# AN HYBRID METAHEURISTIC APPROACH TO THE TRAVELLING SALESMAN PROBLEM


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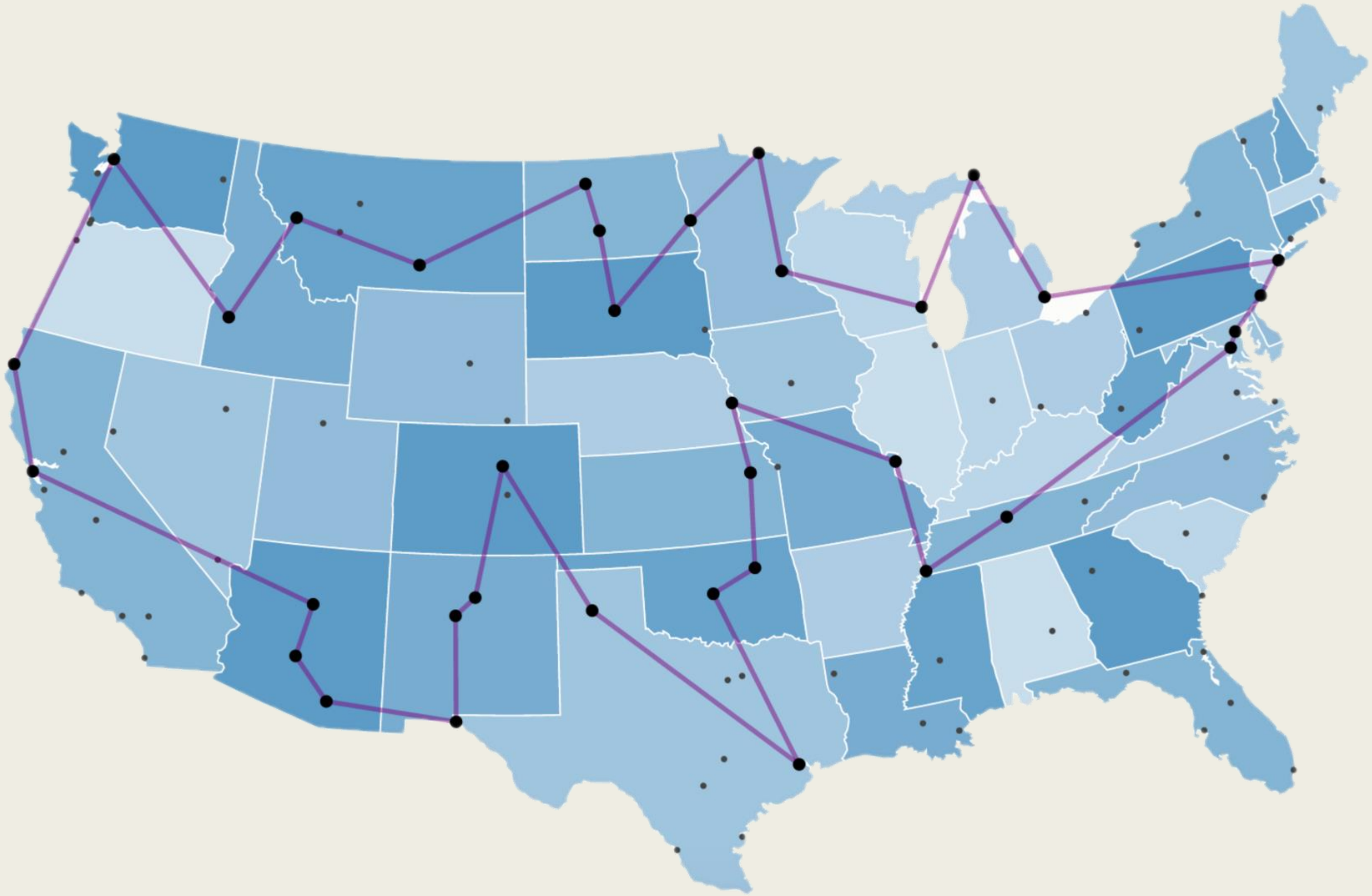
**Decision Models**
Final Project

