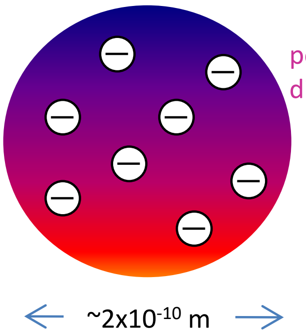
VISUAL PHYSICS ONLINE

THE NUCLEUS OF AN ATOM

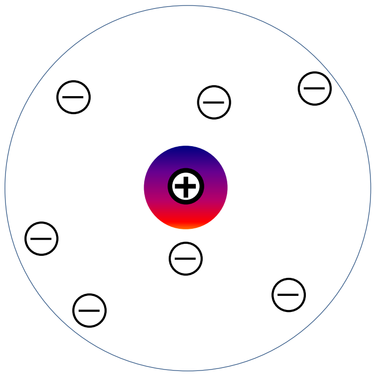
Models of the atom



J. J. Thomson model of the atom (1907)

plum-pudding model: positive charge uniformly distributed over a sphere of radius ~10^{-10} m with the electrons spread out throughout the sphere in such a way that the whole system is stable and electrically neutral.

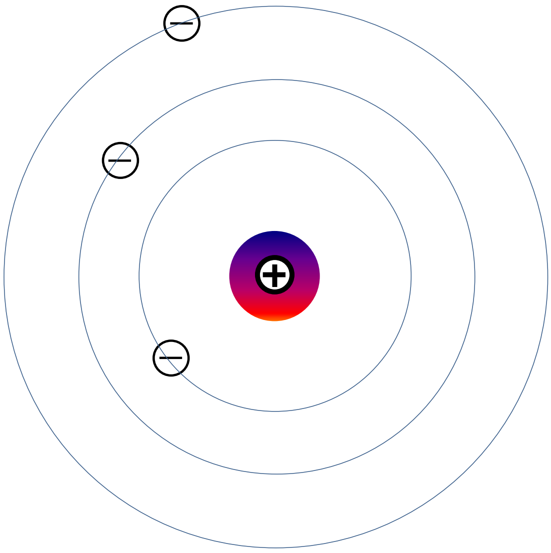
Rutherford model of the atom (1911)



Atom consists of a very tiny but positively charged nucleus containing over 99.9% of the atom's mass, surrounded by electrons some distance away. The electrons would be moving in orbits about the nucleus, much as planets move around the Sun.

Bohr-Rutherford model of the atom (1913)

Bohr proposed a planetary model of the atom based upon the Rutherford model but added the restriction that the electrons can only orbit the nucleus in circles but only with certain radii allowed such that the angular momentum of the electron is quantized.

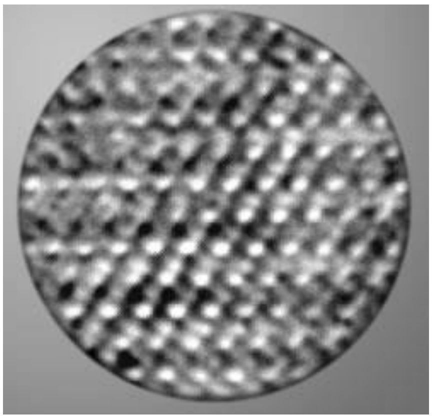


not to scale: nucleus << atom

Seeing atoms

atomic dimensions ~ 10^{-10} m

nuclear dimensions ~ 10^{-15} m



THE NUCLEUS

Until now the nucleus of an atom was considered to be a tiny positively charged lump which contribution to the majority of the mass of an atom and to hold its electrons in place. It was the atomic electrons responsible for the behaviour of matter in bulk, not the nucleus. However, the nucleus turns out to be of supreme importance in the universe.

What is the nucleus made of?

In the periodic table, the elements are listed in order of their **atomic number Z**, with the number of the element defined as the number of electrons in each of its atoms.

1 hydrogen
2 helium
…
92 uranium

The atomic mass of the elements increases with atomic number Z and it was first suggested that all atoms are simply combinations of hydrogen atoms (protons). Thus, a helium atom (Z = 2) should have a nucleus with two protons, a lithium atom (Z = 3) should have a nucleus composed of 3 protons, and so on.

However, atomic masses do not increase in steps of one hydrogen atom mass. Helium atoms weigh about 4 times as much as hydrogen atoms and lithium atoms about 7 times as much as hydrogen atoms. But, atomic masses were very close to exact multiples of the mass of the hydrogen atom!

