Measurements

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Measurements

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- Properties of performance metrics
- Indices of central tendency and variability
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Measurements

Performance metrics in a computer

- A count of how many times an event occurs (e.g. processor initiates an IO request)
- The duration of some time interval (e.g. how long each of these requests take)
- The size of some parameter (e.g. number of bits transmitted and stored)

Throughput is calculated by dividing the count of the number of events that occur in a given interval by the time interval over which the events occurs.

Network throughput is the rate of successfull message delivery over a communication channel (e.g bits/s)

Measurements

(Good) Properties of performance metrics:

- **Linearity**: the value of the metric should be proportional to the actual performance of the system being measured.
- **Reliability**: a metric is *reliable* if system A always outperforms system B when the corresponding values of the metric for both systems indicate that system A should outperform system B.
- **Repeatibility**: a metric is *repeatable* if the same value of the metric is measured each time the same experiment is performed.
- **Easiness of measurement**
- **Consistency:** a consistent metric is one which the units of the metric and its precise definition are the same across different systems
- **Independence:** a metric should be independent of outside influences

Measurements

How to choose a computer?

Investigate computer performance metric with respect to these properties:

- Clock rate
- MIPS (millions of instructions executed per second)
- MFLOPS (millions of floating-point operations executed per second)
- SPEC (System Performance Evaluation Cooperative)
- Execution time of a given application program (CPU-time vs. Wall-clock time)

Measurements

Nondeterministic measurements

- Measurements of the same system may vary significantly due to random events.
- Exmple: CPU-time of a program may be interfered by background operating systems tasks, page and cache mappings, system load, etc. Therefore, program’s executation time is **nondeterministic**.
- A way of dealing with nondeterministic metrics is to repeat the measurements several times and report a value that summarizes the values collected (*indice of central tendency*) and other that indicates how dispersed they are (*variability*).

Measurements

The sample mean

- A **sample mean** refers to the mean value of a sample of *n* measurements and is given by the **arithmetic mean**. It is an estimate of the population mean.

$$\bar{x}_{\mathrm{A}} = \frac{1}{n}\sum_{i=1}^{n} x_i .$$

- The **harmonic mean** is more appropriate for summarizing rate measurements.

$$\bar{x}_{\mathrm{H}} = \frac{n}{\sum_{i=1}^{n} 1/x_i}$$

Measurements

- Sample mean gives equal weight to all measurements and, for this reason, an outlier can have a large influence.

- The **median** is an index of central tendency that reduces the skewing effect of outliers. It consists of ordering all of the n measurements and select the middle value, if n is odd, or the mean of the two middle values, if n is even.

- The **mode** is the value that occurs most frequently and it may not be unique. It is more appropriate to categorical data.

Measurements

Quantify variability

- The **range** is an index of dispersion and is the difference of the maximum and the minimum of the measured values.

$$R_{max} = \max_{i=1..n} x_i - \min_{i=1..n} x_i$$

- The maximum of the absolute values of the difference of each measurement from the mean value is less sensible to extreme values.

$$\Delta_{max} = \max_{i=1..n} |x_i - \overline{x}|$$

Measurements

Quantify variability

- The **sample variance** is the estimate of the variance of the underlying distribution from which the measurements are taken.

$$s^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}$$

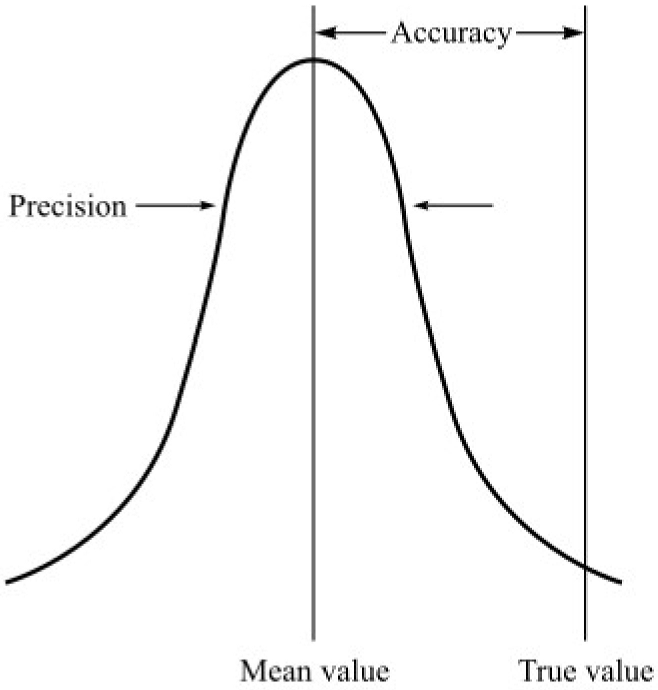
- The **standard deviation** allows the indices of variability and central tendency to be in the same units.

