AN HYBRID METAHEURISTIC APPROACH TO THE TRAVELLING SALESMAN PROBLEM

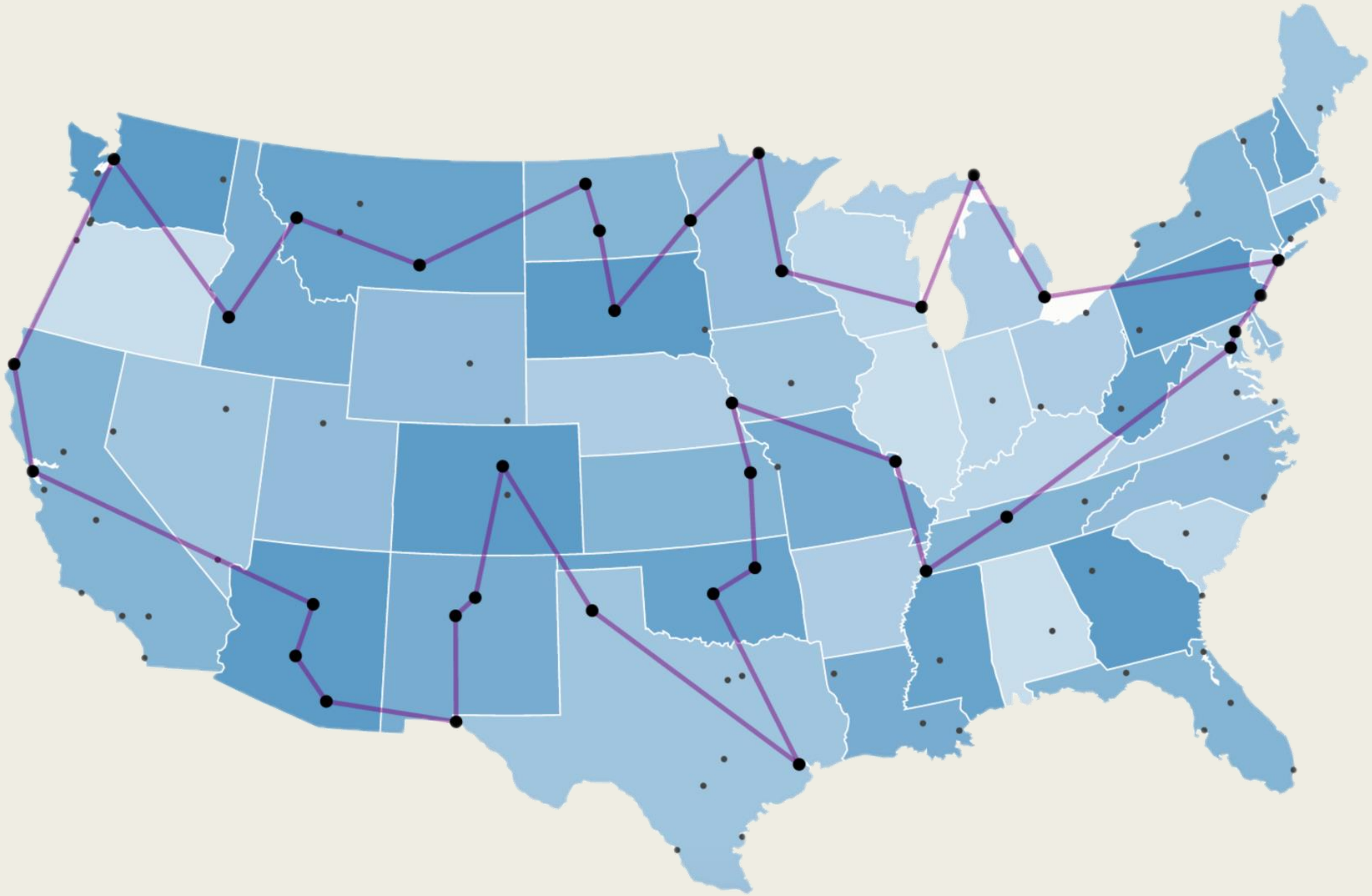
DARIO BERTAZIOLI
FABRIZIO D'INTINOSANTE
MASSIMILIANO PERLETTI

UNIVERSITA' DEGLI STUDI DI MILANO BICOCCA

Decision Models
Final Project

INTRODUCTION

Travelling Salesman Problem

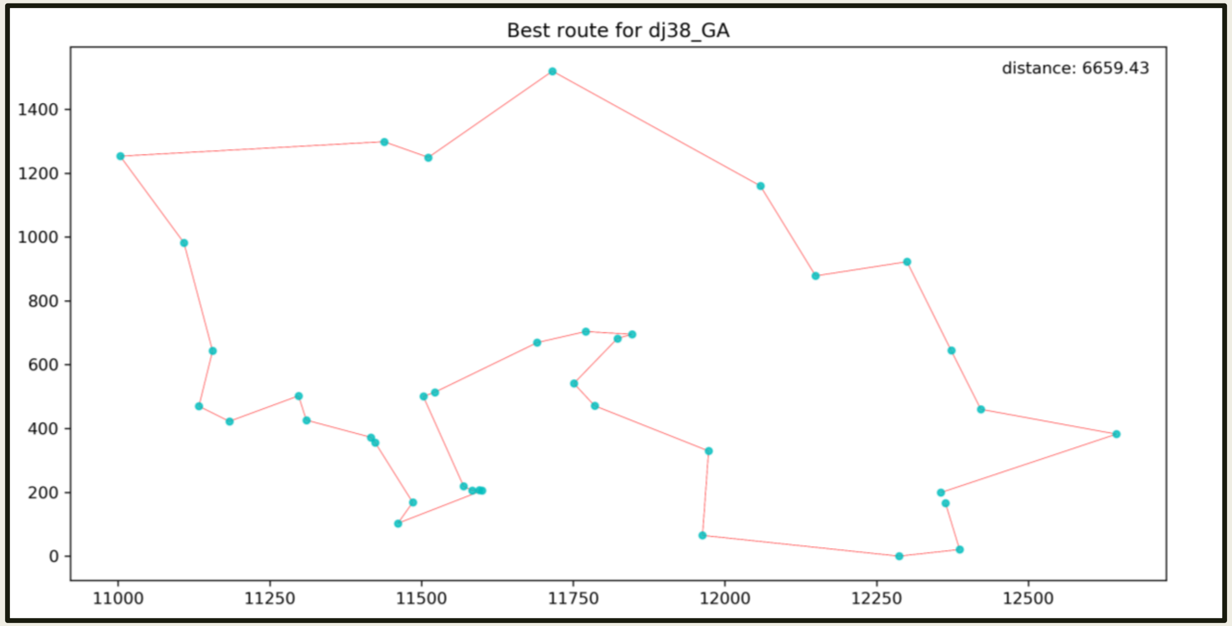


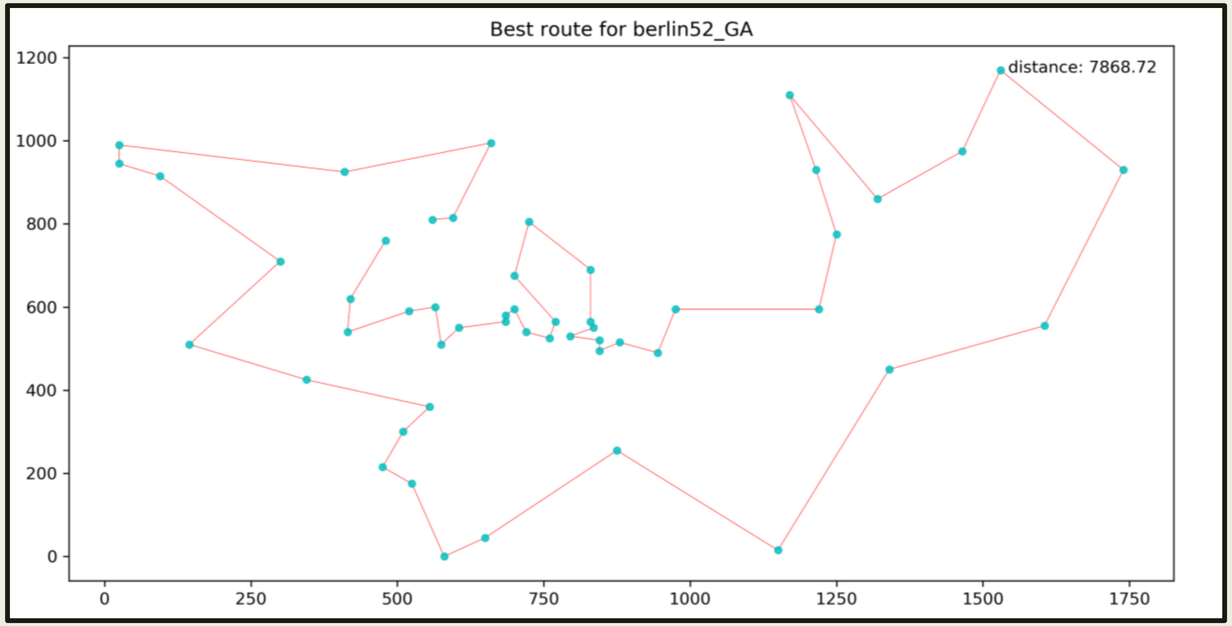
TSP

is an algorithmic problem tasked with finding the shortest route between a set of points and locations that must be visited

DATASETS

- dj38
- berlin52
- ch130
- d198
- pr1002





availables at https://wwwproxy.iwr.uniheidelberg.de/groups/comopt/software/TSPLIB95/tsp/

METHODOLOGICAL APPROACH

ACO – Ant-Q – GA – KGA

ACO FAMILY ALGORITHMS

- inspiration from biology
- accomplish difficult tasks exploiting collaboration
- pheromone as a communication channel