The Proton-Electron Model

It was hypothesized that there are enough protons in each nucleus to provide for the observed atomic mass, with several electrons present whose negative charge would cancel out the "excess" positive charge of the extra protons. However, this is not an acceptable model of the nucleus since too much energy would be required to localize electrons within the nucleus according to the Heisenberg Uncertainty Principle.

Rutherford (1914): an atom such as fluorine (atomic number 9) for example, had a mass equivalent to 19 protons but a charge of only 9 protons → the nucleus contained protons and electrons to balance the charge discrepancy. A fluorine nucleus would therefore contain 19 protons and 10 electrons – a total charge of 9 protons and total mass of 19 protons (electron mass being negligible compared to the proton mass) → a nucleus contained A protons and (A – Z) electrons. This model could explain how α and β particles could be emitted from some radioactive nuclei but problems arose:

- Energies of emitted β particles could not be accurately predicted.
- Quantum number anomalies arose with the spin of electrons and protons within the nucleus.
- Heisenberg's Uncertainty Principle suggested that electrons could not be confined within the nucleus.

Uncertainty Principle $\Delta x \, \Delta p > \dfrac{h}{2\pi}$ $\quad h = 6.63x10^{-34}$ J.s

Uncertainty in position of electron

~ size of nucleus $\quad \Delta x \sim 10^{-15}$ m

Uncertainty in momentum and hence minimum value of momentum of electron

$$\Delta p > \frac{h}{2\pi\,\Delta x} \Rightarrow p > \frac{h}{2\pi\,\Delta x}$$

Minimum kinetic energy of electron

$$E_K = \frac{p^2}{2m_e} \quad m_e = 9.11\times10^{-31}\ \text{kg}$$

$$E_K > \left(\frac{h}{2\pi\,\Delta x}\right)^2 / \left(2\,m_e\right)\ \text{J}$$

$$E_K > \left(\frac{h}{2\pi\,\Delta x}\right)^2 / \left(2\,m_e\,e\right)\ \text{eV} \quad e = 1.602\times10^{-19}\ C$$

$$E_K > 4\times10^{10}\ \text{eV}$$

Energies of electrons in atoms are ~ eV and so electrons can't exist in the nucleus with enormous energies > 10^{10} eV.

Electrons are not a constituent of a nucleus

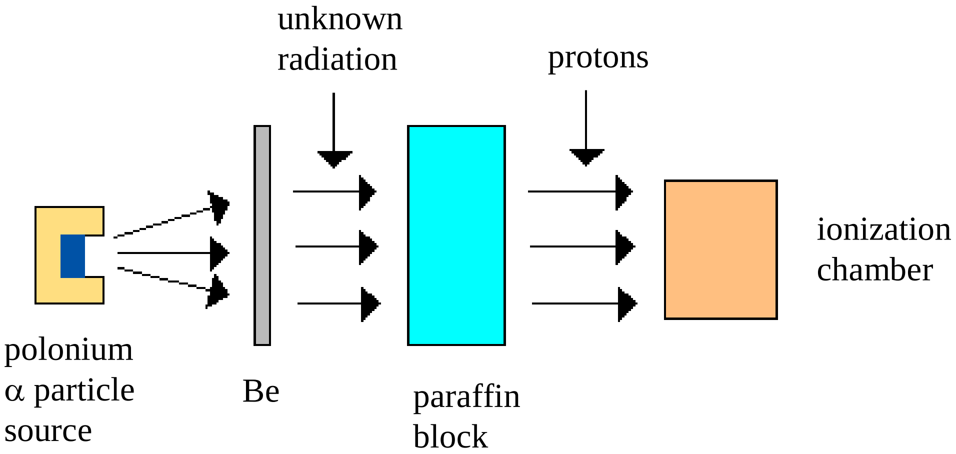
The Neutron

Rutherford (1920) - a proton and an electron within the nucleus might combine together to produce a neutral particle. He named this particle the **neutron**. Experimental difficulties associated with the detection of a neutral particle greatly hindered the research. In 12 years of searching, no such particle was found.

In 1930, two German physicists, Bothe & Becker, bombarded the element beryllium Be with α particles and found a very penetrating form of radiation that was much more energetic than gamma-rays was emitted from Be.

Frederic & Irene Joliot (daughter of Marie Curie) in 1932: although this radiation could pass through thick sheets of lead, it was stopped by water or paraffin wax. They found that large numbers of very energetic protons were emitted from the paraffin when it absorbed the radiation.

