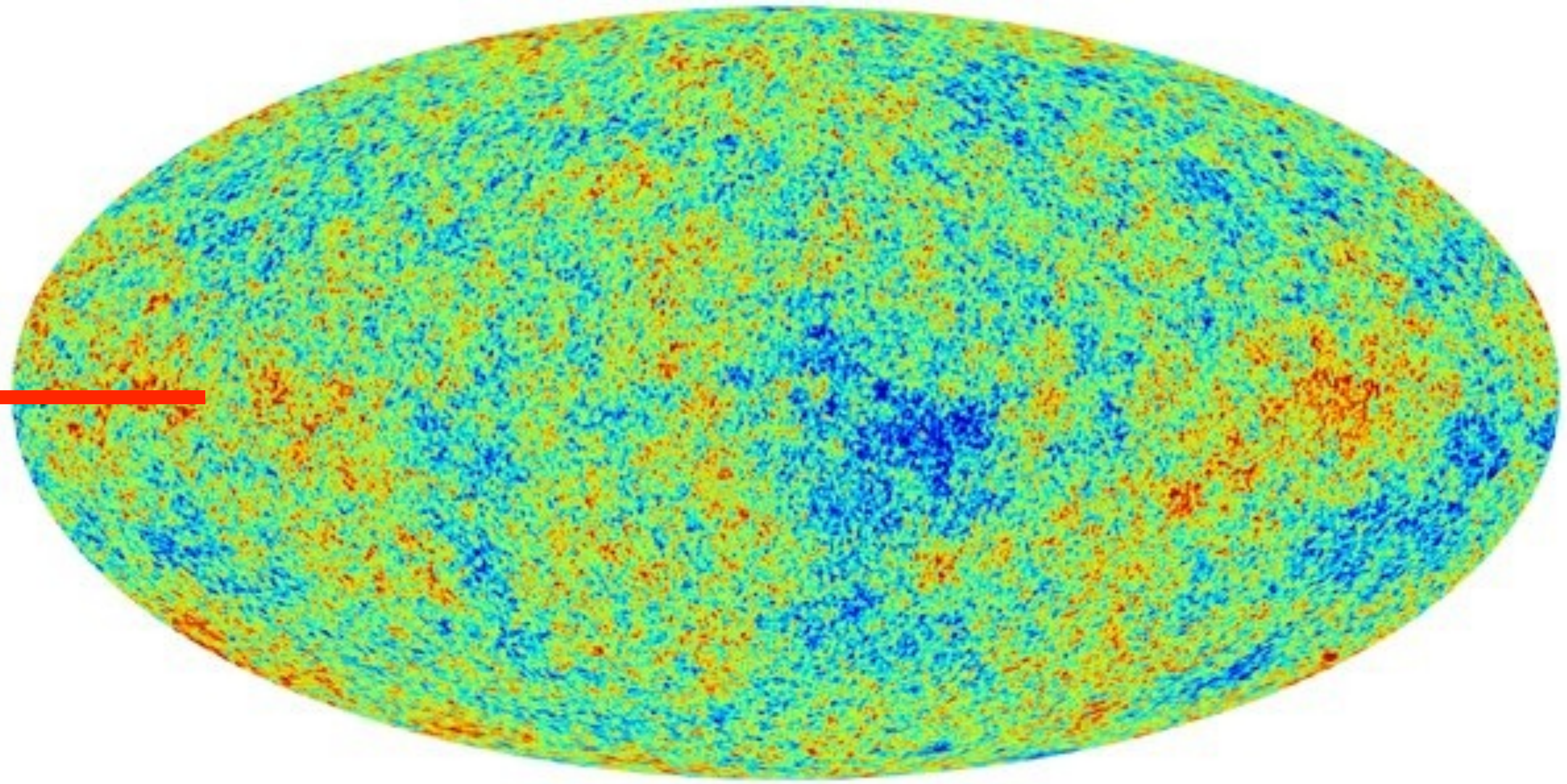
2PCF

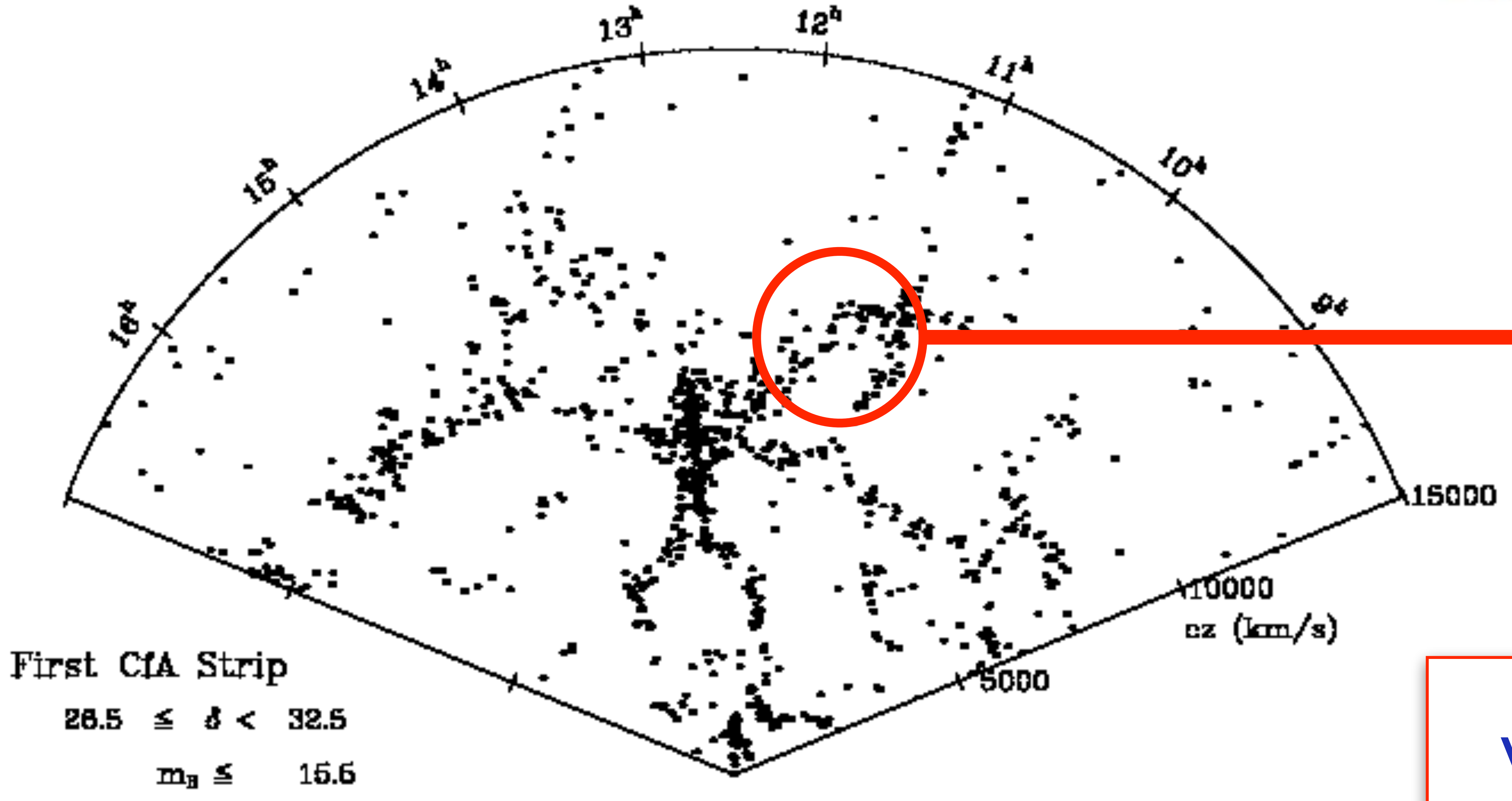
Slides stolen from Emiliano Sefussati

Cosmological perturbations



CMB temperature fluctuations

$$T(\hat{n})$$



$$n_g(\vec{x})$$

number density of galaxies

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We can only study the **statistical properties** of cosmological perturbations