- the more ants follow a specific path,

  the more likely it becomes to be followed
- shortest path problems

Dorigo, Maniezzo, Colorni, Ant System: Optimization by a Colony of Cooperating Agents. In: IEEE Transactions on Systems, Man, and Cybernetics. Vol. 26, No. 1, 1996.

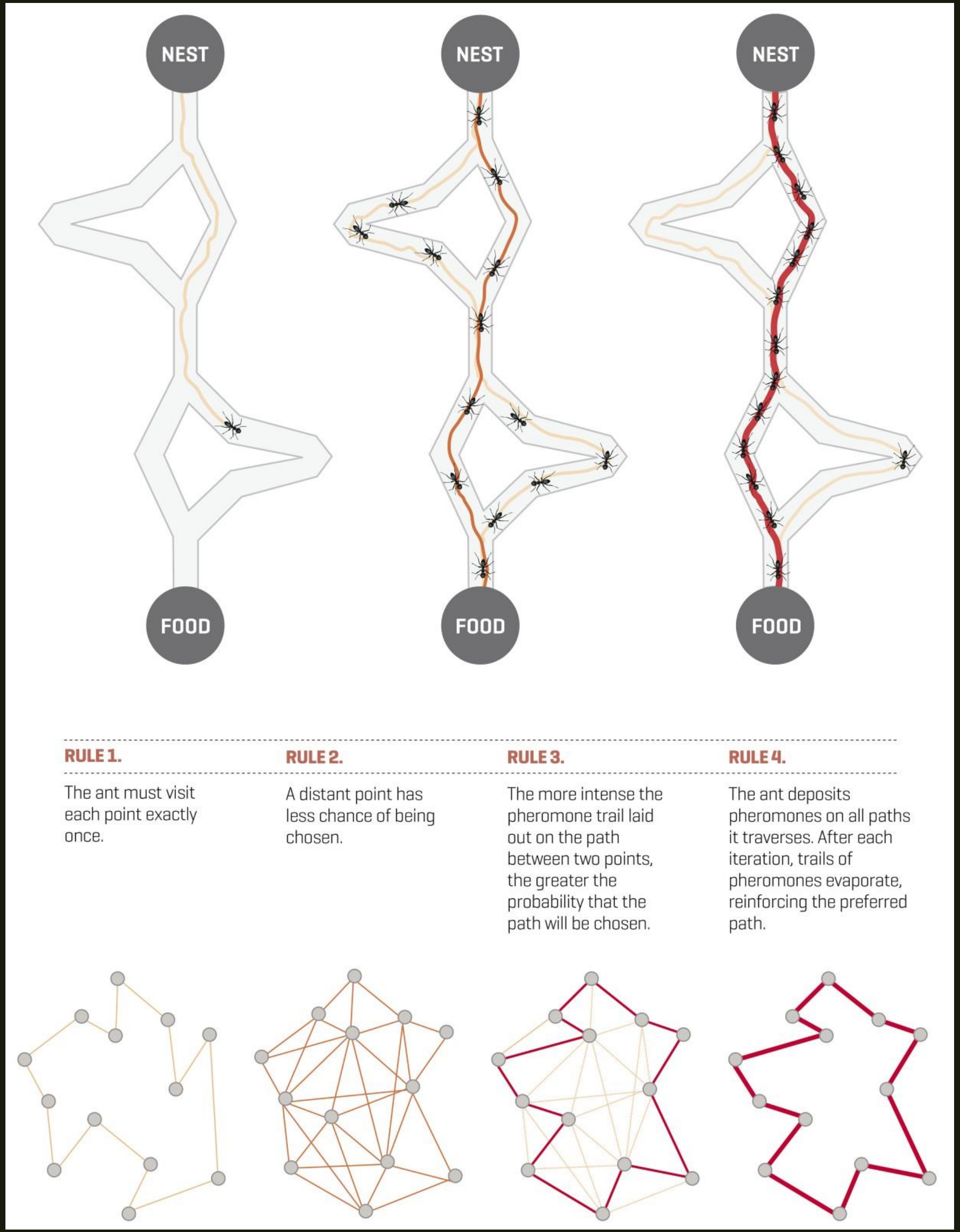
Ant Colony Optimization

- multi-agent collaborative approach
- pheromone evaporation and deposit
  - exploration vs exploitation tradeoff

$$\tau_{ij} = (1-\rho)\tau_{ij} + \Delta\tau_{ij}$$

- artificial ants might have different aims
  - pheromone guided moves vs heuristically driven ones

$$p_{ij}^{k} = \frac{[\tau_{ij}]^{\alpha}[\eta_{ij}]^{\beta}}{\sum_{l \in N_i^k} [\tau_{il}]^{\alpha}[\eta_{il}]^{\beta}} \quad \text{if } \mathrm{j} \in \mathrm{N}_{\mathrm{i}}^{\mathrm{k}}$$



