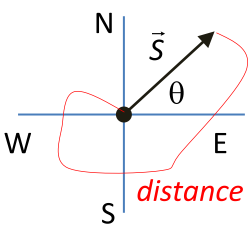
POSITION / LENGTH / DISTANCE / DISPLACEMENT [metre m]

scalar or vector

$L \quad d \quad \Delta d \quad D \quad h \quad H \quad r \quad R \quad a \quad x \quad \Delta x \quad y \quad s \quad s_x \quad s_y$

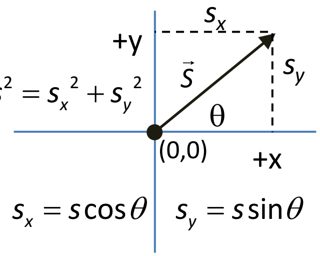
nano $1\ \text{nm} = 10^{-9}\ \text{m}$ / micro $1\ \mu\text{m} = 10^{-6}\ \text{m}$ / $1\ \text{mm} = 10^{-3}\ \text{m}$ / $1\ \text{km} = 10^{3}\ \text{m}$



distance travelled d

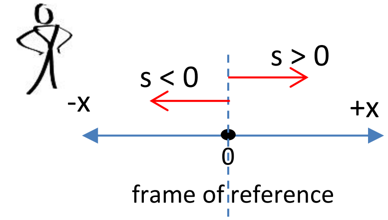
displacement $\vec{S}$

magnitude s / direction θ N of E

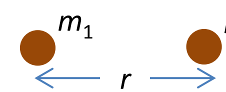


$s^2 = s_x^{\ 2} + s_y^{\ 2}$ $\quad \tan\theta = \dfrac{s_y}{s_x}$

$s_x = s\cos\theta \quad s_y = s\sin\theta$

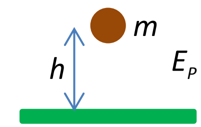


frame of reference



$F = G\dfrac{m_1 m_2}{r^2}$

Gravitation

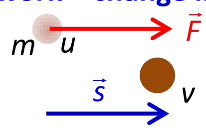


$E_P = mgh$

average speed $\quad v_{avg} = \dfrac{\Delta d}{\Delta t}$

average velocity $\quad \vec{v}_{avg} = \dfrac{\Delta \vec{s}}{\Delta t}$

work = change in KE



$W = F\,s = \tfrac{1}{2}mv^2 - \tfrac{1}{2}mu^2$

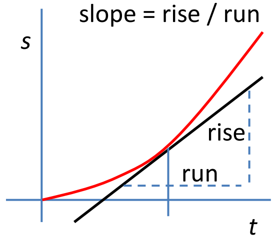
acceleration a = constant

$s = ut + \tfrac{1}{2}at^2 \quad v^2 = u^2 + 2as$

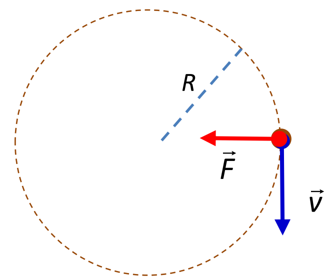
Instantaneous velocity

$v = \dfrac{ds}{dt}$ = slope of tangent s/t graph

slope = rise / run



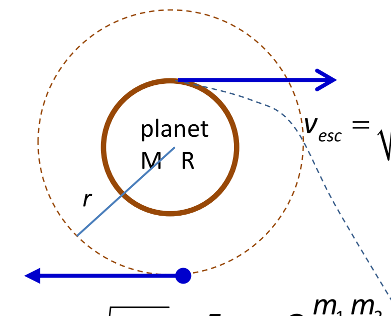
uniform circular motion



$F_c = \dfrac{mv^2}{R} \quad a_c = \dfrac{v^2}{R}$

centripetal force

centripetal acceleration

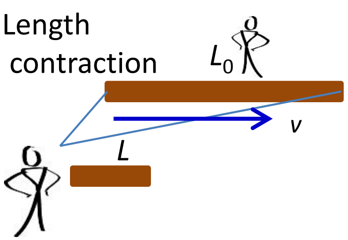


$v_{esc} = \sqrt{\dfrac{2GM}{R}}$

$v_{orb} = \sqrt{\dfrac{GM}{r}} \quad E_P = -G\dfrac{m_1 m_2}{r}$

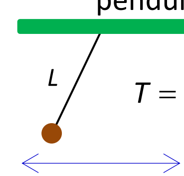
Kepler's 3rd law $\quad \dfrac{T^2}{r^3} = \dfrac{4\pi}{GM}$

Length contraction



$L = L_0\sqrt{1 - v^2/c^2}$

pendulum



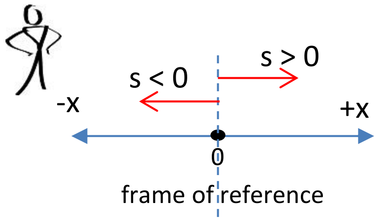
$T = 2\pi\sqrt{\dfrac{L}{g}}$

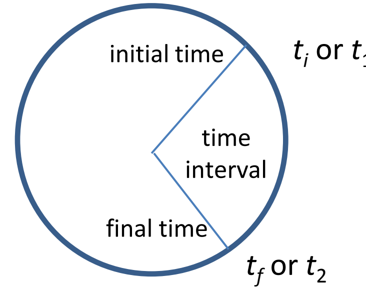
small amplitude only

measure L & $T \Rightarrow g$

TIME TIME INTERVAL [second s] t Δt T

scalar





$\Delta t = t_f - t_i = t_2 - t_1$

acceleration a = constant

$v = u + at \qquad s = ut + \frac{1}{2}at^2$

period T time for **one** complete oscillation or orbit

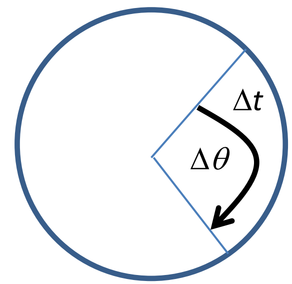
frequency f [hertz Hz]

number of oscillations (orbits) in one second

$$T = \frac{1}{f} = \frac{2\pi}{\omega}$$

angular frequency ω [rad.s^{-1}]

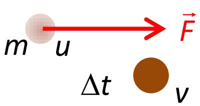
rate at which angle is swept out



$$\omega = 2\pi f = \frac{2\pi}{T}$$

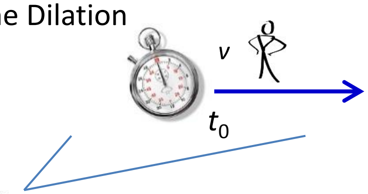
impulse

= change on momentum



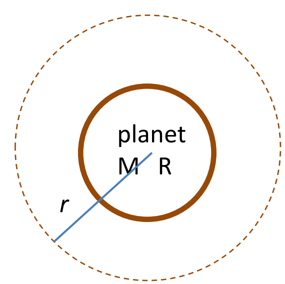
$J = F\,\Delta t = mv - mu$

Time Dilation



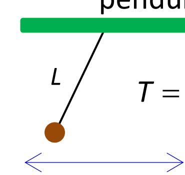
$$t = \frac{t_0}{\sqrt{1 - v^2/c^2}}$$

Kepler's 3rd law



$$\frac{T^2}{r^3} = \frac{4\pi}{GM}$$

pendulum



$$T = 2\pi\sqrt{\frac{L}{g}}$$

small amplitude only

measure L & $T \Rightarrow g$

VELOCITY $[\mathrm{m.s^{-1}}]$

fast

slow

velocity (vector) / speed (scalar)

[1D] $v \quad u \quad v_0 \quad v_1 \quad v_2 \quad v_A \quad V_B$

$v > 0 \qquad v < 0$

convert $\mathrm{m.s^{-1}} \Leftrightarrow \mathrm{km.h^{-1}}$

average $\quad v_{av} = \dfrac{\Delta s}{\Delta t} = \dfrac{s_2 - s_1}{t_2 - t_1}$

instantaneous $\quad \vec{v} = \lim_{\Delta t \to 0}\left\{\dfrac{\Delta \vec{s}}{\Delta t}\right\} = \dfrac{d\vec{s}}{dt}$

$$\Delta v = v_2 - v_1 = \int_{t_1}^{t_2} a\,dt$$

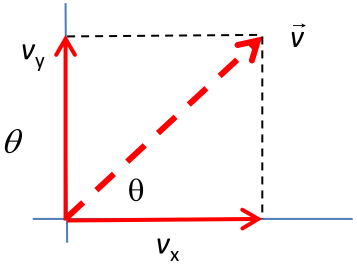
speed of light 3×10^8 m.s-1
speed of sound in air ~ 340 $\mathrm{m.s^{-1}}$

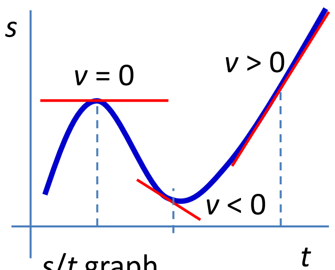
[2D] $\vec{v} \quad v \quad vx \quad vy$

$$v^2 = v_x^{\,2} + v_y^{\,2}$$

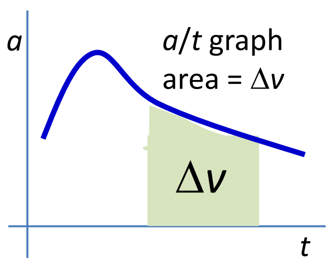
$$vv_x = v\cos\theta \quad \rightarrow v_y = v\sin\theta$$

$$\tan\theta = \frac{v_y}{v_x}$$





s/t graph
slope tangent = Δv



a/t graph
area = Δv

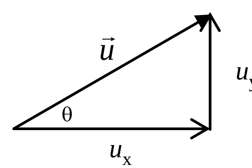
Motion with uniform acceleration

[1D] $v = u + at \quad s = ut + \tfrac{1}{2}at^2 \quad v^2 = u^2 + 2as \quad v_{av} = s/t = (u+v)/2$

[2D] **horizontal motion** $\quad a_x = 0 \quad v_x = u_x \quad s_x = v_x t$

vertical motion $\quad a_y = g \quad v_y = u_y + a_y t \quad s_y = u_y t + \tfrac{1}{2}a_y t^2$

$$v_y^{\,2} = u_y^{\,2} + 2\,a_y s_y$$



$u_x = u\cos\theta \qquad u_y = u\sin\theta$

$\tan\theta = \dfrac{u_y}{u_x} \qquad u^2 = u_x^{\,2} + u_y^{\,2}$

Object of mass m moving with velocity $\vec{v}$

momentum $\quad \vec{p} = m\vec{v} \qquad p = mv$

kinetic energy $\quad E_K = \tfrac{1}{2}mv^2 \qquad m \quad \vec{v}$

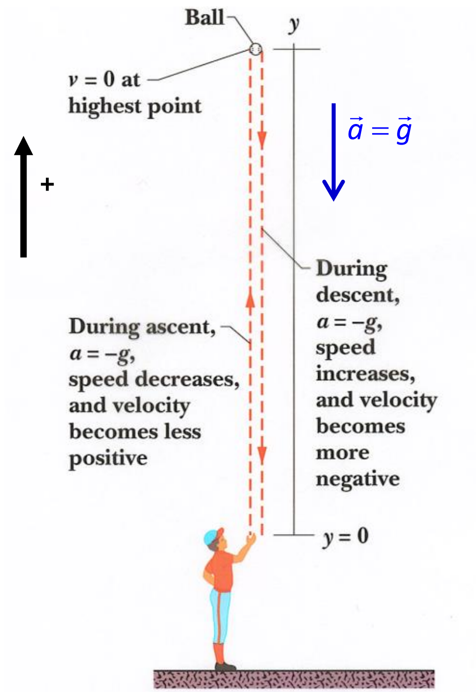
Newton's 1st law $\quad \Sigma\vec{F} = 0 \Rightarrow \vec{a} = 0 \Rightarrow \vec{v} = \text{constant} \Rightarrow$ straight line motion with constant speed

Newton's 2nd law $\quad \vec{a} = \dfrac{\Sigma\vec{F}}{m} \Rightarrow \Delta\vec{v} \Rightarrow \vec{v}$ changes (faster, slower, chnage in direction)

ACCELERATION [m.s^{-2}]

vector $\vec{a}$ a a_{av} a_x a_y g a_c

acceleration due to gravity g = 9.8 m.s^{-2}



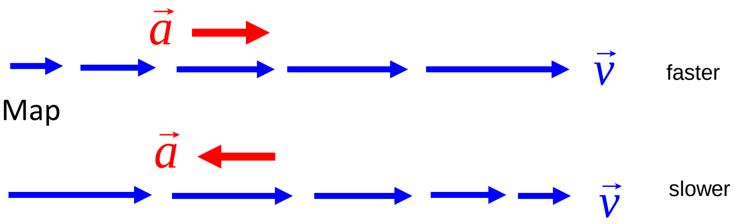
average $$a_{av} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$$

instantaneous $$\vec{a} = \lim_{\Delta t \to 0}\left\{\frac{\Delta \vec{v}}{\Delta t}\right\} = \frac{d\vec{v}}{dt}$$

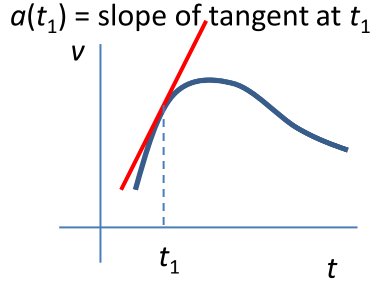
acceleration is the time rate of change of the velocity

* getting faster
* getting slower
* change in direction

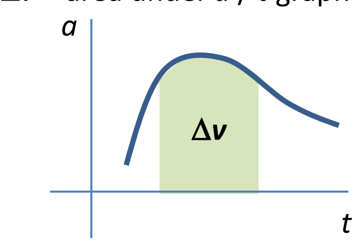
Motion Map



$a(t_1)$ = slope of tangent at t_1



Δv = area under a / t graph



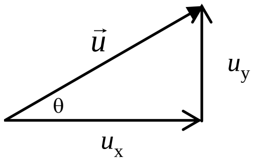
Motion with uniform acceleration

[1D] $v = u + a\,t$ $\quad s = u\,t + ½\,a\,t^2$ $\quad v^2 = u^2 + 2a\,s$ $\quad v_{av} = s / t = (u + v) / 2$

[2D] **horizontal motion** $\quad a_x = 0$ $\quad v_x = u_x$ $\quad s_x = v_x\,t$

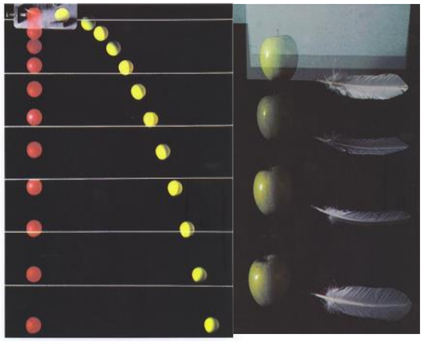
vertical motion $\quad a_y = g$ $\quad v_y = u_y + a_y\,t$ $\quad s_y = u_y\,t + ½\,a_y\,t^2$

$v_y^2 = u_y^2 + 2\,a_y\,s_y$



$u_x = u\cos\theta$ $\qquad u_y = u\sin\theta$

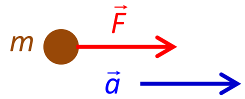
$$\tan\theta = \frac{u_y}{u_x} \qquad u^2 = u_x^{\,2} + u_y^{\,2}$$



Horizontal & Vertical motions are independent

ACCELERATION [m.s^{-2}]

Newton's 1st Law inertia



$\vec{F}_{net} = \sum \vec{F} = 0 \quad \Rightarrow \quad \vec{a} = 0 \quad \Rightarrow$

$\vec{v} = \text{constant} \quad \Rightarrow \quad v = 0 \quad \text{or} \quad \text{constant speed in a straight line}$

Newton's 2nd Law

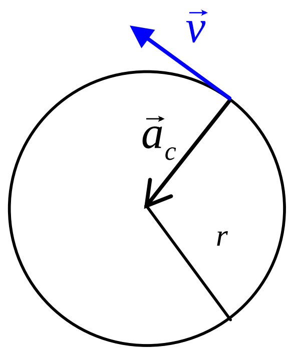
$$\vec{a} = \frac{\vec{F}_{net}}{m} = \frac{\sum \vec{F}}{m}$$

direction of acceleration same as net force acting on object

Centripetal acceleration a_c

always directed towards centre of circle

$$a_c = \frac{v^2}{r}$$



Some of the effects of acceleration we are familiar with include:

(1) Experience of sinking into the seat as a plane accelerates down the runway.

