

# 物理海洋学笔记

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<https://github.com/Cuiyingzhe/Physical-Oceanography-Notes/>

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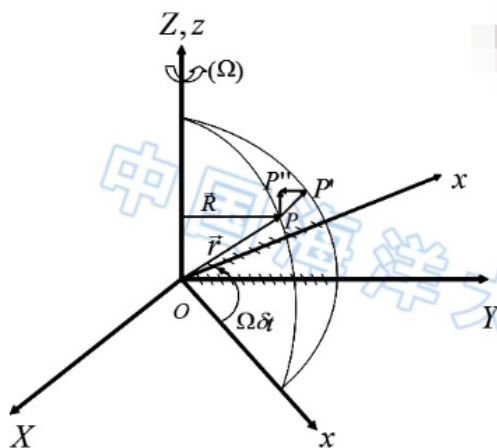
# 1 基本方程

## 1.1 旋转坐标系的速度和加速度

惯性坐标系: 静止的或是匀速直线运动的坐标系, 固定在恒星上的坐标系可以被看成惯性坐标系.

固定在地球上的坐标系: 地球对恒星的加速度主要是由地球自转引起的, 于是可以把地球当作一个对惯性坐标系作纯粹地转运动的物体.

### 1.1.1 旋转坐标系和惯性坐标系中的速度



惯性坐标系  $(XYZ)$  绝对位移:  $p\vec{p}'' = \vec{V}_a \delta t$ ,  $\vec{V}_a$  为绝对速度

旋转坐标系  $(xyz)$  相对位移:  $p'\vec{p}'' = \vec{V} \delta t$ ,  $\vec{V}$  为相对速度

$$\because p\vec{p}'' = p'\vec{p}'' + p\vec{p}'$$

$$\therefore \vec{V}_a \delta t = \vec{V} \delta t + \vec{V}_e \delta t \Rightarrow \vec{V}_a = \vec{V} + \vec{V}_e \text{ (绝对速度等于相对速度与牵连速度的向量和)}$$

$$\text{其中, } \vec{V}_e = \vec{\Omega} \times \vec{r} \Rightarrow \vec{V}_a = \vec{V} + \vec{\Omega} \times \vec{r}$$

$$\Rightarrow \frac{d_a \vec{r}}{dt} = \frac{d\vec{r}}{dt} + \vec{\Omega} \times \vec{r}$$

$$\frac{d_a \vec{A}}{dt} = \frac{d\vec{A}}{dt} + \vec{\Omega} \times \vec{A}$$

### 1.1.2 旋转坐标系和惯性坐标系中的加速度

$$\text{令 } \vec{A} = \vec{V}_a = \vec{V}_e + \vec{V} = \vec{V} + \vec{\Omega} \times \vec{r}$$

$$\begin{aligned} \frac{d\vec{V}_a}{dt} &= \frac{d_a}{dt} (\vec{V} + \vec{V}_e) = \frac{d_a}{dt} (\vec{V} + \vec{\Omega} \times \vec{r}) \\ &= \frac{d}{dt} (\vec{V} + \vec{\Omega} \times \vec{r}) + \vec{\Omega} \times (\vec{V} + \vec{\Omega} \times \vec{r}) \\ &= \frac{d\vec{V}}{dt} + \vec{\Omega} \times \vec{V} + \vec{\Omega} \times \vec{V} + \vec{\Omega} \times (\vec{\Omega} \times \vec{r}) \\ &= \frac{d\vec{V}}{dt} + 2\vec{\Omega} \times \vec{V} - \Omega^2 \vec{R} \end{aligned}$$

## 1.2 作用在海水微团上的外力运动方程的向量形式

压强梯度力:  $\frac{1}{\rho} \nabla p$

分子粘性力 (摩擦力):

$$\begin{cases} F_x = \frac{1}{\rho} \mu \left[ \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right] = \frac{\mu}{\rho} \Delta u \\ F_y = \frac{1}{\rho} \mu \left[ \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right] = \frac{\mu}{\rho} \Delta v \\ F_z = \frac{1}{\rho} \mu \left[ \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right] = \frac{\mu}{\rho} \Delta w \end{cases} \Rightarrow \vec{F} = \frac{\mu}{\rho} \Delta \vec{V} = \gamma \Delta \vec{v}$$

重力 (地球引力与地球自转产生的惯性离心力的合力):  $\vec{g} = -G \frac{M_g}{r^2} \cdot \left( \frac{\vec{r}}{r} \right)$

科氏力:  $-2\vec{\Omega} \times \vec{V}$

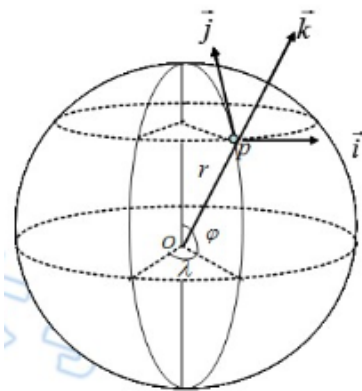
天体引潮力 (受其他天体万有引力与惯性力离心力的合力):  $\vec{F}_M = -G \frac{M_M}{L^2} + G \frac{M_M}{D^2} \cdot \left( \frac{\vec{D}}{D} \right)$

由牛顿第二定律和坐标系转换关系:

$$\begin{cases} \frac{d_a \vec{V}_a}{dt} = \sum_i \vec{F}_i \\ \frac{d_a \vec{A}_a}{dt} = \frac{d\vec{V}}{dt} + 2\vec{\Omega} \times \vec{V} - \Omega^2 \vec{R} \end{cases}$$

$$\Rightarrow \frac{d\vec{V}}{dt} = -\frac{1}{\rho} \nabla P - 2\vec{\Omega} \times \vec{V} + \vec{g} + \nu \Delta \vec{V} + \vec{F}_T$$

### 1.3 运动方程在球坐标系的标量形式



速度:

$$\vec{V} = u\vec{i} + v\vec{j} + w\vec{k}$$

$$\Rightarrow \begin{cases} u = r \cos \varphi \frac{d\lambda}{dt} \\ v = r \frac{d\varphi}{dt} \\ w = \frac{dr}{dt} \end{cases}$$

