

物理海洋学笔记

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<https://github.com/Cuiyingzhe/Physical-Oceanography-Notes/>

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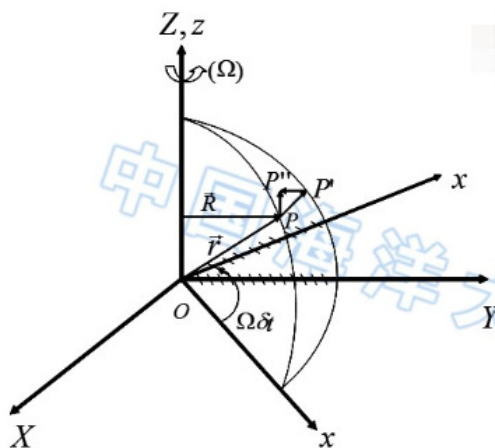
1 基本方程

1.1 旋转坐标系的速度和加速度

惯性坐标系: 静止的或是匀速直线运动的坐标系, 固定在恒星上的坐标系可以被看成惯性坐标系.

固定在地球上的坐标系: 地球对恒星的加速度主要是由地球自转引起的, 于是可以把地球当作一个对惯性坐标系作纯粹地转运动的物体.

1.1.1 旋转坐标系和惯性坐标系中的速度



惯性坐标系(XYZ)绝对位移: $p\vec{p}'' = \vec{V}_a \delta t$, \vec{V}_a 为绝对速度

旋转坐标系(xyz)相对位移: $p'\vec{p}'' = \vec{V} \delta t$, \vec{V} 为相对速度

$\therefore p\vec{p}'' = p'\vec{p}'' + p\vec{p}'$

$\therefore \vec{V}_a \delta t = \vec{V} \delta t + \vec{V}_e \delta t \Rightarrow \vec{V}_a = \vec{V} + \vec{V}_e$ (绝对速度等于相对速度与牵连速度的向量和)

其中, $\vec{V}_e = \vec{\Omega} \times \vec{r} \Rightarrow \vec{V}_a = \vec{V} + \vec{\Omega} \times \vec{r}$

$$\Rightarrow \frac{d_a \vec{r}}{dt} = \frac{d\vec{r}}{dt} + \vec{\Omega} \times \vec{r}$$

$$\frac{d_a \vec{A}}{dt} = \frac{d\vec{A}}{dt} + \vec{\Omega} \times \vec{A}$$

1.1.2 旋转坐标系和惯性坐标系中的加速度

令 $\vec{A} = \vec{V}_a = \vec{V}_e + \vec{V} = \vec{V} + \vec{\Omega} \times \vec{r}$

$$\begin{aligned} \frac{d\vec{V}_a}{dt} &= \frac{d_a}{dt} (\vec{V} + \vec{V}_e) = \frac{d_a}{dt} (\vec{V} + \vec{\Omega} \times \vec{r}) \\ &= \frac{d}{dt} (\vec{V} + \vec{\Omega} \times \vec{r}) + \vec{\Omega} \times (\vec{V} + \vec{\Omega} \times \vec{r}) \\ &= \frac{d\vec{V}}{dt} + \vec{\Omega} \times \vec{V} + \vec{\Omega} \times \vec{V} + \vec{\Omega} \times (\vec{\Omega} \times \vec{r}) \\ &= \frac{d\vec{V}}{dt} + 2\vec{\Omega} \times \vec{V} - \Omega^2 \vec{R} \end{aligned}$$

1.2 作用在海水微团上的外力运动方程的向量形式

压强梯度力: $\frac{1}{\rho} \nabla p$

分子粘性力(摩擦力):

$$\begin{cases} F_x = \frac{1}{\rho} \mu \left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right] = \frac{\mu}{\rho} \Delta u \\ F_y = \frac{1}{\rho} \mu \left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right] = \frac{\mu}{\rho} \Delta v \\ F_z = \frac{1}{\rho} \mu \left[\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right] = \frac{\mu}{\rho} \Delta w \end{cases} \Rightarrow \vec{F} = \frac{\mu}{\rho} \Delta \vec{V} = \gamma \Delta \vec{v}$$

