物理海洋学笔记

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https://github.com/Cuiyingzhe/Physical-Oceanography-Notes/

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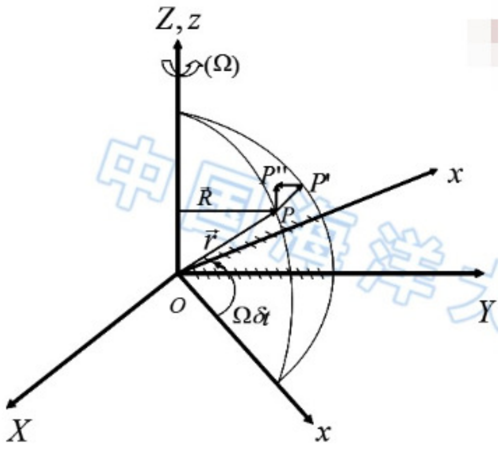
1 基本方程

1.1 旋转坐标系的速度和加速度

惯性坐标系: 静止的或是匀速直线运动的坐标系，固定在恒星上的坐标系可以被看成惯性坐标系.
固定在地球上的坐标系: 地球对恒星的加速度主要是由地球自转引起的，于是可以把地球当作一个对惯性坐标系作纯粹地转运动的物体.

1.1.1 旋转坐标系和惯性坐标系中的速度



惯性坐标系(XYZ)绝对位移：$\vec{pp''} = \vec{V}_a \delta t, \vec{V}_a$为绝对速度
旋转坐标系(xyz)相对位移：$\vec{p'p''} = \vec{V}\delta t, \vec{V}$为相对速度
$\because \vec{pp''} = \vec{p'p''} + \vec{pp'}$
$\therefore \vec{V}_a \delta t = \vec{V}\delta t + \vec{V}_e \delta t \Rightarrow \vec{V}_a = \vec{V} + \vec{V}_e$(绝对速度等于相对速度与牵连速度的向量和)
其中，$\vec{V}_e = \vec{\Omega} \times \vec{r} \Rightarrow \vec{V}_a = \vec{V} + \vec{\Omega} \times \vec{r}$

$$\Rightarrow \frac{d_a \vec{r}}{dt} = \frac{d\vec{r}}{dt} + \vec{\Omega} \times \vec{r}$$

$$\frac{d_a \vec{A}}{dt} = \frac{d\vec{A}}{dt} + \vec{\Omega} \times \vec{A}$$

1.1.2 旋转坐标系和惯性坐标系中的加速度

令$\vec{A} = \vec{V}_a = \vec{V}_e + \vec{V} = \vec{V} + \vec{\Omega} \times \vec{r}$

$$\begin{aligned}
\frac{d\bar{V}_a}{dt} &= \frac{d_a}{dt}\left(\vec{V} + \vec{V}_e\right) = \frac{d_a}{dt}(\vec{V} + \vec{\Omega} \times \vec{r}) \\
&= \frac{d}{dt}(\vec{V} + \vec{\Omega} \times \vec{r}) + \vec{\Omega} \times (\vec{V} + \vec{\Omega} \times \vec{r}) \\
&= \frac{d\vec{V}}{dt} + \vec{\Omega} \times \vec{V} + \vec{\Omega} \times \vec{V} + \vec{\Omega} \times (\vec{\Omega} \times \vec{r}) \\
&= \frac{d\vec{V}}{dt} + 2\vec{\Omega} \times \vec{V} - \Omega^2 \vec{R}
\end{aligned}$$

1.2 作用在海水微团上的外力运动方程的向量形式

压强梯度力：$\frac{1}{\rho}\nabla p$
分子粘性力(摩擦力)：

$$\begin{cases}
F_x = \frac{1}{\rho}\mu\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right] = \frac{\mu}{\rho}\Delta u \\
F_y = \frac{1}{\rho}\mu\left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right] = \frac{\mu}{\rho}\Delta v \\
F_2 = \frac{1}{\rho}\mu\left[\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right] = \frac{\mu}{\rho}\Delta w
\end{cases} \Rightarrow \vec{F} = \frac{\mu}{\rho}\Delta\vec{V} = \gamma\Delta\vec{v}$$

