VISUAL PHYSICS ONLINE

MODULE 5
ADVANCED MECHANICS

GRAVITATIONAL FIELD: FORCES

Newton's Law of Universal Gravitation (attraction between two masses) $|\vec{F}| = F = \dfrac{G m_1 m_2}{r^2}$

Gravitational field strength $\vec{g} = \dfrac{\vec{F}}{m}$

Gravitation field strength surrounding the Earth $g = \dfrac{G\,M_E}{r^2}$

Gravitation field strength at the Earth's surface

$$g = \frac{G\,M_E}{R_E^{\,2}} \sim 9.8\ \text{m.s}^{-2}$$

Weight $F_G = m\,g$

NEWTON'S LAW OF UNIVERSAL GRAVITATION

Newton's Universal Law of Gravitation states that any two objects exert a gravitational force of attraction on each other. The direction of the force is along the line joining the two objects. The magnitude of the force is proportional to the product of the masses of the two objects, and inversely proportional to the square of the distance between them (figure 1, equation 1).

(1) $|\vec{F}| = F = \dfrac{G m_1 m_2}{r^2}$ **Newton's Law of Universal Gravitation**

force F [N]

mass of the two objects m_1 and m_2 [kg]

distance between the centres of the two objects r [m]

Universal Gravitation Constant G = 6.673x10^{-11} N.m^2.kg^{-2}

This equation should **not** be expressed in terms of vectors. $|\vec{F}|$ is the magnitude of the force acting on mass m_1 and mass m_2. The force acting on m_1 is opposite in direction to the force acting on mass m_2.

Figure 1 shows why it better not to use vector notation for the gravitational force.

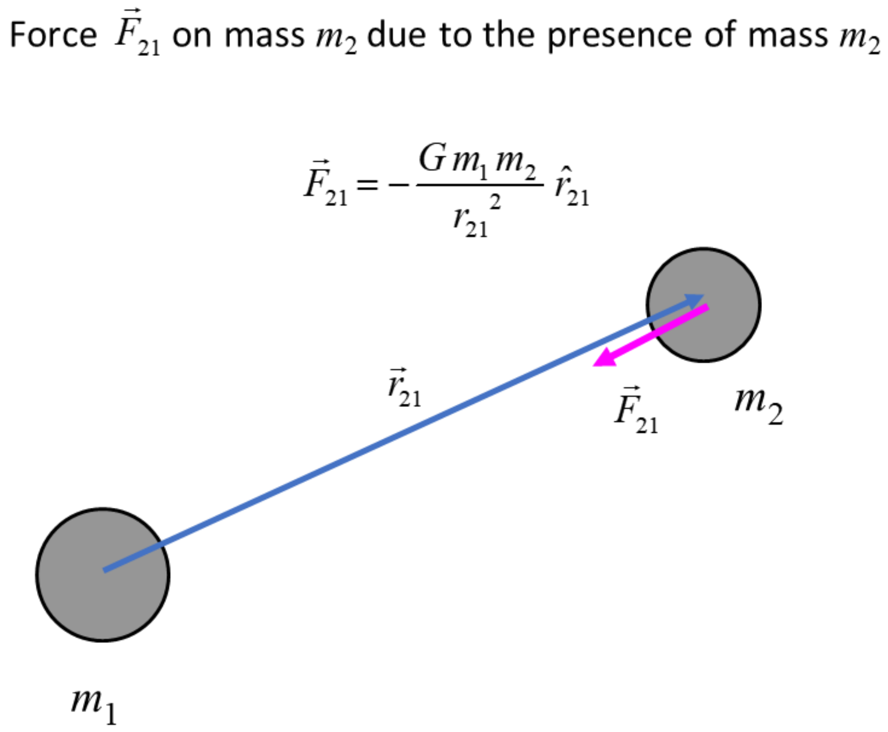


Fig. 1. Gravitational force $\vec{F}_{21}$ on mass on m_2 due to presence of mass m_1 in vector notation.

By 1687, Newton had formulated his ideas and unified centuries of astronomical observations into a coherent theory of gravity in his famous book *Philosophiae Naturalis Principia Mathematica*. A quote from Newton

> *Every particle in the universe attracts every other particle with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.*

For any calculations, we can take the mass of an object being concentrated at a point known as the **centre of mass**. For example, we take the geometric centres to determine the separation distance between two planets.

The variation of the magnitude of the gravitational force with separation distance between two objects is shown in figure (2).