重力(地球引力与地球自转产生的惯性离心力的合力): $\vec{g} = -G \frac{M_g}{r^2} \cdot \left(\frac{\vec{r}}{r} \right)$

科氏力: $-2\vec{\Omega} \times \vec{V}$

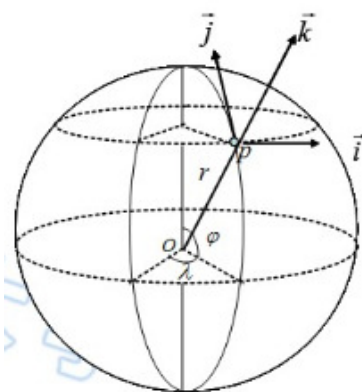
天体引潮力(受其他天体万有引力与惯性力离心力的合力): $\vec{F}_M = -G\frac{M_M}{L^2} + G\frac{M_M}{D^2} \cdot \left(\frac{\vec{D}}{D}\right)$

由牛顿第二定律和坐标系转换关系:

$$\begin{cases} \frac{d_a \vec{V}_a}{dt} = \sum_i \vec{F}_i \\ \frac{d_a \vec{A}_a}{dt} = \frac{d\vec{V}}{dt} + 2\vec{\Omega} \times \vec{V} - \Omega^2 \vec{R} \end{cases}$$

$$\Rightarrow \frac{d\vec{V}}{dt} = -\frac{1}{\rho} \nabla P - 2\vec{\Omega} \times \vec{V} + \vec{g} + \nu \Delta \vec{V} + \vec{F}_T$$

1.3 运动方程在球坐标系的标量形式



速度:

$$\vec{V} = u\vec{i} + v\vec{j} + w\vec{k}$$

$$\Rightarrow \begin{cases} u = r \cos \varphi \frac{d\lambda}{dt} \\ v = r \frac{d\varphi}{dt} \\ w = \frac{dr}{dt} \end{cases}$$

加速度:

$$\begin{aligned} \frac{d\vec{A}}{dt} &= \frac{\frac{\partial \vec{A}}{\partial t} dt + \frac{\partial \vec{A}}{\partial \lambda} d\lambda + \frac{\partial \vec{A}}{\partial \varphi} d\varphi + \frac{\partial \vec{A}}{\partial r} dr}{dt} \\ &= \frac{\partial \vec{A}}{\partial t} + \frac{\partial \vec{A}}{\partial \lambda} \frac{d\lambda}{dt} + \frac{\partial \vec{A}}{\partial \varphi} \frac{d\varphi}{dt} + \frac{\partial \vec{A}}{\partial r} \frac{dr}{dt} \\ &= \frac{\partial \vec{A}}{\partial t} + \frac{u}{r \cos \varphi} \frac{\partial \vec{A}}{\partial \lambda} + \frac{v}{r} \frac{\partial \vec{A}}{\partial \varphi} + w \frac{\partial \vec{A}}{\partial r} \\ \Rightarrow \frac{d}{dt} &= \frac{\partial}{\partial t} + u \frac{\partial}{r \cos \varphi \partial \lambda} + v \frac{\partial}{r \partial \varphi} + w \frac{\partial}{\partial r} \\ \Rightarrow \frac{d}{dt} &= \frac{\partial}{\partial t} + (\vec{V} \cdot \nabla) \\ \Rightarrow \nabla &= \frac{\partial}{r \cos \varphi \partial \lambda} \vec{i} + \frac{\partial}{r \partial \varphi} \vec{j} + \frac{\partial}{\partial r} \vec{k} \\ \Rightarrow \begin{cases} \frac{d\vec{i}}{dt} = \frac{\partial \vec{i}}{\partial t} + u \frac{\partial \vec{i}}{r \cos \varphi \partial \lambda} + v \frac{\partial \vec{i}}{r \partial \varphi} + w \frac{\partial \vec{i}}{\partial r} \\ \frac{d\vec{j}}{dt} = \frac{\partial \vec{j}}{\partial t} + u \frac{\partial \vec{j}}{r \cos \varphi \partial \lambda} + v \frac{\partial \vec{j}}{r \partial \varphi} + w \frac{\partial \vec{j}}{\partial r} \\ \frac{d\vec{k}}{dt} = \frac{\partial \vec{k}}{\partial t} + u \frac{\partial \vec{k}}{r \cos \varphi \partial \lambda} + v \frac{\partial \vec{k}}{r \partial \varphi} + w \frac{\partial \vec{k}}{\partial r} \end{cases} \end{aligned}$$