The Joliots assumed that the radiation must be an extremely energetic form of gamma radiation.



Joliot's Experiment

The English physicist, **James Chadwick** (1932) showed theoretically that gamma rays produced by α particle bombardment of Be would not have sufficient energy to knock protons out of paraffin, and that momentum could not be conserved in such a collision between a gamma ray and a proton. Chadwick repeated the Joliot's experiments many times. He measured the energy of the radiation emitted from the Be and the energies (and therefore the velocities) of the protons coming from the paraffin. On the basis of its great penetrating power, Chadwick proposed that the radiation emitted from the Be was a new type of neutral particle – the **neutron**, as originally proposed by Rutherford.

He then applied the **conservation of energy and momentum laws** to his experimental results and showed that the particles emitted from the Be had to be neutral with about the same mass as the proton. **Chadwick had indeed discovered the neutron**. Chadwick explained that when the neutrons emitted from the Be collided with the light hydrogen nuclei in the paraffin, the neutron came to a sudden stop and the hydrogen nucleus (proton) moved off with the same momentum as the neutron had before the collision.

$$^{4}He_{2} + {}^{9}Be_{4} \rightarrow {}^{12}C_{6} + {}^{1}n_{0}$$

The Proton-Neutron Model

Following Chadwick's discovery of the neutron, a new model of the nucleus was proposed. This model suggests that the nucleus consists of protons and neutrons. Together these particles are called the **nucleons** – particles that make up the nucleus.

nucleon is a generic term for a **proton** or a **neutron**

Nuclear masses

Z Proton number (Atomic Number) → element

N Neutron number

A Mass number $A = Z + N$

Isotopes: nuclei with the same atomic number Z

Isobars: nuclei with the same mass number A

The number of protons in the nucleus is called the **atomic number Z** of the nucleus and corresponds to the position of the nucleus in the Periodic Table of Elements. For example:

hydrogen $^{1}H_{1}$ $^{2}H_{1}$ (deuterium) $^{3}H_{1}$ (tritium)

carbon $^{12}C_{6}$ $^{13}C_{6}$ $^{14}C_{6}$

Energy / Mass units, values and conversion factors

amu (atomic mass unit) = 1 u = 1.66054×10^{-27} kg

1 eV = 1.602×10^{-19} J 1 MeV = 10^{6} eV

$E = mc^2$ $c = 2.99792\times10^{8}$ m.s^{-1}

energy equivalent **1 u**

$E = (1.66054\times10^{-27})(2.99792\times10^{8})^2$ J $= 1.49242\times10^{-10}$ J

$E = 931.494$ MeV

$1\text{ u} \equiv 931.494\text{ MeV/c}^2$

Proton mass

$m_p = 1.67262\times10^{-27}$ kg = 1.0072765 u = 938.3 MeV/c^2

Neutron mass

$m_n = 1.67493\times10^{-27}$ kg = 1.0086649 u = 939.6 MeV/c^2

Electron mass

$m_e = 9.1093897\times10^{-31}$ kg = 0.0005485799 u = 0.511 MeV/c^2

Hydrogen atom $m_H = m_p + m_e = 1.0078250$ u

Charge of electron $q_e = -e = -1.602\times10^{-19}$ C

Charge on proton $q_p = +e = +1.602\times10^{-19}$ C

Charge on neutron $q_n = 0$

Nuclear Radius

We can't talk about the definite size of a nucleus because of the wave-particle duality principle. We can think about the nucleus as a fuzzy ball whose spatial extent can be measured by scattering high speed electrons off nuclei. The size of a nucleus is found to increase in size with mass number *A*

$r = r_o A^{1/3}$ *A* is mass number (number of nucleons)

$r_o = 1.2\times10^{-15}$ m = 1.2 fm

1 fm = 10^{-15} m fm = "femtometre" or "fermi"

STRONG NUCLEAR FORCE

- Does not depend on charge i.e. binding is the same for protons and neutrons.

- It has very short range ~10^{-15} m. A nucleon only interacts with neighbouring nucleons (saturation of nuclear force).

- Nuclear force favours binding of pairs of protons or neutrons with opposite spin. The force is really between quarks as three quarks combine to give either a proton or neutron.

- Nature of NUCLEAR FORCE is not well understood.

