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ELEN 160 Project (Fall 2019)

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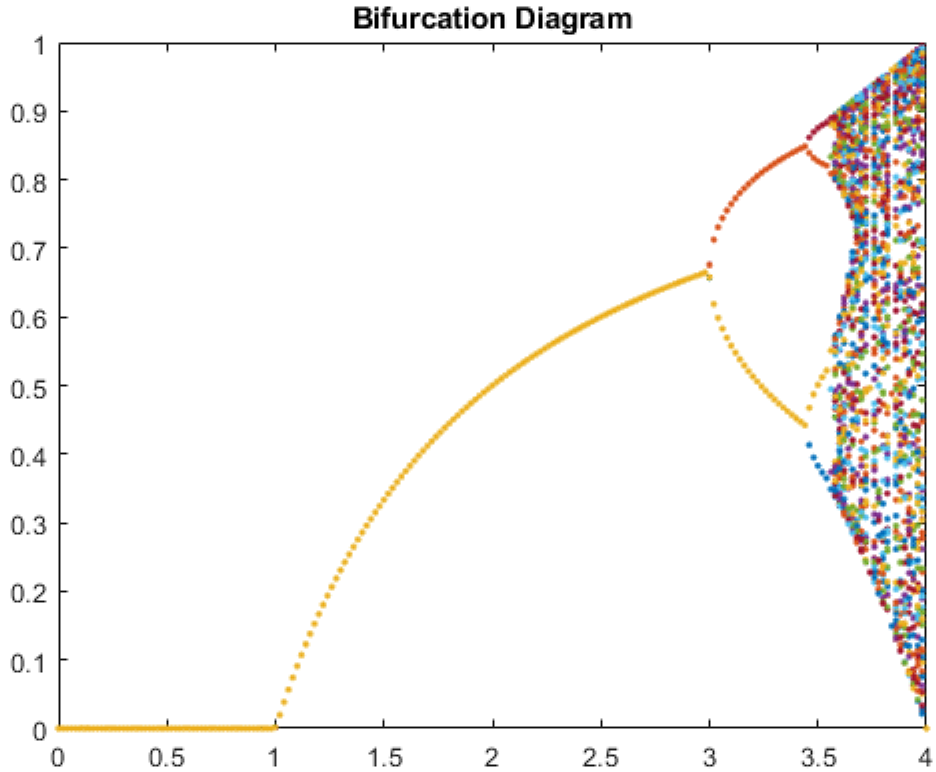
```
clear, clc, close all
```

Problem 1

```
M = []; % M will be our matrix of x(k) where each column corresponds
 to a different p value
kmax = 700;
init = 0.5;
probs = 0:0.02:4;
for p = 0:0.02:4
    res = compute_logistic_map(p, init, kmax); % compute the column
 vector
    M = [M, res];   % Append resulting column vector to matrix
end
M_slice = M(end-100:end,:); % the last 100 or so rows of M
figure;
plot(probs,M_slice,'.');
title('Bifurcation Diagram');

% From approximately 0 <= p < 1, the system converges to 0
% From approximately 1 <= p < 3, the system has a single amplitude
 that varies with p
% From approximately 3 <= p < 3.57, the system displays period
 doubling
% From approximately 3.57 <= p < 4, the system exhibits chaos
```



