

<sup>1</sup> A geometric framework for capturing high-resolution  
<sup>2</sup> insights into conceptual knowledge and learning in  
<sup>3</sup> classroom-like settings

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<sup>5</sup> **Abstract**

We develop a mathematical framework, based on natural language processing models, for tracking and characterizing the acquisition of conceptual knowledge in real-world educational contexts. Our approach embeds course content in a high-dimensional conceptual space, where nearby coordinates reflect similar or related concepts. We test our approach using behavioral data from participants who watched two lecture videos from the Khan Academy platform, interleaved between three short multiple-choice quizzes. We applied our framework to the videos' transcripts and the text of the quiz questions to quantify the conceptual content presented in each moment of video and knowledge probed by each quiz question. We used these embeddings, along with participants' quiz responses, to track how the learners' knowledge changed after watching each video. Our findings demonstrate how a small set of quiz questions may be used to obtain rich and meaningful high-resolution insights into individual students' knowledge, and how it changes over time as they learn.

19

## Introduction

20 Suppose that a teacher had access to a complete “map” of everything a student knew. Defining  
21 what such a map might even look like, let alone how it might be constructed or filled in, is itself a  
22 non-trivial problem. But if a teacher *were* to gain access to such a map, how might it change their  
23 ability to teach that student? Perhaps they might start by checking how well the student knew  
24 the to-be-learned information already, or how much they knew about related concepts. For some  
25 students, they could potentially optimize their teaching efforts to maximize efficiency by focusing  
26 primarily on not-yet-known content. For other students (or other content areas), it might be more  
27 effective to optimize for direct connections between already known content and new material.  
28 Observing how the student’s knowledge changed over time, in response to their teaching, could  
29 also help to guide the teacher towards the most effective strategy for that individual student.

30 Designing and building procedures and tools for mapping out knowledge touches on deep  
31 questions about what it means to learn. For example, how do we acquire conceptual knowledge?  
32 Memorizing course lectures or textbook chapters by rote can lead to the superficial *appearance*  
33 of understanding the underlying content, but achieving true conceptual understanding seems  
34 to require something deeper and richer. Does conceptual understanding entail connecting newly  
35 acquired information to the scaffolding of one’s existing knowledge or experience [6, 10, 13, 14, 57]?  
36 Or weaving a lecture’s atomic elements (e.g., its component words) into a structured network  
37 that describes how those individual elements are related? Conceptual understanding could also  
38 involve building a mental model that transcends the meanings of those individual atomic elements  
39 by reflecting the deeper meaning underlying the gestalt whole [34, 38, 54].

40 The difference between “understanding” and “memorizing,” as framed by researchers in ed-  
41 ucation, cognitive psychology, and cognitive neuroscience [e.g., 19, 25, 30, 38, 54] has profound  
42 analogs in the fields of natural language processing and natural language understanding. For  
43 example, considering the raw contents of a document (e.g., its constituent symbols, letters, and

44 words) might provide some information about what the document is about, just as memorizing  
45 a passage might provide some ability to answer simple questions about it [e.g., whether it might  
46 contain words related to furniture versus physics; 7, 8, 37]. However, modern natural language  
47 processing models [e.g., 9, 11, 45] also attempt to capture the deeper meaning *underlying* those  
48 atomic elements. These models consider not only the co-occurrences of those elements within  
49 and across documents, but also patterns in how those elements appear across different scales (e.g.,  
50 sentences, paragraphs, chapters, etc.), the temporal and grammatical properties of the elements,  
51 and other high-level characteristics of how they are used [39, 40]. According to these models, the  
52 deep conceptual meaning of a document may be captured by a feature vector in a high-dimensional  
53 representation space, where nearby vectors reflect conceptually related documents. A model that  
54 succeeds at capturing an analog of “understanding” is able to assign nearby feature vectors to  
55 two conceptually related documents, *even when the words contained in those documents have very little*  
56 *overlap*.

57 Given these insights, what form might the representation of the sum total of a person’s knowl-  
58 edge take? First, we might require a means of systematically describing or representing the nearly  
59 infinite set of possible things a person could know. Second, we might want to account for potential  
60 associations between different concepts. For example, the concepts of “fish” and “water” might be  
61 associated in the sense that fish live in water. Third, knowledge may have a critical dependency  
62 structure, such that knowing about a particular concept might require first knowing about a set of  
63 other concepts. For example, understanding the concept of a fish swimming in water first requires  
64 understanding what fish and water *are*. Fourth, as we learn, our “current state of knowledge”  
65 should change accordingly. Learning new concepts should both update our characterizations of  
66 “what is known” and also unlock any now-satisfied dependencies of those newly learned concepts  
67 so that they are “tagged” as available for future learning.

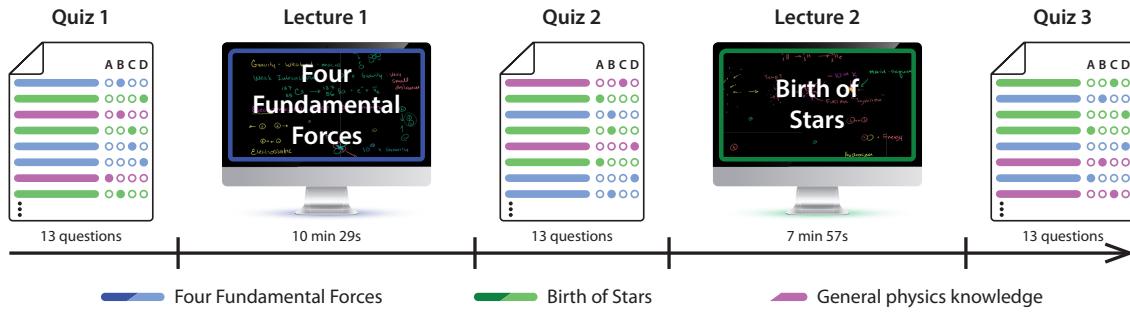
68 Here we develop a framework for modelling how knowledge is acquired during learning. The  
69 central idea is to use text embedding models to define the coordinate systems of two maps: (a) a  
70 *knowledge map* that describes the extent to which each concept is currently known and (b) a *learning*  
71 *map* that describes changes in knowledge over time. Each location on these maps represents

72 a single concept, and the maps' geometries are defined such that related concepts are located  
73 nearby in space. We use this framework to analyze and interpret behavioral data collected from  
74 an experiment that has participants watch and answer multiple choice questions about a series of  
75 recorded course lectures.