重力(地球引力与地球自转产生的惯性离心力的合力)：$\vec{g} = -G\frac{M_g}{r^2} \cdot \left(\frac{\vec{r}}{r}\right)$
科氏力：$-2\vec{\Omega} \times \vec{V}$

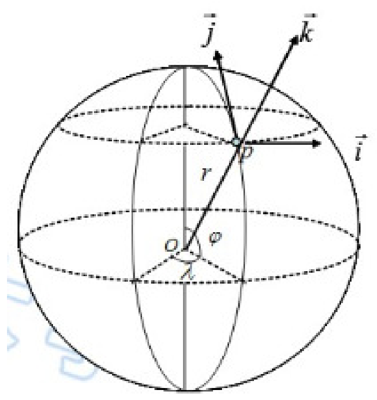
天体引潮力(受其他天体万有引力与惯性力离心力的合力)：$\vec{F_M} = -G\frac{M_M}{L^2} + G\frac{M_M}{D^2}\cdot\left(\frac{\vec{D}}{D}\right)$

由牛顿第二定律和坐标系转换关系：

$$\begin{cases} \frac{d_a\vec{V}_a}{dt} = \sum_i \vec{F}_t \\ \frac{d_a\vec{A}_a}{dt} = \frac{d\vec{V}}{dt} + 2\vec{\Omega}\times\vec{V} - \Omega^2\vec{R} \end{cases}$$

$$\Rightarrow \boxed{\frac{d\vec{V}}{dt} = -\frac{1}{\rho}\nabla P - 2\vec{\Omega}\times\vec{V} + \vec{g} + \nu\Delta\vec{V} + \vec{F_T}}$$

1.3 运动方程在球坐标系的标量形式



速度：

$$\vec{V} = u\vec{i} + v\vec{j} + w\vec{k}$$

$$\Rightarrow \begin{cases} u = r\cos\varphi\frac{d\lambda}{dt} \\ v = r\frac{d\varphi}{dt} \\ w = \frac{dr}{dt} \end{cases}$$

加速度：

$$\begin{aligned} \frac{d\vec{A}}{dt} &= \frac{\frac{\partial\vec{A}}{\partial t}dt + \frac{\partial\vec{A}}{\partial\lambda}d\lambda + \frac{\partial\vec{A}}{\partial\varphi}d\varphi + \frac{\partial\vec{A}}{\partial r}dr}{dt} \\ &= \frac{\partial\vec{A}}{\partial t} + \frac{\partial\vec{A}}{\partial\lambda}\frac{d\lambda}{dt} + \frac{\partial\vec{A}}{\partial\varphi}\frac{d\varphi}{dt} + \frac{\partial\vec{A}}{\partial r}\frac{dr}{dt} \\ &= \frac{\partial\vec{A}}{\partial t} + \frac{u}{r\cos\varphi}\frac{\partial\vec{A}}{\partial\lambda} + \frac{v}{r}\frac{\partial\vec{A}}{\partial\varphi} + w\frac{\partial\vec{A}}{\partial r} \end{aligned}$$

$$\Rightarrow \frac{d}{dt} = \frac{\partial}{\partial t} + u\frac{\partial}{r\cos\varphi\partial\lambda} + v\frac{\partial}{r\partial\varphi} + w\frac{\partial}{\partial r}$$

$$\Rightarrow \boxed{\boldsymbol{\frac{d}{dt} = \frac{\partial}{\partial t} + (\vec{V}\cdot\nabla)}}$$

$$\Rightarrow \boxed{\boldsymbol{\nabla = \frac{\partial}{r\cos\varphi\partial\lambda}\vec{i} + \frac{\partial}{r\partial\varphi}\vec{j} + \frac{\partial}{\partial r}\vec{k}}}$$

