LTspice simulator for complex signals

LTspice simulator, you can download for free: https://www.analog.com/en/design-center/design-tools-and-calculators/ltspice-simulator.html

Periodical continuation of $f(x)$ and its Fourier series:

$$\boxed{f(x)=x^2,\quad x\in[-\pi,\pi]}$$

This function is even and has the period $2l=2\pi$. The reference time $t_0=-\pi$.

$$a_0=\frac{1}{\pi}\int_{-\pi}^{\pi}x^2dx=\frac{2\pi^2}{3}$$

$$\boxed{f(x)=\frac{\pi^2}{3}+4\sum_{k=1}^{\infty}\left[\frac{(-1)^k}{k^2}\cos(kx)\right]}$$

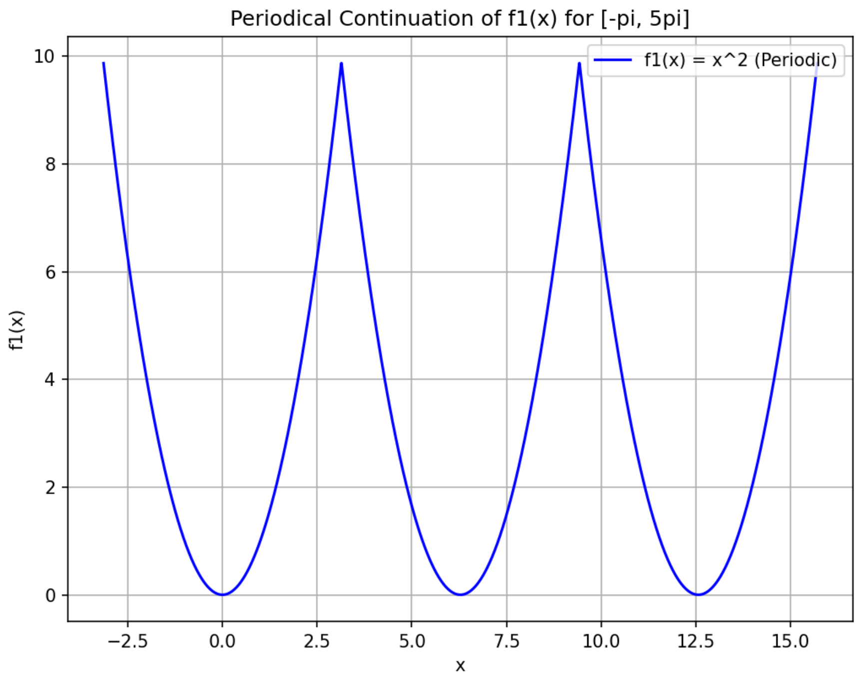
$$b_k\equiv 0$$

$$a_k=\frac{1}{\pi}\int_{-\pi}^{\pi}x^2\cos(kx)dx=x^2\frac{\sin(kx)}{\pi k}\bigg|_{-\pi}^{\pi}-\frac{2}{\pi k}\int_{-\pi}^{\pi}x\sin(kx)dx=$$

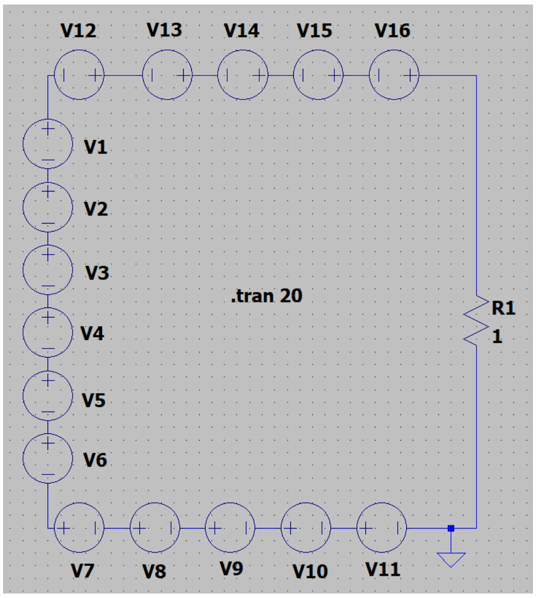
$$=-\frac{2}{\pi k}\int_{-\pi}^{\pi}x\sin(kx)dx=\frac{2x\cos(kx)}{\pi k^2}\bigg|_{-\pi}^{\pi}-\frac{2}{\pi k^2}\int_{-\pi}^{\pi}\cos(kx)dx=$$

