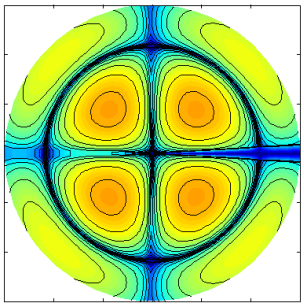


HSC PHYSICS ONLINE

KINEMATICS
PROBLEMS and ANSWERS

P1653

Two tractors labelled A and B are moving around a large field. The centre of the field is taken as the Origin O(0, 0).

Initially tractor A is located at the Origin O(0, 0) while the other tractor B is located a distance of 500 m due East of it.

The tractor A at the Origin moves with a constant speed of 5.00 $m.s^{-1}$ while the other tractor B moves with a constant speed of 2.00 $m.s^{-1}$.

Initial (t = 0 s) velocities of tractors A and B:

Case 1: Tractors A and B are both moving due East.
Case 2: Tractor A is moving East and tractor B is moving West.
Case 3: Tractor A is moving East and tractor B in a SW direction.

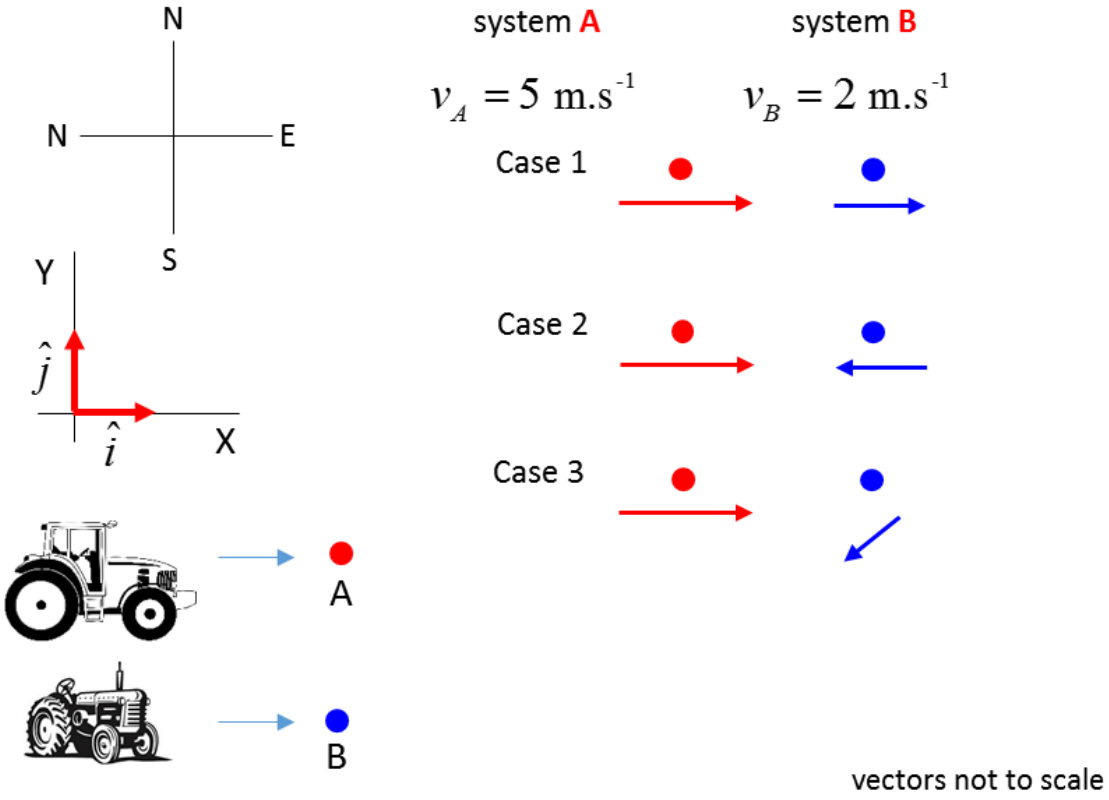
After a time interval of 100 s, calculate the following quantities for Cases 1, 2 and 3:

A. The displacements of the tractors w.r.t. the Origin.
B. The displacement of tractor A w.r.t. tractor B.
C. The displacement of tractor B w.r.t tractor A.
D. Relative velocity of tractor A w.r.t. tractor B.
E. Relative velocity of tractor B w.r.t. tractor A.

