

Table ronde weak lensing calibration

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1 Abstract

Observed ellipticity of one individual galaxy with properties \vec{P} (Eq.10 in Pujol+2018)

$$\vec{e}^{obs} = R(\vec{P})\vec{g} + \vec{a}(\vec{P}) + f(\vec{e}^I)$$

Response (Eq.2 in Pujol+2018)

$$R_{\alpha\beta} = \frac{\partial e_{\alpha}^{obs}}{\partial g_{\beta}} \approx \frac{e_{\alpha}^{obs,+} - e_{\alpha}^{obs,-}}{2\Delta g_{\beta}}$$

Taylor expansion of the exact expression from theory (for $|g| \leq 1$)

$$e = \frac{e^I + g}{1 + g^* e^I} = e^I + g - g^*(e^I)^2 + O(g^2), \quad \text{if } |g^* e^I| < 1 \text{ as } \frac{1}{1+x} = \sum_{k=0}^{\infty} (-1)^k x^k \text{ for } |x| < 1$$

$$\vec{e} = \vec{e}^I + \vec{g}(1 + |\vec{e}^I|^2) - 2(\vec{e}^I \cdot \vec{g})\vec{e}^I + O(g^2)$$

$$\vec{e} = \vec{e}^I + \begin{pmatrix} 1 - (e_1^I)^2 + (e_2^I)^2 & -2e_1^I e_2^I \\ -2e_1^I e_2^I & 1 + (e_1^I)^2 - (e_2^I)^2 \end{pmatrix} \vec{g} = \vec{e}^I + A(\vec{e}^I)\vec{g}$$

Observed ellipticity of one individual galaxy with the next order term

$$\vec{e}^{obs} = R(\vec{P})A(\vec{e}^I)\vec{g} + \vec{a}(\vec{P}) + f(\vec{e}^I)$$

$$\frac{\partial e_{\alpha}^{obs}}{\partial g_{\beta}} = \left[R(\vec{P})A(\vec{e}^I) \right]_{\alpha\beta} = \tilde{R}(\vec{P}, \vec{e}^I)_{\alpha\beta}$$

2 Proof of the mean property

This is the proof of that the mean of the ellipticity estimator e^{obs} is zero. The mean is taken over intrinsic ellipticities e^I with probability density $p(e^I)$.

Proof with exact expression

$$\begin{aligned} \langle e^{obs} \rangle &= \int_{D(\vec{0},1)} e^{obs}(e^I, g) p(e^I) de^I && D(\vec{0},1) \text{ is the disk of center } \vec{0} \text{ and radius } 1 \text{ (hereafter } C(0,1) \text{ the circle)} \\ &= \int_0^1 \left(\int_0^{2\pi} e^{obs}(ye^{2i\phi}, g) d\phi \right) p(y) y dy && e^I = ye^{2i\phi}, p(e^I) \text{ must be polar symmetric} \\ &= \int_0^1 \left(\int_0^{2\pi} \frac{ye^{2i\phi} + g}{1 + g^* ye^{2i\phi}} d\phi \right) p(y) y dy \\ &= \int_0^1 \left(-i \oint_{C(0,1)} \frac{yu + g}{u(1 + g^* yu)} du \right) p(y) y dy && u = e^{2i\phi}, \frac{du}{d\phi} = 2iu, \frac{d\phi}{du} = \frac{-i}{2u} \text{ but as } \phi \in [0, 2\pi), \text{ there are two circles} \\ &= 2\pi g \int_0^1 p(y) y dy && \text{by residue theorem, with poles } u = 0 \text{ (inside) and } u = -1/(g^* y) \text{ (outside)} \\ &= g && \text{imposed by normalization of } p \end{aligned}$$

Proof with Taylor expansion

$$\begin{aligned} \langle e^{obs} \rangle &= \int_0^1 \left(-i \oint_{C(0,1)} \frac{yu + g}{u(1 + g^* yu)} du \right) p(y) y dy \\ &= \int_0^1 \left(-i \oint_{C(0,1)} y + \frac{g}{u} - g^* y^2 u + \sum_{k=2}^{\infty} f_k u^k du \right) p(y) y dy && \text{with } f_k \text{ some factors} \\ &= 2\pi g \int_0^1 p(y) y dy && \text{as } \oint_{C(0,1)} z^m dz = \begin{cases} 2\pi i & m = -1 \\ 0 & \text{otherwise} \end{cases} \\ &= g \end{aligned}$$