$$\Rightarrow \begin{cases} \frac{d\vec{i}}{dt} = \frac{\partial\vec{i}}{\partial t} + u\frac{\partial\vec{i}}{r\cos\varphi\partial\lambda} + v\frac{\partial\vec{i}}{r\partial\varphi} + w\frac{\partial\vec{i}}{\partial r} \\ \frac{\partial\vec{j}}{dt} = \frac{\partial\vec{j}}{\partial t} + u\frac{\partial\vec{j}}{r\cos\varphi\partial\lambda} + v\frac{\partial\vec{j}}{r\partial\varphi} + w\frac{\partial\vec{j}}{\partial r} \\ \frac{d\vec{k}}{dt} = \frac{\partial\vec{k}}{\partial t} + u\frac{\partial\vec{k}}{r\cos\varphi\partial\lambda} + v\frac{\partial\vec{k}}{r\partial\varphi} + w\frac{\partial\vec{k}}{\partial r} \end{cases}$$

$$\frac{d\vec{V}}{dt}=\frac{du}{dt}\vec{i}+\frac{dv}{dt}\vec{j}+\frac{dv}{dt}\vec{k}+u\frac{d\vec{i}}{dt}+v\frac{d\vec{j}}{dt}+w\frac{d\vec{k}}{dt}$$

$$\Rightarrow\boxed{\frac{d\vec{V}}{dt}=\left(\frac{du}{dt}-\frac{uvtg\varphi}{r}+\frac{uw}{r}\right)\vec{i}+\left(\frac{dv}{dt}+\frac{u^2\operatorname{tg}\varphi}{r}+\frac{vw}{r}\right)\vec{j}+\left(\frac{dw}{dt}-\frac{u^2+v^2}{r}\right)}$$

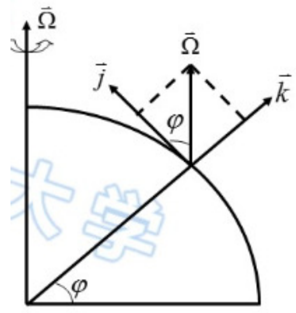
压强梯度力：

$$\frac{1}{\rho}\nabla p=-\frac{1}{\rho}\left(\frac{1}{r\cos\varphi}\frac{\partial p}{\partial\lambda}\vec{i}+\frac{1}{r}\frac{\partial p}{\partial\varphi}\vec{j}+\frac{\partial p}{\partial r}\vec{k}\right)$$

重力：

$$\vec{g}=-g\vec{k}$$

科氏力：



$$\vec{\Omega}=\Omega\sin\varphi\vec{k}+\Omega\cos\varphi\vec{j}$$

$$-2\vec{\Omega}\times\vec{V}=-2\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 0 & \Omega\cos\varphi & \Omega\sin\varphi\\ u & v & w\end{vmatrix}$$

$$=-2[(w\Omega\cos\varphi-v\Omega\sin\varphi)\vec{i}+(u\Omega\sin\varphi)\vec{j}+(-u\Omega\cos\varphi)\vec{k}]$$

$$\Rightarrow -2\vec{\Omega}\times\vec{V}=(fv-\tilde{f}w)\vec{i}-(fu)\vec{j}+(\tilde{f}u)\vec{k}$$