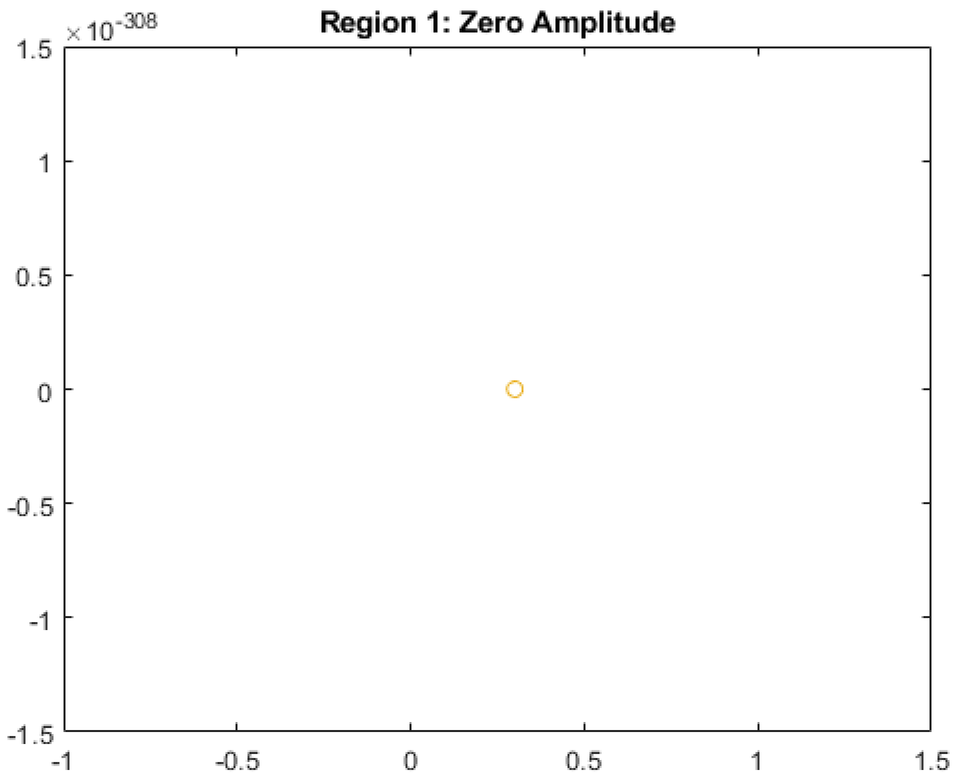
Problem 2

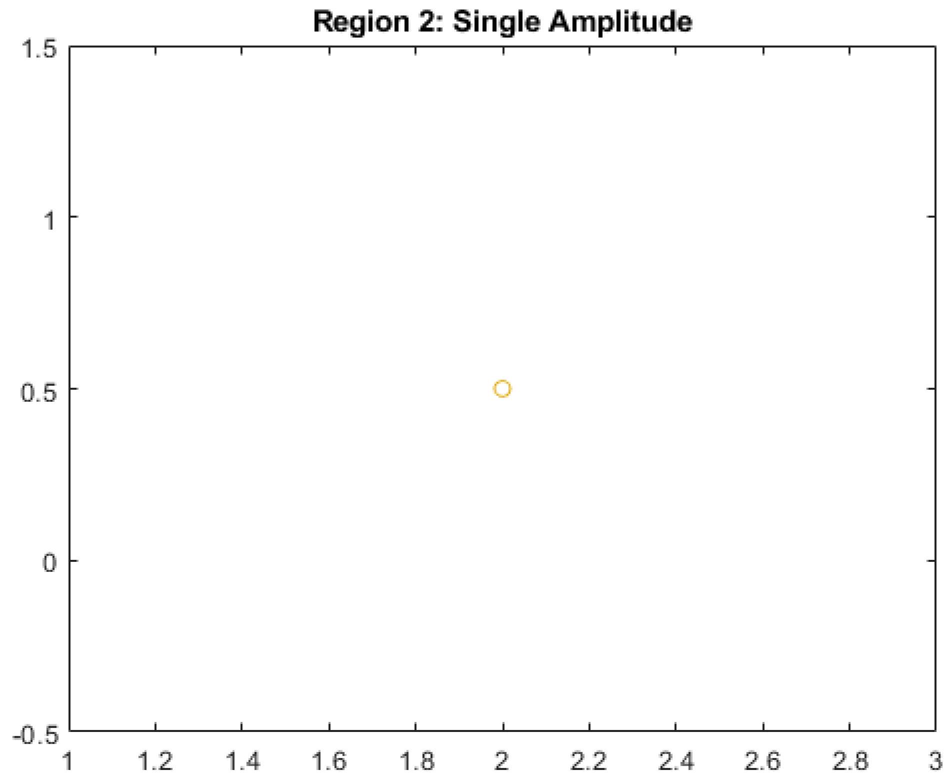
```
% a) Plotting four representative sequences with initial condition 0.5

% When equilibria converge to 0 (initial condition is 0.5)
p = 0.3;
kmax = 700;
init = 0.5;
sequence = compute_logistic_map(p, init, kmax);
sequence_slice = sequence(end-100:end,:); % the last 100 or so rows of
 sequence
figure;
plot(p,sequence_slice,'o')
title('Region 1: Zero Amplitude')

% Single Amplitude (initial condition is 0.5)
p = 2;
kmax = 700;
init = 0.5;
sequence = compute_logistic_map(p, init, kmax);
sequence_slice = sequence(end-100:end,:); % the last 100 or so rows of
 sequence
figure;
plot(p,sequence_slice,'o')
title('Region 2: Single Amplitude')
```

```
% Period Doubling (initial condition is 0.5)
p = 3.25;
kmax = 700;
init = 0.5;
sequence = compute_logistic_map(p, init, kmax);
sequence_slice = sequence(end-100:end,:); % the last 100 or so rows of
 sequence
figure;
plot(p,sequence_slice,'o')
title('Region 3: Period Doubling Starts (Two Amplitudes)')

% Four Amplitudes (initial condition is 0.5)
p = 3.5;
kmax = 700;
init = 0.5;
sequence = compute_logistic_map(p, init, kmax);
sequence_slice = sequence(end-100:end,:); % the last 100 or so rows of
 sequence
figure;
plot(p,sequence_slice,'o')
title('Region 4: Four Amplitudes')

% Chaos (initial condition is 0.5)
p = 3.9;
kmax = 700;
init = 0.5;
sequence = compute_logistic_map(p, init, kmax);
sequence_slice = sequence(end-100:end,:); % the last 100 or so rows of
 sequence
figure;
plot(p,sequence_slice,'o')
title('Region 5: Chaos')

% p > 4 (initial condition is 0.5) Diverges
p = 4.3;
kmax = 700;
init = 0.5;
sequence = compute_logistic_map(p, init, kmax);
sequence_slice = sequence(end-100:end,:); % the last 100 or so rows of
 sequence
% Sequence_slice is all negative infinity
figure;
plot(p,sequence_slice,'o')
title('Region 6: Equilibria diverge')

% b) Now we try with initial condition 1.5

% When equilibria converge to 0 (initial condition is 1.5)
p = 0.3;
kmax = 700;
init = 1.5;
sequence = compute_logistic_map(p, init, kmax);
```

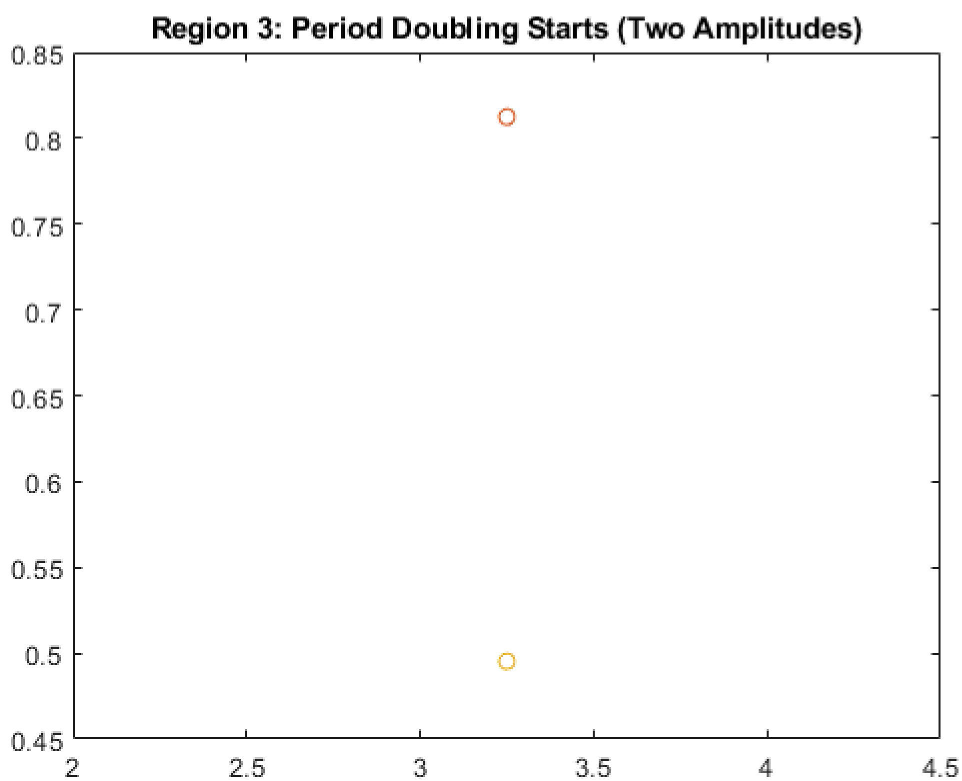
```
sequence_slice = sequence(end-100:end,:); % the last 100 or so rows of
 sequence
figure;
plot(p,sequence_slice,'o')
title('Region 1: Zero Amplitude')

% Single Amplitude (initial condition is 1.5)
p = 2;
kmax = 700;
init = 1.5;
sequence = compute_logistic_map(p, init, kmax);
sequence_slice = sequence(end-100:end,:); % the last 100 or so rows of
 sequence
figure;
plot(p,sequence_slice,'o')
title('Region 2: Diverges')

% For initial condition 1.5, anything that does not converge to 0
 blows up
% to infinity
%
% Period Doubling (initial condition is 1.5)
p = 3.25;
kmax = 700;
init = 1.5;
sequence = compute_logistic_map(p, init, kmax);
sequence_slice = sequence(end-100:end,:); % the last 100 or so rows of
 sequence
figure;
plot(p,sequence_slice,'o')
title('Region 3: Diverges')
% For initial condition 1.5, anything that does not converge to 0
 blows up
% to infinity

% Four Amplitudes (initial condition is 1.5)
p = 3.5;
kmax = 700;
init = 1.5;
sequence = compute_logistic_map(p, init, kmax);
sequence_slice = sequence(end-100:end,:); % the last 100 or so rows of
 sequence
figure;
plot(p,sequence_slice,'o')
title('Region 4: Diverges')
% For initial condition 1.5, anything that does not converge to 0
 blows up
% to infinity

% Chaos (initial condition is 1.5)
p = 3.9;
kmax = 700;
init = 1.5;
sequence = compute_logistic_map(p, init, kmax);
```

```
sequence_slice = sequence(end-100:end,:); % the last 100 or so rows of
 sequence
figure;
plot(p,sequence_slice,'o')
title('Region 5: Chaos (Diverges)')
% For initial condition 1.5, this region diverges

% p > 4 (initial condition is 1.5) Diverges
p = 4.3;
kmax = 700;
init = 1.5;
sequence = compute_logistic_map(p, init, kmax);
sequence_slice = sequence(end-100:end,:); % the last 100 or so rows of
 sequence
% Sequence_slice is all negative infinity as before
figure;
plot(p,sequence_slice,'o')
title('Region 6 (p = 4.3): Equilibria diverge')
```

