

LTspice simulator for complex signals

LTspice simulator, you can download for free: <https://www.analog.com/en/design-center/design-tools-and-calculators/ltspice-simulator.html>

Periodical continuation of $f(x)$ and its Fourier series:

$$f(x) = x^2, \quad x \in [-\pi, \pi]$$

This function is even and has the period $2l = 2\pi$. The reference time $t_0 = -\pi$.

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} x^2 dx = \frac{2\pi^2}{3}$$

$$b_k \equiv 0$$

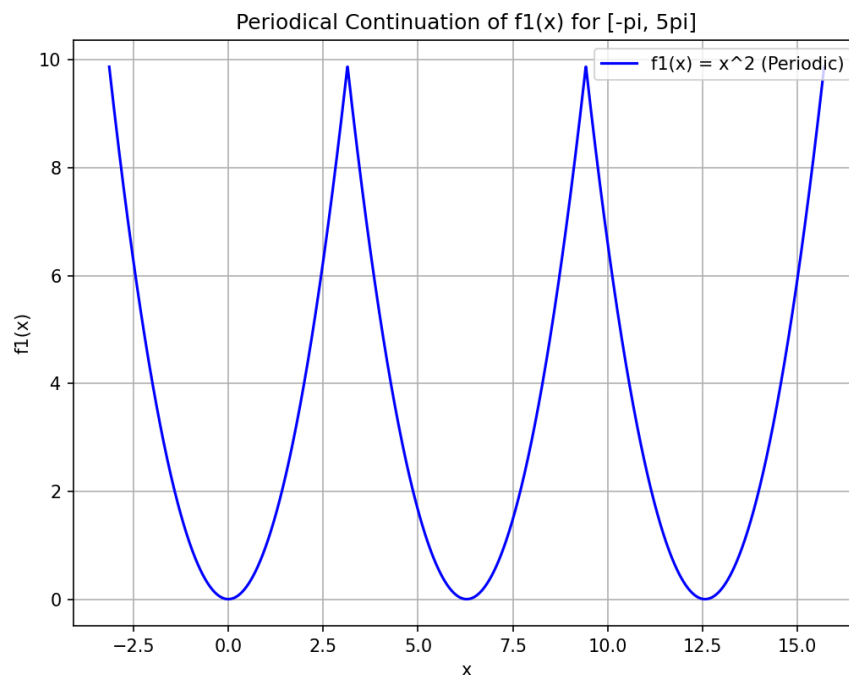
$$a_k = \frac{1}{\pi} \int_{-\pi}^{\pi} x^2 \cos(kx) dx = x^2 \frac{\sin(kx)}{\pi k} \Big|_{-\pi}^{\pi} - \frac{2}{\pi k} \int_{-\pi}^{\pi} x \sin(kx) dx =$$

$$= -\frac{2}{\pi k} \int_{-\pi}^{\pi} x \sin(kx) dx = \frac{2x \cos(kx)}{\pi k^2} \Big|_{-\pi}^{\pi} - \frac{2}{\pi k^2} \int_{-\pi}^{\pi} \cos(kx) dx =$$

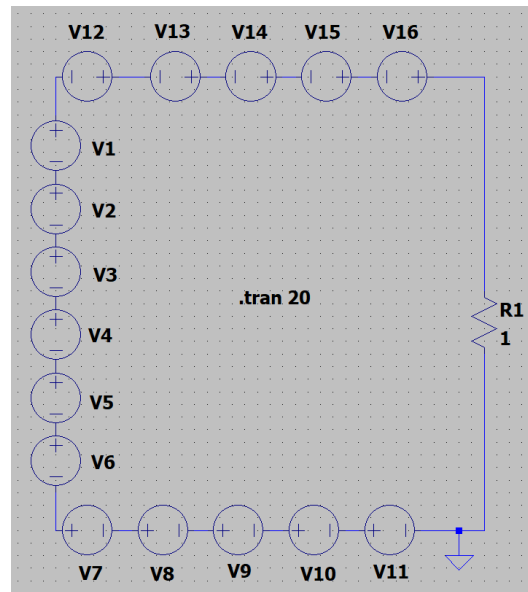
$$= \frac{2x \cos(kx)}{\pi k^2} \Big|_{-\pi}^{\pi} = \frac{4 \cos(k\pi)}{k^2} = \frac{4(-1)^k}{k^2}$$

$$f(x) = \frac{\pi^2}{3} + 4 \sum_{k=1}^{\infty} \left[\frac{(-1)^k}{k^2} \cos(kx) \right]$$