# INTRODUCTION

Travelling Salesman Problem

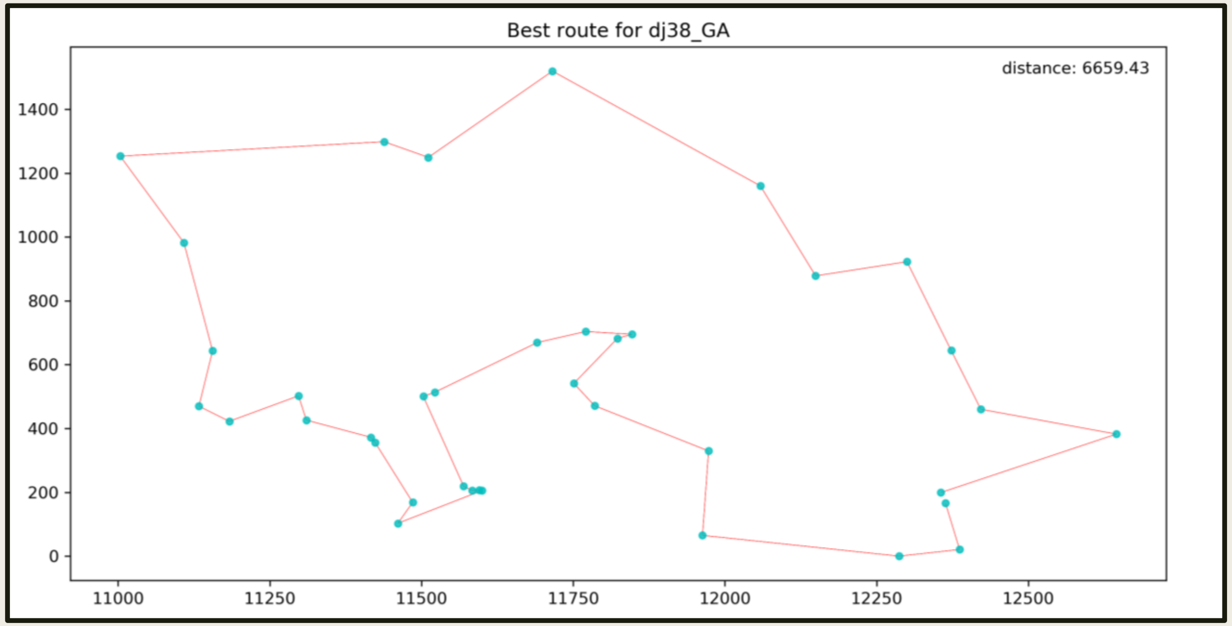


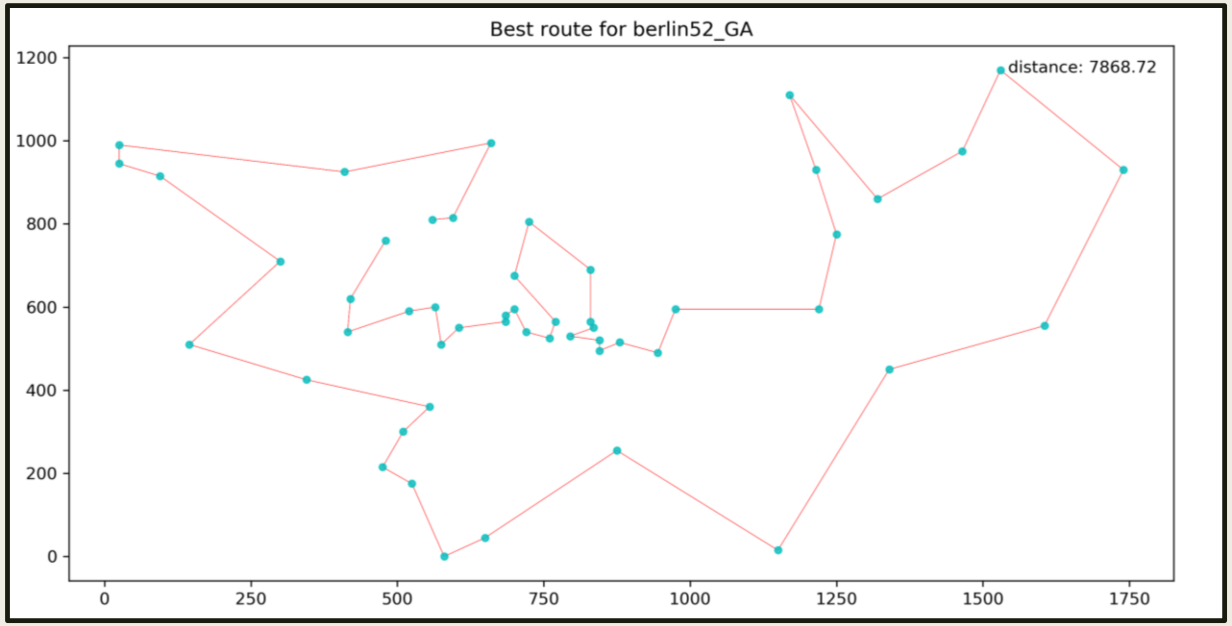
# TSP

is an algorithmic problem tasked with finding the shortest route between a set of points and locations that must be visited

# DATASETS

- dj38
- berlin52
- ch130
- d198
- pr1002




availables at https://wwwproxy.iwr.uniheidelberg.de/groups/comopt/software/TSPLIB95/tsp/

# METHODOLOGICAL APPROACH

ACO – Ant-Q – GA – KGA

# ACO FAMILY ALGORITHMS

- inspiration from biology
- accomplish difficult tasks exploiting collaboration
- pheromone as a communication channel
- the more ants follow a specific path,

  the more likely it becomes to be followed
- shortest path problems

Dorigo, Maniezzo, Colorni, Ant System: Optimization by a Colony of Cooperating Agents. In: IEEE Transactions on Systems, Man, and Cybernetics. Vol. 26, No. 1, 1996.

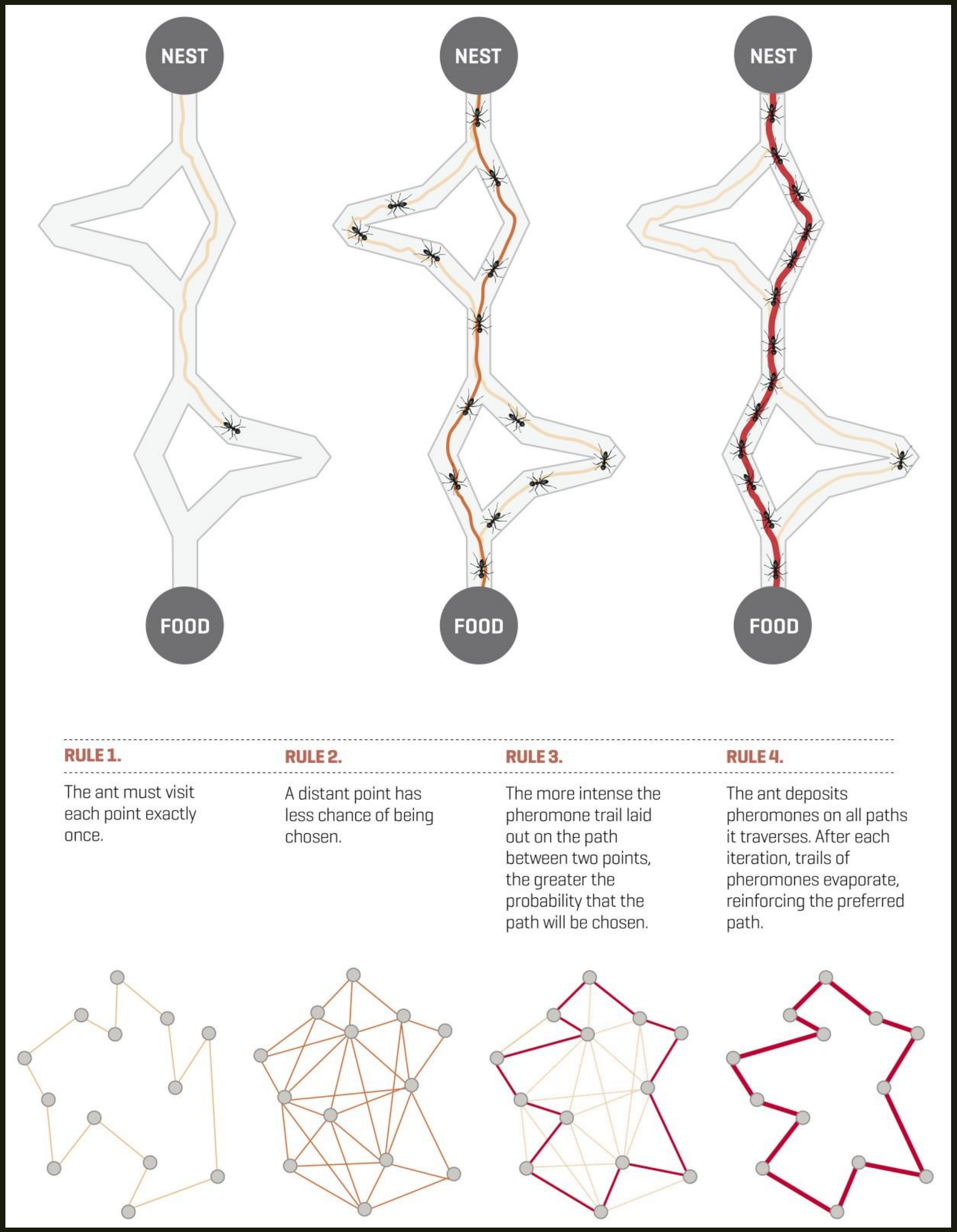
# Ant Colony Optimization

- multi-agent collaborative approach
- pheromone evaporation and deposit
  - exploration vs exploitation tradeoff

$$\tau_{ij} = (1-\rho)\tau_{ij} + \Delta\tau_{ij}$$

- artificial ants might have different aims
  - pheromone guided moves vs heuristically driven ones

$$p_{ij}^{k} = \frac{[\tau_{ij}]^{\alpha}[\eta_{ij}]^{\beta}}{\sum_{l \in N_i^k} [\tau_{il}]^{\alpha}[\eta_{il}]^{\beta}} \quad \text{if } \mathrm{j} \in \mathrm{N_i^k}$$




