DAILY PRACTICE PROBLEMS

PHYSICS

P04

Relative velocity

Topics : Relative Velocity

SECTION - I (Straight Objective Question)

This section contains seven (7) multiple choice questions. Each question has four (4) options (a), (b), (c), (d) out of which ONLY ONE is correct.

1. A boat which rows with a speed of 5 kmh^{-1} in still water crosses a river of width 1 km along the shortest path in 15 min. The velocity of river water is

(a) 1 km h^{-1} (b) 3 km h^{-1} (c) 4 km h^{-1} (d) 5 km h^{-1}

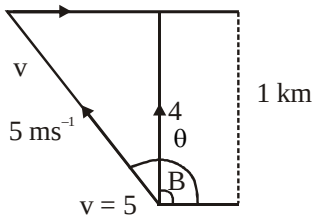
Sol. (b)

$\beta = 90^0$

R = resultant vel

= 4km h^{-1}

θ = angle at which boat rows



$$\tan\beta = \frac{v\sin\theta}{u + v\cos\theta} = \infty$$

$$\cos\theta = -\frac{1}{5}u = -0.2u$$

$$R = \sqrt{u^2 + v^2 + 2uv\cos\theta}$$

$$4 = \sqrt{u^2 + 25 + 10u(0.2u)}$$

$16 = u^2 + 25 - 2u^2 \quad \therefore u^2 = 9$

u = 3 km h^{-1}

2. A train S, 120 m long runs with a velocity 72 kmh^{-1}; while train T 130 m long is running in opposite direction with a velocity 108 kmh^{-1}. The time taken by T to cross train S is

(a) 5 sec (b) 10 sec (c) 25 sec (d) 50 sec

Sol. (a)

Velocity of S - train = 20 ms^{-1}

Velocity of T - train = –30 ms^{-1}

Relative vel. of T w.r.t. s

= – 30 – 20 = – 50 ms^{-1}

Distance = $l_1 + l_2$ = 250 m

$$\text{time } t = \frac{-250\ m}{-50\ ms^{-1}} = 5 \text{sec}$$

3. A man swims in still water of a river of width D with speed v. The river flows with velocity (v/2). He swims making an angle θ with the upstream to cross the river in shortest distance. The time taken to cross the river is

(a) D/V (b) $\frac{D}{v\cos\theta}$ (c) $\frac{D}{v\sin\theta}$ (d) $\frac{D}{\sqrt{2}\ v}$

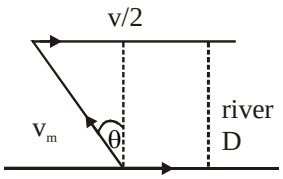
Sol. (c)

v_m = vel. of man = v

v_r = vel of river $= \frac{v}{2}$

$\frac{v}{2} = v \sin\theta \ \therefore \ \sin\theta = \frac{1}{2}$

$\theta = 30^0$ *time* $t_1 = \frac{D}{v\sin\theta} = \frac{2D}{v}$



4. A person aiming to reach the exactly opposite point on the bank of a strean is swimming with a speed of 0.5 ms^{-1} at an angle of 120^0 with teh direction of flow of the stream. The speed of the stream is

(a) 1 ms^{-1} (b) 0.5 ms^{-1} (c) 0.25 ms^{-1} (d) 0.433 ms^{-1}

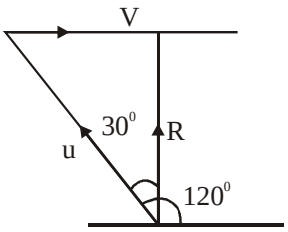
Sol. (c)

v = velocity of stream. Resultant of u and v must be R

u = 0.5 ms^{-1}, R = resultant velocity

v = u sin 30^0

$$= 0.5 \times \frac{1}{2} = 0.25\ ms^{-1}$$



5. A boat crosses a river from port *A* to port *B* which are just on the opposite side. The speed of the water is V_w and that of boat is V_B relative to still water. Assume $V_B = 2V_w$. What is the time taken by the boat, if it has to cross the river directly on the *AB* line

(a) $\frac{2D}{V_B\sqrt{3}}$ (b) $\frac{\sqrt{3D}}{2V_B}$ (c) $\frac{D}{V_B\sqrt{2}}$ (d) $\frac{D\sqrt{2}}{V_B}$

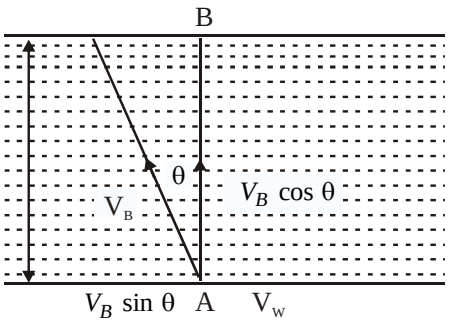
Sol. (a)

