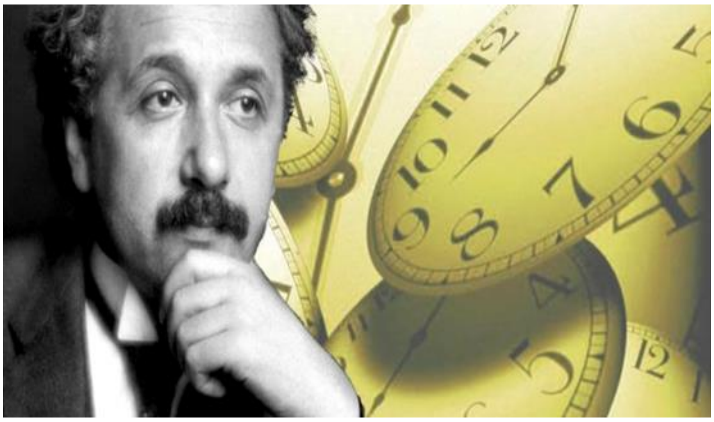
VISUAL PHYSICS ONLINE



LIGHT and SPECIAL RELATIVITY

EXPERIMENTAL VERIFICATION

TIME DILATION

HAFELE-KEATING ATOMIC CLOCK

Video: Hafele-Keating Experiment

An extremely accurate measurement of time can be made using a well-defined electronic transition in the $^{133}Cs_{55}$ atom that has a frequency of 9 192 961 770 Hz.


"During October, 1971, four cesium atomic beam clocks were flown on regularly scheduled commercial jet flights around the world twice, once eastward and once westward, to test Einstein's theory of relativity with macroscopic clocks. From the actual flight paths of each trip, the theory predicted that the flying clocks, compared with reference clocks at the U.S. Naval Observatory, should have lost 40 $\pm$23 nanoseconds during the eastward trip and should have gained 275$\pm$ nanoseconds during the westward trip ... relative to the atomic time scale of the U.S. Naval Observatory, the flying clocks lost 59$\pm$10 nanoseconds during the eastward trip and gained 273$\pm$7 nanosecond during the westward trip, where the errors are the corresponding standard deviations. These results provide an unambiguous empirical resolution of the famous clock "paradox" with macroscopic clocks."

J.C. Hafele and R. E. Keating, Science 177, 166 (1972)

In this experiment, both gravitational time dilation and kinematic time dilation are significant and are in fact of comparable magnitude. Their predicted and measured time dilation effects were as follows.

	East – West [ns]	West -East [ns]
Gravitational	144 $\pm$ 14	179 $\pm$ 18
Kinematic	-184 $\pm$ 18	96 $\pm$ 10
Net effect	-40 $\pm$ 23	275 $\pm$ 21
Observed	-59 $\pm$ 10	273 $\pm$ 7

(1 ns = $1\text{x}10^{-9}$ s)

A negative time indicates that the time on the moving clock is less than the reference clock. The moving clocks lost time (ran slow) on the eastward trip but gained time (ran faster) during the westward trip. This occurs because of the rotation of the Earth, indicating that the flying clocks ticked faster or slower than the reference clocks on Earth. The special theory of relativity is verified with the experimental uncertainties.