其中，$\begin{cases}f=2\Omega sin\varphi\\ \tilde{f}=2\Omega cos\varphi\end{cases}$

$$\Rightarrow\boxed{\begin{cases}\dfrac{du}{dt}=-\dfrac{1}{\rho}\dfrac{\partial p}{r\cos\varphi\partial\lambda}+fv-\tilde{f}w+\dfrac{uv\tan\varphi}{r}-\dfrac{uw}{r}+\gamma(\Delta\vec{v})_\lambda-\dfrac{1}{r\cos\varphi}\dfrac{\partial\phi_T}{\partial\lambda}\\ \dfrac{dy}{dt}=7\dfrac{1}{\rho}\dfrac{\partial p}{r\partial\varphi}-fu-\dfrac{u^2\tan\varphi}{r}-\dfrac{vw}{r}+\gamma(\Delta\bar{v})_\varphi-\dfrac{1}{r}\dfrac{\partial\phi_T}{\partial\varphi}\\ \dfrac{dw}{dt}=-\dfrac{1}{\rho}\dfrac{\partial p}{\partial r}+\tilde{f}u=g+\dfrac{u^2+v^2}{r}+\gamma(\Delta\vec{v})_r-\dfrac{\partial\phi_T}{\partial r}\end{cases}}$$

1.4 直角坐标系的运动方程

略去地球曲率的影响

$$\Rightarrow\boxed{\begin{cases}\dfrac{du}{dt}=-\dfrac{1}{\rho}\dfrac{\partial p}{\partial x}+fv+\tilde{f}w+F_{N\lambda}+F_{T\lambda}\\ \dfrac{dv}{dt}=-\dfrac{1}{\rho}\dfrac{\partial p}{\partial x}-fu+F_{Ny}+F_{Ty}\\ \dfrac{dw}{dt}=-\dfrac{1}{\rho}\dfrac{\partial p}{\partial z}+\tilde{f}u-g+F_{Nz}+F_{Tz}\end{cases}}$$

1.5 海水层流运动的基本方程组

1.5.1 连续方程

$$\frac{\partial\rho}{\partial t}+\nabla\cdot(\rho\vec{V})=0\Leftrightarrow\frac{d\rho}{dt}+\rho\nabla\cdot\vec{V}=0$$

特别地，对于不可压缩流体：

$$\nabla\cdot\vec{V}=0$$

1.5.2 盐量扩散方程

$$
\begin{aligned}
&\overset{\text{盐量增加量}}{\frac{\partial}{\partial t}\iiint_{\tau}\rho s d\tau} = \overset{\text{平流作用}}{-\oiint_{\sigma}\rho s V_n d\sigma} + \overset{\text{分子扩散作用}}{-\oiint_{\sigma} S_n d\sigma} \\
&\iiint_{\tau}\frac{\partial(\rho s)}{\partial t}d\tau = \iiint_{\tau}\nabla\cdot(\rho s\vec{V})d\tau - \iiint_{\tau}\nabla\cdot\vec{S}d\tau \\
\Rightarrow &\frac{\partial(\rho s)}{\partial t} + \nabla\cdot(\rho s\vec{V}) + \nabla\cdot\vec{S} = 0 \\
\Rightarrow &\rho\frac{\partial s}{\partial t} + s\frac{\partial \rho}{\partial t} + s\nabla\cdot(\rho\vec{V}) + \rho\vec{V}\cdot\nabla s + \nabla\cdot\vec{S} = 0 \\
\Rightarrow &\left(\frac{\partial s}{\partial t} + \vec{V}\cdot\nabla s\right) + \frac{s}{\rho}\left[\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\vec{V})\right] = -\frac{1}{\rho}\nabla\cdot\vec{S} \\
\Rightarrow &\frac{\partial s}{\partial t} + \vec{V}\cdot\nabla s = \frac{k}{\rho}\Delta s = k_D\Delta s
\end{aligned}
$$

其中，$k_D = \frac{k}{\rho} \sim 1.1\times10^{-9}\left(\mathrm{m}^2/\mathrm{s}\right)$

1.5.3 热传导方程

与上面类似：

$$
\frac{\partial\theta}{\partial t} + \vec{V}\cdot\nabla\theta = \frac{\kappa}{\rho c_p}\Delta\theta = k_\theta\Delta\theta
$$

其中，$k_\theta = \frac{\kappa}{\rho c_p} \sim 1.4\times10^{-7}\left(\mathrm{m}^2/\mathrm{s}\right)$