ANSWER

Problem category: relative velocity

Visualise the physical situation and think about the displacements and velocities of each tractor.



velocity of tractor A w.r.t. ground $\vec{v}_A$

velocity of tractor B w.r.t. ground $\vec{v}_B$

VIEW animation of the tractors for Cases 1, 2 and 3

Case 1

$$v_{Ax} = 5.00 \text{ m.s}^{-1} \quad v_{Ay} = 0 \text{ m.s}^{-1}$$

$$v_{Bx} = 2.00 \text{ m.s}^{-1} \quad v_{By} = 0 \text{ m.s}^{-1}$$

Event 1 (initial values):

$$t_1 = 0 \quad s_{Ax1} = 0 \text{ m} \quad s_{Ay1} = 0 \text{ m} \quad s_{Bx1} = 500 \text{ m} \quad s_{By1} = 0 \text{ m}$$

Event 2 (final values):

$$t_2 = 100 \text{ s} \quad s_{Ax2} = ? \text{ m} \quad s_{Ay2} = ? \text{ m} \quad s_{Bx2} = ? \text{ m} \quad s_{By2} = ? \text{ m}$$

Since the velocities are constant, the change in displacement is

$$v = \frac{\Delta s}{\Delta t} \Rightarrow \Delta s = v\Delta t$$

$$\Delta t = t_2 - t_1 = (10 - 0) \text{ s} = 100 \text{ s}$$

$$s_2 = s_1 + \Delta s = s_1 + 100v$$

A. Displacements of systems w.r.t. Origin

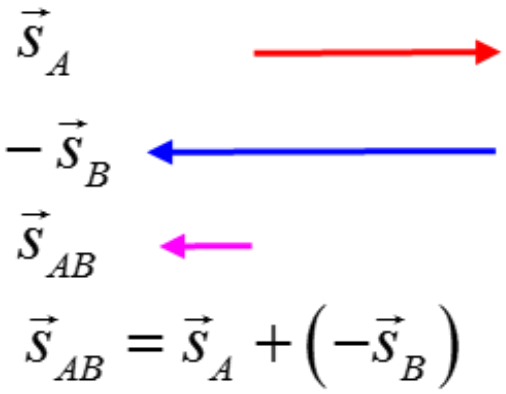
$$s_{Ax2} = s_{Ax1} + 100\, v_{Ax} = 500 \text{ m} \quad s_{Ax2} = 0 \text{ m}$$

$$s_{Bx2} = s_{Bx1} + 100\, v_{Bx} = (500 + 200) \text{ m} = 700 \text{ m} \quad s_{By2} = 0 \text{ m}$$

B. The displacement of tractor A w.r.t. tractor B

$$s_{ABx2} = s_{Ax2} - s_{Bx2} = (500 - 700) \text{ m} = -200 \text{ m}$$

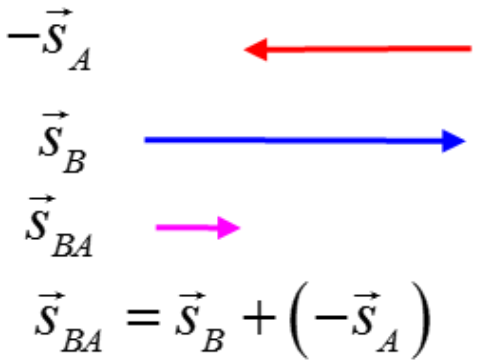
tractor A is 200 m due West of tractor B



C. The displacement of tractor B w.r.t. tractor A

$$s_{BAx2} = s_{Bx2} - s_{Ax2} = (700 - 500)\ \text{m} = +200\ \text{m}$$

tractor B is 200 m due East of tractor A



D. Relative velocity of tractor A w.r.t. tractor B

$$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$$

$$v_{ABx} = v_{Ax} - v_{Bx} = (5.00 - 2.00)\ \text{m.s}^{-1} = 3.00\ \text{m.s}^{-1}$$

$$v_{ABy} = 0\ \text{m.s}^{-1}$$

Tractor B see tractor A approaching from the West at 3.00 m.s^{-1}

E. Relative velocity of tractor B w.r.t. tractor A

$$\vec{v}_{BA} = \vec{v}_B - \vec{v}_A = -\vec{v}_{AB}$$

$$v_{BAx} = v_{Bx} - v_{Ax} = (2.00 - 5.00)\ \text{m.s}^{-1} = +3.00\ \text{m.s}^{-1}$$

$$v_{ABy} = 0\ \text{m.s}^{-1}$$

Tractor A see tractor B approaching from the East at 3.00 m.s^{-1}

Case 2

$$v_{Ax} = 5.00 \text{ m.s}^{-1} \quad v_{Ay} = 0 \text{ m.s}^{-1}$$

$$v_{Bx} = -2.00 \text{ m.s}^{-1} \quad v_{By} = 0 \text{ m.s}^{-1}$$

Event 1 (initial values):

$$t_1 = 0 \quad s_{Ax1} = 0 \text{ m} \quad s_{Ay1} = 0 \text{ m} \quad s_{Bx1} = 500 \text{ m} \quad s_{By1} = 0 \text{ m}$$