From figure, $V_B \sin\theta = V_W$

$$\sin\theta = \frac{V_w}{V_B} = \frac{1}{2} \Rightarrow \theta = 30^0 \qquad [\because V_B = 2V_W]$$

Time taken to cross the river,

$$t = \frac{D}{V_B\cos\theta} = \frac{D}{V_B\cos 30^0} = \frac{2D}{V_B\sqrt{3}}$$



6. A boat moves with a speed of 5 km/h relative to water in a river flowing with a speed of 3 km/h and having a width of 1 km. The minium time taken around a round trip is

(a) 5 min (b) 60 min (c) 20 min (d) 30 min

Sol. (d)

For the round trip he should cross perpendicular to the river $\therefore$ Time for trip to that side $= \frac{1km}{4km/hr} = 0.25hr$ To come back, again he take 0.25 hr to cross the river. Total time is 30 min, he goes to the other bank and come back at the same point.

7. Consider a collection of large number of particles each with speed v. The direction of velocity is randomly distributed in the collection. The magnitude of relative velocity between a pair of collection particles averaged over all the particles is

(a) zero (b) greater than v (c) less than v (d) v

Sol. (b)

Consider two particles with angle θ between their directions of velocities then $V_1 = V_1 - V_2$. Hence magnitude of relative velocity

$$V_1 = \sqrt{V_1^2 + V_2^2 - 2V_1V_2\cos\theta}$$

or $V_1 = 2V \sin\frac{\theta}{2}. V_1 = V_2 = V$ and average value of V_r is

$$\langle V_r \rangle = \frac{\int_0^{2\pi} V_r . d\theta}{\int_0^{2\pi} d\theta} = \frac{1}{2\pi}\int_0^{2\pi} 2V \sin\frac{\theta}{2} = \frac{V}{\pi}\int_0^{2\pi} \sin\frac{\theta}{2} d\theta$$

$$= -\frac{2V}{\pi}\left[\cos\frac{\theta}{2}\right]_0^{2\pi} = -\frac{2V}{\pi}\left[\cos 180^0 - \cos 0\right] = \frac{4V}{\pi} > V.$$

SECTION-II (One or More than correct objective Question)

This section contains three (3) multiple choice questions. Each question has four (4) options (a), (b), (c), (d) out of which ONE OR MORE is/are correct.

8. A river flows at 2 ms^{-1}. The man can swim in still water with velocity of 4 ms^{-1}. The angle at which man must swim with the banks so that he crosses the river in the shortest distance is

(a) 30^0 up stream (b) 30^0 down stream (c) 60^0 up stream (d) 60^0 down stream

Sol. (d)

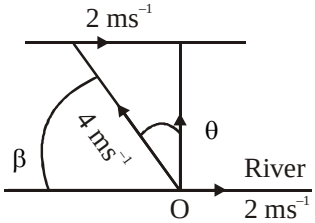
$4 \sin\theta = 2$

$$Sin\theta = \frac{1}{2}$$

$\theta = 30^0$

angle with downstream = $(90 - \theta)$

= 60^0 with downstream



9. A boat crosses the river in short possible route. The speed of the boat in still water is 13 ms^{-1}. The average speed of the boat while crossing the river comes out to be 12 ms^{-1}. The velocity of the river is

(a) 5 ms^{-1} (b) 25 ms^{-1} (c) 1 ms^{-1} (d) 4 ms^{-1}

Sol. (a)

V_b = velocity of boat = 13 ms^{-1}

V_a = average speed = 12 ms^{-1}

V_r = velocity of river $= \sqrt{13^2 - 12^2}$

= 5 ms^{-1}

10. River is flowing with a velocity $\overrightarrow{v_R} = 4\hat{i}$ m/s. A boat is moving with a volocity of $|v_B| = \sqrt{(4)^2 + (2)^2} = 2\sqrt{5}$ m/s relative to river. The width of the river is 100 m along y-direction. Choose the correct alternative(s).

(a) The boatman will cross the river in 25 s.

(b) Absolute velocity of boat man is $2\sqrt{5}$ m/s.

(c) Drift of the boatman along current is 50 m

(d) The boatman can never cross the river.