Mathematically, these are **random fields**

Random fields

If ϕ is a ***random variable*** with Probability Distribution Function (PDF) $\mathcal{P}(\phi)$ we can compute:

$$\langle\phi\rangle = \int d\phi\, \mathcal{P}(\phi)\, \phi$$ mean

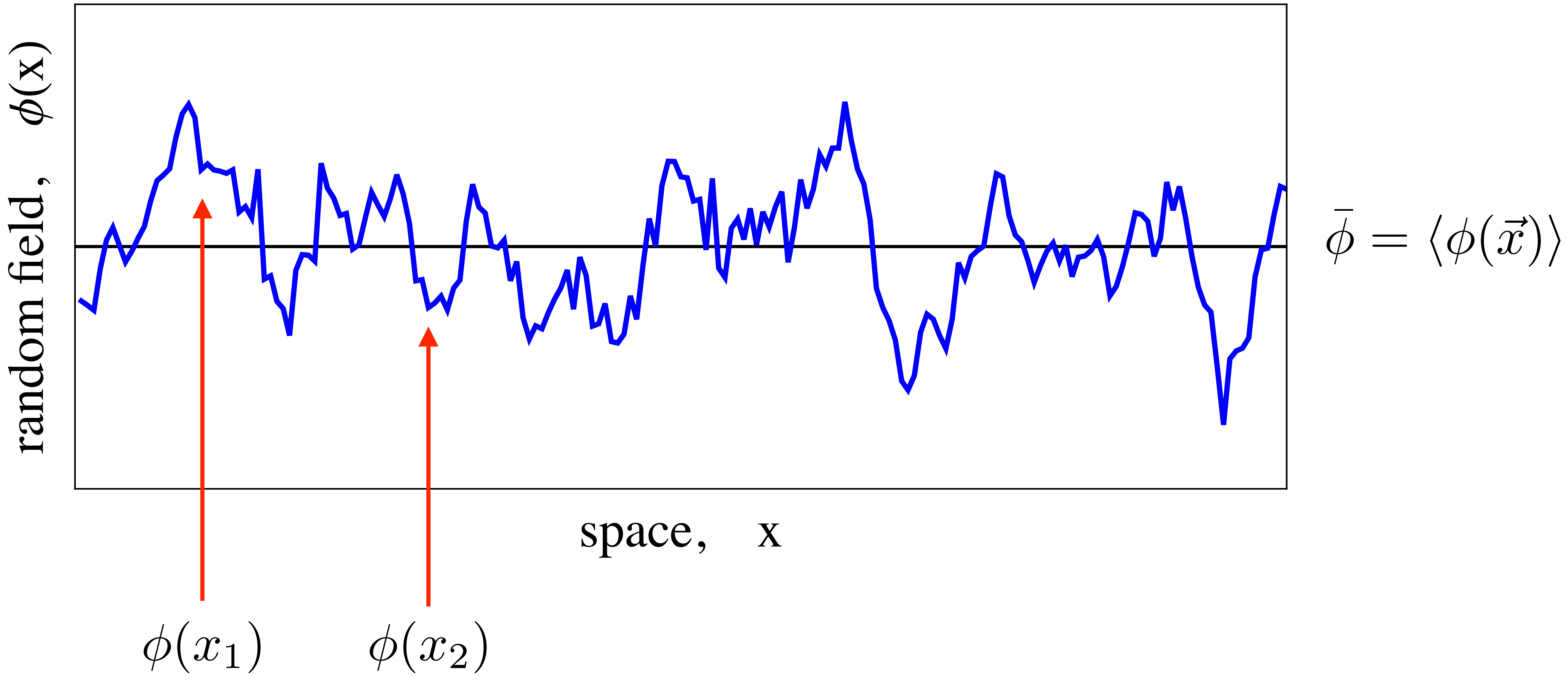
$$\langle\phi^2\rangle = \int d\phi\, \mathcal{P}(\phi)\, \phi^2$$ 2-nd-order moment