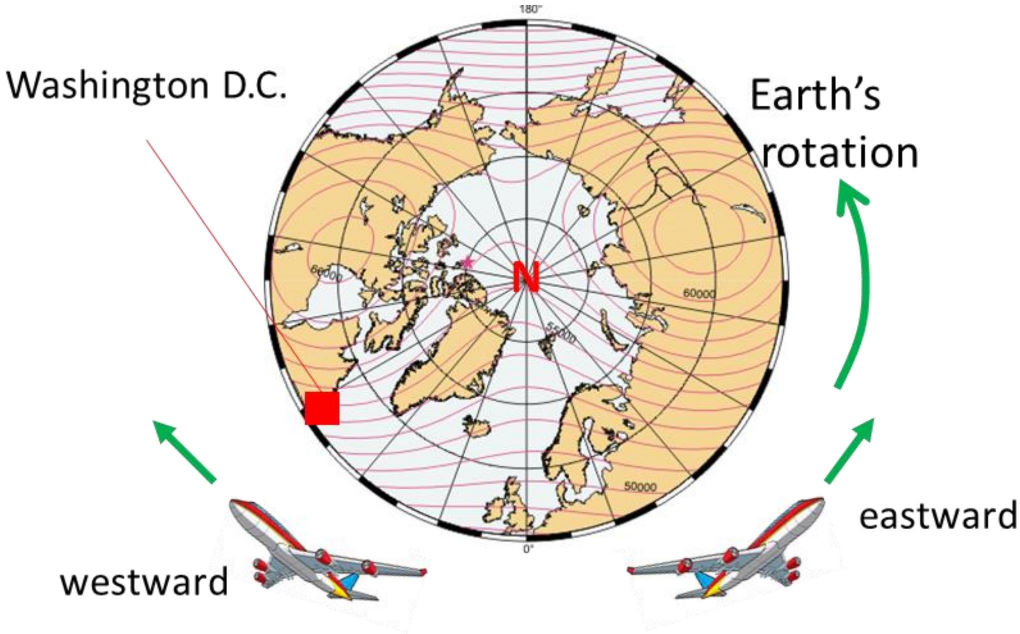


Fig. 2. Two planes take off from Washington D.C. where two atomic clocks are located at the U.S. Naval Observatory. One plane travels around the world in an easterly direction carrying an Cs atomic clock, while another Cs atomic clock is flown in a plane around the world in w westerly direction as the Earth rotates. At the end of the flights the clocks are compared. The results show that the effects of time dilation are correct.

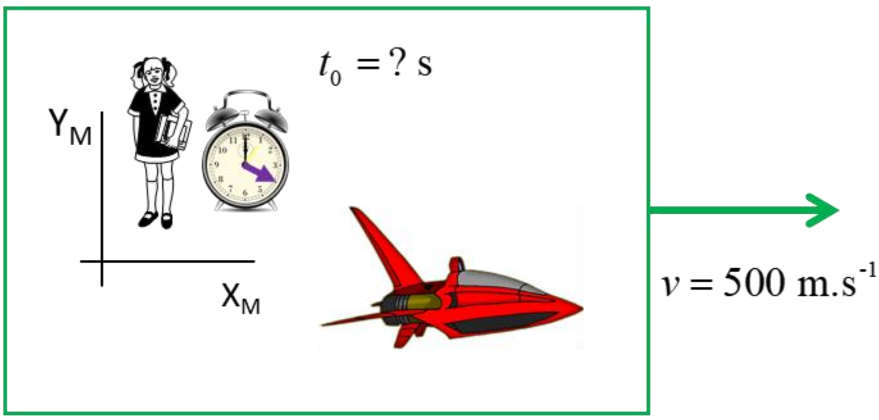
Exercise 1

An aeroplane travels around the around the circumference of the Earth at 500 m.s^{-1}. One clock remains on Earth and another clock is in the moving plane. Ignoring the effects of the Earth's rotation and gravitational effects, estimate the time difference between the two clock for a round-the-world trip.

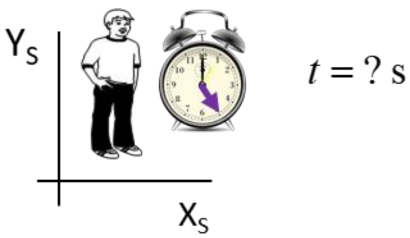
Radius of the Earth R_E = 6371 km

Solution

Mary
moving frame



Steve
fixed frame



$$R_E = 6371\ \text{km} = 6.371 \times 10^3\ \text{m}$$

Steve's system

Time for plane travels around the world

circumference of Earth $\Delta s = 2\pi R_E = 4.003\times 10^7\ \text{m}$

speed of plane $v = 500\ \text{m.s}^{-1}$

flight time $t = \dfrac{\Delta s}{v} = \dfrac{4.003\times 10^7}{500}\ \text{s} = 8.006\times 10^4\ \text{s}$

Steve's observes Mary's moving clock: moving clock run slow according to the time dilation effect

$$t = \frac{t_0}{\sqrt{1-\dfrac{v^2}{c^2}}}$$

The interval for the trip according to Mary's clock is

$$t_0 = t\sqrt{1-\frac{v^2}{c^2}}$$

Calculation of $\sqrt{1-\dfrac{v^2}{c^2}}$

$$v = 500\ \text{m.s}^{-1} \qquad c = 3\times 10^8\ \text{m.s}^{-1}$$

Putting the numbers into a calculator gives $\sqrt{1-\dfrac{v^2}{c^2}} = 1.000$

Since *v* is so small, you must use a power series expansion of the square root.

$$\sqrt{1-\frac{v^2}{c^2}} = \left(1-\frac{v^2}{c^2}\right)^{1/2} \approx 1-\frac{v^2}{2c^2}$$

So, it is best to calculate the time difference $\Delta t = t - t_0$ and not t_0

$$\Delta t = t - t_0 = t\left(\frac{v^2}{2c^2}\right) = \left(8.006\times10^4\right)\left(\frac{500^2}{(2)\left(3\times10^8\right)^2}\right)\text{ s}$$

$$\Delta t = t - t_0 = 1.11\times10^{-7}\ s = 111\text{ns}$$

At the end of the round-the-world trip, Mary's clock would be 111 ns slower than Steve's clock. Mary has aged move slowly than Steve by 111 ns.

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If you have any feedback, comments, suggestions or corrections please email:

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