Data Driven Process Engineering

Mixed Integer Programming

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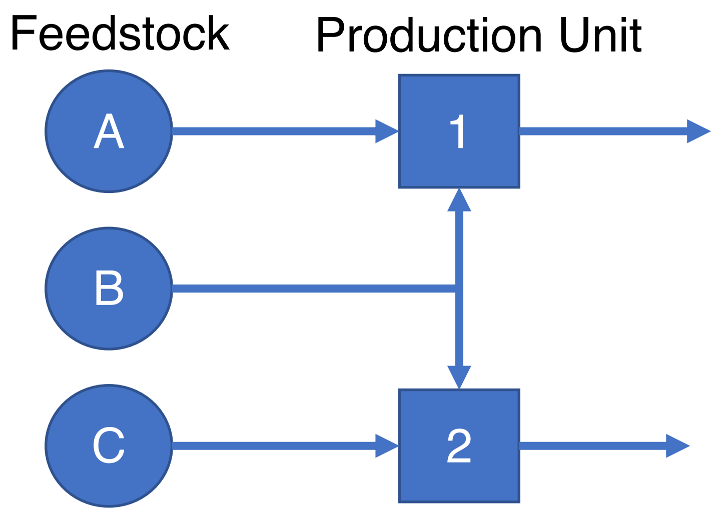
Mixed-Integer Formulations

Learning Objectives

- Construct a Mixed-Integer formulation from problem description
- Explain logical operators and their relationship with binary variables
- Make use of binary variables and constraints to represent choices

Mixed-Integer Formulations

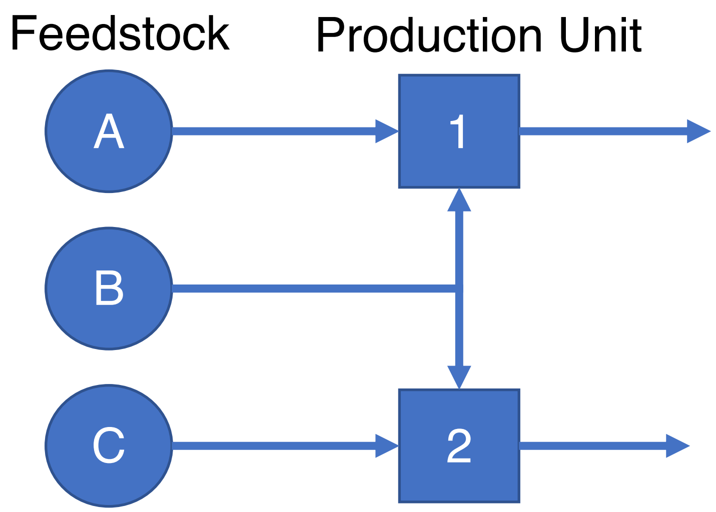


Example: "Blending products including discrete batch sizes"

- Products: 1 and 2
- Each batch 2000 lbs
- Feedstocks: A, B, and C

- Unit 1 (capacity 8000 lb/day) produces product 1 , requires 0.4lb of A and 0.6lb of B
- Unit 2 (capacity 10000 lb/day) produces product 2, requires 0.3lb of B and 0.7lb of C
- Only 6000 lb/day of B is available.

Mixed-Integer Formulations



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- Products: 1 and 2
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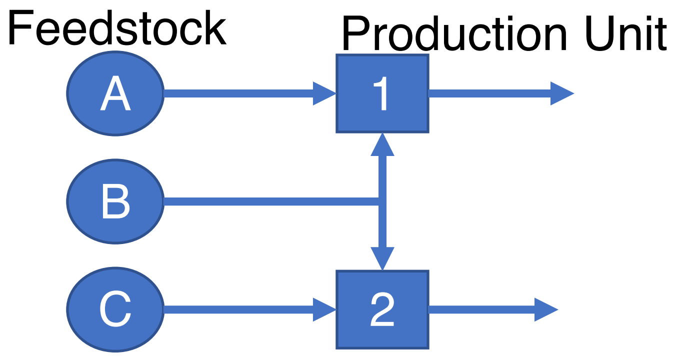
Variables: **Production volume (lb)** x_1, x_2
Volume of feedstock (lb): f_A, f_B, f_C
Number of batches: y_1, y_2

Objective: Maximize Revenue (R)

Mixed-Integer Formulations



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- Unit 2 (capacity 10000 lb) produces product 2, requires 0.3 of B and 0.7 of C
- Only 6000 lb of B is available
- Net Revenue is given by following function: $R = 0.16x_1^{0.7} + 0.2x_2^{0.6}$

Variables:
Production volume (lb) x_1, x_2
Volume of feedstock (lb): f_A, f_B, f_C
Number of batches: y_1, y_2

Objective: Maximize Revenue (R)