Dorigo, Gambardella, Ant Colony System:
A Cooperative Learning Approach to the Travelling Salesman Problem.
In: IEEE Transactions on Evolutionary Computation, Vol.1 No.1, pp.53-66

# Ant-Q

## a reinforcement learning approach

### main idea

- integrate a Q-learning-like rule in the pheromone update procedure

$$\boxed{\tau_{ij} = (1-\alpha)\tau_{ij} + \alpha(\Delta\tau_{ij} + \gamma max_{l \in N_j^k} \tau_{jl})}$$

- and in the single decision process

$$s = \begin{cases} arg\ max_{a \in J_k(s)} [Q(s,a)]^{\alpha} [\eta(s,a)]^{\beta} & \text{if } q \leq q_0 \\ S & \text{otherwise} \end{cases}$$

Gambardella,Dorigo, Ant-Q: A Reinforcement Learning Approach to the Traveling Salesman Problem In: Proc. ML-95, 12th Int. Conf. Machine Learning. Palo Alto, CA, pp. 252-260

# Parallel Implementation Hints

- ACO algorithm are quite memory expensive, but… naturally parallelizable!

- parallel systems *(parent/children)*
  - running on each core
- local pheromone matrices
- global updates every iteration *(parent)*
  - providing each child with an updated mean pheromone matrix
- technically
  - Open MPI (https://www.open-mpi.org)
  - mpi4py (https://mpi4py.readthedocs.io/en/stable)

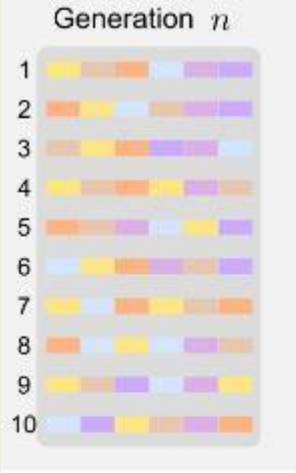
A great context review: M. Pedemonte, S. Nesmachnow, H. Cancela, A survey on parallel ant colony optimization, Applied Soft Computing,Volume 11, Issue 8, 2011, Pages 5181-5197

# GENETIC ALGORITHM

- based on a biological metaphor *(Darwinism)*
- individuals as chromosomes *(genotypes)* into a population
- fitness function to evaluate each individual
- selection operator to choose best individuals *(natural selection)*
- crossover operator as gene transfer
- random mutation at each generation