76 Our primary research goal is to advance our understanding of what it means to acquire deep  
77 real-world conceptual knowledge. Traditional laboratory approaches to studying learning and  
78 memory (e.g., list learning studies) often draw little distinction between memorization and under-  
79 standing. Instead, these studies typically focus on whether information is effectively encoded or  
80 retrieved, rather than whether the information is *understood*. Approaches to studying conceptual  
81 learning, such as category learning experiments, can start to investigate the distinction between  
82 memorization and understanding, often by training participants to distinguish arbitrary or ran-  
83 dom features in otherwise meaningless categorized stimuli. However the objective of real-world  
84 training, or learning from life experiences more generally, is often to develop new knowledge that  
85 may be applied in *useful* ways in the future. In this sense, the gap between modern learning theo-  
86 ries and modern pedagogical approaches and classroom learning strategies is enormous: most of  
87 our theories about *how* people learn are inspired by experimental paradigms and models that have  
88 only peripheral relevance to the kinds of learning that students and teachers actually seek. To help  
89 bridge this gap, our study uses course materials from real online courses to inform, fit, and test  
90 models of real-world conceptual learning. We also provide a “proof of concept” demonstration  
91 of how our models might be used to construct “maps” of what students know, and how their  
92 knowledge changes with training. In addition to helping to visualize knowledge (and changes  
93 in knowledge), we hope that such maps might lead to real-world tools for improving how we  
94 educate.

## 95 Results

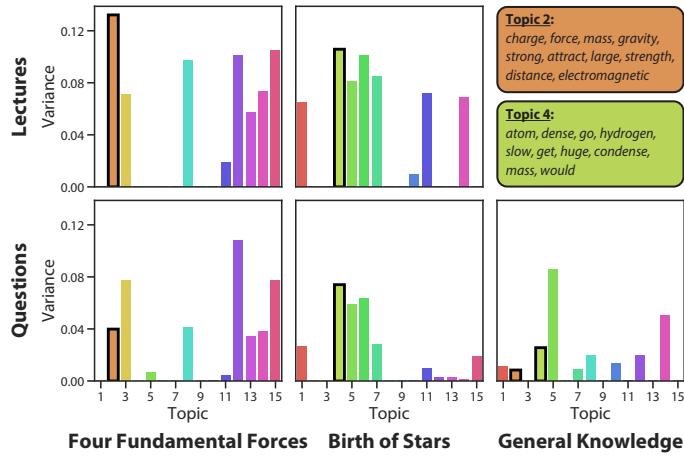
96 At its core, our main modeling approach is based around a simple assumption that we sought to test  
97 empirically: all else being equal, knowledge about a given concept is predictive of knowledge about



**Figure 1: Experimental paradigm.** Participants alternate between answering 13-question multiple choice quizzes and watching two Khan academy videos. Each quiz contains a mix of 5 questions about lecture 1, 5 questions about lecture 2, and 3 general physics knowledge questions. The specific questions reflected on each quiz, and the orders of each quiz's questions, were randomized across participants.

similar or related concepts. From a geometric perspective, this assumption implies that knowledge is fundamentally “smooth.” In other words, as one moves through a space representing someone’s knowledge (where similar concepts occupy nearby coordinates), their “level of knowledge” should change relatively gradually throughout that space. To begin to test this smoothness assumption, we sought to track our participants’ knowledge and how it changed over time in response to training.

We asked our participants to answer questions from several multiple choice quizzes and watch two lecture videos from the *Khan Academy* platform (Fig. 1). One lecture video, entitled *Four Fundamental Forces*, was about the four fundamental forces in physics: gravity, strong and weak interactions, and electromagnetism. The second lecture video, entitled *Birth of Stars*, provides an overview of our current understanding of how stars form. We selected both lessons to be (a) accessible to a broad audience, e.g., by minimizing prerequisite knowledge, (b) largely independent of each other, e.g., so that the two videos focused on different material and did not depend on each other, and (c) related to each other, e.g., so that both videos contained at least *some* similar or overlapping content. The two videos we selected are introductory (i.e., minimizing specific prerequisite knowledge), and are about different primary concepts, but they both also touch on “physics” and “astronomy” themes. We also wrote a set of multiple choice quiz questions that would enable us to test participants’ knowledge about each individual video and about



**Figure 2: Lecture and question topic overlap.** The bar plots display the variability in topic weights across lecture timepoints (top panels) and questions (bottom panels); colors denote topics. The top-weighted words from the most “expressive” (i.e., variable across observations) topics from each lecture are displayed in the upper right (orange: topic 2; yellow-green: topic 4). The top-weighted words from the full set of topics may be found in Table S2.

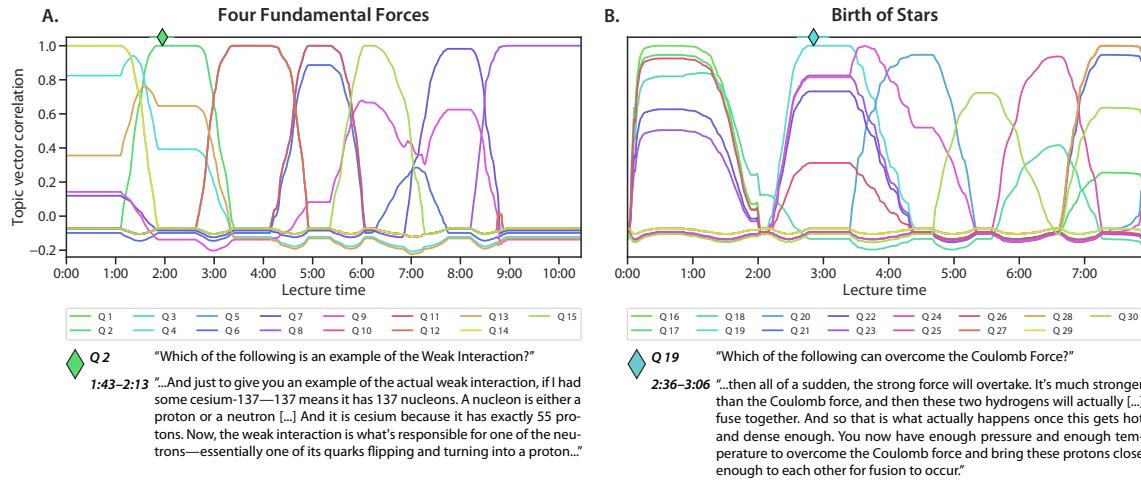
related content not specifically presented in either video (Tab. S1). Participants answered questions randomly drawn from each content area (lecture 1, lecture 2, and general physics knowledge) across each of three quizzes. Quiz 1 was intended to assessed participants’ knowledge before training; quiz 2 assessed knowledge after watching the *Four Fundamental Forces* video (i.e., lecture 1); and quiz 3 assessed knowledge after watching the *Birth of Stars* video (i.e., lecture 2).

We trained a text embedding model using sliding windows of text from the two videos’ transcripts (see *Constructing text embeddings of multiple videos and questions*). We also used the same model (i.e., trained on the videos’ transcripts) to embed the text of each question in our pool. This yielded, for each second of each video, and for each question, a single 15-dimensional topic vector—i.e., a coordinate in a text embedding space (Fig. 7). Intuitively, each dimension of the embedding space corresponds to a “theme” or “topic” reflected in some part(s) of the videos (Tab. S2), and the coordinates in embedding space denote the blend of themes reflected by a particular excerpt of text (e.g., from part of a video’s transcript, from a question, etc.).