1.5.4 热膨胀方程-状态方程

热膨胀方程：

$$
\rho = \underset{\rho_0}{\text{0℃时的海水密度}}(1 - \underset{k}{\text{海水的热膨胀系数}}\theta)
$$

EOS80国际海水状态方程：

$$
\rho(s,t,p) = \rho(s,t,0)\left[1 - \frac{np}{k(s,t,p)}\right]^{-1}
$$

1.6 基本方程的矢量形式和标量形式

矢量形式：

$$
\boxed{\begin{cases}
\frac{d\vec{V}}{dt} = -\frac{1}{\rho}\nabla p - 2\Omega\times\vec{V} + \vec{g} + \gamma\Delta\vec{V} - \nabla\phi_T \\
\frac{\partial\rho}{\partial t} + \rho\nabla\cdot\vec{V} = 0 \\
\frac{\partial s}{\partial t} + \vec{V}\cdot\nabla s = k_D\Delta s \\
\frac{\partial\theta}{\partial t} + \vec{V}\cdot\nabla\theta = k_\theta\Delta\theta \\
\rho = \rho(\theta,s,p)
\end{cases}}
$$

标量形式(直角坐标系)：

$$
\boxed{\begin{cases}
\frac{dv}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial y} - fu + \gamma\Delta v - \frac{\partial\phi_T}{\partial y} \\
\frac{dw}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \tilde{f}u - g + \gamma\Delta w - \frac{\partial\phi_T}{\partial z} \\
\frac{d\rho}{dt} + \rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) = 0 \\
\frac{\partial s}{\partial t} + u\frac{\partial s}{\partial x} + v\frac{\partial s}{\partial y} + w\frac{\partial s}{\partial z} = k_D\left(\frac{\partial^2 s}{\partial x^2} + \frac{\partial^2 s}{\partial y^2} + \frac{\partial^2 s}{\partial z^2}\right) \\
\frac{\partial\theta}{\partial t} + u\frac{\partial\theta}{\partial x} + v\frac{\partial\theta}{\partial y} + w\frac{\partial\theta}{\partial z} = k_\theta\left(\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2} + \frac{\partial^2\theta}{\partial z^2}\right) \\
\rho = \rho(\theta,s,p)
\end{cases}}
$$

1.7 边界条件

无质量交换的运动学边界条件：

$$\frac{\partial F}{\partial t} + \vec{c} \cdot \nabla F = 0$$

例：

(1) 海面($z = \zeta(x, y, t)$)：$\dfrac{\partial \zeta}{\partial t} + \vec{V}_H \cdot \nabla_H \zeta - w = 0$

(2) 海底($z = -h(x, y)$): $\vec{V}_H \cdot \nabla_H h + w = 0$

动力学边界条件：

由牛顿第三定律，在界面法线方向有：

$$(\vec{p}_n)_1 = (\vec{p}_n)_2$$

1.8 ※间平均的基本方程和边界条件(直角坐标系)

连续方程:

$$\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} + \frac{\partial \bar{w}}{\partial z} = 0$$

