Table ronde weak lensing calibration

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1 Abstract

$$R_{\alpha\beta} = \frac{\partial e_\alpha^{obs}}{\partial g_\beta} \approx \frac{e_\alpha^{obs,+} - e_\alpha^{obs,-}}{2\Delta g_\beta}$$

Taylor expansion of the exact expression from theory

$$e = \frac{e^I + g}{1 + g^* e^I} = e^I + g - g^*(e^I)^2 + O(g^2), \quad \text{if } |g^* e^I| < 1$$

$$\vec{e} = \vec{e}^I + \vec{g}(1 + |\vec{e}^I|^2) - 2(\vec{e}^I \cdot \vec{g})\vec{e}^I + O(g^2)$$

$$\vec{e} = \vec{e}^I + \begin{pmatrix} 1 - (e_1^I)^2 + (e_2^I)^2 & -2e_1^I e_2^I \\ -2e_1^I e_2^I & 1 + (e_1^I)^2 - (e_2^I)^2 \end{pmatrix} \vec{g} = \vec{e}^I + A(\vec{e}^I)\vec{g}$$

observed ellipticity of one galaxy with properties $\vec{P}$ (Eq.10 in Pujol+2018)

$$\vec{e}^{obs} = R(\vec{P})\vec{g} + \vec{a}(\vec{P}) + f(\vec{e}^I)$$

idem with the next order term

$$\vec{e}^{obs} = R(\vec{P})A(\vec{e}^I)\vec{g} + \vec{a}(\vec{P}) + f(\vec{e}^I)$$

$$\frac{\partial e_\alpha^{obs}}{\partial g_\beta} = \left[R(\vec{P})A(\vec{e}^I)\right]_{\alpha\beta} = \widetilde{R}(\vec{P}, \vec{e}^I)_{\alpha\beta}$$

2 Proof of the mean property

Proof with exact expression

$$\begin{aligned}
\langle e^{obs} \rangle &= \int_{D(\vec{0},1)} e^{obs}(e^I, g) p(e^I) de^I \\
&= \int_0^1 \left(\int_0^{2\pi} e^{obs}(ye^{2i\phi}, g) d\phi \right) p(y) y dy && e^I = ye^{2i\phi},\ p(e^I) \text{ must be polar symmetric} \\
&= \int_0^1 \left(\int_0^{2\pi} \frac{ye^{2i\phi} + g}{1 + g^* ye^{2i\phi}} d\phi \right) p(y) y dy \\
&= \int_0^1 \left(-i \oint \frac{yu + g}{u(1 + g^* yu)} du \right) p(y) y dy && u = e^{2i\phi},\ \frac{du}{d\phi} = 2iu,\ \frac{d\phi}{du} = \frac{-i}{2u} \text{ but as } \phi \in [0, 2\pi) \text{, there are two circles} \\
&= 2\pi g \int_0^1 p(y) y dy && \text{by residue theorem} \\
&= g && \text{imposed by normalization of } p
\end{aligned}$$

Proof with Taylor expansion

$$\begin{aligned}
\langle e^{obs} \rangle &= \int_0^1 \left(-i \oint \frac{yu + g}{u(1 + g^* yu)} du \right) p(y) y dy \\
&= \int_0^1 \left(-i \oint y + \frac{g}{u} - g^* y^2 u + \sum_{k=2}^{\infty} f_k u^k du \right) p(y) y dy && \text{with } f_k \text{ some factors} \\
&= 2\pi g \int_0^1 p(y) y dy && \text{as } \oint z^m dz = \begin{cases} 2\pi i & m = -1 \\ 0 & \text{otherwise} \end{cases} \\
&= g
\end{aligned}$$