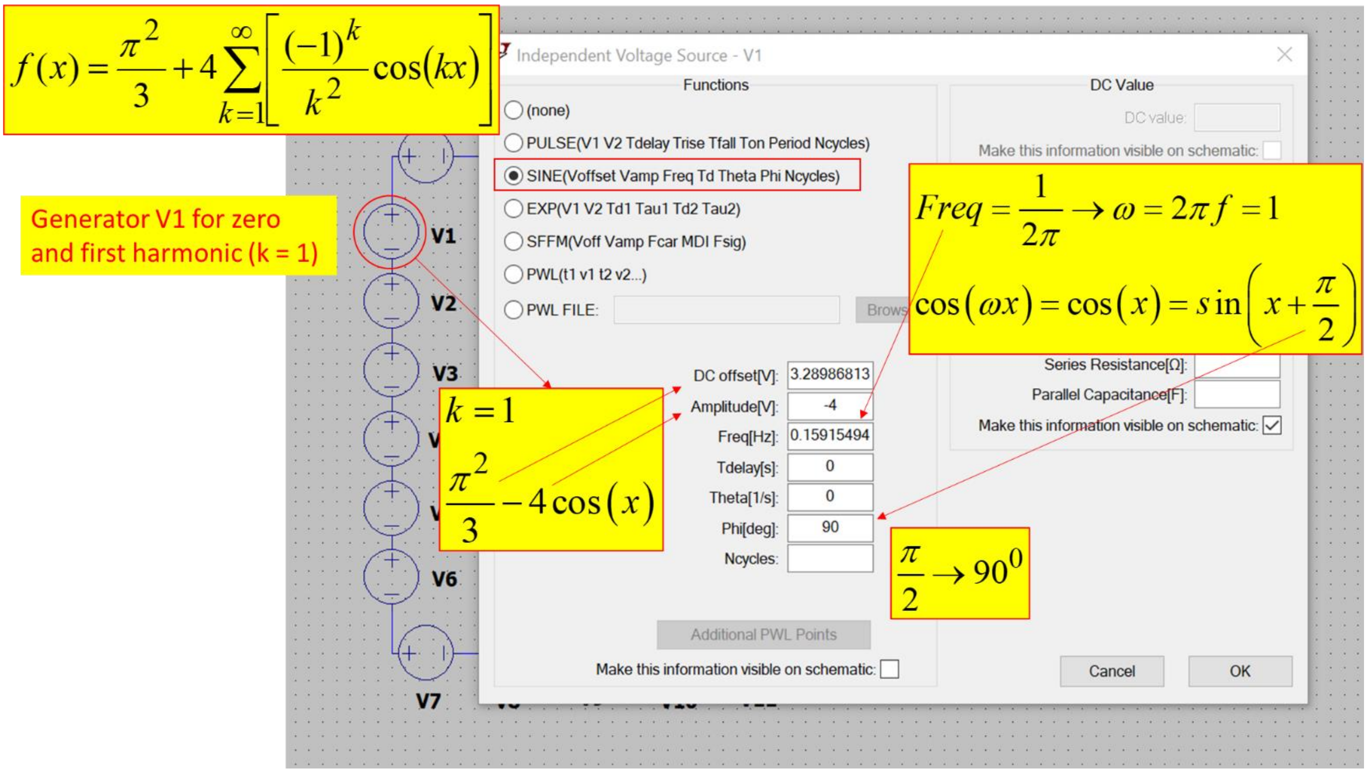
$$=\frac{2x\cos(kx)}{\pi k^2}\bigg|_{-\pi}^{\pi}=\frac{4\cos(k\pi)}{k^2}=\frac{4(-1)^k}{k^2}$$

