Geometric models reveal the hidden structure of conceptual knowledge

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Abstract

We develop a mathematical framework, based on natural language processing models, for tracking and characterizing the acquisition of conceptual knowledge. Our approach embeds each concept in a high dimensional representation space, where nearby coordinates reflect similar or related concepts. We tested our approach using behavioral data collected from a group of college students. In the experiment, we asked the participants to answer sets of quiz questions interleaved between watching two course videos from the Khan Academy platform. We applied our framework to the videos' transcripts, and to text of the quiz questions, to quantify the content of each moment of video and each quiz question. We used these embeddings, along with participants' quiz responses, to track how the learners' knowledge changed after watching each video. Our findings show how a limited set of quiz questions may be used to construct rich and meaningful representations of what each learner knows, and how their knowledge changes over time as they learn.

Keywords: education, learning, knowledge, concepts, natural language processing

Introduction

How do we acquire conceptual knowledge? Memorizing course lectures or textbook chapters by rote can lead to the superficial *appearance* of understanding the underlying content, but achieving true conceptual understanding seems to require something deeper and richer. Does conceptual understanding entail connecting newly acquired information to the scaffolding of one's existing knowledge or experience [1, 5, 7, 8, 19]? Or weaving a lecture's atomic elements (e.g., its component words) into a structured network that describes how those individual elements are related? Conceptual understanding could also involve building a mental model that transcends the meanings of those individual atomic elements by reflecting the deeper meaning underlying the gestalt whole [12, 14, 18].

The difference between "understanding" and "memorizing," as framed by the researchers in education, cognitive psychology, and cognitive neuroscience [9–11, 14, 18] has profound analogs in the fields of natural language processing and natural language understanding. For example, considering the raw contents of a document (e.g., its constituent symbols, letters, and words) might provide some information about what the document is about, just as memorizing a passage might be used to answer simple questions about the passage [e.g., whether it might contain words related to furniture versus physics; 2, 3, 13]. However, modern natural language processing models [e.g., 4, 6, 17] also attempt to capture the deeper meaning *underlying* those atomic elements. These models consider not only the co-occurrences of those elements within and across documents, but also patterns in how those elements appear across different scales (e.g., sentences, paragraphs, chapters, etc.), the temporal and grammatical properties of the elements, and other high-level characteristics of how they are used [15, 16]. According to these models, the deep conceptual meaning of a document may be captured by a feature vector in a high-dimensional representation space, where nearby vectors reflect conceptually related documents. A model that succeeds at capturing an analog of "understanding" is able to assign nearby feature vectors to two conceptually related documents, *even when the words contained in those documents have very little overlap*.

What form might the representation of the sum total of a person's knowledge take? First, we might require a means of systematically describing or representing the nearly infinite set of possible things a person could know. Second, we might want to account for potential associations between different concepts. For example, the concepts of “fish” and “water” might be associated in the sense that fish live in water. Third, knowledge may have a critical dependency structure, such that knowing about a particular concept might require first knowing about a set of other concepts. For example, understanding the concept of a fish swimming in water first requires understanding what fish and water *are*. Fourth, as we learn, our “current state of knowledge” should change accordingly. Learning new concepts should both update our characterizations of “what is known” and should also unlock any now-satisfied dependencies of that newly learned concept so that they are “tagged” as available for future learning.

Here we develop a framework for modelling how knowledge is acquired during learning. The central idea is to use text embedding models to define the coordinate systems of two maps: (a) a *knowledge map* that describes the extent to which each concept is currently known and (b) a *learning map* that describes the extent to which each concept could be learned. Each location on these maps represents a single concept, and the geometries are defined such that related concepts are located nearby in space. We use this framework to analyzing and interpreting behavioral data collected from an experiment that has participants watch and answer conceptual questions about a series of recorded course lectures.

Our primary research goal is to advance our understanding of what it means to acquire deep real-world conceptual knowledge. Traditional laboratory approaches to studying learning and memory (e.g., list learning studies) often draw little distinction between memorization and understanding. Instead, these studies typically focus on whether information is effectively encoded or retrieved, rather than whether the information is *understood*. Approaches to studying conceptual learning, such as category learning experiments, can start to investigate the distinction between memorization and understanding, often by training participants to distinguish arbitrary or random features in otherwise meaningless categorized stimuli. However the objective of real-world training, or learning from life experiences more generally, is often to develop new knowledge that may be applied in *useful* ways in the future. In this sense, the gap between modern learning theories and modern pedagogical approaches and classroom learning strategies is enormous: most of our theories about *how* people learn are inspired by experimental paradigms and models that have only peripheral relevance to the kinds of learning that students and teachers actually seek. To help bridge this gap, our study uses course materials from real online courses to inform, fit, and test models of real-world conceptual learning.

