

¹ Text embedding models yield high-resolution insights
² into conceptual knowledge from short multiple-choice
³ quizzes

⁴ Paxton C. Fitzpatrick¹, Andrew C. Heusser^{1, 2}, and Jeremy R. Manning^{1,*}

¹Department of Psychological and Brain Sciences

Dartmouth College, Hanover, NH 03755, USA

²Akili Interactive Labs

Boston, MA 02110, USA

*Corresponding author: Jeremy.R.Manning@Dartmouth.edu

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Abstract

⁶

We develop a mathematical framework, based on natural language processing models, for tracking and characterizing the acquisition of conceptual knowledge. Our approach embeds each concept in a high-dimensional representation space, where nearby coordinates reflect similar or related concepts. We test our approach using behavioral data from participants who answered small sets of multiple-choice quiz questions interleaved between watching two course videos from the Khan Academy platform. We apply our framework to the videos' transcripts and the text of the quiz questions to quantify the content of each moment of video and each quiz question. We use these embeddings, along with participants' quiz responses, to track how the learners' knowledge changed after watching each video. Our findings show how a small set of quiz questions may be used to obtain rich and meaningful high-resolution insights into what each learner knows, and how their knowledge changes over time as they learn.

¹⁷

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¹⁸ **Introduction**

¹⁹ Suppose that a teacher had access to a complete, tangible “map” of everything a student knows.
²⁰ Defining what such a map might even look like, let alone how it might be constructed or filled in, is
²¹ itself a non-trivial problem. But if a teacher *were* to gain access to such a map, how might it change
²² their ability to teach that student? Perhaps they might start by checking how well the student
²³ knows the to-be-learned information already, or how much they know about related concepts.
²⁴ For some students, they could potentially optimize their teaching efforts to maximize efficiency
²⁵ by focusing primarily on not-yet-known content. For other students (or other content areas), it
²⁶ might be more effective to optimize for direct connections between already known content and
²⁷ new material. Observing how the student’s knowledge changed over time, in response to their
²⁸ teaching, could also help to guide the teacher towards the most effective strategy for that individual
²⁹ student.

³⁰ A common approach to assessing a student’s knowledge is to present them with a set of quiz
³¹ questions, calculate the proportion they answer correctly, and provide them with feedback in the
³² form of a simple numeric or letter grade. While such a grade can provide *some* indication of whether
³³ the student has mastered the to-be-learned material, any univariate measure of performance on a
³⁴ complex task sacrifices certain relevant information, risks conflating underlying factors, and so on.
³⁵ For example, consider the relative utility of the theoretical map described above that characterizes
³⁶ a student’s knowledge in detail, versus a single annotation saying that the student answered 85%
³⁷ of their quiz questions correctly, or that they received a ‘B’. Here, we show that the same quiz data
³⁸ required to compute proportion-correct scores or letter grades can instead be used to obtain far
³⁹ more detailed insights into what a student knew at the time they took the quiz.

⁴⁰ Designing and building procedures and tools for mapping out knowledge touches on deep
⁴¹ questions about what it means to learn. For example, how do we acquire conceptual knowledge?
⁴² Memorizing course lectures or textbook chapters by rote can lead to the superficial *appearance*
⁴³ of understanding the underlying content, but achieving true conceptual understanding seems to
⁴⁴ require something deeper and richer. Does conceptual understanding entail connecting newly

45 acquired information to the scaffolding of one’s existing knowledge or experience [3, 7, 9, 10, 48]?
46 Or weaving a lecture’s atomic elements (e.g., its component words) into a structured network that
47 describes how those individual elements are related [29]? Conceptual understanding could also
48 involve building a mental model that transcends the meanings of those individual atomic elements
49 by reflecting the deeper meaning underlying the gestalt whole [26, 30, 45].

50 The difference between “understanding” and “memorizing,” as framed by researchers in ed-
51 ucation, cognitive psychology, and cognitive neuroscience (e.g., 16, 19, 23, 30, 45), has profound
52 analogs in the fields of natural language processing and natural language understanding. For
53 example, considering the raw contents of a document (e.g., its constituent symbols, letters, and
54 words) might provide some clues as to what the document is about, just as memorizing a pas-
55 sage might provide some ability to answer simple questions about it. However, text embedding
56 models (e.g., 4–6, 8, 11, 28, 37) also attempt to capture the deeper meaning *underlying* those atomic
57 elements. These models consider not only the co-occurrences of those elements within and across
58 documents, but also patterns in how those elements appear across different scales (e.g., sentences,
59 paragraphs, chapters, etc.), the temporal and grammatical properties of the elements, and other
60 high-level characteristics of how they are used [31, 32]. According to these models, the deep
61 conceptual meaning of a document may be captured by a feature vector in a high-dimensional
62 representation space, wherein nearby vectors reflect conceptually related documents. A model
63 that succeeds at capturing an analogue of “understanding” is able to assign nearby feature vectors
64 to two conceptually related documents, *even when the specific words contained in those documents have*
65 *very little overlap.*

66 Given these insights, what form might a representation of the sum total of a person’s knowledge
67 take? First, we might require a means of systematically describing or representing the nearly
68 infinite set of possible things a person could know. Second, we might want to account for potential
69 associations between different concepts. For example, the concepts of “fish” and “water” might be
70 associated in the sense that fish live in water. Third, knowledge may have a critical dependency
71 structure, such that knowing about a particular concept might require first knowing about a set of
72 other concepts. For example, understanding the concept of a fish swimming in water first requires

73 understanding what fish and water *are*. Fourth, as we learn, our “current state of knowledge”
74 should change accordingly. Learning new concepts should both update our characterizations of
75 “what is known” and also unlock any now-satisfied dependencies of those newly learned concepts
76 so that they are “tagged” as available for future learning.

77 Here we develop a framework for modeling how conceptual knowledge is acquired during
78 learning. The central idea behind our framework is to use text embedding models to define the
79 coordinate systems of two maps: a *knowledge map* that describes the extent to which each concept is
80 currently known, and a *learning map* that describes changes in knowledge over time. Each location
81 on these maps represents a single concept, and the maps’ geometries are defined such that related
82 concepts are located nearby in space. We use this framework to analyze and interpret behavioral
83 data collected from an experiment that had participants answer sets of multiple-choice questions
84 about a series of recorded course lectures.

85 Our primary research goal is to advance our understanding of what it means to acquire deep,
86 real-world conceptual knowledge. Traditional laboratory approaches to studying learning and
87 memory (e.g., list-learning studies) often draw little distinction between memorization and under-
88 standing. Instead, these studies typically focus on whether information is effectively encoded or
89 retrieved, rather than whether the information is *understood*. Approaches to studying conceptual
90 learning, such as category learning experiments, can begin to investigate the distinction between
91 memorization and understanding, often by training participants to distinguish arbitrary or random
92 features in otherwise meaningless categorized stimuli [1, 13, 14, 17, 21, 43]. However the objective
93 of real-world training, or learning from life experiences more generally, is often to develop new
94 knowledge that may be applied in *useful* ways in the future. In this sense, the gap between modern
95 learning theories and modern pedagogical approaches that inform classroom learning strategies is
96 enormous: most of our theories about *how* people learn are inspired by experimental paradigms
97 and models that have only peripheral relevance to the kinds of learning that students and teachers
98 actually seek [19, 30]. To help bridge this gap, our study uses course materials from real on-
99 line courses to inform, fit, and test models of real-world conceptual learning. We also provide a
100 demonstration of how our models can be used to construct “maps” of what students know, and

101 how their knowledge changes with training. In addition to helping to visually capture knowledge
102 (and changes in knowledge), we hope that such maps might lead to real-world tools for improving
103 how we educate. Taken together, our work shows that existing course materials and evaluative
104 tools like short multiple-choice quizzes may be leveraged to gain highly detailed insights into what
105 students know and how they learn.