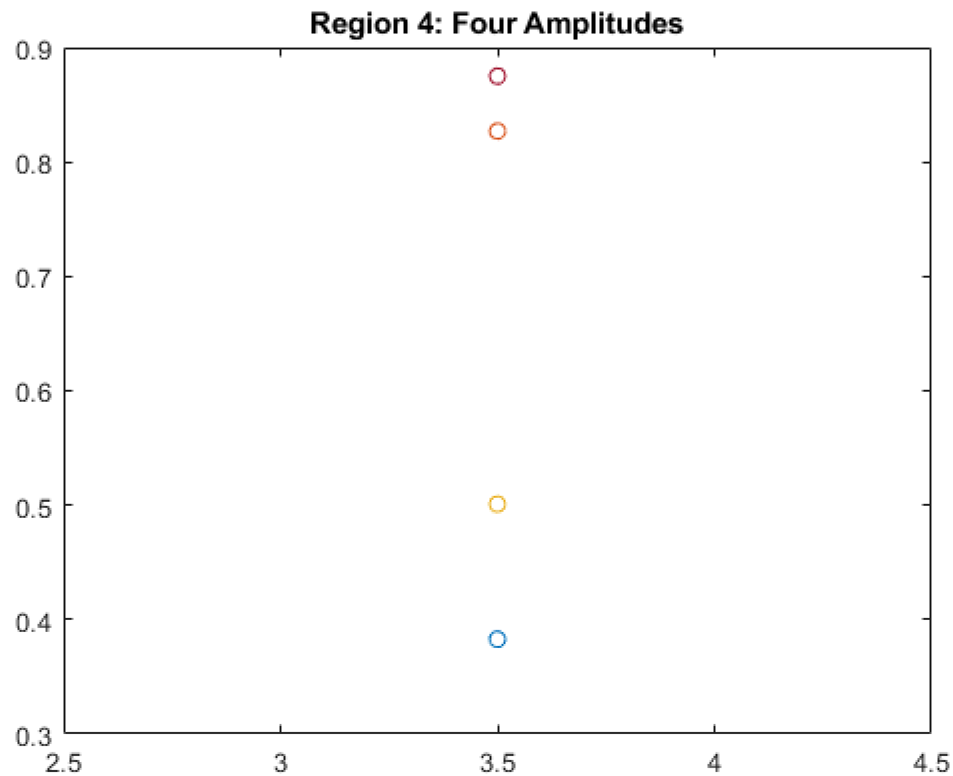


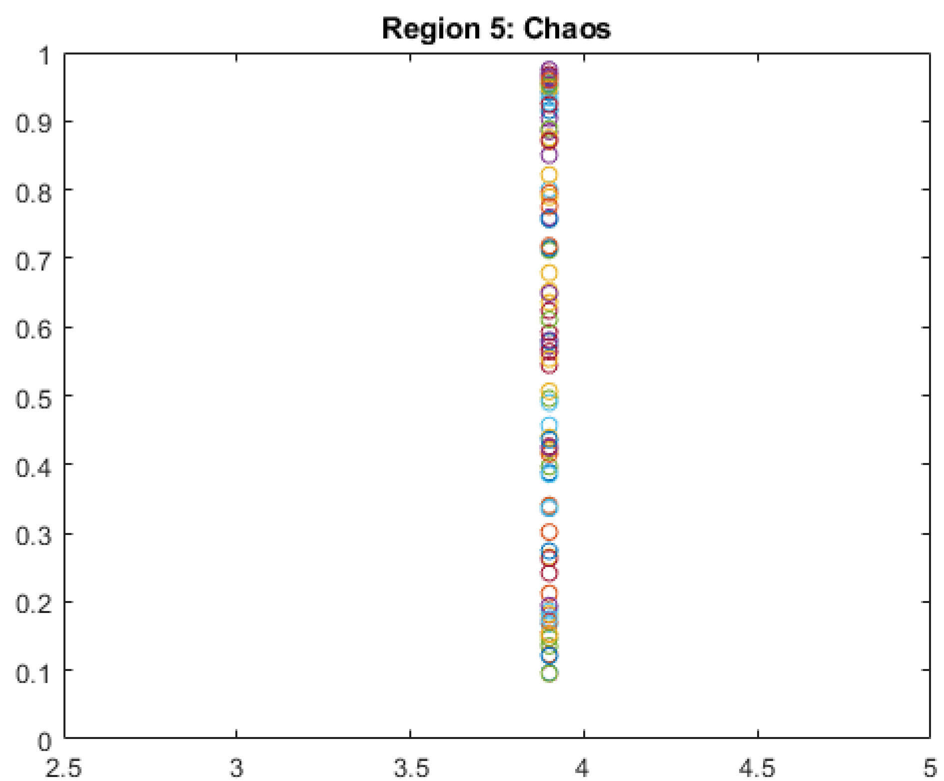


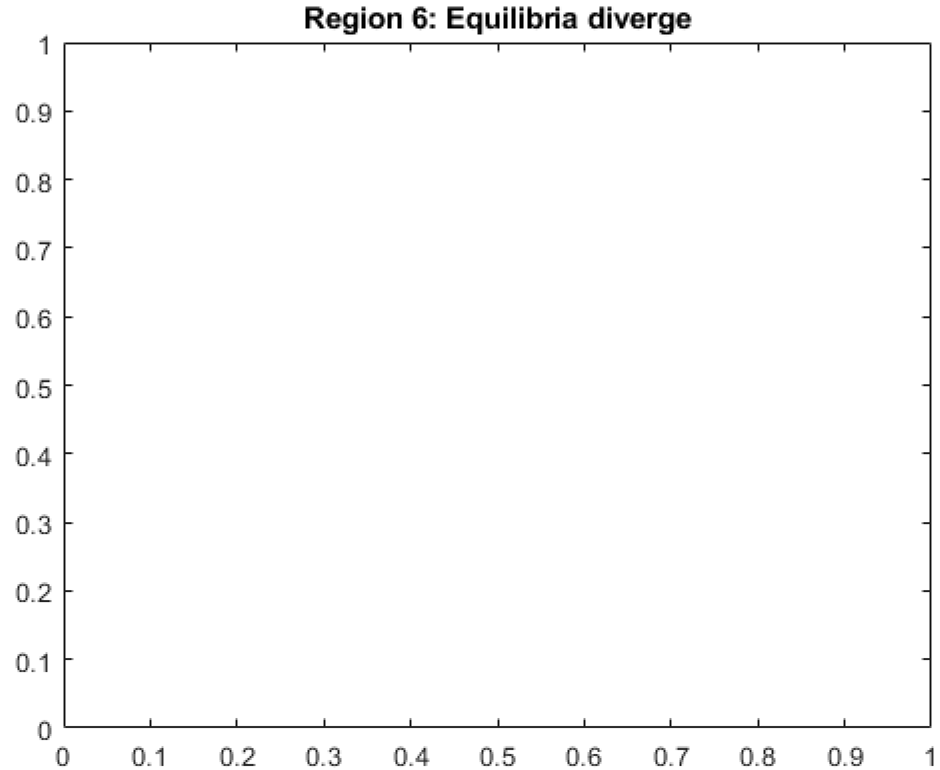
Region 2: Single Amplitude
1.5
1
0.5
0
-0.5
1
1.2
1.4
1.6
1.8
2
2.2
2.4
2.6
2.8
3