加速度:

$$\begin{aligned} \frac{d\vec{A}}{dt} &= \frac{\frac{\partial \vec{A}}{\partial t} dt + \frac{\partial \vec{A}}{\partial \lambda} d\lambda + \frac{\partial \vec{A}}{\partial \varphi} d\varphi + \frac{\partial \vec{A}}{\partial r} dr}{dt} \\ &= \frac{\partial \vec{A}}{\partial t} + \frac{\partial \vec{A}}{\partial \lambda} \frac{d\lambda}{dt} + \frac{\partial \vec{A}}{\partial \varphi} \frac{d\varphi}{dt} + \frac{\partial \vec{A}}{\partial r} \frac{dr}{dt} \\ &= \frac{\partial \vec{A}}{\partial t} + \frac{u}{r \cos \varphi} \frac{\partial \vec{A}}{\partial \lambda} + \frac{v}{r} \frac{\partial \vec{A}}{\partial \varphi} + w \frac{\partial \vec{A}}{\partial r} \\ \Rightarrow \frac{d}{dt} &= \frac{\partial}{\partial t} + u \frac{\partial}{r \cos \varphi \partial \lambda} + v \frac{\partial}{r \partial \varphi} + w \frac{\partial}{\partial r} \\ \Rightarrow \frac{d}{dt} &= \frac{\partial}{\partial t} + (\vec{V} \cdot \nabla) \\ \Rightarrow \nabla &= \frac{\partial}{r \cos \varphi \partial \lambda} \vec{i} + \frac{\partial}{r \partial \varphi} \vec{j} + \frac{\partial}{\partial r} \vec{k} \\ \Rightarrow \begin{cases} \frac{d\vec{i}}{dt} = \frac{\partial \vec{i}}{\partial t} + u \frac{\partial \vec{i}}{r \cos \varphi \partial \lambda} + v \frac{\partial \vec{i}}{r \partial \varphi} + w \frac{\partial \vec{i}}{\partial r} \\ \frac{d\vec{j}}{dt} = \frac{\partial \vec{j}}{\partial t} + u \frac{\partial \vec{j}}{r \cos \varphi \partial \lambda} + v \frac{\partial \vec{j}}{r \partial \varphi} + w \frac{\partial \vec{j}}{\partial r} \\ \frac{d\vec{k}}{dt} = \frac{\partial \vec{k}}{\partial t} + u \frac{\partial \vec{k}}{r \cos \varphi \partial \lambda} + v \frac{\partial \vec{k}}{r \partial \varphi} + w \frac{\partial \vec{k}}{\partial r} \end{cases} \end{aligned}$$

$$\frac{d\vec{V}}{dt} = \frac{du}{dt}\vec{i} + \frac{dv}{dt}\vec{j} + \frac{dw}{dt}\vec{k} + u\frac{d\vec{i}}{dt} + v\frac{d\vec{j}}{dt} + w\frac{d\vec{k}}{dt}$$

$$\Rightarrow \frac{d\vec{V}}{dt} = \left(\frac{du}{dt} - \frac{uv \tan \varphi}{r} + \frac{uw}{r}\right)\vec{i} + \left(\frac{dv}{dt} + \frac{u^2 \tan \varphi}{r} + \frac{vw}{r}\right)\vec{j} + \left(\frac{dw}{dt} - \frac{u^2 + v^2}{r}\right)\vec{k}$$

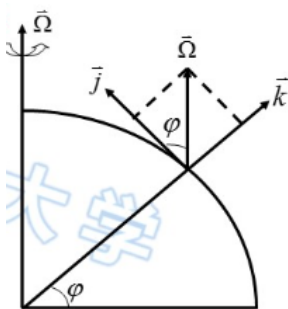
压强梯度力:

$$\frac{1}{\rho}\nabla p = -\frac{1}{\rho}\left(\frac{1}{r \cos \varphi}\frac{\partial p}{\partial \lambda}\vec{i} + \frac{1}{r}\frac{\partial p}{\partial \varphi}\vec{j} + \frac{\partial p}{\partial r}\vec{k}\right)$$

重力:

$$\vec{g} = -g\vec{k}$$

科氏力:



$$\vec{\Omega} = \Omega \sin \varphi \vec{k} + \Omega \cos \varphi \vec{j}$$

$$-2\vec{\Omega} \times \vec{V} = -2 \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & \Omega \cos \varphi & \Omega \sin \varphi \\ u & v & w \end{vmatrix}$$

$$= -2[(w\Omega \cos \varphi - v\Omega \sin \varphi)\vec{i} + (u\Omega \sin \varphi)\vec{j} + (-u\Omega \cos \varphi)\vec{k}]$$

$$\Rightarrow -2\vec{\Omega} \times \vec{V} = (fv - \tilde{f}w)\vec{i} - (fu)\vec{j} + (\tilde{f}u)\vec{k}$$

其中,  $\begin{cases} f = 2\Omega \sin \varphi \\ \tilde{f} = 2\Omega \cos \varphi \end{cases}$

$$\Rightarrow \begin{cases} \frac{du}{dt} = -\frac{1}{\rho r \cos \varphi} \frac{\partial p}{\partial \lambda} + fv - \tilde{f}w + \frac{uv \tan \varphi}{r} - \frac{uw}{r} + \gamma(\Delta \vec{v})_{\lambda} - \frac{1}{r \cos \varphi} \frac{\partial \phi_T}{\partial \lambda} \\ \frac{dv}{dt} = -\frac{1}{\rho r} \frac{\partial p}{\partial \varphi} - fu - \frac{u^2 \tan \varphi}{r} - \frac{vw}{r} + \gamma(\Delta \vec{v})_{\varphi} - \frac{1}{r} \frac{\partial \phi_T}{\partial \varphi} \\ \frac{dw}{dt} = -\frac{1}{\rho} \frac{\partial p}{\partial r} + \tilde{f}u = g + \frac{u^2 + v^2}{r} + \gamma(\Delta \vec{v})_r - \frac{\partial \phi_T}{\partial r} \end{cases}$$