Dorigo, Gambardella, Ant Colony System:
A Cooperative Learning Approach to the Travelling Salesman Problem.
In: IEEE Transactions on Evolutionary Computation, Vol.1 No.1, pp.53-66

Ant-Q
a reinforcement learning approach

main idea

- integrate a Q-learning-like rule in the pheromone update procedure

$$\tau_{ij} = (1-\alpha)\tau_{ij} + \alpha(\Delta\tau_{ij} + \gamma max_{l \in N_j^k} \tau_{jl})$$

- and in the single decision process

$$s = \begin{cases} arg\ max_{a \in J_k(s)} [Q(s,a)]^{\alpha} [\eta(s,a)]^{\beta} & \text{if } q \leq q_0 \\ S & \text{otherwise} \end{cases}$$

Gambardella,Dorigo, Ant-Q: A Reinforcement Learning Approach to the Traveling Salesman Problem In: Proc. ML-95, 12th Int. Conf. Machine Learning. Palo Alto, CA, pp. 252-260

Parallel Implementation Hints

- ACO algorithm are quite memory expensive, but… naturally parallelizable!

- parallel systems *(parent/children)*
  - running on each core
- local pheromone matrices
- global updates every iteration *(parent)*
  - providing each child with an updated mean pheromone matrix
- technically
  - Open MPI (https://www.open-mpi.org)
  - mpi4py (https://mpi4py.readthedocs.io/en/stable)

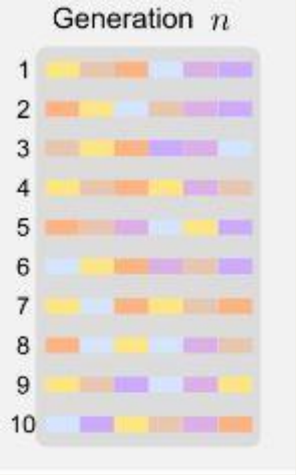
A great context review: M. Pedemonte, S. Nesmachnow, H. Cancela, A survey on parallel ant colony optimization, Applied Soft Computing,Volume 11, Issue 8, 2011, Pages 5181-5197

GENETIC ALGORITHM

- based on a biological metaphor *(Darwinism)*
- individuals as chromosomes *(genotypes)* into a population
- fitness function to evaluate each individual
- selection operator to choose best individuals *(natural selection)*
- crossover operator as gene transfer
- random mutation at each generation

Genetic Algorithm

generate random population
(chromosomes) - Tp



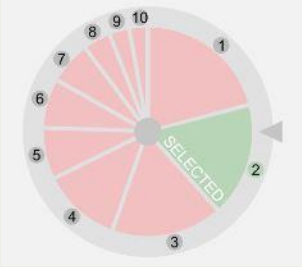
START

evaluation *(rank pop)*

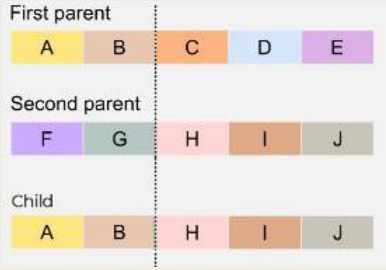
selection mechanism and elitism – *elite_n*

tournament selection

roulette-wheel selection



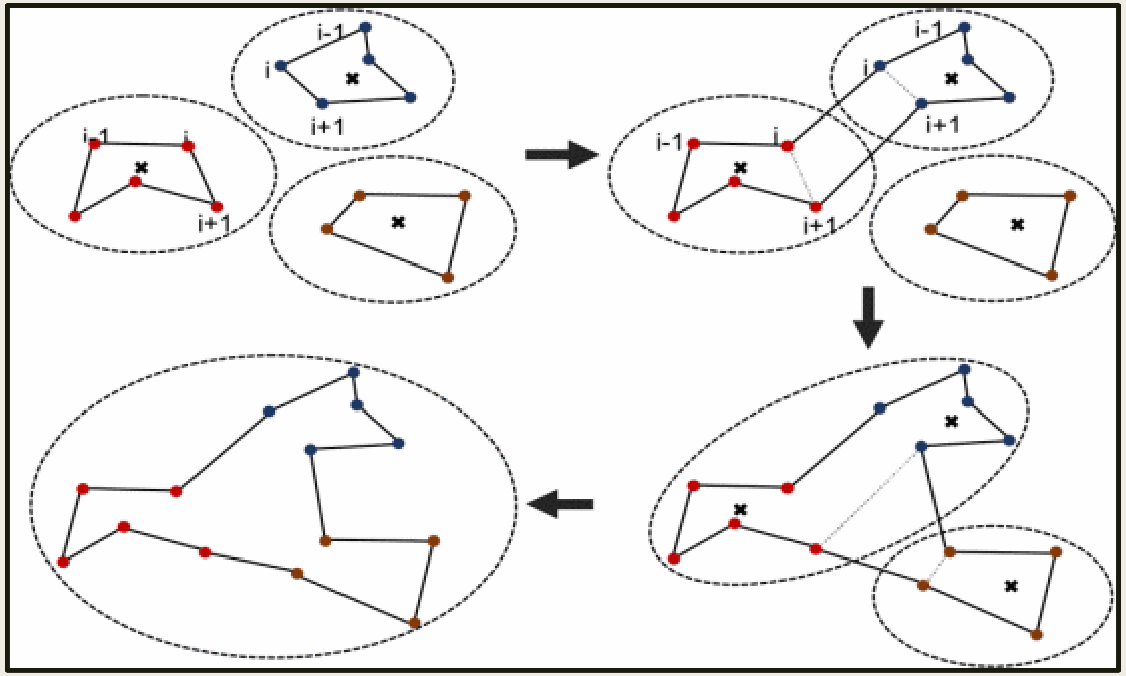
crossover *(single-point)*



mutation - *Tm*

K-means Genetic Algorithm

- hybrid implementation of GA
- transform wide problem into smaller sub-problems (clusters)



THREE STEPS

- create cluster *(k-means clustering algortihms)*
- solve sub-problems with GA *(intra-group evolution operation)*
- reconstruct optimal solution from sub-problems solutions *(inter-group connection)*

N.B. to achieve better performance use GA to order centroids

Test Parameters GA – KGA

G A

5 run

Tp = 300 (pr1002 Tp = 125)

elite_n = 30 (10%) (pr1002 elite_n = 25 (20%))

tournament selection

5k generations

K G A

5 run

Tp = 125

elite_n = 25 (20%)