Event 2 (final values):

$$t_2 = 100 \text{ s} \quad s_{Ax2} = ? \text{ m} \quad s_{Ay2} = ? \text{ m} \quad s_{Bx2} = ? \text{ m} \quad s_{By2} = ? \text{ m}$$

Since the velocities are constant, the change in displacement is

$$v = \frac{\Delta s}{\Delta t} \Rightarrow \Delta s = v\Delta t$$

$$\Delta t = t_2 - t_1 = (10 - 0) \text{ s} = 100 \text{ s}$$

$$s_2 = s_1 + \Delta s = s_1 + 100v$$

A. Displacements of systems w.r.t. Origin

$$s_{Ax2} = s_{Ax1} + 100 \; v_{Ax} = 500 \text{ m} \quad s_{Ax2} = 0 \text{ m}$$

$$s_{Bx2} = s_{Bx1} + 100 \; v_{Bx} = (500 - 200) \text{ m} = 300 \text{ m} \quad s_{By2} = 0 \text{ m}$$

B. The displacement of tractor A w.r.t. tractor B

$$s_{ABx2} = s_{Ax2} - s_{Bx2} = (500 - 300) \text{ m} = 200 \text{ m}$$

tractor A is 200 m due East of tractor B

$$\vec{s}_A \quad \longrightarrow$$

$$-\vec{s}_B \quad \longleftarrow$$

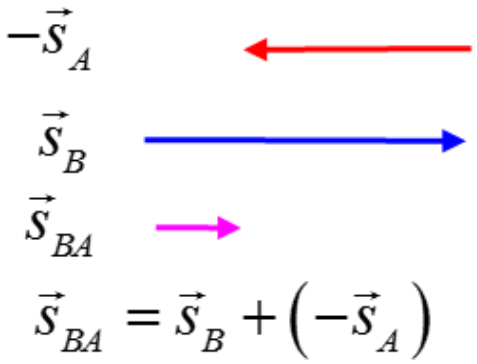
$$\vec{s}_{AB} \quad \longleftarrow$$

$$\vec{s}_{AB} = \vec{s}_A + (-\vec{s}_B)$$

C. The displacement of tractor B w.r.t. tractor A

$$s_{BAx2} = s_{Bx2} - s_{Ax2} = (300 - 500)\ \text{m} = -200\ \text{m}$$

tractor B is 200 m due West of tractor A



D. Relative velocity of tractor A w.r.t. tractor B

$$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$$

$$v_{ABx} = v_{Ax} - v_{Bx} = (5.00 - (-2.00))\ \text{m.s}^{-1} = 7.00\ \text{m.s}^{-1}$$

$$v_{ABy} = 0\ \text{m.s}^{-1}$$

Tractor B see tractor A travelling towards the East at 7.00 m.s^{-1}

E. Relative velocity of tractor B w.r.t. tractor A

$$\vec{v}_{BA} = \vec{v}_B - \vec{v}_A = -\vec{v}_{AB}$$

$$v_{BAx} = v_{Bx} - v_{Ax} = (-2.00 - 5.00)\ \text{m.s}^{-1} = -7.00\ \text{m.s}^{-1}$$

$$v_{ABy} = 0\ \text{m.s}^{-1}$$

Tractor A see tractor B travelling towards the West at 7.00 m.s^{-1}

Case 3

$v_{Ax} = 5.00 \text{ m.s}^{-1} \quad v_{Ay} = 0 \text{ m.s}^{-1}$

$v_{Bx} = -\sqrt{2.00} \text{ m.s}^{-1} = -1.41 \text{ m.s}^{-1} \quad v_{By} = -\sqrt{2.00} \text{ m.s}^{-1} = -1.41 \text{ m.s}^{-1}$

Event 1 (initial values):