Graph of the periodical continuation for three periods:



Simulation in LTspice with 16 generators (harmonics):



$$f(x) = \frac{\pi^2}{3} + 4 \sum_{k=1}^{\infty} \left[\frac{(-1)^k}{k^2} \cos(kx) \right]$$

Generator V1 for zero and first harmonic ($k = 1$)

$$k = 1$$

$$\frac{\pi^2}{3} - 4 \cos(x)$$

Independent Voltage Source - V1

Functions

- ☐ (none)
- ☐ PULSE(V1 V2 Tdelay Trise Tfall Ton Period Ncycles)
- ☒ SINE(Voffset Vamp Freq Td Theta Phi Ncycles)
- ☐ EXP(V1 V2 Td1 Tau1 Td2 Tau2)
- ☐ SFFM(Voff Vamp Fcar MDI Fsig)
- ☐ PWL(t1 v1 t2 v2...)
- ☐ PWL FILE:

DC Value

DC value:

Make this information visible on schematic: ☐

Series Resistance[Ω]:

Parallel Capacitance[F]:

Make this information visible on schematic: ☒

Additional PWL Points

Make this information visible on schematic: ☐

Cancel OK

DC offset[V]: 3.28986813

Amplitude[V]: -4

Freq[Hz]: 0.15915494

Tdelay[s]: 0

Theta[1/s]: 0

Phi[deg]: 90

Ncycles:

$$Freq = \frac{1}{2\pi} \rightarrow \omega = 2\pi f = 1$$

$$\cos(\omega x) = \cos(x) = \sin\left(x + \frac{\pi}{2}\right)$$

$$\frac{\pi}{2} \rightarrow 90^\circ$$

$$f(x) = \frac{\pi^2}{3} + 4 \sum_{k=1}^{\infty} \left[\frac{(-1)^k}{k^2} \cos(kx) \right]$$

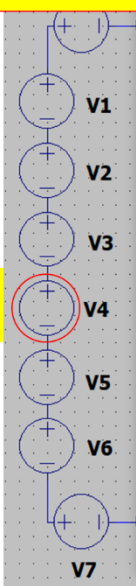
$k = 4$

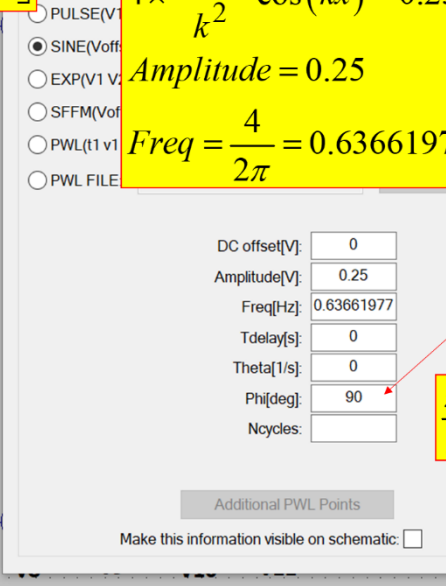
$$4 \times \frac{(-1)^k}{k^2} \cos(kx) = 0.25 \cos(4x) = 0.25 \sin\left(4x + \frac{\pi}{2}\right)$$