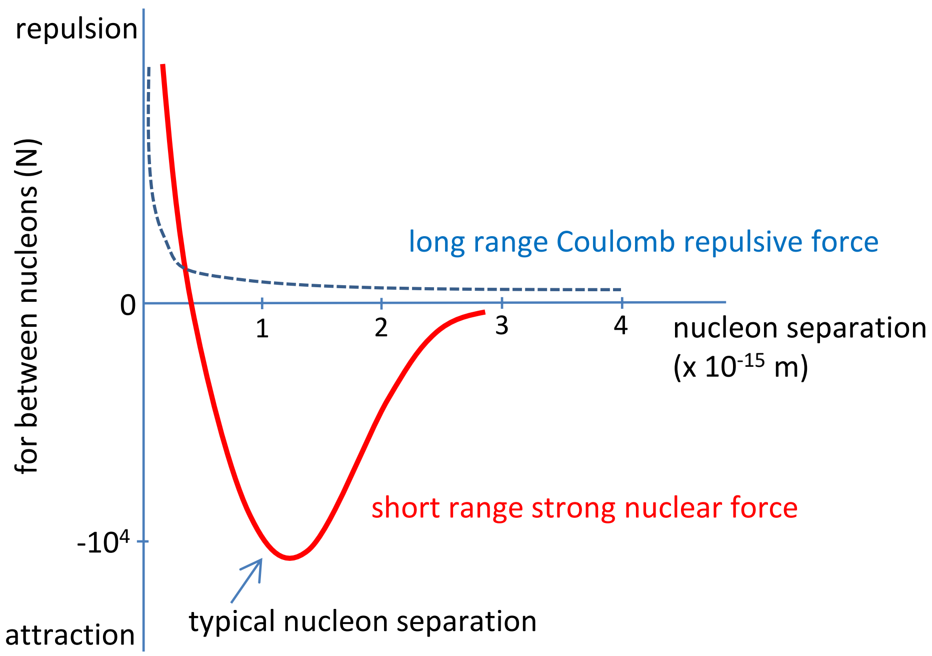


Fig. 1. Strong nuclear force between pairs of nucleons.

A nucleus is composed of a collection of protons and neutrons, but the protons are positively charged and so there is a very large repulsive force between them when they are close together.

Consider two protons with a separation distance *r*

r = 1 fm = 1x10^{-15} m nuclear dimension

m_p = 1.67262x10^{-27} kg

q_p = +1.602x10^{-19} C

Gravitational force between two protons

$$F_G = \frac{G\, m_p\, m_p}{r^2} = 2\times10^{-34}\ \mathrm{N}$$ attractive force

Electrostatic force between two protons

$$F_E = \frac{1}{4\pi\,\varepsilon_0}\frac{e^2}{r^2} = 2\times10^{2}\ \mathrm{N}$$ repulsive

Ratio $\dfrac{F_E}{F_G} = 10^{36}$

Use your calculator to check the numerical values

Even at this very close distance the gravitational force between protons is negligible and the magnitude of the electrostatic force is enormous.

The **strong nuclear force** is an attractive force that acts between all nucleons (protons and neutrons) and at short distances (nuclear dimensions) is greater in magnitude than the electrostatic force acting between the protons. The nuclear force is a much more complicated force than either the electromagnetic force or gravitational force. The strong nuclear force is very strong between a pair of nucleons only if their separation distance is less than 10^{-15}, for distances greater than this, the force is essentially zero.

If a nucleus contains too many or too few neutrons relative to the number of protons, the binding of the nucleus is reduced and the nucleus is unstable and the nucleus will decay into a more stable one (radioactive decay). Nuclei with $A < 40$ tend to be stable when the number of protons equals the number of neutrons. When $A > 40$, the stable nuclei have more neutrons than protons because the increasing number of protons in a nucleus increases the electrostatic repulsive force acting between, making the nuclei more unstable. When $Z > 82$, there are no completely stable nuclei.

There is also a second type of nuclear force, which is called the **weak nuclear force**, and is much weaker in strength than the strong nuclear force. The weak nuclear force is responsible for certain types of radioactive decay known as beta (β) decay.

These two nuclear forces, the strong and the weak, together with the electromagnetic and gravitational forces, comprise the **four** known types of forces acting in nature.

Relative strengths of the four fundamental forces of nature

Strong force	1	short range
Electromagnetic force	1/137	charged particles – inverse square law
Weak force	10^{-9}	
Gravitational	10^{-38}	mass – inverse square law

NUCLEAR BINDING ENERGY

Energy ≡ Mass $E = m\,c^2$

Why is the mass of a nucleus **less** than the combined mass of its nucleons?