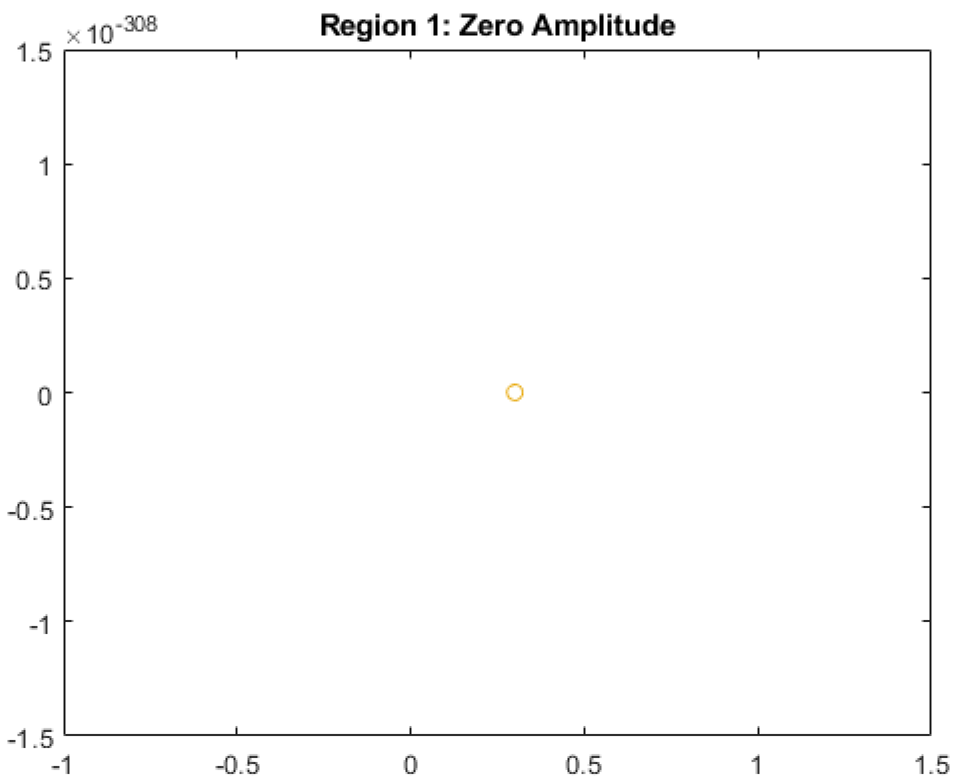
Region 3: Period Doubling Starts (Two Amplitudes)
0.85
0.8
0.75
0.7
0.65
0.6
0.55
0.5
0.45
2
2.5
3
3.5
4
4.5


Region 4: Four Amplitudes
0.9
0.8
0.7
0.6
0.5
0.4
0.3
2.5
3
3.5
4
4.5

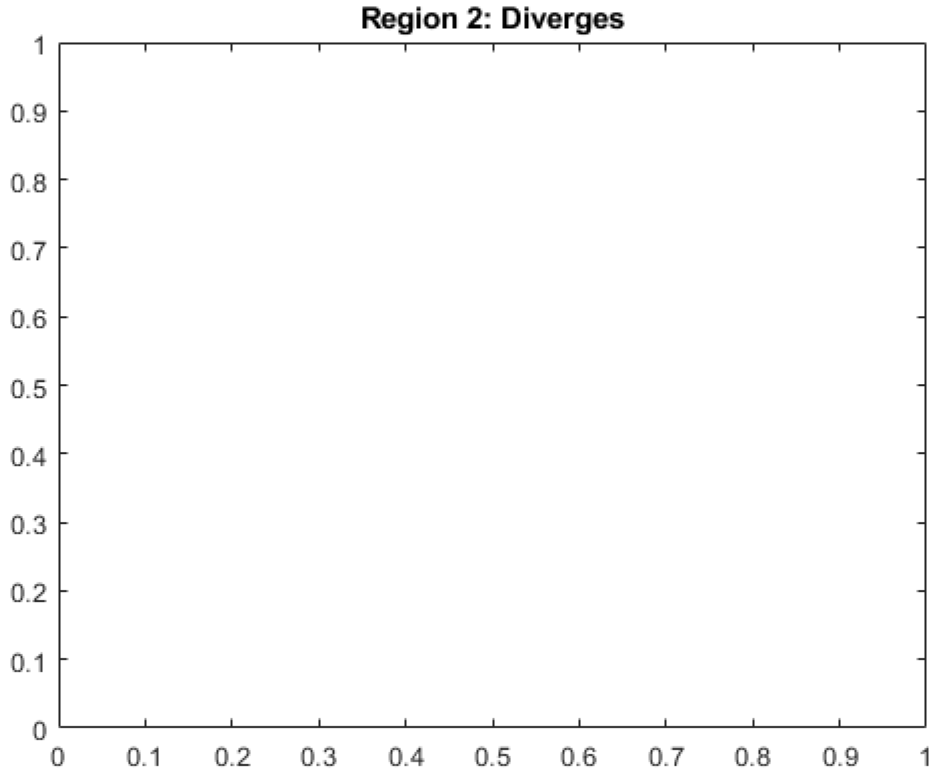

Region 5: Chaos
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
2.5
3
3.5
4
4.5
5

