

Table ronde weak lensing calibration

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August 27, 2019

1 Abstract

Observed ellipticity of one individual galaxy with properties \vec{P} (Eq.10 in Pujol+2018)

$$\vec{e}^{obs} = R(\vec{P})\vec{g} + \vec{a}(\vec{P}) + f(\vec{e}^I)$$

Response (Eq.2 in Pujol+2018)

$$R_{\alpha\beta} = \frac{\partial e_{\alpha}^{obs}}{\partial g_{\beta}} \approx \frac{e_{\alpha}^{obs,+} - e_{\alpha}^{obs,-}}{2\Delta g_{\beta}}$$

Taylor expansion of the exact expression from theory (for $|g| \leq 1$)

$$e = \frac{e^I + g}{1 + g^* e^I} = e^I + g - g^*(e^I)^2 + O(g^2), \quad \text{if } |g^* e^I| < 1 \text{ as } \frac{1}{1+x} = \sum_{k=0}^{\infty} (-1)^k x^k \text{ for } |x| < 1$$

$$\vec{e} = \vec{e}^I + \vec{g}(1 + |\vec{e}^I|^2) - 2(\vec{e}^I \cdot \vec{g})\vec{e}^I + O(g^2)$$

$$\vec{e} = \vec{e}^I + \begin{pmatrix} 1 - (e_1^I)^2 + (e_2^I)^2 & -2e_1^I e_2^I \\ -2e_1^I e_2^I & 1 + (e_1^I)^2 - (e_2^I)^2 \end{pmatrix} \vec{g} = \vec{e}^I + A(\vec{e}^I)\vec{g}$$

Observed ellipticity of one individual galaxy with the next order term

$$\vec{e}^{obs} = R(\vec{P})A(\vec{e}^I)\vec{g} + \vec{a}(\vec{P}) + f(\vec{e}^I)$$

$$\frac{\partial e_{\alpha}^{obs}}{\partial g_{\beta}} = \left[R(\vec{P})A(\vec{e}^I) \right]_{\alpha\beta} = \tilde{R}(\vec{P}, \vec{e}^I)_{\alpha\beta}$$

2 Update 2019

2.1 Predictions

Taylor expansion around $g = 0$

$$e_{\alpha}^{true} = e_{\alpha}^I + A_{\alpha\beta}g_{\beta} + B_{\alpha\beta\gamma}g_{\beta}g_{\gamma} + O(g^3)$$

The mean property $\langle e^{true} \rangle = g$ holds whatever the degree of the expansion (see proof below).

$$\begin{aligned} A_{11} &= 1 - (e_1^I)^2 + (e_2^I)^2 \\ A_{12} = A_{21} &= -2e_1^I e_2^I \\ A_{22} &= 1 - (e_1^I)^2 + (e_2^I)^2 \\ B_{111} &= (e_1^I)^3 - 3e_1^I (e_2^I)^2 - e_1^I \\ B_{112} = B_{121} &= 3(e_1^I)^2 e_2^I - (e_2^I)^3 \\ B_{122} &= -(e_1^I)^3 + 3e_1^I (e_2^I)^2 - e_1^I \\ B_{211} &= -(e_2^I)^3 + 3(e_1^I)^2 e_2^I - e_2^I \\ B_{212} = B_{221} &= 3e_1^I (e_2^I)^2 - (e_1^I)^3 \\ B_{222} &= (e_2^I)^3 - 3(e_1^I)^2 e_2^I - e_2^I \end{aligned}$$

2.2 Observations

Observed ellipticity defined as the true ellipticity with a multiplicative response and an additive bias, both functions of the galaxy physical properties \vec{P}

$$\begin{aligned} e_{\alpha}^{obs} &= R_{\alpha\beta} e_{\beta}^{true} + a_{\alpha} + S \\ &= R_{\alpha\beta} e_{\beta}^I + R_{\alpha\beta} A_{\beta\gamma} g_{\gamma} + R_{\alpha\beta} B_{\beta\gamma\delta} g_{\gamma} g_{\delta} + a_{\alpha} + S \\ &= \tilde{R}_{\alpha\beta} g_{\beta} + \frac{1}{2} \tilde{Q}_{\alpha\beta\gamma} g_{\beta} g_{\gamma} + a_{\alpha} + f(e^I) \end{aligned}$$

with $\tilde{R}_{\alpha\beta} = R_{\alpha i} A_{i\beta}$ (4 independent values) and $\tilde{Q}_{\alpha\beta\gamma} = 2R_{\alpha i} B_{i\beta\gamma}$ (6 independent values)

