ACS6124 Multisensor and Decision Systems
Part I – Multisensor Systems
Lab A: Sensor Signal Estimation

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2023/2024

Aims

- To relate theory and practice of sensor signal estimation and detection in engineering applications.
- To demonstrate the understanding of different design considerations depending on e.g. the availability of prior knowledge.

Objectives

At the end of the lab, you will be able to

- Implement sensor signal estimation schemes based on measurement data and different prior knowledge.
- Design algorithms that improve the estimation as new measurements arrive.

How to use MATLAB files

Download the files from ACS6124 Course Content at Blackboard at Labs/ACS6124 Part I - Multisensor Systems - Laboratory/LabAFiles.zip. Extract and put those files in your working directory and add the corresponding working path to MATLAB.

Pre-lab activity

In the lecture slides on Sequential LMMSE Estimation, we obtained the update rule

$$\hat{x}_{T+1} = \hat{x}_T + \underbrace{\frac{\sigma_x^2}{(T+1)\sigma_x^2 + \sigma^2}\,(y_{T+1} - \hat{x}_T)}_{\text{Scaled version of prediction error: } y_{T+1} - \hat{x}_T} \quad ,$$

based on the assumption $x \sim \mathrm{N}(0, \sigma_x^2)$. Verify that the recursive update for the estimator has the same expression with non-zero expected value in the prior knowledge of x, i.e. $x \sim \mathrm{N}(\mu_x, \sigma_x^2)$.

Sensor Signal Estimation

Imagine you are employed in a wind turbine manufacturing company and have been tasked to design a multisensor signal estimation algorithm for the blade pitching mechanism of the wind turbine. In order to measure the pitch angle $\hat{\omega}$, the blade is equipped with a rotary encoder connected to the blade bearing, where the sensor noise is $\nu \sim \mathrm{N}(0, 9)$.

Task 1: MMSE estimator

Task 1.1: In the template script provided, load in the entire measurement vector `encoder.mat`. You can find the files inside the module's Assignment folder in Blackboard.

Task 1.2: Compute the mean values based on

- the full measurement data,
- only the first 3 elements of the provided measurement vector.

Now assume you have the prior knowledge that the angle is uniformly distributed in the range $21° \leq \hat{\omega} \leq 30°$.

Task 1.3: Truncate the data according to the prior knowledge and compute the mean values again based on

- the full measurement data,
- only the first 3 elements of the provided measurement vector.

Task 1.4: Calculate the MMSE estimation by incorporating the prior knowledge. You can use the following code template

```
fun1 = @(x) (x/sqrt(2*pi*(noise_var)/n_measurements)).*exp(-
    n_measurements/(2*noise_var)*(x-sum(encoder_x)/n_measurements)
    .^2);
fun2 = @(x) (1/sqrt(2*pi*(noise_var)/n_measurements)).*exp(-
    n_measurements/(2*noise_var)*(x-sum(encoder_x)/n_measurements)
    .^2);
mmse_estimation_uniprior = integral(fun1,prior_min,prior_max)/
    integral(fun2,prior_min,prior_max);
```

for the numerical computation of

$$x^* = \frac{\int_{-\Delta}^{\Delta} \frac{x}{\sqrt{2\pi\frac{\sigma^2}{T}}} \exp\left[-\frac{1}{2\frac{\sigma^2}{T}}\left(x - \frac{1}{T}\sum_{k=1}^{T} y_k\right)^2\right] dx}{\int_{-\Delta}^{\Delta} \frac{1}{\sqrt{2\pi\frac{\sigma^2}{T}}} \exp\left[-\frac{1}{2\frac{\sigma^2}{T}}\left(x - \frac{1}{T}\sum_{k=1}^{T} y_k\right)^2\right] dx}.$$

Again, compute your estimates based on

- the full measurement data,
- only the first 3 elements of the provided measurement vector.

Task 1.5: Extend the script and calculate the MMSE estimator in the case when the prior knowledge is Gaussian distributed with a mean value of 25 and variance of 2. As before, consider both scenarios on the availability of measurements and use the same measurement vector `encoder.mat`.

Task 1.6: Observations and reflections.

- First focus on the cases where you only have the first 3 elements of the measurements. How do the MMSE results differ from the mean and the truncated mean? What role does the prior knowledge play in such a difference?
- Next focus on how the estimations change when the full measurement data is considered. Can you explain the pros and cons of using Bayesian estimations?

Task 2: Sequential MMSE estimator

Extend the script and use the appropriate estimator in the case when the prior knowledge is Gaussian distributed with a mean value of 25 and the measurements arrive in a sequential manner. Consider both cases for the prior knowledge of variance

- variance of 0.1
- variance of 4

and compare the estimation results for the same measurement vector `encoder.mat`. You should also compute the running average (taking the mean value at step k for data of all previous steps (i.e. $1, \ldots, k$) and compare the estimations.

Observations and reflections:

- How does the prior knowledge of the mean influence the estimation results?
- What role does the variance play in the Bayesian MMSE estimation? .