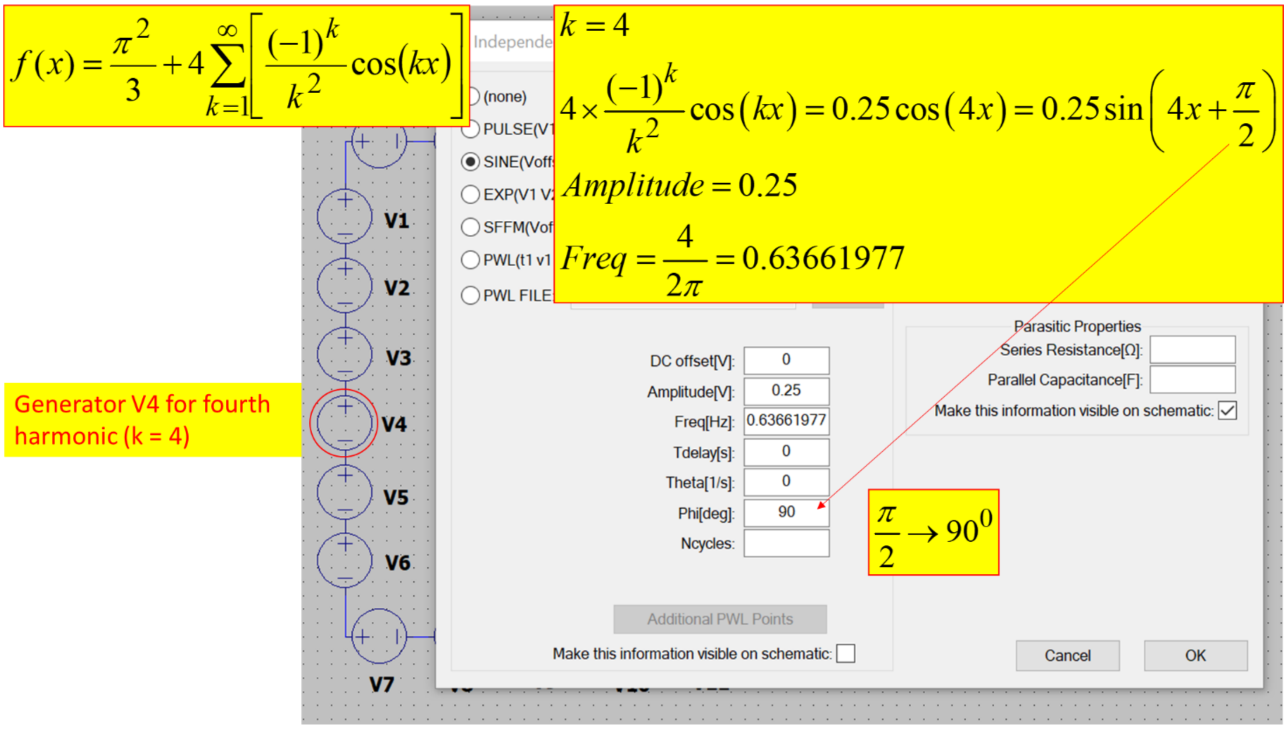
Graph of the periodical continuation for three periods:



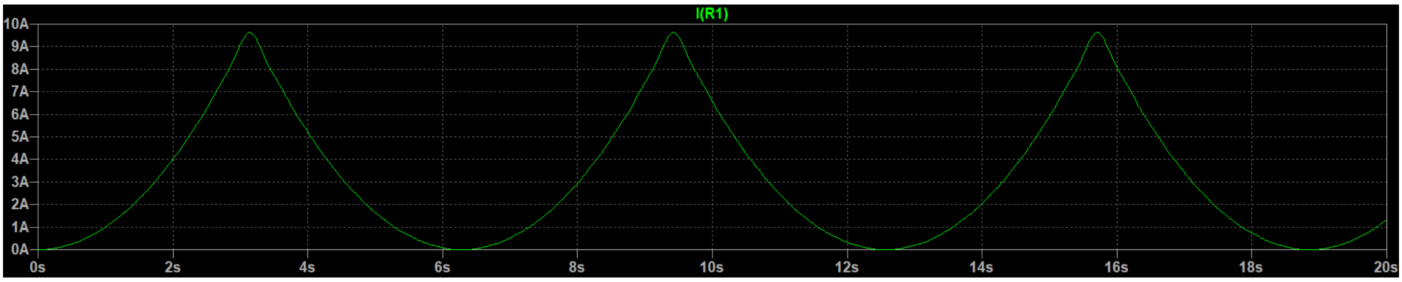
Simulation in LTspice with 16 generators (harmonics):







Current response through $R1 = 1\Omega$:



Conclusion: The shape has been reproduced fairly accurately with 16 harmonics, although the peaks should be slightly sharper.

Periodical continuation of $f(x)$ and its Fourier series:

$$f(x)=\begin{cases}\cos\left(\dfrac{\pi x}{l}\right), & 0\le x\le \dfrac{l}{2}\\ 0, & \dfrac{l}{2}<x<l\end{cases}$$

This function has the period l. The reference time $t_0 = 0$.

$$a_0=\frac{2}{l}\int_0^{l/2}\cos\left(\frac{\pi x}{l}\right)dx=\frac{2}{\pi}\sin\left(\frac{\pi x}{l}\right)\Bigg|_0^{l/2}=\frac{2}{\pi}$$

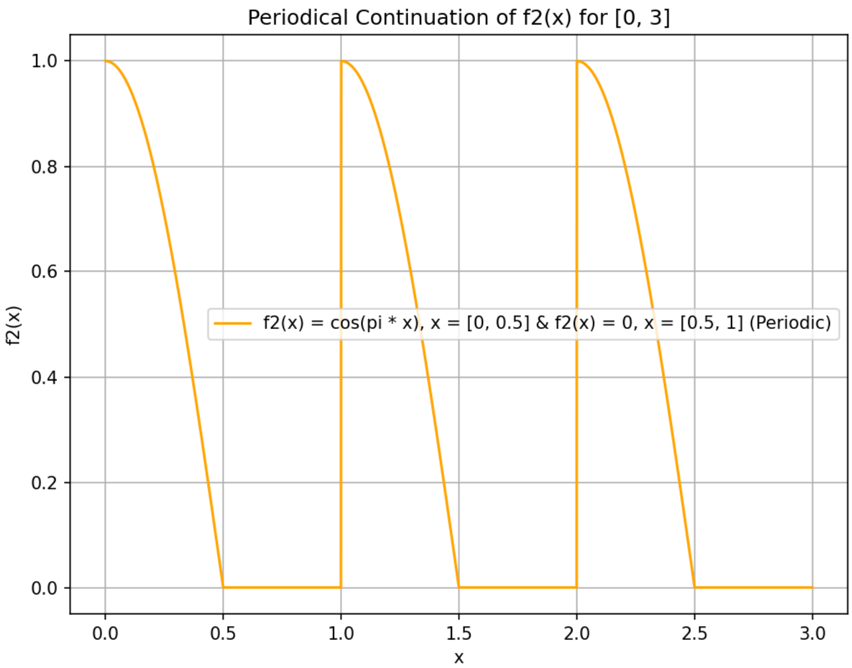
$$a_k=\frac{2}{l}\int_0^{l/2}\cos\left(\frac{\pi x}{l}\right)\cos\left(\frac{2\pi k}{l}x\right)dx\Rightarrow\left\{\frac{\pi x}{l}=s;\quad dx=\frac{l}{\pi}ds\right\}\Rightarrow\frac{2}{\pi}\int_0^{\pi/2}\cos(s)\cos(2ks)ds=$$

