1, 5 questions about lecture 2, and 3 questions about general physics knowledge, and  
<sup>423</sup> (b) each question appear exactly once for each participant. The order of questions on each quiz, and  
<sup>424</sup> the order of answer options for each question, were also randomized. Our experimental protocol  
<sup>425</sup> was approved by the Committee for the Protection of Human Subjects at Dartmouth College. We  
<sup>426</sup> used the experiment to develop and test our computational framework for estimating knowledge  
<sup>427</sup> and learning.

## <sup>428</sup> **Analysis**

### <sup>429</sup> **Constructing text embeddings of multiple lectures and questions**

<sup>430</sup> Following Heusser et al. [27]

<sup>431</sup> We adapted an approach we developed in prior work [27] to embed each moment of the two  
<sup>432</sup> lectures and each question in our pool in a common representational space. Briefly, our approach

433 uses a topic model [Latent Dirichlet Allocation; 8], fit to a corpus of documents to discover a set of  
434  $k$  “topics” or “themes” from their contents. Formally, each topic is defined as a set of weights over  
435 each word in the model’s vocabulary (i.e., the union of all unique words, across all documents,  
436 excluding “stop words.”).

437 We adapted an approach we developed in prior work [27] to embed each moment of the two  
438 lectures and each question in our pool in a common representational space. Briefly, our approach  
439 uses a topic model [Latent Dirichlet Allocation; 8], trained on a set of documents, to discover a  
440 set of  $k$  “topics” or “themes.” Formally, each topic is defined as a set of weights over each word  
441 in the model’s vocabulary (i.e., the union of all unique words, across all documents, excluding  
442 “stop words.”). Conceptually, each topic is intended to give larger weights to words that are  
443 conceptually related or that tend to co-occur in the same documents. After fitting a topic model,  
444 each document in the training set, or any *new* document that contains at least some of the words in  
445 the model’s vocabulary, may be represented as a  $k$ -dimensional vector describing how much the  
446 document (most probably) reflects each topic. (Unless, otherwise noted, we used  $k = 15$  topics.)

447 As illustrated in Figure 2A, we first

448 As illustrated in Figure 2A, we start by building up a corpus of documents using overlapping  
449 sliding windows that span each video’s transcript. Khan Academy provides professionally created,  
450 manual transcriptions of all videos for closed captioning. However, such transcripts would not  
451 be readily available in all contexts to which our framework could potentially be applied. Khan  
452 Academy videos are hosted on the YouTube platform, which additionally provides automated  
453 transcripts

454 Khan Academy videos are hosted on the YouTube platform, and all YouTube videos are run  
455 through Google’s speech-to-text API [26] to derive a timestamped transcript of any detected speech  
456 in the video. The resulting transcripts contain one timestamped row per line, and each line  
457 generally corresponds to a few seconds of spoken content from the video. We defined a sliding  
458 window length of (up to)  $w = 30$  transcript lines, and we assigned each window a timestamp  
459 according to the midpoint between its first and last lines’ timestamps. These sliding windows  
460 ramped up and down in length at the very beginning and end of the transcript, respectively. In

461 other words, the first sliding window covered only the first line from the transcript; the second  
462 sliding window covered the first two lines; and so on. This insured that each line of the transcript  
463 appeared in the same number ( $w$ ) of sliding windows. We treated the text from each sliding window  
464 as a single “document,” and we combined these documents across the two videos’ windows to  
465 create a single training corpus for the topic model. The top words from each of the 15 discovered  
466 topics may be found in Table S2.

467 After fitting a topic model to each videos’ transcripts, we could use the trained model to  
468 transform arbitrary (potentially new) documents into  $k$ -dimensional topic vectors. A convenient  
469 property of these topic vectors is that documents that reflect similar blends of topics (i.e., documents  
470 that reflect similar themes, according to the model) will yield similar (in terms of Euclidean distance,  
471 correlation, etc.) topic vectors. In general, the similarity between different documents’ topic vectors  
472 may be used to characterize the similarity in conceptual content between the documents.

473 We transformed each sliding window’s text into a topic vector, and then used linear interpola-  
474 tion (independently for each topic dimension) to resample the resulting timeseries to once per  
475 second. This yielded a single topic vector for each second of each video. We also used the fitted  
476 model to obtain topic vectors for each question in our pool (Tab. S1). Taken together, we obtained  
477 a *trajectory* for each video, describing its path through topic space, and a single coordinate for each  
478 question (Fig. 2B). Embedding both videos and all of the questions using a common model enables  
479 us to compare the content from different moments of videos, compare the content across videos,  
480 and estimate potential associations between specific questions and specific moments of video.