Results

Discussion

Materials and methods

Participants

We enrolled a total of 50 Dartmouth undergraduate students in our study. Participants received course credit for enrolling. We asked each participant to fill out a demographic survey that included questions about their age, gender, native spoken language, ethnicity, race, hearing, color vision, sleep, coffee consumption, level of alertness, and several aspects of their educational background and prior coursework.

Participants' ages ranged from 18 to 22 years (mean: 19.52 years; standard deviation: 1.09 years). A total of 15 participants reported their gender as male and 35 participants reported their gender as female. A total of 49 participants reported their native language as "English" and 1 reported having another native language. A total of 47 participants reported their ethnicity as "Not Hispanic or Latino" and three reported their ethnicity as "Hispanic or Latino." Participants reported their races as White (32 participants), Asian (14 participants), Black or African American (5 participants), American Indian or Alaska Native (1 participant), and Native Hawaiian or Other Pacific Islander (1 participant). (Note that some participants selected multiple recial categories.)

A total of 49 participants reporting having normal hearing and 1 participant reported having some hearing impairment. A total of 49 participants reported having normal color vision and 1 participant reported being color blind. Participants reported having had, on the night prior to testing, 2 – 4 hours of sleep (1 participant), 4 – 6 hours of sleep (9 participants), 6 – 8 hours of sleep (35 participants), or 8+ hours of sleep (5 participants). They reported having consumed, on the same day and leading up to their testing session, 0 cups of coffee (38 participants), 1 cup of coffee (10 participants), 3 cups of coffee (1 participant), or 4+ cups of coffee (1 participant).

No participants reported that their focus was currently impaired (e.g., by drugs or alcohol). Participants reported their current level of alertness, and we converted their responses to numerical scores as follows: "very sluggish" (-2), "a little sluggish" (-1), "neutral" (0), "fairly alert" (1), and "very alert" (2). Across all participants, a range of alertness levels were reported (range: -2 – 1; mean: -0.10; standard deviation: 0.84).

Particpants reported their undergraduate major(s) as Social Sciences (28 participants), Natural sciences (16), Professional (e.g., pre-med or pre-law; 8 participants), Mathematics and engineering (7 participants), Humanities (4 participants), or Undecided (3 participants). Note that some participants selected multiple categories for their undergraduate major. We also asked participants about the courses they had taken. In total, 46 participants reported having taken at least one Khan academy course in the past or being familiar with the Khan academy, and 4 reported not having taken any Khan academy courses. Of the participants who reported having watched at least one Khan academy course, 1 participant declined to report the number of courses they had watched; 7 participants reported having watched 1–2 courses; 11 reported having watched 3–5 courses; 8 reported having watched 5–10 courses; and 19 reported having watched 10 or more courses. We also asked participants about the specific courses they had watched, categorized under different subject areas. In the "Mathematics" area participants reported having watched videos on AP Calculus AB (21 participants), Precalculus (17 participants), Algebra 2 (14 participants), AP Calculus BC (12 participants), Trigonometry (11 participants), Algebra 1 (10 participants), Geometry (8 participants), Pre-algebra (7 participants), Multivariable Calculus (5 participants), Differential Equations (5 participants, Statistics and Probability (4 participants), AP Statistics (2 participants), Linear Algebra (2 participants), Early Math (1 participant), Arithmetic (1 participant), and other videos not listed in our survey (6 participants). In the "Science and engineering" area participants reported having watched videos on Chemistry, AP Chemistry, or Organic Chemstry (21 participants); Physics, AP Physics I, or AP Physics II (15 participants); Biology, AP Biology; or High school Biology (15 participants); Health and Medicine (1 participant); or other videos not listed in our survey (20 participants). We also asked participants if they had specifically seen the videos used in our experiment. When we asked about the *Four Fundamental Forces* video, 45 participants reported not having watched it before, 1 participant reported that they were not sure if they had watched it before, and 4 participants declined to respond. When we asked about the *Birth of Stars* video, 46 participants reported not having watched it before and 4 participants declined to respond. When we asked participants about non-Khan academy online courses, they reported having watched or taken courses on Mathematics (15 participants), Science and engineering (11 participants), Test preparation (9 participants), Economics and finance (3 participants), Arts and humanities (2 participants), Computing (2 participants), and other categories not listed in our survey (18 participants). Finally, we asked participants about in-person courses they had taken in different subject areas. They reported taking courses in Mathematics (39 participants), Science and engineering (38 participants), Arts and humanities (35 participants), Test preparation (27 participants), Economics and finance (26 participants), Computing (15 participants), College and careers (7 participants), or other courses not listed in our survey (6 participants).

