

## MILLIKAN OIL DROP EXPERIMENT

This experiment is designed to show the quantization of electric charge and allow determination of the elementary charge,  $e$ .

As in Millikan's original experiment, oil drops are sprayed into a region where a uniform electric field can be established, and the motions of drops are studied under the action of the electric field being turned on and off.

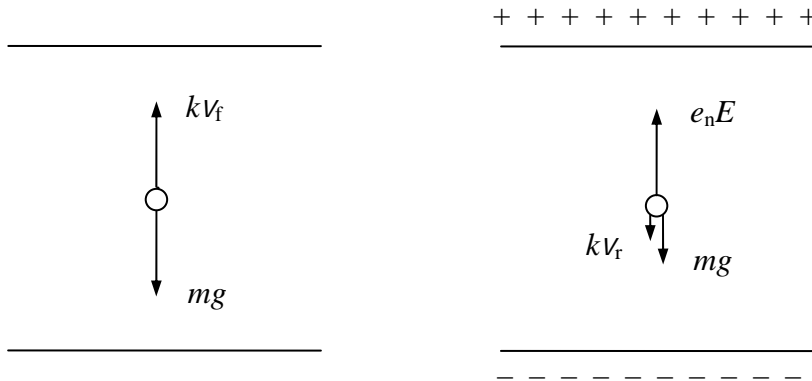
Although this experiment will allow one to measure the total charge on a drop, it is only through an analysis of the data obtained, and a certain degree of experimental skill, that the elementary charge can be determined. By selecting drops which rise and fall slowly, one can be certain that the drop has a small number of excess electrons. A number of such drops should be observed and their respective charges calculated. If the charges on these drops are integral multiples of a certain smallest charge, then this is a good indication of the quantum nature of electricity.

### Theory:

An analysis of the forces acting on a charged droplet will allow determination of its charge.

Figure 1 shows the forces acting on the drop when it is falling in air and has reached its terminal velocity (terminal velocity is reached in a few milliseconds for the droplets used in this experiment). In Figure 1  $v_f$  is the velocity of fall,  $k$  is the coefficient of friction between the air and the drop,  $m$  is the mass of the drop, and  $g$  is the acceleration due to gravity. Since the net force on the drop is zero (constant velocity motion),

$$mg = kv_f \quad (1)$$



**Figure 1**

**Figure 2**

Figure 2 shows the forces acting on the drop when it is rising under the influence of an electric field.  $E$  is the electric field,  $e_n$  is the charge carried by the drop, and  $v_r$  is the velocity of rise. Adding the forces vectorially (and assuming that terminal velocity has been reached) yields:

$$e_n E = mg + kv_r \quad (2)$$

In both cases there is also a small buoyant force exerted by the air on the droplet. The buoyant force is taken into account by substituting  $\rho = \rho_{\text{oil}} - \rho_{\text{air}}$  for  $\rho_{\text{oil}}$  wherever the density of the oil occurs in subsequent equations.

Eliminating  $k$  from equations (1) and (2) and solving for  $e_n$  yields:

$$e_n = \frac{mg(v_f + v_r)}{Ev_f} \quad (3)$$

To eliminate  $m$  from equation (3), one uses the expression for the volume of a sphere:

$$m = \frac{4}{3}\pi a^3 \rho \quad (4)$$

where  $a$  is the radius of the droplet and  $\rho$  is the oil density (corrected for buoyancy).

To calculate  $a$  one employs Stoke's Law, relating the radius of a spherical body to its velocity of fall in a viscous medium, namely

$$a = \sqrt{\frac{9\eta v_f}{2g\rho}} \quad (5)$$

where  $\eta$  is the coefficient of viscosity of air. Substituting equation (5) into equation (4), and then substituting for  $m$  in equation (3) yields

$$e_n = \frac{4}{3}\pi \sqrt{\frac{1}{g\rho} \left(\frac{9\eta}{2}\right)^3} \frac{(v_f + v_r)\sqrt{v_f}}{E} \quad (6)$$

However, for very small drops, the drop radii are on the order of the inter-molecular spacing of the air in which the drops are moving. Thus the assumption, implicit in Stoke's Law, that the air can be treated as a continuous medium, is no longer valid. As determined by Millikan, a correction factor of

$$\left(1 + \frac{b}{pa}\right)^{-3/2} \quad (7)$$

must be included in the expression for  $e_n$  to account for the inhomogeneity of the air. In the correction factor,  $p$  is the atmospheric pressure in cm of Hg,  $a$  is the radius of the drop (in m) as calculated by the uncorrected form of Stoke's Law (equation (5)), and  $b = 6.17 \times 10^{-6}$  m-cm Hg, a constant.