481 **Estimating dynamic knowledge traces**

482 We used the following equation to estimate each participant’s knowledge about timepoint  $t$  of a  
483 given lecture,  $\hat{k}(t)$ :

$$\hat{k}(f(t, L)) = \frac{\sum_{i \in \text{correct}} \text{ncorr}(f(t, L), f(i, Q))}{\sum_{j=1}^N \text{ncorr}(f(t, L), f(j, Q))}, \quad (1)$$

484 where

$$\text{ncorr}(x, y) = \frac{\text{corr}(x, y) - \text{mincorr}}{\text{maxcorr} - \text{mincorr}}, \quad (2)$$

and where  $\text{mincorr}$  and  $\text{maxcorr}$  are the minimum and maximum correlations between any lecture timepoint and question, taken over all timepoints and questions across both lectures and all three question sets. We also define  $f(s, \Omega)$  as the  $s^{\text{th}}$  topic vector from the set of topic vectors  $\Omega$ . Here  $t$  indexes the set of lecture topic vectors,  $L$ , and  $i$  and  $j$  index the topic vectors of questions in the quiz's question set,  $Q$ . Note that "correct" denotes the set of indices of the questions the participant answered correctly on the given quiz.

Intuitively,  $\text{ncorr}(x, y)$  is the correlation between two topic vectors (e.g., the topic vector from one timepoint in a lecture,  $x$ , and the topic vector for one question,  $y$ ), normalized by the minimum and maximum correlations (across all timepoints and questions) to range between 0 and 1, inclusive. Equation 1 then computes the weighted average proportion of correctly answered questions about the content presented at timepoint  $t$ , where the weights are given by the normalized correlations between timepoint  $t$ 's topic vector and the topic vectors for each question. The normalization step (i.e., using  $\text{ncorr}$  instead of the raw correlations) insures that every question (except the least-relevant question) contributes some non-zero amount to the knowledge estimate.

#### **Creating knowledge and learning map visualizations**

An important feature of our approach is that, given a trained text embedding model and participants' quiz performance on each question, we can estimate their knowledge about *any* content expressible by the embedding model— not solely the content explicitly probed by the quiz questions. To visualize these estimates (Figs. 7, S1, S2, S3, S4, and S5), we used UMAP [41] to define a 2D projection of the text embedding space. Sampling the original 100-dimensional space at high resolution to obtain an adequate set of topic vectors spanning the embedding space would be computationally intractable. However, sampling a 2D grid is much more feasible. We defined a rectangle enclosing the 2D projections of the lectures' and quizzes' embeddings, and we sampled points from a regular  $100 \times 100$  grid of coordinates that evenly tiled the enclosing rectangle. We

509 sought to estimate participants' knowledge (and learning—i.e., changes in knowledge) at each of  
510 the resulting 10000 coordinates.

511 To generate our estimates, we placed a set of 39 radial basis functions (RBFs) throughout the  
512 embedding space, centered on the 2D projections for each question (i.e., we included one RBF for  
513 each question). At coordinate  $x$ , the value of an RBF centered on a question's coordinate  $\mu$ , is given  
514 by:

$$\text{RBF}(x, \mu, \lambda) = \exp \left\{ -\frac{\|x - \mu\|^2}{\lambda} \right\}. \quad (3)$$

515 The  $\lambda$  term in the RBF equation controls the “smoothness” of the function, where larger values  
516 of  $\lambda$  result in smoother maps. In our implementation we used  $\lambda = 50$ . Next, we estimated the  
517 “knowledge” at each coordinate,  $x$ , using:

$$\hat{k}(x) = \frac{\sum_{i \in \text{correct}} \text{RBF}(x, q_i, \lambda)}{\sum_{j=1}^N \text{RBF}(x, q_j, \lambda)}. \quad (4)$$

518 Intuitively, Equation 4 computes the weighted proportion of correctly answered questions, where  
519 the weights are given by how nearby (in the 2D space) each question is to the  $x$ . We also defined  
520 *learning maps* as the coordinate-by-coordinate differences between any pair of knowledge maps.  
521 Intuitively, learning maps reflect the *change* in knowledge across two maps.

## 522 Author contributions

523 Conceptualization: PCF, ACH, and JRM. Methodology: PCF, ACH, and JRM. Software: PCF.  
524 Validation: PCF. Formal analysis: PCF. Resources: PCF, ACH, and JRM. Data curation: PCF.  
525 Writing (original draft): JRM. Writing (review and editing): PCF, ACH, and JRM. Visualization:  
526 PCF and JRM. Supervision: JRM. Project administration: PCF. Funding acquisition: JRM.

527 **Data and code availability**

528 All of the data analyzed in this manuscript, along with all of the code for running our experiment  
529 and carrying out the analyses may be found at [https://github.com/ContextLab/efficient-learning-](https://github.com/ContextLab/efficient-learning-khan)  
530 [khan](https://github.com/ContextLab/efficient-learning-khan).

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