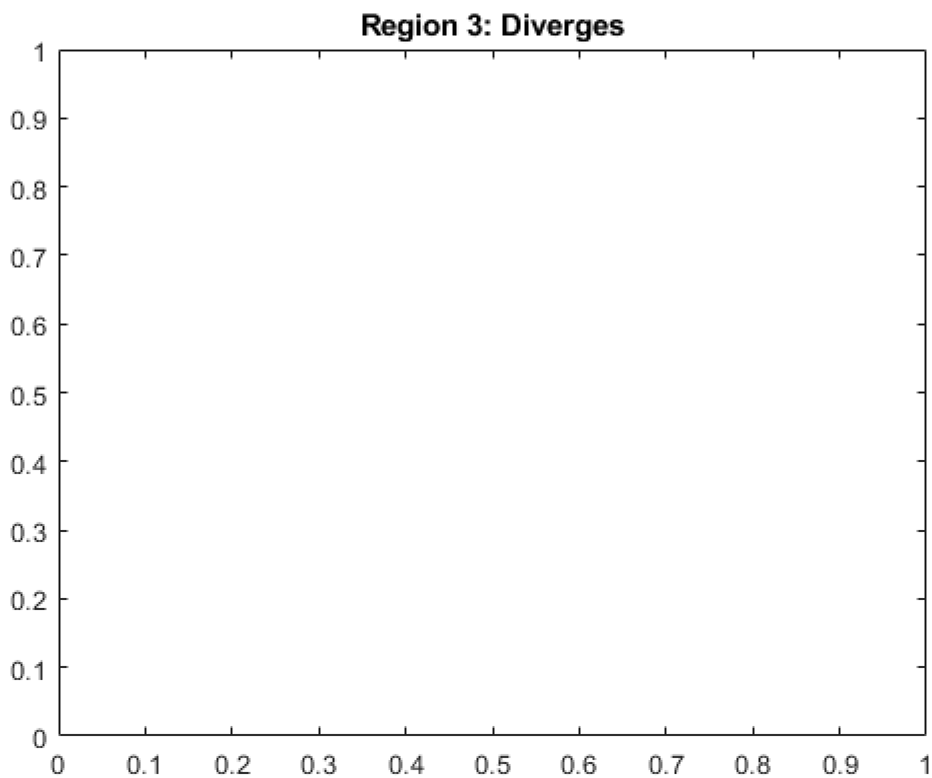


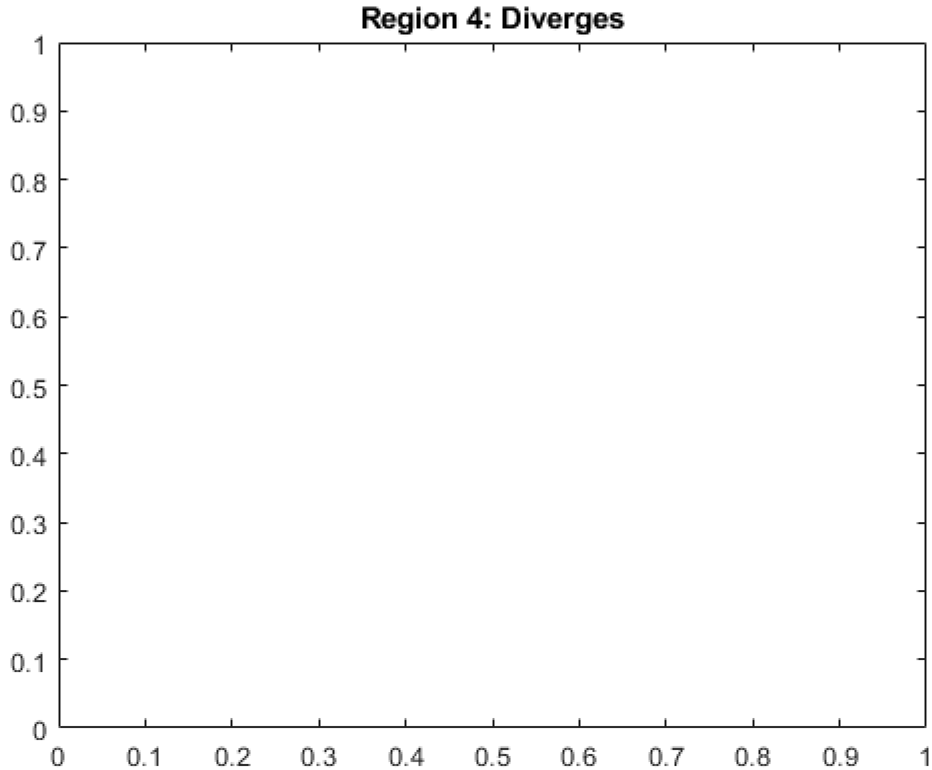
Region 6: Equilibria diverge
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

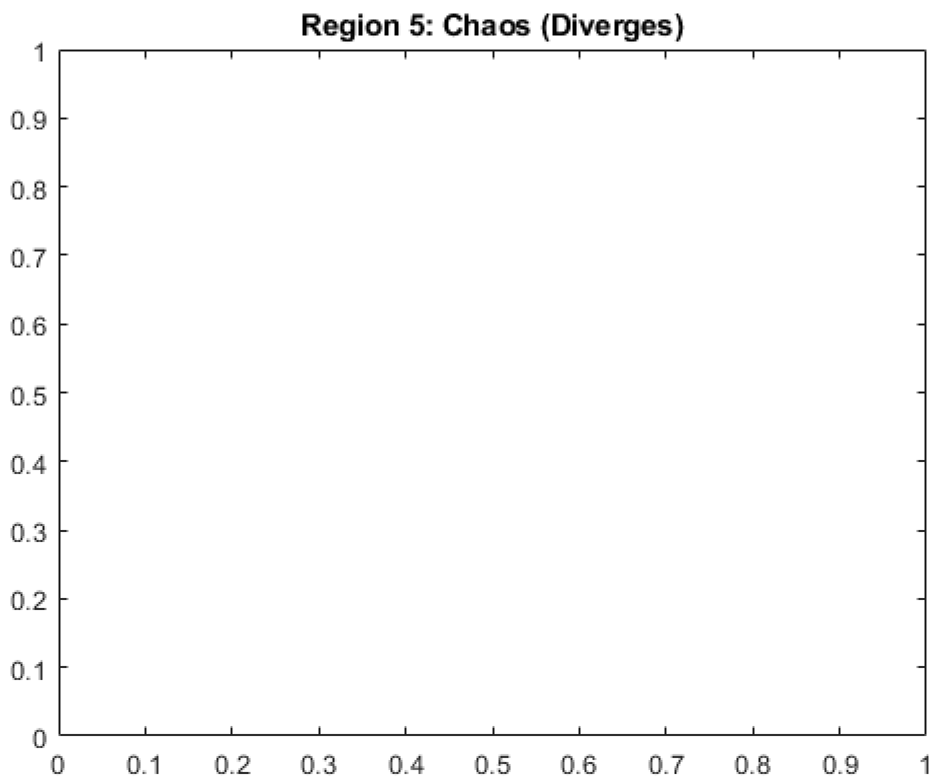


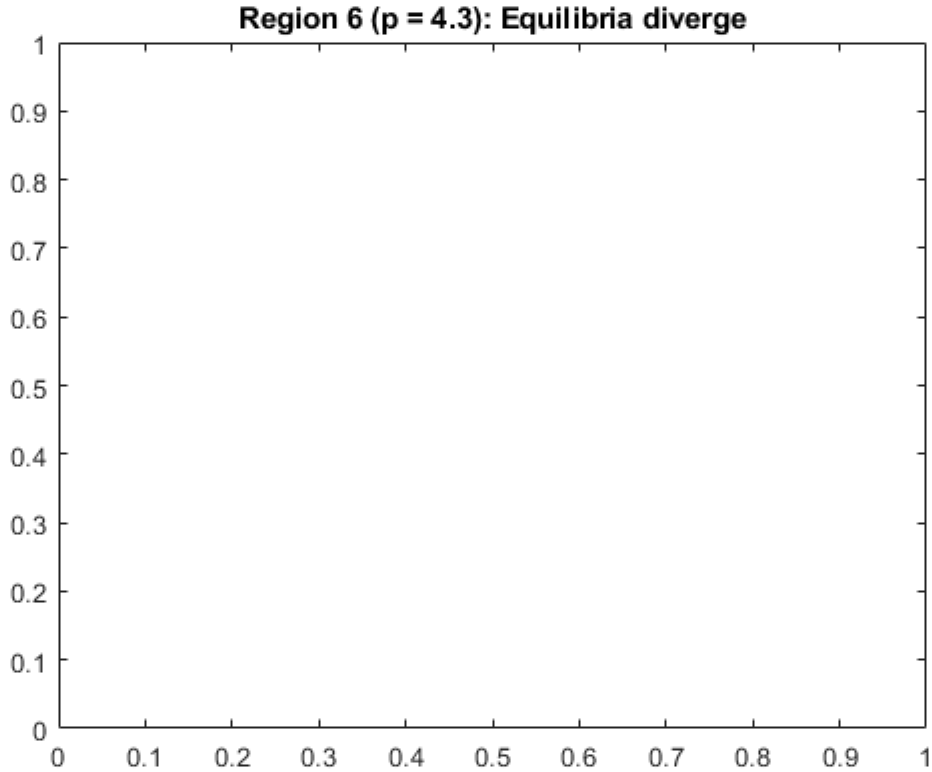
Region 1: Zero Amplitude
×10^-308
1.5
1
0.5
0
-0.5
-1
-1.5
-1
-0.5
0
0.5
1
1.5