The answer is that when nucleons combine to form a nucleus, the total energy of the system (including rest mass energy) remains constant, although mass does not. The difference in mass between the constituents of the nucleus and the nucleus itself appears as additional kinetic energy of the nucleus beyond which the constituents initially possessed. Correspondingly, this same mass deficiency must be made up by an addition of energy to break up a nucleus. Consider a deuteron nucleus $^2H_1$.

Mass of deuteron nucleus	2.014186 u
Mass of proton	1.007593 u
Mass of neutron	1.008982 u
Mass (proton + neutron)	2.016575 u

Mass of deuteron nucleus < mass (proton + neutron)

Mass of nucleus is less than the constituent nucleons. The difference is mass is called the **mass defect**.

Mass defect $\Delta m = (2.016575 - 2.014186)\ \text{u} = 0.002389\ \text{u}$

This mass defect is responsible for the energy that enables the nucleus to "stick together". The binding energy E_B of the deuteron is

$$E_B = \Delta m\, c^2 = (0.002389)(1.660\text{x}10^{-27})(3.0\text{x}10^{8})^2\ \text{J} = 3.57\text{x}10^{-13}\ \text{J}$$

$$E_B = (3.57\text{x}10^{-13} / 1.602\text{x}10^{-19})\ \text{eV} = 2.22\text{x}10^{6}\ \text{eV} = 2.22\ \text{MeV}$$

$1\ \text{amu} = 1\ \text{u} = 1.660\text{x}10^{-27}\ \text{kg}$ $\quad c = 3.0\text{x}10^{8}\ \text{m.s}^{-1}$

$q_e = e = 1.602\text{x}10^{-19}\ \text{C}$ $\quad 1\ \text{eV} = 1.602\text{x}10^{-19}\ \text{J}$

$1\ \text{u} = 931\ \text{MeV/c}^2$

Alternatively $\quad E_B = \Delta m\, c^2 = (0.002389)(931)c^2\ \text{MeV/c}^2 = 2.22\ \text{MeV}$

This number is confirmed by experiments that show that the minimum energy of a gamma ray must be greater than 2.22 MeV to disrupt a deuteron nucleus.

Figure 2 shows a very interesting curve when we plot the binding energy per nucleon (E_B / A) vs mass number A of the nucleus. The curve is surprising regular in shape except for the peak for ^{4}He2. The middle range nuclei have the highest binding energy per nucleon (~ 8.8 MeV / nucleon) and therefore the most stable.

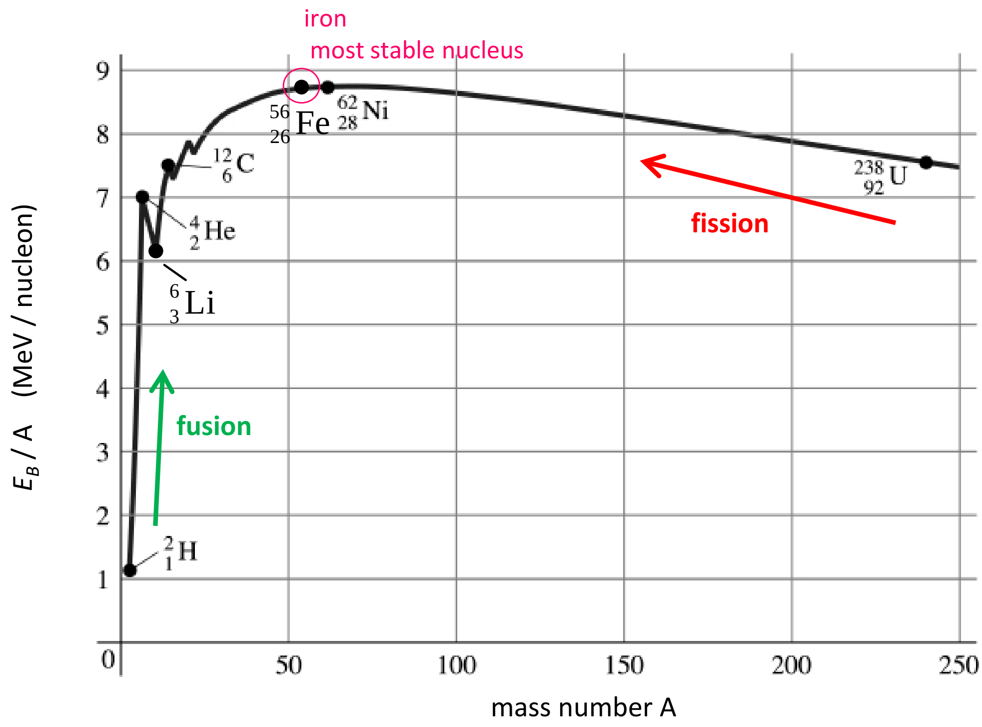


Fig. 2. The binding energy E_B per nucleon vs mass number A. The higher the binding energy per nucleon, the more stable the nucleus. When a heavy nucleus is split into two lighter ones, a process called **fission** occurs and energy is liberated. When two very light nuclei join to form a heavier one, a process called **fusion** occurs and energy is liberated.