The electric field is given by  $E = V/d$  (equation (8)) where  $V$  is the potential difference across the parallel plates separated by a distance  $d$ . Substituting for the electric field and the correction factor in equation (6) yields:

$$e_n = \frac{4}{3} \pi d \sqrt{\frac{1}{\rho g} \left( \frac{9\eta}{2} \right)^3} \left( 1 + \frac{b}{pa} \right)^{-3/2} \frac{(V_f + V_r) \sqrt{V_f}}{V} \quad (9)$$

Another method for determining the charge on the oil drop is to adjust the voltage (and hence the electric field) until the oil drop floats. The charge on the oil drop is then given by

$$e_n = \frac{4}{3} \pi d \sqrt{\frac{1}{\rho g} \left( \frac{9\eta}{2} \right)^3} \left( 1 + \frac{b}{pa} \right)^{-3/2} \frac{V_f^{3/2}}{U} \quad (10)$$

where  $U$  = float voltage.

### Apparatus:

The equipment supplied, as shown in Figure 3, consists of a stand supporting the oil drop chamber, the light, and the measuring microscope; and a power supply for the light and the voltage to produce the electric field in the chamber. In addition, an electronic digital stopwatch is provided for measuring the rise and fall times of the oil drops.

An atomizer is used to produce the oil drops. The nozzle of the atomizer is placed against the two boreholes of the chamber. A quick squirt will fill the chamber with drops which become visible in the viewing area.

When determining the oil drop speeds, the actual distance,  $s$ , travelled by a drop is given by

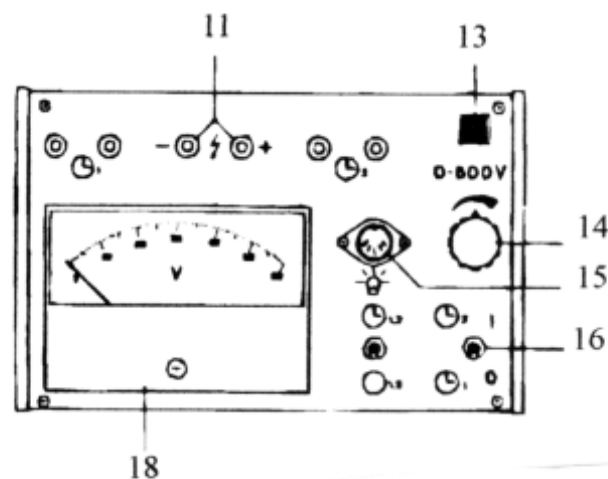
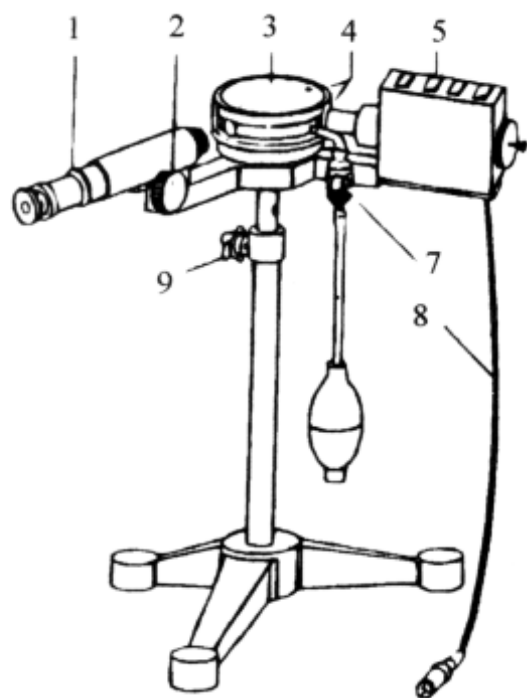
$$s = \frac{y}{2.017} \times 10^{-4} \text{ m} \quad (11)$$

where  $y$  is the number of microscope scale divisions through which the drop moves. (Ask your lab instructor to confirm that 2.017 is the correct number to use in this equation.)