## 1.4 直角坐标系的运动方程

略去地球曲率的影响

$$\Rightarrow \begin{cases} \frac{du}{dt} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + fv + \tilde{f}w + F_{N\lambda} + F_{T\lambda} \\ \frac{dv}{dt} = -\frac{1}{\rho} \frac{\partial p}{\partial y} - fu + F_{Ny} + F_{Ty} \\ \frac{dw}{dt} = -\frac{1}{\rho} \frac{\partial p}{\partial z} + \tilde{f}u - g + F_{Nz} + F_{Tz} \end{cases}$$

## 1.5 海水层流运动的基本方程组

### 1.5.1 连续方程

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{V}) = 0 \Leftrightarrow \frac{d\rho}{dt} + \rho \nabla \cdot \vec{V} = 0$$

特别地, 对于不可压缩流体:

$$\nabla \cdot \vec{V} = 0$$

### 1.5.2 盐量扩散方程

$$\begin{aligned}
& \overset{\text{盐量增加量}}{\frac{\partial}{\partial t} \iiint_{\tau} \rho s d\tau} = - \overset{\text{平流作用}}{\oint_{\sigma} \rho s V_n d\sigma} + \overset{\text{分子扩散作用}}{- \oint_{\sigma} S_n d\sigma} \\
& \iiint_{\tau} \frac{\partial(\rho s)}{\partial t} d\tau = \iiint_{\tau} \nabla \cdot (\rho s \vec{V}) d\tau - \iiint_{\tau} \nabla \cdot \vec{S} d\tau \\
& \Rightarrow \frac{\partial(\rho s)}{\partial t} + \nabla \cdot (\rho s \vec{V}) + \nabla \cdot \vec{S} = 0 \\
& \Rightarrow \rho \frac{\partial s}{\partial t} + s \frac{\partial \rho}{\partial t} + s \nabla \cdot (\rho \vec{V}) + \rho \vec{V} \cdot \nabla s + \nabla \cdot \vec{S} = 0 \\
& \Rightarrow \left( \frac{\partial s}{\partial t} + \vec{V} \cdot \nabla s \right) + \frac{s}{\rho} \left[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{V}) \right] = -\frac{1}{\rho} \nabla \cdot \vec{S} \\
& \Rightarrow \frac{\partial s}{\partial t} + \vec{V} \cdot \nabla s = \frac{k}{\rho} \Delta s = k_D \Delta s
\end{aligned}$$

其中,  $k_D = \frac{k}{\rho} \sim 1.1 \times 10^{-9} \text{ (m}^2/\text{s)}$

### 1.5.3 热传导方程

与上面类似:

$$\frac{\partial \theta}{\partial t} + \vec{V} \cdot \nabla \theta = \frac{\kappa}{\rho c_p} \Delta \theta = k_{\theta} \Delta \theta$$

其中,  $k_{\theta} = \frac{\kappa}{\rho c_p} \sim 1.4 \times 10^{-7} \text{ (m}^2/\text{s)}$

### 1.5.4 热膨胀方程-状态方程

热膨胀方程:

$$\rho = \overset{0^{\circ}\text{C 时的海水密度}}{\rho_0} (1 - \overset{\text{海水的热膨胀系数}}{k} \theta)$$

EOS80 国际海水状态方程:

$$\rho(s, t, p) = \rho(s, t, 0) \left[ 1 - \frac{np}{k(s, t, p)} \right]^{-1}$$

## 1.6 基本方程的矢量形式和标量形式

矢量形式:

$$\left\{ \begin{aligned} \frac{d\vec{V}}{dt} &= -\frac{1}{\rho} \nabla p - 2\Omega \times \vec{V} + \vec{g} + \gamma \Delta \vec{V} - \nabla \phi_T \\ \frac{\partial \rho}{\partial t} + \rho \nabla \cdot \vec{V} &= 0 \\ \frac{\partial s}{\partial t} + \vec{V} \cdot \nabla s &= k_D \Delta s \\ \frac{\partial \theta}{\partial t} + \vec{V} \cdot \nabla \theta &= k_{\theta} \Delta \theta \\ \rho &= \rho(\theta, s, p) \end{aligned} \right.$$

标量形式 (直角坐标系):

$$\left\{ \begin{aligned} \frac{dv}{dt} &= -\frac{1}{\rho} \frac{\partial p}{\partial y} - fu + \gamma \Delta v - \frac{\partial \phi_T}{\partial y} \\ \frac{dw}{dt} &= -\frac{1}{\rho} \frac{\partial p}{\partial z} + \tilde{f}u - g + \gamma \Delta w - \frac{\partial \phi_T}{\partial z} \\ \frac{d\rho}{dt} + \rho \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) &= 0 \\ \frac{\partial s}{\partial t} + u \frac{\partial s}{\partial x} + v \frac{\partial s}{\partial y} + w \frac{\partial s}{\partial z} &= k_D \left( \frac{\partial^2 s}{\partial x^2} + \frac{\partial^2 s}{\partial y^2} + \frac{\partial^2 s}{\partial z^2} \right) \\ \frac{\partial \theta}{\partial t} + u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} + w \frac{\partial \theta}{\partial z} &= k_{\theta} \left( \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2} \right) \\ \rho &= \rho(\theta, s, p) \end{aligned} \right.$$

## 1.7 边界条件

无质量交换的运动学边界条件：

$$\frac{\partial F}{\partial t} + \vec{c} \cdot \nabla F = 0$$

例：

$$(1) \text{ 海面 } (z = \zeta(x, y, t)): \frac{\partial \zeta}{\partial t} + \vec{V}_H \cdot \nabla_H \zeta - w = 0$$

$$(2) \text{ 海底 } (z = -h(x, y)): \vec{V}_H \cdot \nabla_H h + w = 0$$

动力学边界条件：

由牛顿第三定律，在界面法线方向有：

$$(\vec{p}_n)_1 = (\vec{p}_n)_2$$

## 1.8 \* 时间平均的基本方程和边界条件 (直角坐标系)

连续方程：

$$\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} + \frac{\partial \bar{w}}{\partial z} = 0$$

