Indian Institute of Technology Indore
MA 204 Numerical methods
(Spring Semester 2022)
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Tutorial Sheet 2

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1. Solve the system

$$\begin{aligned} 2x_1 \ +x_2 \ +x_3 &= 10 \\ 3x_1+2x_2+3x_3 &= 18 \\ x_1+4x_2+9x_3 &= 16 \end{aligned}$$

by the Gauss-Jordan method.

2. The *symmetric* $n \times n$ Pascal matrix, denoted by pascal(n), is the matrix having $(i,j)^{\text{th}}$ entry as $\binom{i+j}{i}$, where $i, j = 0, 1, 2, 3, \ldots, n-1$ and the notation $\binom{a}{b}$ denotes the binomial coefficient. Note that the counting of elements of this matrix starts from 0 (not from 1); for instance, the element in the first row and first column is at $(0,0)^{\text{th}}$ position, the element in the first row and second column is at $(0,1)^{\text{th}}$ position, and so on. Write down pascal(4), and decompose it into LU-form using the Gauss elimination.

3. (a) What matrix E puts the matrix

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 4 & 2 \\ 6 & 3 & 5 \end{bmatrix}$$

into an upper triangular form $EA = U$?

   (b) Compute E^{-1} without using the determinants and cofactors.

   (c) Verify that E^{-1} is a lower triangular matrix and that $A = LU$.

4. (a) Which number c leads to zero in the second pivot position while factorizing the matrix

$$A = \begin{bmatrix} 1 & c & 0 \\ 2 & 4 & 1 \\ 3 & 5 & 1 \end{bmatrix}$$

into LU form? Note that, for this c, $A = LU$ is not possible; nevertheless, a row exchange is needed to obtain the factorization in the form $PA = LU$, where P is a permutation matrix. Find this P, L and U.

   (b) (Continuing from the previous part) Which c produces zero in the third pivot position? Observe that row exchanges in this case cannot help and the elimination fails.

5. Determine the LU factorization for matrix

$$A = \begin{bmatrix} 1 & 1 & 0 & 3 \\ 2 & 1 & -1 & 1 \\ 3 & -1 & -1 & 2 \\ -1 & 2 & 3 & -1 \end{bmatrix}$$

using Doolittle's method, and use this factorization to solve the system

$$\begin{aligned}
x_1 + x_2 \phantom{{}+3x_3} + 3x_4 &= 8, \\
2x_1 + x_2 - x_3 + x_4 &= 7, \\
3x_1 - x_2 - x_3 + 2x_4 &= 14, \\
-x_1 + 2x_2 + 3x_3 - x_4 &= -7.
\end{aligned}$$

6. When a real symmetric $n \times n$ matrix has one of the following four properties, it has them all:

   (i) All n eigenvalues are positive.

   (ii) All n upper left determinants are positive.

   (iii) All n pivots are positive.

   (iv) $\boldsymbol{x}^\top A\boldsymbol{x} > 0$ for all $\boldsymbol{x} \neq \boldsymbol{0}$, i.e., the matrix A is *positive definite.*

   (a) Verify all the four properties for the matrix

   $$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}.$$

   (b) Use any appropriate property (whichever is the easiest to apply) from the above properties to answer the following. For what values of a, b and c, the matrices

   $$A = \begin{bmatrix} 2 & -1 & a \\ -1 & 2 & -1 \\ a & -1 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & b & 4 \\ 3 & 4 & 5 \end{bmatrix} \quad \text{and} \quad C = \begin{bmatrix} c & 1 & 1 \\ 1 & c & 1 \\ 1 & 1 & c \end{bmatrix}$$

   are positive definite?

7. (a) For each vector $\boldsymbol{x} \in \mathbb{R}^n$, prove that

   $$\|\boldsymbol{x}\|_\infty \leq \|\boldsymbol{x}\|_2 \leq \sqrt{n}\,\|\boldsymbol{x}\|_\infty.$$

   (b) Prove the triangular inequality for the ℓ_∞-norm, i.e., for $\boldsymbol{x}, \boldsymbol{y} \in \mathbb{R}^n$, prove that

   $$\|\boldsymbol{x} + \boldsymbol{y}\|_\infty \leq \|\boldsymbol{x}\|_\infty + \|\boldsymbol{y}\|_\infty.$$