First derivatives (4 equations):

$$\frac{\partial e_{\alpha}^{\text{obs}}}{\partial g_{\beta}} = \tilde{R}_{\alpha\beta} + \frac{1}{2} \left(\tilde{Q}_{\alpha\beta\gamma} g_{\gamma} + \tilde{Q}_{\alpha\gamma\beta} g_{\beta} \right)$$

Second derivatives (6 equations):

$$\frac{\partial^2 e_{\alpha}^{\text{obs}}}{\partial g_{\beta} \partial g_{\gamma}} = \frac{1}{2} \left(\tilde{Q}_{\alpha\beta\gamma} + \tilde{Q}_{\alpha\gamma\beta} \right)$$

Finite differences for first derivatives, with 2 points in the (g_1, g_2) plane:

$$\frac{\partial e_{\alpha}^{\text{obs}}}{\partial g_{\beta}} \approx \frac{e_{\alpha}^{\text{obs}}(+\Delta g_{\beta}) - e_{\alpha}^{\text{obs}}(-\Delta g_{\beta})}{2\Delta g_{\beta}}$$

Finite differences for second derivatives, with 5 points in the (g_1, g_2) plane:

$$\frac{\partial^2 e_{\alpha}^{\text{obs}}}{\partial g_{\beta} \partial g_{\gamma}} \approx \frac{e_{\alpha}^{\text{obs}}(+\Delta g_{\beta}) + e_{\alpha}^{\text{obs}}(-\Delta g_{\beta}) + e_{\alpha}^{\text{obs}}(+\Delta g_{\gamma}) + e_{\alpha}^{\text{obs}}(-\Delta g_{\gamma}) - 4e_{\alpha}^{\text{obs}}(0)}{4\Delta g_{\beta} \Delta g_{\gamma}}$$

3 Proof of the mean property

This is the proof of that the mean of the ellipticity estimator e^{true} is zero. The mean is taken over intrinsic ellipticities e^I with probability density $p(e^I)$.

Proof with exact expression

$$\begin{aligned} \langle e^{\text{true}} \rangle &= \int_{D(\vec{0}, 1)} e^{\text{true}}(e^I, g) p(e^I) de^I && D(\vec{0}, 1) \text{ is the disk of center } \vec{0} \text{ and radius 1 (hereafter } C(0, 1) \text{ the circle)} \\ &= \int_0^1 \left(\int_0^{2\pi} e^{\text{true}}(ye^{2i\phi}, g) d\phi \right) p(y) y dy && e^I = ye^{2i\phi}, p(e^I) \text{ must be polar symmetric} \\ &= \int_0^1 \left(\int_0^{2\pi} \frac{ye^{2i\phi} + g}{1 + g^* ye^{2i\phi}} d\phi \right) p(y) y dy \\ &= \int_0^1 \left(-i \oint_{C(0,1)} \frac{yu + g}{u(1 + g^* yu)} du \right) p(y) y dy && u = e^{2i\phi}, \frac{du}{d\phi} = 2iu, \frac{d\phi}{du} = \frac{-i}{2u} \text{ but as } \phi \in [0, 2\pi), \text{ there are two circles} \\ &= 2\pi g \int_0^1 p(y) y dy && \text{by residue theorem, with poles } u = 0 \text{ (inside) and } u = -1/(g^* y) \text{ (outside)} \\ &= g && \text{imposed by normalization of } p \end{aligned}$$

Proof with Taylor expansion

$$\begin{aligned} \langle e^{\text{true}} \rangle &= \int_0^1 \left(-i \oint_{C(0,1)} \frac{yu + g}{u(1 + g^* yu)} du \right) p(y) y dy \\ &= \int_0^1 \left(-i \oint_{C(0,1)} y + \frac{g}{u} - g^* y^2 u + \sum_{k=2}^{\infty} f_k u^k du \right) p(y) y dy && \text{with } f_k \text{ some factors} \\ &= 2\pi g \int_0^1 p(y) y dy && \text{as } \oint_{C(0,1)} z^m dz = \begin{cases} 2\pi i & m = -1 \\ 0 & \text{otherwise} \end{cases} \\ &= g \end{aligned}$$