Region 2: Diverges
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1


Region 3: Diverges
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1


Region 4: Diverges
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1


Region 5: Chaos (Diverges)
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

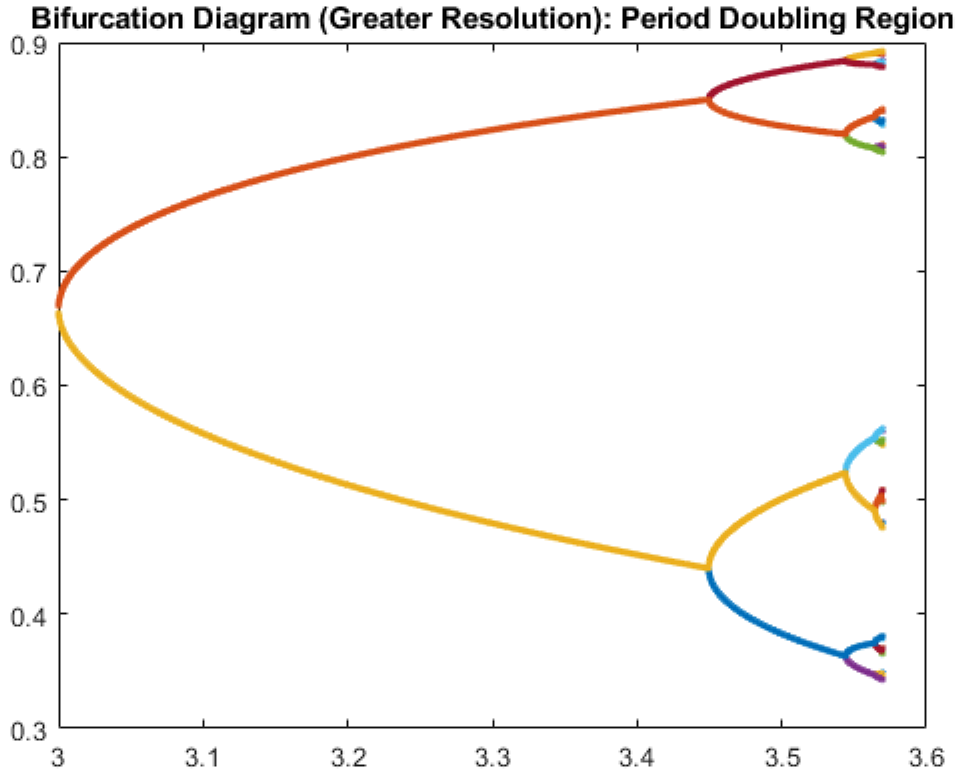


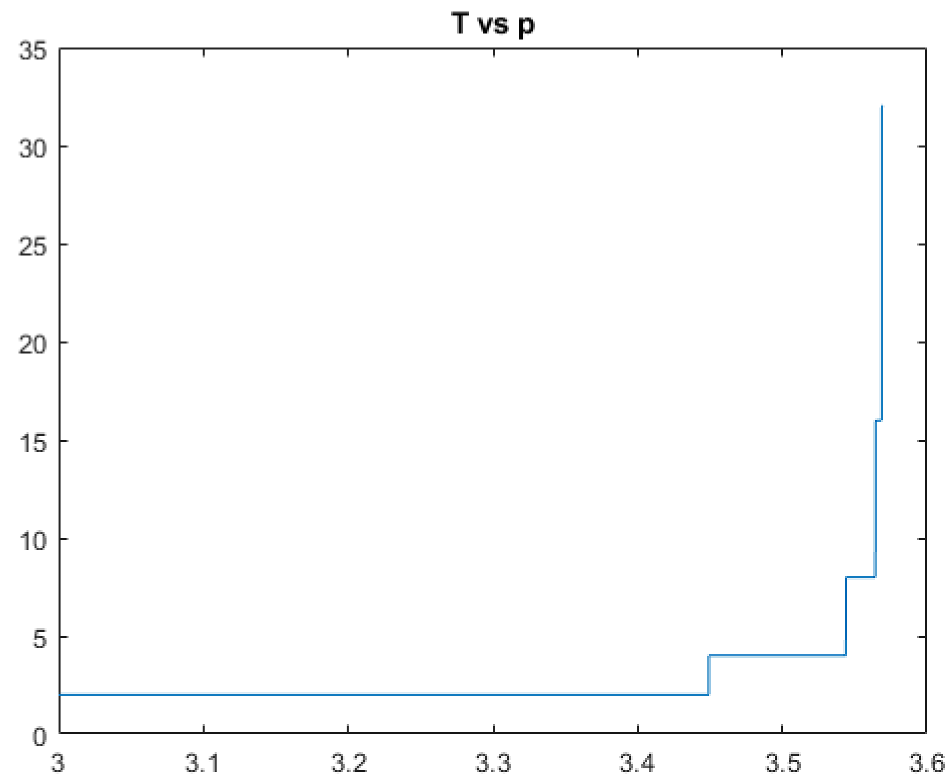
Problem 3

```
M = [];
kmax = 5000;
init = 0.5;
probs = 3:0.0001:3.5697;
for p = 3:0.0001:3.5697
    res = compute_logistic_map(p, init, kmax);
    M = [M, res];
end
M_slice = M(end-100:end,:); % the last 100 or so rows of M
figure;
plot(probs,M_slice,'.');
title('Bifurcation Diagram (Greater Resolution): Period Doubling
 Region');
[row_num_M,col_num_M] = size(M_slice);

Ts = [];
deltas = [];
for i=1:col_num_M
    [T,delta] = compute_delta(M_slice(:,i));
    Ts = [Ts, T];
end
figure
plot(probs,Ts)
```

```
title('T vs p')

periods = [2, 4, 8, 16, 32];
ws = [];
for i=[2,4,8,16,32]
    indices = find(Ts==i);
    min_index = min(indices);
    max_index = max(indices);
    p_min = probs(min_index);
    p_max = probs(max_index);
    w = p_max-p_min;
    ws = [ws w];
    fprintf('T = %.4f \t p_min = %.4f \t p_max = %.4f \t w = %.4f\n',
 i, p_min, p_max, w)
end

F = [];
for i = 1:length(ws)-1
    ratio = ws(i)/ws(i+1);
    F = [F ratio];
end

% List of ratios: F1, F2, F3, and F4
F

% All ratios are between 4.6 and 4.8, and converging to about 4.7 or
 so. We
% know that the actual Feigenbaum constant is 4.669 so this checks
 out.
```