运动方程：

$$\begin{cases} \frac{\partial \bar{u}}{\partial t} + \bar{u} \frac{\partial \bar{u}}{\partial x} + \bar{v} \frac{\partial \bar{u}}{\partial y} + \bar{w} \frac{\partial \bar{u}}{\partial z} = -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial x} + f\bar{v} - \bar{f}\bar{w} + \gamma \Delta \bar{u} - \frac{\partial \bar{\phi}_T}{\partial x} + \frac{\partial}{\partial x} \left( A_{x\alpha} \frac{\partial \bar{u}}{\partial x} \right) + \frac{\partial}{\partial y} \left( A_{xy} \frac{\partial \bar{u}}{\partial y} \right) + \frac{\partial}{\partial z} \left( A_{xz} \frac{\partial \bar{u}}{\partial z} \right) \\ \frac{\partial \bar{v}}{\partial t} + \bar{u} \frac{\partial \bar{v}}{\partial x} + \bar{v} \frac{\partial \bar{v}}{\partial y} + \bar{w} \frac{\partial \bar{v}}{\partial z} = -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial y} - f\bar{u} + \gamma \Delta \bar{v} - \frac{\partial \bar{\phi}_T}{\partial y} + \frac{\partial}{\partial x} \left( A_{yx} \frac{\partial \bar{v}}{\partial x} \right) + \frac{\partial}{\partial y} \left( A_{yy} \frac{\partial \bar{v}}{\partial y} \right) + \frac{\partial}{\partial z} \left( A_{yz} \frac{\partial \bar{v}}{\partial z} \right) \\ \frac{\partial \bar{w}}{\partial t} + \bar{u} \frac{\partial \bar{w}}{\partial x} + \bar{v} \frac{\partial \bar{w}}{\partial y} + \bar{w} \frac{\partial \bar{w}}{\partial z} = -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial z} + \bar{f}\bar{u} - g + \gamma \Delta \bar{w} - \frac{\partial \bar{\phi}_T}{\partial z} + \frac{\partial}{\partial x} \left( A_{2x} \frac{\partial \bar{w}}{\partial x} \right) + \frac{\partial}{\partial y} \left( A_{zy} \frac{\partial \bar{w}}{\partial y} \right) + \frac{\partial}{\partial z} \left( A_{zz} \frac{\partial \bar{w}}{\partial z} \right) \end{cases}$$

盐量扩散方程：

$$\frac{\partial \bar{s}}{\partial t} + \bar{u} \frac{\partial \bar{s}}{\partial x} + \bar{v} \frac{\partial \bar{s}}{\partial y} + \bar{w} \frac{\partial \bar{s}}{\partial z}$$

热传导方程：

$$\frac{\partial \bar{\theta}}{\partial t} + \bar{u} \frac{\partial \bar{\theta}}{\partial x} + \bar{v} \frac{\partial \bar{\theta}}{\partial y} + \bar{w} \frac{\partial \bar{\theta}}{\partial z} = k_\theta \Delta \bar{\theta} + \frac{\partial}{\partial x} \left( K_{\theta x} \frac{\partial \bar{\theta}}{\partial x} \right) + \frac{\partial}{\partial y} \left( K_{\theta y} \frac{\partial \bar{\theta}}{\partial y} \right) + \frac{\partial}{\partial z} \left( K_{\theta z} \frac{\partial \bar{\theta}}{\partial z} \right)$$

状态方程：

$$\bar{\rho} = \bar{\rho}(\bar{s}, \bar{\theta}, \bar{p})$$

## 1.9 铅直向平均的基本方程

$$\begin{aligned} \frac{\partial}{\partial x} [(h + \zeta) \langle u \rangle] + \frac{\partial}{\partial y} [(h + \zeta) \langle v \rangle] - \left[ u|_\zeta \frac{\partial \zeta}{\partial x} + v|_\zeta \frac{\partial \zeta}{\partial y} - w|_\zeta \right] - \left[ u|_{-h} \frac{\partial h}{\partial x} + v|_{-h} \frac{\partial h}{\partial y} + w|_{-h} \right] = 0 \\ \frac{\partial \zeta}{\partial t} + \frac{\partial [(h + \zeta) \langle u \rangle]}{\partial x} + \frac{\partial [(h + \zeta) \langle v \rangle]}{\partial y} = 0 \end{aligned}$$

## 1.10 尺度分析

$$\text{Rossby 数 } \text{Ro} = \frac{U}{FL} \begin{cases} \gg 1: \text{平流非线性项比 Coriolis 力重要, 大尺度运动} \\ = 1: \text{平流非线性项与 Coriolis 力同等重要} \\ \ll 1: \text{平流非线性项可以忽略, 小尺度运动} \end{cases}$$

$$\text{水平 Ekman 数 } E_l = \frac{A_l}{FL^2} \text{ 水平湍流摩擦项与 Coriolis 力比值}$$

$$\text{垂直 Ekman 数 } E_z = \frac{A_z}{FD^2} \text{ 垂直湍流摩擦项与 Coriolis 力比值}$$

准静力近似  $f$  平面近似  $\beta$  平面近似 Boussinesq 近似

## 2 海流

### 2.1 地转流

地转流：不考虑海面风的作用，远离沿岸的大洋中部大尺度、准水平、定常的海水流动。

产生原因：海水受热力和动力因素导致压力 (和密度) 在水平方向分布不均匀。

$$p = p_a + \rho gh \quad \rho \begin{cases} \neq \rho_0 \Rightarrow \text{梯度流} \\ = \rho_0 \Rightarrow \text{倾斜流} \end{cases}$$

#### 2.1.1 梯度流

假定和方程

(1) 在相当长一段时间里海面温度变化和降水蒸发变化都不大，于是可以认为已形成的海水密度场、温度场和盐度场近似于定常，从而相应的海水运动也近似于定常： $\frac{\partial}{\partial t} = 0$ 。

(2) 海洋深而宽广，在远离海岸及海底的大洋中部海区，大尺度运动： $Ro \ll 1$ 。

(3) 不考虑海底摩擦及边界摩擦的影响，且海面无风力作用，则流动属一种无摩擦流动： $E_l, E_z \ll 1$ 。

(4)  $\beta$  平面近似准静力近似

$x$  方向基本方程：

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + fv + \frac{\partial}{\partial x} \left( A_{xx} \frac{\partial u}{\partial x} \right) + \frac{\partial}{\partial y} \left( A_{xy} \frac{\partial u}{\partial y} \right) + \frac{\partial}{\partial z} \left( A_{xz} \frac{\partial u}{\partial z} \right)$$

$$\text{假定 (1)} \Rightarrow \frac{\partial u}{\partial t} = 0$$

$$\text{假定 (2)} \Rightarrow u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = 0$$

$$\text{假定 (3)} \Rightarrow \frac{\partial}{\partial x} \left( A_{xx} \frac{\partial u}{\partial x} \right) + \frac{\partial}{\partial y} \left( A_{xy} \frac{\partial u}{\partial y} \right) + \frac{\partial}{\partial z} \left( A_{xz} \frac{\partial u}{\partial z} \right) = 0$$