106 Results

107 At its core, our main modeling approach is based around a simple assumption that we sought to
108 test empirically: all else being equal, knowledge about a given concept is predictive of knowledge
109 about similar or related concepts. From a geometric perspective, this assumption implies that
110 knowledge is fundamentally “smooth.” In other words, as one moves through a space representing
111 an individual’s knowledge (where similar concepts occupy nearby coordinates), their “level of
112 knowledge” should change relatively gradually. To begin to test this smoothness assumption, we
113 sought to track participants’ knowledge and how it changed over time in response to training.
114 Two overarching goals guide our approach. First, we want to gain detailed insights into what
115 learners know at different points in their training. For example, rather than simply reporting on
116 the proportions of questions participants answer correctly (i.e., their overall performance), we seek
117 estimates of their knowledge about a variety of specific concepts. Second, we want our approach to
118 be potentially scalable to large numbers of diverse concepts, courses, and students. This requires
119 that the conceptual content of interest be discovered *automatically*, rather than relying on manually
120 produced ratings or labels.

121 We asked participants in our study to complete brief multiple-choice quizzes before, between,
122 and after watching two lecture videos from the Khan Academy [?] platform (Fig. 1). The first
123 lecture video, entitled *Four Fundamental Forces*, discussed the four fundamental forces in physics:
124 gravity, strong and weak interactions, and electromagnetism. The second, entitled *Birth of Stars*,
125 provided an overview of our current understanding of how stars form. We selected these particular
126 lectures to satisfy three general criteria. First, we wanted both lectures to be accessible to a broad

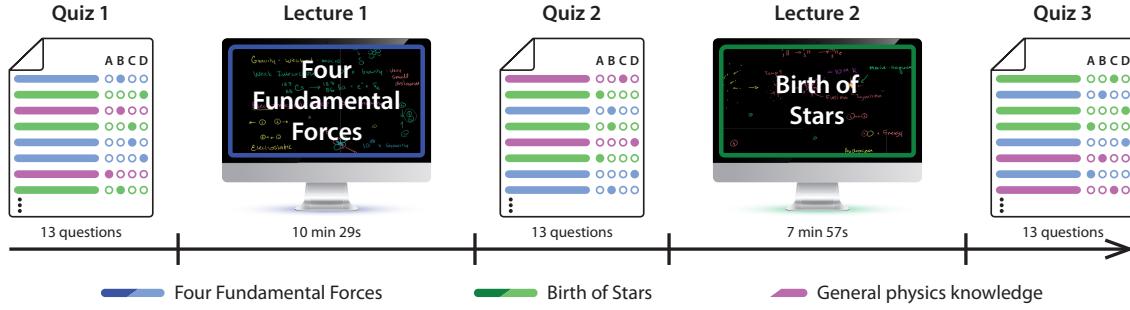


Figure 1: Experimental paradigm. Participants alternate between completing three 13-question multiple-choice quizzes and watching two Khan Academy lectures. Each quiz contains a mix of 5 questions about Lecture 1, 5 questions about Lecture 2, and 3 questions about general physics knowledge. The specific questions reflected on each quiz, and the orders of each quiz's questions, were randomized across participants.

audience (i.e., with minimal prerequisite knowledge) so as to limit the impact of prior training on participants' abilities to learn from the lectures. To this end, we selected two introductory videos that were intended to be viewed at the start of students' training in their respective content areas. Second, we wanted the two lectures to have some related content, so that we could test our approach's ability to distinguish similar conceptual content. To this end, we chose two videos from the same Khan Academy course domain, "Cosmology and Astronomy." Third, we sought to minimize dependencies and specific overlap between the videos. For example, we did not want participants' abilities to understand one video to (directly) influence their abilities to understand the other. To satisfy this last criterion, we chose videos from two different lecture series (Lectures 1 and 2 were from the "Scale of the Universe" and "Stars, Black Holes, and Galaxies" series, respectively).

We also wrote a set of multiple-choice quiz questions that we hoped would enable us to evaluate participants' knowledge about each individual lecture, along with related knowledge about physics concepts not specifically presented in either video (see Supp. Tab. 1 for the full list of questions in our stimulus pool). Participants answered questions randomly drawn from each content area (Lecture 1, Lecture 2, and general physics knowledge) on each of the three quizzes. Quiz 1 was intended to assess participants' "baseline" knowledge before training, Quiz 2 assessed knowledge after watching the *Four Fundamental Forces* video (i.e., Lecture 1), and Quiz 3 assessed knowledge after watching the *Birth of Stars* video (i.e., Lecture 2).



Figure 2: Modeling course content. **A. Building a document pool from sliding windows of text.** We decompose each lecture’s transcript into a series of overlapping sliding windows. The full set of transcript snippets (across all windows) may be treated as a set of “documents” for training a text embedding model. **B. Constructing lecture content trajectories.** After training the model on the sliding windows from both lectures, we transform each lecture into a “trajectory” through text embedding space by joining the embedding coordinates of successive sliding windows parsed from its transcript. **C. Embedding multiple lectures and questions in a shared space.** We apply the same model (trained on the two lectures’ windows) to both lectures, along with the text of each question in our pool (Supp. Tab. 1), to project them into a shared text embedding space. This results in one trajectory per lecture and one coordinate for each question. Here, we have projected the 15-dimensional embeddings onto their first 3 principal components for visualization.

145 To study in detail how participants’ conceptual knowledge changed over the course of the
 146 experiment, we first sought to model the conceptual content presented to them at each moment
 147 throughout each of the two lectures. We adapted an approach we developed in prior work [20]
 148 to identify the latent themes in the lectures using a topic model [5]. Briefly, topic models take
 149 as input a collection of text documents, and learn a set of “topics” (i.e., latent themes) from their
 150 contents. Once fit, a topic model can be used to transform arbitrary (potentially new) documents
 151 into sets of “topic proportions,” describing the weighted blend of learned topics reflected in their
 152 texts. We parsed automatically generated transcripts of the two lectures into overlapping sliding
 153 windows, where each window contained the text of the lecture transcript from a particular time
 154 span. We treated the set of text snippets (across all of these windows) as documents to fit the
 155 model (Fig. 2A; see *Constructing text embeddings of multiple lectures and questions*). Transforming the
 156 text from every sliding window with the model yielded a number-of-windows by number-of-topics
 157 (15) topic-proportions matrix describing the unique mixture of broad themes from both lectures
 158 reflected in each window’s text. Each window’s “topic vector” (i.e., column of the topic-proportions
 159 matrix) is analogous to a coordinate in a 15-dimensional space whose axes are topics discovered

160 by the model. Within this space, each lecture’s sequence of topic vectors (i.e., corresponding to its
161 transcript’s overlapping text snippets across sliding windows) forms a *trajectory* that captures how
162 its conceptual content unfolds over time (Fig. 2B). We resampled these trajectories to a resolution
163 of one topic vector for each second of video (i.e., 1 Hz).