$$\langle\phi^n\rangle = \int d\phi\, \mathcal{P}(\phi)\, \phi^n$$ n-th-order moment

$$\sigma_\phi^2 = \langle\phi^2\rangle - \langle\phi\rangle^2$$ variance

Random fields

If $\phi(\vec{x})$ is a ***random field*** we can also compute **correlation functions**



two-point function $\qquad \langle\phi(x_1)\phi(x_2)\rangle = \langle\phi(x_1)\rangle\langle\phi(x_2)\rangle + \langle\phi(x_1)\phi(x_2)\rangle_c$

three-point function

$$\langle\phi(x_1)\phi(x_2)\phi(x_3)\rangle = \langle\phi(x_1)\rangle\langle\phi(x_2)\rangle\langle\phi(x_3)\rangle + \\ +\langle\phi(x_1)\phi(x_2)\rangle_c\,\langle\phi(x_3)\rangle + \text{perm.} + \\ +\langle\phi(x_1)\phi(x_2)\phi(x_3)\rangle_c$$

...

n-point function $\qquad \langle\phi(x_1)\phi(x_2)\ldots\phi(x_n)\rangle$

The distribution of galaxies

The galaxy number density and its perturbations as random fields

$$\boxed{n_g(\vec{x}) \equiv \bar{n}_g \left[1 + \delta_g(\vec{x})\right]}$$

galaxy number density

mean galaxy number

$$\boxed{\delta_g(\vec{x}) \equiv \frac{n_g(\vec{x}) - \bar{n}_g}{\bar{n}_g}}$$

galaxy overdensity
or density contrast

N.B. $\langle \delta_g(\vec{x}) \rangle \equiv 0$

$\delta_g(\vec{x}) \geq -1$

The galaxy two-point correlation function

What is the probability of finding two galaxies in the volume elements dV_1 and dV_2 ?

$$dP = dV_1\, dV_2 \,\langle\, n_g(\vec{x}_1)\, n_g(\vec{x}_2)\,\rangle$$

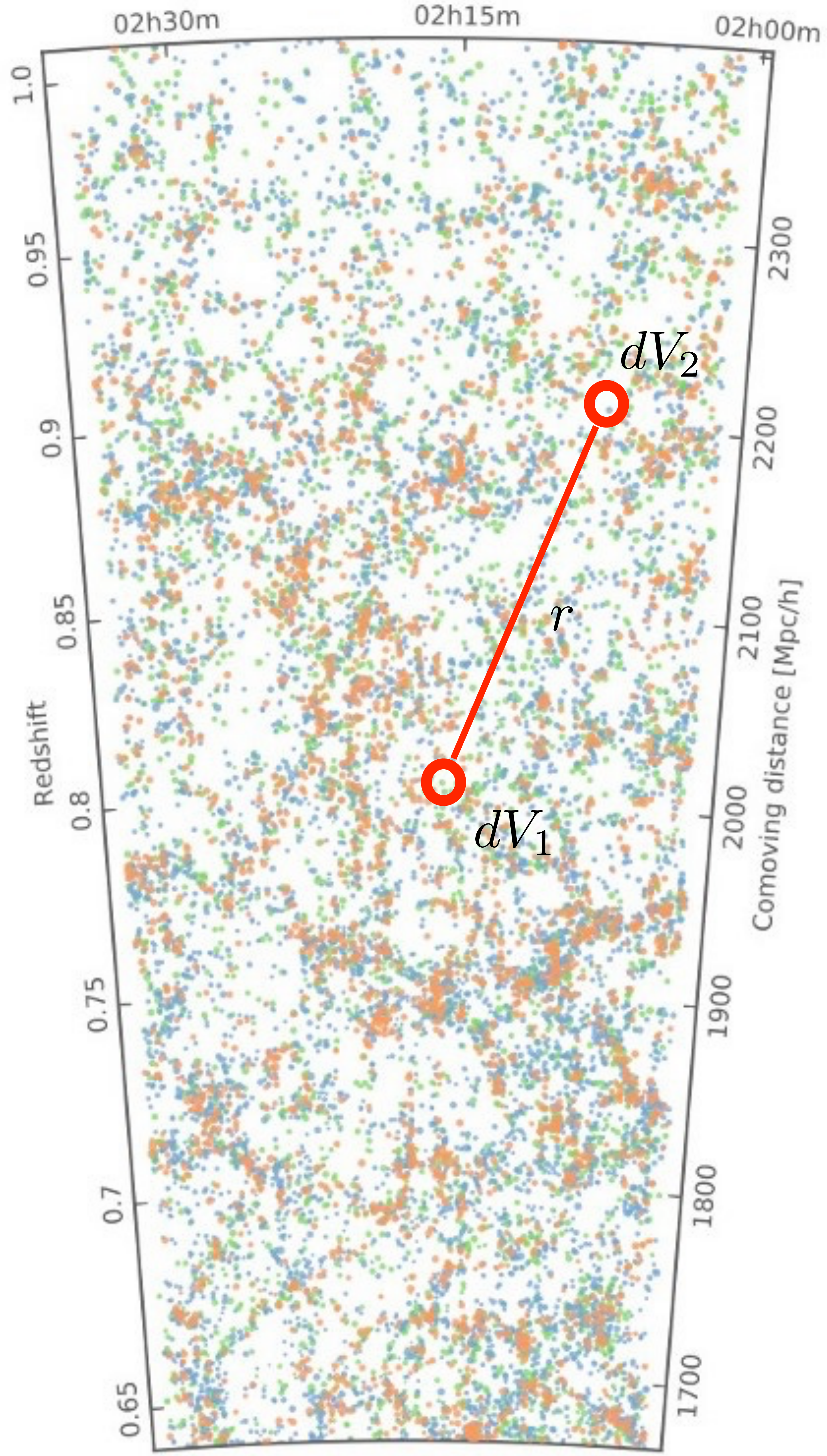
$$= dV_1\, dV_2\, \bar{n}_g^2\, [1 + \langle\, \delta_g(\vec{x}_1)\, \delta_g(\vec{x}_2)\,\rangle]$$