运动方程：

$$\begin{cases} \dfrac{\partial \bar{u}}{\partial t} + \bar{u}\dfrac{\partial \bar{u}}{\partial x} + \bar{v}\dfrac{\partial \bar{u}}{\partial y} + \bar{w}\dfrac{\partial \bar{u}}{\partial z} = -\dfrac{1}{\rho}\dfrac{\partial \bar{p}}{\partial x} + f\bar{v} - \tilde{f}w + \gamma\Delta\bar{u} - \dfrac{\partial \bar{\phi}_T}{\partial x} + \dfrac{\partial}{\partial x}\left(A_{x\alpha}\dfrac{\partial \bar{u}}{\partial x}\right) + \dfrac{\partial}{\partial y}\left(A_{xy}\dfrac{\partial \bar{u}}{\partial y}\right) + \dfrac{\partial}{\partial z}\left(A_{xz}\dfrac{\partial \bar{u}}{\partial z}\right) \\ \dfrac{\partial \bar{v}}{\partial t} + \vec{u}\dfrac{\partial \bar{v}}{\partial x} + \vec{v}\dfrac{\partial \bar{v}}{\partial y} + \vec{w}\dfrac{\partial \bar{v}}{\partial z} = -\dfrac{1}{\rho}\dfrac{\partial \bar{p}}{\partial y} - f\bar{u} + \gamma\Delta\bar{v} - \dfrac{\partial \bar{\phi}_T}{\partial y} + \dfrac{\partial}{\partial x}\left(A_{yx}\dfrac{\partial \bar{v}}{\partial x}\right) + \dfrac{\partial}{\partial y}\left(A_{yy}\dfrac{\partial \bar{v}}{\partial y}\right) + \dfrac{\partial}{\partial z}\left(A_{yz}\dfrac{\partial \bar{v}}{\partial z}\right) \\ \dfrac{\partial \bar{w}}{\partial t} + \bar{u}\dfrac{\partial \bar{w}}{\partial x} + \bar{v}\dfrac{\partial \bar{w}}{\partial y} + \bar{w}\dfrac{\partial \bar{w}}{\partial z} = -\dfrac{1}{\rho}\dfrac{\partial \bar{p}}{\partial z} + \tilde{f}\bar{u} - g + \gamma\Delta\bar{w} - \dfrac{\partial \bar{\phi}_T}{\partial z} + \dfrac{\partial}{\partial x}\left(A_{2x}\dfrac{\partial \bar{w}}{\partial x}\right) + \dfrac{\partial}{\partial y}\left(A_{zy}\dfrac{\partial \bar{w}}{\partial y}\right) + \dfrac{\partial}{\partial z}\left(A_{zz}\dfrac{\partial \bar{w}}{\partial z}\right) \end{cases}$$

盐量扩散方程：

$$\frac{\partial \bar{s}}{\partial t} + \bar{u}\frac{\partial \bar{s}}{\partial x} + \bar{v}\frac{\partial \bar{s}}{\partial y} + \bar{w}\frac{\partial \bar{s}}{\partial z}$$

热传导方程：

$$\frac{\partial \bar{\theta}}{\partial t} + \vec{u}\frac{\partial \bar{\theta}}{\partial x} + \vec{v}\frac{\partial \bar{\theta}}{\partial y} + \vec{w}\frac{\partial \bar{\theta}}{\partial z} = k_\theta \Delta\bar{\theta} + \frac{\partial}{\partial x}\left(K_{\theta_x}\frac{\partial \bar{\theta}}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_{\theta y}\frac{\partial \bar{\theta}}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_{\theta z}\frac{\partial \bar{\theta}}{\partial z}\right)$$

状态方程：

$$\bar{\rho} = \bar{\rho}(\bar{s}, \bar{\theta}, \bar{p})$$

1.9 铅直向平均的基本方程

$$\frac{\partial}{\partial x}[(h+\zeta)\langle u\rangle] + \frac{\partial}{\partial y}[(h+\zeta)\langle v\rangle] - \left[u|_\zeta \frac{\partial \zeta}{\partial x} + v|_\zeta \frac{\partial \zeta}{\partial y} - w|_\zeta\right] - \left[u|_{-h}\frac{\partial h}{\partial x} + v|_{-h}\frac{\partial h}{\partial y} + w|_{-h}\right] = \mathbf{0}$$

$$\frac{\partial \zeta}{\partial t} + \frac{\partial[(h+\zeta)\langle u\rangle]}{\partial x} + \frac{\partial[(h+\zeta)\langle v\rangle]}{\partial y} = 0$$

1.10 尺度分析

Rossby数 Ro=$\dfrac{U}{FL}$ $\begin{cases} \gg 1: \text{平流非线性项比Coriolis力重要,大尺度运动} \\ = 1: \text{平流非线性项与Coriolis力同等重要} \\ \ll 1: \text{平流非线性项可以忽略,小尺度运动} \end{cases}$