164 We hypothesized that a topic model trained on transcripts of the two lectures should also capture
165 the conceptual knowledge probed by each quiz question. If indeed the topic model could capture
166 information about the deeper conceptual content of the lectures (i.e., beyond surface-level details
167 such as particular word choices), then we should be able to recover a correspondence between each
168 lecture and questions *about* each lecture. Importantly, such a correspondence could not solely arise
169 from superficial text matching between lecture transcripts and questions, since the lectures and
170 questions used different words. Simply comparing the average topic weights from each lecture and
171 question set (averaging across time and questions, respectively) reveals a striking correspondence
172 (Supp. Fig. 2). Specifically, the average topic weights from Lecture 1 are strongly correlated with the
173 average topic weights from Lecture 1 questions ($r(13) = 0.809, p < 0.001, 95\% \text{ CI} = [0.633, 0.962]$), and the average topic weights from Lecture 2 are strongly correlated with the
174 average topic weights from Lecture 2 questions ($r(13) = 0.728, p = 0.002, 95\% \text{ CI} = [0.456, 0.920]$).
175 At the same time, the average topic weights from the two lectures are *negatively* correlated with
176 their non-matching question sets (Lecture 1 video vs. Lecture 2 questions: $r(13) = -0.547, p = 0.035,$
177 $95\% \text{ CI} = [-0.812, -0.231]$; Lecture 2 video vs. Lecture 1 questions: $r(13) = -0.612, p = 0.015, 95\%$
178 $\text{CI} = [-0.874, -0.281]$), indicating that the topic model also exhibits some degree of specificity. The
179 full set of pairwise comparisons between average topic weights for the lectures and question sets
180 is reported in Supplementary Figure 2.
181

182 Another, more sensitive, way of summarizing the conceptual content of the lectures and ques-
183 tions is to look at *variability* in how topics are weighted over time and across different questions
184 (Fig. 3). Intuitively, the variability in the expression of a given topic relates to how much “infor-
185 mation” [15] the lecture (or question set) reflects about that topic. For example, suppose a given
186 topic is weighted on heavily throughout a lecture. That topic might be characteristic of some
187 aspect or property of the lecture *overall* (conceptual or otherwise), but unless the topic’s weights



Figure 3: Lecture and question topic overlap. A. Topic weight variability. The bar plots display the variance of each topic's weight across lecture timepoints (top row) and questions (bottom row); colors denote topics. The top-weighted words from the most “expressive” (i.e., variable across observations) topic from each lecture are displayed in the upper right (orange: topic 2; yellow-green: topic 4). The top-weighted words from the full set of topics may be found in Supplementary Table 2. **B. Relationships between topic weight variability.** Pairwise correlations between the distributions of topic weight variance for each lecture and question set. Each row and column corresponds to a bar plot in Panel A.

changed in meaningful ways over time, the topic would be a poor indicator of any *specific* conceptual content in the lecture. We therefore also compared the variances in topic weights (across time or questions) between the lectures and questions. The variability in topic expression (over time and across questions) was similar for the Lecture 1 video and questions ($r(13) = 0.824, p < 0.001, 95\% \text{ CI} = [0.696, 0.973]$) and the Lecture 2 video and questions ($r(13) = 0.801, p < 0.001, 95\% \text{ CI} = [0.539, 0.958]$). Simultaneously, as reported in Figure 3B, the variability in topic expression across *different* videos and lecture-specific questions (i.e., Lecture 1 video vs. Lecture 2 questions; Lecture 2 video vs. Lecture 1 questions) were negatively correlated, and neither video's topic variability was reliably correlated with the topic variability across general physics knowledge questions. Taken together, the analyses reported in Figure 3 and Supplementary Figure 2 indicate that a topic model fit to the videos' transcripts can also reveal correspondences (at a coarse scale) between the lectures and questions.

While an individual lecture may be organized around a single broad theme at a coarse scale, at a finer scale, each moment of a lecture typically covers a narrower range of content. Given

202 the correspondence we found between the variability in topic expression across moments of each
203 lecture and questions from its corresponding set (Fig. 3), we wondered whether the text embedding
204 model might additionally capture these conceptual relationships at a finer scale. For example, if a
205 particular question asks about the content from one small part of a lecture, we wondered whether
206 the text embeddings could be used to automatically identify the “matching” moment(s) in the
207 lecture. To explore this, we computed the correlation between each question’s topic weights and the
208 topic weights for each second of its corresponding lecture, and found that each question appeared
209 to be temporally specific (Fig. 4). In particular, most questions’ topic vectors were maximally
210 correlated with a well-defined (and relatively narrow) range of timepoints from their corresponding
211 lectures, and the correlations fell off sharply outside of that range. We also qualitatively examined
212 the best-matching intervals for each question by comparing the question’s text to the text of
213 the most-correlated parts of the lectures. Despite that the questions were excluded from the
214 text embedding model’s training set, in general we found (through manual inspection) a close
215 correspondence between the conceptual content that each question probed and the content covered
216 by the best-matching moments of the lectures. Two representative examples are shown at the
217 bottom of Figure 4.

218 The ability to quantify how much each question is “asking about” the content from each moment
219 of the lectures could enable high-resolution insights into participants’ knowledge. Traditional
220 approaches to estimating how much a student “knows” about the content of a given lecture entail
221 computing the proportion of correctly answered questions. But if two students receive identical
222 scores on an exam, might our modeling framework help us to gain more nuanced insights into the
223 *specific* content that each student has mastered (or failed to master)? For example, a student who
224 misses three questions that were all about the same concept (e.g., concept *A*) will have gotten the
225 same *proportion* of questions correct as another student who missed three questions about three
226 *different* concepts (e.g., *A*, *B*, and *C*). But if we wanted to help these two students fill in the “gaps” in
227 their understandings, we might do well to focus specifically on concept *A* for the first student, but
228 to also add in materials pertaining to concepts *B* and *C* for the second student. In other words, raw
229 “proportion-correct” measures may capture *how much* a student knows, but not *what* they know.



Figure 4: Which parts of each lecture are captured by each question? Each panel displays timeseries plots showing how each question’s topic vector correlates with each video timepoint’s topic vector (Panel A.: correlations for the *Four Fundamental Forces* lecture and associated questions; Panel B.: correlations for the *Birth of Stars* lecture and associated questions). The colors denote question identities. The diamonds in each panel denote the moment of peak correlation between the indicated question and the lecture trajectory. The associated questions’ text and snippets of the lectures’ transcripts from the surrounding 30 seconds, are displayed at the bottom of the figure.

230 We wondered whether our modeling framework might enable us to (formally and automatically)
 231 infer participants’ knowledge at the scale of individual concepts (e.g., as captured by a single
 232 moment of a lecture).

233 We developed a simple formula (Eqn. 1) for using a participant’s responses to a small set of
 234 multiple-choice questions to estimate how much the participant “knows” about the concept re-
 235 flected by any arbitrary coordinate, x , in text embedding space (e.g., the content reflected by any
 236 moment in a lecture they had watched; see *Estimating dynamic knowledge traces*). Essentially, the
 237 estimated knowledge at coordinate x is given by the weighted average proportion of quiz questions
 238 the participant answered correctly, where the weights reflect how much each question is “about” the
 239 content at x . When we apply this approach to estimate the participant’s knowledge about the con-
 240 tent presented in each moment of each lecture, we can obtain a detailed timecourse describing how
 241 much “knowledge” the participant has about any part of the lecture. As shown in Figure 5A and C,
 242 we can apply this approach separately for the questions from each quiz participants took through-

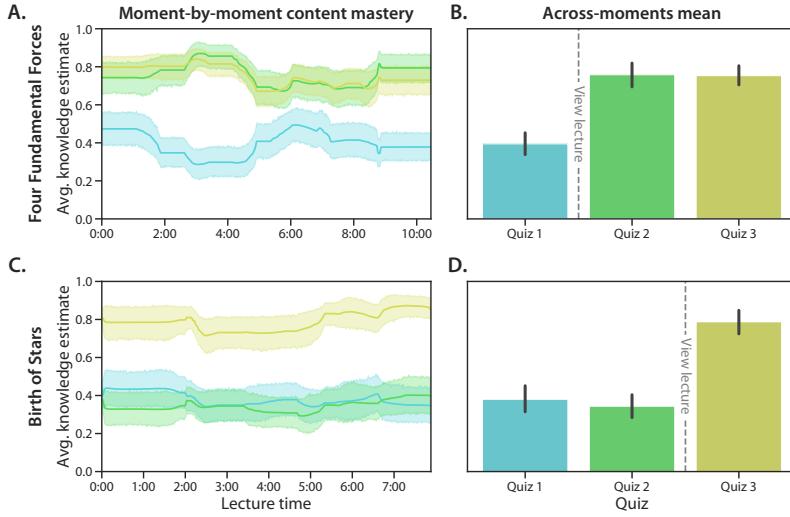


Figure 5: Estimating moment-by-moment knowledge acquisition. **A. Moment-by-moment knowledge about the *Four Fundamental Forces*.** Each trace displays the weighted proportion of correctly answered questions about the content reflected in each moment of the lecture (see *Estimating dynamic knowledge traces*), using responses from a single quiz’s color). The traces are averaged across participants. **B. Average estimated knowledge about the *Four Fundamental Forces*.** Each bar displays the across-timepoint average knowledge, estimated using the responses to one quiz’s questions. **C. Moment-by-moment knowledge about the *Birth of Stars*.** The panel is in the same format as Panel A, but here the knowledge estimates are for the moment-by-moment content of the *Birth of Stars* lecture. **D. Average estimated knowledge about the *Birth of Stars*.** The panel is in the same format as Panel B, but here the knowledge estimates are for the content of the *Birth of Stars* lecture. All panels: error ribbons and error bars denote 95% confidence intervals, estimated across participants.

out the experiment. From just a few questions per quiz (see *Estimating dynamic knowledge traces*), we obtain a high-resolution snapshot (at the time each quiz was taken) of what the participants knew about any moment’s content, from either of the two lectures they watched (comprising a total of 1,100 samples across the two lectures).

