# An AGI architecture in vector space

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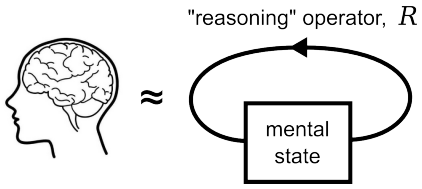
**Abstract.** This is a draft.

## 1 Main idea

The main idea is to regard "thinking" as a **dynamical system** operating on **mental states**:

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For example, a mental state could be the following set of propositions:

- I am in my room, writing a paper for AGI-16.
- I am in the midst of writing the first sentence, "The main idea is..."
- I am about to write an infinitive phrase "to regard..."

Thinking is the process of **transitioning** from one mental state to another.

By representing a cognitive state as a vector $\boldsymbol{x} \in X$ where $X$ is the cognitive state-space, the reasoning operator $R$ as a map $X \to X$, we would have at disposal all the tools available in vector space such as:

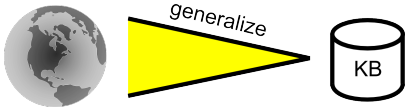
- numerical optimization (including gradient descent)
- differential equations governing time evolution
- dynamical systems theory, control theory (eg. adaptive filters)
- Lie algebra and $C^*$-algebra of continuous operators
- matrix theory, iteration and fixed-point theory
- dynamic programming (aka. reinforcement learning)
- neural networks and deep learning ... etc.

This is also partly inspired by the success of Google's PageRank and Word2Vec [] algorithms.

## 2 Relation to Logic-based AI (LBAI)

In this paper we would jump back and forth between the logic-based view and the dynamical state-space view.

LBAI can be viewed as the compression of a world model into a knowledge-base (KB) of logic formulas:

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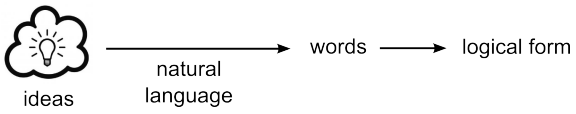
The world model is *generated* combinatorially from the set of logic formulas, vaguely reminiscent of a "basis" in vector space. The generative process in logic is much more complicated, contributing to its high *compressive* ability on the one hand, and the *complexity* of learning such formulas on the other hand.

In LBAI the knowledge representation structure is built (*fixed*) from the bottom up:

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but is it valid (or profitable) to assume that our mental representations are *isomorphic* to such logical structures? Or drastically different?

Humans are good at designing symbolic structures, but we don't know how to design *neural* representations which are more or less opaque to us. Perhaps we could use a neural network acting recurrently on the state vector to **induce** an internal representation of mental space. "*Induced by what*," you ask? By the very structure of the neural network itself. In other words, forcing a neural network to *approximate* the ideal operator $R^*$.

From an abstract point of view, we require:

- $R$ be an endomorphism: $X \to X$
- $R$ has a learning algorithm: $R \xmapsto{A} R^*$

$R$ would contain all the knowledge of the KB, so we expect it to be "large" (eg. having a huge number of parameters). We also desire $R$ to possess a **hierarchical** structure because hierarchies are computationally very efficient. A multi-layer perceptron (MLP) seems to be a good candidate, as it is just a bunch of numbers (weight matrices $W$) interleaved by non-linear activation functions:

$$R(\boldsymbol{x}) = \textcircled{\scriptsize$\int$}(W_1\textcircled{\scriptsize$\int$}(W_2 ... \textcircled{\scriptsize$\int$}(W_L\boldsymbol{x}))) \tag{4}$$

where $L$ is the number of layers. MLPs would be our starting point to explore more design options.

In 1991 Siegelmann and Sontag [2] proved that recurrent neural networks (RNNs) can emulate any Turing machine. In 1993 James Lo [1] proved that RNNs can universally approximate any non-linear dynamical system.

The idea of $R$ as an operator acting on the state is inspired by the "consequence operator" in logic, usually denoted as Cn:

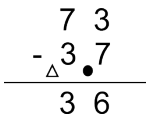
$$\mathrm{Cn}(\Gamma) = \{ \text{ set of propositions that entails from } \Gamma \} \tag{5}$$

but the function of $R$ can be broader than logical entailment. We could use $R$ to perform the following functions which are central to LBAI:

- **deduction** – forward- and backward-chaining
- **abduction** – finding explanations
- **inductive learning**

**Example 1:** primary-school arithmetic

A recurrent neural network is a *much more powerful* learning machine than a feed-forward network, even if they look the same superficially.

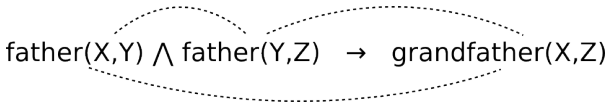



As an example, consider the way we perform 2-digit subtraction in primary school. This is done in two steps, and we put a dot on paper to mark "carry-over". The use of the paper is analogous to the "tape" in a Turing machine – the ability to use short-term memory allows us to perform much more complex mental tasks.