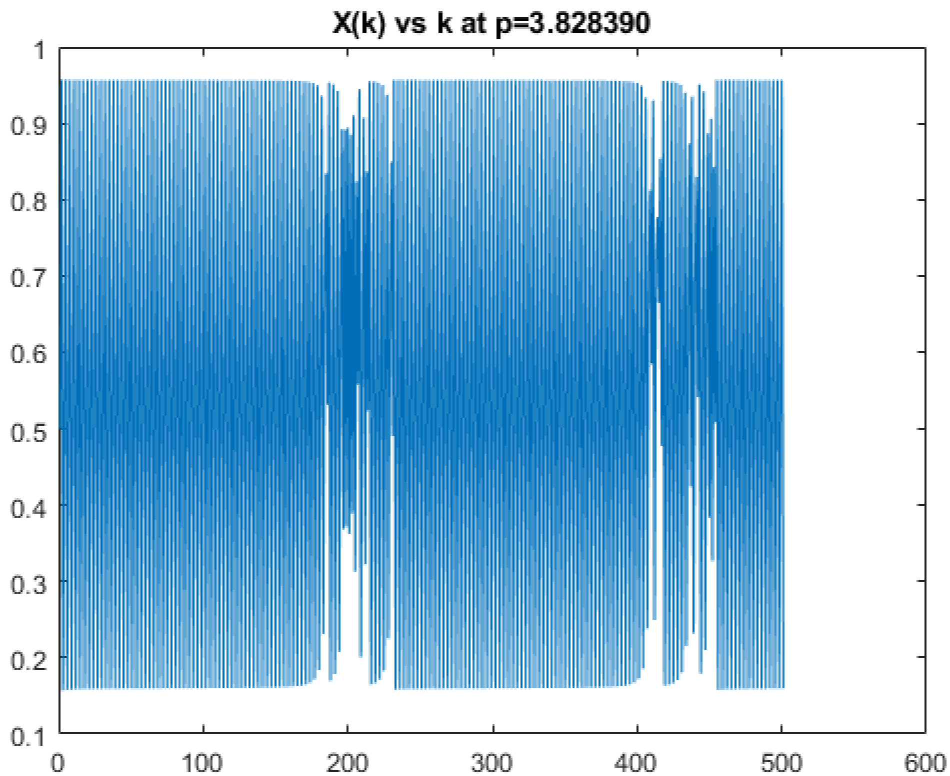
```
T = 2.0000   p_min = 3.0000   p_max = 3.4491   w = 0.4491
T = 4.0000   p_min = 3.4492   p_max = 3.5439   w = 0.0947
T = 8.0000   p_min = 3.5440   p_max = 3.5643   w = 0.0203
T = 16.0000   p_min = 3.5644   p_max = 3.5687   w = 0.0043
T = 32.0000   p_min = 3.5688   p_max = 3.5697   w = 0.0009

F =

    4.7423    4.6650    4.7209    4.7778
```

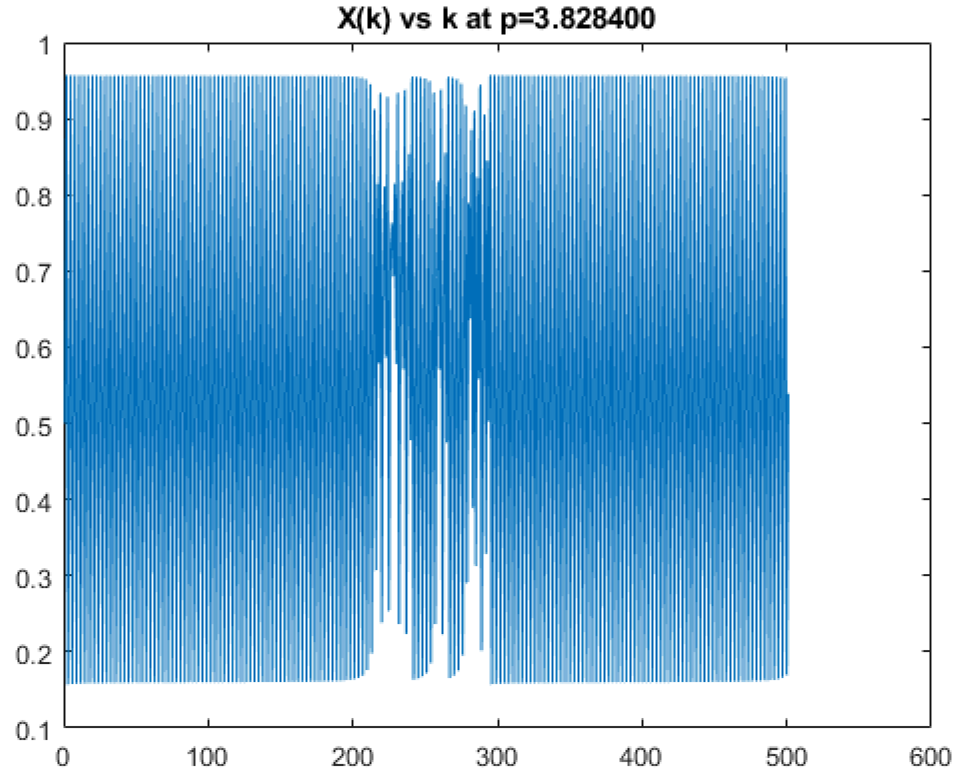

Bifurcation Diagram (Greater Resolution): Period Doubling Region
0.9
0.8
0.7
0.6
0.5
0.4
0.3
3
3.1
3.2
3.3
3.4
3.5
3.6