Experiment

We hand-selected two roughly 10-minute course videos from the Khan Academy platform: *The Four Fundamental Forces* (an introduction to gravity, electromagnetism, the weak nuclear force, and the strong nuclear force; duration: 10 minutes and 29 seconds) and *Birth of Stars* (an introduction to how stars are formed; duration: 7 minutes and 57 seconds). We hand-wrote 39 multiple choice questions: 15 about the conceptual content of *The Four Fundamental Forces*, another 15 about the conceptual content of *Birth of Stars*, and 9 other questions that tested for general conceptual knowledge about basic physics (covering material that was not presented in either video). The full set of questions may be found in Table S1.

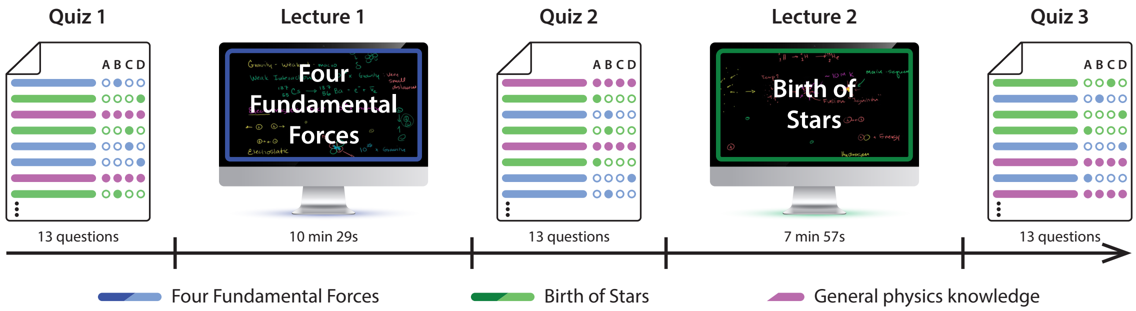

Figure 1: Experimental paradigm. Participants alternate between answering 13-question multiple choice quizzes and watching two Khan academy videos. Each quiz contains a mix of 5 questions about lecture 1, 5 questions about lecture 2, and 3 general physics knowledge questions. The specific questions reflected on each quiz, and the orders of each quiz's questions, were randomized across participants.

Participants began the main experiment by answering a battery of 13 randomly selected questions (chosen from the full set of 39). Then they watched the *The Four Fundamental Forces* video. Next, they answered a second set of 13 questions (chosen at random from the remaining 26 questions). Fourth, participants watch the *Birth of Stars* video, and finally they answered the remaining 13 questions. Our experimental procedure is diagramed in Figure 1. We used the experiment to develop and test our computational framework for estimating knowledge and learning maps.

Analysis

Constructing text embeddings of multiple videos and questions

Estimating dynamic knowledge traces

Estimating held-out conceptual knowledge

Creating knowledge and learning map visualizations

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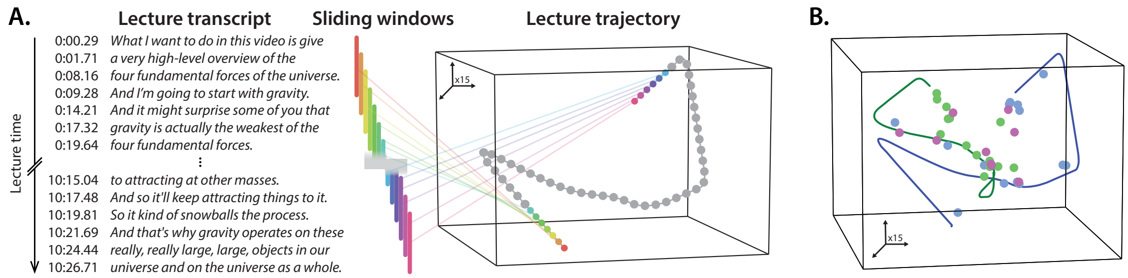

Figure 2: Constructing video content *trajectories*. A. Building a document pool from sliding windows of text. We decompose each video's transcript into a series of overlapping sliding windows. The set of transcript snippets (across all windows) may be treated as a set of "documents" for training a text embedding model. After training a text embedding model using the two videos' sliding windows, along with the text from each question in our pool (Tab. S1), we construct "trajectories" through text embedding space by joining the embedding coordinates of successive sliding windows from each video. **B. Embedding multiple videos and questions.** Applying the same text embedding approach to each video, along with the text of each question, results in one trajectory per video and one embedding coordinate (dot) per question (blue: Four Fundamental Forces; green: Birth of Stars; pink: general physics knowledge).

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