Given that we trained the text embedding model using the video transcripts, we wondered whether the questions that were (ostensibly, by design) “about” the content of each lecture would

131 “match up” correctly with the lectures. In other words, we hoped that the text embeddings would  
132 capture something about the deeper conceptual content of the lectures, beyond surface details such  
133 as exact wording choices. If so, when we embed *new* text outside of the model’s training set, we  
134 should see a correspondance between the embeddings of the training data (i.e., snippets of text  
135 from the lectures’ transcripts) and other text that reflects related concepts (e.g., questions *about* each  
136 lecture). Further, although the content from any given moment from a lecture might stray from the  
137 average content (across all timepoints), we hoped that *variability* in each topic’s expression over  
138 timepoints within a lecture would match up with the variability in topic expressions for questions  
139 about that lecture. Intuitively, the variability in the expression of a given topic relates to how  
140 much “information” [18] the lecture (or questions) reflect about that topic. When we compared the  
141 variability in topic weights across each lecture’s timepoints with the variability in topic weights  
142 across each question set, we found a strong correspondence (Fig. 2). The most variable topics from  
143 the *Four Fundamental Forces* lecture, and questions about that lecture, are 2, 3, 8, 12, 13, 14, and 15.  
144 The most variable topics from the *Birth of Stars* lecture, and questions about that lecture, are 1, 4,  
145 5, 6, and 7. This strong overlap between the lectures and questions specifically about each lecture  
146 indicates that the topic model captures some of the underlying conceptual content.

147 Although a single lecture may be organized around a single broad theme at a coarse scale, at a  
148 finer scale each moment of a lecture typically covers a narrower range of content. We wondered  
149 whether a text embedding model trained on the lectures’ transcripts might capture some of this  
150 finer scale content. For example, if a particular question asks about the content from one small  
151 part of a lecture, we wondered whether our text embedding model could be used to automatically  
152 identify the “matching” moment(s) in the lecture. When we correlated each question’s topic vector  
153 with the topic vectors for each second of the lectures, we found some evidence that each question is  
154 temporally specific (Fig. 3). In particular, most questions’ topic vectors were maximally correlated  
155 with a well-defined (and relatively narrow) range of timepoints from their corresponding lectures,  
156 and the correlations fell off sharply outside of that range. We also examined the best-matching  
157 intervals for each question qualitatively by comparing the text of the question to the text of the most-  
158 correlated parts of the lectures. Despite that the questions were excluded from the text embedding

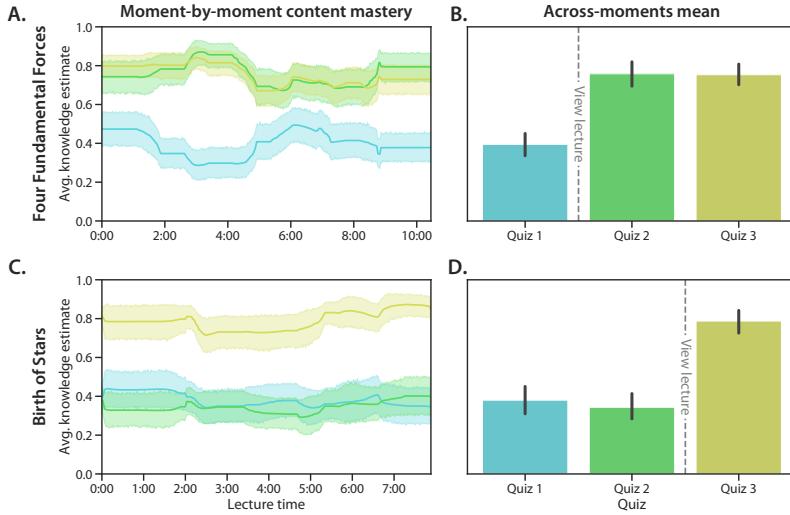


**Figure 3: Which parts of each lecture are captured by each question?** Each panel displays timeseries plots showing how each question’s topic vector correlates with each video timepoint’s topic vector (Panel A.: correlations for the *Four Fundamental Forces* lecture and associated questions; Panel B.: correlations for the *Birth of Stars* lecture and associated questions). The colors denote question identities. The diamonds in each panel denote the moment of peak correlation between the indicated questions, in the indicated lectures. The associated questions’ text, and snippets of the lectures’ transcripts in the best-matching sliding windows, are displayed at the bottom of the figure.

159 model’s training set, in general we found (through manual inspection) a close correspondence  
160 between the conceptual content that each question covered and the content covered by the best-  
161 matching moments of the lectures. Two representative examples are shown at the bottom of  
162 Figure 3.

163 The ability to quantify how much each question is “asking about” the content from each moment  
164 of the lectures could enable high-resolution insights into participants’ knowledge. Traditional  
165 approaches to estimating how much a student “knows” about the content of a given lecture entail  
166 computing the proportion of correctly answered questions. But if two students receive identical  
167 scores on an exam, might our modeling framework help us to gain more nuanced insights into the  
168 *specific* content that each student has mastered (or failed to master)? For example, a student who  
169 misses three questions that were all about the same concept (e.g., concept *A*) will have gotten the  
170 same *proportion* of questions correct as another student who missed three questions about three  
171 *different* concepts (e.g., *A*, *B*, and *C*). But if we wanted to fill in the “gaps” in the two students’  
172 understandings, we might do well to focus on concept *A* for the first student, but to also add in  
173 materials pertaining to concepts *B* and *C* for the second student.

174 We developed a simple formula (Eqn. 1) for using a participant’s responses to a small set  
175 of multiple choice questions to estimate how much the participant “knows” about the concept  
176 reflected by any arbitrary coordinate,  $x$ , in text embedding space (e.g., the content reflected by  
177 any moment in a lecture they had watched; see *Estimating dynamic knowledge traces*). Essentially,  
178 the estimated knowledge at the coordinate is given by the weighted average proportion of quiz  
179 questions the participant answered correctly, where the weights reflect how much each question  
180 is “about” the content at  $x$ . When we apply this approach to estimate the participant’s knowledge  
181 about the content presented in each moment of each lecture, we can obtain a detailed timecourse  
182 describing how much “knowledge” the participant has about any part of the lecture. As shown  
183 in Figure 4, we can also apply this approach separately for the questions from each quiz the  
184 participants took throughout the experiment. From just 13 questions per quiz, we obtain a high-  
185 resolution snapshot (at the time each quiz was taken) of what the participants knew about any  
186 moment’s content, from either of the two lectures they watched (comprising a total of 1106 samples



**Figure 4: Estimating moment-by-moment knowledge acquisition.** **A. Moment-by-moment knowledge about the *Four Fundamental Forces*.** Each trace displays the weighted proportion of correctly answered questions about the content reflected in each moment of the lecture (see *Estimating dynamic knowledge traces*), using responses from one quiz’s color). The traces are averaged across participants. **B. Average estimated knowledge about the *Four Fundamental Forces*.** Each bar displays the across-timepoint average knowledge, estimated using the responses to one quiz’s questions. **C. Moment-by-moment knowledge about the *Birth of Stars*.** The panel is in the same format as Panel A, but here the knowledge estimates are for the moment-by-moment content of the *Birth of Stars* lecture. **D. Average estimated knowledge about the *Birth of Stars*.** The panel is in the same format as Panel B, but here the knowledge estimates are for the content of the *Birth of Stars* lecture. All panels: error ribbons and error bars denote 95% confidence intervals, estimated across participants.