**NOTE:** *The optics of the microscope cause image inversion.  $\therefore$  Drops appear to rise under gravity and fall when the electric field is on.*

For the apparatus provided:

$$\begin{aligned} \eta &= 1.83 \times 10^{-5} \text{ N}\cdot\text{s}/\text{m}^2 \\ d &= 6.00 \times 10^{-3} \text{ m} \\ \rho &= 874 \text{ kg}/\text{m}^3 \end{aligned}$$



- ① Measuring microscope with micrometer eyepiece
- ② Knurled knob for microscope adjustment
- ③ Millikan chamber (plate capacitor) with acrylic glass cover
- ④ Socket pair to connect the d. c. voltage for the plate capacitor (can be tapped from socket pair ⑦, adjustable via knob ⑫)
- ⑤ Illumination device
- ⑦ Oil atomizer with rubber ball in resilient holder
- ⑧ Connecting cable for lamp voltage (from multiple socket ⑬)
- ⑨ Screw for height adjustment (to adapt the microscope to the eye level of the experimenter)

- ⑦ Socket pair to connect the Millikan chamber
  - ⑬ Mains pilot lamp
  - ⑫ Rotary knob for 0 to 600 V capacitor voltage setting
  - ⑬ Multiple socket to connect the illumination device
  - ⑮ On/off switch for 0 to 600 V
  - ⑪ Voltmeter 0 to 600 V to indicate the voltage for the capacitor plates of the Millikan chamber set by rotary knob ⑫
- Mains switch, fine-wire fused and mains connector are at the instrument rear.

Figure 3

Substituting these values into equations (5), (9), and (10) yields

$$a = 9.80 \times 10^{-5} \sqrt{V_f} \text{ m} \quad (12)$$

$$e_n = 2.03 \times 10^{-10} \left( 1 + \frac{6.17 \times 10^{-6} \text{ m} \cdot \text{cmHg}}{pa} \right)^{-3/2} \frac{(V_f + V_r) \sqrt{V_f}}{V} \quad (13)$$

$$e_n = 2.03 \times 10^{-10} \left( 1 + \frac{6.17 \times 10^{-6} \text{ m} \cdot \text{cmHg}}{pa} \right)^{-3/2} \frac{v_f^{3/2}}{U} \quad (14)$$

### Procedure and Experiment:

Record the atmospheric pressure in cm Hg. All quantities other than the atmospheric pressure are to be expressed in SI units.

For each of 10 oil drops, measure 4 sets of FIELD ON (500 V) and FIELD OFF times and distances. (For FIELD ON the drops are actually rising but appear to fall, for FIELD OFF the drops are actually falling but appear to rise.)

**Also, for each drop, measure the float potential.** (The float potential is the applied voltage for which the drop has negligible vertical motion.)

To ensure that the drops are small and have a small number of excess charges, choose drops which move slowly both when the field is on and when it is off.

*For best results, ensure that some of the measured drops are moving at rise and fall speeds such that it takes about 15 seconds to move 10 microscope divisions with the field on or off. It is also important that some of the drops move faster than this with the field on or off, so that you obtain some drops with more than one excess charge.*

Data for each drop should be recorded as follows:

Drop	Trial	FIELD ON		FIELD OFF		Float Potential (± __ V)
		Time (± __ s)	Divisions (± __)	Time (± __ s)	Divisions (± __)	
1	1					
	2					
	3					
	4					
2	1					
	etc.					

Using the average rise and fall speeds calculate the charge on each drop. Using the average fall speed and the float potential calculate the charge on each drop. (Note that the subscripts in the equations refer to actual (rather than apparent) drop motion.)

An Excel spreadsheet is available to aid with the presentation and analysis of the data. Note that you must do a complete set of sample calculations for one of the oil drops by hand.

The spreadsheet is obtained from the lab manual web page:

<http://physics.usask.ca/~bzulkosk/modphyslab/phys251.htm> :

1. Load the spreadsheet *millikan.xls*.
2. If you are using a university computer, once the file has loaded save it to your home directory (most likely on the *h:* drive) by selecting **Save As...** from the **File** menu.

3. Enter the time error in cell D4, the division error in cell D5, and the atmospheric pressure and its error in cells J4 and L4.
4. Enter the oil drop data in the appropriate cells of columns C, D, H, I, M, and O.
5. Remember to periodically save your work. (Select **S**ave from the **F**ile menu.)
6. Print the spreadsheet by selecting **P**rint... from the **F**ile menu. Unless you have collected data for more than 10 drops you only need to print pages 1 and 2. Therefore, in 'Page Range' select 'Page(s) From 1 To 2' and click on **O**K.
7. ***Before leaving the lab, have your rough data and printed output signed by one of the instructors.***

By careful analysis of the results for each method (rise/fall method and float method), verify the quantum nature of electric charge (i.e. the existence of an elementary charge). Calculate the value of the elementary charge and compare to the accepted value of  $1.602 \times 10^{-19}$  C.

**References:**

- R.A. Millikan, The Electron, Chicago, University of Chicago Press  
L. Page, Introduction to Theoretical Physics, New York, Van Nostrand, Chpt. 6