# Genetic Algorithm

Generate random population
*(chromosomes)*



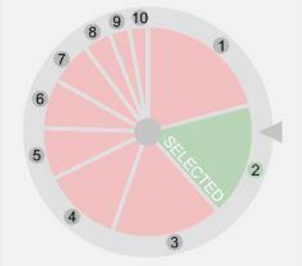

START

Evaluation *(rank pop)*


selection mechanism and elitism

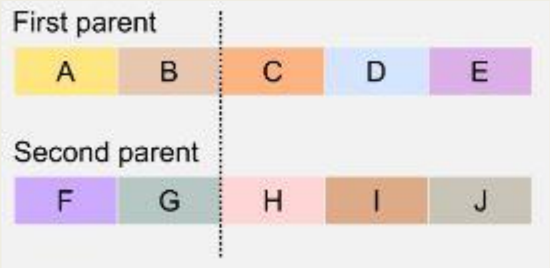
tournament selection

roulette-wheel selection

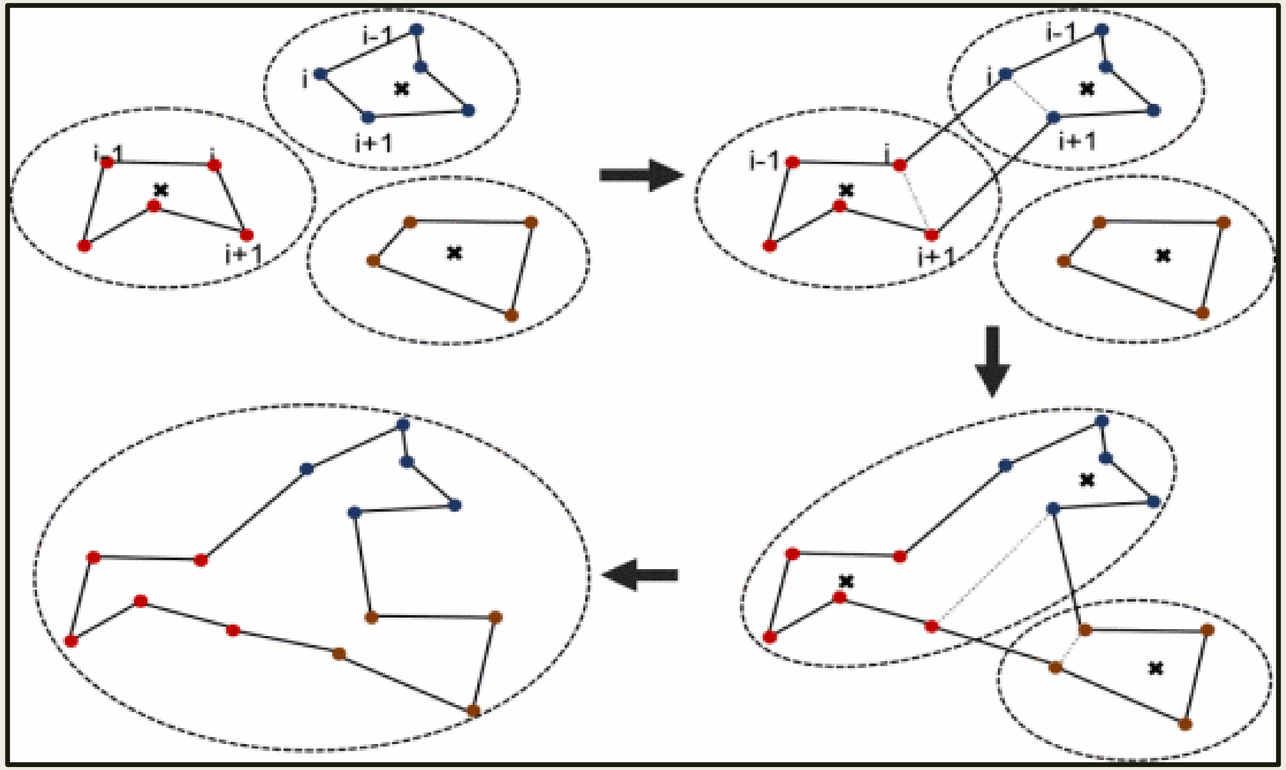



mutation


crossover *(single-point)*



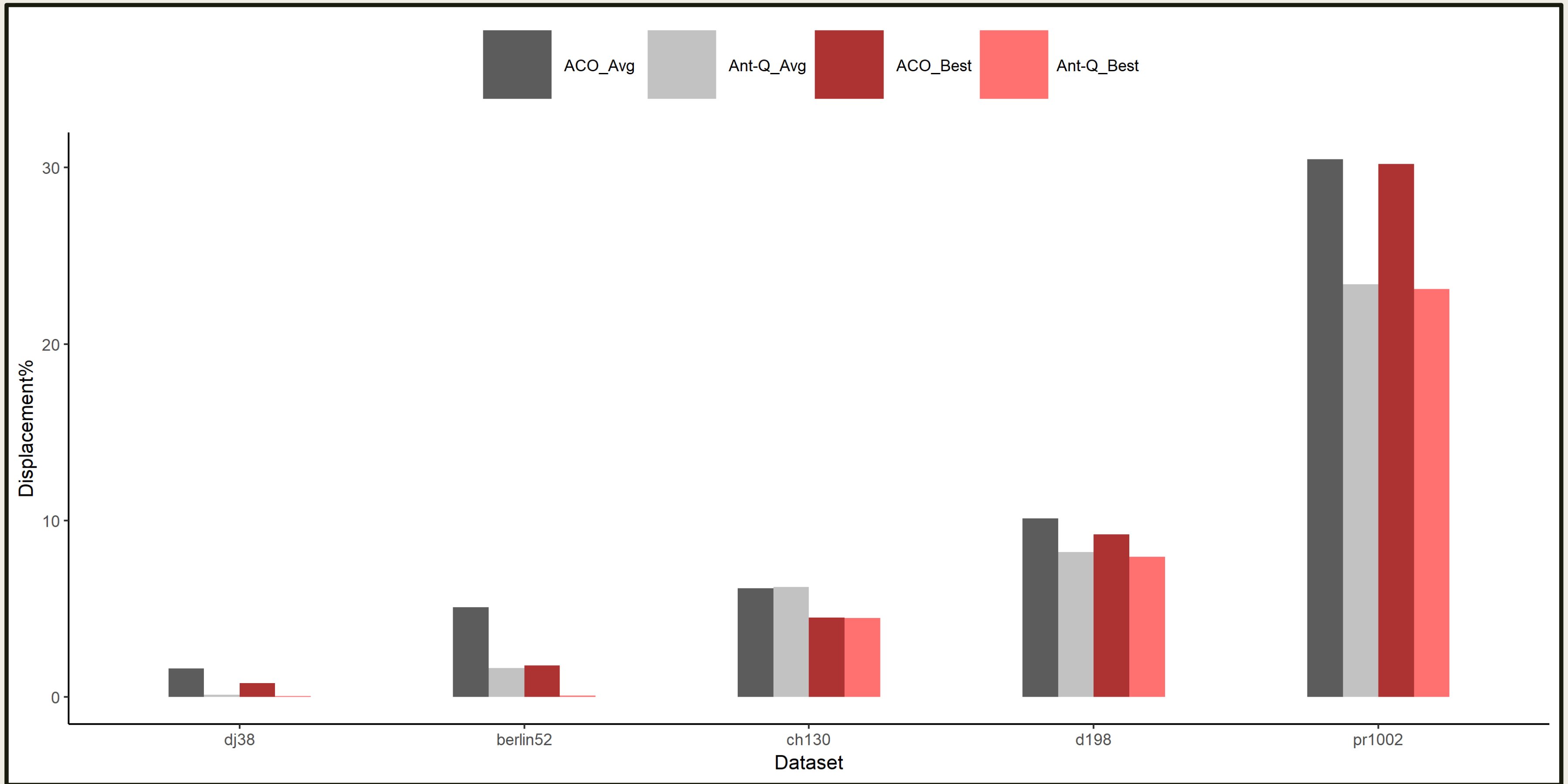
# K-means Genetic Algorithm




- hybrid implementation of GA
- transform wide problem into smaller sub-problems *(clusters)*
- solve sub-problems with GA
- reconstruct optimal solution from sub-problems solutions

# RESULTS

# ACO – ANT-Q




| Dataset | ACO | | | | | | |
|---|---|---|---|---|---|---|---|
| | OptimalTour | Mean | SD | Best | % Avg Dist | % Best Dist | Avg time (s) |
| dj38 (1000 it) | 6656 | 6763.03 | 75.16 | 6708.04 | 1.61 | 0.78 | 75 |
| berlin52 (1000 it) | 7542 | 7925.39 | 162.00 | 7677.66 | 5.08 | 1.80 | 320 |
| ch130 (500 it) | 6110 | 6487.19 | 74.20 | 6385.46 | 6.17 | 4.51 | 9240 |
| d198 (100 it) | 15780 | 17376.23 | 105.81 | 17235.44 | 10.12 | 9.22 | 6540 |
| pr1002 (10 it) | 259054 | 337976.61 | 833.96 | 337309.96 | 30.47 | 30.21 | 6180 |

