Indian Institute of Technology Indore
MA 204 Numerical methods
(Spring Semester 2022)
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Tutorial Sheet 1

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1. Solve the system

$$\begin{aligned} 4x_1+3x_2+2x_3+x_4 &= 1 \\ 3x_1+4x_2+3x_3+2x_4 &= 1 \\ 2x_1+3x_2+4x_3+3x_4 &= -1 \\ x_1+2x_2+3x_3+4x_4 &= -1 \end{aligned}$$

   by the Gauss elimination.

2. The exact solution of the system

$$\begin{aligned} 0.003000x_1 + 59.14x_2 &= 59.17 \\ 5.291x_1 - 6.130x_2 &= 46.78 \end{aligned}$$

   is $x_1 = 10$ and $x_2 = 1$. Solve this system by the Gauss elimination (a) *without pivoting* and (b) *with partial pivoting* using four-digit arithmetic with rounding. Observe the error in the solution in both the cases.

3. Verify that the exact solution of the system

$$\begin{aligned} 6x_1 + 2\,x_2 + 2\,x_3 &= -2 \\ 2x_1 + \frac{2}{3}x_2 + \frac{1}{3}x_3 &= 1 \\ x_1 + 2\,x_2 - x_3 &= 0 \end{aligned}$$

   is $x_1 = 13/5$, $x_2 = -19/5$, $x_3 = -5$. Solve this system by the Gauss elimination (a) *without pivoting* and (b) *with partial pivoting* using four-digit floating point arithmetic. Observe the error in the solution in both the cases.

4. For which three numbers k, the matrix

$$A = \begin{bmatrix} 2 & k & k \\ k & k & k \\ 8 & 7 & k \end{bmatrix}$$

   is not invertible and why? Are there any other values of k for which the matrix A is not invertible?

5. Prove that the matrix

$$A = \begin{bmatrix} a & b & b \\ a & a & b \\ a & a & a \end{bmatrix}$$

   is invertible if $a \neq 0$ and $a \neq b$. Assuming that these conditions are fulfilled, find A^{-1} without using the determinants and co-factors.
   (Hint: Apply the Gauss elimination, and identify the pivots.)

6. Compute the inverse of the following matrix without using the determinants and co-factors.

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}.$$

7. Write down all 3×3 permutation matrices.

8. Suppose a linear system $Ax = b$ is to be solved by Gauss elimination method, where A is $n \times n$ matrix, and x and b are column vectors of order $n \times 1$ of unknowns and known values respectively. Show that

   (a) the number of multiplications/divisions operations $\frac{2n^3+3n^2-5n}{6}$ is required in forward elimination.

   (b) the number of additions/subtractions operations $\frac{n^3-n}{3}$ is required in forward elimination.

   (c) the number of multiplications/divisions operations $\frac{n^2+n}{2}$ is required in backward substitution.

   (d) the number of additions/subtractions operations $\frac{n^2-n}{2}$ is required in backward substitution.