$$\frac{d\vec{V}}{dt} = \frac{du}{dt}\vec{i} + \frac{dv}{dt}\vec{j} + \frac{dw}{dt}\vec{k} + u\frac{d\vec{i}}{dt} + v\frac{d\vec{j}}{dt} + w\frac{d\vec{k}}{dt}$$

$$\Rightarrow \frac{d\vec{V}}{dt} = \left(\frac{du}{dt} - \frac{uv \tan \varphi}{r} + \frac{uw}{r}\right)\vec{i} + \left(\frac{dv}{dt} + \frac{u^2 \tan \varphi}{r} + \frac{vw}{r}\right)\vec{j} + \left(\frac{dw}{dt} - \frac{u^2 + v^2}{r}\right)\vec{k}$$

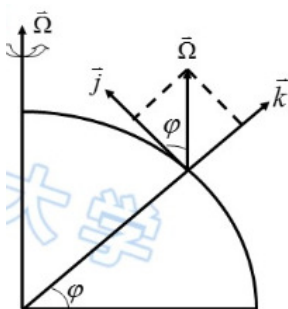
压强梯度力:

$$\frac{1}{\rho} \nabla p = -\frac{1}{\rho} \left(\frac{1}{r \cos \varphi} \frac{\partial p}{\partial \lambda} \vec{i} + \frac{1}{r} \frac{\partial p}{\partial \varphi} \vec{j} + \frac{\partial p}{\partial r} \vec{k} \right)$$

重力:

$$\vec{g} = -g\vec{k}$$

科氏力:



$$\vec{\Omega} = \Omega \sin \varphi \vec{k} + \Omega \cos \varphi \vec{j}$$

$$-2\vec{\Omega} \times \vec{V} = -2 \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & \Omega \cos \varphi & \Omega \sin \varphi \\ u & v & w \end{vmatrix}$$

$$= -2[(w\Omega \cos \varphi - v\Omega \sin \varphi)\vec{i} + (u\Omega \sin \varphi)\vec{j} + (-u\Omega \cos \varphi)\vec{k}]$$

$$\Rightarrow -2\vec{\Omega} \times \vec{V} = (fv - \tilde{f}w)\vec{i} - (fu)\vec{j} + (\tilde{f}u)\vec{k}$$

其中, $\begin{cases} f = 2\Omega \sin \varphi \\ \tilde{f} = 2\Omega \cos \varphi \end{cases}$

$$\Rightarrow \begin{cases} \frac{du}{dt} = -\frac{1}{\rho r \cos \varphi} \frac{\partial p}{\partial \lambda} + fv - \tilde{f}w + \frac{uv \tan \varphi}{r} - \frac{uw}{r} + \gamma(\Delta \vec{v})_\lambda - \frac{1}{r \cos \varphi} \frac{\partial \phi_T}{\partial \lambda} \\ \frac{dv}{dt} = -\frac{1}{\rho r} \frac{\partial p}{\partial \varphi} - fu - \frac{u^2 \tan \varphi}{r} - \frac{vw}{r} + \gamma(\Delta \vec{v})_\varphi - \frac{1}{r} \frac{\partial \phi_T}{\partial \varphi} \\ \frac{dw}{dt} = -\frac{1}{\rho} \frac{\partial p}{\partial r} + \tilde{f}u = g + \frac{u^2 + v^2}{r} + \gamma(\Delta \vec{v})_r - \frac{\partial \phi_T}{\partial r} \end{cases}$$