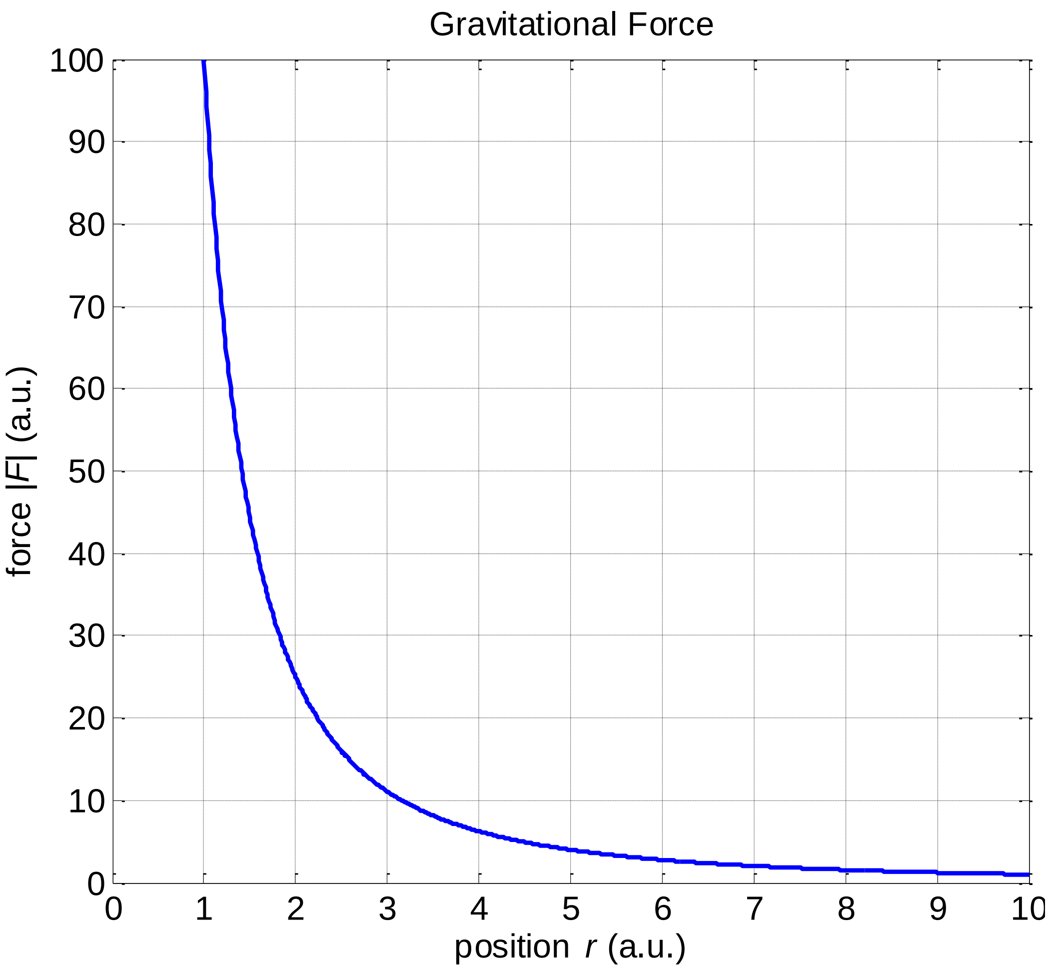


Fig.2. Gravitation force as a function of separation distance. The graph shows the inversely proportional relationship between force and separation distance.

Figure 3 shows how the force is dependent upon the masses of the two objects and their separation distance.