水平Ekman数 $\mathrm{E_l} = \dfrac{A_l}{FL^2}$ 水平湍流摩擦项与Coriolis力比值

垂直Ekman数 $\mathrm{E_z} = \dfrac{A_z}{FD^2}$ 垂直湍流摩擦项与Coriolis力比值

准静力近似 f平面近似 β平面近似 Boussinesq近似

2 海流

2.1 地转流

地转流：不考虑海面风的作用，远离沿岸的大洋中部大尺度、准水平、定常的海水流动.
产生原因：海水受热力和动力因素导致压力(和密度)在水平方向分布不均匀.

$$p = p_a + \rho g h \quad \rho \begin{cases} \neq \rho_0 \Rightarrow \text{梯度流} \\ = \rho_0 \Rightarrow \text{倾斜流} \end{cases}$$

2.1.1 梯度流

假定和方程

(1) 在相当长一段时间里海面温度变化和降水蒸发变化都不大，于是可以认为已形成的海水密度场、温度场和盐度场近似于定常，从而相应的海水运动也近似于定常：$\dfrac{\partial}{\partial t} = 0$.
(2) 海洋深而宽广，在远离海岸及海底的大洋中部海区，大尺度运动：$\mathrm{Ro} \ll 1$.
(3) 不考虑海底摩擦及边界摩擦的影响，且海面无风力作用，则流动属一种无摩擦流动：$\mathrm{E_l}, \mathrm{E_z} \ll 1$.
(4) β平面近似准静力近似
x方向基本方程：

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + fv + \frac{\partial}{\partial x}\left(A_{xx}\frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(A_{xy}\frac{\partial u}{\partial y}\right) + \frac{\partial}{\partial z}\left(A_{xz}\frac{\partial u}{\partial z}\right)$$

假定(1)$\Rightarrow \dfrac{\partial u}{\partial t} = 0$

假定(2)$\Rightarrow u\dfrac{\partial u}{\partial x} + v\dfrac{\partial u}{\partial y} + w\dfrac{\partial u}{\partial z} = 0$

假定(3)$\Rightarrow \dfrac{\partial}{\partial x}\left(A_{xx}\dfrac{\partial u}{\partial x}\right) + \dfrac{\partial}{\partial y}\left(A_{xy}\dfrac{\partial u}{\partial y}\right) + \dfrac{\partial}{\partial z}\left(A_{xz}\dfrac{\partial u}{\partial z}\right) = 0$

可得梯度流的控制方程：

$$\begin{cases} 0 = -\dfrac{1}{\rho}\dfrac{\partial p}{\partial x} + fv \\ 0 = -\dfrac{1}{\rho}\dfrac{\partial p}{\partial y} - fu \\ 0 = -\dfrac{1}{\rho}\dfrac{\partial p}{\partial z} - g \\ \dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} + \dfrac{\partial w}{\partial z} = 0 \\ u\dfrac{\partial \theta}{\partial x} + v\dfrac{\partial \theta}{\partial y} + w\dfrac{\partial \theta}{\partial z} = 0 \\ u\dfrac{\partial s}{\partial x} + v\dfrac{\partial s}{\partial y} + w\dfrac{\partial s}{\partial z} = 0 \\ \rho = \rho(s, \theta) \end{cases}$$

特征

水平速度：

$$\begin{cases} 0 = -\dfrac{1}{\rho}\dfrac{\partial p}{\partial x} + fv & (1) \\ 0 = -\dfrac{1}{\rho}\dfrac{\partial p}{\partial y} - fu & (2) \end{cases}$$

$$\Longrightarrow \begin{cases} v = \dfrac{1}{\rho f}\dfrac{\partial p}{\partial x} \\ u = -\dfrac{1}{\rho f}\dfrac{\partial p}{\partial y} \end{cases}$$