$$=\frac{\sin((2k+1)s)}{\pi(2k+1)}\Bigg|_0^{\pi/2}+\frac{\sin((2k-1)s)}{\pi(2k-1)}\Bigg|_0^{\pi/2}=\frac{\sin\left(\frac{(2k+1)\pi}{2}\right)}{\pi(2k+1)}+\frac{\sin\left(\frac{(2k-1)\pi}{2}\right)}{\pi(2k-1)}=\frac{(-1)^k}{\pi(2k+1)}+\frac{(-1)^{k+1}}{\pi(2k-1)}=\frac{2(-1)^{k+1}}{\pi(4k^2-1)}$$

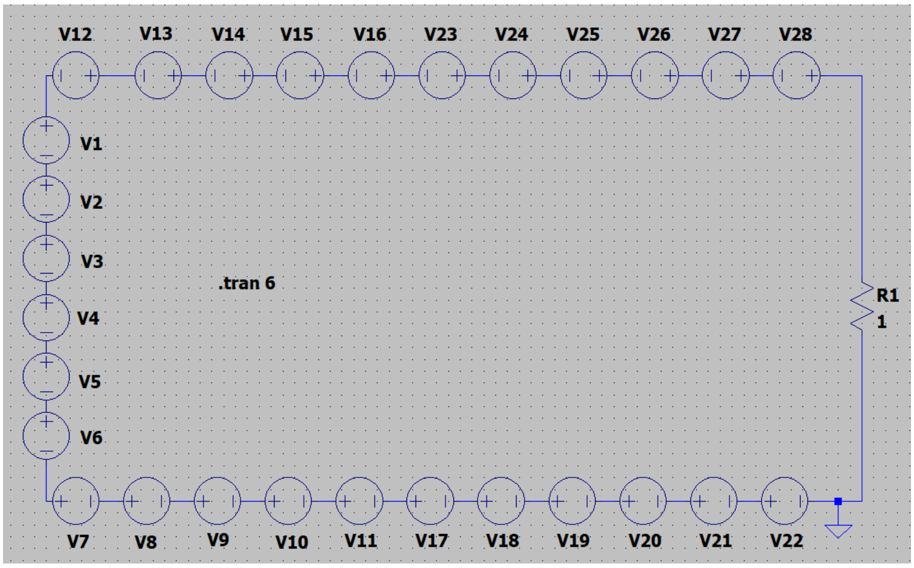
$$b_k=\frac{2}{l}\int_0^{l/2}\cos\left(\frac{\pi x}{l}\right)\sin\left(\frac{2\pi k}{l}x\right)dx\Rightarrow\left\{\frac{\pi x}{l}=s;\quad dx=\frac{l}{\pi}ds\right\}\Rightarrow\frac{2}{\pi}\int_0^{\pi/2}\cos(s)\sin(2ks)ds=-\frac{\cos((2k+1)s)}{\pi(2k+1)}\Bigg|_0^{\pi/2}-\frac{\cos((2k-1)s)}{\pi(2k-1)}\Bigg|_0^{\pi/2}=\frac{4k}{\pi(4k^2-1)}$$

$$f(x)=\frac{1}{\pi}+\frac{2}{\pi}\sum_{k=1}^{\infty}\left[\frac{(-1)^{k+1}}{(4k^2-1)}\cos\left(\frac{2k\pi}{l}x\right)+\frac{2k}{(4k^2-1)}\sin\left(\frac{2k\pi}{l}x\right)\right]$$

Graph of the periodical continuation for three periods:

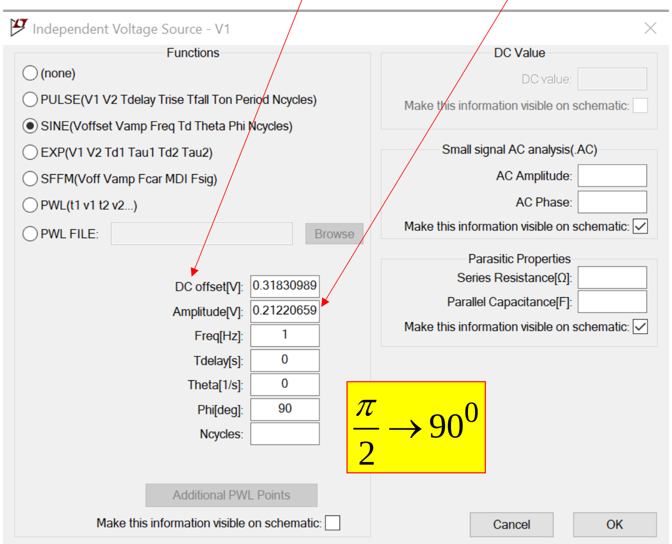


Simulation in LTspice with 8 and 14 generators (harmonics):

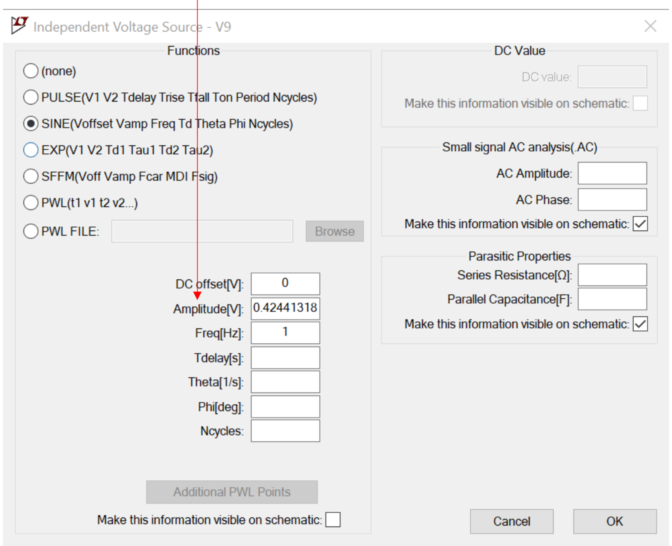


$$f(x) = \frac{1}{\pi} + \frac{2}{\pi}\sum_{k=1}^{\infty}\left[\frac{(-1)^{k+1}}{(4k^2-1)}\cos\left(\frac{2k\pi}{l}x\right) + \frac{2k}{(4k^2-1)}\sin\left(\frac{2k\pi}{l}x\right)\right]$$