We did a simple experiment to train a neural network to perform primary-school subtraction. The operator is learned easily if we train the two steps *separately*. The challenge is to find an algorithm that can learn **multi-step** operations by itself.

**Example 2:** variable binding in predicate logic

The following formula in predicate logic defines the "grandfather" relation:

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In LBAI, logic possesses additional structure:

- **truth values** (eg. P(rain tomorrow) = 0.7)
- **propositional structure** (eg. conjunction: $A \wedge B$)
- **sub-propositional structure** (eg. predication: loves(john, mary) )
- **subsumption structure** (eg. dog $\subseteq$ animal)

These structures can be "transplanted" to the vector space $X$ via:

- **truth values:** an extra dimension conveying the "strength" of states
- **propositional structure:** eg. conjunction as vector addition,

$$A \wedge B = \boldsymbol{x}_A + \boldsymbol{x}_B + ... \tag{7}$$

but we have to avoid linear dependencies ("clashing") such as:

$$\boldsymbol{x}_3 = a_1\boldsymbol{x}_1 + a_2\boldsymbol{x}_2 \tag{8}$$

This would force the vector space dimension to become very high.

- **sub-propositional structure:** eg. tensor products as composition of concept atoms:

$$\text{loves(john, pete)} = \overrightarrow{john} \otimes \overrightarrow{love} \otimes \overrightarrow{pete} \tag{9}$$

- **subsumption structure:** eg. define the positive cone $C$ such that

$$\text{animal} \supseteq \text{dog} \quad \Leftrightarrow \quad \overrightarrow{animal} - \overrightarrow{dog} \in C \tag{10}$$

But the more logical structure we add to $X$, the more it will resemble logic, and this whole exercise becomes pointless. Remember our original goal is to try something different from logic, by *relaxing* what defines a logical structure. So we would selectively add features to $X$.

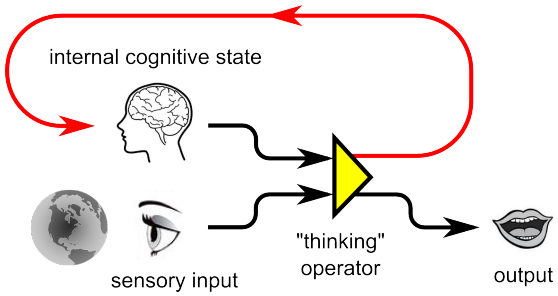
## 3 Misc points

- If sigmoid is replaced by polynomial, universal approximating property may be retained.
- Banach fixed point theorem does not apply because $R$ in general need not be contractive. Question is whether $R$ necessarily converges to fixed points and the answer is no.
- If reasoning operator $R$ is continuous, the flow of the dynamical system is governed by an autonomous differential equation. Poincare-Bendixson only applies to dynamical systems on the plane, and is irrelevant to systems whose phase space has dimension $\geq 3$, or to discrete dynamical systems.
- Time can be discrete or continuous.
- Goal is to find minimizer of error (ie, to approximate a function given some input-output data points). The (finite) set of local minima can be solved via setting $\frac{\partial R}{\partial W} = 0$. The number of local minima can be calculated as: ? McClelland paper.
- If operator is discontinuous, what advantages can be gained?

What I want to do now is to determine if $R$ implemented as a deep network is sufficient to model human-level reasoning.

## 4 Architecture

First, cartoon version:

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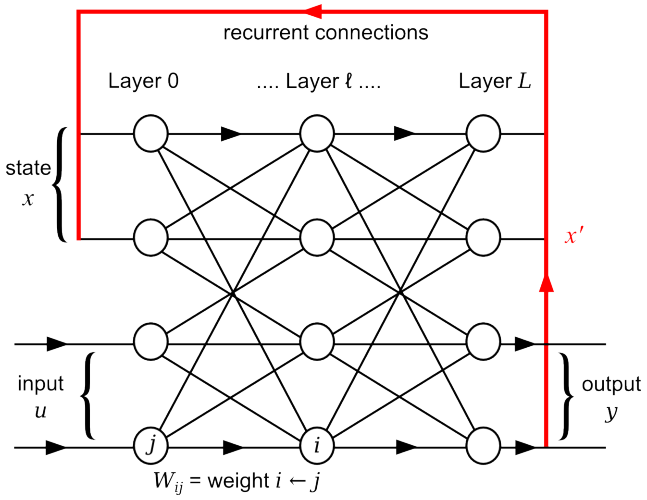


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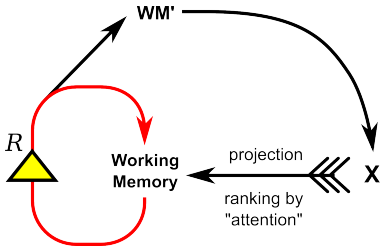
TO-DO: The state space $X$ may be too large and we may need an **attention mechanism** to select some parts of $X$ for processing by $R$. This is the notion of **working memory** in cognitive science.