(2) "flutter" in our stomach when a lift suddenly speeds up or slows down.

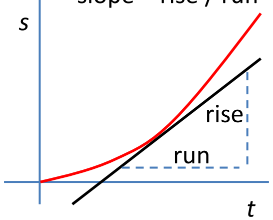
(3) "being thrown side ways" in a car going around a corner too quickly. $\Leftarrow$ *Newton's 1st Law - inertia*

Motion Graphs

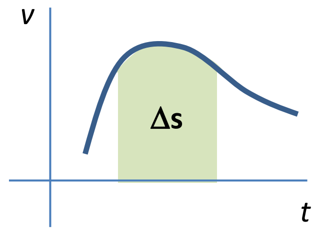
Instantaneous velocity

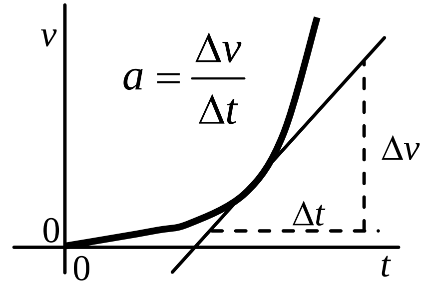
$v = \dfrac{ds}{dt}$ = slope of tangent $s\,/\,t$ graph

slope = rise / run

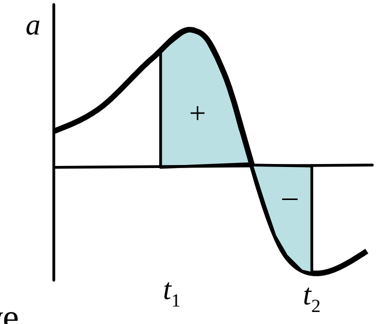


Δs = area under v / *t* graph



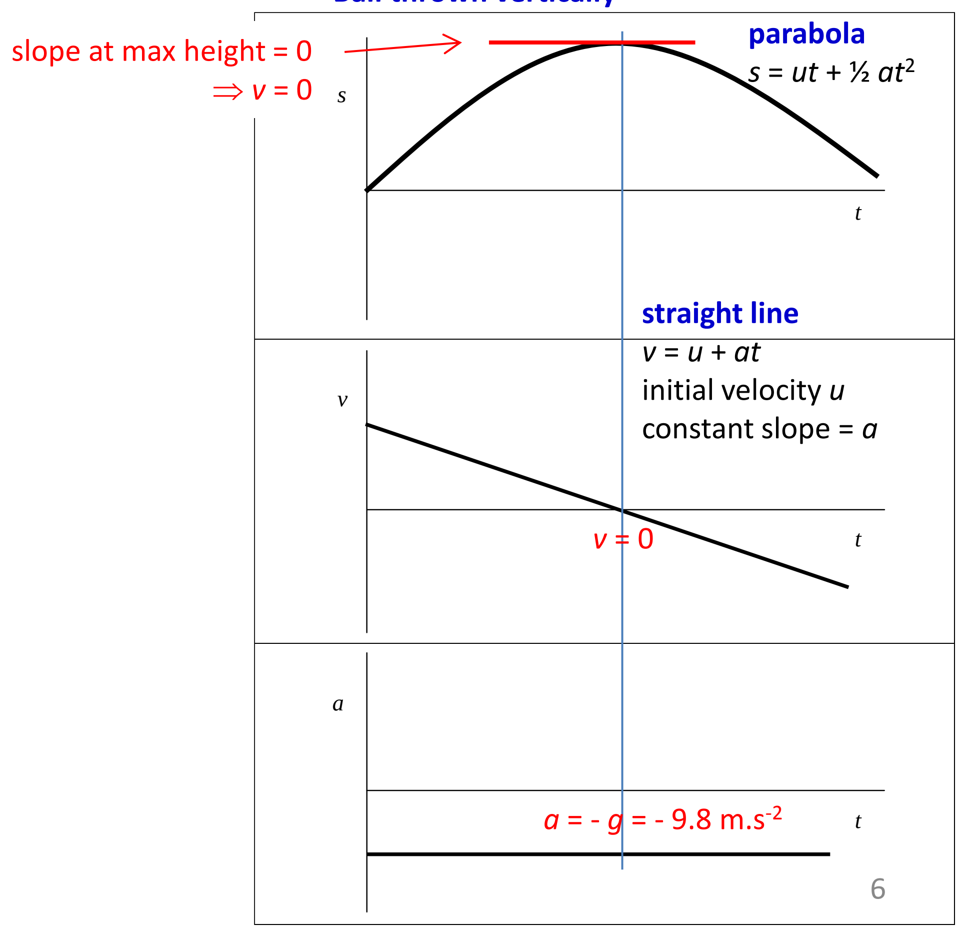


a = slope of tangent to v/t curve

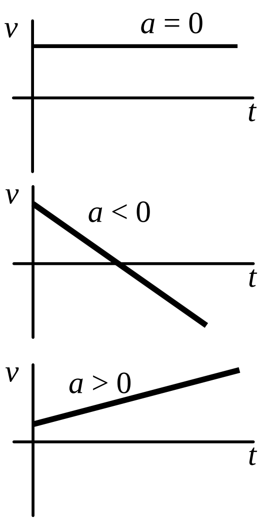


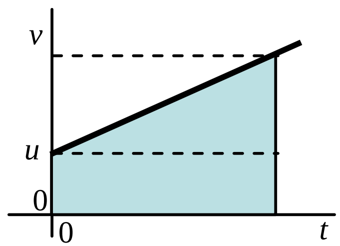
Δv = area under a/t curve

Ball thrown vertically



uniform acceleration ***a*** **= constant**





$v = u + a\,t$ *(straight line)*

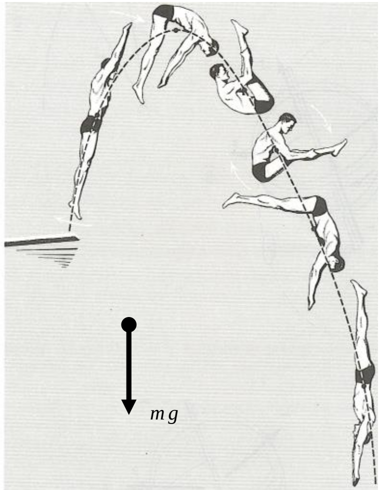
a = *constant (constant slope)*

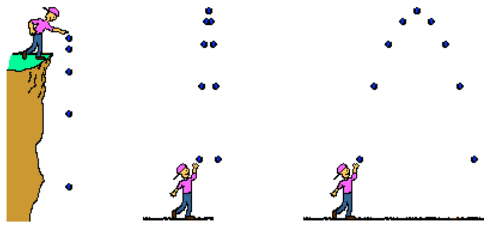
s = *area rectangle + triangle*

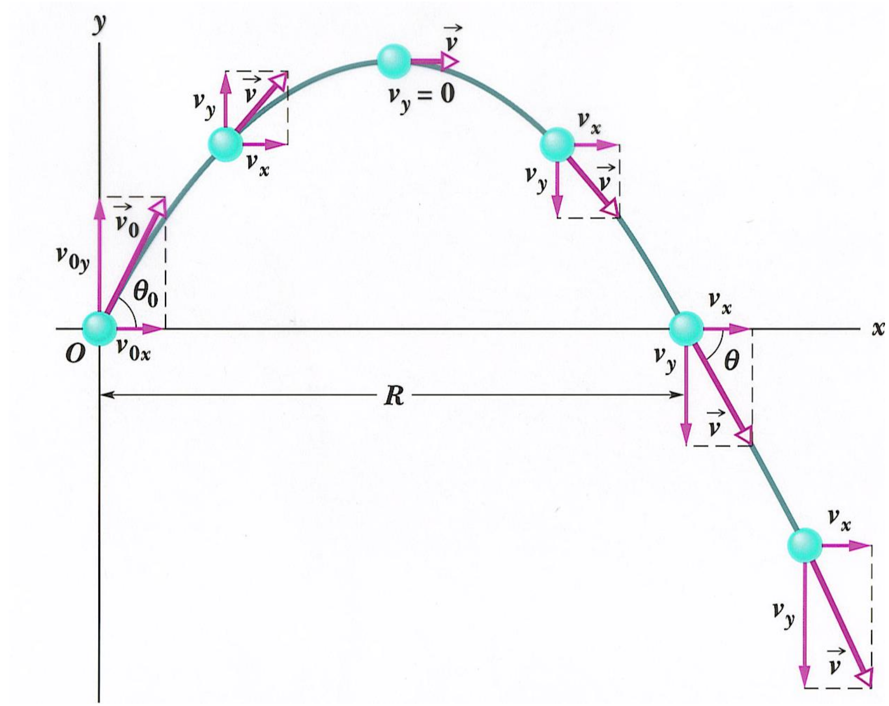
$\quad = u\,t + ½\,t\,(v - u)$

$s = u\,t + ½\,a\,t^2$

PROJECTILE MOTION







Galileo's analysis of projectile motion:

Projectile motion was consisted of both horizontal and vertical components - these components were independent of each other occurred simultaneously perpendicular to each other.

Vertical acceleration was the same for all falling objects if air resistance is disregarded.

Trajectory of a projectile is a parabola.

A motion of an object is relative to its frame of reference and an object has the motion of its inertial frame of reference.
He tested this in his Crow's Nest experiment - It was thought that, if a ship was moving at a constant speed, and a ball was dropped from the crows nest, it would fall behind the ship and into the sea as the ship would have moved. Instead, it fell straight down onto the ship as if it hadn't moved.

Problem

A volcano that is 3300 m above sea level erupts and sends rock fragments hurling into the sea 9.4 km away. If the fragments were ejected at an angle of 35^o, what was their initial speed?

Solution

Identify / setup

v_0 = ? m.s^{-1}

θ = 35°

x_0 = 0 $\quad$ y_0 = 0

x = 9400 m $\quad$ y = -3300 m

a_x = 0 $\quad$ a_y = -9.8 m.s^{-2}

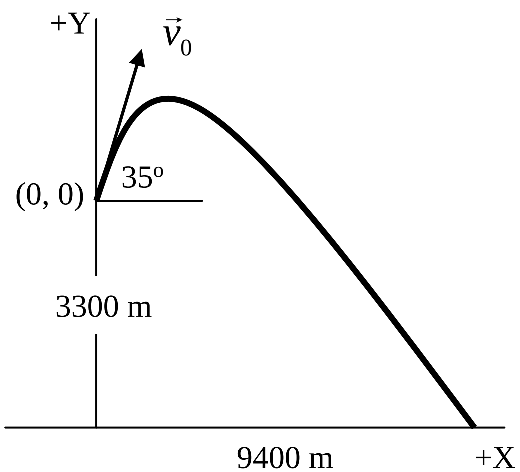
$v_{0x} = v_0\cos\theta$ $\quad$ $v_{0y} = v_0\sin\theta$

Equation for uniformly accelerated motion

$$v = v_0 + at \qquad s = v_0 t + \frac{1}{2}at^2$$

$$v^2 = v_0^2 + 2as \qquad \bar{v} = \frac{u+v}{2} = \frac{s}{t}$$

how to approach the problem



Execute

X motion

$$x = v_{0x}t = v_0\cos\theta\, t$$

$$t = \frac{x}{v_0\cos\theta}$$

Y Motion

$$y = v_{0y}t + \frac{1}{2}a_y t^2$$

$$y = v_{0y}\sin\theta\, t + \frac{1}{2}a_y t^2$$

Eliminate t to find equation for v_0

$$y = x\frac{v_0\sin\theta}{v_0\cos\theta} + \frac{1}{2}a_y\frac{x^2}{v_0^{\,2}\cos^2\theta}$$

$$\left(y - x\tan\theta\right) = \frac{a_y x^2}{2\cos^2\theta}\frac{1}{v_0^{\,2}}$$

$$v_0 = \sqrt{\left(\frac{a_y x^2}{2\cos^2\theta\left(y - x\tan\theta\right)}\right)}$$

$$v_0 = \sqrt{\left(\frac{(-9.8)(9400)^2}{(2)\left(\cos^2 35^o\right)\left(-3300 - 9400\tan 35^o\right)}\right)}$$

$$v_0 = 255\ \text{m.s}^{-1}$$

$$v_0 = (255)\left(10^{-3}\right)\left(3.6\times10^3\right)\ \text{km.h}^{-1}$$

$$v_0 = 920\ \text{km.h}^{-1}$$

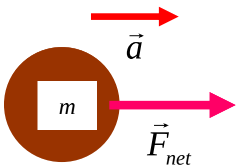
Evaluate

The rocks in a volcanic explosion can be thrown out at enormous speeds.

FORCE [newton N]

F F_G W F_N N F_f f T R vector $\vec{F}$ ***push / pull / interaction between objects***

vector



Newton's 1[st] Law inertia

$$\vec{F}_{net} = \sum \vec{F} = 0 \quad \Rightarrow \quad \vec{a} = 0 \quad \Rightarrow$$

$$\vec{v} = \text{constant} \quad \Rightarrow$$

$v = 0$ or constant speed in a straight line

Newton's 2[nd] Law $\vec{a} = \dfrac{\vec{F}_{net}}{m} = \dfrac{\sum \vec{F}}{m}$

direction of acceleration
same as net force acting on object

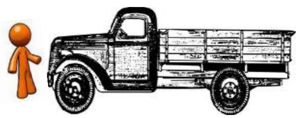
$$\vec{F}_{net} = \sum \vec{F} = m\,\vec{a} = m\frac{d\,\vec{v}}{dt} = \frac{d\,\vec{p}}{dt}$$

Newton's 3[rd] Law

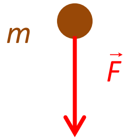
Force between two positively charged objects is repulsive

$$\vec{F}_{AB} = -\vec{F}_{BA}$$

Person is hit by a speeding truck, the magnitude of the forces experienced by the person & truck are the same.