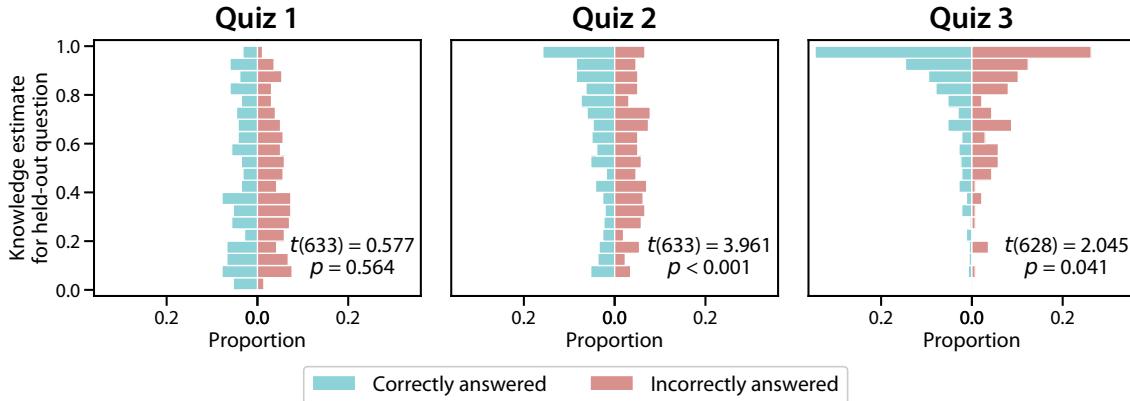
187 across the two lectures).

188 Of course, even though the timecourses in Figure 4A and C provide detailed *estimates* about  
 189 participants’ knowledge, those estimates are only *useful* to the extent that they accurately reflect what  
 190 participants actually know. As one sanity check, we anticipated that the knowledge estimates  
 191 should show a content-specific “boost” in participants’ knowledge after watching each lecture.  
 192 In other words, if participants learn about each lecture’s content when they watch each lecture,  
 193 the knowledge estimates should reflect that. After watching the *Four Fundamental Forces* lecture,  
 194 participants should show more knowledge for the content of that lecture than they had before,  
 195 and that knowledge should persist for the remainder of the experiment. Specifically, knowledge  
 196 about that lecture’s content should be relatively low when estimated using Quiz 1 responses,  
 197 but should increase when estimated using Quiz 2 or 3 responses (Fig. 4B). Indeed, we found

198 that participants' estimated knowledge about the content of the *Four Fundamental Forces* was  
199 substantially higher on Quiz 2 versus Quiz 1 ( $t(49) = 8.764, p < 0.001$ ) and on Quiz 3 versus Quiz  
200 1 ( $t(49) = 10.519, p < 0.001$ ). We found no reliable differences in estimated knowledge about  
201 that lecture's content on Quiz 2 versus 3 ( $t(49) = 0.160, p = 0.874$ ). Similarly, we hypothesized  
202 (and subsequently confirmed) that participants should show more estimated knowledge about the  
203 content of the *Birth of Stars* lecture after (versus before) watching it (Fig. 4D). Specifically, since  
204 participants watched that lecture after taking Quiz 2 (but before Quiz 3), we hypothesized that their  
205 knowledge estimates should be relatively low on Quizzes 1 and 2, but should show a "boost" on  
206 Quiz 3. Consistent with this prediction, we found no reliable differences in estimated knowledge  
207 about the *Birth of Stars* lecture content on Quizzes 1 versus 2 ( $t(49) = 1.013, p = 0.316$ ), but the  
208 estimated knowledge was substantially higher on Quiz 3 versus 2 ( $t(49) = 10.561, p < 0.001$ ) and  
209 Quiz 3 versus 1 ( $t(49) = 8.969, p < 0.001$ ).

210 If we are able to accurately estimate a participant's knowledge about the content tested by a  
211 given question, the estimated knowledge should have some predictive information about whether  
212 the participant is likely to answer the question correctly or incorrectly. For each question in turn, for  
213 each participant, we used Equation 1 to estimate (using all *other* questions from the same quiz, from  
214 the same participant) the participant's knowledge at the held-out question's embedding coordinate.  
215 For each quiz, we grouped these estimates into two distributions: one for the estimated knowledge  
216 at the coordinates of each *correctly* answered question, and another for the estimated knowledge at  
217 the coordinates of each *incorrectly* answered question (Fig. 5). We then used independent samples  
218 *t*-tests to compare the means of these distributions of estimated knowledge.

219 For the initial quizzes participants took (prior to watching either lecture), participants' estimated  
220 knowledge tended to be low overall, and relatively unstructured (Fig. 5, left panel). When we held  
221 out individual questions and estimated their knowledge at the held-out questions' embedding  
222 coordinates, we found no reliable differences in the estimates when the held-out question had  
223 been correctly versus incorrectly answered ( $t(633) = 0.577, p = 0.564$ ). After watching the first  
224 video, estimated knowledge for held-out correctly answered questions (from the second quiz;  
225 Fig. 5, middle panel) exhibited a positive shift relative to held-out incorrectly answered questions