While the timecourses in Figure 5A and C provide detailed *estimates* about participants’ knowledge, these estimates are of course only *useful* to the extent that they accurately reflect what participants actually know. As one sanity check, we anticipated that the knowledge estimates should reflect a content-specific “boost” in participants’ knowledge after watching each lecture. In other words, if participants learn about each lecture’s content when they watch each lecture, the knowledge estimates should capture that. After watching the *Four Fundamental Forces* lecture, participants should exhibit more knowledge for the content of that lecture than they had before,

and that knowledge should persist for the remainder of the experiment. Specifically, knowledge about that lecture’s content should be relatively low when estimated using Quiz 1 responses, but should increase when estimated using Quiz 2 or 3 responses (Fig. 5B). Indeed, we found that participants’ estimated knowledge about the content of the *Four Fundamental Forces* was substantially higher on Quiz 2 versus Quiz 1 ($t(49) = 8.764, p < 0.001$) and on Quiz 3 versus Quiz 1 ($t(49) = 10.519, p < 0.001$). We found no reliable differences in estimated knowledge about that lecture’s content on Quiz 2 versus 3 ($t(49) = 0.160, p = 0.874$). Similarly, we hypothesized (and subsequently confirmed) that participants should show greater estimated knowledge about the content of the *Birth of Stars* lecture after (versus before) watching it (Fig. 5D). Specifically, since participants watched that lecture after taking Quiz 2 (but before Quiz 3), we hypothesized that their knowledge estimates should be relatively low on Quizzes 1 and 2, but should show a “boost” on Quiz 3. Consistent with this prediction, we found no reliable differences in estimated knowledge about the *Birth of Stars* lecture content on Quizzes 1 versus 2 ($t(49) = 1.013, p = 0.316$), but the estimated knowledge was substantially higher on Quiz 3 versus 2 ($t(49) = 10.561, p < 0.001$) and Quiz 3 versus 1 ($t(49) = 8.969, p < 0.001$).

If we are able to accurately estimate a participant’s knowledge about the content tested by a given question, our estimates of their knowledge should carry some predictive information about whether the participant is likely to answer that question correctly or incorrectly. We developed a statistical approach to test this claim. For each question, in turn, we used Equation 1 to estimate each participant’s knowledge at the given question’s embedding space coordinate, using all *other* questions that participant answered on the same quiz. For each quiz, we grouped these estimates into two distributions: one for the estimated knowledge at the coordinates of *correctly* answered questions, and another for the estimated knowledge at the coordinates of *incorrectly* answered questions (Fig. 6). We then used independent samples t -tests to compare the means of these distributions of estimated knowledge.

For the initial quizzes participants took (prior to watching either lecture), participants’ estimated knowledge tended to be low overall, and relatively unstructured (Fig. 6, left panel). When we held out individual questions and estimated their knowledge at the held-out questions’ embedding

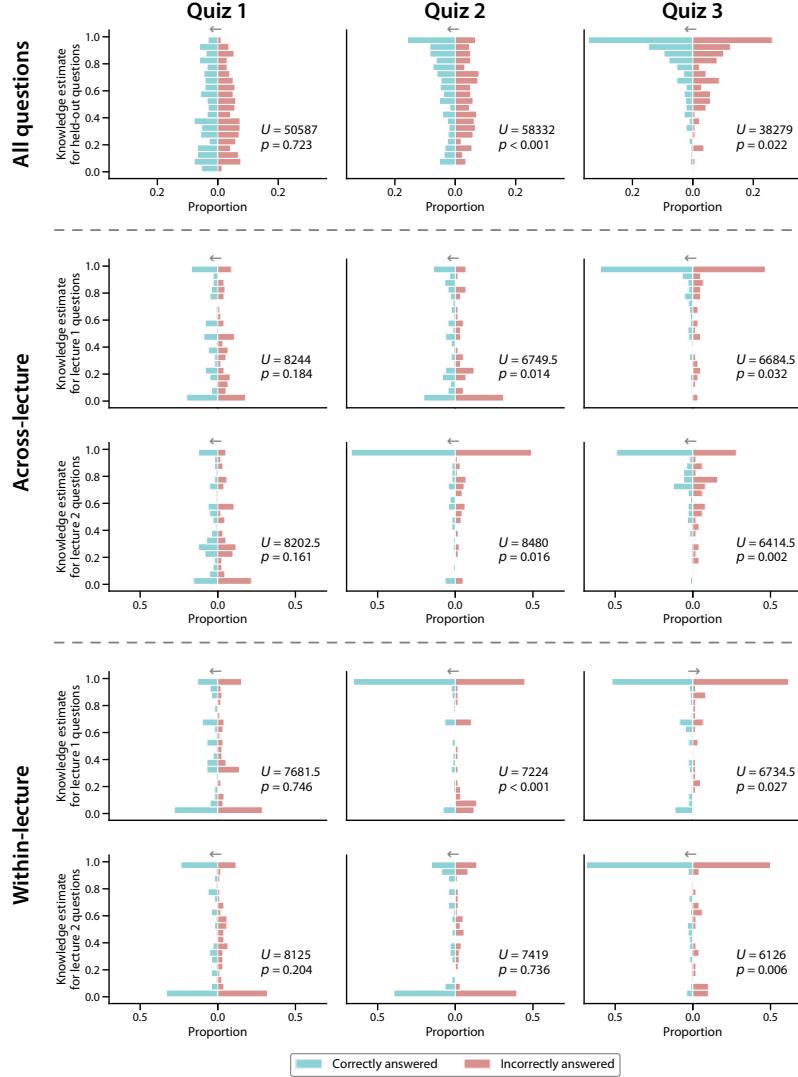


Figure 6: Estimating knowledge at the embedding coordinates of held-out questions. Separately for each quiz (panel), we plot the distributions of predicted knowledge at the embedding coordinates of each held-out correctly (blue) or incorrectly (red) answered question. The t -tests reported in each panel are between the distributions of estimated knowledge at the coordinates of correctly versus incorrectly answered held-out questions.

coordinates, we found no reliable differences in the estimates when the held-out question had been correctly versus incorrectly answered ($t(633) = 0.577$, $p = 0.564$). This reflects a floor effect: when knowledge is low everywhere, there is little signal to differentiate between what is known versus unknown. After watching the first lecture, estimated knowledge for held-out correctly answered questions (from the second quiz; Fig. 6, middle panel) exhibited a positive shift relative to held-out incorrectly answered questions ($t(633) = 3.961$, $p < 0.001$). This second quiz provides the maximally sensitive test for our knowledge predictions, since (if knowledge is estimated accurately) participants' Quiz 2 responses should demonstrate specific knowledge about Lecture 1 content, but knowledge about Lecture 2 and general physics concepts should be roughly unchanged from before they watched Lecture 1. After watching the second lecture, estimated knowledge (from the third quiz; Fig. 6, right panel) for *all* questions exhibited a positive shift. However, the estimated knowledge for held-out correctly answered questions remained greater than that for held-out incorrectly answered questions ($t(628) = 2.045$, $p = 0.041$). This third contrast reflects a ceiling effect: when knowledge is relatively high everywhere, the signal differentiating what is known versus unknown is relatively weak. Taken together, this set of analyses demonstrates that our knowledge prediction framework is most informative when participants exhibit variability in their knowledge of the content captured by the text embedding model.