Weight $F_G = W = m\,g$

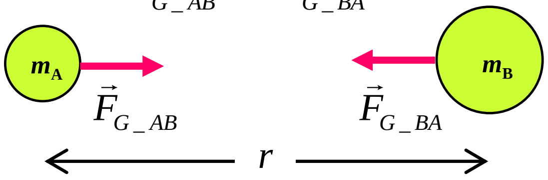


$g = |g| = 9.8\ \text{m.s}^{-2}$

weight of an object is due to the force acting on it in a gravitational field (attraction between object and planet)

Newton's Law of Gravitation

$$\vec{F}_{G\_AB} = -\vec{F}_{G\_BA}$$

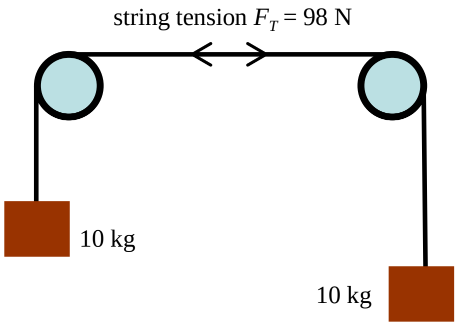


$$F_G = \frac{G\,m_A\,m_B}{r^2}$$

Universal gravitational constant
$G = 6.6742 \times 10^{-11}\ \text{N.m}^2.\text{kg}^{-2}$

At the surface of the Earth $g = \dfrac{G\,M_E}{{R_E}^2}$

Tension $F_T = T$

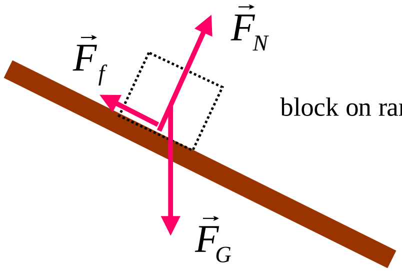


FORCE [newton N]

F F_G W F_N N F_f f T R vector $\vec{F}$ ***push / pull / interaction between objects***

Normal force $F_N = N$

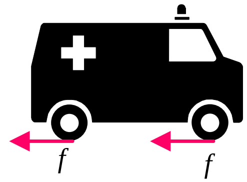
acts at right angles to a surface



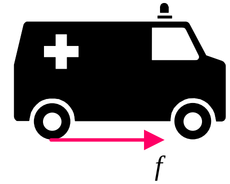
block on ramp

Friction force $F_f = f$

acts along (parallel) surface

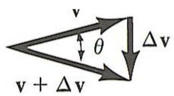


ambulance stopping: friction between wheels & road

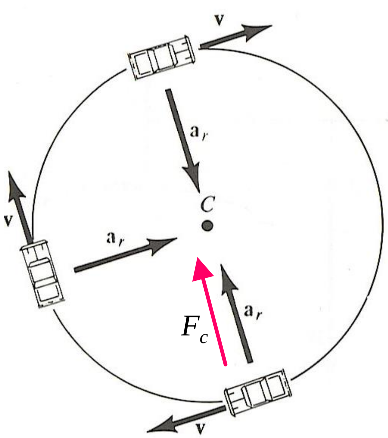


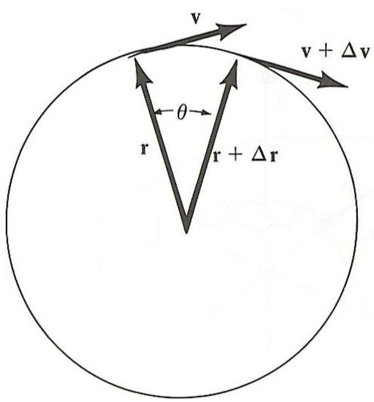
ambulance acceleration: friction between drive wheels & road

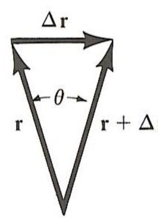
Centripetal force F_c



$$F_c = \frac{mv^2}{r}$$







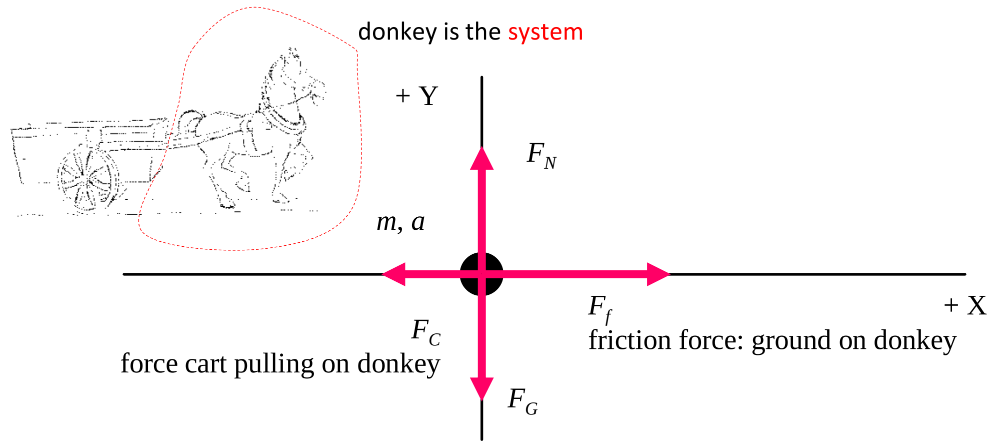
FORCE [newton N]

$F \quad F_G \quad W \quad F_N \quad N \quad F_f \quad f \quad T \quad R$ vector $\vec{F}$ ***push / pull / interaction between objects***

Free Body Diagram

Take the donkey to be the system:

– consider only the forces acting on the donkey

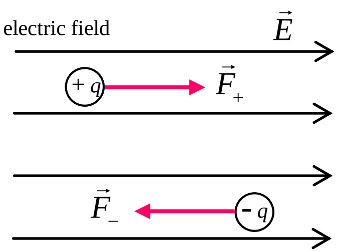


$$\sum F_y = F_N - F_G = 0 \qquad a_y = 0$$

Newton's 2nd Law

$$\sum F_x = F_f - F_C = m\,a_x = m\,a$$

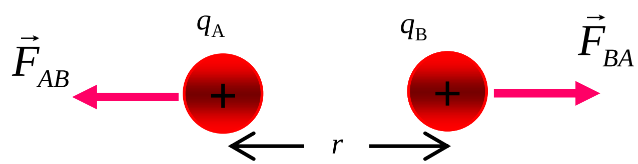
Force *F* on a charged particle *q* in an electric field *E*



$$\vec{F} = q\,\vec{E}$$

Coulomb's Law: force between two pint charges q_A & q_B

Force between two positively charged objects is repulsive



$$\vec{F}_{AB} = -\vec{F}_{BA}$$

magnitude of force between charges:

$$F = k\frac{q_A\,q_B}{r^2} = \frac{1}{4\pi\varepsilon_o}\,\frac{q_A\,q_B}{r^2}$$

$k = 9.0\times10^{9}$ N.m^2.C^{-2}

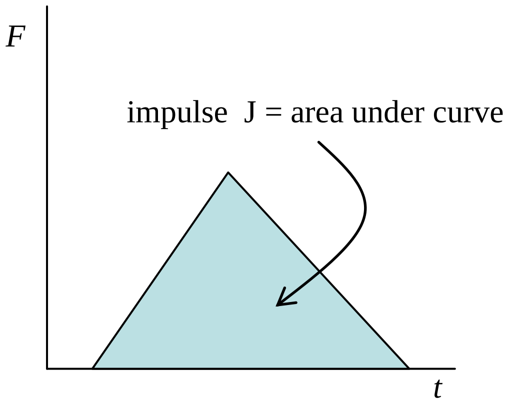
permittivity of free space

$\varepsilon_o = 8.854\times10^{-12}$ C^2.N^{-1}.m^{-2}

Charges of the same sign repel and of opposite sign attract

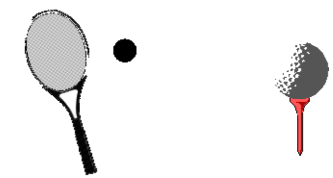
MOMENTUM p P IMPULSE J [N.s kg.m.s^{-1}]

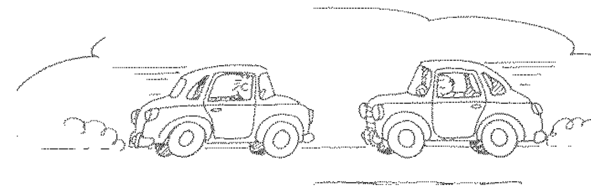
vector



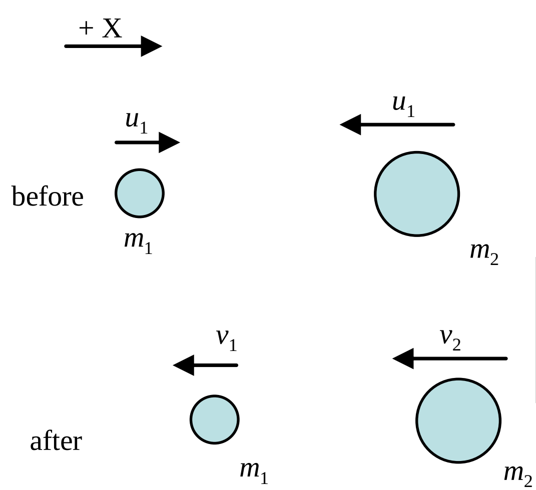
$$\vec{p} = m\vec{v} \qquad p = mv$$

$$\vec{J} = \int_{t_1}^{t_2} \vec{F}_{net}\, dt = \vec{F}_{net\_avg} \Delta t = m\vec{v}_2 - m\,\vec{v}_1$$





For a system in which the net force acting on it is zero, then the total momentum of the system does not change: Newton's 3rd Law $\Rightarrow$ ***Law of Conservation of Momentum***. This law is useful for making predictions when collisions or explosions occur.



Conservation of momentum
(initial momentum = final momentum

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

Elastic collision: conservation of energy

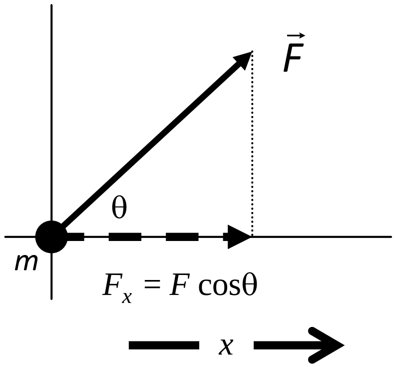
$$\tfrac{1}{2} m_1 u_1^{\,2} + \tfrac{1}{2} m_2 u_2^{\,2} = \tfrac{1}{2} m_1 v_1^{\,2} + \tfrac{1}{2} m_2 v_2^{\,2}$$

Inelastic collision: non conservation of energy

$$\tfrac{1}{2} m_1 u_1^{\,2} + \tfrac{1}{2} m_2 u_2^{\,2} > \tfrac{1}{2} m_1 v_1^{\,2} + \tfrac{1}{2} m_2 v_2^{\,2}$$

KINETIC ENERGY K KE E_K / WORK W [joule J]

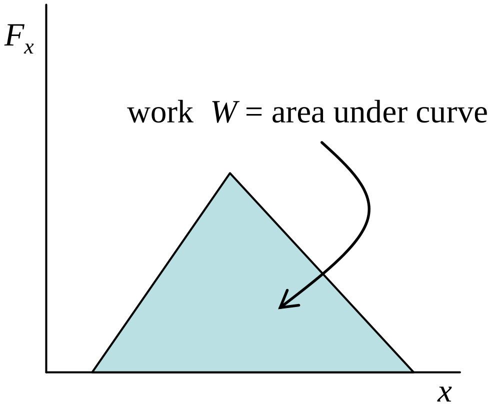
scalar



Work done W by a single force $\vec{F}$ acting on an object of mass m

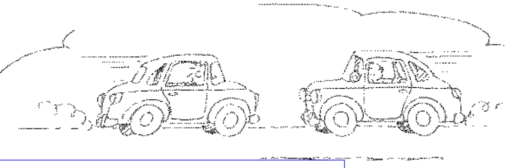
$$W_x = \int_{x_1}^{x_2} F_x\, dx = F_{x\_avg}\, \Delta x$$

moving object has kinetic energy $E_K = \frac{1}{2} m\, v^2$



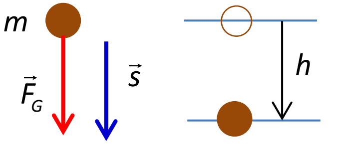
Net force acting over a displacement on object does work on object and changes its kinetic energy: **Net Work = Change in kinetic energy**

More than a single force acting on the object then: $W_{net} = \sum_i W_i = \frac{1}{2} m\, v_2^{\;2} - \frac{1}{2} m\, v_1^{\;2}$



Freely falling object: Work done W by the gravitational force F_G as an object falls a height h near the surface of the Earth

$W = F\, s \cos\theta$ $F = F_G = m\, g$ $s = h$ $\theta = 0$ $\boldsymbol{W = m\, g\, h}$



Gravitational potential energy $U_G = E_p = - W$

Loss in GPE $\boldsymbol{E_p = - m\, g\, h}$

Numerical example

In golf, a typical contact time is 1.0 ms. If a 45 g ball leaves the club at a speed of 240 km.h^{-1}, estimate the average force exerted by the club on the ball.

Solution

Identify / Setup

Δt = 1.0 ms = 10^{-3} s

m = 45 g = 45×10^{-3} kg

v_1 = 0 m.s^{-1}

v_2 = 240 km.h^{-1} = (240)(103)/(3.6×10^3) m.s^{-1}

v_2 = 67 m.s^{-1}

F_{avg} = ? N

Impulse = change in momentum

$$J = \int \vec{F}_{net}\, dt = \vec{F}_{net\_avg} \Delta t = m\vec{v}_2 - m\vec{v}_1$$

how to approach the problem

Execute

$$F_{avg}\, \Delta t = m\, v_2 - m\, v_1$$

$$F_{avg} = \frac{m\, v_2}{\Delta t} = \frac{(45\times10^{-3})(67)}{10^{-3}}\ \text{N}$$

$$F_{avg} = 3.0\times10^{3}\ \text{N}$$

Evaluate

units ok

significant figures (2) ok

Answers approximately equivalent to the mass of 60 people of mass 50 kg

Numerical example

A boy pushes a toy car of mass 250 g, initially at rest with a horizontal force of 5.0 N through a distance of 1.2 m. How much work is done on the car? What is the final speed of the car?