excess probability

We now make the assumption of
***statistical* homogeneity and isotropy**

$$\xi(|\vec{x}_1 - \vec{x}_2|) \equiv \langle\, \delta_g(\vec{x}_1)\, \delta_g(\vec{x}_2)\,\rangle$$

the two-point correlation function $\xi(r)$
only depends on the distance $r = |\vec{x}_1 - \vec{x}_2|$
between the two points

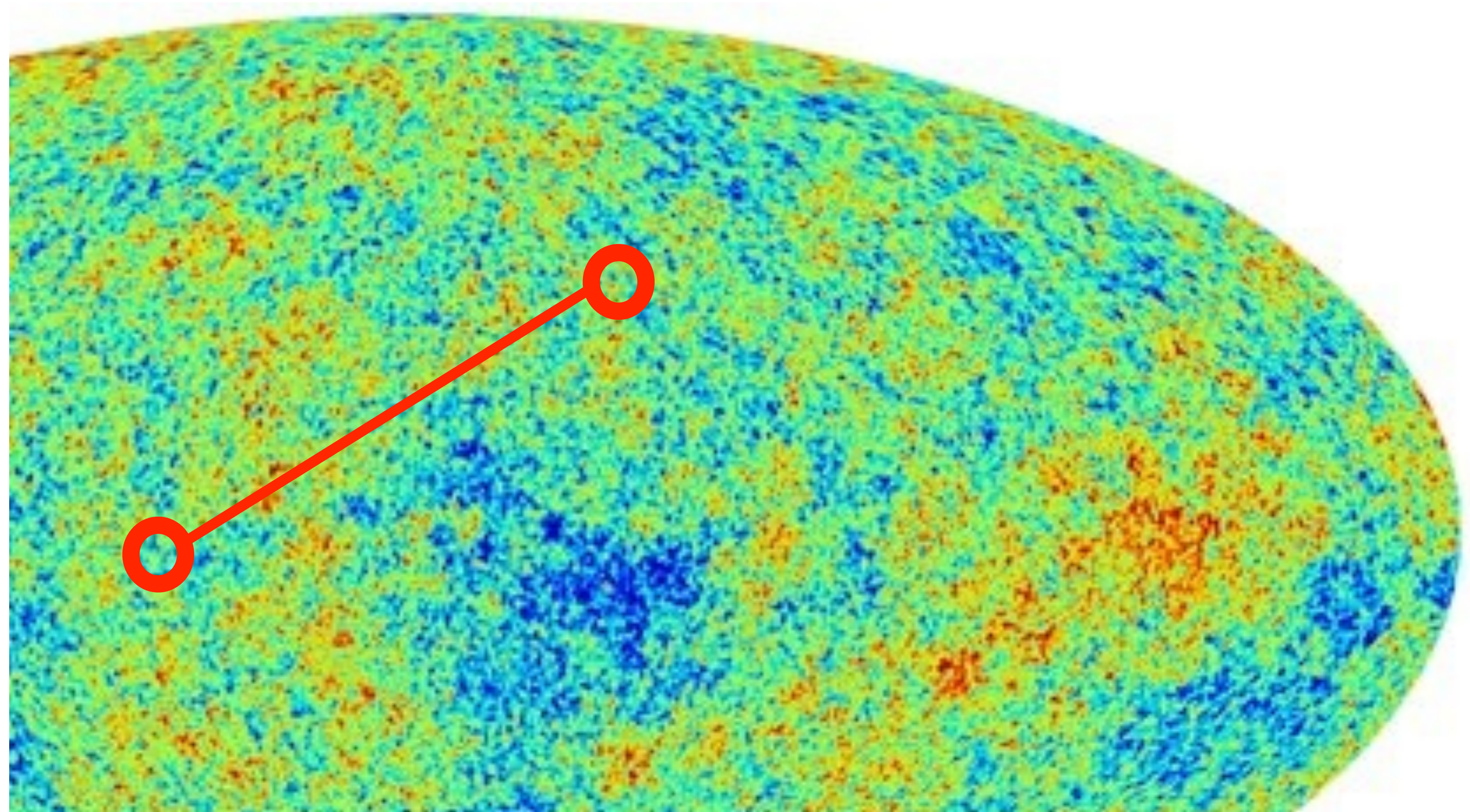


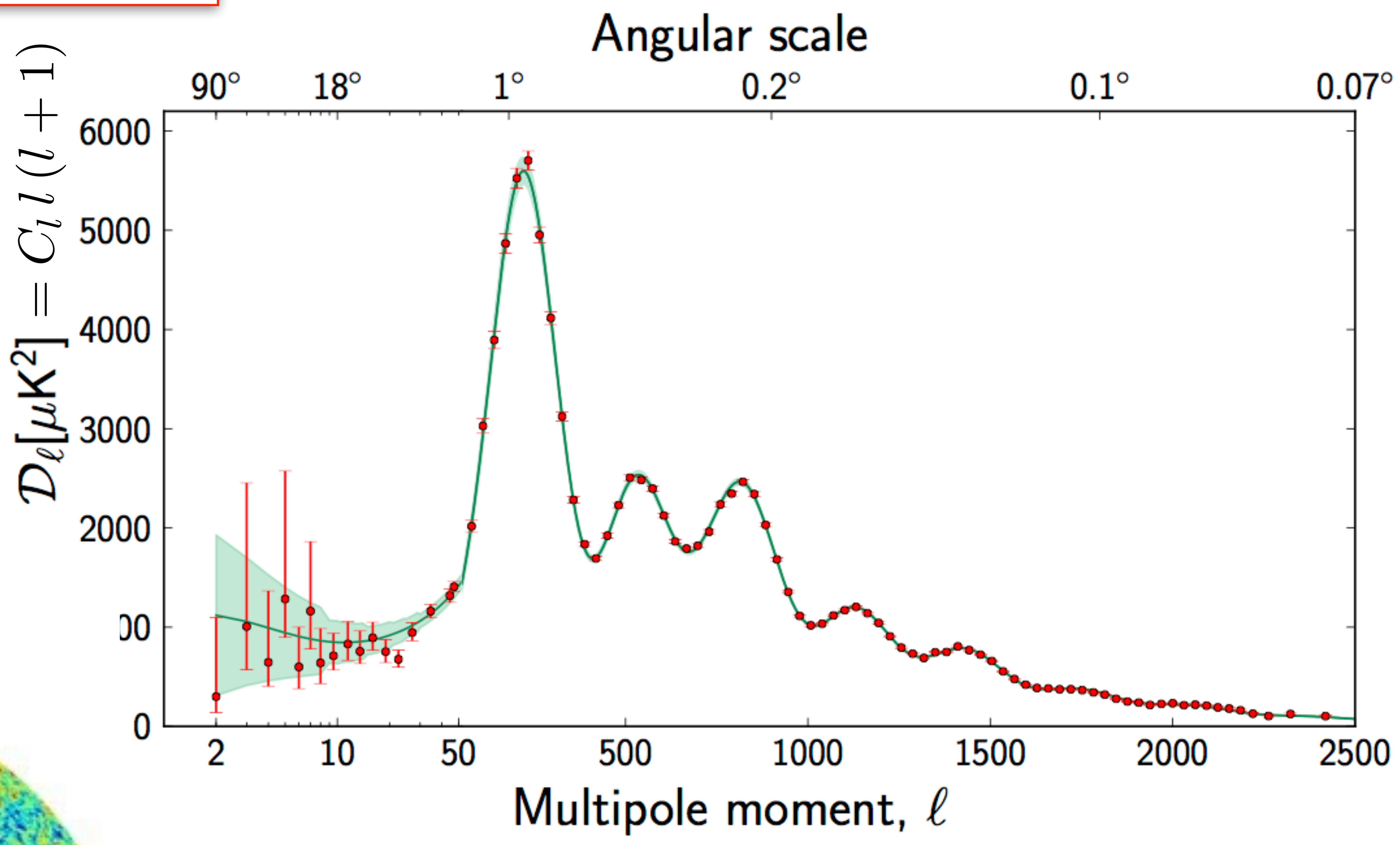
Gaussian and non-Gaussian random fields

The statistical properties of a Gaussian random field are completely characterised by its 2-point correlation function. All higher-order, *connected* correlation functions are vanishing

$$\delta_T(\hat{n}) \equiv \frac{T(\hat{n}) - \bar{T}}{\bar{T}}$$

$$\mathcal{P}[\delta_T(\hat{n})] = \frac{1}{\sqrt{2\pi\sigma_T^2}} e^{-\frac{1}{2}\frac{\delta_T^2}{\sigma_T^2}}$$