1.4 直角坐标系的运动方程

略去地球曲率的影响

$$\Rightarrow \begin{cases} \frac{du}{dt} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + fv + \tilde{f}w + F_{N\lambda} + F_{T\lambda} \\ \frac{dv}{dt} = -\frac{1}{\rho} \frac{\partial p}{\partial y} - fu + F_{Ny} + F_{Ty} \\ \frac{dw}{dt} = -\frac{1}{\rho} \frac{\partial p}{\partial z} + \tilde{f}u - g + F_{Nz} + F_{Tz} \end{cases}$$

1.5 海水层流运动的基本方程组

1.5.1 连续方程

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{V}) = 0 \Leftrightarrow \frac{d\rho}{dt} + \rho \nabla \cdot \vec{V} = 0$$

特别地, 对于不可压缩流体:

$$\nabla \cdot \vec{V} = 0$$

1.5.2 盐量扩散方程

$$\begin{aligned}
& \begin{array}{ccc} \text{盐量增加量} & \text{平流作用} & \text{分子扩散作用} \\ \frac{\partial}{\partial t} \iiint_{\tau} \rho s d\tau = - \oiint_{\sigma} \rho s V_n d\sigma + & - \oiint_{\sigma} S_n d\sigma \end{array} \\
& \iiint_{\tau} \frac{\partial(\rho s)}{\partial t} d\tau = \iiint_{\tau} \nabla \cdot (\rho s \vec{V}) d\tau - \iiint_{\tau} \nabla \cdot \vec{S} d\tau \\
& \Rightarrow \frac{\partial(\rho s)}{\partial t} + \nabla \cdot (\rho s \vec{V}) + \nabla \cdot \vec{S} = 0 \\
& \Rightarrow \rho \frac{\partial s}{\partial t} + s \frac{\partial \rho}{\partial t} + s \nabla \cdot (\rho \vec{V}) + \rho \vec{V} \cdot \nabla s + \nabla \cdot \vec{S} = 0 \\
& \Rightarrow \left(\frac{\partial s}{\partial t} + \vec{V} \cdot \nabla s \right) + \frac{s}{\rho} \left[\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{V}) \right] = -\frac{1}{\rho} \nabla \cdot \vec{S} \\
& \Rightarrow \frac{\partial s}{\partial t} + \vec{V} \cdot \nabla s = \frac{k}{\rho} \Delta s = k_D \Delta s
\end{aligned}$$

其中, $k_D = \frac{k}{\rho} \sim 1.1 \times 10^{-9} \text{ (m}^2/\text{s)}$

1.5.3 热传导方程

与上面类似:

$$\frac{\partial \theta}{\partial t} + \vec{V} \cdot \nabla \theta = \frac{\kappa}{\rho c_p} \Delta \theta = k_{\theta} \Delta \theta$$

其中, $k_{\theta} = \frac{\kappa}{\rho c_p} \sim 1.4 \times 10^{-7} \text{ (m}^2/\text{s)}$

1.5.4 热膨胀方程-状态方程

热膨胀方程:

$$\rho = \rho_0 \left(1 - \frac{\theta}{k} \right)$$

0°C时的海水密度 海水的膨胀系数

EOS80国际海水状态方程:

$$\rho(s, t, p) = \rho(s, t, 0) \left[1 - \frac{np}{k(s, t, p)} \right]^{-1}$$

1.6 基本方程的矢量形式和标量形式

矢量形式:

$$\left\{ \begin{array}{l} \frac{d\vec{V}}{dt} = -\frac{1}{\rho} \nabla p - 2\Omega \times \vec{V} + \vec{g} + \gamma \Delta \vec{V} - \nabla \phi_T \\ \frac{\partial \rho}{\partial t} + \rho \nabla \cdot \vec{V} = 0 \\ \frac{\partial s}{\partial t} + \vec{V} \cdot \nabla s = k_D \Delta s \\ \frac{\partial \theta}{\partial t} + \vec{V} \cdot \nabla \theta = k_{\theta} \Delta \theta \\ \rho = \rho(\theta, s, p) \end{array} \right.$$