Solution

Identify / Setup

m = 0.250 g = 0.25 kg

F_x = 5.0 N

x = 1.2 m

W = ? J

v_2 = ? $\mathrm{m.s^{-1}}$ v_1 = 0 $\mathrm{m.s^{-1}}$

Work done produces a change in KE

$$W = F_x \Delta x = \tfrac{1}{2} m v_2^2 - \tfrac{1}{2} m v_1^2$$

how to approach the problem

Execute

$$W = F_x x = (5.0)(1.2)\ \mathrm{J} = \underline{6.0\ \mathrm{J}}$$

$$W = \tfrac{1}{2} m v_2^2 \qquad v_2 = \sqrt{\frac{2W}{m}} = \sqrt{\frac{(2)(6)}{0.25}}\ \mathrm{m.s^{-1}} = \underline{6.9\ \mathrm{m.s^{-1}}}$$

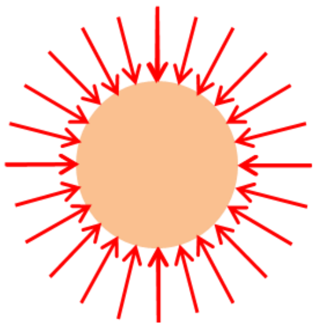
Evaluate

units ok

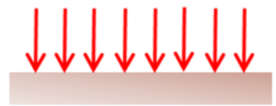
significant figures (2) ok

Answers seems OK

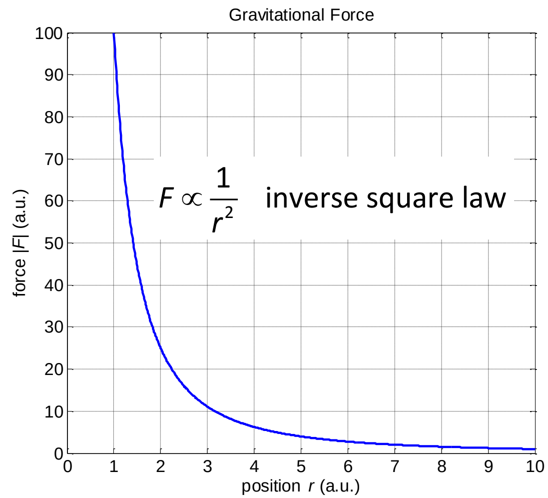
GRAVITATION



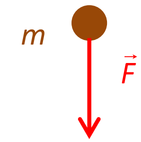
Gravitational field surrounding the planet increases towards the surface as shown by the increase in the density of the field lines.



Near the surface of a planet the gravitational field lines are approximately uniformly spaced hence we can assume a uniform gravitational field strength.

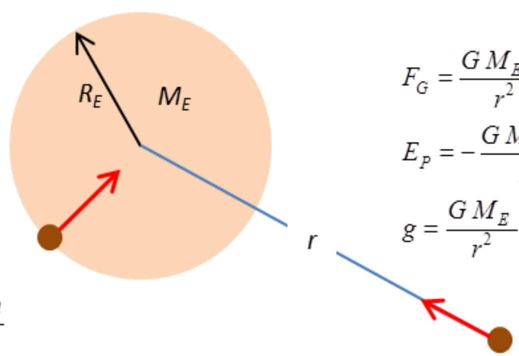


Weight $F_G = W = m\,g$



$g = |g| = 9.8\ \text{m.s}^{-2}$

weight of an object is due to the force acting on it in a gravitational field (attraction between object and planet)



$$F_G = \frac{G\,M_E\,m}{R_E^{\,2}}$$

$$E_P = -\frac{G\,M_E\,m}{R_E}$$

$$g = \frac{G\,M_E}{R_E^{\,2}}$$

at the Earth's surface

$g = 9.8\ \ \text{m.s}^{-2}$

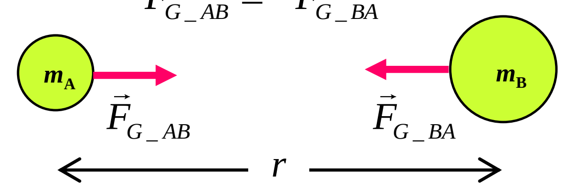
$$F_G = \frac{G\,M_E\,m}{r^2}$$

$$E_P = -\frac{G\,M_E\,m}{r}$$

$$g = \frac{G\,M_E}{r^2}$$

Newton's Law of Gravitation

$$\vec{F}_{G\_AB} = -\vec{F}_{G\_BA}$$



$$F_G = \frac{G\,m_A\,m_B}{r^2}$$

Universal gravitational constant
$G = 6.6742\times10^{-11}\ \ \text{N.m}^2.\text{kg}^{-2}$

At the surface of the Earth $\quad g = \dfrac{G\,M_E}{R_E^{\,2}}$

$$g_{planet} = \frac{F}{m} = \frac{G\,M_{planet}\,m}{m\,R_{planet}^{\,2}} = \frac{G\,M_{planet}}{R_{planet}^{\,2}}$$

GRAVITATIONAL POTENTIAL ENERGY E_P U U_G [joule J]

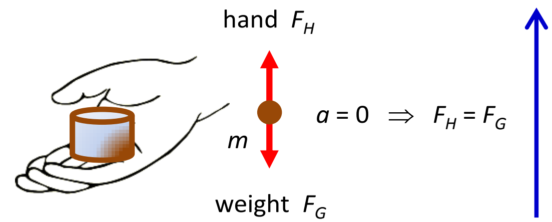
Lifting an object vertically produces an increase in gravitational potential energy of the system of the object and the Earth.

Work done = Increase in gravitational potential energy

$$W = F_H\, h = F_G\, h = m\, g\, h$$

$$W = \Delta E_P$$

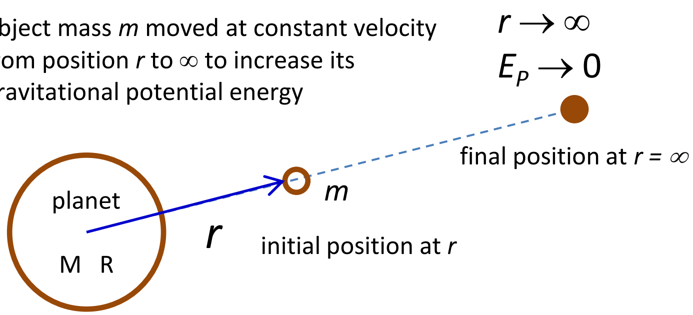
$$\Delta E_P = m\, g\, h$$



Object raised through a vertical displacement h at a constant velocity

near the Earth's surface: g = constant $g = 9.8\ \text{m.s}^{-2}$ (positive number)

object mass m moved at constant velocity from position r to ∞ to increase its gravitational potential energy

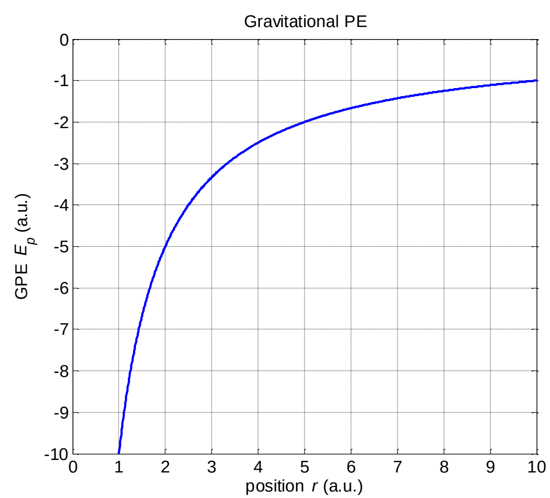


Work done W on object in moving it from r to $r = \infty$

$$W = \int_r^\infty F\, dr' = \int_r^\infty F_G\, dr' = G\, M_E\, m \int_r^\infty \frac{dr'}{r'^2} = G\, M_E\, m \left[\frac{-1}{r'}\right]_r^\infty = \frac{G\, M_E\, m}{r}$$

$$W = \Delta E_P = E_P(\infty) - E_P(r) = 0 - E_P(r) = -E_P(r)$$

$$\boxed{E_P(r) = -\frac{G\, M_E\, m}{r}}$$



$r \to \infty$

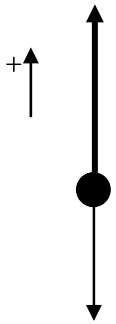
$E_P \to 0$

gravitational potential energy E_p increases as the distance r from the centre of the planet increases

MOTION OF ROCKETS

Forces experienced by astronauts during take-off

Rocket accelerating upwards



normal reaction force acting on astronaut

F_N (eg measured by set of bathroom scales)

+

astronaut
mass m
acceleration a

weight $F_G = m\,g$

Newton's 2nd Law

$\Sigma F = F_N - F_G = F_N - m\,g = m\,a$

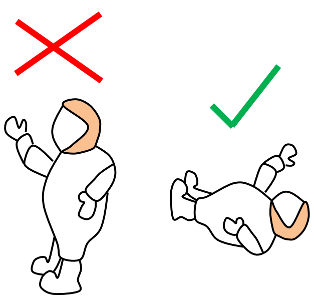
apparent weight $F_N = m\,g + m\,a$

$$g\text{-force} = \frac{m\,g + m\,a}{m\,g} > 1$$

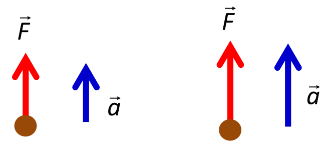
you experience g-forces when going up & down a elevator

Greater the acceleration of the rocket – the greater the g-force experienced by the astronaut

$$g\text{-force} = \frac{\text{apparent weight}}{\text{actual weight}} = \frac{mg + ma}{mg} = \frac{g + a}{g}$$



It is must safer for an astronaut to lie in a crouching position rather than standing up because the body can tolerate larger g-forces. In the crouching position g-force(max) ~ 20g

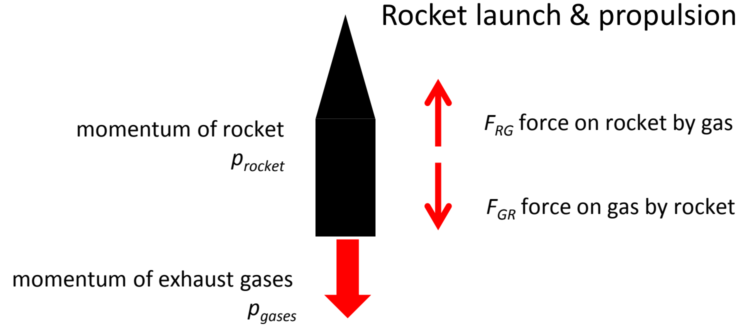


LIFT OFF
v increasing

RE-ENTRY
v decreasing

Alan Shepard – first man in space
g-force (lift off) ~ 6 g
g-force (re-entry) ~ 12 g

Rocket launch & propulsion



Newton's third law: $F_{RG} = -F_{GR}$

Forces act for time interval $\rightarrow$ impulse:

$$F_{RG}\,\Delta t = -F_{GR}\,\Delta t$$

Impulse = Change in momentum:

$$\Delta p_R = -\Delta p_G \qquad \Delta(mv)_{rocket} = -\Delta(mv)_{gases}$$

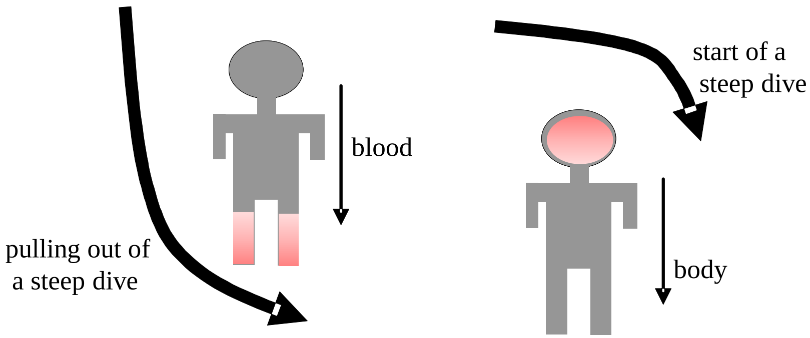
Momentum is conserved: $\Delta p_R + \Delta p_G = 0$

Gases expelled
$\Rightarrow$ mass of the rocket decreases
$\Rightarrow$ acceleration increases

$$a = \frac{\sum F}{m}$$

MOTION OF ROCKETS

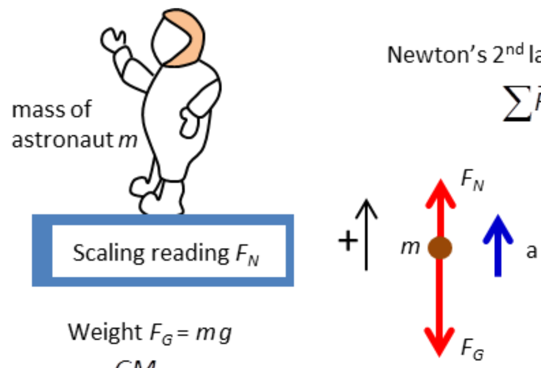
Pilots experience large g-forces
when entering and pulling out of a steep dive



Newton's 1st law: blood keeps moving

Newton's 1st law: blood stays put

Force experienced by an astronaut during a space flight



Newton's 2nd law applied to astronaut

$$\sum \vec{F} = m\vec{a}$$

$$\sum F = F_N - F_G = F_N - mg = ma$$

$$F_N = m\,a + m\,g$$

Weight $F_G = m\,g$

$g = \dfrac{GM_E}{r^2}$ *g is a positive number*

(1) v = constant → $a = 0$ → $F_N = m\,g$

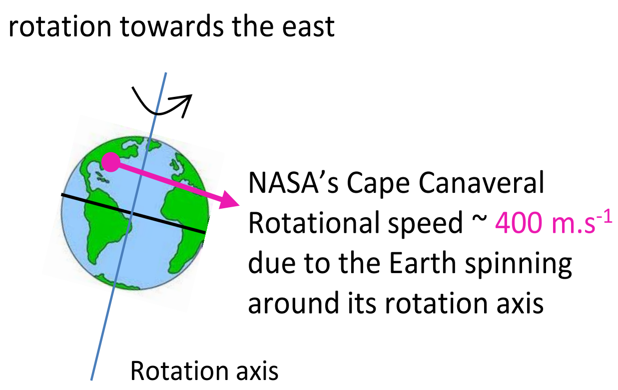
(2) v increasing → $a > 0$ → $F_N = m\,g + m\,a > m\,g$ apparent weight > weight

(3) v decreasing → $a < 0$ → $F_N = m\,g - m\,|a| < m\,g$ apparent weight < weight

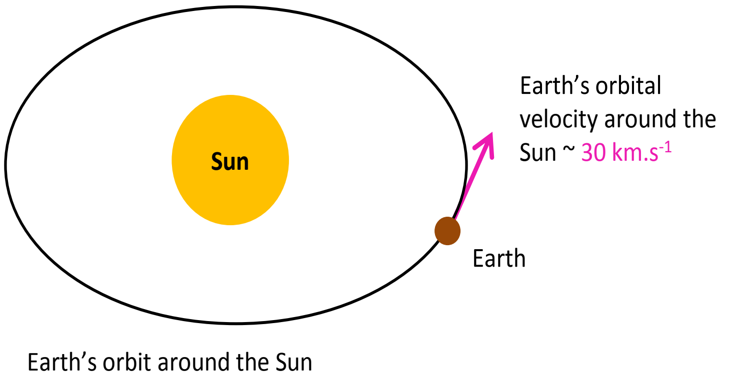
(4) Free fall $a = -g$ → $F_N = 0$ → apparent weight = 0 **weightless**