Knowledge estimates need not be limited to the content of the lectures. As illustrated in Figure 8, our general approach to estimating knowledge from a small number of quiz questions may be extended to *any* content, given its text embedding coordinate. To visualize how knowledge "spreads" through text embedding space to content beyond the lectures participants watched, we first fit a new topic model to the lectures' sliding windows with (up to) $k = 100$ topics. Conceptually, increasing the number of topics used by the model functions to increase the "resolution" of the embedding space, providing a greater ability to estimate knowledge for content that is highly similar to (but not precisely the same as) that contained in the two lectures. Aside from increasing the number of topics from 15 to 100, all other procedures and model parameters were carried over from the preceding analyses. As in our other analyses, we resampled each lecture's topic trajectory to 1 Hz and projected each question into a shared text embedding space.

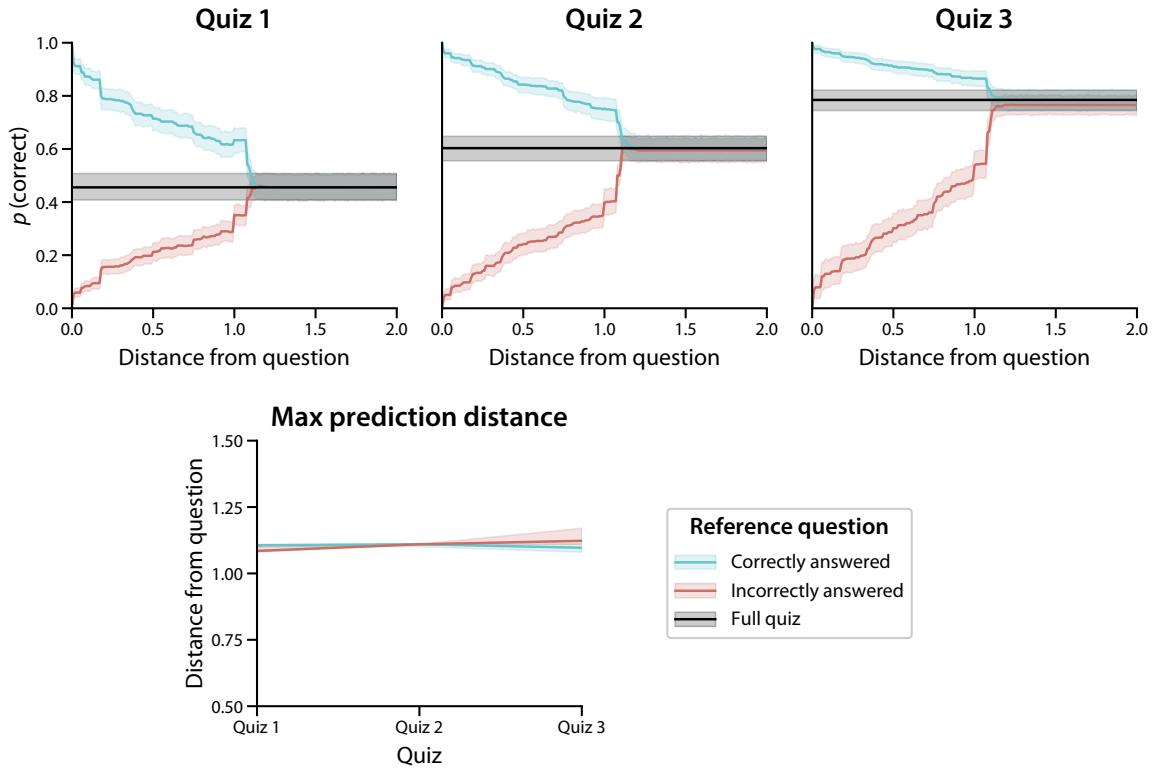


Figure 7: Caption title. Caption content.

310 We projected the resulting 100-dimensional topic vectors (for each second of video and each quiz
 311 question) onto a shared 2-dimensional plane (see *Creating knowledge and learning map visualizations*).
 312 Next, we sampled points from a 100×100 grid of coordinates that evenly tiled a rectangle enclos-
 313 ing the 2D projections of the videos and questions. We used Equation 4 to estimate participants'
 314 knowledge at each of these 10,000 sampled locations, and averaged these estimates across par-
 315 ticipants to obtain an estimated average *knowledge map* (Fig. 8A). Intuitively, the knowledge map
 316 constructed from a given quiz's responses provides a visualization of how "much" participants
 317 knew about any content expressible by the fitted text embedding model at the point in time when
 318 they completed that quiz.

319 Several features of the resulting knowledge maps are worth noting. The average knowledge
 320 map estimated from Quiz 1 responses (Fig. 8A, leftmost map) shows that participants tended to

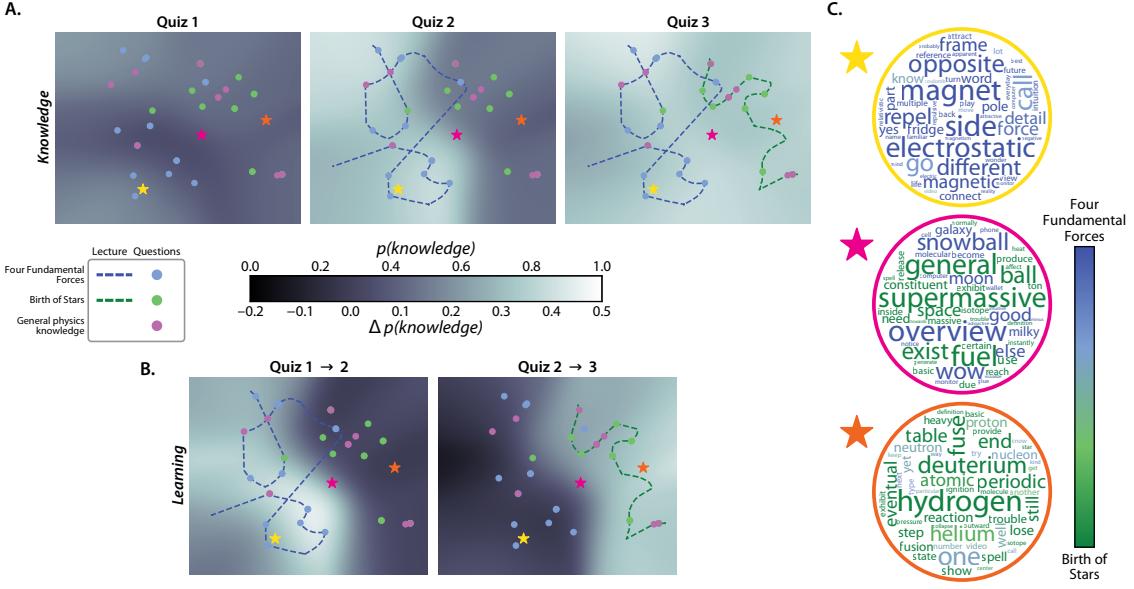


Figure 8: Mapping out the geometry of knowledge and learning. **A. Average “knowledge maps” estimated using each quiz.** Each map displays a 2D projection of the estimated knowledge about the content reflected by *all* regions of topic space (see *Creating knowledge and learning map visualizations*). The topic trajectories of the two lectures are indicated by dotted lines (blue: Lecture 1; green: Lecture 2), and the coordinates of each question are indicated by dots (light blue: Lecture 1-related; light green: Lecture 2-related; purple: general physics knowledge). Each map reflects an average across all participants. For individual participants’ maps, see Supplementary Figures 3, 4, and 5. **B. Average “learning maps” estimated between each successive pair of quizzes.** The learning maps follow the same general format as the knowledge maps in Panel A, but here the shading at each coordinate indicates the *difference* between the corresponding coordinates in the indicated pair of knowledge maps—i.e., how much the estimated knowledge “changed” between the two quizzes. Each map reflects an average across all participants. For individual participants’ maps, see Supplementary Figures 6 and 7. **C. Word clouds for sampled points in topic space.** Each word cloud displays the weighted blend of words underlying the topic proportions represented at the corresponding colored star’s location on the maps. In each word cloud, the words’ relative sizes correspond to their relative weights at the starred location, and their colors indicate their relative weights in the *Four Fundamental Forces* (blue) versus *Birth of Stars* (green) lectures, on average, across all timepoints’ topic vectors.