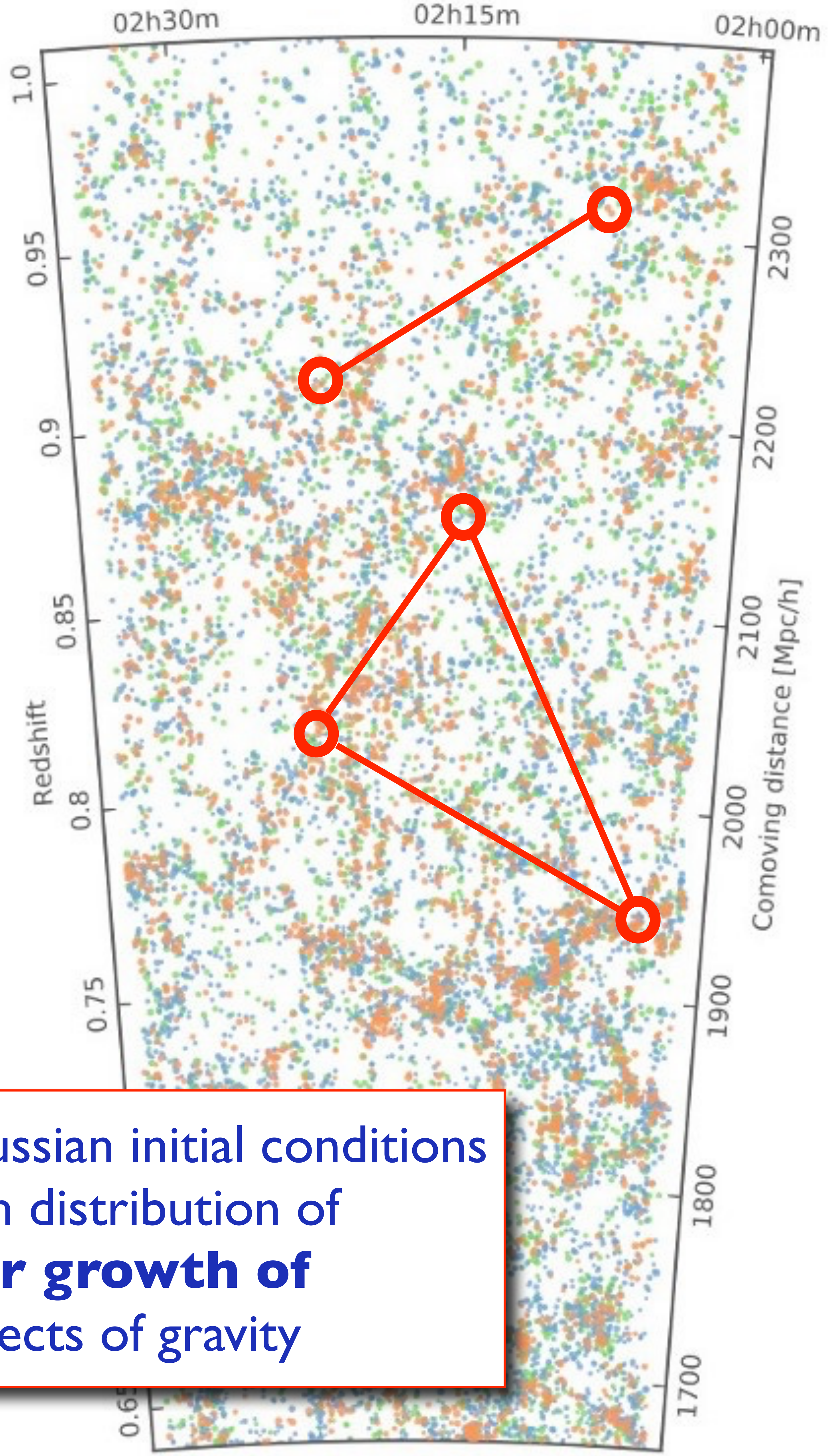


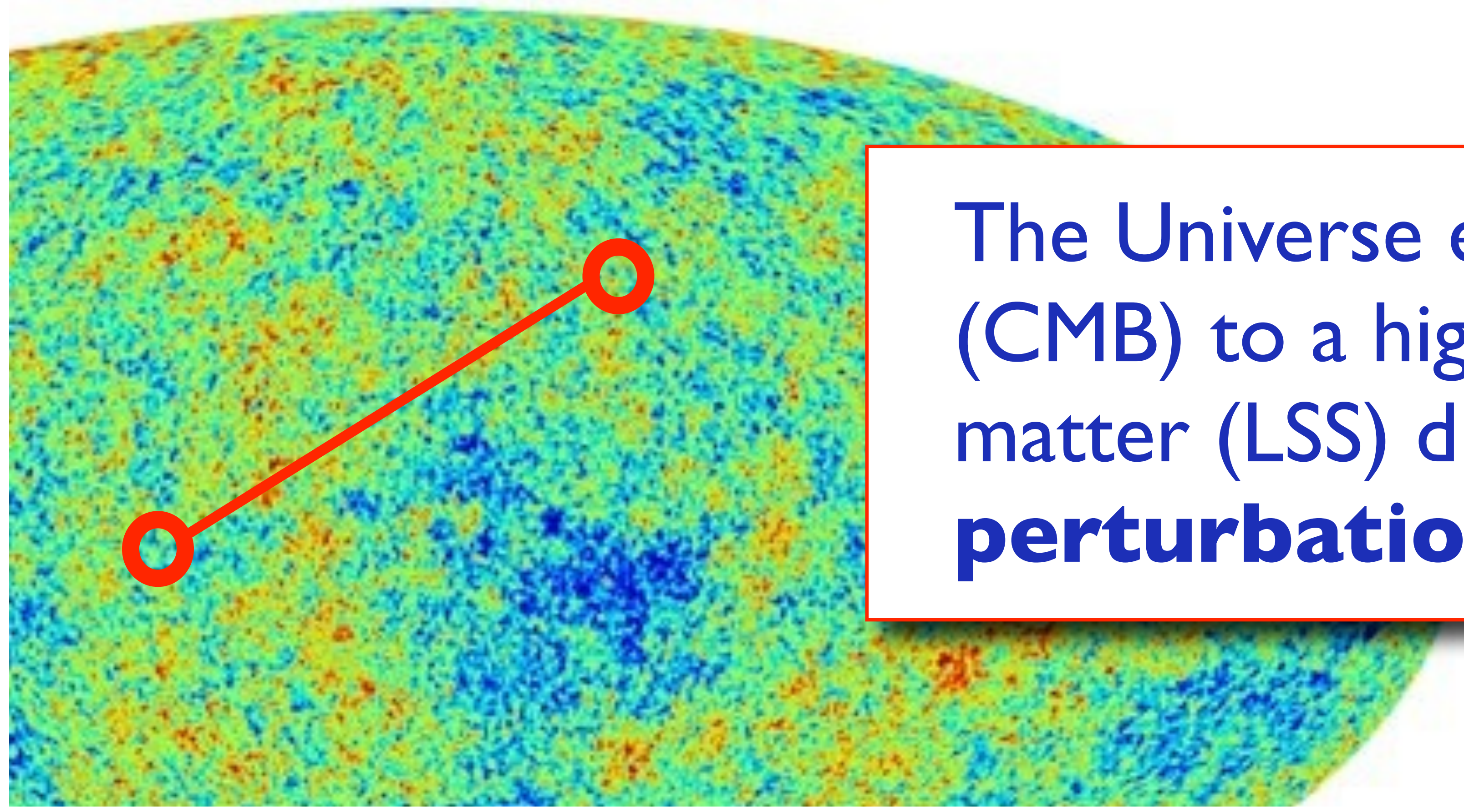
Perturbations in the CMB are one of the best examples of Gaussian random field