Sol. (a, b, c)

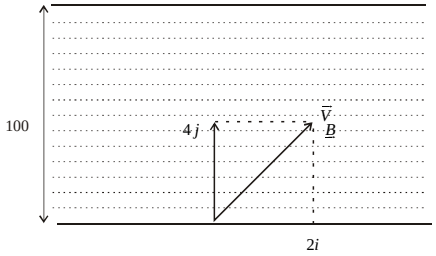
Absolute velocity of boatman is

$\vec{v}_B = \vec{v}_{BR} + \vec{v}_B = \vec{v}_R$

$= (-2\hat{i} + 4\hat{j}) + (4\hat{i})$

$= 2\hat{i} + 4\hat{j}$

Time t = $\frac{100}{4} = 25\ s$



Drift x = (2) (25) = 50 m

$|v_B| = \sqrt{(4)^2 + (2)^2} = 2\sqrt{5}$ m/s

SECTION - III (Assertion & Reasoning Type Question)

This section contains three (3) assertion & reasoning type questions. Each question has four (4) options (a), (b), (c), (d) out of which ONLY ONE is correct.

Statement based Questions :

For the following question, choose the correct answer from the codes (a), (b), (c) and (d) defined as follows ;

(a) Statement I is true, Statement II is true; Statement II is correct explanation for Statement I.
(b) Statement I is true, Statement I is true; Statement II is not correct explanation of Statement I
(c) Statement I is true; Statement II is false.
(d) Statement I is false; statement II is true.

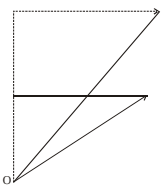
11. STATEMENT-1

For an observer looking out through the windows of a fast moving train, the near by objects appear to move in opposite direction to the train. While distant appear to be stationary

STATEMENT-2

If the observer and the object are moving with velocities V_1 and V_2 respectively with reference to a laboratory frame the velocities of the object with respect to the observer is $V_2 - V_1$

Sol. (b) Both statements are true but reason is not correct explanation of assertion. Distant objects motion depends on the agle subtended at eye. It is illustrated in fig.



12. STATEMENT – 1

The relative velocity between any two bodies moving in opposite direction is equal to the sum of the velocities of two bodies.()

STATEMENT – 2

Some times relative velocity between two bodies is equal to the difference in velocities of the two bodies.

Sol. (b) When two bodies are moving in opposite direction relative velocity between then is equal to sum of the velocities of bodies ($V_1 = V_1 – (V_2) = V_1 + V_2$). But if the bodies are moving in the same direction their relative velocity is equal to difference in velocities of bodies ($V_r = V_1 – V_2$).

13. STATEMENT – 1

A body, what ever its motion, is always at rest in a frame of reference which is fixed to the body itself.

STATEMENT – 2

The releative velocity of a body with respect to itself is zero.

Sol. (a) A body has no relative motion with respect to itself. Hence in frame of reference attached to body itself its velocity is zero.

SECTION - IV (Passage Based Question)

This section contains one (1) paragraph based upon the paragraph three(3) multiple choice questions have to be answered. Each question has four(4) options (a), (b), (c), (d) out of which ONLY ONE is correct.

Two parallel rail tracks run north south. Train *A* moves north with a speed of 54 km/h and train *B* moves south with a speed of 90 km/ h.

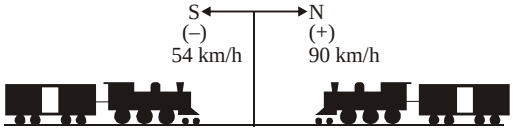
14. The relative velocity of B with respect to A is

(a) – 90 km/h (b) –144 km/h (c) –132 km/h (d) none of these

Sol. (b)

Let $v_A = 54$ km/h

then $v_B = – 90$ km/h



$\therefore$ Relative velocity of *B* with respect to *A*

$\vec{v}_{BA} = –90 – 54$

$= –144$ km/h

The velocity of train *B* with respect to train *A* appears 144 km/h along south.

15. The relative velocity of ground with respect to *B* is

(a) – 90 km/h (b) –144 km/h (c) –70 km/h (d) –60 km/h

Sol. (a)

Relative velocity of ground with respect to train *B*

$= v_{ground} – v_B$

$= 0 – 90 = – 90$ km/h

To train *B*, the ground appears to move with a speed of 90 km/h along south.

16. The velocity of a monkey running on the roof of the train *A* against its motion (with a velocity of 18 km/ h with respect to the train *A*) as observed by a man standing on the ground is

(a) 33 km/h (b) 36 km/h (c) 99 km/h (d) zero

Sol. (b)

The velocity of monkey with respect to train *A*

$[v_{monkey}]_{train\ A} = [v_{monkey}]_{ground} - [v_{train\ A}]_{ground}$

or $[v_{monkey}]_{ground} = [v_{monkey}]_{train\ A} + [v_{train\ A}]_{ground}$

= 36 km/h.