321 have relatively little knowledge about any parts of the text embedding space (i.e., the shading is
322 relatively dark everywhere). The knowledge map estimated from Quiz 2 responses shows a marked
323 increase in knowledge on the left side of the map (around roughly the same range of coordinates
324 traversed by the *Four Fundamental Forces* lecture, indicated by the dotted blue line). In other words,
325 participants' estimated increase in knowledge is localized to conceptual content that is nearby (i.e.,
326 related to) the content from the lecture they watched prior to taking Quiz 2. This localization is
327 non-trivial: these knowledge estimates are informed only by the embedded coordinates of the
328 *quiz questions*, not by the embeddings of either lecture (see Eqn. 4). Finally, the knowledge map
329 estimated from Quiz 3 responses shows a second increase in knowledge, localized to the region
330 surrounding the embedding of the *Birth of Stars* lecture participants watched immediately prior to
331 taking Quiz 3.

332 Another way of visualizing these content-specific increases in knowledge after participants
333 viewed each lecture is displayed in Figure 8B. Taking the point-by-point difference between the
334 knowledge maps estimated from responses to a successive pair of quizzes yields a *learning map*
335 that describes the *change* in knowledge estimates from one quiz to the next. These learning maps
336 highlight that the estimated knowledge increases we observed across maps were specific to the
337 regions around the embeddings of each lecture, in turn.

338 Because the 2D projection we used to construct the knowledge and learning maps is invertible,
339 we may gain additional insights into these maps' meaning by reconstructing the original high-
340 dimensional topic vector for any location on the map we are interested in. For example, this could
341 serve as a useful tool for an instructor looking to better understand which content areas a student
342 (or a group of students) knows well (or poorly). As a demonstration, we show the top-weighted
343 words from the blends of topics reconstructed from three example locations on the maps (Fig. 8C):
344 one point near the *Four Fundamental Forces* embedding (yellow), a second point near the *Birth of*
345 *Stars* embedding (orange), and a third point between the two lectures' embeddings (pink). As
346 shown in the word clouds in the panel, the top-weighted words at the example coordinate near the
347 *Four Fundamental Forces* embedding tended to be weighted more heavily by the topics expressed
348 in that lecture. Similarly, the top-weighted words at the example coordinate near the *Birth of Stars*

349 embedding tended to be weighted more heavily by the topics expressed in *that* lecture. And the
350 top-weighted words at the example coordinate between the two lectures' embeddings show a
351 roughly even mix of words most strongly associated with each lecture.

352 Discussion

353 We developed a computational framework that uses short multiple-choice quizzes to gain nuanced
354 insights into what learners know and how their knowledge changes with training. First, we show
355 that our approach can automatically match the conceptual knowledge probed by individual quiz
356 questions to the corresponding moments in lecture videos when those concepts were presented
357 (Fig. 4). Next, we demonstrate how we can estimate moment-by-moment "knowledge traces"
358 that reflect the degree of knowledge participants have about each video's time-varying content,
359 and capture temporally specific increases in knowledge after viewing each lecture (Fig. 5). We
360 also show that these knowledge estimates can generalize to held-out questions (Fig. 6). Finally,
361 we use our framework to construct visual maps that provide snapshot estimates of how much
362 participants know about any concept within the scope of our text embedding model, and how
363 much their knowledge of those concepts changes with training (Fig. 8).

364 Over the past several years, the global pandemic has forced many educators to suddenly
365 adapt to teaching remotely [25, 38, 47, 50]. This change in world circumstances is happening
366 alongside (and perhaps accelerating) geometric growth in the availability of high-quality online
367 courses from platforms such as Khan Academy [?], Coursera [51], EdX [27], and others [44].
368 Continued expansion of the global internet backbone and improvements in computing hardware
369 have also facilitated improvements in video streaming, enabling videos to be easily shared and
370 viewed by increasingly large segments of the world's population. This exciting time for online
371 course instruction provides an opportunity to re-evaluate how we, as a global community, educate
372 ourselves and each other. For example, we can ask: what defines an effective course or training
373 program? Which aspects of teaching might be optimized and/or augmented by automated tools?
374 How and why do learning needs and goals vary across people? How might we lower barriers of

375 access to a high-quality education?

376 Alongside these questions, there is a growing desire to extend existing theories beyond the
377 domain of lab testing rooms and into real classrooms [24]. In part, this has led to a recent
378 resurgence of “naturalistic” or “observational” experimental paradigms that attempt to better
379 reflect more ethologically valid phenomena that are more directly relevant to real-world situations
380 and behaviors [39]. In turn, this has brought new challenges in data analysis and interpretation. A
381 key step towards solving these challenges will be to build explicit models of real-world scenarios
382 and how people behave in them (e.g., models of how people learn conceptual content from real-
383 world courses, as in our current study). A second key step will be to understand which sorts of
384 signals derived from behaviors and/or other measurements (e.g., neurophysiological data; 2, 12, 36,
385 40, 41) might help to inform these models. A third major step will be to develop and employ reliable
386 ways of evaluating the complex models and data that are a hallmark of naturalistic paradigms.

387 Beyond specifically predicting what people *know*, the fundamental ideas we develop here also
388 relate to the notion of “theory of mind” of other individuals [18, 22, 35]. Considering others’ unique
389 perspectives, prior experiences, knowledge, goals, etc., can help us to more effectively interact and
390 communicate [42, 46, 49]. One could imagine future extensions of our work (e.g., analogous to
391 the knowledge and learning maps shown in Fig. 8), that attempt to characterize how well-aligned
392 different people’s knowledge bases or backgrounds are. In turn, this might be used to model how
393 knowledge (or other forms of communicable information) flows not just between teachers and
394 students, but between friends having a conversation, individuals on a first date, participants at
395 a business meeting, doctors and patients, experts and non-experts, political allies or adversaries,
396 and more. For example, the extent to which two people’s knowledge maps “match” or “align” in
397 a given region of text embedding space might serve as a predictor of how effectively they will be
398 able to communicate about the corresponding conceptual content.

399 Ultimately, our work suggests a rich new line of questions about the geometric “form” of
400 knowledge, how knowledge changes over time, and how we might map out the full space of
401 what an individual knows. Our finding that detailed estimates about knowledge may be obtained
402 from short quizzes shows one way that traditional approaches to evaluation in education may be

403 extended. We hope that these advances might help pave the way for new approaches to teaching
404 or delivering educational content that are tailored to individual students' learning needs and goals.

405 Materials and methods

406 Participants

407 We enrolled a total of 50 Dartmouth undergraduate students in our study. Participants received
408 optional course credit for enrolling. We asked each participant to complete a demographic survey
409 that included questions about their age, gender, native spoken language, ethnicity, race, hearing,
410 color vision, sleep, coffee consumption, level of alertness, and several aspects of their educational
411 background and prior coursework.

412 Participants' ages ranged from 18 to 22 years (mean: 19.52 years; standard deviation: 1.09
413 years). A total of 15 participants reported their gender as male and 35 participants reported their
414 gender as female. A total of 49 participants reported their native language as "English" and 1
415 reported having another native language. A total of 47 participants reported their ethnicity as
416 "Not Hispanic or Latino" and three reported their ethnicity as "Hispanic or Latino." Participants
417 reported their races as White (32 participants), Asian (14 participants), Black or African American
418 (5 participants), American Indian or Alaska Native (1 participant), and Native Hawaiian or Other
419 Pacific Islander (1 participant). (Note that some participants selected multiple racial categories.)