MOTION OF ROCKETS

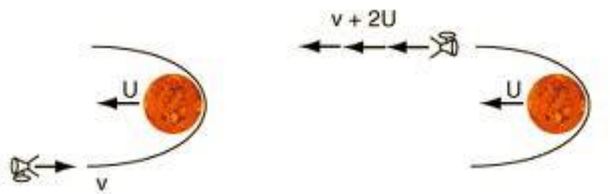
Rocket launch & orbital motion of the Earth Rockets are launched in an easterly direction to get a velocity booster as a result of the Earth spinning about its axis of rotation



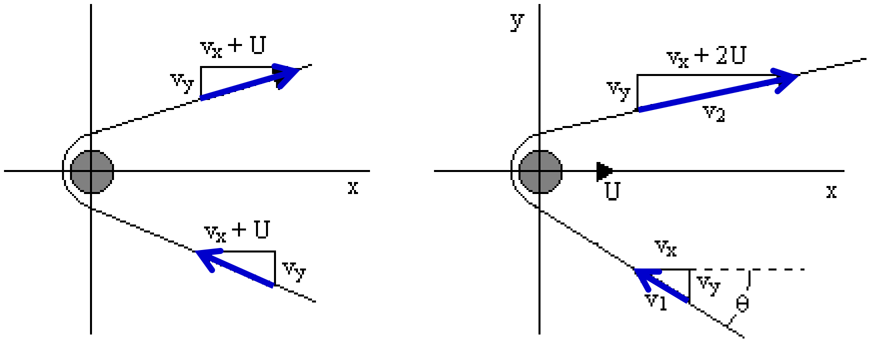
Earth's orbital motion around the Sun can be helpful in launching rockets to planets in our Solar System



SLINGSHOT EFFECT - performed to achieve an increase in speed and/or a change of direction of a spacecraft as it travels around a planet. As it approaches, it is caught by the gravitational field of the planet, and swings around it, the speed acquired throws the spacecraft back out again, away from the planet. By controlling the approach, the outcome of the manoeuvre can be manipulated and the spacecraft gain some of the planet's momentum, relative to the Sun.



Simple view from solar reference frame: energy & momentum are conserved

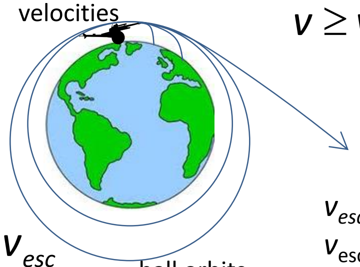


Planet's Rest Frame   Solar Reference Frame

http://www.physics.usyd.edu.au/teach_res/hsp/u5/t5_slingshot.htm

MOTION OF ROCKETS - ESCAPE VELOCITY v_{esc}

Cannon ball launched from top of a mountain with increasing velocity



v_{esc}(Earth) = 11 km.s^{-1}
v_{esc}(Moon) = 2.4 km.s^{-1}
v_{esc}(Sun) = 620 km.s^{-1}

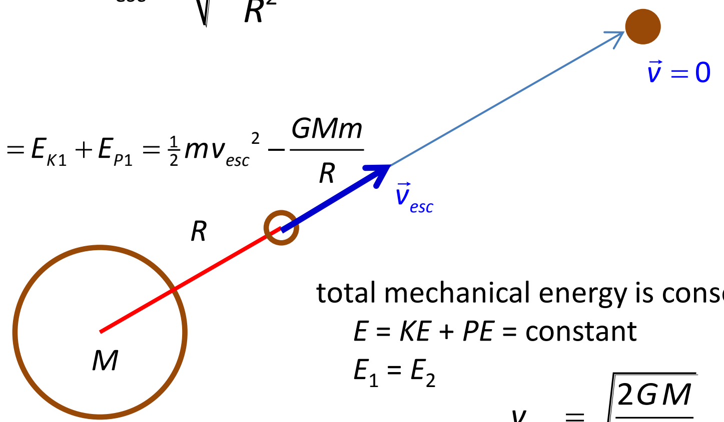
orbital velocity $$v_{orb} = \sqrt{\frac{GM}{R^2}} = \frac{1}{\sqrt{2}} v_{esc}$$

The **escape velocity v_{esc}** for a rocket fired from a planet or moon (mass M, radius R)

$$v_{esc} = \sqrt{\frac{2GM}{R^2}}$$

$$E_2 = E_{K2} + E_{P2} = 0 + 0 = 0$$

$$E_1 = E_{K1} + E_{P1} = \tfrac{1}{2} m v_{esc}{}^2 - \frac{GMm}{R}$$



total mechanical energy is conserved

$E = KE + PE =$ constant

$E_1 = E_2$

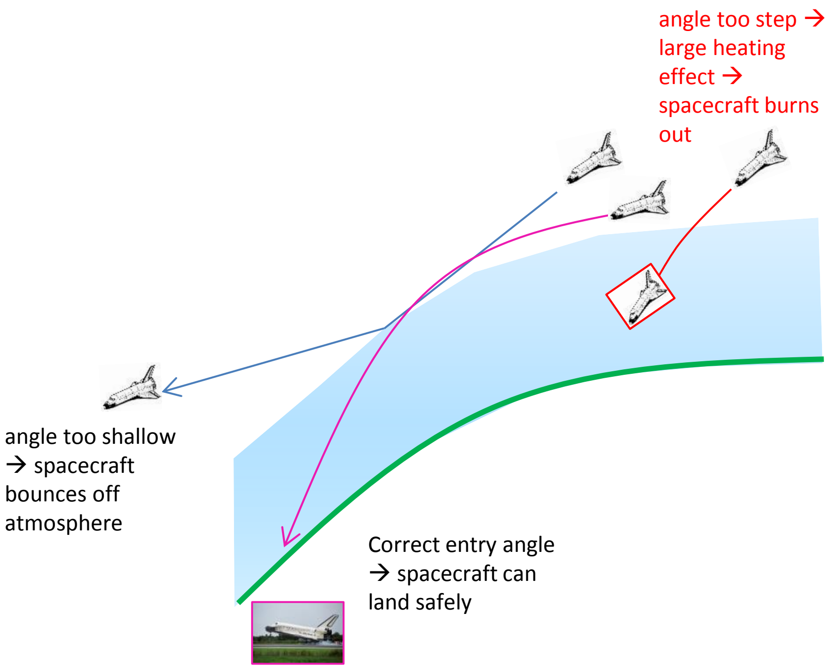
$$v_{esc} = \sqrt{\frac{2GM}{R^2}}$$

Safe re-entry and landing of rockets or spacecraft

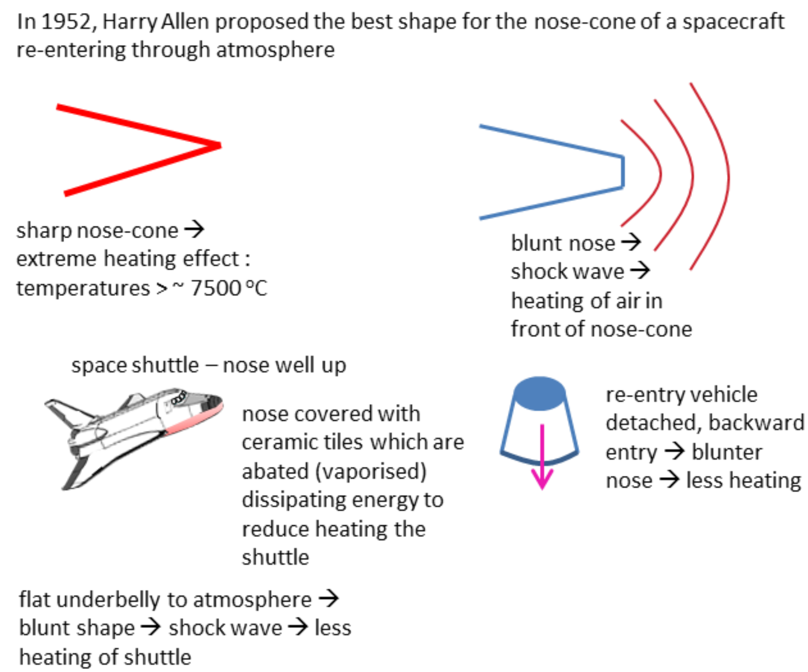
The safe return of a spacecraft into the Earth's atmosphere and subsequent descent to Earth requires consideration of two main issues:

(1) How to handle the **intense heat generated** as the spacecraft enters the Earth's atmosphere.

(2) How to keep the **g-forces** of deceleration within safe limits.



For a safe landing of a spacecraft it must enter the atmosphere in the correct range of angles.



Spacecraft re-entry and the heating effect in passing through the atmosphere.

Safe re-entry and landing of rockets or spacecraft

What Can Go Wrong?

- If the **angle of re-entry** is too shallow, the spacecraft may skip off the atmosphere. The commonly cited analogy is a rock skipping across a pond. If the angle of entry is too steep, the spacecraft will burn up due to the heat of re-entry.

- Because of collisions with air particles and the huge deceleration, a huge amount of **thermal energy** is produced from friction. The space shuttle must be able to withstand these temperatures. It uses a covering of insulating tiles which are made of glass fibres but are about 90% air. This gives them excellent thermal insulating properties and also conserves mass. The tile construction is denser near the surface to make the tiles more resistant to impact damage, but the surface is also porous. Damage to the space shuttle Columbia's heat shield is thought to have caused its disintegration and the loss of seven astronauts on 1st February 2003. Investigators believe that the scorching air of re-entry penetrated a cracked panel on the left wing and melted the metal support structures inside.

- **Large g-forces** are experienced by astronauts as the space shuttle decelerates and re-enters the Earth's atmosphere. Astronauts are positioned in a transverse position with their backs towards the Earth's surface as g-forces are easier for humans to tolerate in these positions. Supporting the body in as many places as possible also helps to increase tolerance.

- There is an **ionisation blackout** for the space shuttle of about 16 minutes where no communication is possible. This is because as thermal energy builds up, air becomes ionised forming a layer around the spacecraft. Radio signals cannot penetrate this layer of ionised particles.

MOTION OF SATELLITES

Satellites are placed in one of several different types of orbit depending on the nature of their mission

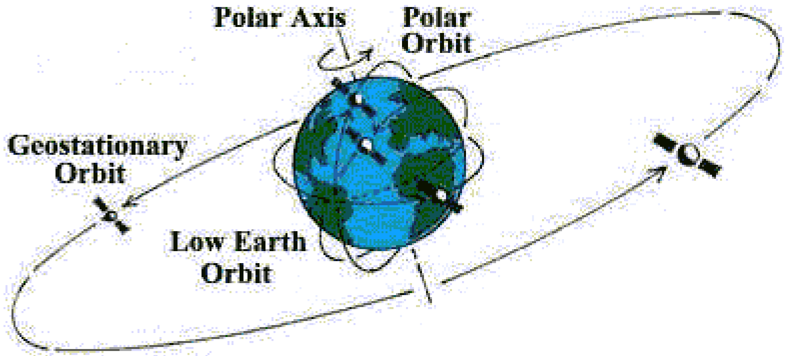
Low Earth Orbit (**LEO**)

- Radius: 200 km to 2000 km above the Earth's surface.
- Period: 60 min 90 minutes. Frequent coverage of specific or varied locations on the Earth's surface.
- Small field of view.
- Orbits less than 400 km are difficult to maintain due to atmospheric drag and subsequent orbital decay.
- Types of satellites: military applications, Earth observation, weather monitoring , shuttle missions.
- With the exception of the lunar flights of the Apollo program, all human spaceflights have taken place in LEO.
- All manned space stations and the majority of artificial satellites in LEO.

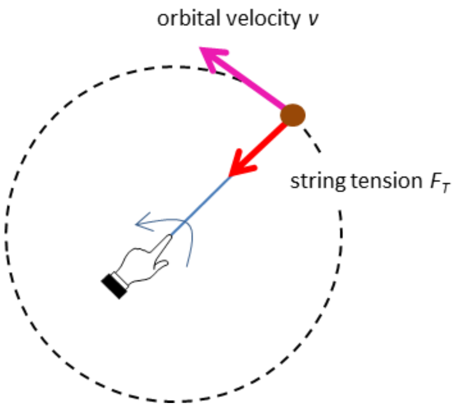
Orbital decay - reduction in the height of an object's orbit over time due to the drag of the atmosphere on the object. All satellites in low Earth orbits are subject to some degree of atmospheric drag that will eventually decay their orbit and limit their lifetimes. Even at 1000 km, as 'thin' as the atmosphere is, it is still sufficiently dense to slow the satellite down over a period of time.

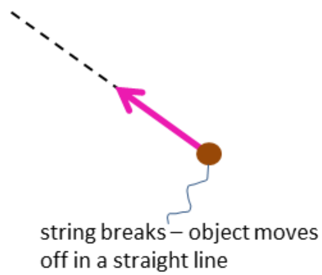
Geostationary Orbit (GEO)

- Circular orbit in the Earth's equatorial plane, any point on which revolves about the Earth in the same direction and with the same period as the Earth's rotation.
- Useful because they cause a satellite to appear stationary with respect to a fixed point on the rotating Earth. As a result an antenna can point in a fixed direction and maintain a link with the satellite.
- The satellite orbits in the direction of the Earth's rotation, at an altitude of approximately 35,786 km above ground. This altitude is significant because it produces an orbital period equal to the Earth's period of rotation, known as the sidereal day.
- These orbits allow for the tracking of stationary point on Earth.
- Have the largest field of view.
- Applications include communications, mass-media and weather monitoring.

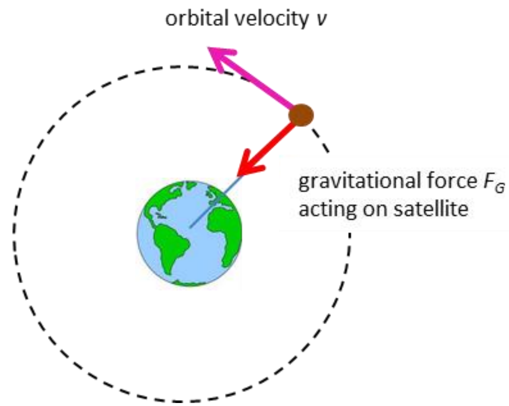


MOTION OF SATELLITES uniform circular motion





A force (**centripetal force F_c**) acting towards the centre of a circle is necessary for an object to rotate in a circular path.



Satellite is acted upon by the gravitation force between the Earth and the satellite. The centripetal force is the gravitational force.