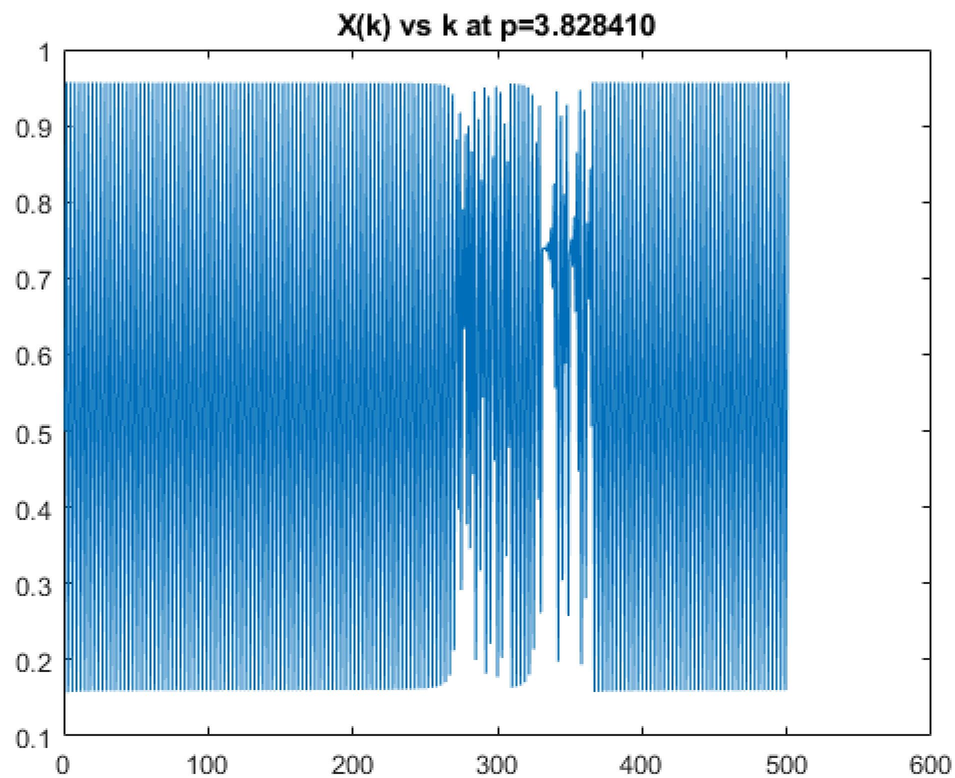

T vs p
35
30
25
20
15
10
5
0
3
3.1
3.2
3.3
3.4
3.5
3.6

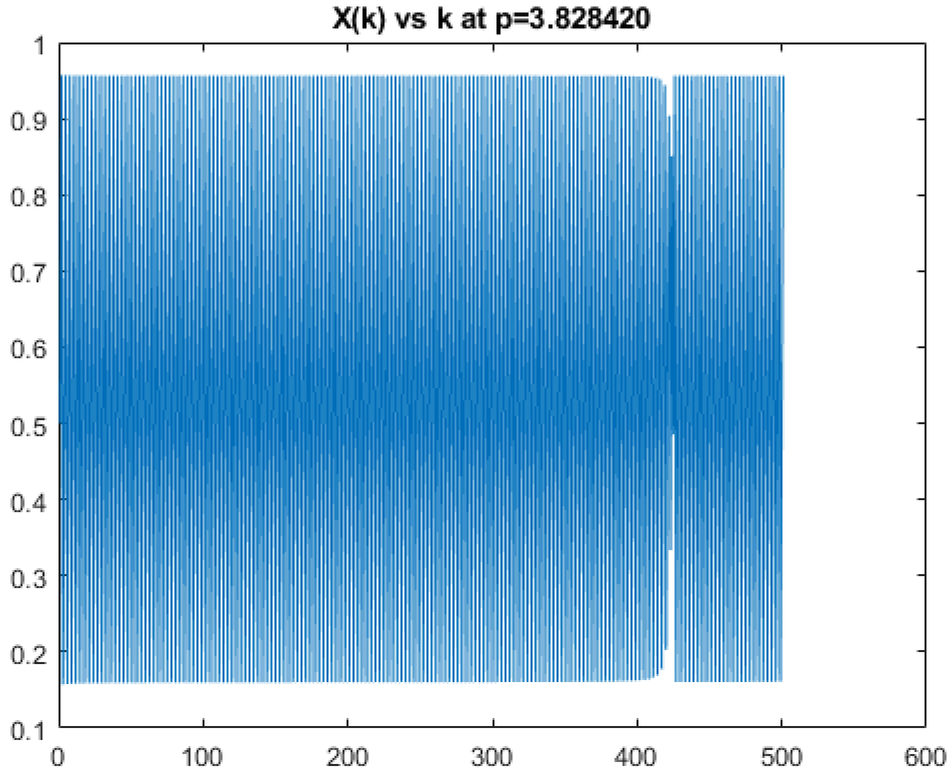
Problem 4

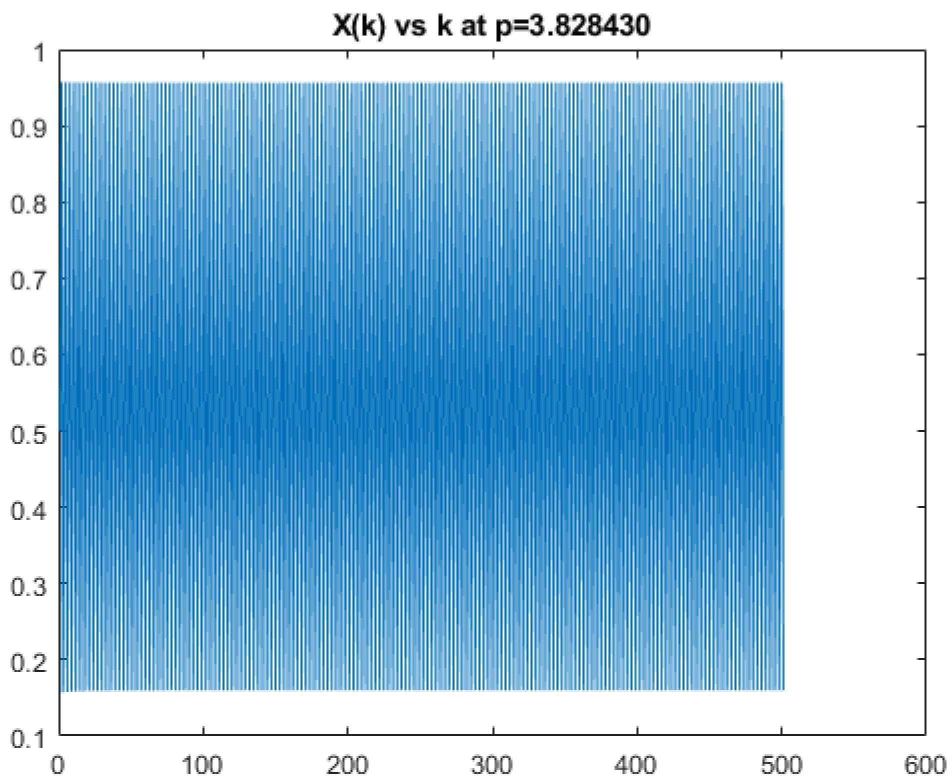
```
kmax = 500;
init = 0.5;
for p = 3.82839:0.00001:3.82843
    res = compute_logistic_map(p, init, kmax);
    figure
    plot(res)
    title(sprintf('X(k) vs k at p=%f',p))
end

% At around 3.82841, we find a brief "disruption"
```




X(k) vs k at p=3.828400
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
100
200
300
400
500
600

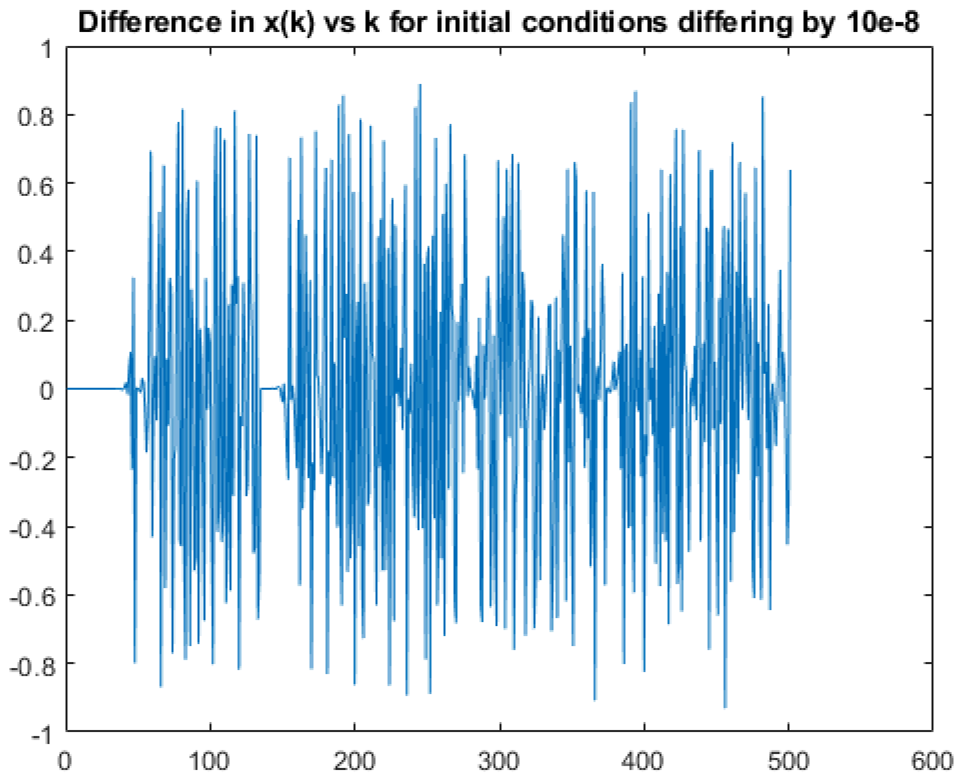

X(k) vs k at p=3.828410
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
100
200
300
400
500
600


X(k) vs k at p=3.828420
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
100
200
300
400
500
600


X(k) vs k at p=3.828430
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
100
200
300
400
500
600

Problem 5

```
kmax = 500;
init1 = 0.5;
init2 = 0.5 + 10e-8;

p = 3.95;

res1 = compute_logistic_map(p, init1, kmax);
res2 = compute_logistic_map(p, init2, kmax);

final_res = res2 - res1;
figure;
plot(final_res)
title('Difference in x(k) vs k for initial conditions differing by
 10e-8')

% At around k = 42, the two solutions become visibly distinct
```



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