Definition of Complex Numbers

We begin by defining the symbol i, called **the imaginary unit,**[1] to have the property

$$i^2 = -1.$$

Thus, we could also call i the **square root of** -1 and denote it $\sqrt{-1}$. Of course, i is not a real number; no real number has a negative square.

DEFINITION 1

A **complex number** is an expression of the form

$$a + bi \qquad \text{or} \qquad a + ib,$$

where a and b are *real numbers*, and i is the imaginary unit.

For example, $3+2i$, $\frac{7}{2}-\frac{2}{3}i$, $i\pi = 0+i\pi$, and $-3 = -3+0i$ are all complex numbers. The last of these examples shows that every real number can be regarded as a complex number. (We will normally use $a + bi$ unless b is a complicated expression, in which case we will write $a + ib$ instead. Either form is acceptable.)

It is often convenient to represent a complex number by a single letter; w and z are frequently used for this purpose. If a, b, x, and y are real numbers, and

$$w = a + bi \qquad \text{and} \qquad z = x + yi,$$

then we can refer to the complex numbers w and z. Note that $w = z$ if and only if $a = x$ and $b = y$. Of special importance are the complex numbers

$$0 = 0 + 0i, \qquad 1 = 1 + 0i, \qquad \text{and} \qquad i = 0 + 1i.$$

DEFINITION 2

If $z = x + yi$ is a complex number (where x and y are real), we call x the **real part** of z and denote it $\operatorname{Re}(z)$. We call y the **imaginary part** of z and denote it $\operatorname{Im}(z)$:

$$\operatorname{Re}(z) = \operatorname{Re}(x + yi) = x, \qquad \operatorname{Im}(z) = \operatorname{Im}(x + yi) = y.$$

Note that both the real and imaginary parts of a complex number are real numbers:

$$\operatorname{Re}(3 - 5i) = 3 \qquad\qquad \operatorname{Im}(3 - 5i) = -5$$