$$s = \sqrt{s^2} = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}}.$$

Measurements

Characteristics of measurement tools

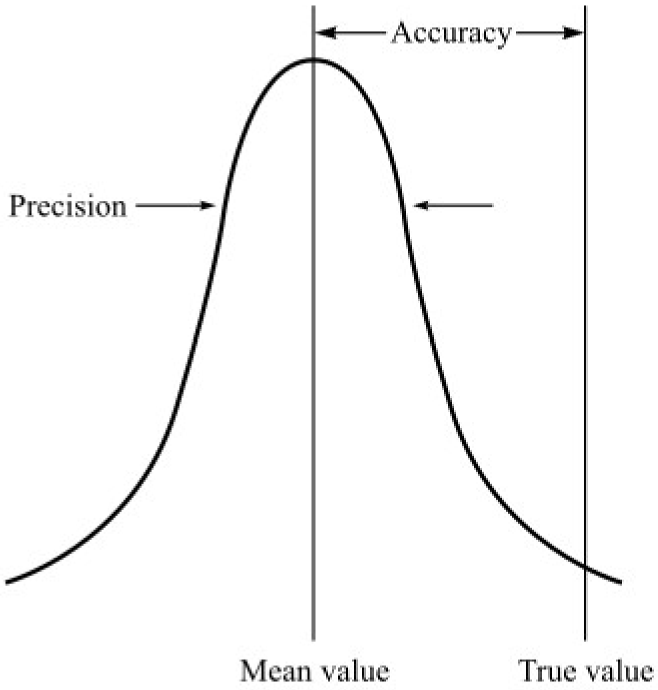
- **Accuracy**: absolute difference between a measured value and the corresponding reference value (usually, an agreed-upon standard)
- **Precision**: closeness of measurements to each other.
- **Resolution**: smallest incremental change that can be detected and displayed



Measurements

Errors in measurements

- **Systematic errors** are due to an experimental "mistake” that introduces bias into the measurements. They affect the accuracy of the measurements.
- **Random errors** are unpredictable, nondeterministic and need not be controllable. They are unbiased since a random error has an equal probability of either increasing or decreasing a measurement. They afect the precision of the measurements.



Measurements

A gaussian model of errors

Assume that a single source of random error can change the value measured for x by $+E$ or $-E$ with equal probability

Error	Measured value	Probability
$-E$	$x - E$	$1/2$
$+E$	$x + E$	$1/2$

Measurements

A gaussian model of errors

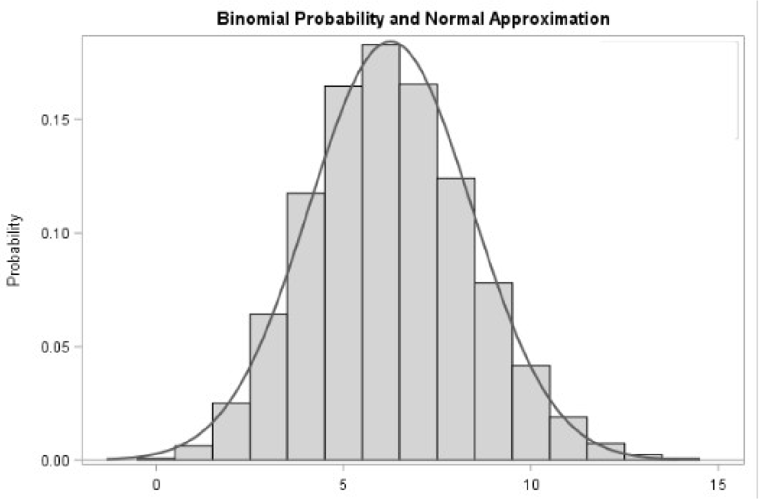
Assume that a **two** sources of random error can change the value measured for *x* by *+E* or *–E* with equal probability

Error 1	Error 2	Measured value	Probability
$-E$	$-E$	$x-2E$	$1/4$
$-E$	$+E$	x	$1/4$
$+E$	$-E$	x	$1/4$
$+E$	$+E$	$x+2E$	$1/4$

Measurements

A gaussian model of errors

- This produces a binomial distribution for the possible measurements – for large *n*, this distribution approximates a **normal distribution**.



Measurements

Recap:

- There are several properties that metrics should have.
- For a nondeterministic metric, indices of central tendency and variability must be calculated.
- Sample mean estimates the true mean and sample median is less sensible to outliers.
- Depending of the metric and the application at hand, one has to choose between the arithmetic mean, the harmonic mean and the geometric mean
- Variability is calculated with sample variance or standard deviation
- A measurement tool is characterized by accuracy, precision and resolution
- Errors in measurements are due to systematic errors, which affect accuracy, and random errors, which affect precision. Under some assumptions, the random errors follow a normal distribution.

Measurements

References:

- D.J.Lilja, *Measuring computer performance*, Cambridge University Press, 2002 (see chapters 2 to 4)
- J.E. Smith, Characterizing computer performance with a single number, Communications of the ACM, 1202-1206, 1988.