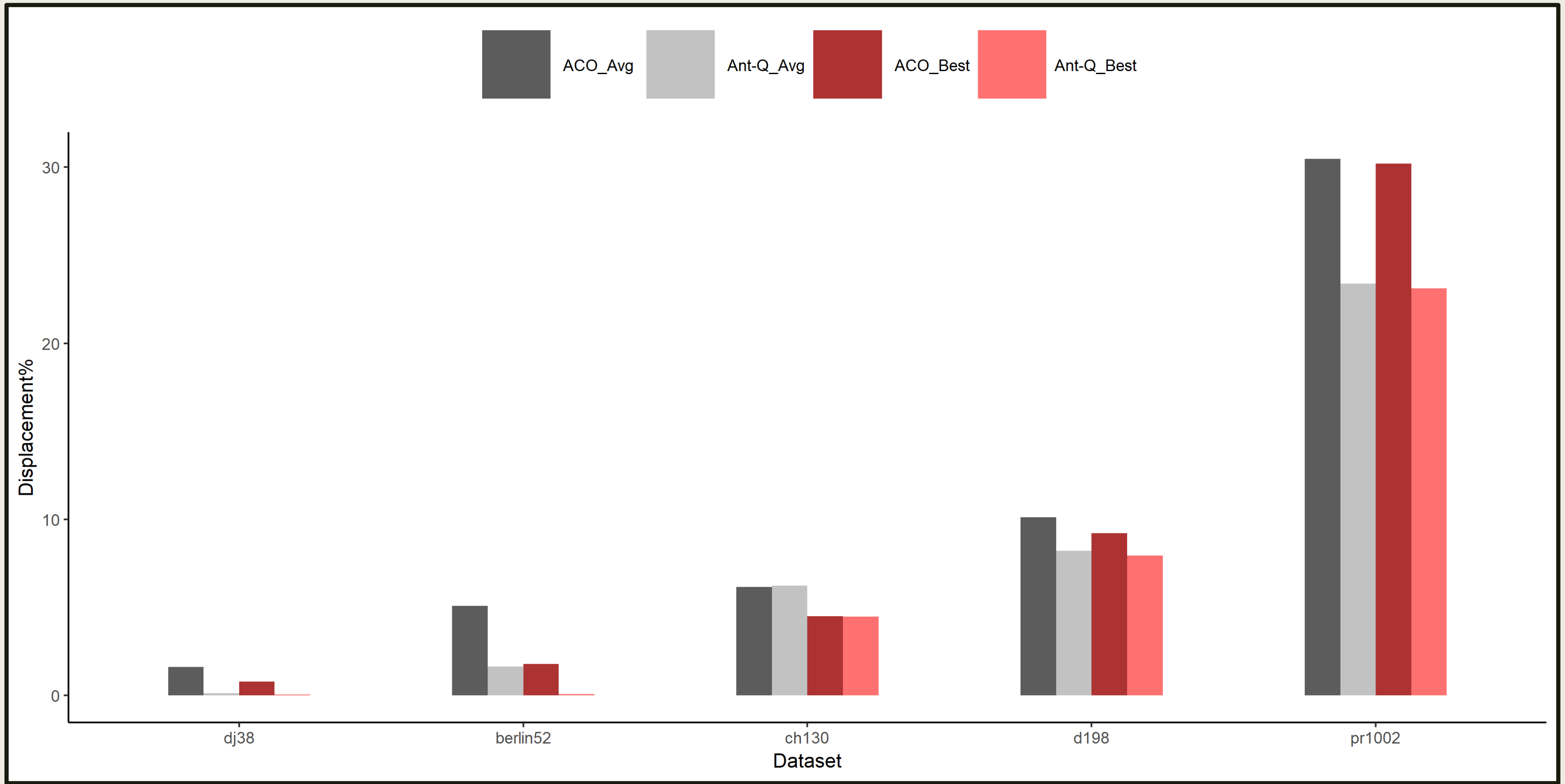
roulette-wheel selection

1k generations (intra-group) + 1k generations (to order centroids)

K cluster = 2, 3, 8, 30, 60 k (initial dataset order, slide 5)