Gaussian and non-Gaussian random fields

The statistical properties of a Gaussian random field are completely characterised by its 2-point correlation function. All higher-order, *connected* correlation functions are vanishing

all other random fields are non-Gaussian!





The Universe evolves from Gaussian initial conditions (CMB) to a highly non-Gaussian distribution of matter (LSS) due to **nonlinear growth of perturbations** under the effects of gravity

Ergodic hypothesis

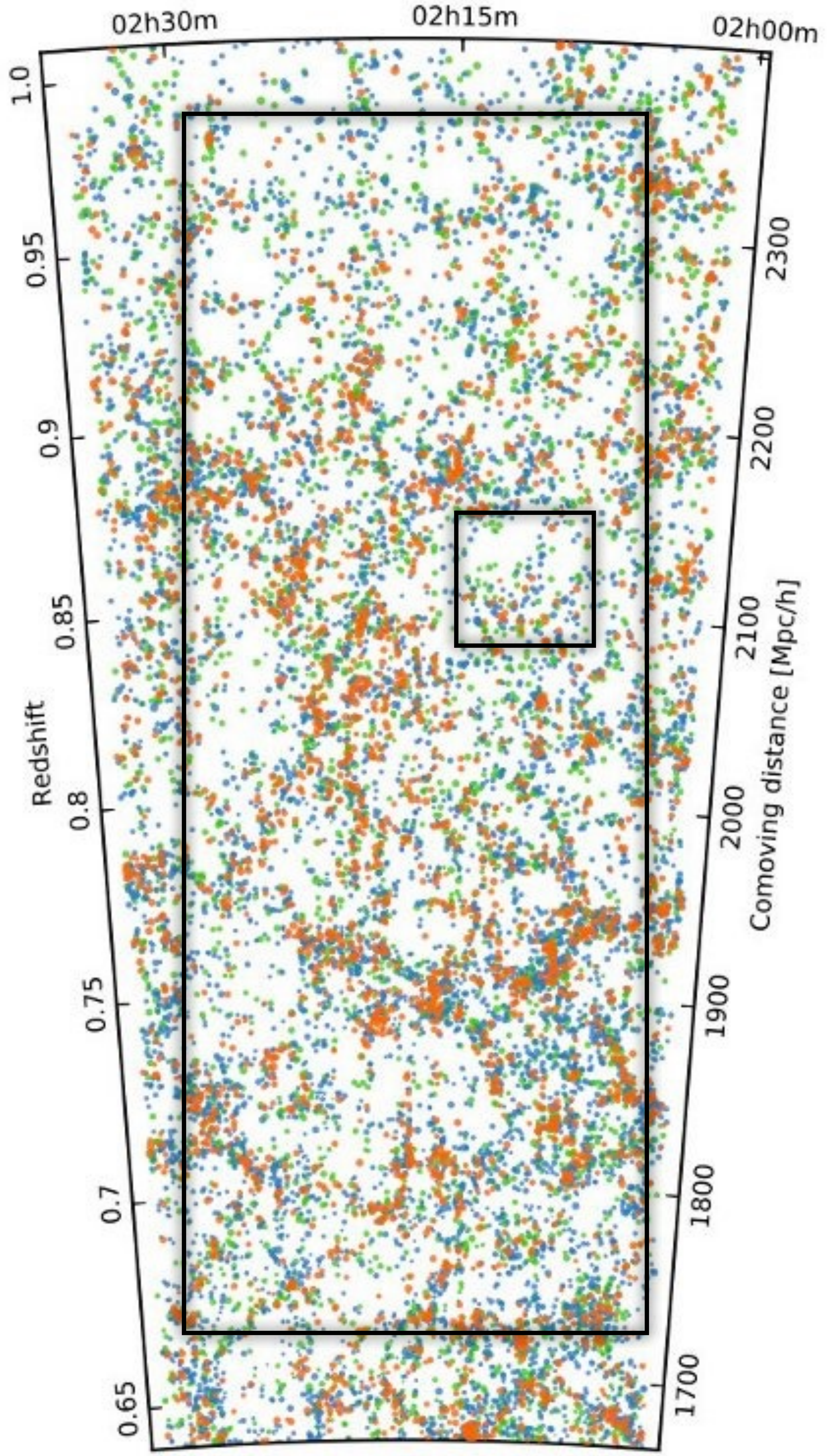
Expectation values, in principle, are to be intended as *ensemble averages*, i.e. averages over many "realisations of the Universe" …

… but we only have one Universe!