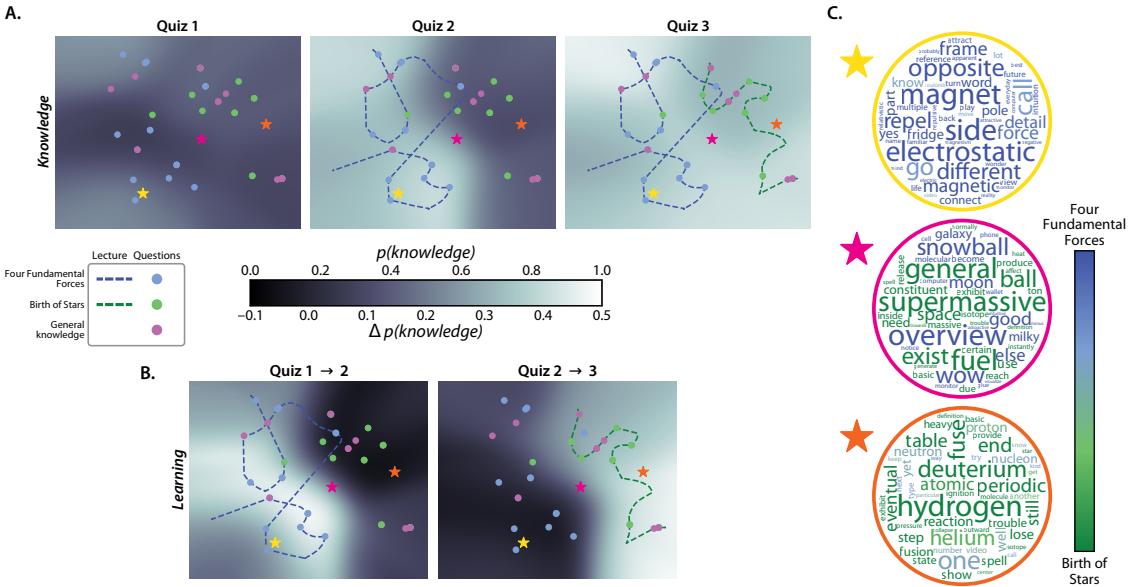


**Figure 5: Estimating knowledge at the embedding coordinates of held-out questions.** Separately for each quiz (panel), we plot the distributions of predicted knowledge at the embedding coordinates of each held out correctly (blue) or incorrectly (red) answered question. The  $t$ -tests reported in each panel are between the distributions of estimated knowledge at the coordinates of correctly versus incorrectly answered held-out questions.

( $t(633) = 3.961, p < 0.001$ ). After watching the second video, estimated knowledge (from the third quiz; Fig. 5, right panel) for *all* questions exhibited a positive shift. However, the increase in estimated knowledge for held-out correctly answered questions was larger than for held-out incorrectly answered questions ( $t(628) = 2.045, p = 0.041$ ).

Knowledge estimates need not be limited to the content of the lectures. As illustrated in Figure 6, our general approach to estimating knowledge from a small number of quiz questions may be applied to *any* content, given its text embedding coordinate. To visualize how knowledge “spreads” through text embedding space to content beyond the lectures participants watched, we first fit a new topic model to the lectures’ sliding windows with  $k = 100$  topics. We hoped that increasing the number of topics from 15 to 100 might help us to generalize the knowledge predictions. (Aside from increasing the number of topics from 15 to 100, all other procedures and model parameters were carried over from the preceding analyses.) As in our other analyses, we resampled each lecture’s topic trajectory to 1 Hz and also projected each question into a shared text embedding space.

We projected the resulting 100-dimensional topic vectors (for each second of video and for each question) into a shared 2-dimensional space (see *Creating knowledge and learning map visualizations*).



**Figure 6: Mapping out the geometry of knowledge and learning.** **A.** Average “knowledge maps” estimated using each quiz. Each map displays a 2D projection of the estimated knowledge about the content reflected by *all* regions of topic space (see *Creating knowledge and learning map visualizations*). The topic trajectories of each lecture and the coordinates of each question are indicated by dotted lines and dots. Each map reflects an average across all participants. For individual participants’ maps, see Figures S1, S2, and S3. **B.** Average “learning maps” estimated between each successive pair of quizzes. The learning maps are in the same general format as the knowledge maps in Panel A, but each coordinate in the learning maps indicates the *difference* between the corresponding coordinates in the indicated *pair* of knowledge maps—i.e., how much the estimated knowledge “changed” across the two quizzes. Each map reflects an average across all participants. For individual participants’ maps, see Figures S4 and S5. **C.** Word clouds for sampled points in topic space. Each word cloud displays the relative weights of each word reflected by the blend of topics represented at the locations of the stars in the maps. The words’ colors indicate how much each word is weighted on average across all timepoints’ topic vectors in the *Four Fundamental Forces* (blue) and *Birth of Stars* (green) videos, respectively.

242 Next, we sampled points evenly from a  $100 \times 100$  grid of coordinates that evenly tiled a rectangle  
243 enclosing the 2D projections of the videos and questions. We used Equation 4 to estimate partici-  
244 pants' knowledge at each of these 10K sampled locations, and we averaged these estimates across  
245 participants to obtain an estimated average *knowledge map* (Fig. 6). Intuitively, the knowledge map  
246 constructed from a given quiz's responses provides a visualization of how "much" participants  
247 know about any content expressible by the fitted text embedding model.

248 Several features of the resulting knowledge maps are worth noting. The average knowledge  
249 map estimated from Quiz 1 responses (Fig. 6, leftmost map) shows that participants tended to  
250 have relatively little knowledge about any parts of the text embedding space (i.e., the shading  
251 is relatively dark everywhere). The knowledge map estimated from Quiz 2 responses shows a  
252 marked increase in knowledge on the left side of the map (around roughly the same range of  
253 coordinates covered by the *Four Fundamental Forces* lecture, indicated by the dotted blue line).  
254 In other words, participants' estimated increase in knowledge is localized to conceptual content  
255 that is nearby (i.e., related to) the content from the lecture they watched prior to taking Quiz  
256 2. This localization is non-trivial: the knowledge estimates are informed only by the embedded  
257 coordinates of the *quiz questions*, not by the embeddings of either lecture (Eqn. 4). Finally, the  
258 knowledge map estimated from Quiz 3 responses shows a second increase in knowledge, localized  
259 to the region surrounding the embedding of the *Birth of Stars* lecture participants watched prior to  
260 taking Quiz 3.

261 Another way of visualizing these content-specific increases in knowledge (apparently driven  
262 by watching each lecture) is displayed in Figure 6B. Taking the point-by-point difference between  
263 the knowledge maps estimated from responses to a successive pair of quizzes yields a *learning map*  
264 that describes the *change* in knowledge estimates from one quiz to the next. These learning maps  
265 highlight that the estimated knowledge increases we observed across maps were specific to the  
266 regions around the embeddings of each lecture in turn.

267 Because the 2D projection we used to construct the knowledge and learning maps is (partially)  
268 invertible, we may gain additional insights into the estimates by reconstructing the original high-  
269 dimensional topic vectors for any point(s) in the maps we are interested in. For example, this

270 could serve as a useful tool for an instructor looking to better understand which content areas  
271 a student (or a group of students) knows well (or poorly). As a demonstration, we show the  
272 top-weighted words from the blends of topics reconstructed from three example locations on the  
273 maps (Fig. 6C): one point near the *Four Fundamental Forces* embedding (yellow); a second point  
274 near the *Birth of Stars* embedding (orange), and a third point somewhere in between the two  
275 lectures' embeddings (pink). As shown in the word clouds in the Panel, the top-weighted words  
276 at the example coordinate near the *Four Fundamental Forces* embedding also tended to be weighted  
277 heavily by the topics expressed in that lecture. Similarly, the top-weighted words at the example  
278 coordinate near the *Birth of Stars* embedding tended to be weighted most heavily by the topics  
279 expressed in *that* lecture. And the top-weighted words at the example coordinate between the  
280 two lectures' embeddings show a roughly even mix of words most strongly associated with each  
281 lecture.