Amplitude = 0.25

$$Freq = \frac{4}{2\pi} = 0.63661977$$

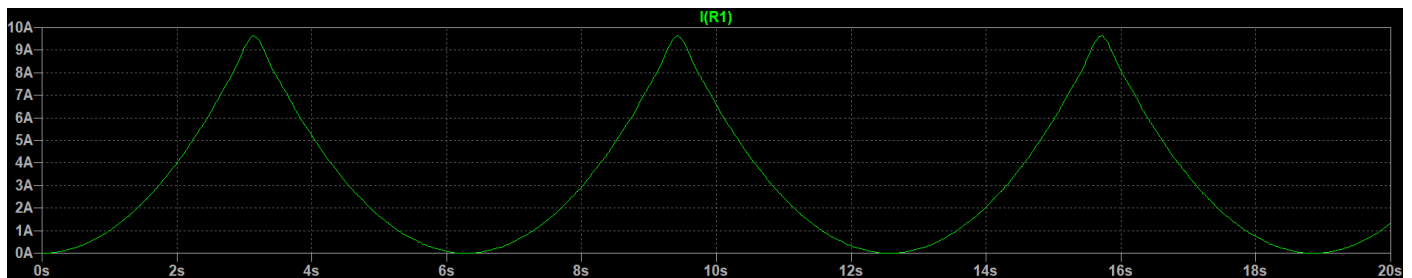
Generator V4 for fourth harmonic (k = 4)





$\frac{\pi}{2} \rightarrow 90^\circ$

Current response through $R1 = 1\Omega$:



Conclusion: The shape has been reproduced fairly accurately with 16 harmonics, although the peaks should be slightly sharper.

Periodical continuation of $f(x)$ and its Fourier series:

$$f(x) = \begin{cases} \cos\left(\frac{\pi x}{l}\right), & 0 \leq x \leq \frac{l}{2} \\ 0, & \frac{l}{2} < x < l \end{cases}$$

This function has the period l . The reference time $t_0 = 0$.

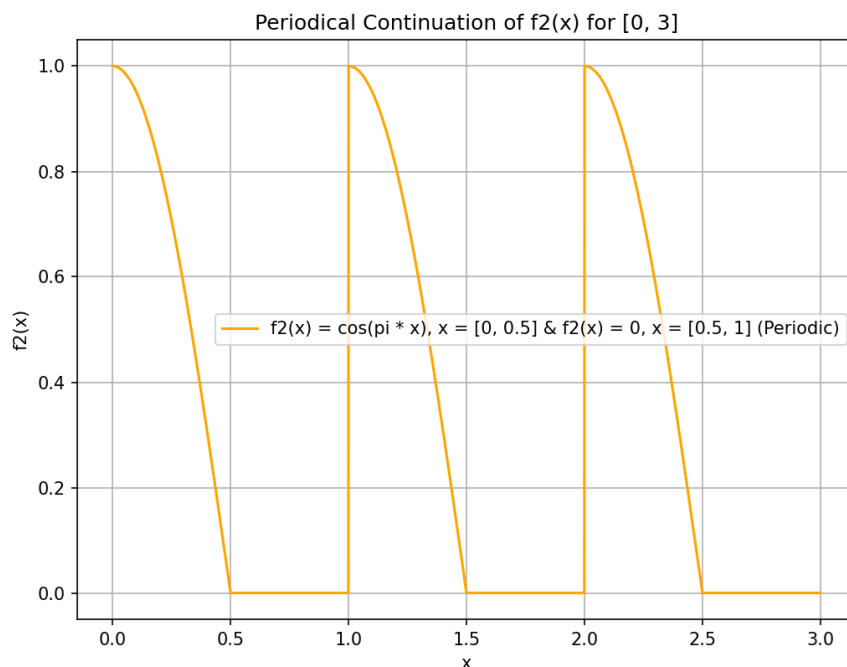
$$a_0 = \frac{2}{l} \int_0^{l/2} \cos\left(\frac{\pi x}{l}\right) dx = \frac{2}{\pi} \sin\left(\frac{\pi x}{l}\right) \Big|_0^{l/2} = \frac{2}{\pi}$$

