Quickly Estimating Powers-of-Two

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Woven Planet

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About me

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- Co-organizer of the Munich C++ User Group
- Currently working as a Runtime Framework Engineer for Woven Planet

$$2^{23} =?$$

$$2^{23} =?$$

$$2^{36} =?$$

$$2^{23} =?$$

$$2^{36} =?$$

$$2^{128} =?$$

Small Powers of Two

- $2^1 = 2$
- $2^2 = 4$
- $2^3 = 8$
- $2^4 = 16$
- $2^5 = 32$
- $2^6 = 64$
- $2^7 = 128$
- $2^8 = 256$
- $2^9 = 512$

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- $2^7 = 128$ (Number of ASCII characters)
- $2^8 = 256$
- $2^9 = 512$

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- $2^7 = 128$
- $2^8 = 256$ (Number of states in an 8 bit `char`)
- $2^9 = 512$

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Filling the gaps for small powers of two

$2^9 = ???$

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- $2^9 = 2^{(8+1)}$

Filling the gaps for small powers of two

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- $2^9 = 2^{(8+1)} = 2 * 2^8$

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$2^6 = ???$

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- $2^6 = 2^{2*3}$

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- $2^6 = 2^{2*3} = (2^3)^2$

Filling the gaps for small powers of two

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$2^5 = ???$

Filling the gaps for small powers of two

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- $2^6 = 2^{2*3} = (2^3)^2 = 8^2 = 64$

$2^5 = ???$

- $2^5 = 2^{(2+3)} = 2^2 * 2^3 = 4 * 8 = 32$

Easy, but impractical for larger powers.

$$2^{10} = 1024$$

$$2^{10} = 1024 \approx 1000$$

Two to the $(n * 10)$

$$2^{10} = 1024 \approx 10^3$$

Two to the $(n * 10)$

$$
\begin{aligned}
2^{10} &= 1024 && \approx 10^{3} \\
2^{20} &= 1024^{2} && \approx 10^{2*3} && = 10^{6}
\end{aligned}
$$

Two to the $(n * 10)$

$$
\begin{aligned}
2^{10} &= 1024 &&\approx 10^{3} \\
2^{20} &= 1024^{2} &&\approx 10^{2*3} &&= 10^{6} \\
2^{30} &= 1024^{3} &&\approx 10^{3*3} &&= 10^{9}
\end{aligned}
$$

Two to the $(n * 10)$

$$
\begin{aligned}
2^{10} &= 1024 && \approx 10^{3} \\
2^{20} &= 1024^{2} && \approx 10^{2*3} && = 10^{6} \\
2^{30} &= 1024^{3} && \approx 10^{3*3} && = 10^{9} \\
2^{40} &= 1024^{4} && \approx 10^{4*3} && = 10^{12} \\
2^{50} &= 1024^{5} && \approx 10^{5*3} && = 10^{15} \\
2^{60} &= 1024^{6} && \approx 10^{6*3} && = 10^{18}
\end{aligned}
$$

Two to the $(n * 10)$

$$
\begin{array}{llll}
2^{10} & = 1024 & \approx 10^{3} & \text{(Kilo)} \\
2^{20} & = 1024^{2} & \approx 10^{2*3} = 10^{6} & \text{(Mega)} \\
2^{30} & = 1024^{3} & \approx 10^{3*3} = 10^{9} & \text{(Giga)} \\
2^{40} & = 1024^{4} & \approx 10^{4*3} = 10^{12} & \text{(Tera)} \\
2^{50} & = 1024^{5} & \approx 10^{5*3} = 10^{15} & \text{(Peta)} \\
2^{60} & = 1024^{6} & \approx 10^{6*3} = 10^{18} & \text{(Exa)}
\end{array}
$$

Two to the $(n * 10)$

$$
\begin{aligned}
2^{10} &= 1024 && \approx 10^{3} && && \text{(Kilo)} \\
2^{20} &= 1024^{2} && \approx 10^{2*3} && = 10^{6} && \text{(Mega)} \\
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2^{60} &= 1024^{6} && \approx 10^{6*3} && = 10^{18} && \text{(Exa)}
\end{aligned}
$$