RESULTS

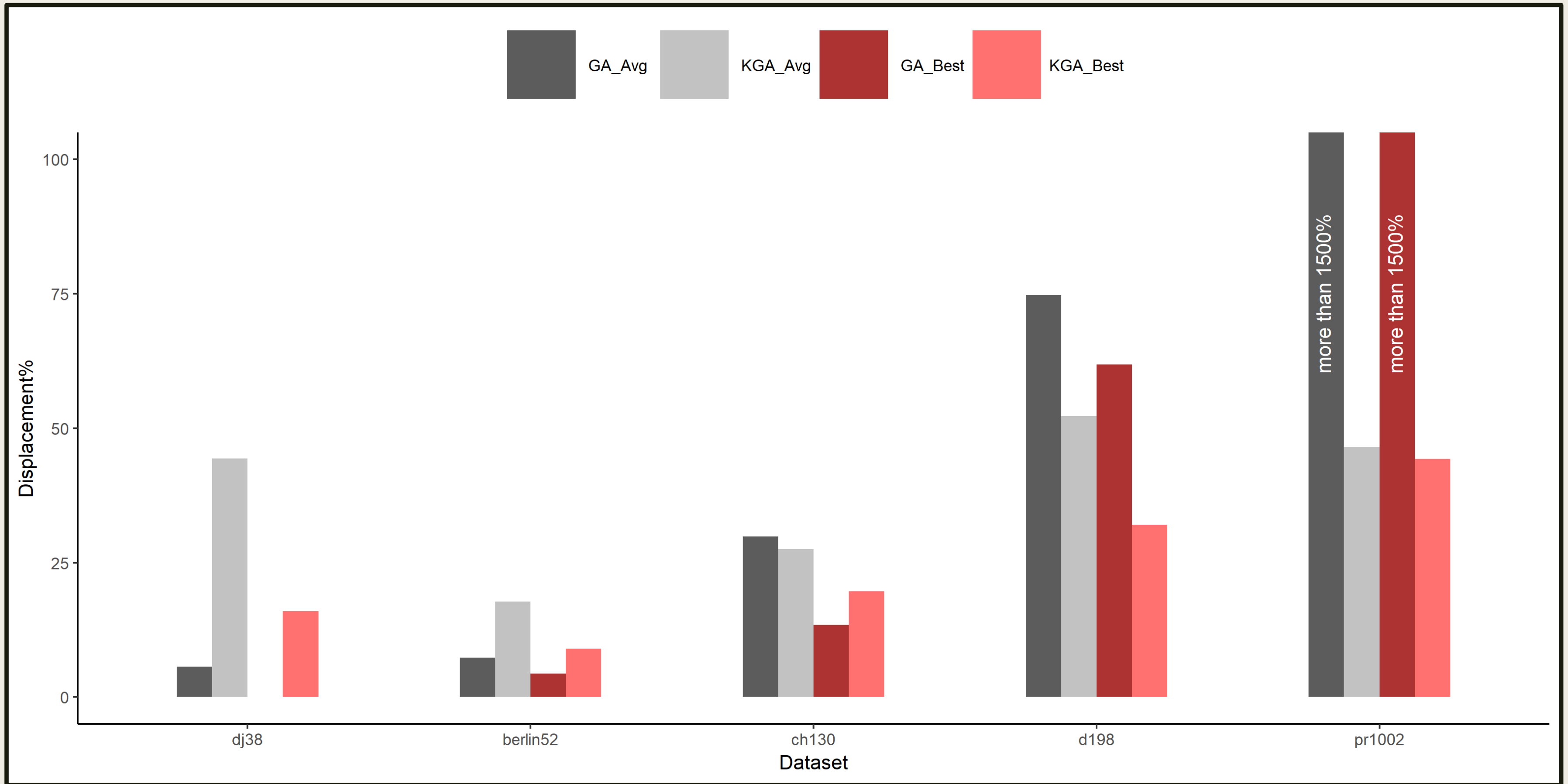
ACO – ANT-Q



Dataset	ACO						
	OptimalTour	Mean	SD	Best	% Avg Dist	% Best Dist	Avg time (s)
dj38 (1000 it)	6656	6763.03	75.16	6708.04	1.61	0.78	75
berlin52 (1000 it)	7542	7925.39	162.00	7677.66	5.08	1.80	320
ch130 (500 it)	6110	6487.19	74.20	6385.46	6.17	4.51	9240
d198 (100 it)	15780	17376.23	105.81	17235.44	10.12	9.22	6540
pr1002 (10 it)	259054	337976.61	833.96	337309.96	30.47	30.21	6180

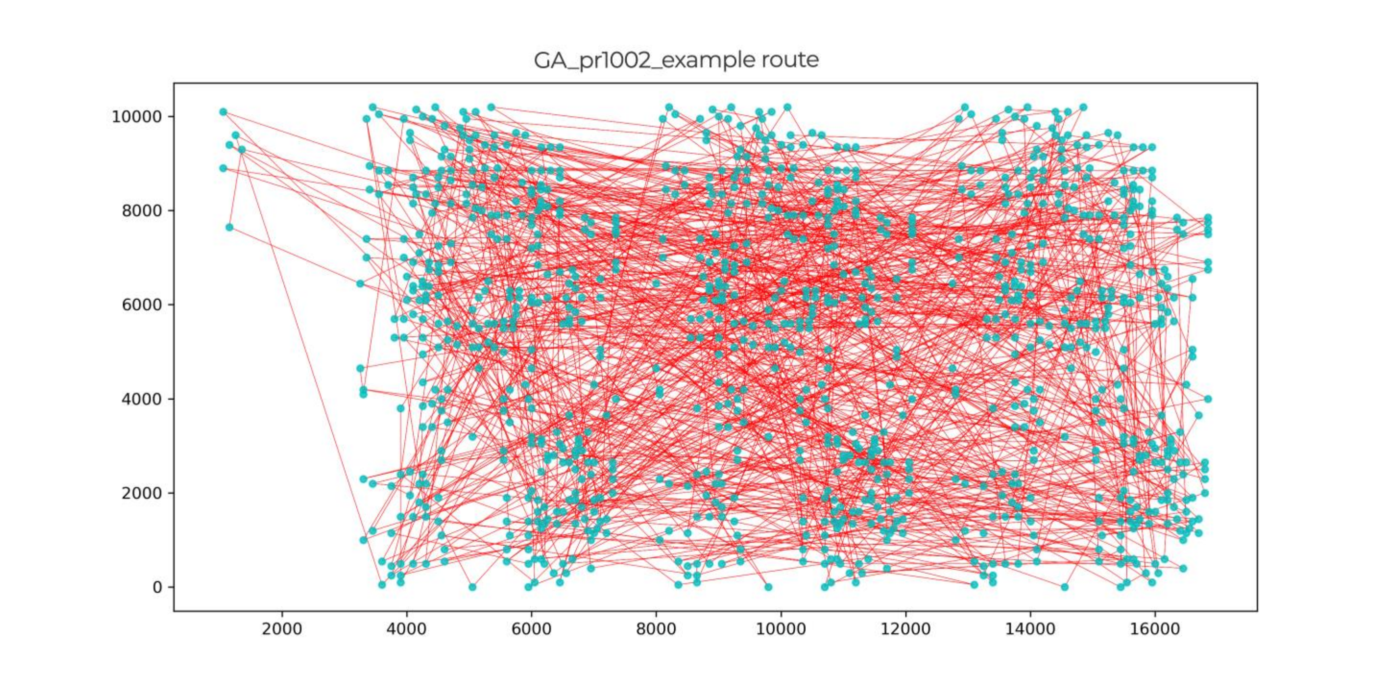
Dataset	Ant-Q						
	OptimalTour	Mean	SD	Best	% Avg Dist	% Best Dist	Avg time (s)
dj38 (1000 it)	6656	6663.99	4.16	6659.43	0.12	0.05	37
berlin52 (1000 it)	7542	7666.17	113.87	7548.99	1.65	0.09	170
ch130 (500 it)	6110	6491.31	70.01	6383.42	6.24	4.47	8400
d198 (100 it)	15780	17075.81	39.69	17032.75	8.21	7.94	2880
pr1002 (10 it)	259054	319646.49	1009.39	318960.00	23.39	23.12	6120

