AUSTRALIAN CURRICULUM PHYSICS

GETTING STARTED WITH PHYSICS

NUMBERS, MATHEMATICS AND EQUATIONS

An integral part to the understanding of our physical world is the use of **mathematical models** which can be used to explain physical phenomena and be used to make predictions. The heart of a mathematical model are its equations. It is surprising that even a simple equation such $\vec{F} = m\vec{a}$ is really a very complex statement. The equal sign simply means the number of the left is equal in magnitude to the number on the right. On the left hand side $\vec{F}$ represents a net force but the product ma on the right does not.

BASIC MATHEMATICS

Using your calculator

You will need to be competent in using your calculator for a wide range of scientific calculations involving exponentials, log functions and trig functions (degrees and radians), and be able to display numbers in scientific notation.

Significant figures

A measurement is the result some process of observation or experiment. The aim of any measurement is to estimate the "true" value of some physical quantity. However, we can never know the "true" value and so there is always some **uncertainty** associated with the measurement (except for some simple counting processes).

A rough method of indicating the degree of uncertainty is through quoting the correct number of **significant figures**. The usual convention is to quote no more than one uncertain figure. So when you write down a number, ***the last figure in that number should be the one that is in doubt***.

Rules for assigning significance to a digit

- Digits other than zero are always significant.
- Final zeros after a decimal point are always significant.
- Zeros between two other significant digits are always significant.
- Zeros at the end of a number maybe ambiguous in counting the number of significant figures.

Example 1
In a lab activity, four students calculated the mass of a brass block. The measurements they recorded were

M_1 = 2000.2041578 g
M_2 = 2002 g
M_3 = 2000 g
M_4 = 2000.2 g

Measurement 1 is given to 11 significant figures, the recording of such a measurement is ridiculous. The mass could not be calculated to this number of significant figures.
Measurement 2 has 4 significant figures.
Measurement 4 has 5 significant figures.

But, what about measurement 3 – it is ambiguous. The best way to clearly indicate the correct number of significant figures is to write the number in **scientific notation** with one digit to the left of the decimal place, so for measurement 3, we could write

$M_3 = 2 \times 10^3$ g (**1** significant figure)
$M_3 = 2.0 \times 10^3$ g (**2** significant figures)
$M_3 = 2.00 \times 10^3$ g (**3** significant figures)
$M_3 = 2.000 \times 10^3$ g (**4** significant figures)

A measurement such as 2000 g has an ambiguous number of significant figures, but without any other information, you can assume that it has 4 significant figures in doing a calculation.

Example 2
Remember, the last digit is usually the one in doubt. For example, you probably know your height to a few centimetres and you could write it as
h = 1.7**3** m **3** is uncertain.

For a tall friend of yours, you can only guess their height and so you would record
h = 1.**9** m **9** is the doubtful number.

Example 3
(a) 0.00341 (3 significant figures since $0.00341 = 3.41 \times 10^{-3}$).
(b) 2.0040×10^4 (5 significant figures).
(c) 2.004 (4 significant figures).

Example 4
Counting numbers 101, 102, 103, … have an unlimited number of significant figures(sf).

Example 5 addition or subtraction
160.45 + 6.73223 ⇒ 160.45 + 6.73 = 167.21

Example 6   multiplication or division
In multiplication and division, the result should have no more significant figures than the number having the fewest number of sf. For example, 0.00172 × 120.46. 0.000172 has only 3 significant digits, and 120.46 has 5. So according to the rule the product answer could only be expressed with 3 significant digits.

$0.00172 \times 120.46 =$ **0.207**

Example 7 square root
The root or power of a number should have as many significant figures as the number itself. $\sqrt{3.142} = 1.773$

Recording a measurement of a physical quantity

The best way to record a measurement is to use one of the following formats:

(1) name of physical quantity, symbol (often used with subscript) = value (± uncertainty)   unit

(2) name of physical quantity, symbol (often used with subscript) = value   unit

For the final recording of your measurement, the **last digit in any number is the one which in doubt**.

Small and large values should always be written in scientific notation.

Example 1
The measurement of Ian's waist can be recorded as

L = 0.87 m   the digit is 7 is in doubt
$L = 8.7\times10^2$ mm   the digit is 7 is in doubt (870 mm is misleading)
L = 0.875 m   the digit is 5 is in doubt
$L = 8.75\times10^2$ mm   the digit is 5 is in doubt
$L = (0.87 \pm 0.01)$ m   uncertainty is ± 0.01 m (10 mm)
$L = (8.7 \pm 0.1)\times10^2$ mm   uncertainty is ± 0.01 m (10 mm)
$L = (0.875 \pm 0.005)$ m   uncertainty is ± 0.005 m (5 mm)
$L = (8.75 \pm 0.05)\times10^2$ mm
uncertainty is ± 0.005 m (5 mm)
L = 0.8764 mm
incorrect – can not measure waist to < 1 mm