可得梯度流的控制方程：

$$\begin{cases} 0 = -\frac{1}{\rho} \frac{\partial p}{\partial x} + fv \\ 0 = -\frac{1}{\rho} \frac{\partial p}{\partial y} - fu \\ 0 = -\frac{1}{\rho} \frac{\partial p}{\partial z} - g \\ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \\ u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} + w \frac{\partial \theta}{\partial z} = 0 \\ u \frac{\partial s}{\partial x} + v \frac{\partial s}{\partial y} + w \frac{\partial s}{\partial z} = 0 \\ \rho = \rho(s, \theta) \end{cases}$$

特征

水平速度：

$$\begin{cases} 0 = -\frac{1}{\rho} \frac{\partial p}{\partial x} + fv \\ 0 = -\frac{1}{\rho} \frac{\partial p}{\partial y} - fu \end{cases} \quad (1)$$

$$\Rightarrow \begin{cases} v = \frac{1}{\rho f} \frac{\partial p}{\partial x} \\ u = -\frac{1}{\rho f} \frac{\partial p}{\partial y} \end{cases} \quad (2)$$

- (1) 水平速度和压强梯度成正比；
- (2) 与密度和科氏参数成反比；
- (3) 地转关系在赤道不成立 ( $f = 0$ ).

垂向速度：

$$\begin{aligned}
 \frac{\partial(1) \times \rho}{\partial y} - \frac{\partial(2) \times \rho}{\partial x} &\Leftrightarrow \frac{\partial(\rho f v)}{\partial v} + \frac{\partial(\rho f u)}{\partial x} = 0 \\
 &\Leftrightarrow f \rho \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) + f \left( u \frac{\partial \rho}{\partial x} + v \frac{\partial \rho}{\partial y} + w \frac{\partial \rho}{\partial z} \right) - f \rho \frac{\partial w}{\partial z} - f w \frac{\partial \rho}{\partial z} + \beta \rho v = 0 \\
 &\Leftrightarrow f \rho \stackrel{=0}{\boxed{\nabla \vec{V}}} + f \vec{V} \cdot \nabla \rho - f \rho \frac{\partial w}{\partial z} - f w \frac{\partial \rho}{\partial z} + \beta \rho v = 0
 \end{aligned} \tag{3}$$

$$\vec{V} \cdot \nabla \rho = \vec{V} \cdot \nabla \rho(s, \theta) = \vec{V} \cdot \left( \nabla_s \frac{\partial \rho}{\partial s} + \nabla_\theta \frac{\partial \rho}{\partial \theta} \right) = 0$$

$$(3) \Leftrightarrow f \left( \rho \frac{\partial w}{\partial z} + w \frac{\partial \rho}{\partial z} \right) = \beta \rho v \stackrel{\text{尺度分析}}{\Rightarrow} f \frac{\partial w}{\partial z} = \beta v \stackrel{\text{尺度分析}}{\Rightarrow} W = \frac{\beta D}{F} U \sim 2 \times 10^{-4} U$$

垂向流速比水平流速小得多，地转流为准水平运动.

### 运动特性

$$(1) \times u + (2) \times v \Leftrightarrow u \frac{\partial p}{\partial x} + v \frac{\partial p}{\partial y} = 0 \Leftrightarrow \vec{V}_H \cdot \nabla_H p = 0$$

- (1) 梯度流平行于等压线；
- (2) 北半球，流向右侧为高压，南半球相反；

### 密度特性

$$\frac{\partial(1) \times \rho}{\partial y} - \frac{\partial(2) \times \rho}{\partial x} \Leftrightarrow f \rho \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) + f \left( u \frac{\partial \rho}{\partial x} + v \frac{\partial \rho}{\partial y} \right) + \rho u \frac{\partial f}{\partial x} + \rho v \frac{\partial f}{\partial y} = 0 \Leftrightarrow \vec{V}_H \cdot \nabla_H \rho = 0$$

- (1) 梯度流近似平行于等密线；
- (2) 在北半球，流向右侧密度小；
- (3) 等压面倾斜与等密面倾斜方向相反.

### 温盐特性

忽略垂向运动：

$$\vec{V}_H \cdot \nabla_H \theta = 0$$

$$\vec{V}_H \cdot \nabla_H s = 0$$

- (1) 梯度流平行于等温线和等盐线；
- (2) 在北半球，流向右侧温度高，盐度低.

### 2.1.2 倾斜流

假定和方程 (1) 海水密度为常数；

(2) 水平方向的压强梯度是由海面倾斜引起的.

$$\Rightarrow p = p_a + \int_z^\zeta \rho g dz = p_a + \rho g(\zeta - z)$$

倾斜流的控制方程：

$$\begin{cases} f v = g \frac{\partial \zeta}{\partial x} \\ f u = -g \frac{\partial \zeta}{\partial y} \end{cases} \tag{4}$$

$$\tag{5}$$



性质：

$$(4) \times u - (5) \times v \Leftrightarrow u \frac{\partial \zeta}{\partial x} + v \frac{\partial \zeta}{\partial y} = 0 \Leftrightarrow \vec{V}_H \cdot \nabla \zeta = 0$$

- (1) 倾斜流平行于等水位线；
- (2) 在北半球，流向右侧水位高；
- (3) 倾斜流从表至底流速流向相同，压强梯度相同。

## 2.2 Ekman 漂流

由恒速定常的风长时间驱动大尺度、均匀密度的海洋，所产生的处于稳定状态的海流。

### 2.2.1 无限深海漂流

#### 物理背景

Ekman 的老师 Nansen 在海洋调查时发现，冰山不是顺风漂移，而是沿着风向右方  $20^\circ \sim 40^\circ$  的方向移动。Ekman 在 1905 年研究了这种现象并提出风海流理论 [1]。