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## 5 Deep Recurrent Learning

The learning algorithm for $R$ is central to our system. $R$ learns to recognize input-output pairs $(\boldsymbol{x}_0, \boldsymbol{x}^*)$. What makes it special is that $R$ is allowed to iterate

a *flexible* number of times before outputting an answer. In feed-forward learning we simply learn single-pass recognition, whereas in common recurrent learning we train against a *fixed* time sequence. Here, the time delay between input and output is allowed to stretch arbitrarily.

Suppose the recurrent network $R$ iterates $n$ times:

$$\boldsymbol{x}_{t+1} = \overbrace{R \circ R \circ ...}^{n}(\boldsymbol{x}) \tag{14}$$

As $n \to \infty$, we get the continuous-time version (a differential equation):

$$\frac{d\boldsymbol{x}(t)}{dt} = \mathfrak{R}(\boldsymbol{x}(t)) \tag{15}$$

We could run the network $R$ for a long enough time $T$ such that it is highly likely to reach an equilibrium point. Then:

$$\boldsymbol{x}_T = \int_0^T \mathfrak{R}(\boldsymbol{x}(t))dt \tag{16}$$

and the error:

$$\mathscr{E} = \boldsymbol{x}^* - \boldsymbol{x}_T \tag{17}$$

where $\boldsymbol{x}^*$ is the target value which is independent of time.

$$\begin{aligned} \frac{\partial \mathscr{E}}{\partial \boldsymbol{W}} &= -\frac{\partial}{\partial \boldsymbol{W}} \int_0^T \mathfrak{R}(\boldsymbol{x}(t))dt \\ &= -\frac{\partial}{\partial \boldsymbol{W}} \int_0^T \mathfrak{R}(W_1 \mathfrak{R}(W_2 ... \mathfrak{R}(W_L \boldsymbol{x}(t)))dt \end{aligned} \tag{18}$$

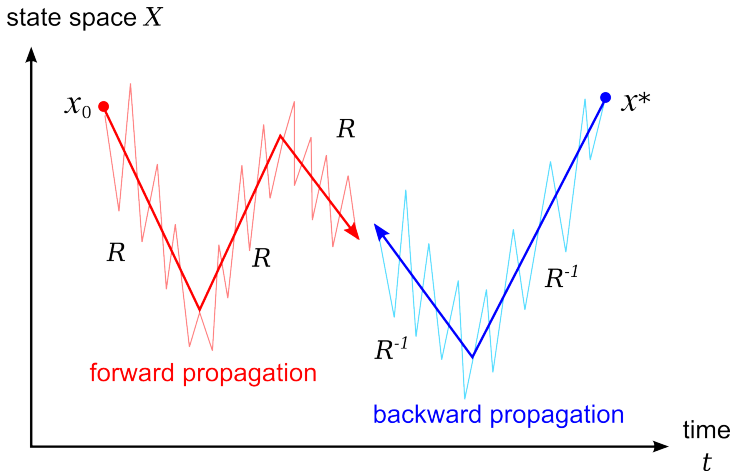
When there are many layers or if the recurrence is too long, back-prop learning becomes ineffective due to the **vanishing gradient** problem. One solution is to use the **rectifier** activation function:

$$\mathfrak{R}(x) = \begin{cases} x, & \text{if } x \geq 0 \\ 0, & \text{otherwise} \end{cases} \tag{19}$$

Since its derivative is piecewise constant, it does not suffer from the vanishing gradient problem.

### 5.1 Forward-backward Algorithm

This is inspired by forward- and backward-chaining in LBAI. We propagate the state vector from both the initial state $\boldsymbol{x}_0$ as well as the final state $\boldsymbol{x}^*$. This bi-directional propagation is added with noise and repeated many times, thus implementing a **stochastic local search**:

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When the forward and backward states get close enough, a successful path is found, and we record the gap and the noises along the path, and use them to train $R$ so that this new path would be recognized.

## Acknowledgements

In a forum discussion with Ben Goertzel dated 25 June 2014 on the AGI mailing-list: (artificial-general-intelligence @googlegroups.com), YKY asked: Why bother with neural networks, which typically require many neurons to encode data, when logic-based AI can represent a proposition with just a few symbols? Ben's insight is that neural networks are capable of learning its own representations, and their learning algorithms are relatively fast. We have been working on "neo-classical" logic-based AI for a long time, and begin to realize that inductive learning in logic (based on combinatorial search in a symbolic space) is perhaps *the bottleneck* in the entire logic-based paradigm. So we try to look for alternatives that might enable learning to be faster, though we would still emphasize that logic-based AI remains a viable approach to AGI.

## References

1. Lo. Dynamical system identification by recurrent multilayer perceptrons. *Proceedings of the 1993 World Congress on Neural Networks*, 1993.
2. Siegelmann and Sontag. Turing computability with neural nets. *Applied Mathematics Letters, vol 4, p77-80*, 1991.