Estimating larger powers of two

2^{32}

Estimating larger powers of two

$$2^{32} = 2^2 * 2^{30}$$

Estimating larger powers of two

$$2^{32} = 2^2 * 2^{30} \approx 4 * 10^9$$

Estimating larger powers of two

$$2^{32} = 2^2 * 2^{30} \approx 4 * 10^9$$

$$2^{23}$$

Estimating larger powers of two

$$2^{32} = 2^2 * 2^{30} \approx 4 * 10^9$$

$$2^{23} = 2^3 * 2^{20}$$

Estimating larger powers of two

$$2^{32} = 2^2 * 2^{30} \approx 4 * 10^9$$

$$2^{23} = 2^3 * 2^{20} \approx 8 * 10^6$$

Estimating larger powers of two

$$2^{32} = 2^2 * 2^{30} \approx 4 * 10^9$$

$$2^{23} = 2^3 * 2^{20} \approx 8 * 10^6$$

$$2^{36}$$

Estimating larger powers of two

$$2^{32} = 2^2 * 2^{30} \approx 4 * 10^9$$

$$2^{23} = 2^3 * 2^{20} \approx 8 * 10^6$$

$$2^{36} = 2^6 * 2^{30}$$

Estimating larger powers of two

$$2^{32} = 2^2 * 2^{30} \approx 4 * 10^9$$

$$2^{23} = 2^3 * 2^{20} \approx 8 * 10^6$$

$$2^{36} = 2^6 * 2^{30} \approx 64 * 10^9$$

Estimating larger powers of two

$$2^{32} = 2^2 * 2^{30} \approx 4 * 10^9$$

$$2^{23} = 2^3 * 2^{20} \approx 8 * 10^6$$

$$2^{36} = 2^6 * 2^{30} \approx 64 * 10^9 = 6.4 * 10^{10}$$

Estimating larger powers of two

$$2^{32} = 2^2 * 2^{30} \approx 4 * 10^9$$

$$2^{23} = 2^3 * 2^{20} \approx 8 * 10^6$$

$$2^{36} = 2^6 * 2^{30} \approx 64 * 10^9 = 6.4 * 10^{10}$$

$$2^{128}$$

Estimating larger powers of two

$$2^{32} = 2^2 * 2^{30} \approx 4 * 10^9$$

$$2^{23} = 2^3 * 2^{20} \approx 8 * 10^6$$

$$2^{36} = 2^6 * 2^{30} \approx 64 * 10^9 = 6.4 * 10^{10}$$

$$2^{128} = 2^8 * 2^{120}$$

Estimating larger powers of two

$$2^{32} = 2^2 * 2^{30} \approx 4 * 10^9$$

$$2^{23} = 2^3 * 2^{20} \approx 8 * 10^6$$

$$2^{36} = 2^6 * 2^{30} \approx 64 * 10^9 = 6.4 * 10^{10}$$

$$2^{128} = 2^8 * 2^{120} \approx 256 * 10^{36}$$

Estimating larger powers of two

$$2^{32} = 2^2 * 2^{30} \approx 4 * 10^9$$

$$2^{23} = 2^3 * 2^{20} \approx 8 * 10^6$$

$$2^{36} = 2^6 * 2^{30} \approx 64 * 10^9 = 6.4 * 10^{10}$$

$$2^{128} = 2^8 * 2^{120} \approx 256 * 10^{36} = 2.56 * 10^{38}$$

Relative error of the estimation

- The relative error of $2^{10} \approx 1000$ is 2.34%.

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- The relative error of $2^{10} \approx 1000$ is 2.34%.
- The relative error *only* increases on the ten steps:

2^{10}	2.34%	2^{70}	15.30%	2^{130}	26.53%
2^{20}	4.63%	2^{80}	17.28%	2^{140}	28.25%
2^{30}	6.87%	2^{90}	19.22%	2^{150}	29.94%
2^{40}	9.05%	2^{100}	21.11%	2^{160}	31.58%
2^{50}	11.18%	2^{110}	22.96%	2^{170}	33.18%
2^{60}	13.26%	2^{120}	24.77%	2^{180}	34.75%

Relative error of the estimation

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2^{40}	9.05%	2^{100}	21.11%	2^{160}	31.58%
2^{50}	11.18%	2^{110}	22.96%	2^{170}	33.18%
2^{60}	13.26%	2^{120}	24.77%	2^{180}	34.75%

- Mind you, for 2^{128} this is merely the difference between $2.56 * 10^{38}$ and $3.40 * 10^{38}$.

$$2^{x*10+y} = 2^y * 2^{x*10} \approx 2^y * 10^{3*x}$$