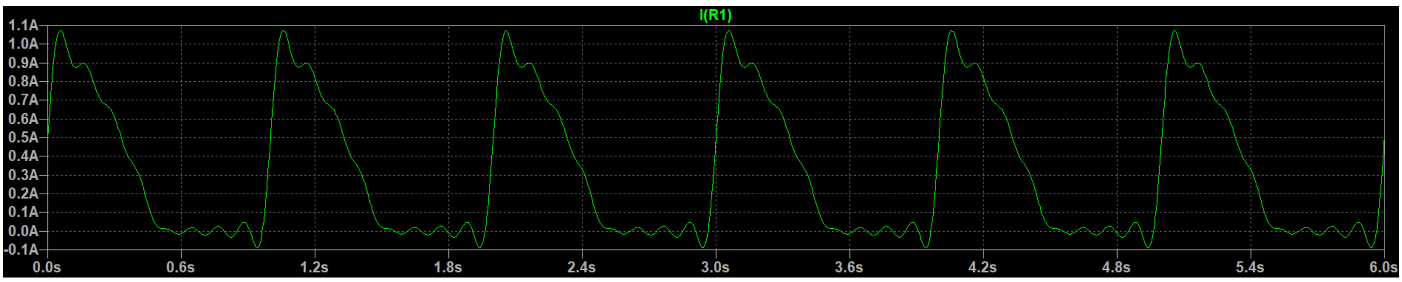
$$Freq = k \text{ for } l = 1$$



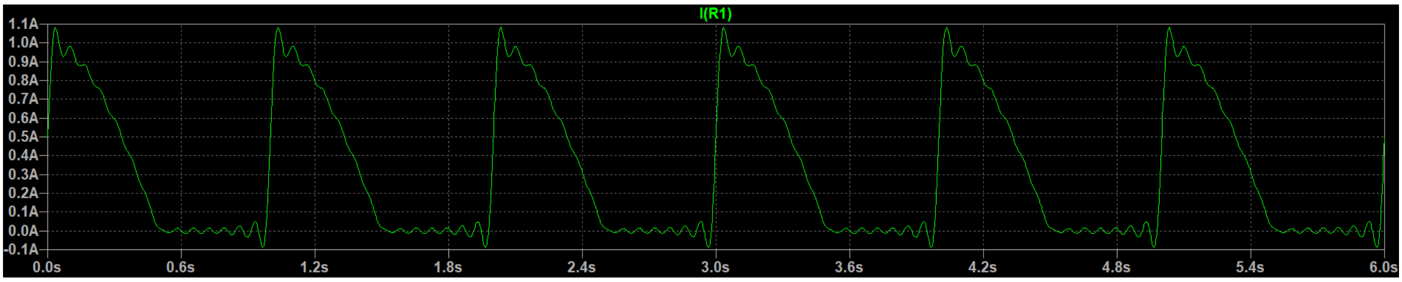
$$\frac{\pi}{2} \rightarrow 90^0$$



Current response through $R1 = 1\Omega$ for 8 harmonics ($l = 1$):



Current response through $R1 = 1\Omega$ for 14 harmonics ($l = 1$):



Conclusion: In this function, two complications arise: jumps and flat areas that are challenging to replicate using oscillating harmonics. The resulting ripple is unavoidable; however, as the number of generators (harmonics) increases, its frequency will rise while its amplitude diminishes.

Python code for drawing functions:

```python
import numpy as np
import matplotlib.pyplot as plt

# Define the functions
def f1(x):
    return x**2

def f2(x):
    result = np.zeros_like(x)  # Create an array to store results with the same shape as x
    mask = (0.0 <= x) & (x <= 0.5)  # Boolean mask for the condition

    result[mask] = np.cos(np.pi * x[mask])  # Calculate only for elements satisfying the condition
    return result

# Define the ranges for both functions
x_values_f1 = np.linspace(-np.pi, np.pi, 1000)  # [-pi, pi] for f1(x)
x_values_f2 = np.linspace(0, 1, 1000)  # [0, 1] for f2(x)

# Calculate periodical continuations for both functions
f1_extended = np.tile(f1(x_values_f1), 3)  # Extend f1(x) for six periods
f2_extended = np.tile(f2(x_values_f2), 3)  # Extend f2(x) for three periods

# Generate x-values for the extended range for both functions
x_extended_f1 = np.linspace(-np.pi, 5 * np.pi, len(f1_extended))  # Generate x-values for extended range for f1(x)
x_extended_f2 = np.linspace(0, 3, len(f2_extended))  # Generate x-values for extended range for f2(x)

# Plotting f1(x) and its periodical continuation for [-pi, 5pi]
plt.figure(figsize=(8, 6))

# Plotting f1(x)
plt.plot(x_extended_f1, f1_extended, label='f1(x) = x^2 (Periodic)', color='blue')
plt.title('Periodical Continuation of f1(x) for [-pi, 5pi]')
plt.xlabel('x')
plt.ylabel('f1(x)')
plt.legend()
plt.grid(True)
plt.show()

# Plotting f2(x) and its periodical continuation for [0, 3]
plt.figure(figsize=(8, 6))

# Plotting f2(x) = exp(-x)
plt.plot(x_extended_f2, f2_extended, label='f2(x) = cos(pi * x), x = [0, 0.5] & f2(x) = 0, x = [0.5, 1] (Periodic)', color='orange')
plt.title('Periodical Continuation of f2(x) for [0, 3]')
plt.xlabel('x')
plt.ylabel('f2(x)')
```

```
plt.legend()
plt.grid(True)
plt.show()

```