## 282 Discussion

283 Teaching, like effective writing and speaking, is fundamentally about empathy [1, 44, 59]. Great  
284 teachers consider students' interests [12, 60], backgrounds [15, 47, 53], and working memory capac-  
285 ities [2], and flexibly optimize their teaching strategies within those constraints [4, 22, 28]. In the  
286 classroom, empathizing with students also means maintaining open lines of communication [65] by  
287 fostering an environment in which all students feel comfortable speaking up if they have an excit-  
288 ing new idea, or if they are having trouble understanding something [20, 61]. In-person instruction  
289 also often entails dynamic student-teacher and student-student interactions. These in-person in-  
290 teractions can provide the instructor with valuable information about students' understanding of  
291 the course material, beyond what they can glean solely from exams or assignments [17, 24, 62].  
292 In turn, this can allow the instructor to adapt their teaching approaches on-the-fly according to  
293 students' questions and behaviors. But what does great teaching look like in asynchronous online  
294 courses, when the instructor typically prepares course lectures and materials without knowing  
295 who will ultimately be learning from them? Can the empathetic side of teaching be automated

296 and scaled?

297 The notion of empathy also related to “theory of mind” of other individuals [21, 29, 42].  
298 Considering others’ unique perspectives, prior experiences, knowledge, goals, etc., can help us  
299 to more effectively interact and communicate [51, 55, 58]. The knowledge and learning maps  
300 we estimate in our study (Fig. 6) hint at one potential form that an automated “empathetic”  
301 teacher might take. We imagine automated content delivery systems that adapt lessons on the  
302 fly according to continually updated estimates of what students know and how quickly they are  
303 learning different conceptual content [e.g., building on ideas such as 3, 23, 36, 64, and others].

304 Over the past several years, the global pandemic has forced many educators to teach re-  
305 motely [32, 46, 56, 63]. This change in world circumstances is happening alongside (and perhaps  
306 accelerating) geometric growth in the availability of high quality online courses on platforms such  
307 as Khan Academy [33], Coursera [66], EdX [35], and others [52]. Continued expansion of the global  
308 internet backbone and improvements in computing hardware have also facilitated improvements  
309 in video streaming, enabling videos to be easily downloaded and shared by large segments of the  
310 world’s population. This exciting time for online course instruction provides an opportunity to  
311 re-evaluate how we, as a global community, educate ourselves and each other. For example, we  
312 can ask: what makes an effective course or training program? Which aspects of teaching might be  
313 optimized or automated? How and why do learning needs and goals vary across people? How  
314 might we lower barriers to achieving a high quality education?

315 Alongside these questions, there is a growing desire to extend existing theories beyond the  
316 domain of lab testing rooms and into real classrooms [31]. In part, this has led to a recent  
317 resurgence of “naturalistic” or “observational” experimental paradigms that attempt to better  
318 reflect more ethologically valid phenomena that are more directly relevant to real-world situations  
319 and behaviors [48]. In turn, this has brought new challenges in data analysis and interpretation. A  
320 key step towards solving these challenges will be to build explicit models of real-world scenarios  
321 and how people behave in them (e.g., models of how people learn conceptual content from real-  
322 world courses, as in our current study). A second key step will be to understand which sorts  
323 of signals derived from behaviors and/or other measurements [e.g., neurophysiological data; 5,

324 16, 43, 49, 50] might help to inform these models. A third major step will be to develop and  
325 employ reliable ways of evaluating the complex models and data that are a hallmark of naturalistic  
326 paradigms.

327 Ultimately, our work suggests a new line of questions regarding the future of education:  
328 which aspects of teaching can be optimized and/or automated? The social benefits of face-to-face  
329 instruction, such as social interactions, friendships, and emotional support, cannot (and perhaps  
330 should not) be fully replaced by an automated computer-based system. Nor can modern computer  
331 systems experience emotional empathy in the human sense of the word. On the other hand,  
332 perhaps it is possible to separate out the social aspects of classroom instruction from the purely  
333 learning-related aspects. Our study shows that text embedding models can uncover detailed  
334 insights into students' knowledge and how it changes over time during learning. We hope that  
335 these advances might help pave the way for new ways of teaching or delivering educational content  
336 that are tailored to individual students' learning needs and goals.

## 337 Materials and methods

### 338 Participants

339 We enrolled a total of 50 Dartmouth undergraduate students in our study. Participants received  
340 course credit for enrolling. We asked each participant to fill out a demographic survey that included  
341 questions about their age, gender, native spoken language, ethnicity, race, hearing, color vision,  
342 sleep, coffee consumption, level of alertness, and several aspects of their educational background  
343 and prior coursework.

344 Participants' ages ranged from 18 to 22 years (mean: 19.52 years; standard deviation: 1.09  
345 years). A total of 15 participants reported their gender as male and 35 participants reported their  
346 gender as female. A total of 49 participants reported their native language as "English" and 1  
347 reported having another native language. A total of 47 participants reported their ethnicity as  
348 "Not Hispanic or Latino" and three reported their ethnicity as "Hispanic or Latino." Participants

349 reported their races as White (32 participants), Asian (14 participants), Black or African American  
350 (5 participants), American Indian or Alaska Native (1 participant), and Native Hawaiian or Other  
351 Pacific Islander (1 participant). (Note that some participants selected multiple racial categories.)

352 A total of 49 participants reporting having normal hearing and 1 participant reported having  
353 some hearing impairment. A total of 49 participants reported having normal color vision and 1  
354 participant reported being color blind. Participants reported having had, on the night prior to  
355 testing, 2–4 hours of sleep (1 participant), 4–6 hours of sleep (9 participants), 6–8 hours of sleep (35  
356 participants), or 8+ hours of sleep (5 participants). They reported having consumed, on the same  
357 day and leading up to their testing session, 0 cups of coffee (38 participants), 1 cup of coffee (10  
358 participants), 3 cups of coffee (1 participant), or 4+ cups of coffee (1 participant).

359 No participants reported that their focus was currently impaired (e.g., by drugs or alcohol).  
360 Participants reported their current level of alertness, and we converted their responses to numerical  
361 scores as follows: “very sluggish” (-2), “a little sluggish” (-1), “neutral” (0), “fairly alert” (1), and  
362 “very alert” (2). Across all participants, a range of alertness levels were reported (range: -2 – 1;  
363 mean: -0.10; standard deviation: 0.84).