(1) 水平速度和压强梯度成正比；
(2) 与密度和科氏参数成反比；
(3) 地转关系在赤道不成立($f=0$).
垂向速度：

$$\begin{aligned}
\frac{\partial(1)\times\rho}{\partial y}-\frac{\partial(2)\times\rho}{\partial x}\Leftrightarrow\frac{\partial(\rho f v)}{\partial v}+\frac{\partial(\rho f u)}{\partial x}=0 \\
&\Leftrightarrow f\rho\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)+f\left(u\frac{\partial\rho}{\partial x}+v\frac{\partial\rho}{\partial y}+w\frac{\partial\rho}{\partial z}\right)-f\rho\frac{\partial w}{\partial z}-fw\frac{\partial\rho}{\partial z}+\beta\rho v=0 \\
&\Leftrightarrow f\rho\overset{=0}{\boxed{\nabla\vec{V}}}+f\vec{V}\cdot\nabla\rho-f\rho\frac{\partial w}{\partial z}-fw\frac{\partial\rho}{\partial z}+\beta\rho v=0 \qquad (3)
\end{aligned}$$

$$\vec{V}\cdot\nabla\rho=\vec{V}\cdot\nabla\rho(s,\theta)=\vec{V}\cdot\left(\nabla s\frac{\partial\rho}{\partial s}+\nabla\theta\frac{\partial\rho}{\partial\theta}\right)=0$$

$$(3)\Leftrightarrow f\left(\rho\frac{\partial w}{\partial z}+w\frac{\partial\rho}{\partial z}\right)=\beta\rho v\overset{\text{尺度分析}}{\Rightarrow}f\frac{\partial w}{\partial z}=\beta v\overset{\text{尺度分析}}{\Rightarrow}W=\frac{\beta D}{F}U\sim 2\times10^{-4}U$$

垂向流速比水平流速小得多，地转流为准水平运动.

运动特性

$$(1)\times u+(2)\times v\Leftrightarrow u\frac{\partial p}{\partial x}+v\frac{\partial p}{\partial y}=0\Leftrightarrow\vec{V}_H\cdot\nabla_H p=0$$

(1) 梯度流平行于等压线；
(2) 北半球，流向右侧为高压，南半球相反；

密度特性

$$\frac{\partial(1)\times\rho}{\partial y}-\frac{\partial(2)\times\rho}{\partial x}\Leftrightarrow f\rho\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)+f\left(u\frac{\partial\rho}{\partial x}+v\frac{\partial\rho}{\partial y}\right)+\rho u\frac{\partial f}{\partial x}+\rho v\frac{\partial f}{\partial y}=0\Leftrightarrow\vec{V}_H\cdot\nabla_H\rho=0$$

(1) 梯度流近似平行于等密线；
(2) 在北半球，流向右侧密度小；
(3) 等压面倾斜与等密面倾斜方向相反.

温盐特性
忽略垂向运动：

$$\begin{aligned}\vec{V}_H\cdot\nabla_H\theta=0\\ \vec{V}_H\cdot\nabla_H s=0\end{aligned}$$

(1) 梯度流平行于等温线和等盐线；
(2) 在北半球，流向右侧温度高，盐度低.

2.1.2 倾斜流

假定和方程 (1) 海水密度为常数；
(2) 水平方向的压强梯度是由海面倾斜引起的.

$$\Rightarrow p=p_a+\int_z^\zeta\rho g dz=p_a+\rho g(\zeta-z)$$

倾斜流的控制方程：

$$\begin{cases}fv=g\dfrac{\partial\zeta}{\partial x} & (4)\\ fu=-g\dfrac{\partial\zeta}{\partial y} & (5)\end{cases}$$

性质：

$$(4) \times u - (5) \times v \Leftrightarrow u\frac{\partial \zeta}{\partial x} + v\frac{\partial \zeta}{\partial y} = 0 \Leftrightarrow \vec{V}_H \cdot \nabla \zeta = 0$$

(1) 倾斜流平行于等水位线；

(2) 在北半球，流向右侧水位高；

(3) 倾斜流从表至底流速流向相同，压强梯度相同.