SECTION - V (Matrix Match Type)

This section contains one (1) question. It contains statements given in two columns, which have to be matched. Statements in column I are labelled as A, B, C and D whereas statements in column II are labelled as p, q, r and s. The answers to these questions have to be appropriately bubbled as illustrated in the following example. If the correct matches are A → p, r ; B → q, s; C → r and D → q, then the correctly bubbled matrix will look like the following:

	p	q	r	s
A	●	○	●	○
B	○	●	○	●
C	○	○	●	○
D	○	●	○	○

17. Match the following columns.

Column I	**Column II**
(a) It is raining verticaly at $2\sqrt{3}$ m·s^{-1}. If wind blows horizontally at 2 m·s^{-1}, . The apparent velocity of rainfall is —	**(p)** 20 m·s^{-1}
(b) A boat is moving at 12 m·s^{-1}towards north and another boat is moving at $12\sqrt{2}$ m·s^{-1} towards north-west. Find the relative velocity of the second boat with respect to the first is equal to —	**(q)** 150 m·s^{-1}
(c) From the top of a tower, two particles are dropped at an interval of 2s. Find the relative velocity and relative acceleration of the particles during the fall. Acceleration due to gravity = 10 m·s^{-2}.	**(r)** 4 m·s^{-1}
(d) A police jeep is chasing with velocity of 12.5 m·s^{-1} a thief in another jeep moving with velocity 42.5 m·s^{-1}. Police fires a bullet with muzzle velocity of 180 m·s^{-1}. The velocity with which it will strike the car of the thief is —	**(s)** 12 m·s^{-1}

Sol. **(a) → (r) ; (b) → (s) ; (c) → (p) ; (d) → (q)**

(d) Effective speed of the bullet

= speed of bullet + speed of police jeep

= 180 m·s^{-1} + 12.5 m·s^{-1} = (180 + 12.5) m·s^{-1} = 192.5 m·s^{-1}

speed of thief's jeep = 42.5 m·s^{-1}

Velocity of bullet w.r.t thief's car = 192.5 – 42.5 = 150 m·s^{-1}

SECTION - VI (Integer Type)

This section contains three (3) questions. The answer to each of the question is a single digit integer, ranging from 0 to 9.

18. Velocity of a boat in still water is 5 km h^{-1}. The boat takes 15 min to cross a river of width 1 km perpendicularly. The velocity of current is —

Sol. The boat crosses a river of width 1 km in 15 min or in $\frac{1}{4}h$.

The resultant of the velocity of the boat and current is *w* (say) and $w = \frac{1}{1/4} = 4\ km\ h^{-1}$

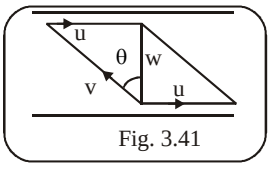


Fig. 3.41

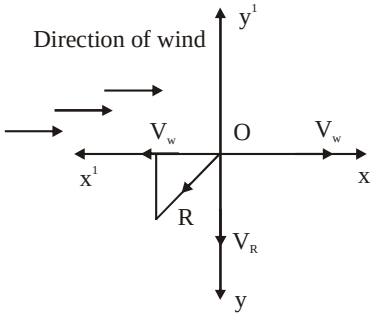
Let the velocity of the boat in still water be $\vec{v}$ and the velocity of current be $\vec{u}$. From Fig. 3.41.
$v^2 = u^2 + w^2$ *or,* $u^2 = v^2 + w^2$

or, $u = \sqrt{v^2 - w^2} = \sqrt{5^2 - 4^2} = 3\ km.h^{-1}$.

19. It is raining vertically at $2\sqrt{3}$ m. s^{-1}. If wind blows horizontally at 2 m. s^{-1}, what will be the apparent velocity and direction of rainfall?

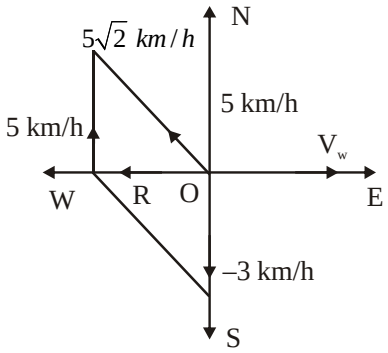
Sol. $V_R = 2\sqrt{3}$ .ms^{-1}
$V_W = 2ms^{-1}$

Let the resault velocity be R $\quad \therefore R = \sqrt{\left(2\sqrt{3}\right)^2 + 2^2} = \sqrt{12+4} = 4\ m/s$



20. A boat is moving at 5 km. h^{-1} towards north and another boat is moving at $5\sqrt{2}$ km. h^{-1} towards north-west. Find the relative velocity of the second boat with respect to the first.

Sol. $\therefore$ Required relative velocity of the second boat w.r. to the first boat is R



$\therefore R^2 + 5^2 = \left(5\sqrt{2}\right)^2 \quad \therefore R^2 = 50 - 25 = 25 \quad \therefore R = 5$