#### 假定

无限深海 Ekman 漂流中用到了以下假定：

- 1) 海洋无限广阔，海洋无限深。

即无侧边界效应，仅有垂直湍流所生水平湍流摩擦力，并假定垂直湍流粘滞系数  $A_z$  为常量。由于海洋无限深， $z \rightarrow \infty, \vec{V} = 0$

- 2) 定常均匀风场长时间作用。

即运动的基本参量与时间和水平坐标无关且海面无升降、无水平压强梯度。

- 3) 密度分布均匀， $\rho$  为常数，不考虑热盐性质。

- 4) 采用  $f$  平面近似。

#### 方程推导

#### 控制方程和边界条件

首先给出一般的控制方程：

$$\begin{cases} \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + fv + A_l \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + A_z \frac{\partial^2 u}{\partial z^2} \\ \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial y} - fu + A_l \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + A_z \frac{\partial^2 v}{\partial z^2} \\ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \end{cases} \quad (6)$$

由假定 1)， $A_l \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) = 0$ ， $A_l \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) = 0$ ；

由假定 2)， $\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = 0$ ， $\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = 0$ ；

由假定 3)， $-\frac{1}{\rho} \frac{\partial p}{\partial x} = 0$ ， $-\frac{1}{\rho} \frac{\partial p}{\partial y} = 0$

则 (6) 化为：

$$\begin{cases} 0 = fv + A_z \frac{\partial^2 u}{\partial z^2} \end{cases} \quad (7a)$$

$$\begin{cases} 0 = -fu + A_z \frac{\partial^2 v}{\partial z^2} \end{cases} \quad (7b)$$

$$\begin{cases} \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \end{cases} \quad (7c)$$

不失一般性地，假定风力仅沿  $y$  方向作用，即  $\tau_x = 0, \tau_y = \text{const.}$  再结合假定 1)，控制方程的边界条件为：

$$\begin{cases} z = 0, \rho A_z \frac{\partial u}{\partial z} = 0 \\ z = 0, \rho A_z \frac{\partial v}{\partial z} = -\tau_y \end{cases} \quad (8a)$$

$$\begin{cases} z = 0, \rho A_z \frac{\partial v}{\partial z} = -\tau_y \\ z = \infty, u = v = 0 \end{cases} \quad (8b)$$

$$\begin{cases} z = \infty, u = v = 0 \end{cases} \quad (8c)$$

方程求解

$$(7a) + (7b) \times i \Leftrightarrow A_z \frac{\partial^2(u + iv)}{\partial z^2} = if(u + iv)$$

令  $W = u + iv$ ，得：

$$A_z \frac{\partial^2 W}{\partial z^2} = ifW \Rightarrow \frac{\partial^2 W}{\partial z^2} = \frac{(1+i)^2 \Omega \sin \varphi}{A_z} W$$

令  $a = \sqrt{\Omega \sin \varphi / A_z}$ ， $j^2 = (1+i)^2 a^2$ ，得：

$$\frac{d^2 W}{dz^2} - j^2 W = 0 \quad (9)$$

(9) 式通解为： $W = Ae^{jz} + Be^{-jz}$

结合边界条件： $(8a) + (8b) \times i \Rightarrow z = 0, \rho A_z \frac{\partial W}{\partial z} = -\tau_y, z \rightarrow \infty, W = 0$

$$z \rightarrow \infty \Rightarrow A = 0, W = Be^{-jz}; z = 0, \rho A_z \frac{\partial W}{\partial z} \Big|_{z=0} = \rho A_z \frac{\partial (Be^{-jz})}{\partial z} \Big|_{z=0} = -\tau_y, \Rightarrow B = \tau_y / (j\rho A_z)$$

因此，方程的解为：

$$W = \frac{\tau_y}{j\rho A_z} e^{-jz} = \frac{i\tau_y}{(1+i)a\rho A_z} e^{-(1+i)az} = \frac{e^{i\frac{\pi}{2}}\tau_y}{\sqrt{2}e^{i\frac{\pi}{4}}a\rho A_z} e^{-(1+i)az}$$

令  $D_0 = \pi/a$ ，得到最终解的形式为：

$$W = \frac{\tau_y}{\sqrt{2}a\rho A_z} e^{-\frac{\pi}{D_0}z + i(\frac{\pi}{4} - \frac{\pi}{D_0}z)} \quad (10)$$

物理性质

运动速度

在海面 ( $z = 0$ ) 处， $W_0 = \frac{\tau_y}{\sqrt{2}a\rho A_z} e^{i\frac{\pi}{4}}$ 。大小为  $|W_0| = \frac{\tau_y}{\sqrt{2}a\rho A_z}$ ，方向与  $x$  轴成  $45^\circ$ ，即与风向向右偏  $45^\circ$ 。

在任意深度处， $|W_z| = \frac{\tau_y}{\sqrt{2}a\rho A_z} e^{-\frac{\pi}{D_0}z}$ ，方向为  $\frac{\pi}{4} - \frac{\pi}{D_0}z$ ，即流速随深度增加呈指数形式减小，流向随深度的增加而逐渐向右偏。

在摩擦深度  $z = D_0$  处， $|W_{D_0}| = \frac{\tau_y e^{-\pi}}{\sqrt{2}a\rho A_z} = e^{-\pi}|W_0| = 0.043|W_0|$ ，方向  $-\frac{3}{4}\pi$ ，即与  $x$  轴成  $-135^\circ$ ，与表面流向正好相反。

**Ekman 螺旋和 Ekman 螺线**

根据速度的垂向分布，表层流速最大，流向偏向风向的右方  $45^\circ$ ；随深度增加，流速逐渐减小，流向逐渐右偏；到摩擦深度，流速是表面流速的 4.3%，流向与表面流向相反，运动可以忽略。连连接各层流速的矢量端点，构成艾克曼螺旋；艾克曼螺旋在平面上的投影，称为艾克曼螺线。