420 A total of 49 participants reporting having normal hearing and 1 participant reported having
421 some hearing impairment. A total of 49 participants reported having normal color vision and 1
422 participant reported being color blind. Participants reported having had, on the night prior to
423 testing, 2–4 hours of sleep (1 participant), 4–6 hours of sleep (9 participants), 6–8 hours of sleep (35
424 participants), or 8+ hours of sleep (5 participants). They reported having consumed, on the same
425 day and leading up to their testing session, 0 cups of coffee (38 participants), 1 cup of coffee (10
426 participants), 3 cups of coffee (1 participant), or 4+ cups of coffee (1 participant).

427 No participants reported that their focus was currently impaired (e.g., by drugs or alcohol).

428 Participants reported their current level of alertness, and we converted their responses to numerical
429 scores as follows: “very sluggish” (-2), “a little sluggish” (-1), “neutral” (0), “fairly alert” (1), and
430 “very alert” (2). Across all participants, a range of alertness levels were reported (range: -2–1;
431 mean: -0.10; standard deviation: 0.84).

432 Participants reported their undergraduate major(s) as “social sciences” (28 participants), “natural
433 sciences” (16 participants), “professional” (e.g., pre-med or pre-law; 8 participants), “mathematics
434 and engineering” (7 participants), “humanities” (4 participants), or “undecided” (3 participants). Note
435 that some participants selected multiple categories for their undergraduate major(s).
436 We also asked participants about the courses they had taken. In total, 45 participants reported hav-
437 ing taken at least one Khan Academy course in the past, and 5 reported not having taken any Khan
438 Academy courses. Of those who reported having watched at least one Khan Academy course,
439 7 participants reported having watched 1–2 courses, 11 reported having watched 3–5 courses, 8
440 reported having watched 5–10 courses, and 19 reported having watched 10 or more courses. We
441 also asked participants about the specific courses they had watched, categorized under different
442 subject areas. In the “Mathematics” area, participants reported having watched videos on AP
443 Calculus AB (21 participants), Precalculus (17 participants), Algebra 2 (14 participants), AP Cal-
444 culus BC (12 participants), Trigonometry (11 participants), Algebra 1 (10 participants), Geometry
445 (8 participants), Pre-algebra (7 participants), Multivariable Calculus (5 participants), Differential
446 Equations (5 participants), Statistics and Probability (4 participants), AP Statistics (2 participants),
447 Linear Algebra (2 participants), Early Math (1 participant), Arithmetic (1 participant), and other
448 videos not listed in our survey (5 participants). In the “Science and engineering” area, participants
449 reported having watched videos on Chemistry, AP Chemistry, or Organic Chemistry (21 partic-
450 ipants); Physics, AP Physics I, or AP Physics II (18 participants); Biology, AP Biology; or High
451 school Biology (15 participants); Health and Medicine (1 participant); or other videos not listed
452 in our survey (5 participants). We also asked participants whether they had specifically seen the
453 videos used in our experiment. Of the 45 participants who reported having having taken at least
454 one Khan Academy course in the past, 44 participants reported that they had not watched the *Four*
455 *Fundamental Forces* video, and 1 participant reported that they were not sure whether they had

456 watched it. All participants reported that they had not watched the *Birth of Stars* video. When
457 we asked participants about non-Khan Academy online courses, they reported having watched
458 or taken courses on Mathematics (15 participants), Science and engineering (11 participants), Test
459 preparation (9 participants), Economics and finance (3 participants), Arts and humanities (2 partic-
460 ipants), Computing (2 participants), and other categories not listed in our survey (17 participants).
461 Finally, we asked participants about in-person courses they had taken in different subject areas.
462 They reported taking courses in Mathematics (38 participants), Science and engineering (37 par-
463 ticipants), Arts and humanities (34 participants), Test preparation (27 participants), Economics
464 and finance (26 participants), Computing (14 participants), College and careers (7 participants), or
465 other courses not listed in our survey (6 participants).

466 Experiment

467 We hand-selected two course videos from the Khan Academy platform: *Four Fundamental Forces*
468 (an introduction to gravity, electromagnetism, the weak nuclear force, and the strong nuclear force;
469 duration: 10 minutes and 29 seconds) and *Birth of Stars* (an introduction to how stars are formed;
470 duration: 7 minutes and 57 seconds). We then hand-created 39 multiple-choice questions: 15 about
471 the conceptual content of *Four Fundamental Forces* (i.e., Lecture 1), 15 about the conceptual content
472 of *Birth of Stars* (i.e., Lecture 2), and 9 questions that tested for general conceptual knowledge about
473 basic physics (covering material that was not presented in either video). The full set of questions
474 and answer choices may be found in Supplementary Table 1.

475 Over the course of the experiment, participants completed three 13-question multiple-choice
476 quizzes: the first before viewing Lecture 1, the second between Lectures 1 and 2, and the third
477 after viewing Lecture 2 (see Fig. 1). The questions appearing on each quiz, for each participant,
478 were randomly chosen from the full set of 39, with the constraints that (a) each quiz contain
479 exactly 5 questions about Lecture 1, 5 questions about Lecture 2, and 3 questions about general
480 physics knowledge, and (b) each question appear exactly once for each participant. The orders of
481 questions on each quiz, and the orders of answer options for each question, were also randomized.
482 Our experimental protocol was approved by the Committee for the Protection of Human Subjects

483 at Dartmouth College. We used this experiment to develop and test our computational framework
484 for estimating knowledge and learning.

485 **Analysis**

486 **Constructing text embeddings of multiple lectures and questions**

487 We adapted an approach we developed in prior work [20] to embed each moment of the two
488 lectures and each question in our pool in a common representational space. Briefly, our approach
489 uses a topic model (Latent Dirichlet Allocation; 5), trained on a set of documents, to discover a set
490 of (up to) k “topics” or “themes.” Formally, each topic is defined as a distribution of weights over
491 each word in the model’s vocabulary (i.e., the union of all unique words, across all documents,
492 excluding “stop words.”). Conceptually, each topic is intended to give larger weights to words that
493 are semantically related or tend to co-occur in the same documents. After fitting a topic model,
494 each document in the training set, or any *new* document that contains at least some of the words
495 in the model’s vocabulary, may be represented as a k -dimensional vector describing how much
496 the document (most probably) reflects each topic. To select an appropriate k for our model, we
497 identified the minimum number of topics that yielded at least one “unused” topic (i.e., in which
498 all words in the vocabulary were assigned uniform weights) after training. This indicated that
499 the number of topics was sufficient to capture the set of latent themes present in the two lectures
500 (from which we constructed our document corpus, as described below). We found this value to
501 be $k = 15$ topics. The distribution of weights over words in the vocabulary for each discovered
502 topic is shown in Supplementary Figure 1, and each topic’s top-weighted words may be found in
503 Supplementary Table 2.

504 As illustrated in Figure 2A, we start by building up a corpus of documents using overlapping
505 sliding windows that span each video’s transcript. Khan Academy provides professionally created,
506 manual transcriptions of all videos for closed captioning. However, such transcripts would not
507 be readily available in all contexts to which our framework could potentially be applied. Khan
508 Academy videos are hosted on the YouTube platform, which additionally provides automated

509 captions. We opted to use these automated transcripts (which, in prior work, we have found to be
510 of sufficiently near-human quality to yield reliable data in behavioral studies; 52) when developing
511 our framework in order to make it more directly extensible and adaptable by others in the future.