364 Participants reported their undergraduate major(s) as Social Sciences (28 participants), Natural  
365 sciences (16), Professional (e.g., pre-med or pre-law; 8 participants), Mathematics and engineering  
366 (7 participants), Humanities (4 participants), or Undecided (3 participants). Note that some par-  
367 ticipants selected multiple categories for their undergraduate major. We also asked participants  
368 about the courses they had taken. In total, 46 participants reported having taken at least one Khan  
369 academy course in the past or being familiar with the Khan academy, and 4 reported not having  
370 taken any Khan academy courses. Of the participants who reported having watched at least one  
371 Khan academy course, 1 participant declined to report the number of courses they had watched;  
372 7 participants reported having watched 1–2 courses; 11 reported having watched 3–5 courses; 8  
373 reported having watched 5–10 courses; and 19 reported having watched 10 or more courses. We  
374 also asked participants about the specific courses they had watched, categorized under different  
375 subject areas. In the “Mathematics” area participants reported having watched videos on AP  
376 Calculus AB (21 participants), Precalculus (17 participants), Algebra 2 (14 participants), AP Cal-

377 calculus BC (12 participants), Trigonometry (11 participants), Algebra 1 (10 participants), Geometry  
378 (8 participants), Pre-algebra (7 participants), Multivariable Calculus (5 participants), Differential  
379 Equations (5 participants), Statistics and Probability (4 participants), AP Statistics (2 participants),  
380 Linear Algebra (2 participants), Early Math (1 participant), Arithmetic (1 participant), and other  
381 videos not listed in our survey (6 participants). In the “Science and engineering” area participants  
382 reported having watched videos on Chemistry, AP Chemistry, or Organic Chemistry (21 partic-  
383 ipants); Physics, AP Physics I, or AP Physics II (15 participants); Biology, AP Biology; or High  
384 school Biology (15 participants); Health and Medicine (1 participant); or other videos not listed in  
385 our survey (20 participants). We also asked participants if they had specifically seen the videos  
386 used in our experiment. When we asked about the *Four Fundamental Forces* video, 45 participants  
387 reported not having watched it before, 1 participant reported that they were not sure if they had  
388 watched it before, and 4 participants declined to respond. When we asked about the *Birth of*  
389 *Stars* video, 46 participants reported not having watched it before and 4 participants declined to  
390 respond. When we asked participants about non-Khan academy online courses, they reported  
391 having watched or taken courses on Mathematics (15 participants), Science and engineering (11  
392 participants), Test preparation (9 participants), Economics and finance (3 participants), Arts and  
393 humanities (2 participants), Computing (2 participants), and other categories not listed in our  
394 survey (18 participants). Finally, we asked participants about in-person courses they had taken in  
395 different subject areas. They reported taking courses in Mathematics (39 participants), Science and  
396 engineering (38 participants), Arts and humanities (35 participants), Test preparation (27 partic-  
397ipants), Economics and finance (26 participants), Computing (15 participants), College and careers  
398 (7 participants), or other courses not listed in our survey (6 participants).

## 399 **Experiment**

400 We hand-selected two course videos from the Khan Academy platform: *Four Fundamental Forces*  
401 (an introduction to gravity, electromagnetism, the weak nuclear force, and the strong nuclear force;  
402 duration: 10 minutes and 29 seconds) and *Birth of Stars* (an introduction to how stars are formed;  
403 duration: 7 minutes and 57 seconds). We hand-wrote 39 multiple choice questions: 15 about the

404 conceptual content of *Four Fundamental Forces*, another 15 about the conceptual content of *Birth*  
405 *of Stars*, and 9 other questions that tested for general conceptual knowledge about basic physics  
406 (covering material that was not presented in either video). The full set of questions may be found  
407 in Table S1.

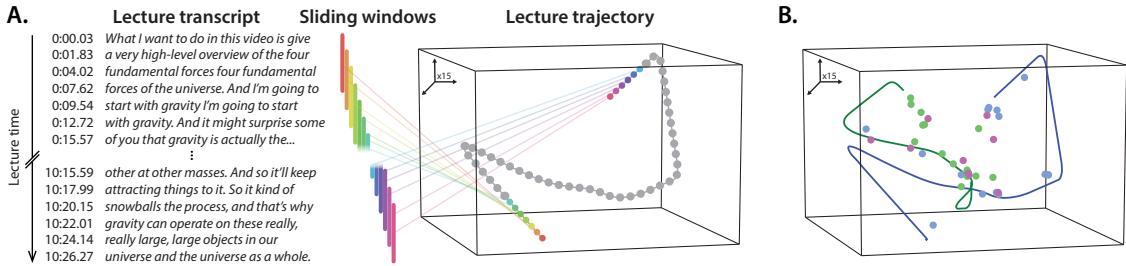
408 Participants began the main experiment by answering a battery of 13 randomly selected ques-  
409 tions (chosen from the full set of 39). Then they watched the *The Four Fundamental Forces* video.  
410 Next, they answered a second set of 13 questions (chosen at random from the remaining 26 ques-  
411 tions). Fourth, participants watch the *Birth of Stars* video, and finally they answered the remaining  
412 13 questions. Our experimental procedure is diagramed in Figure 1. We used the experiment to  
413 develop and test our computational framework for estimating knowledge and learning.

## 414 Analysis

### 415 Constructing text embeddings of multiple videos and questions

416 We extended an approach developed by [27] to construct text embeddings for each moment of  
417 each lecture, and of each question in our pool. Briefly, our approach uses a topic model [8], trained  
418 on a set of documents, to discover a set of  $k$  “topics” or “themes.” Formally, each topic is defined  
419 as a set of weights over each word in the model’s vocabulary (i.e., the union of all unique words,  
420 across all documents, excluding “stop words.”). Conceptually, each topic is intended to give larger  
421 weights to words that are conceptually related or that tend to co-occur in the same documents.  
422 After fitting a topic model, each document in the training set, or any *new* document that contains at  
423 least some of the words in the model’s vocabulary, may be represented as a  $k$ -dimensional vector  
424 describing how much the document (most probably) reflects each topic. (Unless, otherwise noted,  
425 we used  $k = 15$  topics.)

426 As illustrated in Figure 7A, we start by building up a corpus of documents using overlapping  
427 sliding windows that span each video’s transcript. Khan Academy videos are hosted on the  
428 YouTube platform, and all YouTube videos are run through Google’s speech-to-text API [26] to  
429 derive a timestamped transcript of any detected speech in the video. The resulting transcripts



**Figure 7: Constructing video content trajectories.** **A. Building a document pool from sliding windows of text.** We decompose each video’s transcript into a series of overlapping sliding windows. The set of transcript snippets (across all windows) may be treated as a set of “documents” for training a text embedding model. After training a text embedding model using the two videos’ sliding windows, along with the text from each question in our pool (Tab. S1), we construct “trajectories” through text embedding space by joining the embedding coordinates of successive sliding windows from each video. **B. Embedding multiple videos and questions.** Applying the same text embedding approach to each video, along with the text of each question, results in one trajectory per video and one embedding coordinate (dot) per question (blue: *Four Fundamental Forces*; green: *Birth of Stars*; pink: general physics knowledge). Here we have projected the 15-dimensional embeddings into a 3D space using Uniform Manifold Approximation and Projection [UMAP; 41].