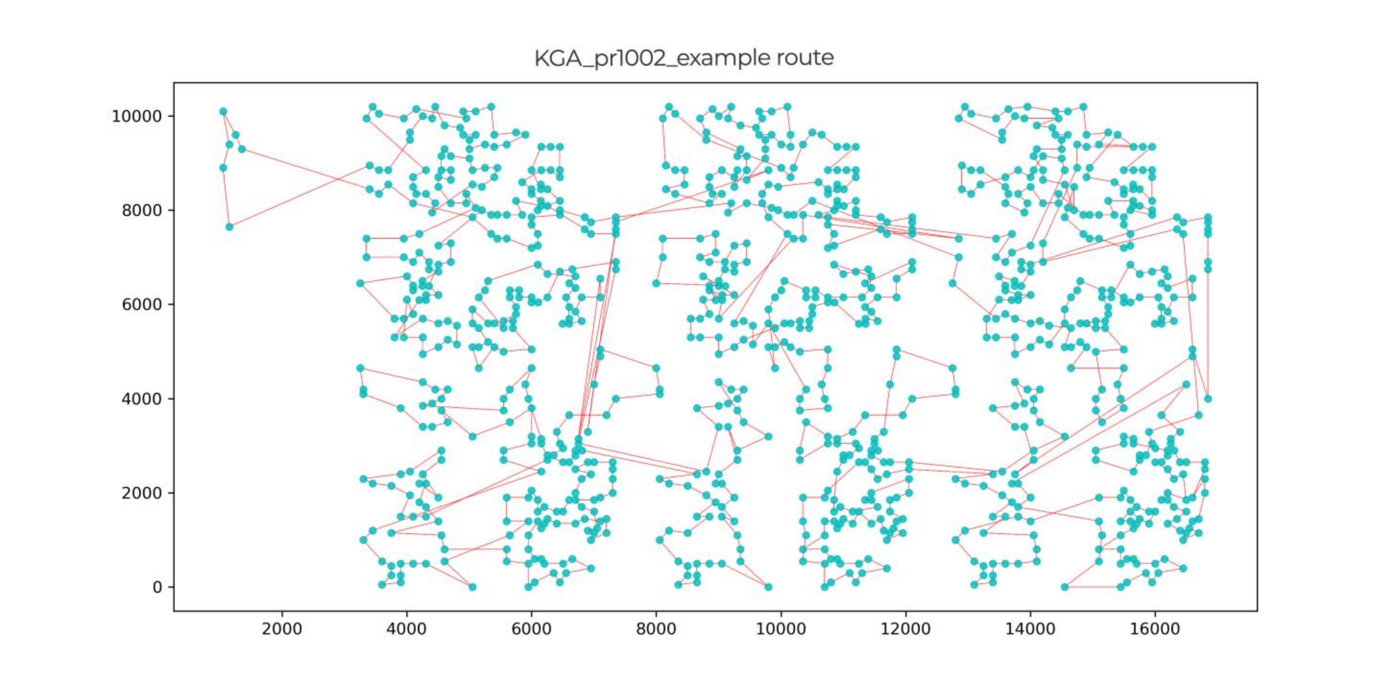
GA – KGA



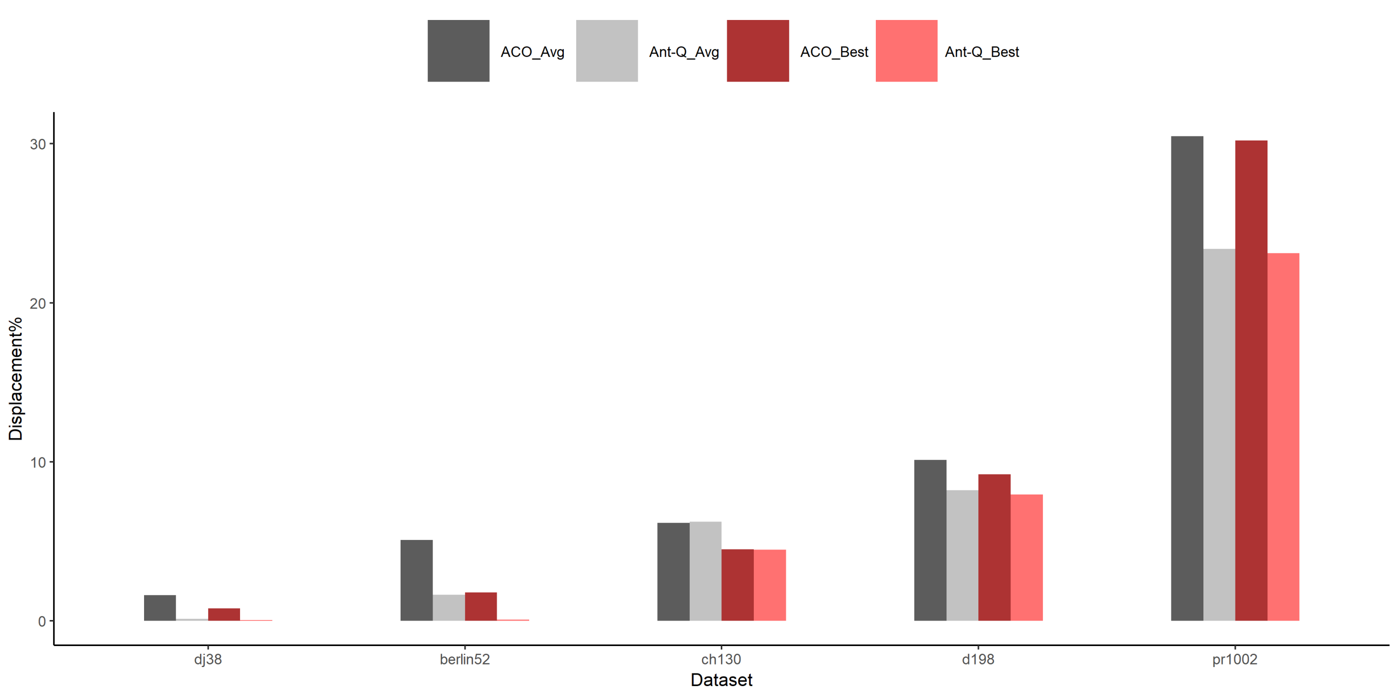
Dataset	GA						
	OptimalTour	Mean	SD	Best	% Avg Dist	% Best Dist	Avg time (s)
dj38	6656	7032.91	574.34	6659.43	5.66	0.05	263
berlin52	7542	8095.35	167.27	7868.72	7.34	4.33	286
ch130	6110	7933.02	1041.27	6929.81	29.84	13.42	435
d198	15780	27579.73	1869.54	25537.85	74.78	61.84	604
pr1002	259045	4666413.09	12608.57	4652556.08	1701.39	1696.04	2641

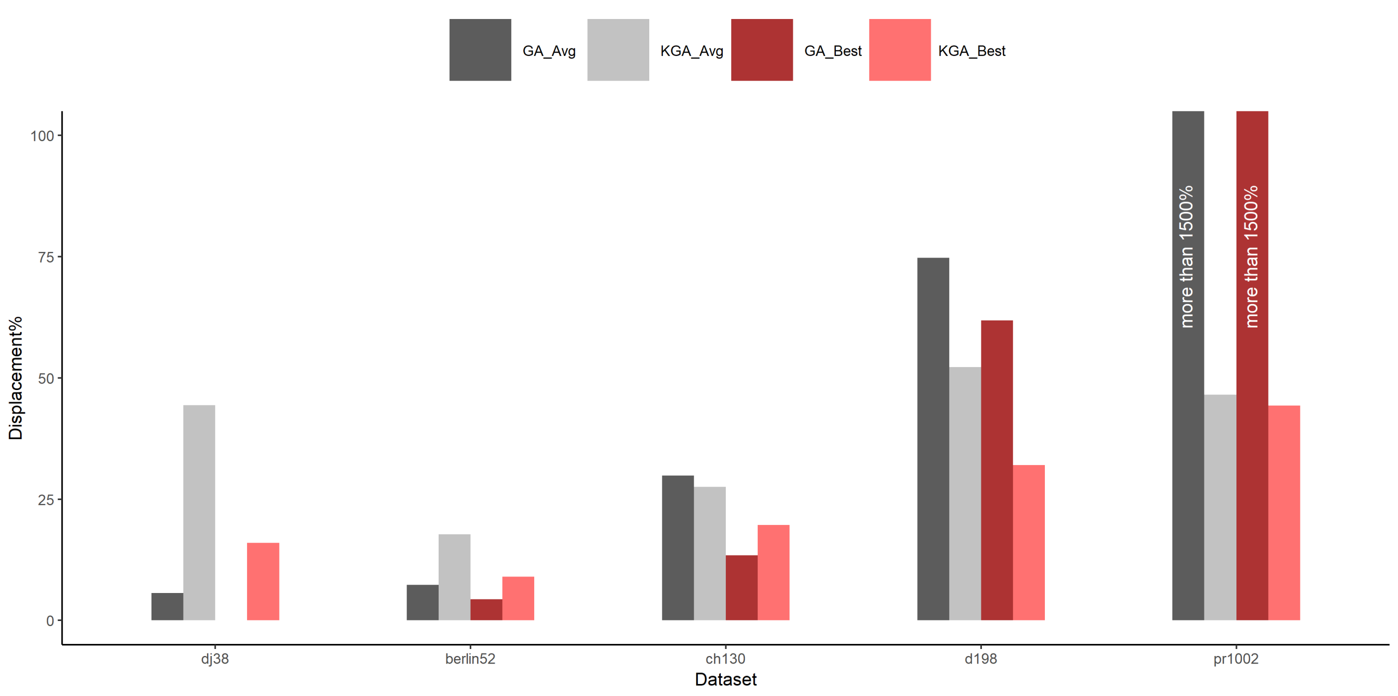
Dataset	KGA						
	OptimalTour	Mean	SD	Best	% Avg Dist	% Best Dist	Avg time (s)
dj38	6656	9608.77	1817.02	7720.03	44.36	15.99	140
berlin52	7542	8879.59	446.69	8218.95	17.74	8.98	188
ch130	6110	7790.85	312.17	7314.08	27.51	19.71	409
d198	15780	24023.86	3066.94	20830.94	52.24	32.01	1413
pr1002	259045	379495.87	6343.57	373865.26	46.50	44.32	4456