标量形式(直角坐标系):

$$\left\{ \begin{array}{l} \frac{dv}{dt} = -\frac{1}{\rho} \frac{\partial p}{\partial y} - fu + \gamma \Delta v - \frac{\partial \phi_T}{\partial y} \\ \frac{dw}{dt} = -\frac{1}{\rho} \frac{\partial p}{\partial z} + \tilde{f}u - g + \gamma \Delta w - \frac{\partial \phi_T}{\partial z} \\ \frac{d\rho}{dt} + \rho \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) = 0 \\ \frac{\partial s}{\partial t} + u \frac{\partial s}{\partial x} + v \frac{\partial s}{\partial y} + w \frac{\partial s}{\partial z} = k_D \left(\frac{\partial^2 s}{\partial x^2} + \frac{\partial^2 s}{\partial y^2} + \frac{\partial^2 s}{\partial z^2} \right) \\ \frac{\partial \theta}{\partial t} + u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} + w \frac{\partial \theta}{\partial z} = k_{\theta} \left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2} \right) \\ \rho = \rho(\theta, s, p) \end{array} \right.$$

1.7 边界条件

无质量交换的运动学边界条件：

$$\frac{\partial F}{\partial t} + \vec{c} \cdot \nabla F = 0$$

例：

$$(1) \text{ 海面}(z = \zeta(x, y, t)) : \frac{\partial \zeta}{\partial t} + \vec{V}_H \cdot \nabla_H \zeta - w = 0$$

$$(2) \text{ 海底}(z = -h(x, y)) : \vec{V}_H \cdot \nabla_H h + w = 0$$

动力学边界条件：

由牛顿第三定律，在界面法线方向有：

$$(\vec{p}_n)_1 = (\vec{p}_n)_2$$

1.8 *间平均的基本方程和边界条件(直角坐标系)

连续方程：

$$\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} + \frac{\partial \bar{w}}{\partial z} = 0$$

运动方程：

$$\begin{cases} \frac{\partial \bar{u}}{\partial t} + \bar{u} \frac{\partial \bar{u}}{\partial x} + \bar{v} \frac{\partial \bar{u}}{\partial y} + \bar{w} \frac{\partial \bar{u}}{\partial z} = -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial x} + f\bar{v} - \tilde{f}w + \gamma \Delta \bar{u} - \frac{\partial \bar{\phi}_T}{\partial x} + \frac{\partial}{\partial x} \left(A_{x\alpha} \frac{\partial \bar{u}}{\partial x} \right) + \frac{\partial}{\partial y} \left(A_{xy} \frac{\partial \bar{u}}{\partial y} \right) + \frac{\partial}{\partial z} \left(A_{xz} \frac{\partial \bar{u}}{\partial z} \right) \\ \frac{\partial \bar{v}}{\partial t} + \bar{u} \frac{\partial \bar{v}}{\partial x} + \bar{v} \frac{\partial \bar{v}}{\partial y} + \bar{w} \frac{\partial \bar{v}}{\partial z} = -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial y} - f\bar{u} + \gamma \Delta \bar{v} - \frac{\partial \bar{\phi}_T}{\partial y} + \frac{\partial}{\partial x} \left(A_{yx} \frac{\partial \bar{v}}{\partial x} \right) + \frac{\partial}{\partial y} \left(A_{yy} \frac{\partial \bar{v}}{\partial y} \right) + \frac{\partial}{\partial z} \left(A_{yz} \frac{\partial \bar{v}}{\partial z} \right) \\ \frac{\partial \bar{w}}{\partial t} + \bar{u} \frac{\partial \bar{w}}{\partial x} + \bar{v} \frac{\partial \bar{w}}{\partial y} + \bar{w} \frac{\partial \bar{w}}{\partial z} = -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial z} + \tilde{f}\bar{u} - g + \gamma \Delta \bar{w} - \frac{\partial \bar{\phi}_T}{\partial z} + \frac{\partial}{\partial x} \left(A_{2x} \frac{\partial \bar{w}}{\partial x} \right) + \frac{\partial}{\partial y} \left(A_{zy} \frac{\partial \bar{w}}{\partial y} \right) + \frac{\partial}{\partial z} \left(A_{zz} \frac{\partial \bar{w}}{\partial z} \right) \end{cases}$$