Representing Choices in Design

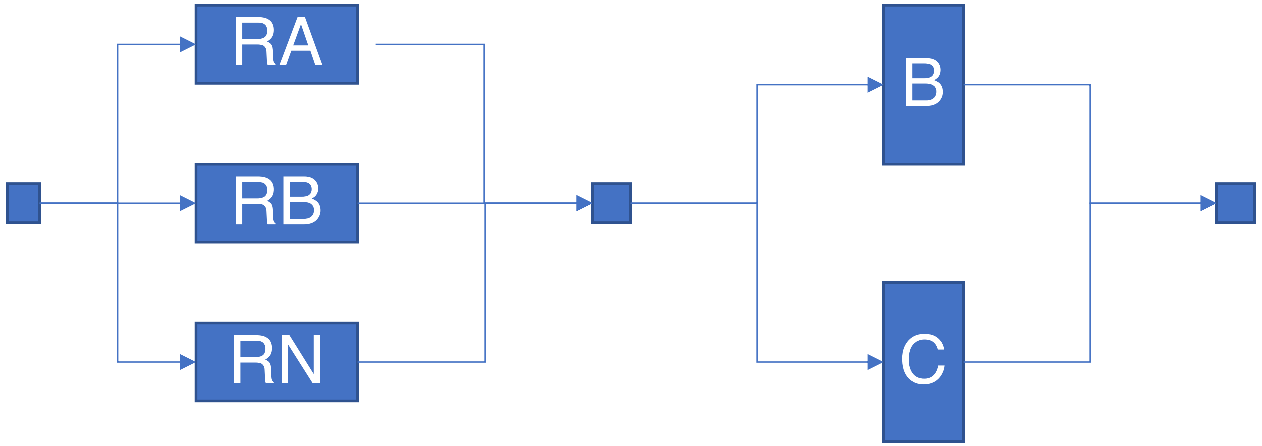
Examples:

"You must choose a reactor"

"If you choose the reactor RA then you must choose one of separator B or C but not both"

"If you choose this separator the feed must be liquid"

Flowsheet that contains ALL POSSIBLE alternatives: "Flowsheet SUPERSTRUCTURE"



"Word Statements of Structure" → Logical Expressions → MILP Constraints

Simple Constraints with Binary Variables

$y_i = [0, 1]$ => zero OR one, no values in between. "Switch on/off"

"At least one of the set S must be chosen." $\sum_{i \in S} y_i \geq 1$

"At most one of the set S must be chosen." $\sum_{i \in S} y_i \leq 1$

"Only one of the set S must be chosen." $\sum_{i \in S} y_i = 1?$

"If Reactor A is chosen then its volume (V_A) must be between 5000L and 50,000L otherwise it is zero" $5000 y_A \leq V_A \leq 50{,}000 y_A$

It can get very complex…

"Either one of {1,2,3,4} or one of {5,6,7,8} must be chosen but not one of both."

$$y_1 + y_2 + y_3 + y_4 = 1 - (y_5 + y_6 + y_7 + y_8)$$

Logical Representations Using Binaries

Logical Operators:
(logical decisions you make every day, and that you will need to make for design)

$\wedge$	AND
$\vee$	OR
$\Rightarrow$	IMPLIES
$\neg$	NOT
$\otimes$	EXCLUSIVE OR*

*either one, but not both nor none

$$(P_1 \vee P_2) \wedge (P_3 \vee P_4)$$

P1 or P2 AND P3 or P4

$$(P_1 \vee P_2) \Rightarrow P_3$$

P1 OR P2 IMPLIES P3

Logical Representation

How to translate the operators over variables into TRUE FALSE values

Truth Tables

$Y_1 \vee Y_2$

$\vee$	T	F
T	T	T
F	T	F

$Y_1 \wedge Y_2$

$\wedge$	T	F
T	T	F
F	F	F

$Y_1 \otimes Y_2$

$\otimes$	T	F
T	F	T
F	T	F

$Y_1 \Rightarrow Y_2$

$\Rightarrow$	T	F
T	T	F
F	T	T

These logical representations can ALL be written as binary-based constraints!!!

Logical Representation as MILP

How to I capture these truth tables in MILP Constraints?

Use BINARY variables for the logical variables, y =1 for TRUE and y=0 for FALSE.

$Y_1 \vee Y_2$	$y_1 + y_2 \geq 1$	y_1 or y_2 or both can be 1
$Y_1 \otimes Y_2$	$y_1 + y_2 = 1$	Only y_1 or y_2 can be 1
$Y_1 \wedge Y_2$	$y_1 \geq 1$ $y_2 \geq 1$	Both must be 1. Also as: $y_1 + y_2 \geq 2$ $y_1 = 1, y_2 = 1$
$Y_1 \Rightarrow Y_2$	$y_2 \geq y_1$	y_2 must be 1 if y_1 is 1

Logical Representation as MILP

Example – word statement

"Reactor A selected implies Separator B or C must be selected."

Represent the choice of the Reactor A as $Y_A = \begin{cases} TRUE \\ FALSE \end{cases}$

Reactor A is selected

Define Y_B and Y_c similarly $\quad Y_A \Rightarrow (Y_B \vee Y_C)$

Example - logical statement

How many total combinations of Y_A, Y_B, Y_C? → $2^{no\_vars} = 2^3 = 8$

$Y_A = 0, Y_B = 0, Y_C = 0$
$Y_A = 1, Y_B = 0, Y_C = 0$
$Y_A = 0, Y_B = 1, Y_C = 0$
$Y_A = 0, Y_B = 0, Y_C = 1$

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Example – MILP Constraint

Summary

- Mixed-Integer formulations are needed when a “choice” needs to be made, and many more

- Every logical statement can be represented by mathematical equations that are part of formulation

- Correct mathematical representation leads to solutions without enumeration