512 We fetched these automated transcripts using the `youtube-transcript-api` Python package [?]
513]. The transcripts consisted of one timestamped line of text for every few seconds (mean: 2.34 s;
514 standard deviation: 0.83 s) of spoken content in the video (i.e., corresponding to each individual
515 caption that would appear on-screen if viewing the lecture via YouTube, and when those lines
516 would appear). We defined a sliding window length of (up to) $w = 30$ transcript lines, and
517 assigned each window a timestamp corresponding to the midpoint between the timestamps for its
518 first and last lines. These sliding windows ramped up and down in length at the beginning and
519 end of each transcript, respectively. In other words, each transcript's first sliding window covered
520 only its first line, the second sliding window covered the first two lines, and so on. This ensured
521 that each line from the transcripts appeared in the same number (w) of sliding windows. After
522 performing various standard text preprocessing (e.g., normalizing case, lemmatizing, removing
523 punctuation and stop-words), we treated the text from each sliding window as a single "document,"
524 and combined these documents across the two videos' windows to create a single training corpus
525 for the topic model.

526 After fitting a topic model to the two videos' transcripts, we could use the trained model to
527 transform arbitrary (potentially new) documents into k -dimensional topic vectors. A convenient
528 property of these topic vectors is that documents that reflect similar blends of topics (i.e., documents
529 that reflect similar themes, according to the model) will yield similar coordinates (in terms of
530 correlation, cosine similarity, Kullback-Leibler divergence, Euclidean distance, or other geometric
531 measures). In general, the similarity between different documents' topic vectors may be used to
532 characterize the similarity in conceptual content between the documents.

533 We transformed each sliding window's text into a topic vector, and then used linear interpo-
534 lation (independently for each topic dimension) to resample the resulting timeseries to one vector
535 per second. We also used the fitted model to obtain topic vectors for each question in our pool (see
536 Supp. Tab. 1). Taken together, we obtained a *trajectory* for each video, describing its path through

topic space, and a single coordinate for each question (Fig. 2C). Embedding both videos and all of the questions using a common model enables us to compare the content from different moments of videos, compare the content across videos, and estimate potential associations between specific questions and specific moments of video.

541 Estimating dynamic knowledge traces

542 We used the following equation to estimate each participant’s knowledge about timepoint t of a
 543 given lecture, $\hat{k}(t)$:

$$\hat{k}(f(t, L)) = \frac{\sum_{i \in \text{correct}} \text{ncorr}(f(t, L), f(i, Q))}{\sum_{j=1}^N \text{ncorr}(f(t, L), f(j, Q))}, \quad (1)$$

544 where

$$\text{ncorr}(x, y) = \frac{\text{corr}(x, y) - \text{mincorr}}{\text{maxcorr} - \text{mincorr}}, \quad (2)$$

545 and where mincorr and maxcorr are the minimum and maximum correlations between any lecture
 546 timepoint and question, taken over all timepoints in the given lecture, and all five questions *about*
 547 that lecture appearing on the given quiz. We also define $f(s, \Omega)$ as the s^{th} topic vector from the set
 548 of topic vectors Ω . Here t indexes the set of lecture topic vectors, L , and i and j index the topic
 549 vectors of questions used to estimate the knowledge trace, Q . Note that “correct” denotes the set
 550 of indices of the questions the participant answered correctly on the given quiz.

551 Intuitively, $\text{ncorr}(x, y)$ is the correlation between two topic vectors (e.g., the topic vector from one
 552 timepoint in a lecture, x , and the topic vector for one question, y), normalized by the minimum and
 553 maximum correlations (across all timepoints t and questions Q) to range between 0 and 1, inclusive.
 554 Equation 1 then computes the weighted average proportion of correctly answered questions about
 555 the content presented at timepoint t , where the weights are given by the normalized correlations
 556 between timepoint t ’s topic vector and the topic vectors for each question. The normalization
 557 step (i.e., using ncorr instead of the raw correlations) insures that every question contributes some
 558 non-negative amount to the knowledge estimate.

559 **Creating knowledge and learning map visualizations**

560 An important feature of our approach is that, given a trained text embedding model and partic-
561 ipants’ quiz performance on each question, we can estimate their knowledge about *any* content
562 expressible by the embedding model—not solely the content explicitly probed by the quiz ques-
563 tions, or even appearing in the lectures. To visualize these estimates (Fig. 8, Supp. Figs. 3, 4, 5, 6,
564 and 7), we used Uniform Manifold Approximation and Projection (UMAP; 33, 34) to construct a
565 2D projection of the text embedding space. Sampling the original 100-dimensional space at high
566 resolution to obtain an adequate set of topic vectors spanning the embedding space would be
567 computationally intractable. However, sampling a 2D grid is trivial.

568 At a high level, the UMAP algorithm obtains low-dimensional embeddings by minimizing
569 the cross-entropy between the pairwise (clustered) distances between the observations in their
570 original (e.g., 100-dimensional) space and the pairwise (clustered) distances in the low-dimensional
571 embedding space (in our approach, the embedding space is 2D). In our implementation, pairwise
572 distances in the original high-dimensional space were defined as 1 minus the correlation between
573 each pair of coordinates, and pairwise distances in the low-dimensional embedding space were
574 defined as the Euclidean distance between each pair of coordinates.

575 In our application, all of the coordinates we embedded were topic vectors, whose elements
576 are always non-negative and sum to one. Although UMAP is an invertible transformation at
577 the embedding locations of the original data, other locations in the embedding space will not
578 necessarily follow the same implicit “rules” as the original high-dimensional data. For example,
579 inverting an arbitrary coordinate in the embedding space might result in negative-valued vectors,
580 which are incompatible with the topic modeling framework. To protect against this issue, we
581 log-transformed the topic vectors prior to embedding them in the 2D space. When we inverted
582 the embedded vectors (e.g., to estimate topic vectors or word clouds, as in Fig. 8C), we passed
583 the inverted (log-transformed) values through the exponential function to obtain a vector of non-
584 negative values, and normalized them to sum to one.

585 After embedding both lectures’ topic trajectories and the topic vectors of every question, we

586 defined a rectangle enclosing the 2D projections of the lectures' and quizzes' embeddings. We then
587 sampled points from a regular 100×100 grid of coordinates that evenly tiled this enclosing rectangle.
588 We sought to estimate participants' knowledge (and learning, i.e., changes in knowledge) at each
589 of the resulting 10,000 coordinates.

590 To generate our estimates, we placed a set of 39 radial basis functions (RBFs) throughout the
591 embedding space, centered on the 2D projections for each question (i.e., we included one RBF for
592 each question). At coordinate x , the value of an RBF centered on a question's coordinate μ , is given
593 by:

$$\text{RBF}(x, \mu, \lambda) = \exp \left\{ -\frac{\|x - \mu\|^2}{\lambda} \right\}. \quad (3)$$

594 The λ term in the RBF equation controls the "smoothness" of the function, where larger values
595 of λ result in smoother maps. In our implementation we used $\lambda = 50$. Next, we estimated the
596 "knowledge" at each coordinate, x , using:

$$\hat{k}(x) = \frac{\sum_{i \in \text{correct}} \text{RBF}(x, q_i, \lambda)}{\sum_{j=1}^N \text{RBF}(x, q_j, \lambda)}. \quad (4)$$

597 Intuitively, Equation 4 computes the weighted proportion of correctly answered questions, where
598 the weights are given by how nearby (in the 2D space) each question is to the x . We also defined
599 *learning maps* as the coordinate-by-coordinate differences between any pair of knowledge maps.
600 Intuitively, learning maps reflect the *change* in knowledge across two maps.

601 Author contributions

602 Conceptualization: PCF, ACH, and JRM. Methodology: PCF, ACH, and JRM. Software: PCF.
603 Validation: PCF. Formal analysis: PCF. Resources: PCF, ACH, and JRM. Data curation: PCF.
604 Writing (original draft): JRM. Writing (review and editing): PCF, ACH, and JRM. Visualization:
605 PCF and JRM. Supervision: JRM. Project administration: PCF. Funding acquisition: JRM.

606 **Data and code availability**

607 All of the data analyzed in this manuscript, along with all of the code for running our experiment
608 and carrying out the analyses may be found at [https://github.com/ContextLab/efficient-learning-](https://github.com/ContextLab/efficient-learning-khan)
609 [khan](#).

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