盐量扩散方程：

$$\frac{\partial \bar{s}}{\partial t} + \bar{u} \frac{\partial \bar{s}}{\partial x} + \bar{v} \frac{\partial \bar{s}}{\partial y} + \bar{w} \frac{\partial \bar{s}}{\partial z}$$

热传导方程：

$$\frac{\partial \bar{\theta}}{\partial t} + \bar{u} \frac{\partial \bar{\theta}}{\partial x} + \bar{v} \frac{\partial \bar{\theta}}{\partial y} + \bar{w} \frac{\partial \bar{\theta}}{\partial z} = k_\theta \Delta \bar{\theta} + \frac{\partial}{\partial x} \left(K_{\theta x} \frac{\partial \bar{\theta}}{\partial x} \right) + \frac{\partial}{\partial y} \left(K_{\theta y} \frac{\partial \bar{\theta}}{\partial y} \right) + \frac{\partial}{\partial z} \left(K_{\theta z} \frac{\partial \bar{\theta}}{\partial z} \right)$$

状态方程：

$$\bar{\rho} = \bar{\rho}(\bar{s}, \bar{\theta}, \bar{p})$$

1.9 铅直向平均的基本方程

$$\begin{aligned} \frac{\partial}{\partial x} [(h + \zeta) \langle u \rangle] + \frac{\partial}{\partial y} [(h + \zeta) \langle v \rangle] - \left[u|_\zeta \frac{\partial \zeta}{\partial x} + v|_\zeta \frac{\partial \zeta}{\partial y} - w|_\zeta \right] - \left[u|_{-h} \frac{\partial h}{\partial x} + v|_{-h} \frac{\partial h}{\partial y} + w|_{-h} \right] &= 0 \\ \frac{\partial \zeta}{\partial t} + \frac{\partial [(h + \zeta) \langle u \rangle]}{\partial x} + \frac{\partial [(h + \zeta) \langle v \rangle]}{\partial y} &= 0 \end{aligned}$$

1.10 尺度分析

$$\text{Rossby数 } \text{Ro} = \frac{U}{FL} \begin{cases} \gg 1 : \text{平流非线性项比Coriolis力重要, 大尺度运动} \\ = 1 : \text{平流非线性项与Coriolis力同等重要} \\ \ll 1 : \text{平流非线性项可以忽略, 小尺度运动} \end{cases}$$

$$\text{水平Ekman数 } E_l = \frac{A_l}{FL^2} \text{ 水平湍流摩擦项与Coriolis力比值}$$

$$\text{垂直Ekman数 } E_z = \frac{A_z}{FD^2} \text{ 垂直湍流摩擦项与Coriolis力比值}$$

准静力近似 f 平面近似 β 平面近似 Boussinesq近似

2 海流

2.1 地转流

地转流：不考虑海面风的作用，远离沿岸的大洋中部大尺度、准水平、定常的海水流动。

产生原因：海水受热力和动力因素导致压力(和密度)在水平方向分布不均匀。

$$p = p_a + \rho gh \quad \rho \begin{cases} \neq \rho_0 \Rightarrow \text{梯度流} \\ = \rho_0 \Rightarrow \text{倾斜流} \end{cases}$$

2.1.1 梯度流