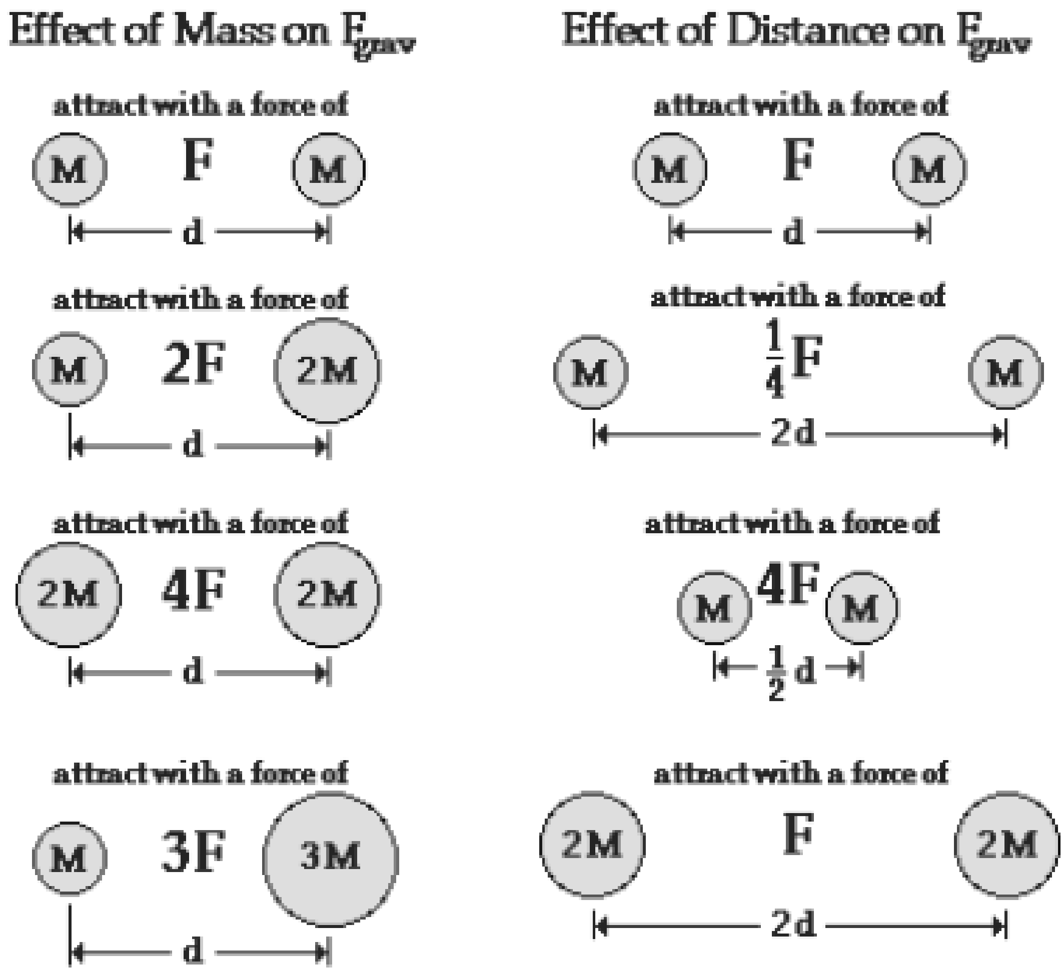


Fig. 3. Gravitational force dependence on mass and distance.

Newton's Law of Universal Gravitation plays a major role in calculations for the motion of rockets, satellites and the motion of planets around the Sun and the Moon around the Earth.

GRAVIATIONAL FIELD

A gravitational field surrounds every object that has mass, and this field permeates all of space. A second object of mass m experiences a gravitational force F in this field. A field is a region of space where an object experiences a force. For example, a charged particle will experience a force in an electric field and a moving charged particle will experience a force in a magnetic field.

The strength of the gravitational field is given by the **gravitational field strength** g (also called the acceleration due to gravity) where

(2) $$\vec{g} = \frac{\vec{F}}{m}$$ **gravitational field strength**

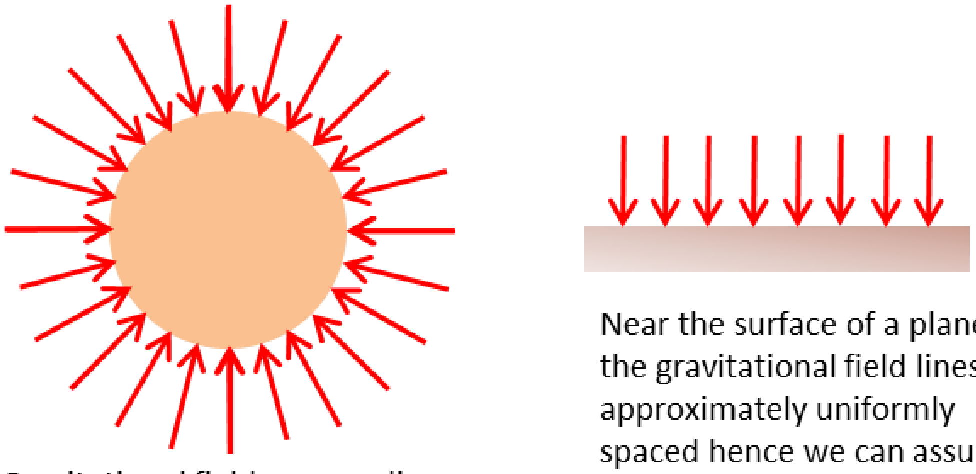
The S.I. unit for the gravitational field strength g is $N.kg^{-1}$ or $m.s^{-2}$.