Wide tours comparison (GA – KGA)





Compare performances
(ACO – Ant-Q – GA – KGA)

CONCLUSIONS

KEY POINTS

EFFECTIVE IMPROVEMENT OF THE HYBRIDIZED APPROACH WITH RESPECT TO THE CLASSIC VERSION OF EACH ALGORITHM

ANT FAMILY – BEST PERFORMANCES ON SMALL/MEDIUM SIZED PROBLEMS

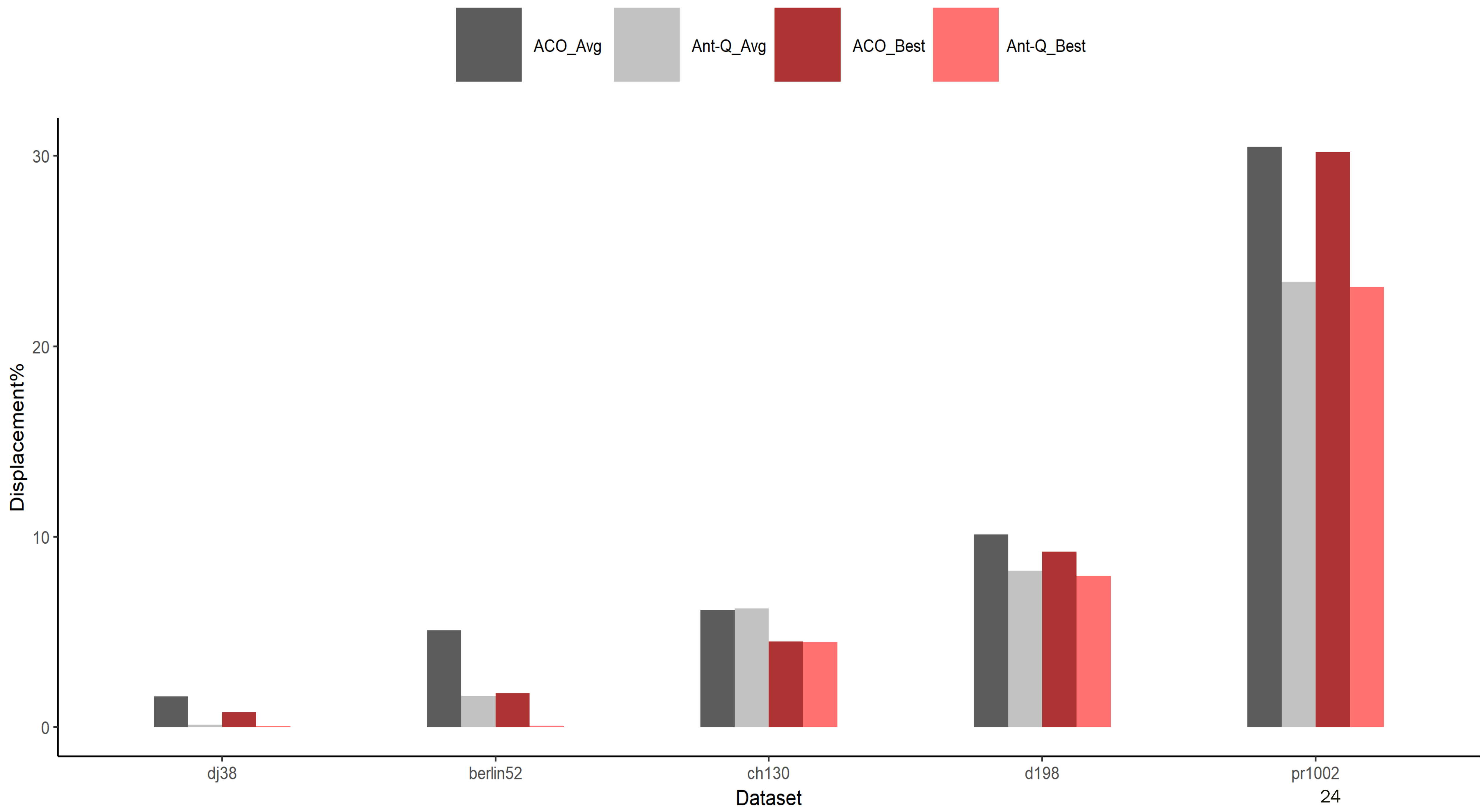
KGA – MORE PERFORMANT ON LARGE DATASETS

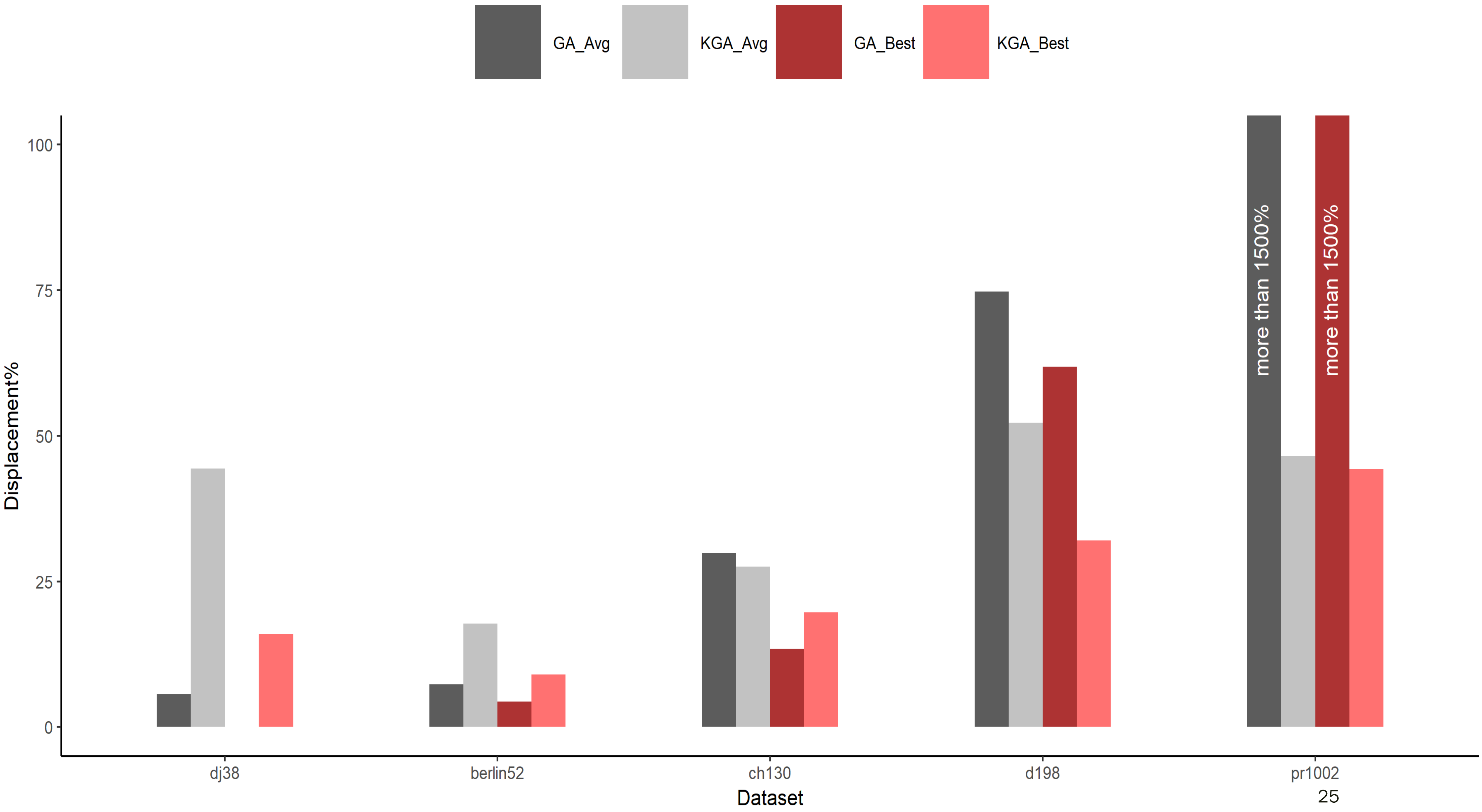
ACO AND ANT-Q SLOWER ON HIGHER DIMENSIONS BUT FIND BETTER SOLUTIONS THAN EA

FUTURE WORK

it could be interesting to further combine Ant-Q and KGA

THANK YOU


ACO_Avg
Ant-Q_Avg
ACO_Best
Ant-Q_Best
Displacement%
0
10
20
30
dj38
berlin52
ch130
d198
pr1002
Dataset


GA_Avg
KGA_Avg
GA_Best
KGA_Best
Displacement%
100
75
50
25
0
more than 1500%
more than 1500%
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Algorithm Ant Colony Optimization

Main Algorithm

initialize best_dist and best_path to None
for generation **in** generations:
  create n_ants artificial ants
  for one_ant **in** ants:
    make a single ant path (see Make path)
    compute the path length
    update best_dist and best_path
  update the pheromone matrix
  (local update only for child processes, according to Eq. (10))
  every a certain n of iterations:
    update the global pheromone matrix
    (shared in MPI environment among master&child.)
return best_dist, best_sol

Make path

start from a vertex
add start vertex to visited nodes
for each remaining vertex:
  list the neighbours
  list the not yet visited neighb
  calculate the probability of choosing a vertex (according to Eq. (7))
  choose the vertex according to probability
  add the chosen vertex to the visited list
  return the chosen vertex id

Local update pheromone matrix

for ant **in** ant_colony :
  for each vertex of one_ant_path :
    increase pheromone_matrix between current and next vertex of $\Delta\tau$
    (according to Eq. (10))

Global update pheromone matrix (parallelism)

gather from MPI env all the pheromone matrices
if process is the parent process (rank==0):
  for each element average over the n_cores matrices.
broadcast obtained pheromone matrix to the other processes

Algorithm Ant-Q algorithm

Main Algorithm

initialize best_dist and best_path to None
for generation **in** generations:
  create n_ants artificial ants