$$\begin{aligned} a_k &= \frac{2}{l} \int_0^{l/2} \cos\left(\frac{\pi x}{l}\right) \cos\left(\frac{2\pi k}{l} x\right) dx \Rightarrow \left\{ \frac{\pi x}{l} = s; \quad dx = \frac{l}{\pi} ds \right\} \Rightarrow \frac{2}{\pi} \int_0^{\pi/2} \cos(s) \cos(2ks) ds = \\ &= \frac{\sin((2k+1)s)}{\pi(2k+1)} \Big|_0^{\pi/2} + \frac{\sin((2k-1)s)}{\pi(2k-1)} \Big|_0^{\pi/2} = \frac{\sin\left(\frac{(2k+1)\pi}{2}\right)}{\pi(2k+1)} + \frac{\sin\left(\frac{(2k-1)\pi}{2}\right)}{\pi(2k-1)} = \frac{(-1)^k}{\pi(2k+1)} + \frac{(-1)^{k+1}}{\pi(2k-1)} = \frac{2(-1)^{k+1}}{\pi(4k^2-1)} \end{aligned}$$

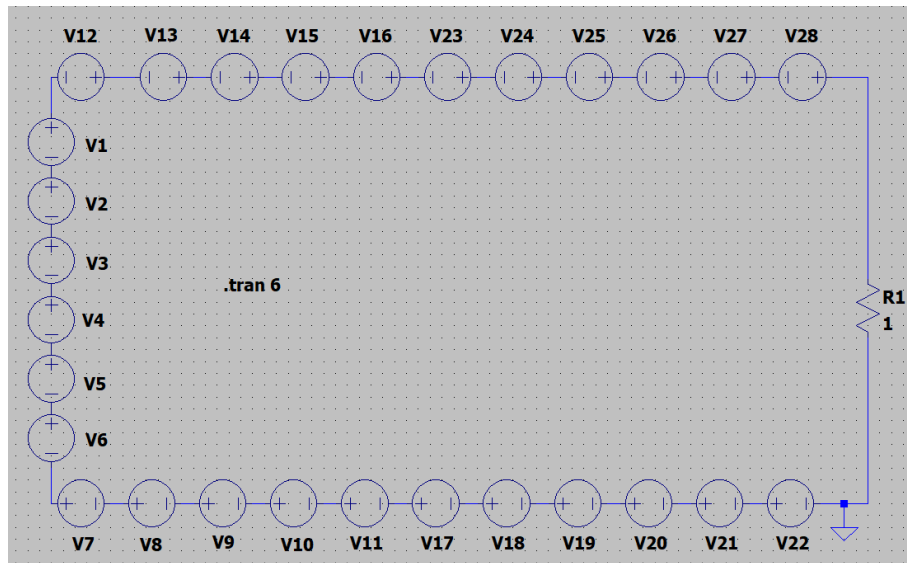
$$b_k = \frac{2}{l} \int_0^{l/2} \cos\left(\frac{\pi x}{l}\right) \sin\left(\frac{2\pi k}{l} x\right) dx \Rightarrow \left\{ \frac{\pi x}{l} = s; \quad dx = \frac{l}{\pi} ds \right\} \Rightarrow \frac{2}{\pi} \int_0^{\pi/2} \cos(s) \sin(2ks) ds = -\frac{\cos((2k+1)s)}{\pi(2k+1)} \Big|_0^{\pi/2} - \frac{\cos((2k-1)s)}{\pi(2k-1)} \Big|_0^{\pi/2} = \frac{4k}{\pi(4k^2-1)}$$

$$f(x) = \frac{1}{\pi} + \frac{2}{\pi} \sum_{k=1}^{\infty} \left[\frac{(-1)^{k+1}}{(4k^2-1)} \cos\left(\frac{2k\pi}{l} x\right) + \frac{2k}{(4k^2-1)} \sin\left(\frac{2k\pi}{l} x\right) \right]$$

Graph of the periodical continuation for three periods:

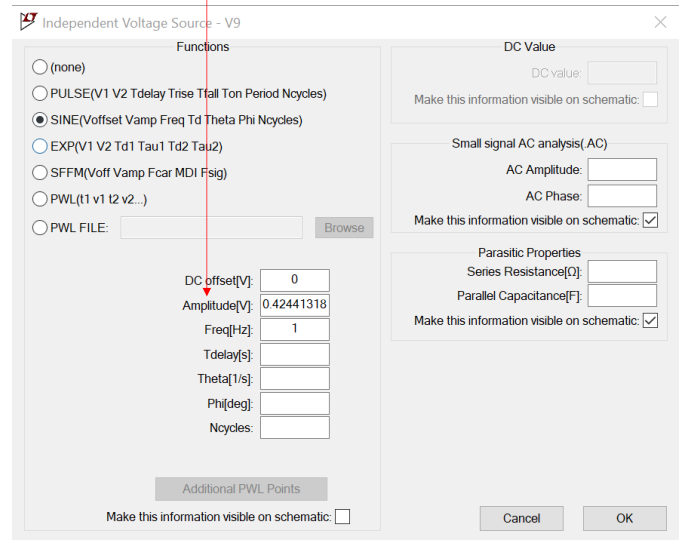
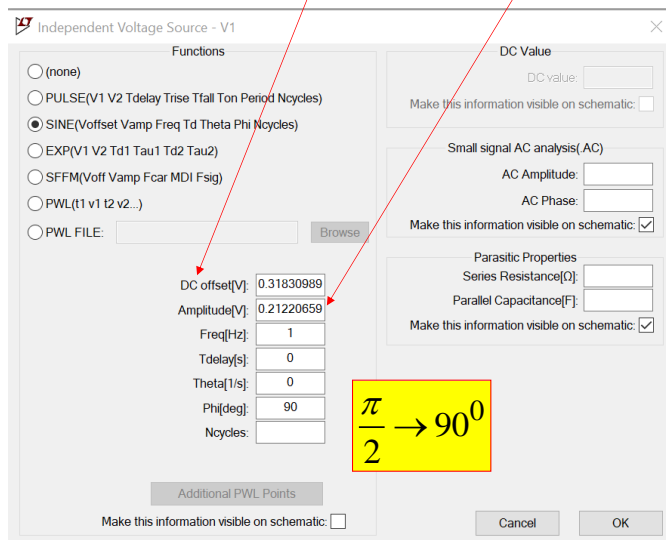


Simulation in LTspice with 8 and 14 harmonics (16 or 28 sin and cos generators):

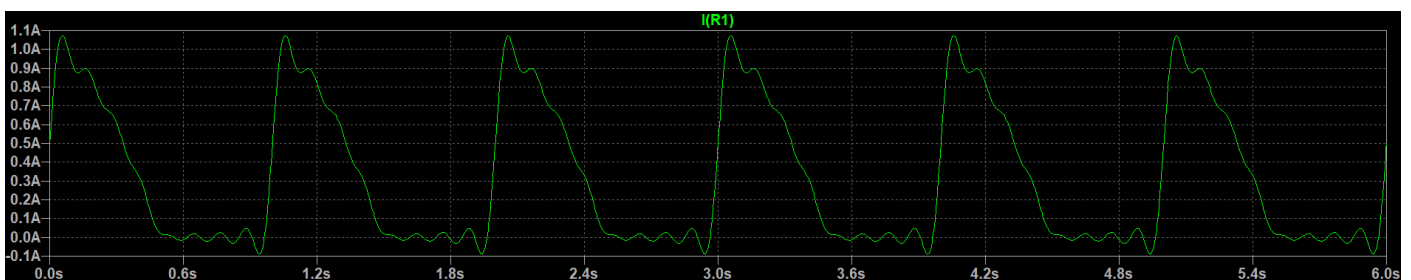


$$f(x) = \frac{1}{\pi} + \frac{2}{\pi} \sum_{k=1}^{\infty} \left[\frac{(-1)^{k+1}}{(4k^2 - 1)} \cos\left(\frac{2k\pi}{l}x\right) + \frac{2k}{(4k^2 - 1)} \sin\left(\frac{2k\pi}{l}x\right) \right]$$