$t_1 = 0 \quad s_{Ax1} = 0 \text{ m} \quad s_{Ay1} = 0 \text{ m} \quad s_{Bx1} = 500 \text{ m} \quad s_{By1} = 0 \text{ m}$

Event 2 (final values):

$t_2 = 100 \text{ s} \quad s_{Ax2} = ? \text{ m} \quad s_{Ay2} = ? \text{ m} \quad s_{Bx2} = ? \text{ m} \quad s_{By2} = ? \text{ m}$

Since the velocities are constant, the change in displacement is

$$v = \frac{\Delta s}{\Delta t} \Rightarrow \Delta s = v \Delta t$$

$$\Delta t = t_2 - t_1 = (10 - 0) \text{ s} = 100 \text{ s}$$

$$s_2 = s_1 + \Delta s = s_1 + 100 v$$

A. Displacements of systems w.r.t. Origin

$s_{Ax2} = s_{Ax1} + 100 \, v_{Ax} = 500 \text{ m} \quad s_{Ax2} = 0 \text{ m}$

$s_{Bx2} = s_{Bx1} + 100 \, v_{Bx} = (500 - 141) \text{ m} = 359 \text{ m}$

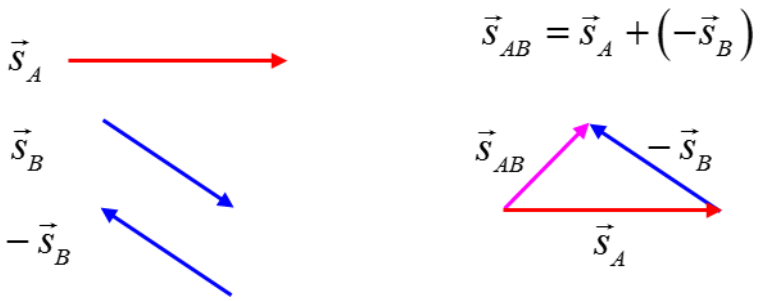
$s_{By2} = s_{By1} + 100 \, v_{By} = (0 - 141) \text{ m} = -141 \text{ m}$

B. The displacement of tractor A w.r.t. tractor B

$s_{ABx2} = s_{Ax2} - s_{Bx2} = (500 - 359) \text{ m} = 141 \text{ m}$

$s_{ABy2} = s_{Ay2} - s_{By2} = (0 - (-141)) \text{ m} = 141 \text{ m}$

Tractor A is 200 m in a NE direction from tractor B

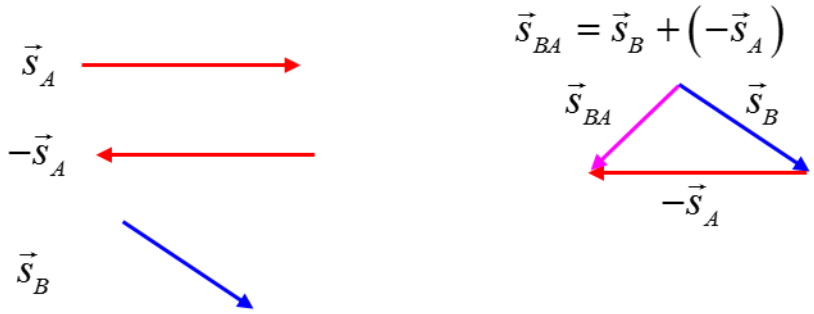


C. The displacement of tractor B w.r.t. tractor A

$$s_{BAx2} = s_{Bx2} - s_{Ax2} = -141 \text{ m}$$

$$s_{BAy2} = s_{By2} - s_{Ay2} = -141 \text{ m}$$

Tractor B is 200 m in a SW direction from tractor A



D. Relative velocity of tractor A w.r.t. tractor B

$$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$$

$$v_{ABx} = v_{Ax} - v_{Bx} = \left(5.00 - (-1.41)\right) \text{ m.s}^{-1} = 6.41 \text{ m.s}^{-1}$$

$$v_{ABy} = v_{Ax} - v_{Bx} = \left(0 - (-1.41)\right) \text{ m.s}^{-1} = 1.41 \text{ m.s}^{-1}$$

Tractor B see tractor A travelling at 6.56 m.s^{-1} in the direction of 12.1° N of E

E. Relative velocity of tractor B w.r.t. tractor A

$$\vec{v}_{BA} = \vec{v}_B - \vec{v}_A = -\vec{v}_{AB}$$

Tractor A see tractor B travelling at 6.56 m.s^{-1} in the direction of 12.1° W of S