The gravitational field strength g_{planet} at the surface a planet can be determined from Newton's Law of Universal Gravitation (equations 1 and 2). Consider an object of mass m at the surface of a planet where R_{planet} is its radius and M_{planet} is the mass of the planet.

$$g_{planet} = \frac{F}{m} = \frac{G M_{planet}\, m}{m R_{planet}^{\;2}}$$

(3) $$g_{planet} = \frac{G M_{planet}}{{R_{planet}}^2}$$

The gravitational force can be visualised by a pattern of **gravitational field lines**. The arrow shows the direction of a force on a mass placed at that point in the field and the density of the field lines is proportional to the gravitational field strength. Figure (4) shows the gravitational field surrounding a planet and in a region close to the surface of the planet.



Gravitational field surrounding the planet increases towards the surface as shown by the increase in the density of the field lines.

Near the surface of a planet the gravitational field lines are approximately uniformly spaced hence we can assume a uniform gravitational field strength.

Fig. 4. Gravitational field surrounding a planet.

The gravitational field strength (or the acceleration due to gravity) for the Earth (mass M_E and radius R_E) is determined by considering the gravitational force acting of an object of mass m and distance r from the centre of the Earth and Newton's Second Law

$$F = \frac{G\,M_E\,m}{r^2} = ma = m\,g$$

$$g = \frac{G\,M_E}{r^2}$$

At the Earth's surface, we take the gravitational field to be constant where $r = R_E$ and the value for the acceleration due to gravity or gravitational field strength is

(4) $$g = \frac{G\,M_E}{R_E^{\,2}}$$

Earth's mass $M_E = 5.97\times10^{24}$ kg

Earth's radius $R_E = 6.38\times10^{6}$ m

$\Rightarrow g = 9.7871$ m.s^{-2}

The value of g for calculations is often taken as 10 m.s^{-2}, 9.8 m.s^{-2} or 9.81 m.s^{-2}.

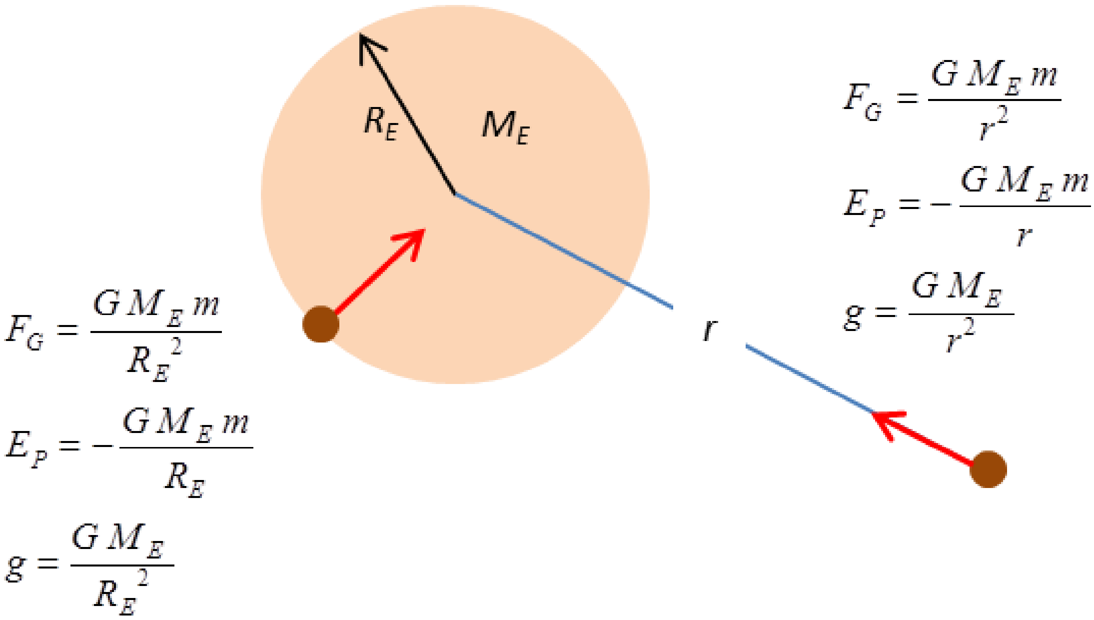


Fig. 5. Gravitational field surrounding the Earth. The magnitudes of the gravitational force, gravitational potential energy and the gravitational field strength (acceleration due to gravity) all depend upon the distance between the centre of the Earth and object within the gravitational field.