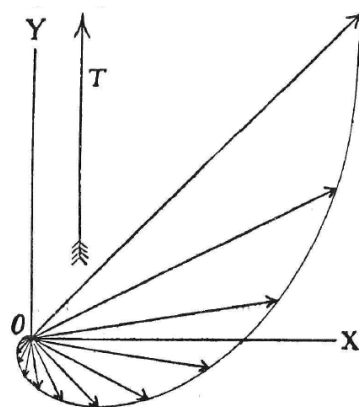


图 1 Ekman 螺线 [1]

水平体积输运

体积输运：

$$\begin{aligned}
 S &= \int_0^\infty W dz \\
 &= \frac{\tau_y}{\sqrt{2}a\rho A_z} e^{i\frac{\pi}{4}} \int_0^\infty e^{-\frac{\pi}{D_0}(1+i)z} dz \\
 &= \frac{\tau_y}{\sqrt{2}a\rho A_z} e^{i\frac{\pi}{4}} \left[ -\frac{D_0}{\pi} \frac{1}{(1+i)} \right] e^{-\frac{\pi}{D_0}(1+i)z} \Big|_0^\infty \\
 &= \frac{\tau_y}{2\Omega \sin \varphi \rho} = \frac{\tau_y}{f\rho}
 \end{aligned}$$

可以发现，得到的输运结果只有实部，没有虚部，说明体积输运方向为  $x$  轴正向，即在北半球水体向风向右侧  $90^\circ$  输运。

## 2.2.2 有限深海漂流

假定

有限深海 Ekman 漂流中用到了以下假定：

1) 海区无限广阔、有限深，远离海岸。

即无侧边界效应，仅有垂直湍流所生水平湍流摩擦力，并假定垂直湍流粘滞系数  $A_z$  为常量。由于海洋有限深， $z \rightarrow h, \vec{V} = 0$

2) 定常均匀风场长时间作用。

即运动的基本参量与时间和水平坐标无关且海面无升降、无水平压强梯度。

3) 密度分布均匀， $\rho$  为常数，不考虑热盐性质。

4) 采用  $f$  平面近似。

控制方程和边界条件：

$$\begin{cases}
 0 = fv + A_z \frac{\partial^2 u}{\partial z^2} \\
 0 = -fu + A_z \frac{\partial^2 v}{\partial z^2} \\
 \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \\
 z = 0 : \rho A_z \frac{\partial u}{\partial z} = 0, \rho A_z \frac{\partial v}{\partial z} = -\tau_y \\
 z \rightarrow h : u = v = 0
 \end{cases}$$

方程求解 令  $\zeta = h - z$ ，定解问题化为：

$$\begin{cases}
 -fv = A_z \frac{\partial^2 u}{\partial \zeta^2} \\
 fu = A_z \frac{\partial^2 v}{\partial \zeta^2}
 \end{cases} \quad (11)$$

$$\begin{cases}
 \zeta = h : \rho A_z \frac{\partial u}{\partial \zeta} = 0, \rho A_z \frac{\partial v}{\partial \zeta} = \tau_y \\
 \zeta \rightarrow 0 : u = v = 0
 \end{cases} \quad (12)$$

令  $W = u + iv, \tau = \tau_x + i\tau_y$ ，控制方程：

$$(11) + (12) \times i \Leftrightarrow \frac{d^2 W}{d\zeta^2} - j^2 W = 0$$

边界条件：

$$\zeta = h : \rho A_z \frac{\partial W}{\partial \zeta} = \tau$$

$$\zeta = 0 : W = 0$$

与无限深海漂流解法类似，解得：

$$W = \frac{(1+i)\tau_y}{2a\rho A_z} \frac{sh(1+i)a\xi}{ch(1+i)ah}$$

物理性质

与水深的关系

(1)  $h \gg 2D_0$  时，有限深海漂流流速流向与无限深海相同；(2) 水深越浅，流向随深度增加右偏（北半球）越缓慢；(3) 从上层到下层的流速矢量越是趋近风矢量的方向。

体积输运

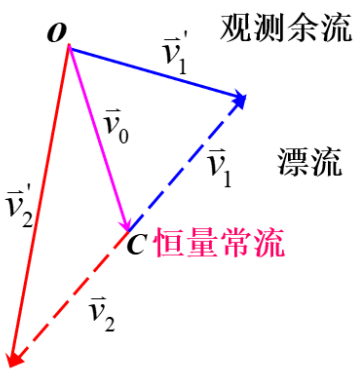
- (1) 在  $x, y$  方向（平行和垂直风向）都有输送；
- (2) 运输方向为风向右端， $\pm 90^\circ$  之间：

$$S_x > 0; 0 < h < D_0, ah < \pi, S_y > 0; D_0 < h < 2D_0, \pi < ah < 2\pi, S_y < 0; h > 2D_0, S_y = 0$$

2.2.3 漂流分离

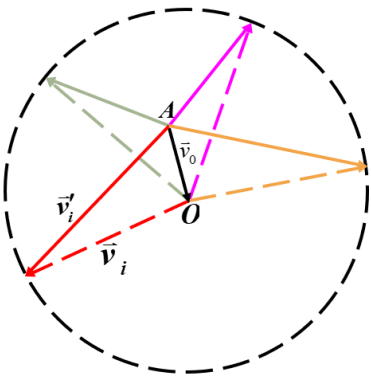
利用风速大小相等、方向相反的两组观测余流分离漂流

※ 余流 = 漂流 + 恒量常流



利用一组风速大小相等、方向不同的实测余流分离漂流

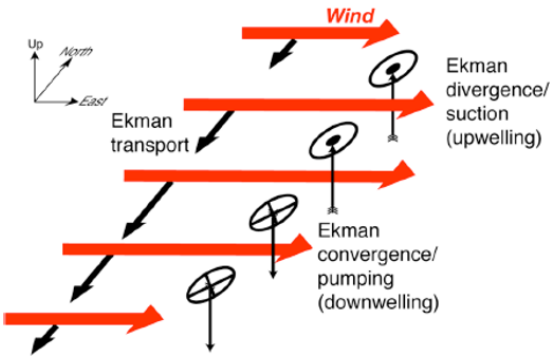
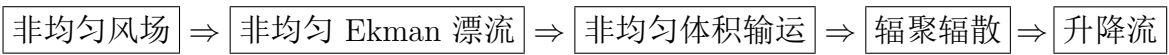
※ 漂流速度矢量端点落在同一圆周上