| Dataset | Ant-Q | | | | | | |
|---|---|---|---|---|---|---|---|
| | OptimalTour | Mean | SD | Best | % Avg Dist | % Best Dist | Avg time (s) |
| dj38 (1000 it) | 6656 | 6663.99 | 4.16 | 6659.43 | 0.12 | 0.05 | 37 |
| berlin52 (1000 it) | 7542 | 7666.17 | 113.87 | 7548.99 | 1.65 | 0.09 | 170 |
| ch130 (500 it) | 6110 | 6491.31 | 70.01 | 6383.42 | 6.24 | 4.47 | 8400 |
| d198 (100 it) | 15780 | 17075.81 | 39.69 | 17032.75 | 8.21 | 7.94 | 2880 |
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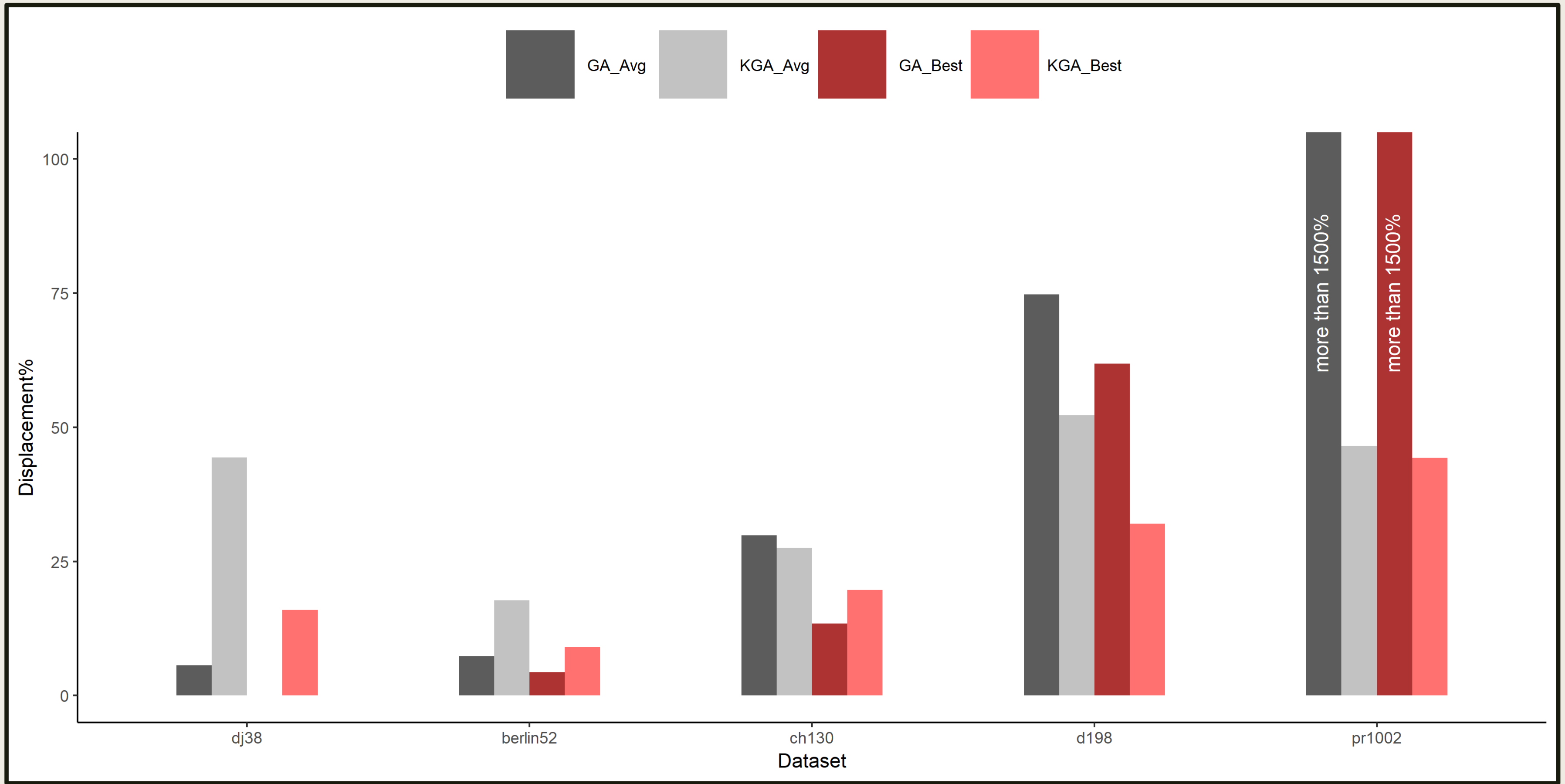
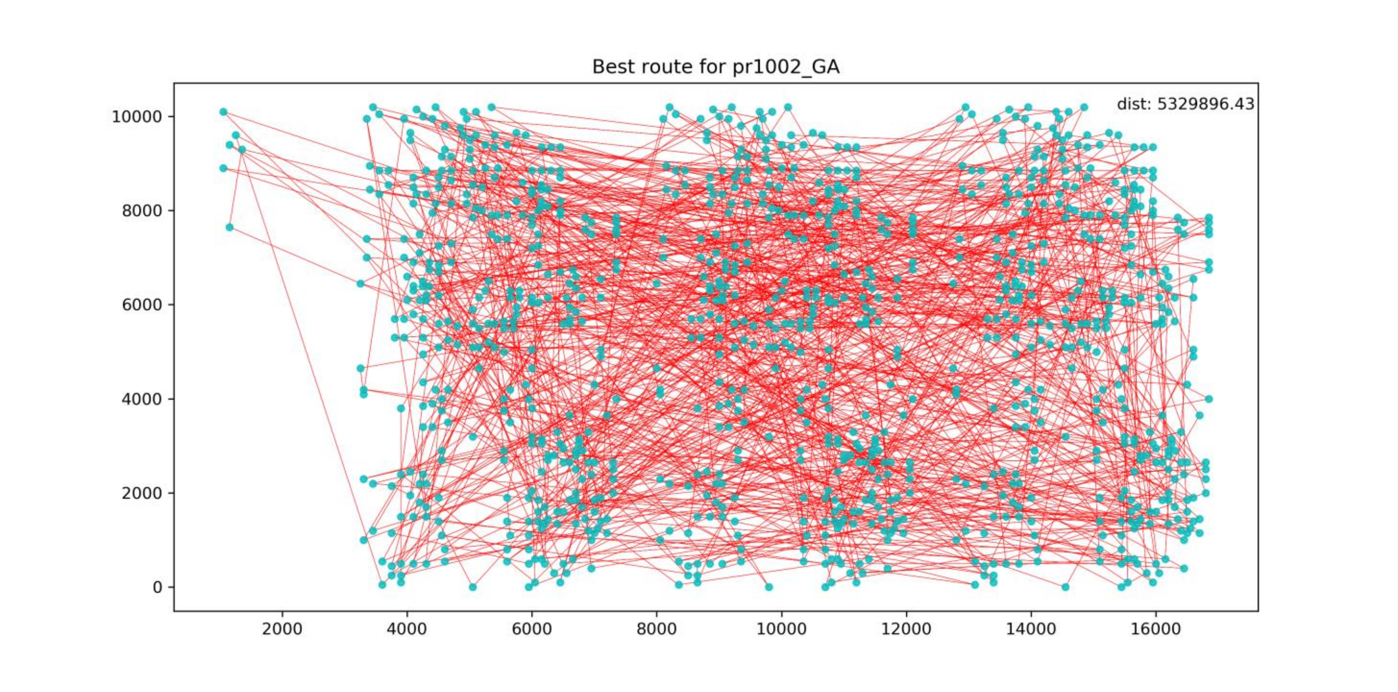
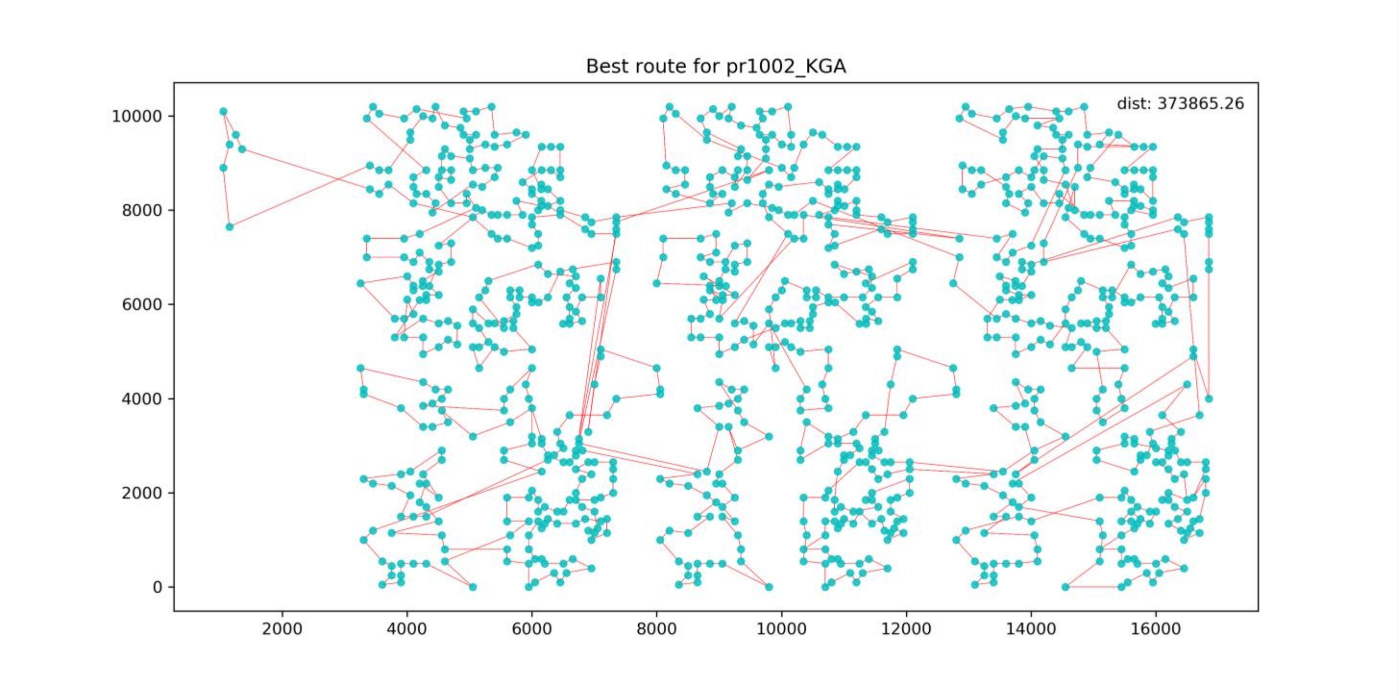
# GA – KGA