430 contain one timestamped row per line, and each line generally corresponds to a few seconds of  
 431 spoken content from the video. We defined a sliding window length of (up to)  $w = 30$  transcript  
 432 lines, and we assigned each window a timestamp according to the midpoint between its first  
 433 and last lines’ timestamps. These sliding windows ramped up and down in length at the very  
 434 beginning and end of the transcript, respectively. In other words, the first sliding window covered  
 435 only the first line from the transcript; the second sliding window covered the first two lines; and  
 436 so on. This insured that each line of the transcript appeared in the same number ( $w$ ) of sliding  
 437 windows. We treated the text from each sliding window as a single “document,” and we combined  
 438 these documents across the two videos’ windows to create a single training corpus for the topic  
 439 model. The top words from each of the 15 discovered topics may be found in Table S2.

440 After fitting a topic model to each videos’ transcripts, we could use the trained model to  
 441 transform arbitrary (potentially new) documents into  $k$ -dimensional topic vectors. A convenient  
 442 property of these topic vectors is that documents that reflect similar blends of topics (i.e., documents  
 443 that reflect similar themes, according to the model) will yield similar (in terms of Euclidean distance,  
 444 correlation, etc.) topic vectors. In general, the similarity between different documents’ topic vectors

445 may be used to characterize the similarity in conceptual content between the documents.

446 We transformed each sliding window’s text into a topic vector, and then used linear interpolation  
447 (independently for each topic dimension) to resample the resulting timeseries to once per  
448 second. This yielded a single topic vector for each second of each video. We also used the fitted  
449 model to obtain topic vectors for each question in our pool (Tab. S1). Taken together, we obtained  
450 a *trajectory* for each video, describing its path through topic space, and a single coordinate for each  
451 question (Fig. 7B). Embedding both videos and all of the questions using a common model enables  
452 us to compare the content from different moments of videos, compare the content across videos,  
453 and estimate potential associations between specific questions and specific moments of video.

454 **Estimating dynamic knowledge traces**

455 We used the following equation to estimate each participant’s knowledge about timepoint  $t$  of a  
456 given lecture,  $\hat{k}(t)$ :

$$\hat{k}(f(t, L)) = \frac{\sum_{i \in \text{correct}} \text{ncorr}(f(t, L), f(i, Q))}{\sum_{j=1}^N \text{ncorr}(f(t, L), f(j, Q))}, \quad (1)$$

457 where

$$\text{ncorr}(x, y) = \frac{\text{corr}(x, y) - \text{mincorr}}{\text{maxcorr} - \text{mincorr}}, \quad (2)$$

458 and where mincorr and maxcorr are the minimum and maximum correlations between any lecture  
459 timepoint and question, taken over all timepoints and questions across both lectures and all three  
460 question sets. We also define  $f(s, \Omega)$  as the  $s^{\text{th}}$  topic vector from the set of topic vectors  $\Omega$ . Here  
461  $t$  indexes the set of lecture topic vectors,  $L$ , and  $i$  and  $j$  index the topic vectors of questions in the  
462 quiz’s question set,  $Q$ . Note that “correct” denotes the set of indices of the questions the participant  
463 answered correctly on the given quiz.

464 Intuitively,  $\text{ncorr}(x, y)$  is the correlation between two topic vectors (e.g., the topic vector from one  
465 timepoint in a lecture,  $x$ , and the topic vector for one question,  $y$ ), normalized by the minimum and  
466 maximum correlations (across all timepoints and questions) to range between 0 and 1, inclusive.  
467 Equation 1 then computes the weighted average proportion of correctly answered questions about

468 the content presented at timepoint  $t$ , where the weights are given by the normalized correlations  
469 between timepoint  $t$ 's topic vector and the topic vectors for each question. The normalization  
470 step (i.e., using ncorr instead of the raw correlations) insures that every question (except the  
471 least-relevant question) contributes some non-zero amount to the knowledge estimate.

472 **Creating knowledge and learning map visualizations**

473 An important feature of our approach is that, given a trained text embedding model and partic-  
474 ipants' quiz performance on each question, we can estimate their knowledge about *any* content  
475 expressible by the embedding model— not solely the content explicitly probed by the quiz ques-  
476 tions. To visualize these estimates (Figs. 6, S1, S2, S3, S4, and S5), we used UMAP [41] to define a  
477 2D projection of the text embedding space. Sampling the original 100-dimensional space at high  
478 resolution to obtain an adequate set of topic vectors spanning the embedding space would be  
479 computationally intractable. However, sampling a 2D grid is much more feasible. We defined a  
480 rectangle enclosing the 2D projections of the lectures' and quizzes' embeddings, and we sampled  
481 points from a regular  $100 \times 100$  grid of coordinates that evenly tiled the enclosing rectangle. We  
482 sought to estimate participants' knowledge (and learning—i.e., changes in knowledge) at each of  
483 the resulting 10000 coordinates.

484 To generate our estimates, we placed a set of 39 radial basis functions (RBFs) throughout the  
485 embedding space, centered on the 2D projections for each question (i.e., we included one RBF for  
486 each question). At coordinate  $x$ , the value of an RBF centered on a question's coordinate  $\mu$ , is given  
487 by:

$$\text{RBF}(x, \mu, \lambda) = \exp\left\{-\frac{\|x - \mu\|^2}{\lambda}\right\}. \quad (3)$$

488 The  $\lambda$  term in the RBF equation controls the “smoothness” of the function, where larger values  
489 of  $\lambda$  result in smoother maps. In our implementation we used  $\lambda = 50$ . Next, we estimated the  
490 “knowledge” at each coordinate,  $x$ , using:

$$\hat{k}(x) = \frac{\sum_{i \in \text{correct}} \text{RBF}(x, q_i, \lambda)}{\sum_{j=1}^N \text{RBF}(x, q_j, \lambda)}. \quad (4)$$

491 Intuitively, Equation 4 computes the weighted proportion of correctly answered questions, where  
492 the weights are given by how nearby (in the 2D space) each question is to the  $x$ . We also defined  
493 *learning maps* as the coordinate-by-coordinate differences between any pair of knowledge maps.  
494 Intuitively, learning maps reflect the *change* in knowledge across two maps.

## 495 **Author contributions**

496 Conceptualization: PCF, ACH, and JRM. Methodology: PCF, ACH, and JRM. Software: PCF.  
497 Validation: PCF. Formal analysis: PCF. Resources: PCF, ACH, and JRM. Data curation: PCF.  
498 Writing (original draft): JRM. Writing (review and editing): PCF, ACH, and JRM. Visualization:  
499 PCH and JRM. Supervision: JRM. Project administration: PCH. Funding acquisition: JRM.

## 500 **Data and code availability**

501 All of the data analyzed in this manuscript, along with all of the code for running our experiment  
502 and carrying out the analyses may be found at <https://github.com/ContextLab/efficient-learning-khan>.  
503

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