Suppose that a uranium $^{235}U_{92}$ nucleus with a binding energy per nucleon of 7.6 MeV / nucleon is split into two lighter nuclei and therefore they will have a higher binding energy per nucleon of about 0.8 MeV / nucleon higher than that of the uranium nucleus. Then the energy released in this fission reaction for each uranium nucleus is about

$$(0.8 \text{ MeV / nucleon})(235 \text{ nucleons}) = 190 \text{ MeV}.$$

This is an enormous amount of energy which is released as the kinetic energy of the two lighter nuclei. It is about 10^8 times the amount of energy released in a chemical reaction (~ 1 eV). This splitting a heavy nucleus into lighter ones is called **fission**.

If two lighter elements combine into a heavier nucleus, again an enormous amount of energy is released. For two deuteron nuclei ($^2H_1$ + $^2H_1$) to combine to give a helium nucleus ($^4He_2$), the energy liberated is about 23 MeV for this single nuclear reaction. The combining of lighter nuclei to give a heavier nucleus is called **fusion**.

The experimentally measured mass of any nucleus is less than the sum of the masses of its constituent protons and neutrons.

proton mass m_p = 1.00728 u

neutron mass m_n = 1.00867 u

electron mass m_e = 0.00055 u.

1 u = 1.6602x10^{-27} kg = 931.5 MeV/c^2

1 eV = 1.602x10^{-19} J 1 MeV = 1x10^6 eV

Consider the isotope of the element $^{35}Cl_{17}$

Z = 17 N = 18 A = 35

mass of atom m_{Cl} = 34.980175 u

Combined mass of the constituent particles

mass of 17 protons = 17 $\times$ 1.00728 u = 17.12376 u

mass of 18 neutrons = 18 $\times$ 1.00867 u = 18.15606 u

<u>mass of 17 electrons = 17 $\times$ 0.00055 u = 0.00935 u</u>

Combined Mass = 35.28917 u

mass defect Δm = mass constituents - mass of atom

= (35.28917 - 34.980175) u = 0.309 u = 5.13$\times$10^{-28} kg

This small mass has been converted into the **binding energy** of the nucleus (the energy holding the nucleus together). The mass defect of a nucleus can therefore be defined as the mass equivalent of the binding energy of the nucleus.

Binding energy

$E_B = m\,c^2$

$E_B = (0.309)\ (931.494)\ c^2\ \ \text{MeV}/c^2 = 287.8\ \text{MeV}$

$E_B = (5.13\times10^{-28})(3\times10^{8})^2\ \ \text{J} = 4.617\times10^{-11}\ \ \text{J} = 288.2\ \ \text{MeV}$

$E_B/\text{A} = (288.2\ /\ 35)\ \text{MeV/nucleon} = 8.23\ \ \text{MeV / nucleon}$

The binding energy of the nucleus is the energy needed to separate the nucleus into its constituent parts. When the nucleons come together to form the nucleus, they release the binding energy. If we take the total binding energy of a nucleus and divide it by the total number of nucleons in the nucleus, we get a very good measure of how tightly each individual nucleon is held in the nucleus. This binding energy per nucleon figure is a very good measure of the stability of the particular nucleus. The higher the binding energy per nucleon, the more stable the nucleus. Figure 2 shows the basic shape of the binding energy per nucleon versus mass number graph. Note that the binding energy per nucleon is low for low mass number nuclei. This is because in such nuclei each nucleon is not uniformly surrounded and thus does not experience the full effects of the strong nuclear force.

Most nuclei have binding energy per nucleon values between 7 and 9 MeV, with the highest value being that for $^{56}Fe_{26}$. For very high mass number nuclei the electrostatic repulsive forces between the protons result in a gradual decrease in binding energy per nucleon values.

VISUAL PHYSICS ONLINE

If you have any feedback, comments, suggestions or corrections please email:

Ian Cooper School of Physics University of Sydney

ian.cooper@sydney.edu.au