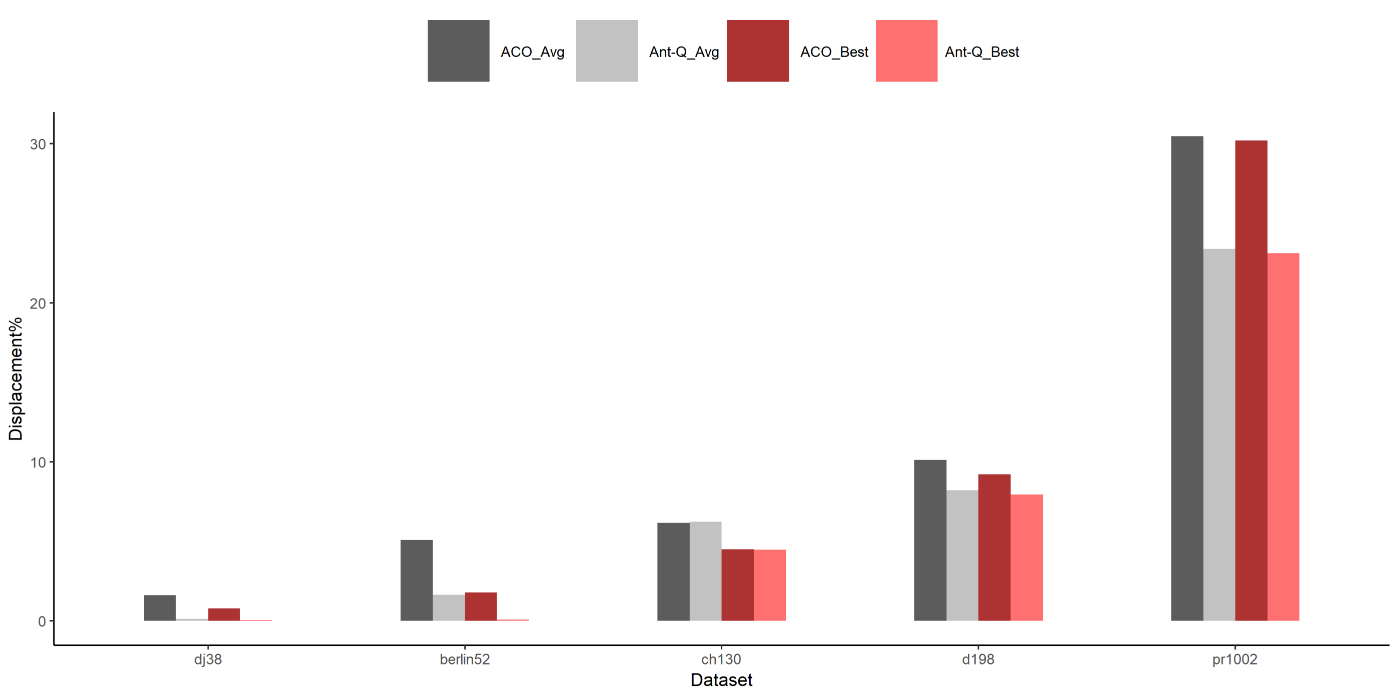

| Dataset | GA | | | | | | |
|---|---|---|---|---|---|---|---|
| | OptimalTour | Mean | SD | Best | % Avg Dist | % Best Dist | Avg time (s) |
| dj38 | 6656 | 7032.91 | 574.34 | 6659.43 | 5.66 | 0.05 | 263 |
| berlin52 | 7542 | 8095.35 | 167.27 | 7868.72 | 7.34 | 4.33 | 286 |
| ch130 | 6110 | 7933.02 | 1041.27 | 6929.81 | 29.84 | 13.42 | 435 |
| d198 | 15780 | 27579.73 | 1869.54 | 25537.85 | 74.78 | 61.84 | 604 |
| pr1002 | 259045 | 4666413.09 | 12608.57 | 4652556.08 | 1701.39 | 1696.04 | 2641 |

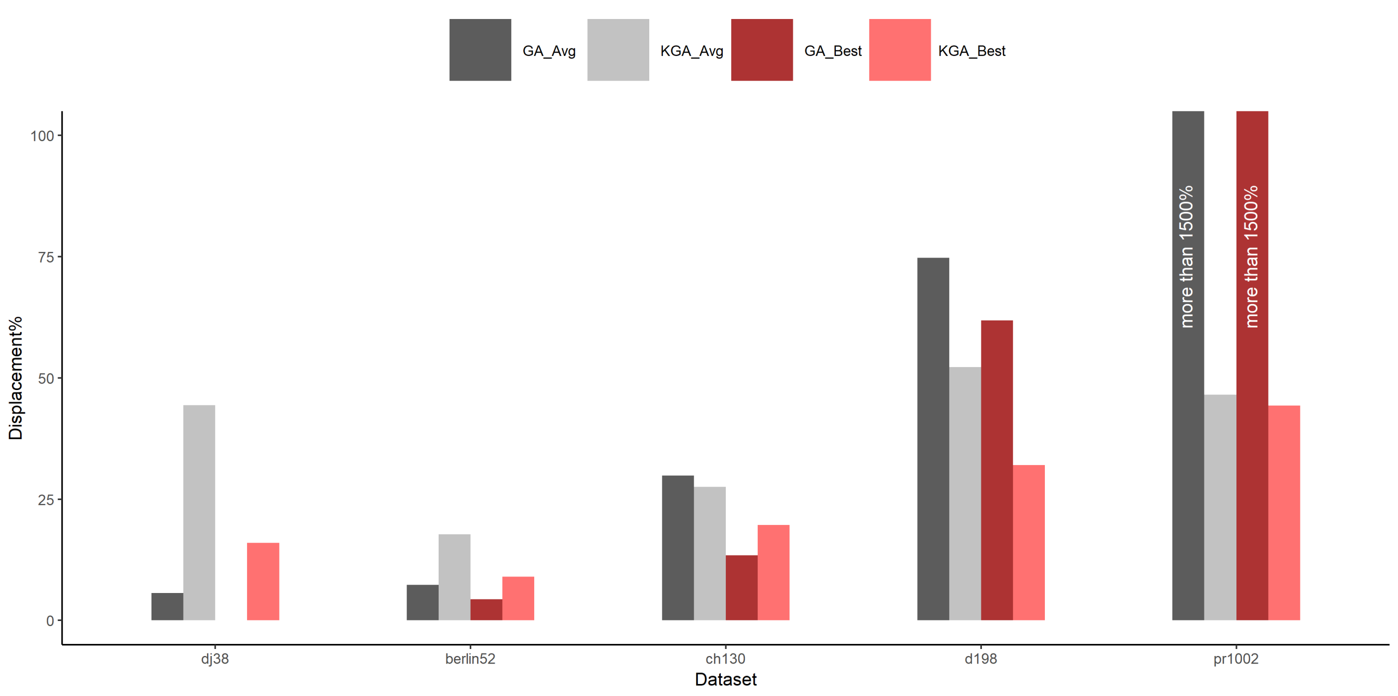
| Dataset | KGA | | | | | | |
|---|---|---|---|---|---|---|---|
| | OptimalTour | Mean | SD | Best | % Avg Dist | % Best Dist | Avg time (s) |
| dj38 | 6656 | 9608.77 | 1817.02 | 7720.03 | 44.36 | 15.99 | 140 |
| berlin52 | 7542 | 8879.59 | 446.69 | 8218.95 | 17.74 | 8.98 | 188 |
| ch130 | 6110 | 7790.85 | 312.17 | 7314.08 | 27.51 | 19.71 | 409 |
| d198 | 15780 | 24023.86 | 3066.94 | 20830.94 | 52.24 | 32.01 | 1413 |
| pr1002 | 259045 | 379495.87 | 6343.57 | 373865.26 | 46.50 | 44.32 | 4456 |