We assume the **ergodic hypothesis: ensemble averages are equal to spatial averages**

$$\langle \phi(\vec{x}) \rangle \equiv \int d\phi\, \phi\, \mathcal{P}(\phi) = \frac{1}{V} \int_V d^3x\, \phi(\vec{x})$$

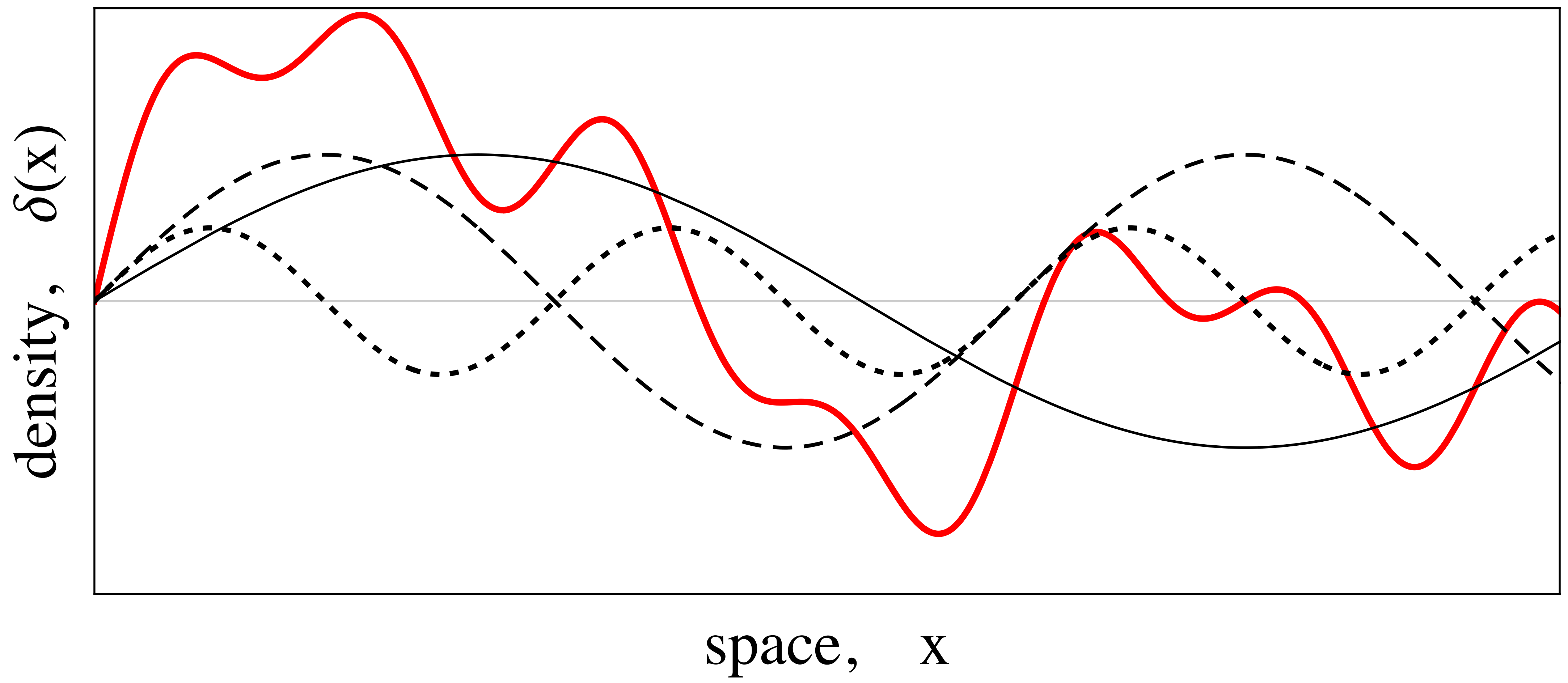
We should make sure, however, that the observed volume correspond to a "fair sample" of the Universe



Fourier space

Theoretical predictions for the matter correlation functions are performed in **Fourier space**

Fourier analysis naturally separates perturbations at different scales:



$$\delta_{\vec{k}} = \int \frac{d^3x}{(2\pi)^3} e^{-i\vec{k}\cdot\vec{x}} \delta(\vec{x})$$

$$\delta(\vec{x}) = \int d^3k e^{i\vec{k}\cdot\vec{x}} \delta_{\vec{k}}$$

- Since $\delta(\vec{x})$ is a random field $\delta_{\vec{k}}$ is also a random field
- Since $\delta(\vec{x})$ is real $\delta^*_{\vec{k}} = \delta_{-\vec{k}}$

Fourier space: correlation functions

The 2-point function in Fourier space: the **power spectrum**

$$\langle\, \delta_{\vec{k}_1}\, \delta_{\vec{k}_2} \rangle = \delta_D(\vec{k}_1 + \vec{k}_2)\, P(k_1)$$

homogeneity & isotropy

$$P(k) = \int \frac{d^3x}{(2\pi)^3}\, e^{i\,\vec{k}\cdot\vec{x}} \xi(x)$$

The power spectrum is the *Fourier Transform* of the 2-point correlation function

The power spectrum is a measure of the amplitude of perturbations as a function of scale

$$\Delta(k) \equiv 4\pi\, k^3\, P(k)$$

adimensional power spectrum

$$\sigma_\delta^2 \equiv \langle \delta^2(\vec{x}) \rangle = 4\pi \int dk\, k^2\, P(k) = \int \frac{dk}{k}\, \Delta(k)$$