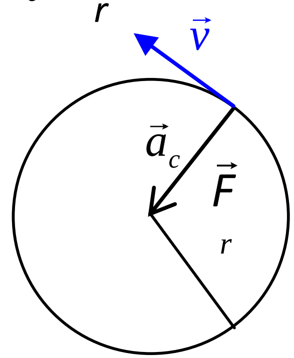
For a mass m, moving at a speed v, in uniform circular motion of radius r, the net force acting on is called the **centripetal force F_c** and its magnitude is given by

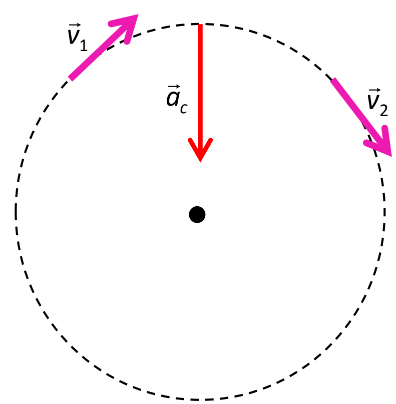
$$F_c = \frac{mv^2}{r}$$

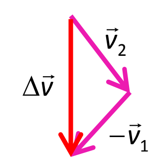
Centripetal force $\rightarrow$ change in direction of the object as its speed is constant. The resulting acceleration due to the change in direction is the **centripetal acceleration a_c** and its magnitude is

$$a_c = \frac{v^2}{r}$$

Direction of the centripetal force and centripetal acceleration is towards the **centre of the circle** (centripetal means 'centre seeking').







$$\Delta\vec{v} = \vec{v}_2 - \vec{v}_1 = \vec{v}_2 + \left(-\vec{v}_1\right)$$

change in velocity Δv directed towards the centre of the circle → acceleration a_c directed towards the centre of the circle

Orbital velocity v_{orb}

To place a satellite of mass m into a stable Earth orbit at a particular radius r, the launch must give it both an initial vertical and horizontal component of velocity relative to the Earth's surface. The satellite will eventually turn so that it is travelling horizontal to the Earth's surface. At this radius r, the force of gravity F_G provides the acceleration needed to keep the object moving in a circle, but a particular orbital velocity is also required to keep the object in a stable orbit – orbital velocity v_{orb}. To calculate that orbital velocity, we equate the centripetal force F_c and gravitational force F_G.

$$\frac{m v_{orb}^{2}}{r} = \frac{G M_E m}{r^2}$$

$$v_{orb}^{2} = \frac{G M_E}{r} \longrightarrow v_{orb} = \sqrt{\frac{G M_E}{r}}$$

Orbital velocity of a satellite as it orbits around the Earth only depends on:

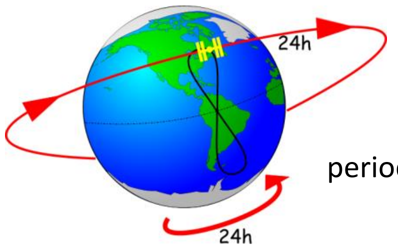
- mass of the Earth M_E
- radius of the orbit r

Altitude is the only variable that determines the orbital velocity required for a specific orbit around the Earth. Greater the radius of that orbit, the lower that velocity v_{orb}

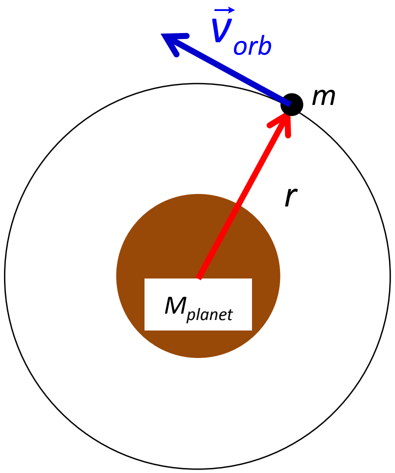
$$r \uparrow \;\Rightarrow\; v_{orb} \downarrow$$

Orbital velocity around other planets $M_E \rightarrow M_{planet}$

$$\boxed{v_{orb} = \sqrt{\frac{G M_{planet}}{r}}}$$



Geostationary Orbit



$$\vec{F}_c = \vec{F}_G$$

net force is always directed to centre of orbit

MOTION OF SATELLITES How do the planets move ? Kepler's Laws of Motion

The motion of a planet is governed by the **Law of Universal Gravitation**

$$F = G\, M_S\, m / r^2$$

where G is the **Universal Gravitational Constant**, M_S is the mass of the Sun, m is the mass of the planet and r is the distance from the Sun to the planet.

$G = 6.67\times10^{-11}$ N.m^2.kg^2

$M_S = 2.0\times10^{30}$ kg

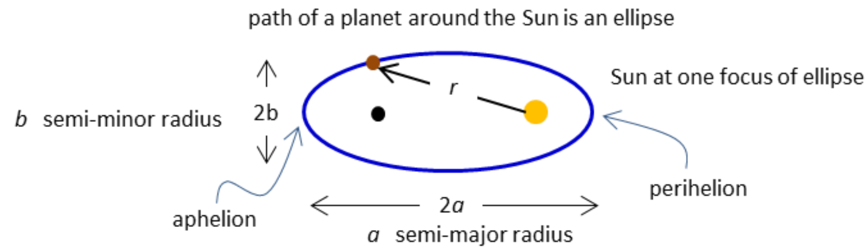
Kepler's Laws of Planetary Motion

1 The path of each planet around the Sun is an **ellipse** with the Sun at one focus.

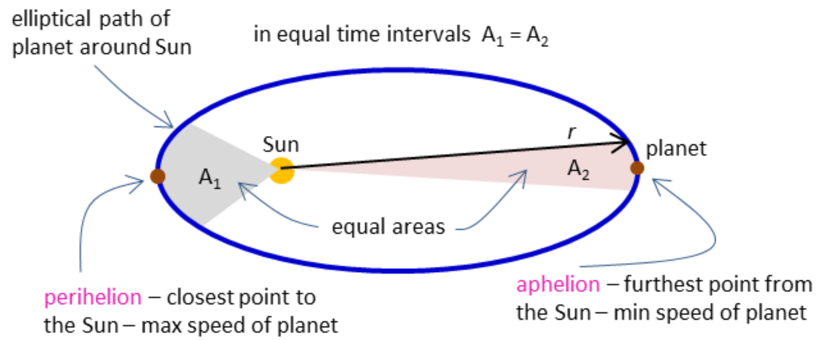
2 Each planet moves so that all imaginary lines drawn from the Sun to the planet **sweeps out equal areas in equal periods of time**.

3 The ratio of the squares of the periods of revolution of planets is equal to the ratio of the cubes of their orbital radii (mean distance from the Sun or length of semi-major axis, a)

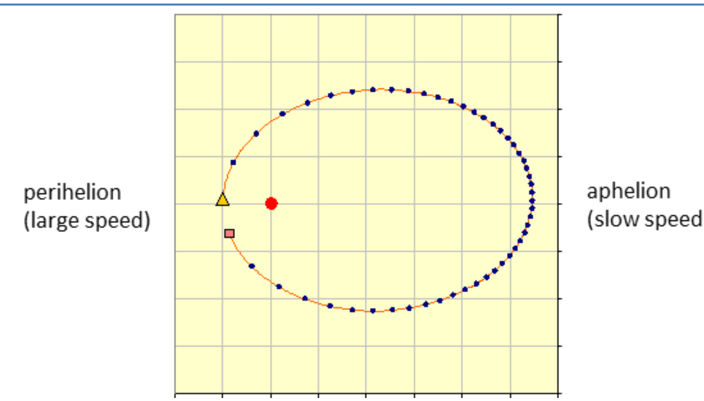
$$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{r_1}{r_2}\right)^3 \quad \text{or} \quad T^2 = 4\pi^2\left(\frac{a^3}{G M_S}\right)$$



Kepler's 1st law: path of a planet around the Sun is an **ellipse.**



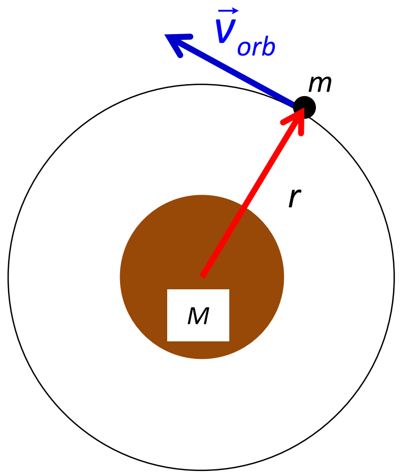
Kepler's 2nd law: Planet moves so that an imaginary line drawn from the Sun to the planet sweeps out equal areas in equal periods of time.



Computer simulation of the motion of a planet around the Sun.

http://www.physics.usyd.edu.au/teach_res/hsp/u5/ag_kepler1.htm

MOTION OF SATELLITES Kepler's Laws of Periods



$$\vec{F}_c = \vec{F}_G$$

net force is always directed to centre of orbit

$$F_C = \frac{m v_{orb}^2}{r} \qquad F_G = \frac{G m M}{r^2}$$

$$F_C = F_G \quad \Rightarrow \quad v_{orb} = \sqrt{\frac{G M}{r}}$$

time for one complete orbit: **period** T

distance travelled in one orbit: circumference $2\pi r$

$$v_{orb} = \sqrt{\frac{G M}{r}} \qquad v_{orb} = \frac{\Delta s}{\Delta t} = \frac{2\pi r}{T} = \sqrt{\frac{G M}{r}}$$

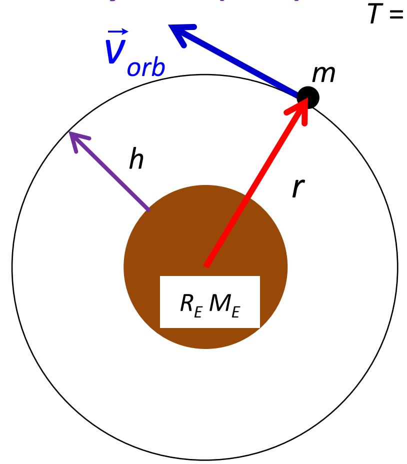
Rearranging gives **Kepler's Law of Periods**

$$\boxed{\frac{r^3}{T^2} = \frac{G M}{4\pi^2}} \qquad \boxed{v_{orb} = \sqrt{\frac{G M}{r}}}$$

Geostationary orbit (GEO)

period

T = 24 h = (24)(3.6x10^3) s = 8.6x10^4 s



R_E = 6.38x10^6 m

M_E = 5.97x10^{24} kg

G = 6.673x10^{-11} N.m^2.kg^{-1}

v_{orb} = ? m.s^{-1}

r = ? m

height about Earth's surface

h = ? m $\quad h = r - R_E$

$$\frac{r^3}{T^2} = \frac{G M}{4\pi^2} \longrightarrow r = \left(\frac{G M_E T^2}{4\pi^2}\right)^{\frac{1}{3}}$$

$$v_{orb} = \sqrt{\frac{G M_E}{r}}$$

Putting in the numbers

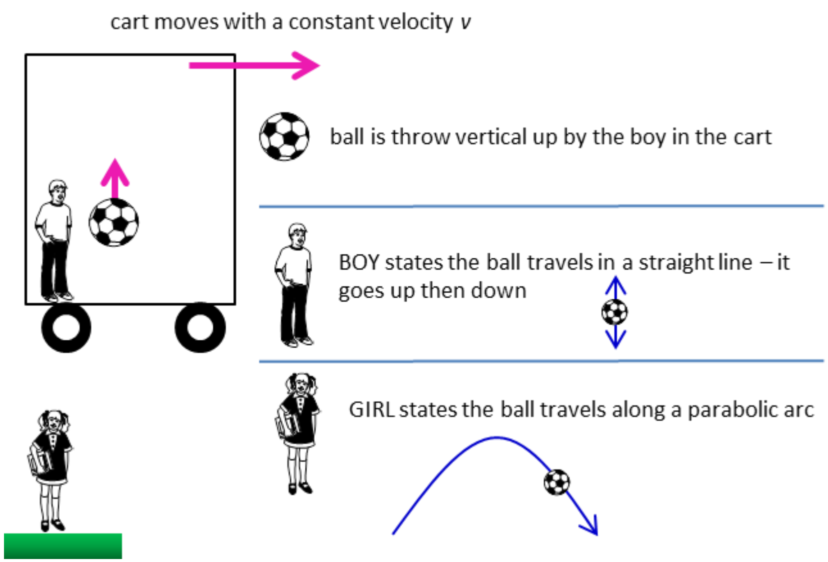
r = 4.22x10^7 m

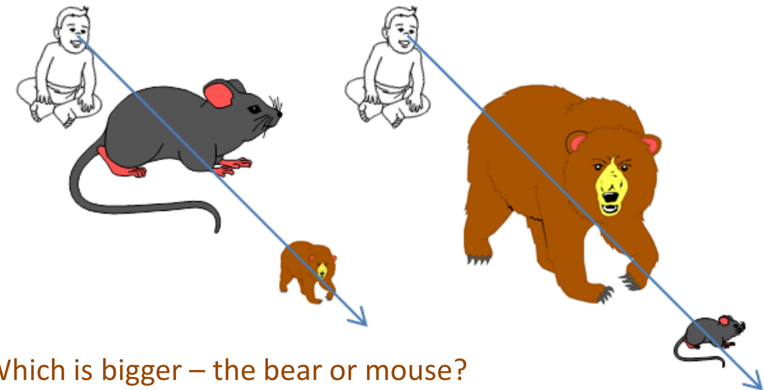
h = 3.59x10^7 m = 36x10^3 km

v_{orb} = 3.07x10^3 m.s^{-1} =1.11x10^4 km.h^{-1}

EINSTEIN THEORY OF SPECIAL RELATIVITY: SPACE & TIME

FRAMES OF REFERENCE





Which is bigger – the bear or mouse?
– depends upon the location of the observer

What is the trajectory of the ball?
– it is relative – it depends upon the motion of the observer

Inertial frame of reference

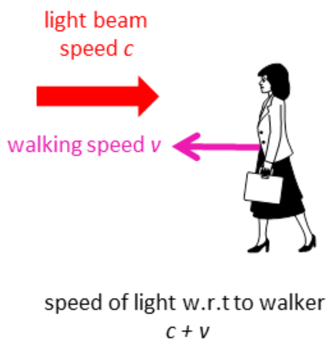
* Fame of reference with constant velocity.
* Is a non-accelerating frame of reference.
* Law of inertia holds.
* Newton's laws of motion hold.
* No fictitious forces arise.

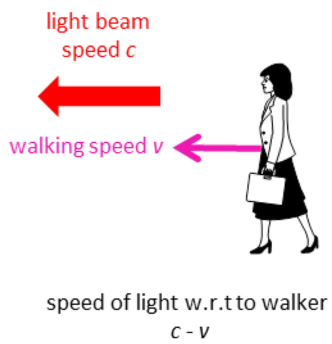
Non-Inertial Frames of Reference

* Does not have a constant velocity. It is accelerating.
* The frame could be travelling in a straight line, but be speeding up or slowing down.
* The frame could be travelling along a curved path at a steady speed.
* The frame could be travelling along a curved path and also speeding up or slowing down.