# Compare wide tour (GA – KGA)




# Compare performances

(ACO – Ant-Q –GA – KGA)

# CONCLUSIONS

# KEY POINTS

EFFECTIVE IMPROVEMENT OF THE HYBRIDIZED APPROACH WITH RESPECT TO THE CLASSIC VERSION OF EACH ALGORITHM

ANT FAMILY – BEST PERFORMANCES ON SMALL/MEDIUM SIZED PROBLEMS

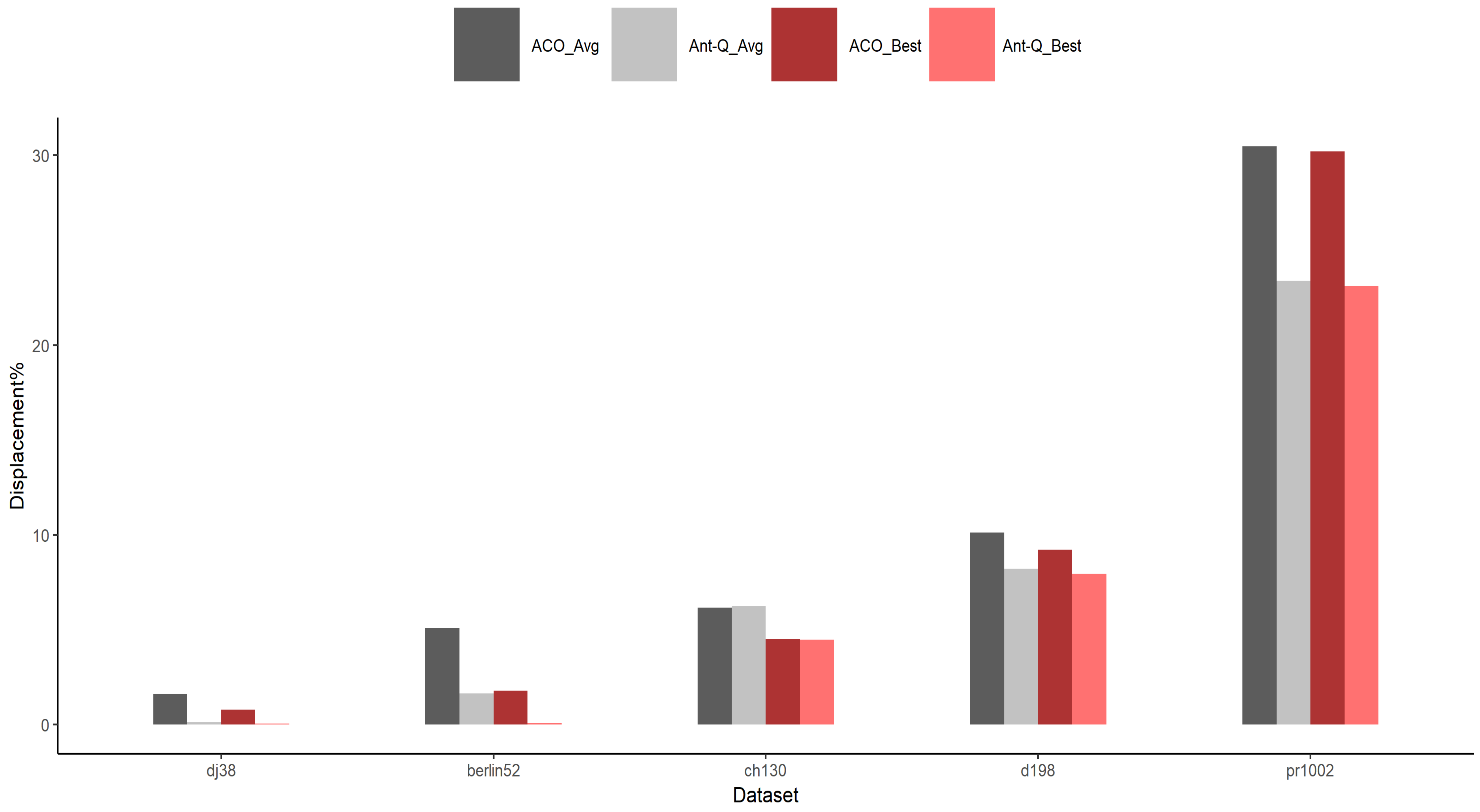
KGA – MORE PERFROMANT ON LARGE DATASETS

ACO AND ANT-Q SLOWER ON HIGHER DIMENSIONS BUT FIND BETTER SOLUTIONS THAN EA

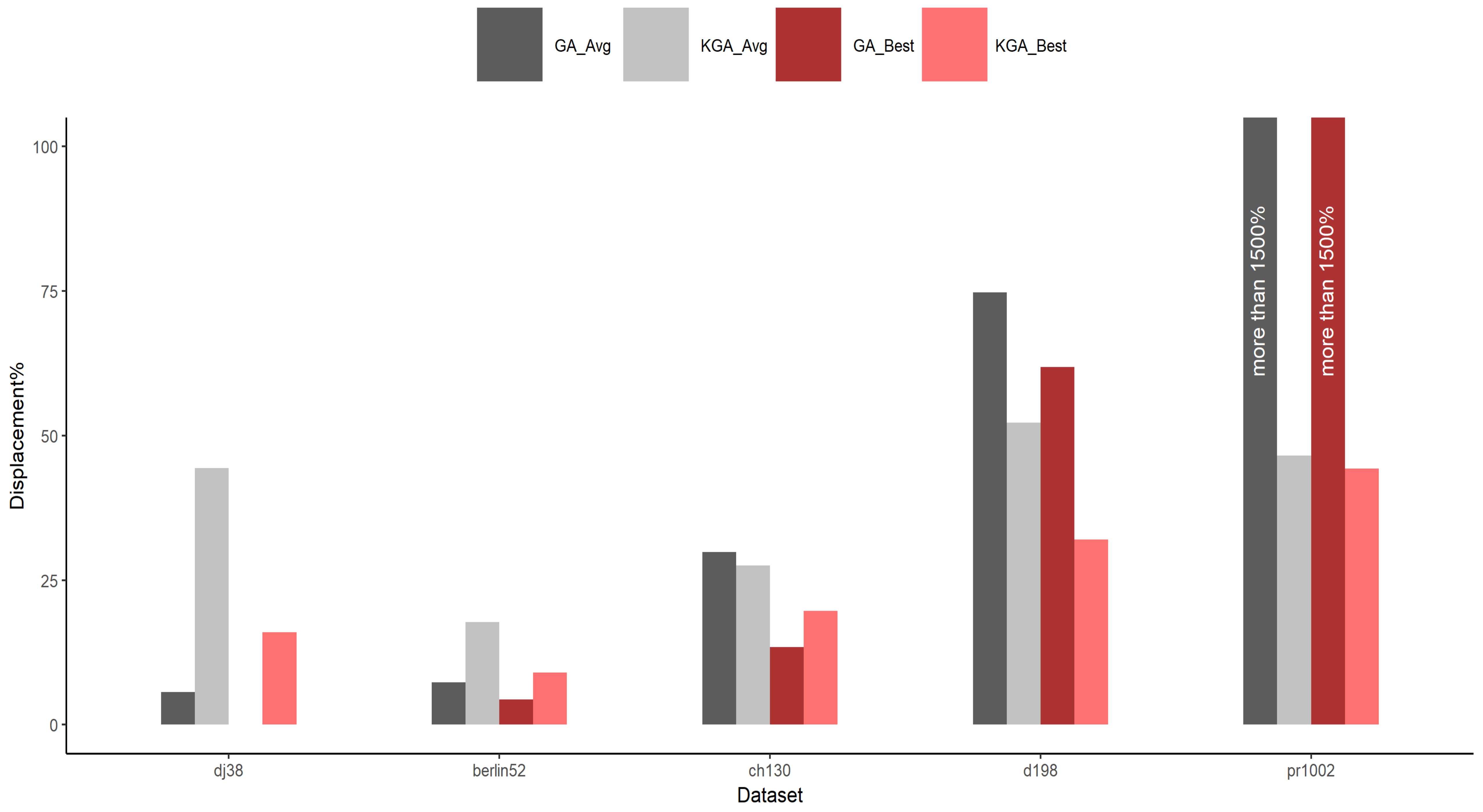
# FUTURE WORK

it could be interesting to further combine Ant-Q and KGA

# THANK YOU



ACO_Avg
Ant-Q_Avg
ACO_Best
Ant-Q_Best
Displacement%
0
10
20
30
dj38
berlin52
ch130
d198
pr1002
Dataset



GA_Avg
KGA_Avg
GA_Best
KGA_Best
Displacement%
100
75
50
25
0
more than 1500%
more than 1500%
dj38
berlin52
ch130
d198
pr1002
Dataset

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| pr1002 | 259045 | 379495.87 | 6343.57 | 373865.26 | 46.50 | 44.32 | 4456 |

**Algorithm** Ant Colony Optimization

**Main Algorithm**

```
initialize best_dist and best_path to None
for generation in generations:
    create n_ants artificial ants
    for one_ant in ants:
        make a single ant path (see Make path)
        compute the path length
        update best_dist and best_path
    update the pheromone matrix
    (local update only for child processes, according to Eq. (10))
    every a certain n of iterations:
        update the global pheromone matrix
        (shared in MPI environment among master&child.)
return best_dist, best_sol
```

**Make path**

```
start from a vertex
add start vertex to visited nodes
for each remaining vertex:
    list the neighbours
    list the not yet visited neighb
    calculate the probability of choosing a vertex (according to Eq. (7))
    choose the vertex according to probability
    add the chosen vertex to the visited list
    return the chosen vertex id
```

**Local update pheromone matrix**

```
for ant in ant_colony :
    for each vertex of one_ant_path :
        increase pheromone_matrix between current and next vertex of Δτ
        (according to Eq. (10))
```

**Global update pheromone matrix (parallelism)**

```
gather from MPI env all the pheromone matrices
if process is the parent process (rank==0):
    for each element average over the n_cores matrices.
broadcast obtained pheromone matrix to the other
processes
```

**Algorithm** Ant-Q algorithm

**Main Algorithm**

```
initialize best_dist and best_path to None