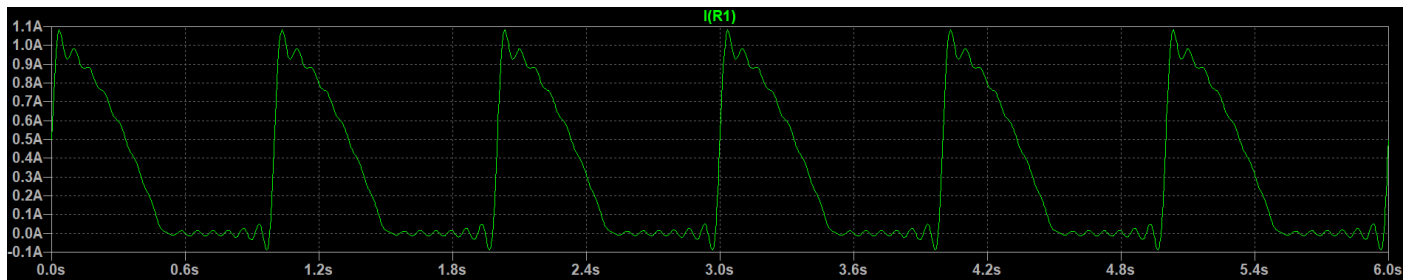
$Freq = k$ for $l = 1$



Current response through $R1 = 1\Omega$ for 8 harmonics ($l = 1$):



Current response through $R1 = 1\Omega$ for 14 harmonics ($l = 1$):



Conclusion: In this function, two complications arise: jumps and flat areas that are challenging to replicate using oscillating harmonics. The resulting ripple is unavoidable; however, as the number of generators (harmonics) increases, its frequency will rise while its amplitude diminishes.

Python code for drawing functions:

```

1. import numpy as np
2. import matplotlib.pyplot as plt
3.
4. # Define the functions
5. def f1(x):
6.     return x**2
7.
8. def f2(x):
9.     result = np.zeros_like(x) # Create an array to store results with the same shape as x
10.    mask = (0.0 <= x) & (x <= 0.5) # Boolean mask for the condition
11.
12.    result[mask] = np.cos(np.pi * x[mask]) # Calculate only for elements satisfying the condition
13.    return result
14.
15. # Define the ranges for both functions
16. x_values_f1 = np.linspace(-np.pi, np.pi, 1000) # [-pi, pi] for f1(x)
17. x_values_f2 = np.linspace(0, 1, 1000) # [0, 1] for f2(x)
18.
19. # Calculate periodical continuations for both functions
20. f1_extended = np.tile(f1(x_values_f1), 3) # Extend f1(x) for six periods
21. f2_extended = np.tile(f2(x_values_f2), 3) # Extend f2(x) for three periods
22.
23. # Generate x-values for the extended range for both functions
24. x_extended_f1 = np.linspace(-np.pi, 5 * np.pi, len(f1_extended)) # Generate x-values for extended range for f1(x)
25. x_extended_f2 = np.linspace(0, 3, len(f2_extended)) # Generate x-values for extended range for f2(x)
26.
27. # Plotting f1(x) and its periodical continuation for [-pi, 5pi]
28. plt.figure(figsize=(8, 6))
29.
30. # Plotting f1(x)
31. plt.plot(x_extended_f1, f1_extended, label='f1(x) = x^2 (Periodic)', color='blue')
32. plt.title('Periodical Continuation of f1(x) for [-pi, 5pi]')
33. plt.xlabel('x')
34. plt.ylabel('f1(x)')
35. plt.legend()
36. plt.grid(True)
37. plt.show()
38.
39. # Plotting f2(x) and its periodical continuation for [0, 3]
40. plt.figure(figsize=(8, 6))
41.
42. # Plotting f2(x) = exp(-x)
43. plt.plot(x_extended_f2, f2_extended, label='f2(x) = cos(pi * x), x = [0, 0.5] & f2(x) = 0, x = [0.5, 1] (Periodic)', color='orange')
44. plt.title('Periodical Continuation of f2(x) for [0, 3]')
45. plt.xlabel('x')
46. plt.ylabel('f2(x)')

```

```
47. plt.legend()  
48. plt.grid(True)  
49. plt.show()  
50.
```