EINSTEIN THEORY OF SPECIAL RELATIVITY: SPACE & TIME

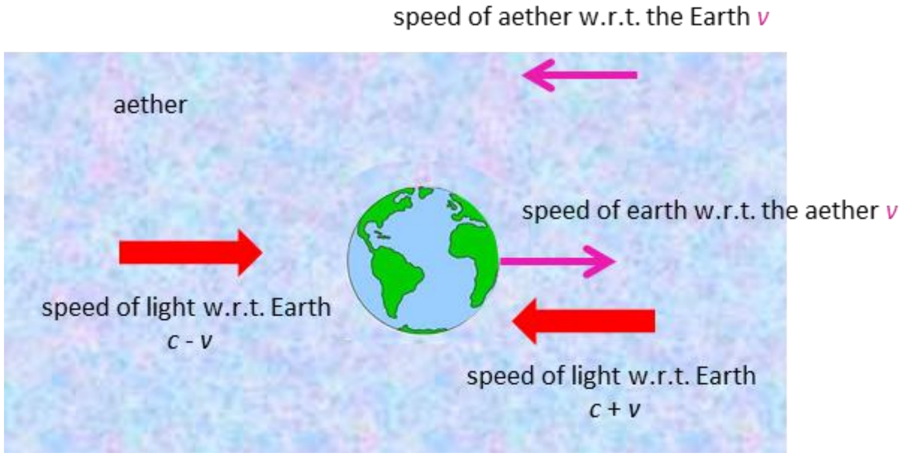
AETHER MODEL FOR THE TRANSMISSION OF LIGHT





Classical picture for the speed of light. The speed of light is relative to the motion of the observer, and so the speed of light is $c+v$ or $c-v$. But this is not correct. The correct answer, is that the person will measure the speed of light to be the constant value ***c*** and it does not matter how fast or slower they are approaching or receding from the light beam or the speed of the light source.

It seemed inconceivable to 19th Century physicists that light and other electromagnetic waves, in contrast to all other kinds of waves, could propagate without a medium. It seemed to be a logical step to postulate such a medium, called the **aether** (or **ether**), even though it was necessary to assume unusual properties for it, such as zero density and perfect transparency, to account for its undetectability. This aether was assumed to fill all space and to be the medium with respect to which electromagnetic waves propagate with the speed c. It followed, using Newtonian relativity, that an observer moving through the aether with velocity v would measure a velocity for a light beam of ($c + v$). ***If the aether exists, an observer on Earth should be able to measure changes in the velocity of light due to the Earth's motion through the aether***. The **Michelson-Morley experiment** attempted to do just this.

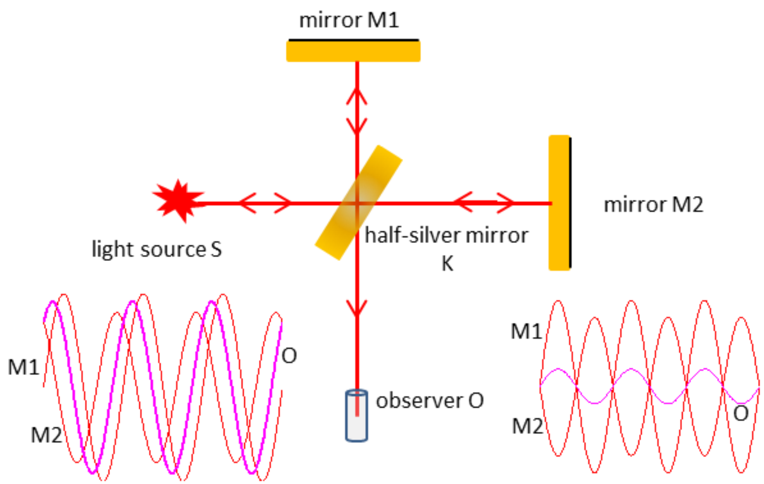


AETHER – proposed medium for the propagation of electromagnetic waves

Property of aether	Evidence
Fills space, permeates all matter	light travels everywhere
Stationary	light travels in straight lines
Transparent	can't see it
Extremely low density	can't be detected
Great elasticity	medium must be elastic otherwise energy dissipated

EINSTEIN THEORY OF SPECIAL RELATIVITY: SPACE & TIME

MICHELSON – MORLEY EXPERIMENT



partial constructive interference of the waves from the two mirrors

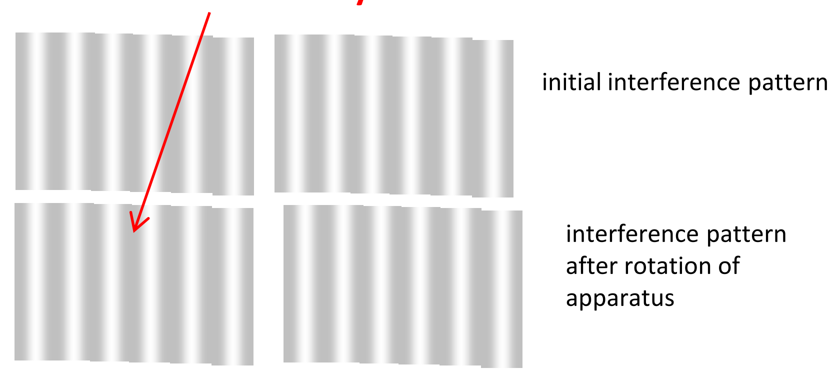
partial destructive interference of the waves from the two mirrors

The Michelson-Morley experiment is an excellent example of a critical experiment in science - the fact that no motion of the Earth relative to the aether was detected suggested quite strongly that the aether hypothesis was incorrect and that no aether (absolute) reference frame existed for electromagnetic phenomena – this opened the way for a whole new way of thinking that was to be proposed by Albert Einstein in his Theory of Special Relativity. The **null** result of the Michelson-Morley experiment was such a blow to the aether hypothesis in particular and to theoretical physics in general that the experiment was repeated by many scientists over more than 50 years. **A null result has always been obtained**.

The light from reflected by the two mirrors produces an interference pattern at the location of the observer.

As the Earth revolves around the Sun and spins on its axis, the direction of the light beams varies with the direction of flow through the aether, their relative velocities would alter and thus the difference in time required for each beam to reach O would alter. This would result in a change in the interference pattern as the apparatus was rotated (changes in the patterns of bright and dark fringes).

A null result has always been obtained



NULL result:
no shift in fringe pattern

Expected result:
slight shift in position of fringes

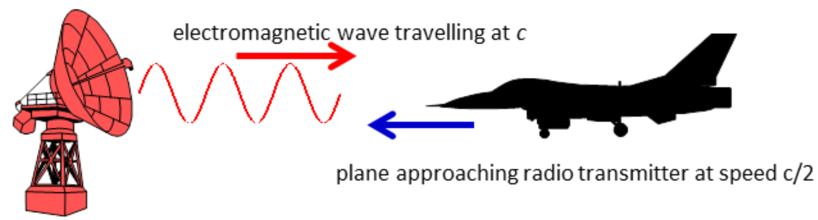
EINSTEIN THEORY OF SPECIAL RELATIVITY: SPACE & TIME

In 1905, **Albert Einstein** (1879 – 1955) published his famous paper entitled: “On the Electrodynamics of Moving Bodies”, in which he proposed his two postulates of relativity and from these derived his Special Relativity Theory.

1. **The Principle of Relativity** – All the laws of physics are the same in all inertial reference frames – no preferred inertial frame exists.

2. **The Principle of the Constancy of the Speed of Light** – the speed of light in free space has the same value *c*, in all inertial frames, regardless of the velocity of the observer or the velocity of the source emitting the light. The speed of light is constant no matter what are the speeds of the transmitter or receiver.

constant *c* $\Rightarrow$
both space and time must be relative quantities



Measured speed of electromagnetic wave *v* w.r.t observer in jet aircraft

Newtonian physics	$v = c + c/2 = 3c/2$	✗
Einstein: special relativity	$v = c$	✓

Einstein concluded - if we accept that the **principle of relativity** can never be violated, then

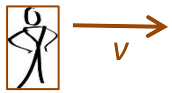
1 The aether model must be wrong.
2 The speed of light is constant regardless of the motion of the observer.

In order to satisfy, **speed of light is constant**, he made a revolutionary statement: it is not the speed of light that is changing, but **time**. Stationary observers and the moving observers perceive **space and time differently**. In classical physics space and time are constants and motion is defined by them. In Einstein's physics it is the speed of light that is constant and space and time change to accommodate this. Using these ideas, Einstein put forward his Special Theory of Relativity

1 All motion is relative — the principle of relativity holds in all situations.
2 The speed of light is constant regardless of the observer's frame of reference.
3 The aether is not needed to explain light, and, in fact it does not exist.

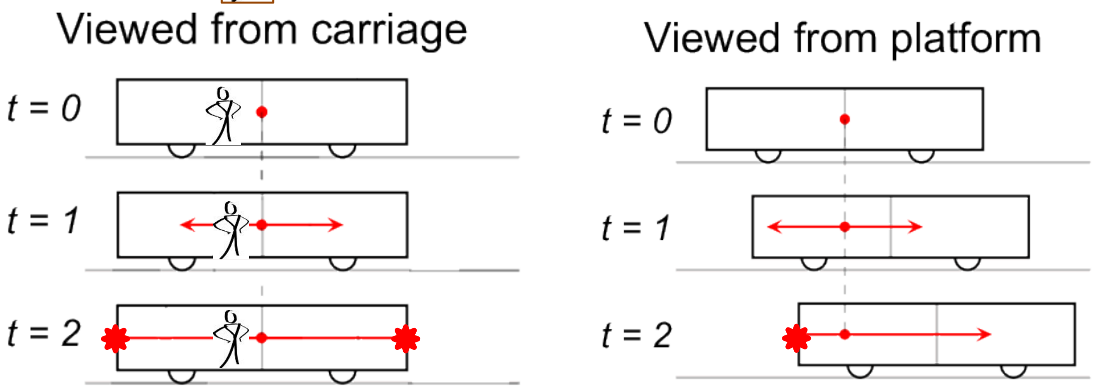
EINSTEIN THEORY OF SPECIAL RELATIVITY: SPACE & TIME

Relativity of simultaneity - whether two spatially separated events occur at the same time is not absolute, but depends on the observer's reference frame. - it is impossible to say in an absolute sense whether two distinct events occur at the same time if those events are separated in space.



Observer in train observes train at rest $v = 0$

Ground based observer sees train go past at speed v



Light beams travel from centre of train and when they hit the ends of the carriage a light flash is given out.

Train observer sees flashes when light reaches the ends of the carriage simultaneously.

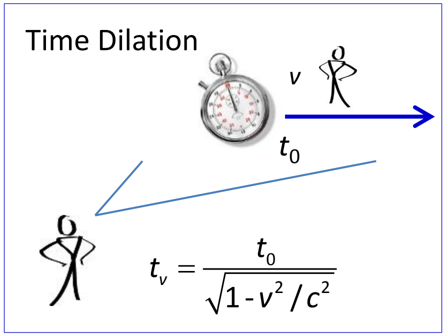
Ground based observer sees a flash at the back of the carriage before the flash at the front of the carriage.

http://faraday.physics.utoronto.ca/GeneralInterest/Harrison/SpecRel/Flash/Simultaneity.html

EINSTEIN: SPECIAL RELATIVITY – SPACE AND TIME

SPEED OF LIGHT IS CONSTANT – INDEPENDENT OF THE MOTION OF SOURCE OR OBSERVER $c = 3.0 \times 10^{8}\ \mathrm{m.s^{-1}}$ $\Rightarrow$

TIME DILATION



t_v time interval measured by observing moving clock
t_0 time interval measured by observing stationary clock

Time is a relative quantity: different observers can measure different time intervals between the occurrence of two events. This arises because the **speed of light is a constant and independent of the motion of the source of light or the motion of an observer**.

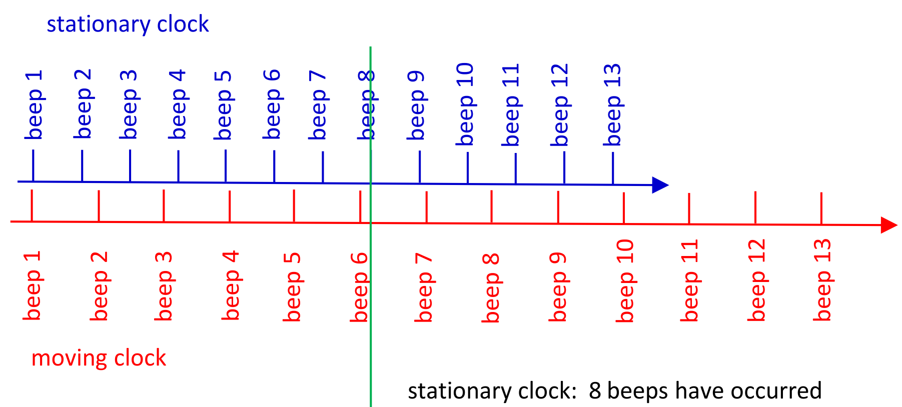
An observer watching a moving clock sees the passage of time on the moving clock to be slower than the passage of time on their own clock.

Time intervals are **not absolute**. This is a violation of a fundamental concepts in Newtonian physics where time is an absolute quantity.

proper time t_0 – the time interval between two events occurring at the same point in space w.r.t. a clock at rest w.r.t. that point.

dilated time intervals t_v – they are the time intervals on moving clocks w.r.t. a stationary observer.