  for each ant:
    make a single ant path (see Make path)
    compute the path length
    update best_dist and best_path
    update pheromone matrix with delayed rewards
    (according to Eq. (14))
  update the global pheromone matrix
  (shared in MPI environment among master&child.)
return best_dist, best_sol

Make path

start from a vertex
add start vertex to visited nodes
for each remaining vertex:
  list the neighbours
  list the not yet visited neighb
  generate a random number $q \in \{0, 1\}$
  if $q < q_0$: (threshold)
    select next vertex according to Eq. (12)
  else:
    calculate the probability of choosing a vertex
    (according to Eq. (7))
    choose the vertex according to probability
  add the chosen vertex to the visited list
  give local rewards (local update pheromone matrix)
  return the chosen vertex id

Local update pheromone matrix

for ant **in** ant_colony :
  for each vertex of one_ant_path :
    increase pheromone_matrix between current
    and next vertex of a $\Delta\tau$ (according to Eq. (11))

Global update pheromone matrix (parallelism)

gather from MPI env all the pheromone matrices
if process is the parent process (rank==0):
  for each element average over the n_cores matrices.
broadcast obtained pheromone matrix to the other processes

Algorithm Genetic Algorithm

```
procedure Genetic(Tm, Tp, elite_n, Selection, MaxGen)
  Pop ← GeneratePopulation(Tp)
  Pop ← Evaluation(Pop)
  for i = 1 ... MaxGen do
    Pop ← Selection(elite_n from Pop)
    Pop ← Crossover(Pop)
    With probability Tm do:
    Pop ← Mutation(Pop)
  end for
    return the best solution in Pop
end procedure
```

Algorithm K-Means Algorithm

```
Set the K cluster centers randomly;
repeat
  for each vertex do
    Calculate distance measure to each cluster;
    Assign it to the closest cluster;
  end
  recompute the cluster centers positions;
until stop criteria are met;
```

Algorithm KGA

```
input an TSP;
K-Means is adopted to cluster the TSP into k sub-problems
For each sub-prob i = 1 to k, do:
  repeat
    GA procedure
  until stop criteria are met
  Output shortest path for sub-problem i;
End
Seek for the best combining seq S with GA
Combine all those shortest path into one tour
Output the shortest whole travelling tour.
```