Experiment – Determination of the acceleration due to gravity

The period of oscillation of a simple pendulum which vibrates with a small amplitude only depends upon the length of the pendulum and the acceleration due to gravity (equation 5)

(5) $$T = 2\pi\sqrt{\frac{L}{g}}$$ small angles only

Perform you own experiment to measure g by varying the length L of the pendulum and measuring its period T.

Hint: Plot a graph of T^2 vs L.

Web activity and spreadsheet activity

Use the web to find the mass and radius of all the planets, the Sun and Moon. Enter the data into a spreadsheet and calculate the acceleration due to gravity at their surfaces and compare their value with g at the surface of the Earth.

What affects the value of *g* – variations of *g* from 9.8 $m.s^{-2}$?

Why ?

- Changes in height of the Earth's surface – mountains & valleys
- Distribution of mass near the Earth's surface - dense mineral deposits
- Rotation of the Earth

  (Earth 'fatter' at the equator $g_{\text{pole}} > g_{\text{equator}}$)
- Earth not a perfect sphere
- 9.782 $m.s^{-2}$ (equator) $\leq g \leq$ 9.832 $m.s^{-2}$ (pole)
- altitude h $g = \dfrac{G M_E}{(r_E + h)^2}$

Location	g ($m.s^{-2}$)	Altitiude (km)	g ($m.s^{-2}$)
Equator	9.780	0	9.81
Sydney	9.797	1 000	7.33
Melbourne	9.800	5 000	3.08
South Pole	9.832	10 000	1.49

WEIGHT

In physics, weight is a measurement of the gravitational force acting on an object. Near the surface of the Earth, the acceleration due to gravity is approximately constant; this means that an object's weight is roughly proportional to its mass.

The weight of an object of mass m is

(6) $F_G = m\,g$ **weight**

- The weight of an object is the same as the gravitational force between the object and its central mass.
- The weight of an object varies according to the gravitational field strength (acceleration) that is affected by distance from the centre of a planet.
- Mass is not affected by gravitational acceleration while weight is.

The weight of an object can be found by measuring the extension or compression of a spring since its extension and compression are proportional to the mass of the object.

It some textbooks weight is defined as the force exerted on a support as shown in figure (5).

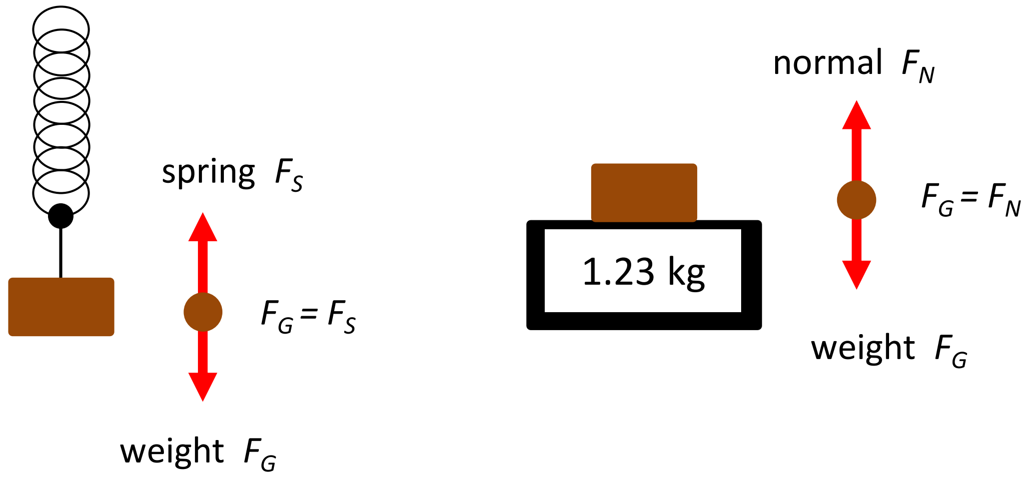


Fig. 5. Weight of an object can be found by measuring the extension or compression of a spring.

Note: The equations given in the Physics Stage 6 Syllabus are **incorrect**

$$\vec{F} = -\frac{GMm}{\vec{r}^{\,2}} \qquad \vec{g} = \frac{GM}{\vec{r}^{\,2}}$$

you simply cannot divide by a vector

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http://www.physics.usyd.edu.au/teach_res/hsp/sp/spHome.htm

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