All time intervals measured on moving clocks are longer compared with the stationary clock → **moving clocks run slower**



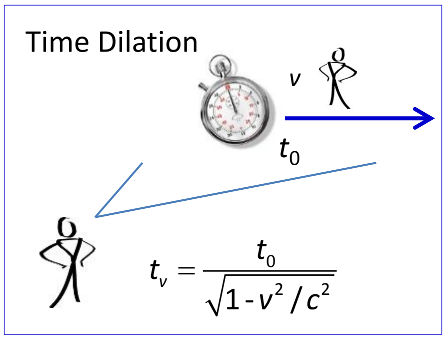
stationary clock: 8 beeps have occurred
moving clock: 6 beeps have occurred

$\Rightarrow$ **moving clocks run slow**

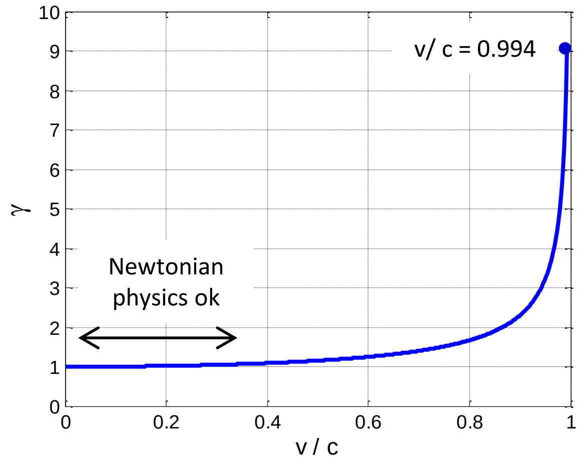
EINSTEIN: SPECIAL RELATIVITY – SPACE AND TIME

SPEED OF LIGHT IS CONSTANT – INDEPENDENT OF THE MOTION OF SOURCE OR OBSERVER $c = 3.0\times10^{8}\ \mathrm{m.s^{-1}}$ $\Rightarrow$

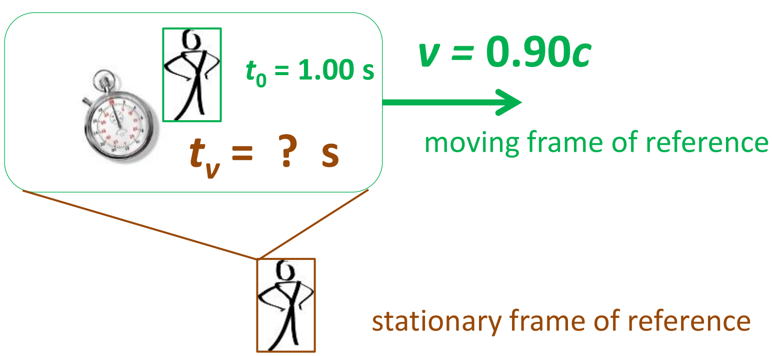
TIME DILATION



$$\gamma = \frac{1}{\sqrt{1-\dfrac{v^2}{c^2}}}$$



Consider a train with velocity $v = 0.90c$ w.r.t. a stationary frame of reference. In the **stationary frame of reference**, the duration of an event was **1.00 s**. What would be the duration of the event as measured by an **observer watching the moving clock**?



$$t_v = \frac{t_0}{\sqrt{1-\dfrac{v^2}{c^2}}} = \frac{1}{\sqrt{1-\dfrac{(0.90c^2)}{c^2}}}\ \mathrm{s} = 2.29\ \mathrm{s}$$

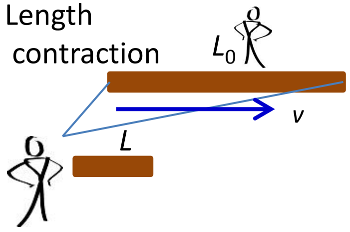
To an **observer on Earth**, the time taken for the event is **2.29 s**. The **observer in the train,** measures a time interval of only **1.00 s**. The Earth observer sees that the train clock has slowed down. It is essential that you understand that this is not an illusion. It makes no sense to ask which of these times is the "real" time. Since no preferred reference frame exists all times are as real as each other. They are the real times seen for the event by the respective observers. Time dilation tells us that a moving clock runs slower than a clock at rest by a factor of 1/√{1 – (v2/c2)}.

This result, can be generalised beyond clocks to include all physical, biological and chemical processes. The Theory of Relativity predicts that all such processes occurring in a moving frame will slow down relative to a stationary clock.

EINSTEIN: SPECIAL RELATIVITY – SPACE AND TIME

SPEED OF LIGHT IS CONSTANT – INDEPENDENT OF THE MOTION OF SOURCE OR OBSERVER $c = 3.0 \times 10^{8}\ \text{m.s}^{-1}$ ⇒

LENGTH CONTRACTION



$$L_v = L_0\sqrt{1 - v^2/c^2}$$

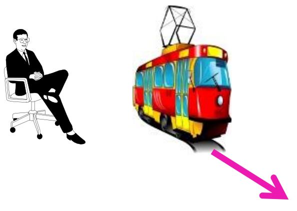
L measured by observer in stationary frame of reference by observing moving object.

L_v is the **proper length** as measured by an observer who is at rest to the object.

train at rest w.r.t. observer

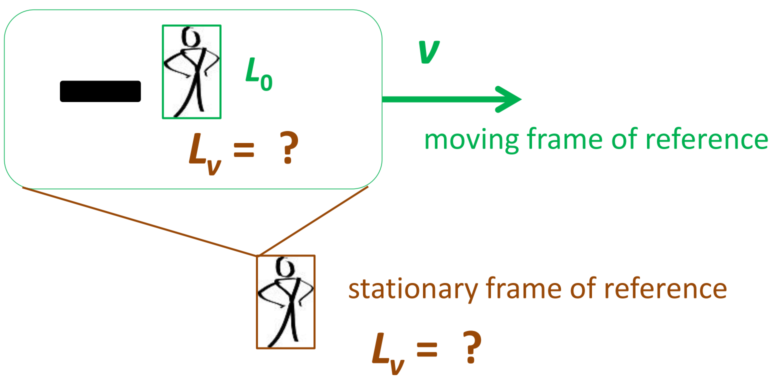
How long is a train? It depends on the relative motion of the observer and the train.

train in motion w.r.t. observer



train is **shorter in direction in motion** but just as high and wide as it was at rest

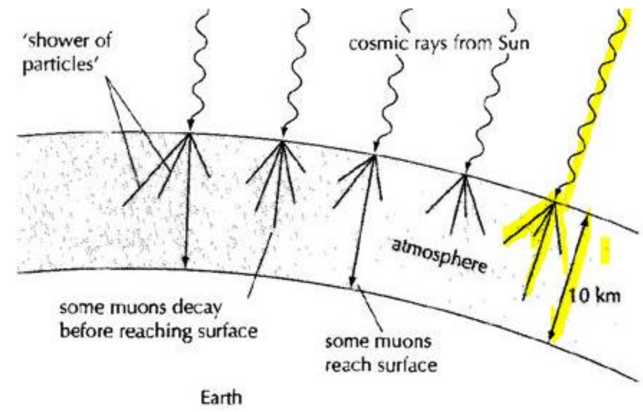
This is a real difference in length of the object when it is motion relative to an observer. For a person in the train, there is no contraction in length.



EINSTEIN: SPECIAL RELATIVITY – SPACE AND TIME

LENGTH CONTRACTION / TIME DILATION / MUON DECAY

Muons are unstable particles with a rest mass of 207 times that of an electron and a charge of $\pm 1.6\text{x}10^{-19}$ C. Muons decay exponentially into electrons or positrons with a **half-life of $t_{1/2} = 1.56\text{x}10^{-6}$ s** as measured in their frame of reference. When high energy particles such as protons called cosmic rays enter the atmosphere from outer space, they interact with air molecules in the upper atmosphere at a height of about **10 km**, creating a cosmic ray shower of particles including **muons** that reach the Earth's surface. The muons created in these cosmic ray showers travel at **v = 0.98c** w.r.t to the Earth.



Exponential decay:
At $t = 0$ N_0 particles
After time t, N particles remaining
decay constant $\lambda = \log_e(2) / t_{1/2}$

$$N = N_0 \, e^{-\lambda t}$$

Newtonian (classical) point of view
Half-life $t_{1/2} = 1.56\text{x}10^{-6}$ s
Decay constant $\lambda = \log_e(2) / t_{1/2} = 4.44\text{x}10^{5}\ \text{s}^{-1}$
Speed of muons $v = 0.98c = (0.98)(3.0\text{x}10^{8})\ \text{m.s}^{-1} = 2.94\text{x}10^{8}\ \text{m.s}^{-1}$
Distance travelled by muons to reach Earth's surface $= 10\text{x}10^{3}$ m
Time to reach Earth's surface $t = 10\text{x}10^{3} / 2.94\text{x}10^{8}$ s $= 3.40\text{x}10^{-5}$ s

Percentage of muons reaching Earth's surface
$= (100)(N/N_0) = 100\ e^{-\lambda t}$
$= (100)\{\exp[(-(4.44\text{x}10^{5})(3.4\text{x}10^{-5})]\} =$ **0.00 %**

Hence, from a Newtonian point of view, most muons would not be able to reach the Earth's surface from the upper atmosphere where they are produced. However, experiments show that a large number of muons do reach the Earth's surface in cosmic ray showers.

Special relativity – length contraction
Muon's frame of reference, the distance from upper atmosphere to Earth's surface contracted
$L_0 = 10\text{x}10^{3}$ m $v = 0.98c$ $L_v = ?$ m

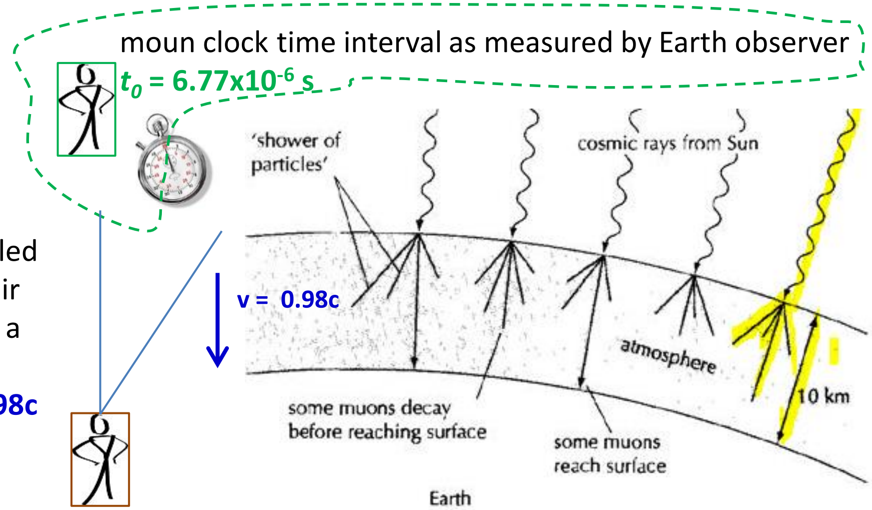
$$L_v = L_0\sqrt{1 - v^2/c^2} = \left(10x10^3\right)\sqrt{1 - 0.98^2}\ \text{m} = 1.99x10^3\ \text{m}$$

Time for muons to reach Earth's surface
$t = (1.99\text{x}10^{3} / 2.94\text{x}10^{8})$ s $= 6.77\text{x}10^{-6}$ s

Percentage of muons reaching Earth's surface
$= (100)(N/N_0) = 100\ e^{-\lambda t}$
$= (100)\{\exp[(-(4.44\text{x}10^{5})(6.77\text{x}10^{-6})]\} =$ **5 %**

Many many more muons can reach the Earth's surface then predicted by Newtonian physics $\Rightarrow$ have to reject Newtonian physics and accept Einstein's postulates: space and time are not absolute quantities.

MUON DECAY



Earth based clock time interval as measured by Earth observer **t_v = 3.40x10^{-5} s**

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Distance travelled by muons to reach Earth's surface = 10x10^3 m
Time to reach Earth's surface $t = 10\text{x}10^3 / 2.94\text{x}10^8\ \text{s} = 3.40\text{x}10^{-5}$ s

Percentage of muons reaching Earth's surface
= $(100)(N/N_0) = 100\ e^{-\lambda t}$
= $(100)\{\exp[(-(4.44\text{x}10^5)(3.4\text{x}10^{-5})]\}$ = **0.00 %**

Hence, from a Newtonian point of view, most muons would not be able to reach the Earth's surface from the upper atmosphere where they are produced. However, experiments show that a large number of muons do reach the Earth's surface in cosmic ray showers.

Special relativity – time dilation
Earth observer: muons travels a distance 10 km at a speed 0.98c.
Time to reach Earth's surface
$t_v = 10\text{x}10^3 / 2.94\text{x}10^8\ \text{s} = 3.40\text{x}10^{-5}$ s

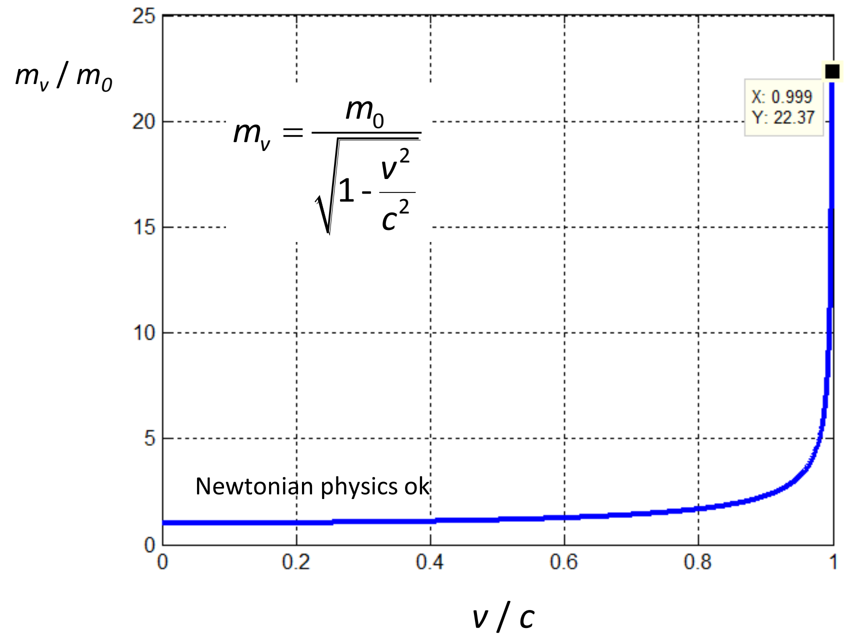
Moving clock's run slow – observed time interval on muon's clock is t_0

$$t_0 = t\sqrt{1 - v^2/c^2} = \left(3.4\text{x}10^{-5}\right)\sqrt{1 - 0.98^2}\ \text{s} = 6.77\text{x}10^{-6}\ \text{s}$$

Percentage of muons reaching Earth's surface
= $(100)(N/N_0) = 100\ e^{-\lambda t}$
= $(100)\{\exp[(-(4.44\text{x}10^5)(6.77\text{x}10^{-6})]\}$ = **5 %**

Same answer as using length contract
Note: had to find t_0 and not t_v.

RELATIVISTIC MASS



Newtonian mechanics, if a force is applied, an object will accelerate and its velocity will increase indefinitely.
Special Relativity, as the velocity approaches the speed of light, the same force produces less and less acceleration, gradually reducing to zero. This is because the mass of the object is increasing and acceleration and mass are inversely proportional when the force is constant ($F = m\,a$). If this did not happen the velocity would become infinite.

Where does this extra mass come from? The applied force is still doing work ($W = F\,s$) so the object is gaining kinetic energy (since v is increasing). This additional energy is converted into mass according to the equation $E = m\,c^2$, rather than continually increasing the velocity of the object.

$$m_v = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

m_v = measured mass of the moving object
m_o = measured mass of the object at rest (**rest mass**)
v = relative velocity between the observed object and the observer
c = speed of light
The mass of an object is a **relative quantity**, it depends on the relative velocity of the object w.r.t. an observer.

MASS ENERGY $E = m c^2$

Einstein's famous equation

$E = m c^2$ $\Rightarrow$ **equivalence of mass and energy**

Energy can be converted into mass and vice versa:

* A particle and its antiparticle collide, all the mass is converted into energy.

* Mass is converted into energy in a nuclear fission reaction.

* When a body gives off energy E in the form of radiation, its mass decreases by an amount equal to E/c^2.

In Special Relativity, the Law of Conservation of Energy and the Law of Conservation of Mass have been replaced by the Law of Conservation of Mass-Energy.

When mass increases as a body gains velocity effectively limits all man-made objects to travel at speeds approaching the speed of light. The closer a body gets to the speed of light, the more massive it becomes. The more massive it becomes, the more energy that has to be used to give it the same acceleration. To accelerate the body up to the speed of light would require an infinite amount of energy. Clearly, this places a limit on both the speed that can be attained by a spacecraft and therefore the time it takes to travel from one point in space to another.