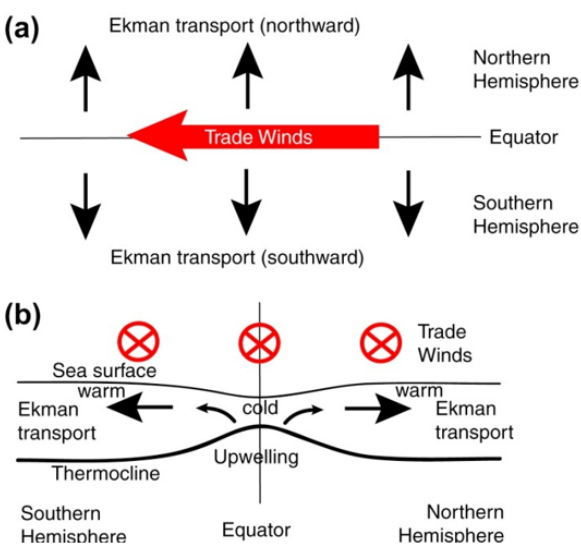
2.2.4 升降流

由不均匀风场或风场和地形配合产生的“较强烈”的铅直向流动。

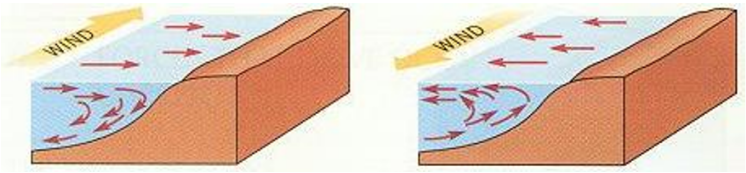
物理背景



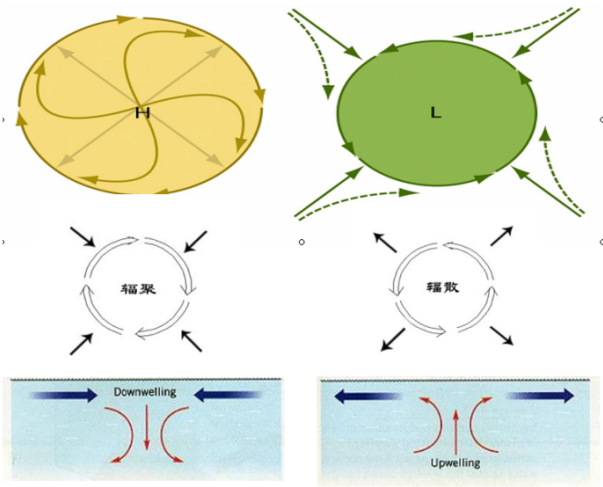
赤道附近的升降流



顺 (沿) 岸风产生的升降流



气旋和反气旋产生的海洋升降流



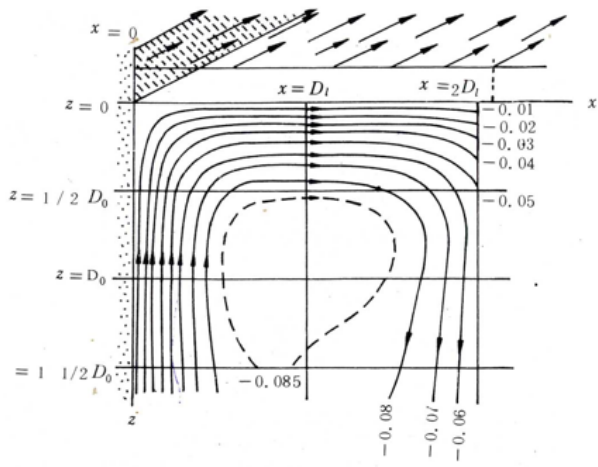
假定

- (1)  $\rho$  为常数;
- (2) 直线风系, 风仅沿  $x$  方向有变化; 风区内为恒定的均匀风场; 风区外无风;  $\frac{\partial}{\partial y} = 0$
- (3) 定常风场;  $\frac{\partial}{\partial t} = 0$
- (4) 大尺度;  $Ro \ll 1$
- (5) 有限深度.  $h \geq 2D_0$

### 控制方程及边界条件

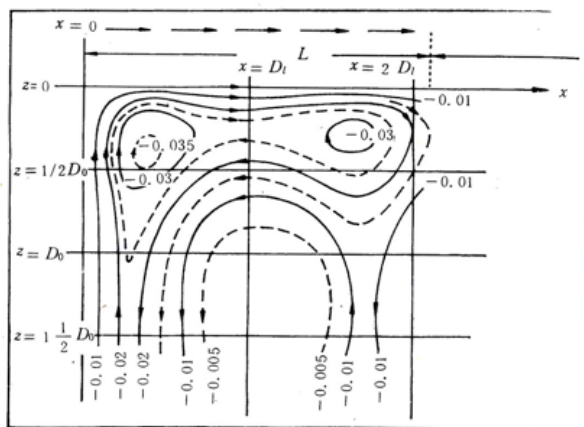
$$\begin{cases} A_l \frac{\partial^2 u}{\partial x^2} + A_z \frac{\partial^2 u}{\partial z^2} + f v + g \frac{\partial \zeta}{\partial x} = 0 \\ A_l \frac{\partial^2 v}{\partial x^2} + A_z \frac{\partial^2 v}{\partial z^2} - f u = 0 \\ \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 \end{cases}$$

## 结果讨论



- (1) 近岸产生上升流  $x \leq 0.5D_l$ ;
- (2) 风区外延附近下降流  $x = 2D_l$ ;
- (3) 上升流来自  $z = 1.5D_0$  或更深;
- (4) 最大  $w$  出现在  $z = D_0$ ;
- (5) 上层离岸流, 下层向岸流, 构成一个循环.

若风向与海岸成  $\theta$  角:



- (1) 三个升降流系统：两个顺时针，一个逆时针；
- (2) 大顺时针循环；
- (3)  $\theta = 21.5^\circ$  时，升降流达最大强度；
- (4) 纬度越低，升降流越强.

## 参考文献

- [1] V. W. Ekman, *On the Influence of the Earth's Rotation on Ocean-Currents*, vol. 2. University Microfilms INC, 01 1905.