Example 2
Ian's height, $h_1 = 1.70$ m
Jan's height, $h_2 = (1.70 \pm 0.02)$ m
Ian's mass, $m_1 = 65.2$ kg
Jan's mass, $m_2 = (7.13 \pm 0.05) \times 10^3$ g
speed of light, $c = 3.000\times10^8$ m.s^{-1}
charge on electron, $e = 1.602\times10^{-19}$ C

Quadratic equation **you should be able to solve a quadratic equation**

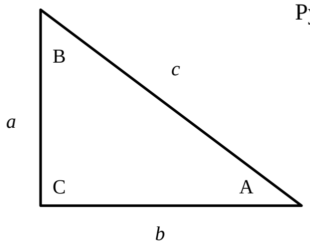
$$ax^2 + bx + c = 0 \qquad \Rightarrow \qquad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Trigonometry

For a right angle triangle with sides *a*, *b* and *c* (hypotenuse) and with angles A, B and C = 90°

$$\sin A = \frac{a}{c} \quad \cos A = \frac{b}{c} \quad \tan A = \frac{a}{b}$$

$$\tan A = \frac{\sin A}{\cos A} \qquad \sin^2 A + \cos^2 A = 1$$



Pythagorean theorem $c^2 = a^2 + b^2$

Area of triangle = ½ *a b*

Geometric formulas

- Circumference of a circle (radius r), $C = 2\pi r$
- Area of a circle (radius r), $A = \pi r^2$
- Area of sphere (radius r), $A = 4\pi r^2$
- Volume of sphere (radius r), $V = (4/3)\pi r^3$
- Area of rectangle (sides L_1 and L_2), $A = L_1 L_2$
- Volume of block (sides L_1, L_2 and L_3) $V = L_1 L_2 L_3$
- Volume of a right cylinder (height h and radius r) $V = \pi r^2 h$

Rearranging an equation

A very importance and essential skills is to be able to rearrange an equation. This can be difficult but if you follow a well define procedure you will be able to master this skill. Always rewrite the equation with the quantity you want on the left hand side of the equals sign and then perform a series of mathematical operations to both sides of the equation. If you have any difficulties, then do the operations step by step.

Example 1

Find an expression for T from the equation

$$P = \varepsilon \sigma A \left(T^4 - T_o^4\right)$$

Solution

Identify / Setup
T = ?   on left hand side of equation
Rearrange equation step by step

Execute

$$\varepsilon \sigma A\left(T^4 - T_o^{\,4}\right) = P$$

$$\left(T^4 - T_o^{\,4}\right) = \frac{P}{\varepsilon \sigma A}$$

$$T^4 = T_o^{\,4} + \frac{P}{\varepsilon \sigma A}$$

$$T = \left(T_o^{\,4} + \frac{P}{\varepsilon \sigma A}\right)^{1/4}$$

Evaluate
T on left ok T_o term ok

Changing units

It is often necessary to convert one set of units into another. This can be done by reducing the conversion to a simple algebraic problem. The following examples will illustrate how to do this.

Example 1
A car is travelling at a speed of 165 km.h^{-1}. What is the speed of the car in m.s^{-1} ?
Identify / Setup
Basic conversions

1 km = 10^3 m   1 h = (60)(60) s = 3.6×10^3 s
1 km. h^{-1} = (10^3) / (3.6×10^3) m.s^{-1}

Execute
165 km.h^{-1} = (165) (10^3) / (3.6×10^3) m.s^{-1} = 45.8 m.s^{-1}
Evaluate
answer should be smaller, ok

Example 2

The density of a liquid was 1.8 $g.mL^{-1}$. What is the density in $kg.m^{-3}$?

Solution

Identify / Setup

Basic conversions

$$1\ g = 10^{-3}\ kg$$
$$1\ mL = 1\ cm^3$$
$$1\ cm = 10^{-2}\ m$$
$$1\ cm^3 = (10^{-2})^3\ cm^3 = 10^{-6}\ m^3$$
$$1\ g.mL^{-1} = (10^{-3}) / (10^{-6})\ kg.m^{-3}$$

Execute

$$1.8\ g.mL^{-1} = (1.8)\ (10^{-3}) / (10^{-6})\ kg.m^{-3} = 1.8\times10^{3}\ kg.m^{-3}$$

Evaluate

Answer should be bigger e.g. density of water 1 $g.mL^{-1}$ or 1000 $kg.m^{-3}$ ok

EQUATION TEMPLATES

One needs to develop a method for getting behind the complexity of equations. One way of doing this is constructing ***equation templates*** to gain a better understanding of equations. An equation template involves completing a detailed summary of the "story' that the equation tells along the lines:

- State what the symbols represent (meaning & interpretation), S.I. units, other units, typical values, vector or scalar, positive or negative quantity.
- A visualisation of what the equation is about, is it a definition or a law, when is it applicable, comments and an interpretation.
- Alternative forms of the equation.
- Graphical representations of the equation.
- Numerical examples.

Examples of equation templates will often be given throughout the web notes.