for generation in generations:
    create n_ants artificial ants
    for each ant:
        make a single ant path (see Make path)
        compute the path length
        update best_dist and best_path
        update pheromone matrix with delayed rewards
        (according to Eq. (14))
    update the global pheromone matrix
    (shared in MPI environment among master&child.)
return best_dist, best_sol
```

**Make path**

```
start from a vertex
add start vertex to visited nodes
for each remaining vertex:
    list the neighbours
    list the not yet visited neighb
    generate a random number q ∈ {0, 1}
    if q < q0: (threshold)
        select next vertex according to Eq. (12)
    else:
        calculate the probability of choosing a vertex
        (according to Eq. (7))
        choose the vertex according to probability
    add the chosen vertex to the visited list
    give local rewards (local update pheromone matrix)
    return the chosen vertex id
```

**Local update pheromone matrix**

```
for ant in ant_colony :
    for each vertex of one_ant_path :
        increase pheromone_matrix between current
        and next vertex of a Δτ (according to Eq. (11))
```

**Global update pheromone matrix (parallelism)**

```
gather from MPI env all the pheromone matrices
if process is the parent process (rank==0):
    for each element average over the n_cores matrices.
broadcast obtained pheromone matrix to the other
processes
```

**Algorithm** Genetic Algorithm

```
procedure Genetic(Tm, Tp, elite_n, Selection, MaxGen)
  Pop ← GeneratePopulation(Tp)
  Pop ← Evaluation(Pop)
  for i = 1 ... MaxGen do
    Pop ← Selection(elite_n from Pop)
    Pop ← Crossover(Pop)
    With probability Tm do:
    Pop ← Mutation(Pop)
  end for
   return the best solution in Pop
end procedure
```

**Algorithm** K-Means Algorithm

```
Set the K cluster centers randomly;
repeat
  for each vertex do
    Calculate distance measure to each cluster;
    Assign it to the closest cluster;
  end
  recompute the cluster centers positions;
until stop criteria are met;
```

**Algorithm** KGA

```
input an TSP;
K-Means is adopted to cluster the TSP into k sub-problems
For each sub-prob i = 1 to k, do:
  repeat
    GA procedure
  until stop criteria are met
  Output shortest path for sub-problem i;
End
Seek for the best combining seq S with GA
Combine all those shortest path into one tour
Output the shortest whole travelling tour.
```